Traitor Tracing with $N^{1/3}$-Size Ciphertexts and $O(1)$-Size Keys from k-Lin

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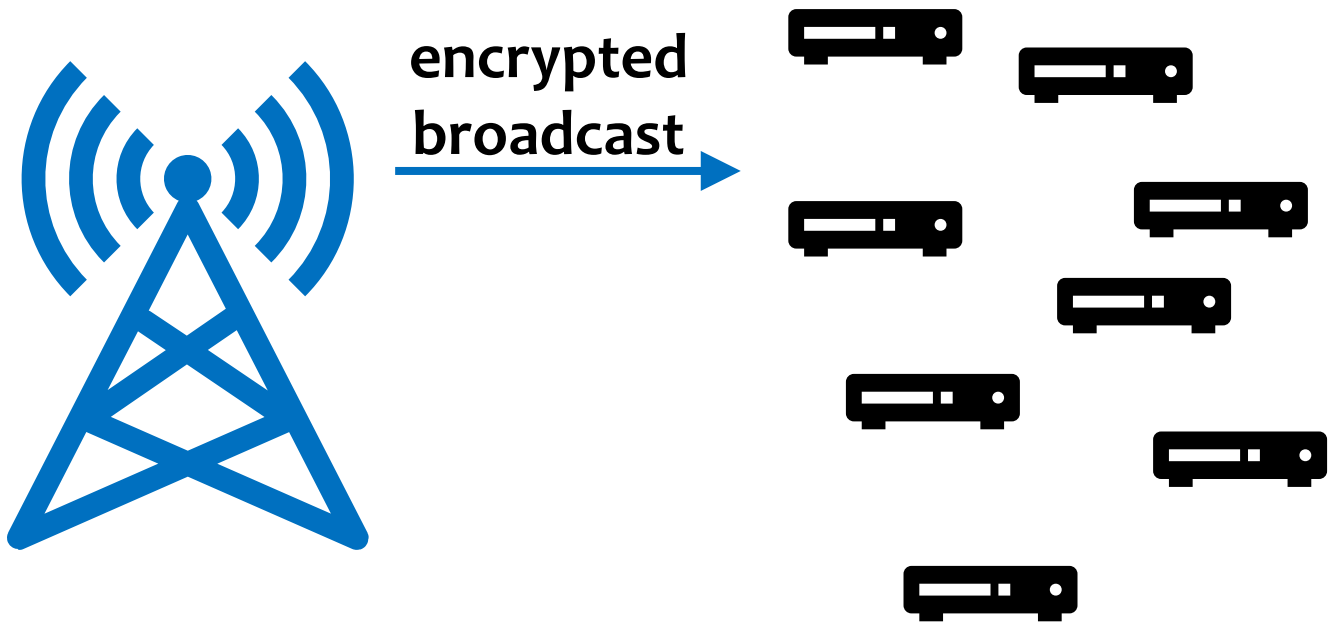
UNIVERSITY of WASHINGTON

Hoeteck Wee ✉

NTTResearch

Traitor Tracing [CFN94]



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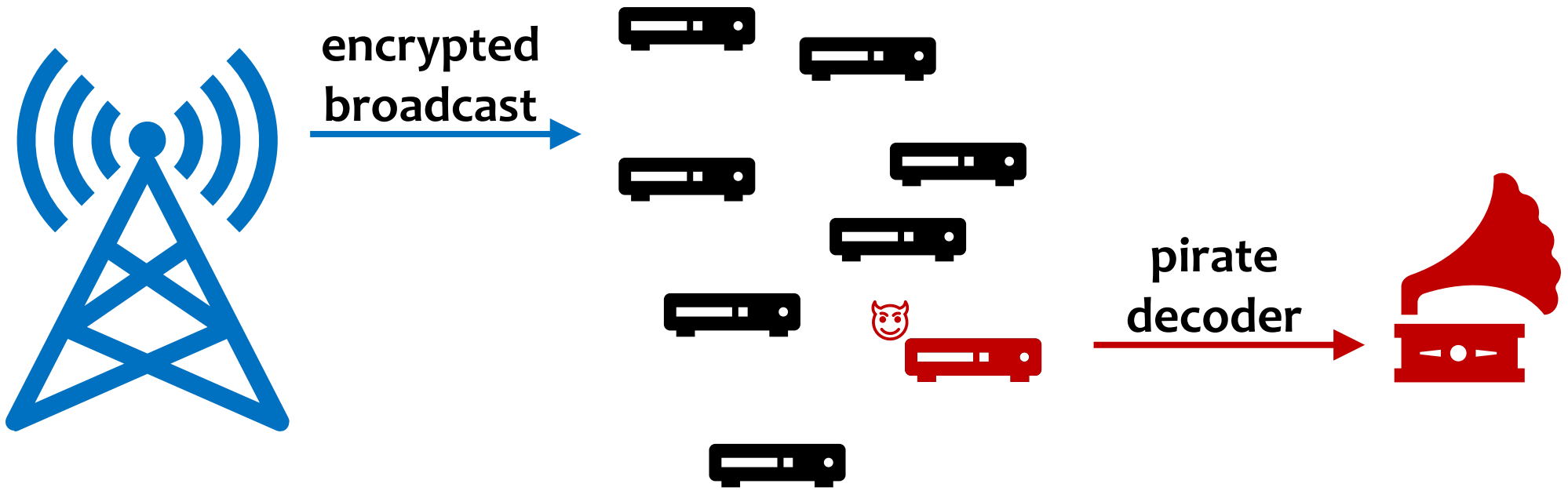

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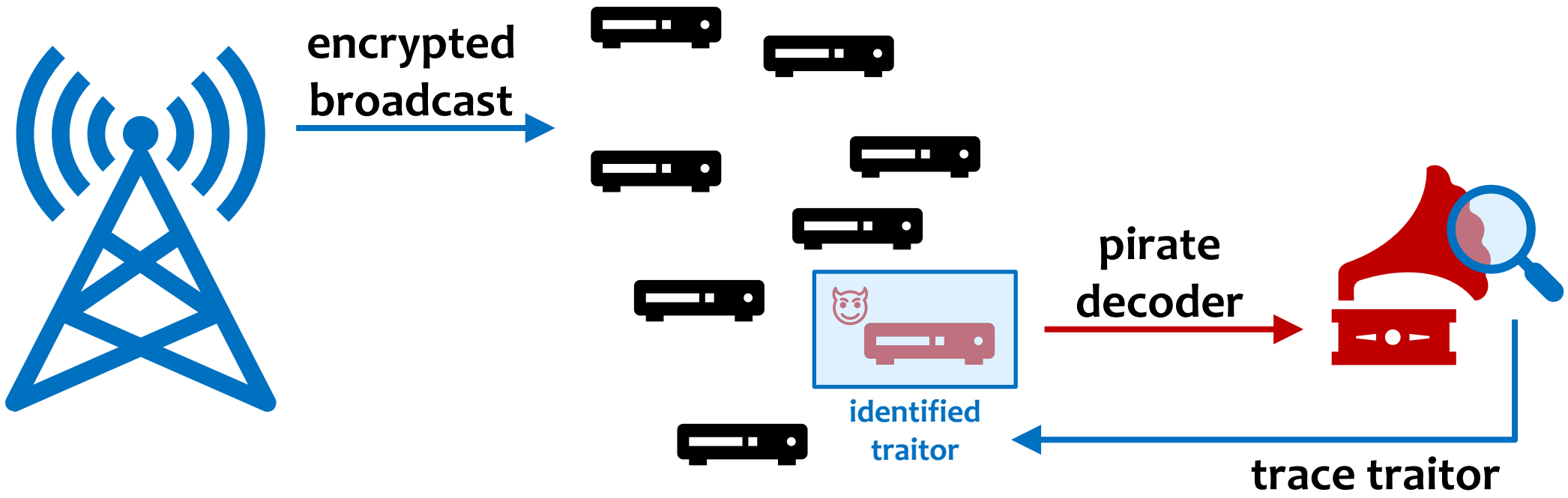

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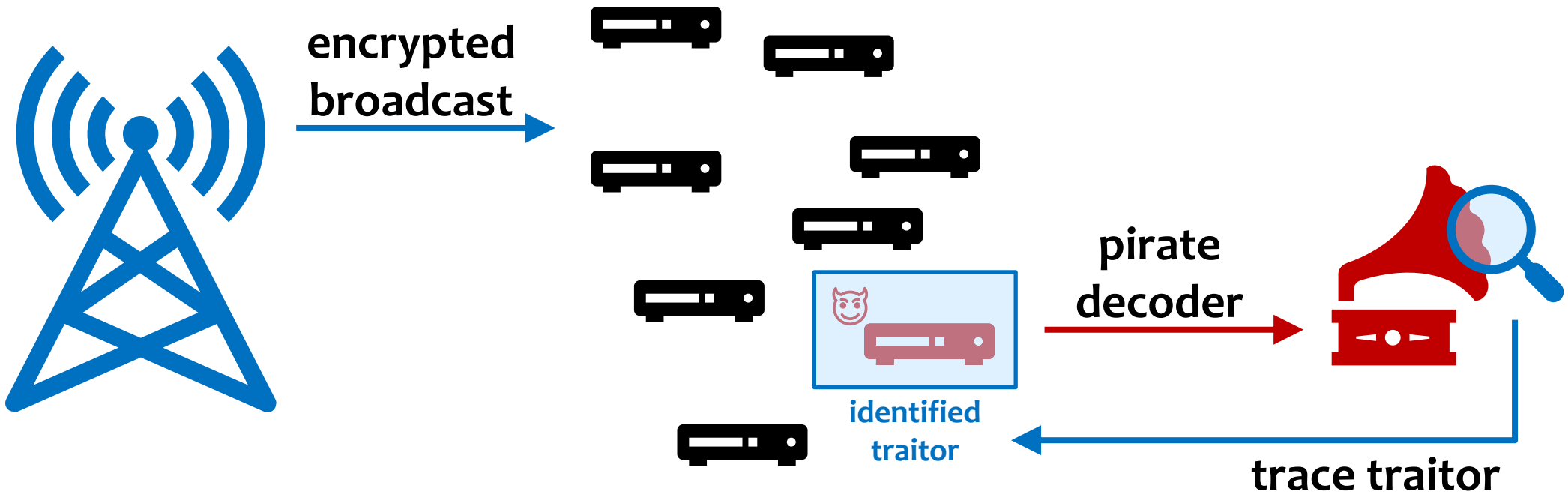

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- **Never** accuses an **innocent** user.

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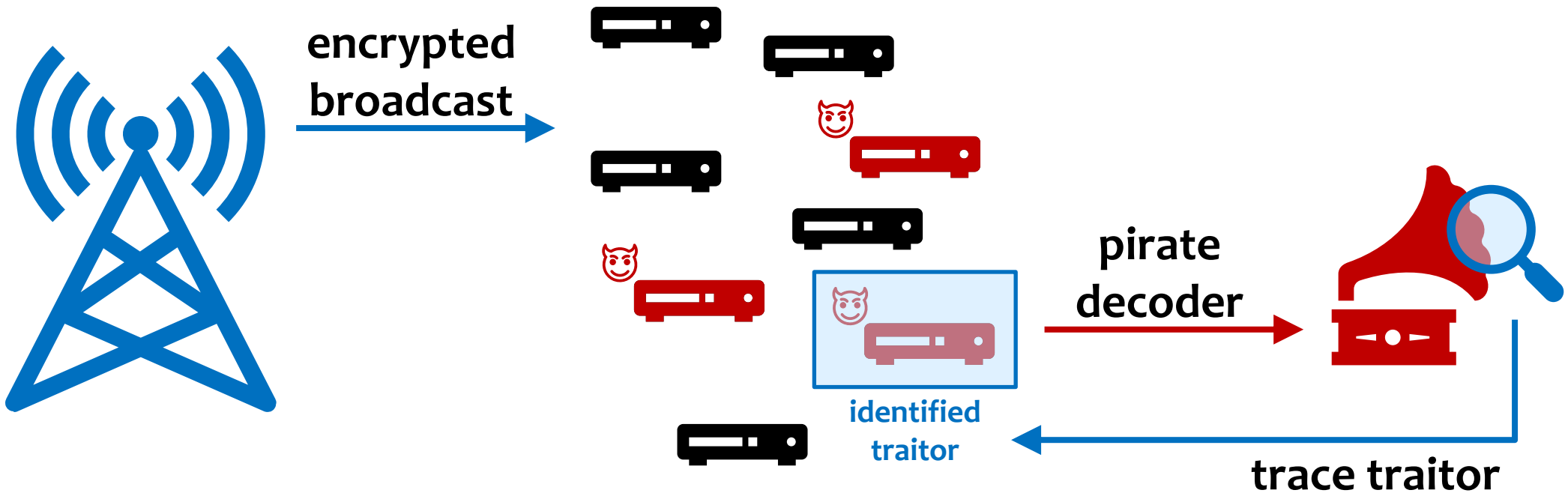

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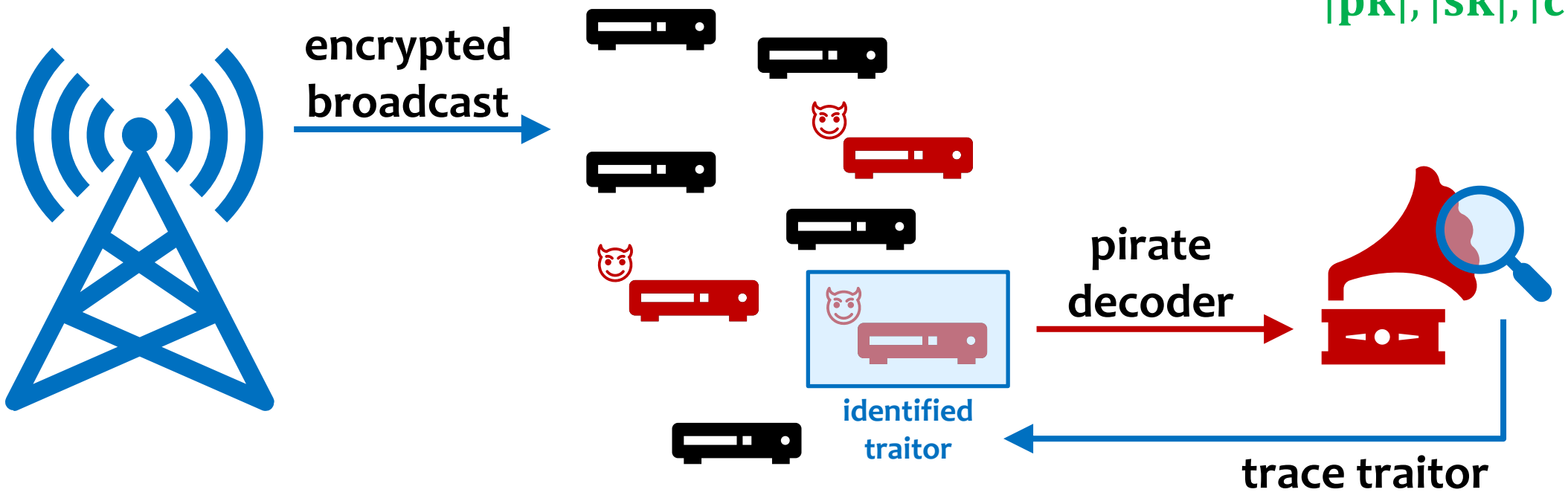
- Even if **arbitrarily** many users **collude**.

Traitor Tracing [CFN94]

minimize communication/storage overhead

$|pk|, |sk|, |ct|$



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- $\mathrm{Trace}^D(\mathrm{pk}, \mathrm{tk}, 1^{1/\varepsilon}) \to i^* \in [N] \cup \{\perp\}$

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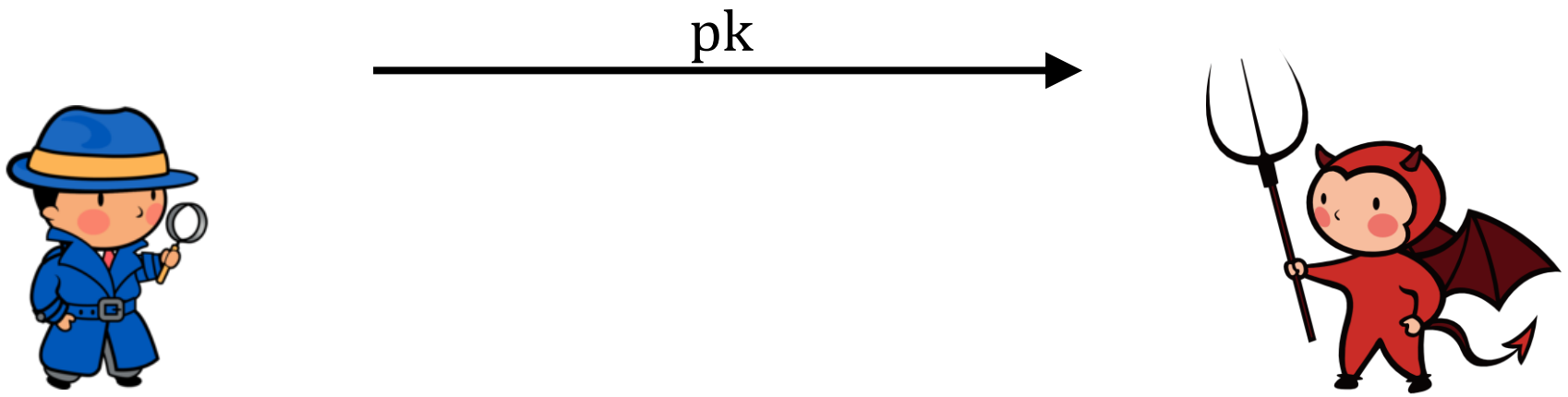

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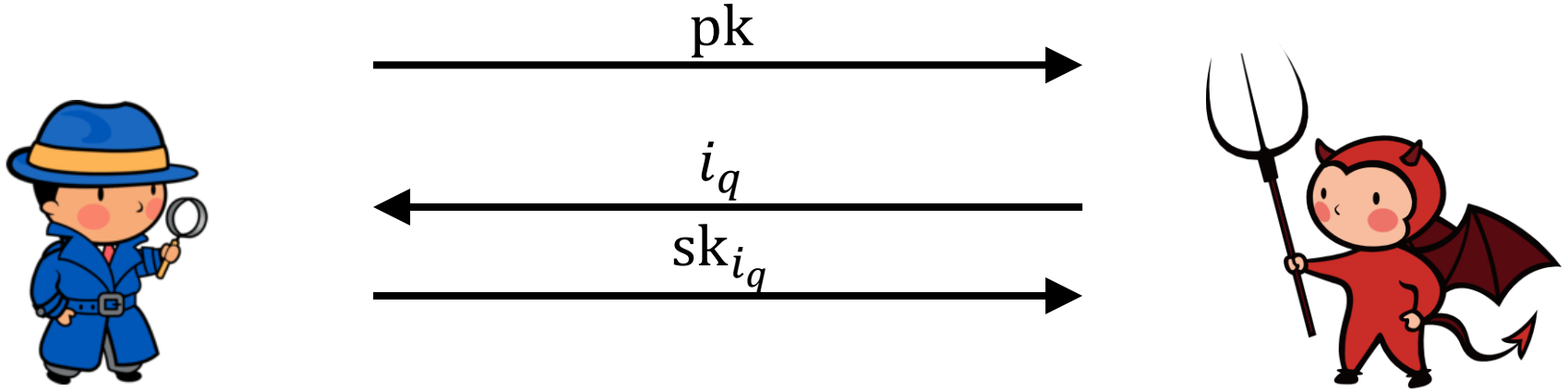

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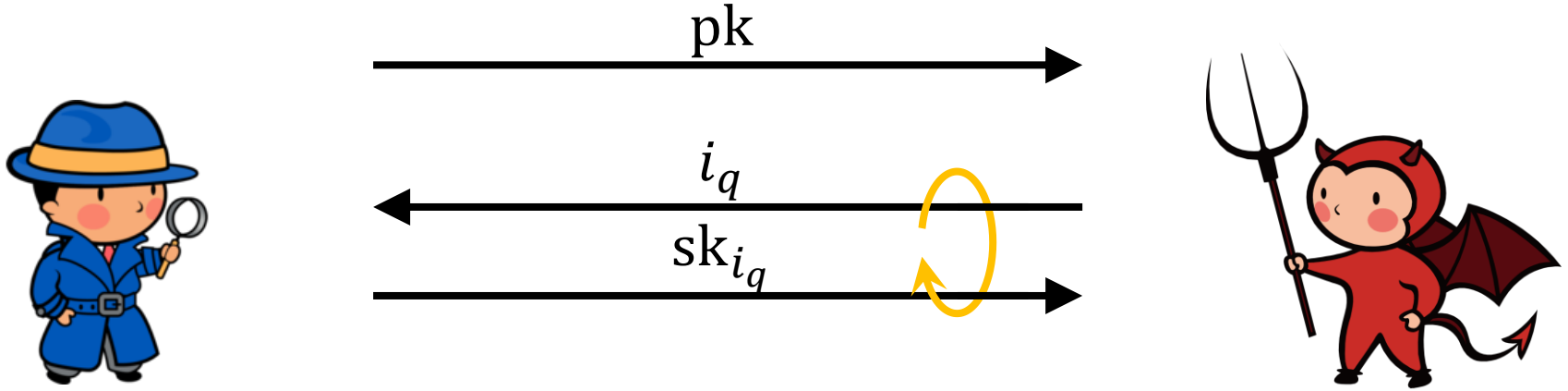

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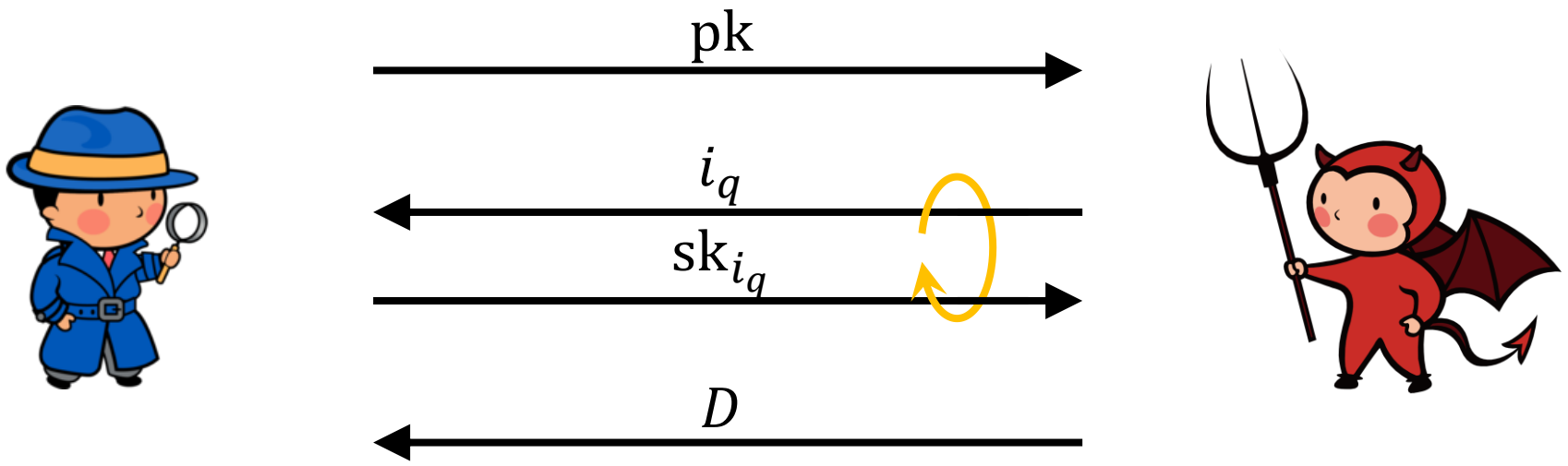

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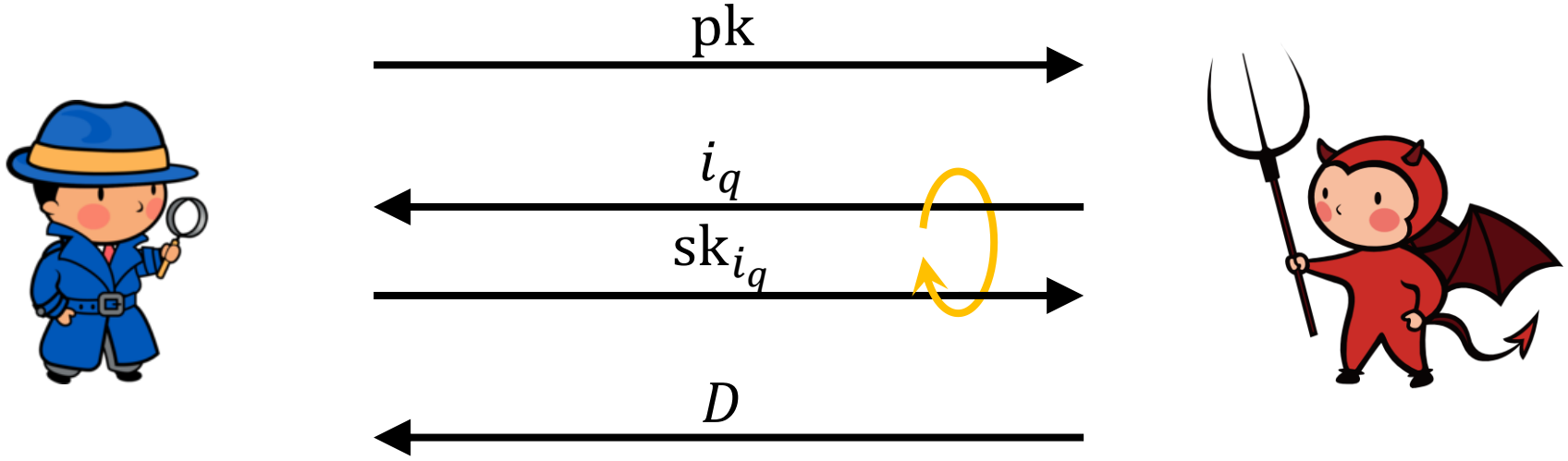

$$\varepsilon^* \stackrel{\text{def}}{=} \Pr\left[\begin{matrix} b, k_0 \leftarrow \$, \$ \\ (\mathrm{ct}, k_1) \leftarrow \mathrm{Enc}(\mathrm{pk}) \end{matrix} : D(\mathrm{ct}, k_b) = b\right] - \frac{1}{2}$$

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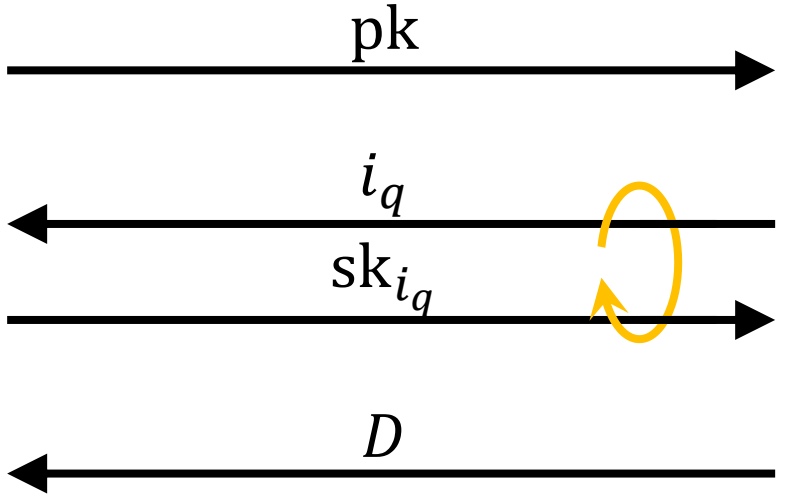

wins if

either $i^* \notin \{i_q\} \cup \{\perp\}$,

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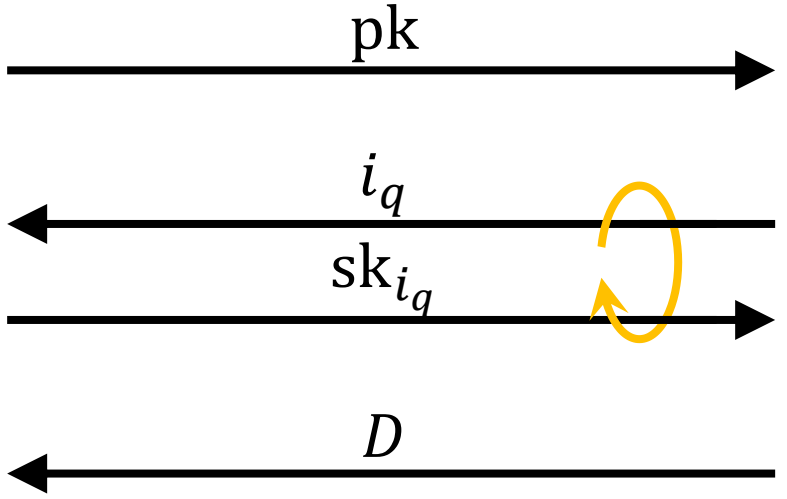

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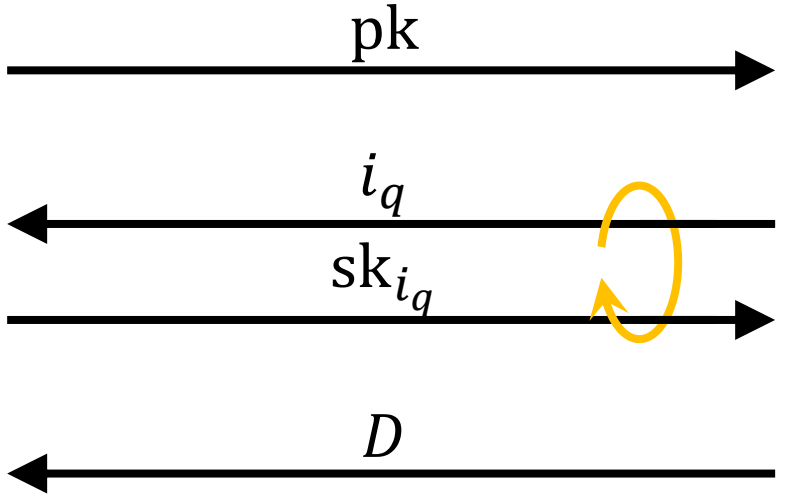

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Pr[ wins] ***negligible*** for all ***polynomial*** $N, 1/\varepsilon$

Traitor Tracing Schemes with Optimal Sizes

	$\|pk\|$	$\|sk\|$	$\|ct\|$	**tracing**	**assumption**
BZ14	$O(1)$			public	$i\mathcal{O}$
GKW18				private	LWE
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- **deployment-friendly**

pairing

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- **exploration of techniques**

Existing Traitor Tracing Schemes from Pairing

	$\lvert pk \rvert$	$\lvert sk \rvert$	$\lvert ct \rvert$	**tracing**	**assumption**
BSW06 + Z20	1	1	N	public	IBE
BN08/BP08 + Z20	1	N^2	1	private	IBE
BSW06	$\sqrt{N}$	1	$\sqrt{N}$	private	composite
BSW06 + W20	$\sqrt{N}$	1	$\sqrt{N}$	public	bi-k-Lin

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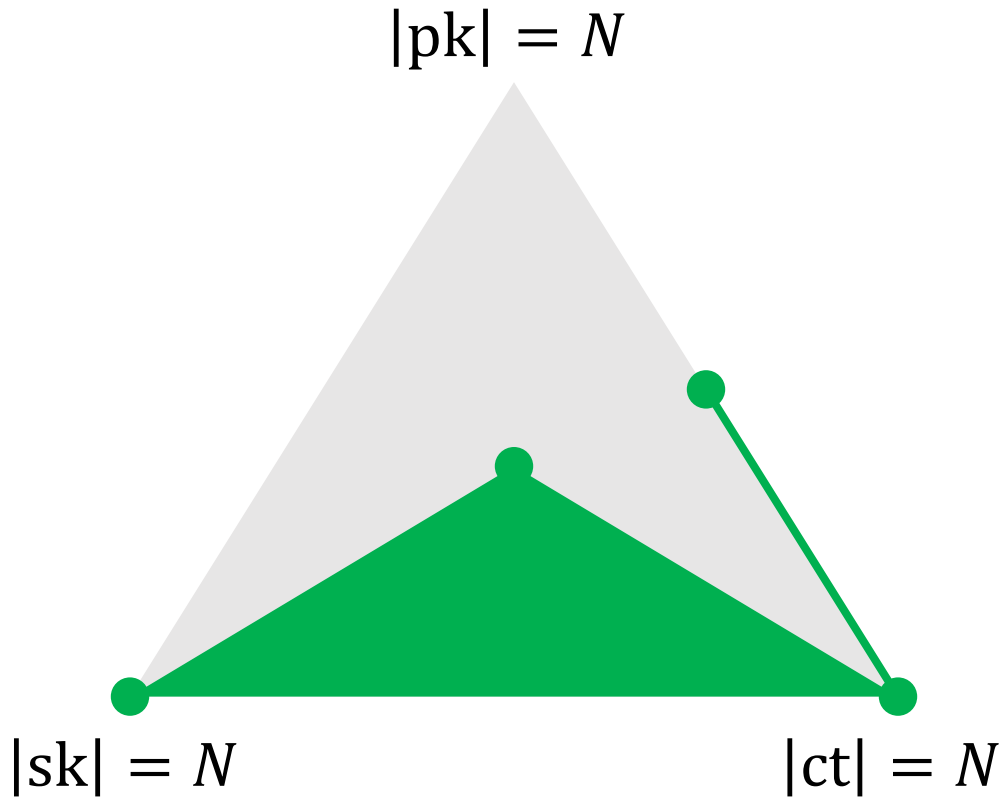
But $\lvert pk \rvert \times \lvert sk \rvert \times \lvert ct \rvert \geq N$ for all schemes so far!

Traitor Tracing from Pairing: Trade-Off Simplices

$|\mathrm{pk}| \times |\mathrm{sk}| \times |\mathrm{ct}| = N$ **simplex** [Z20]



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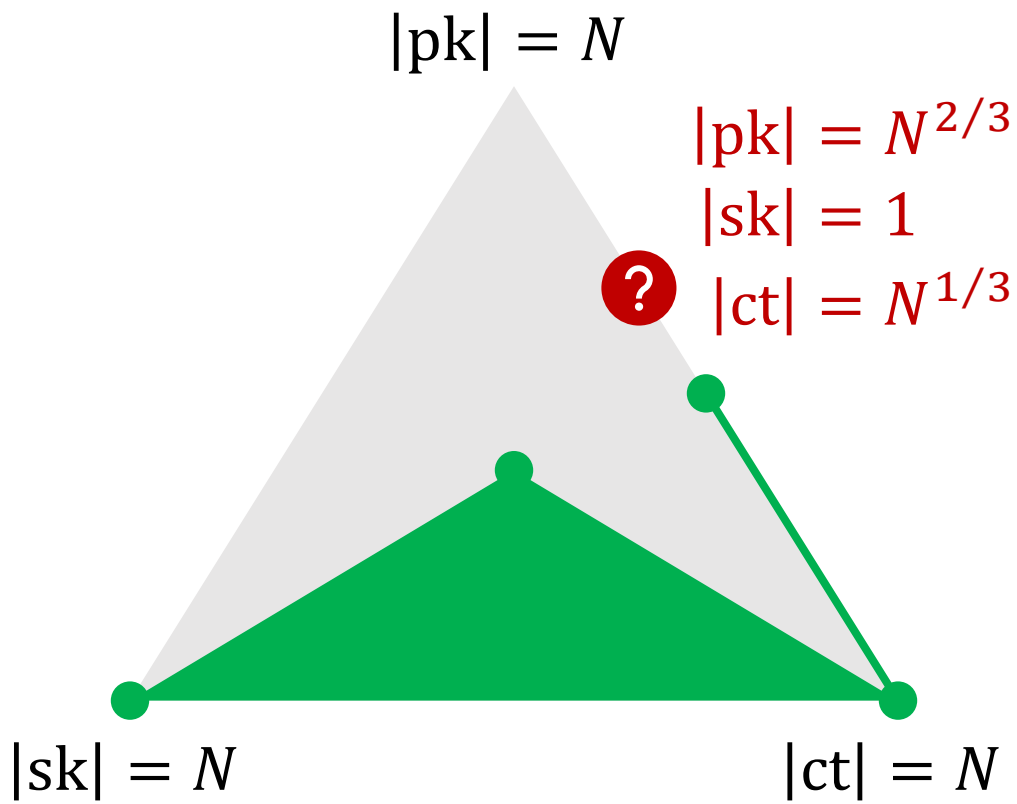

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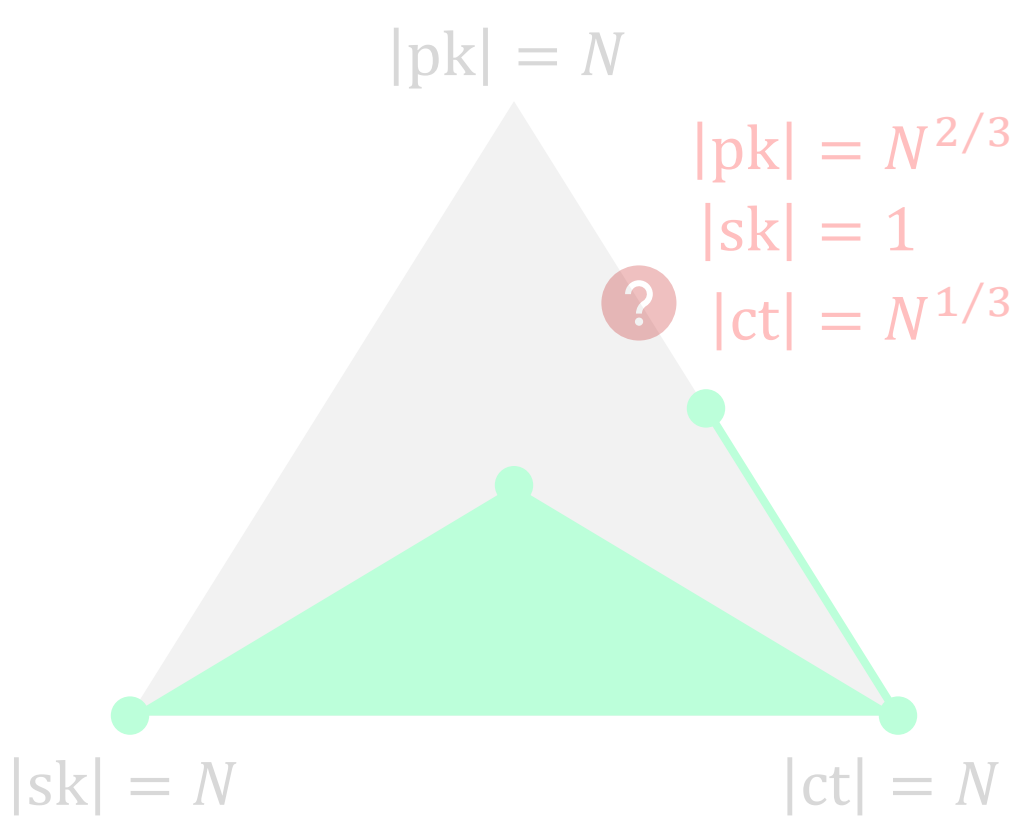

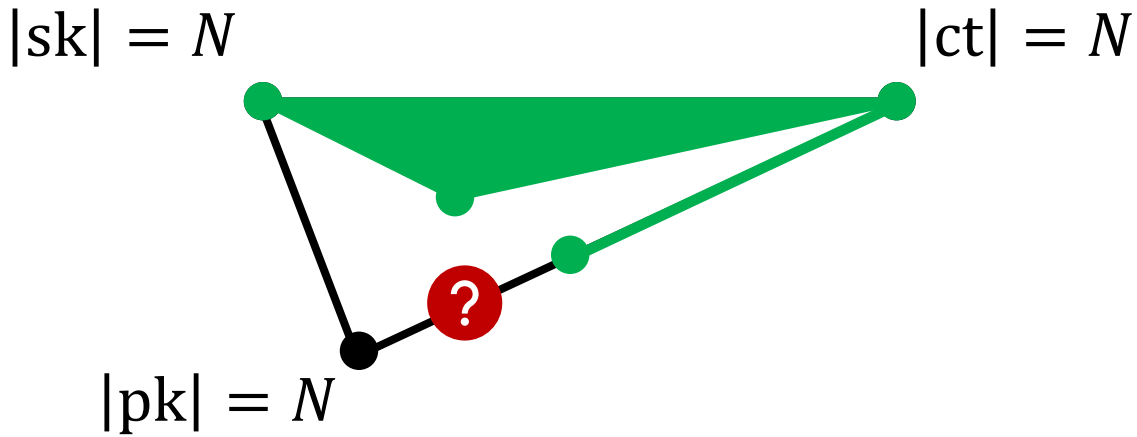

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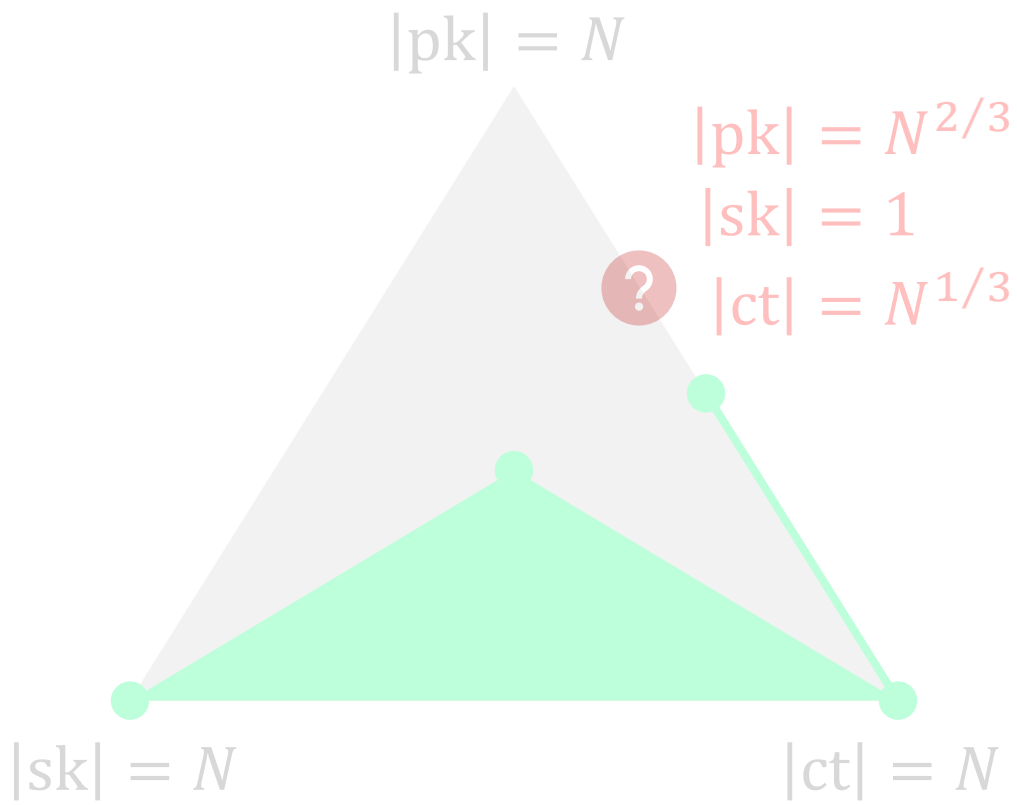
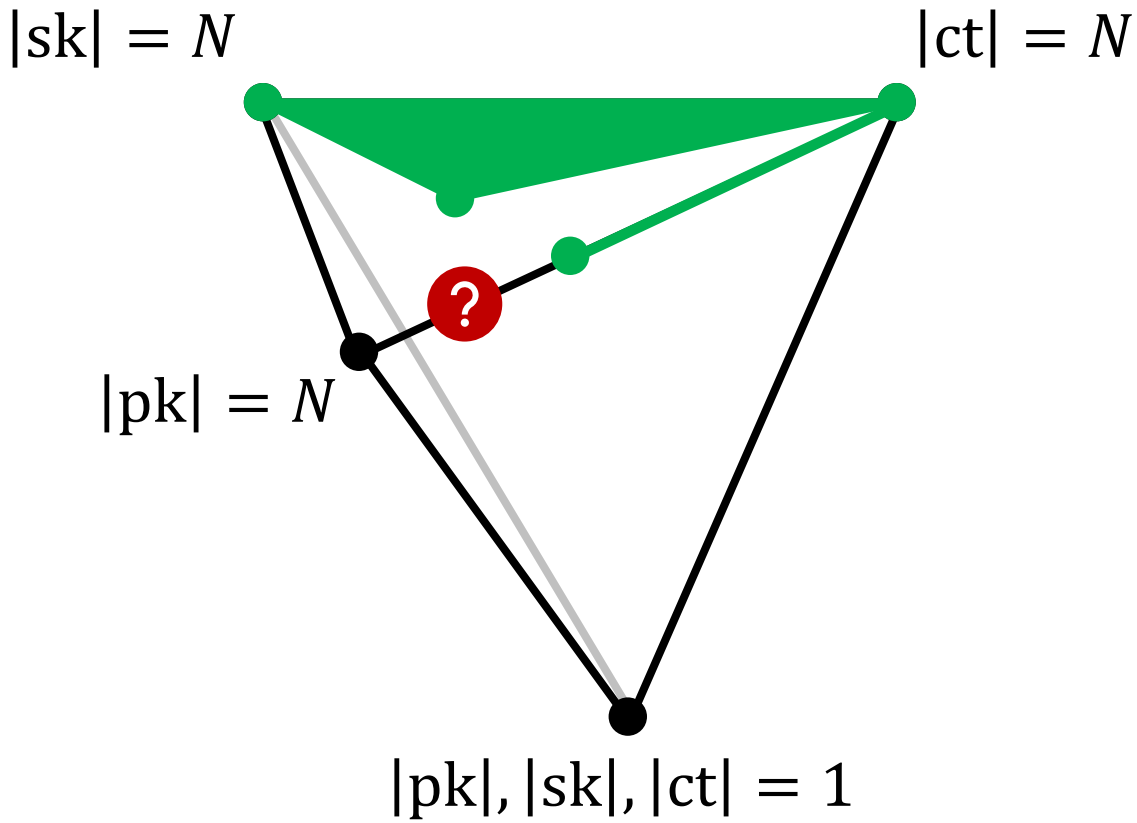


$1 \leq |\mathrm{pk}| \times |\mathrm{sk}| \times |\mathrm{ct}| \leq N$ **simplex**



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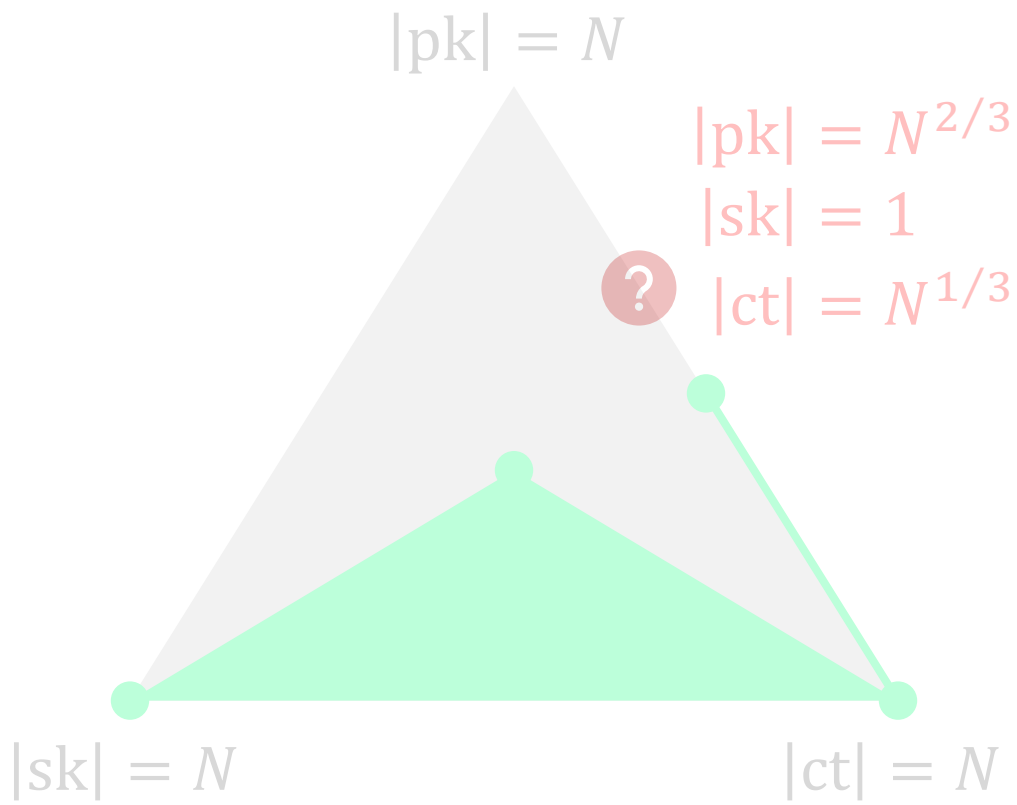

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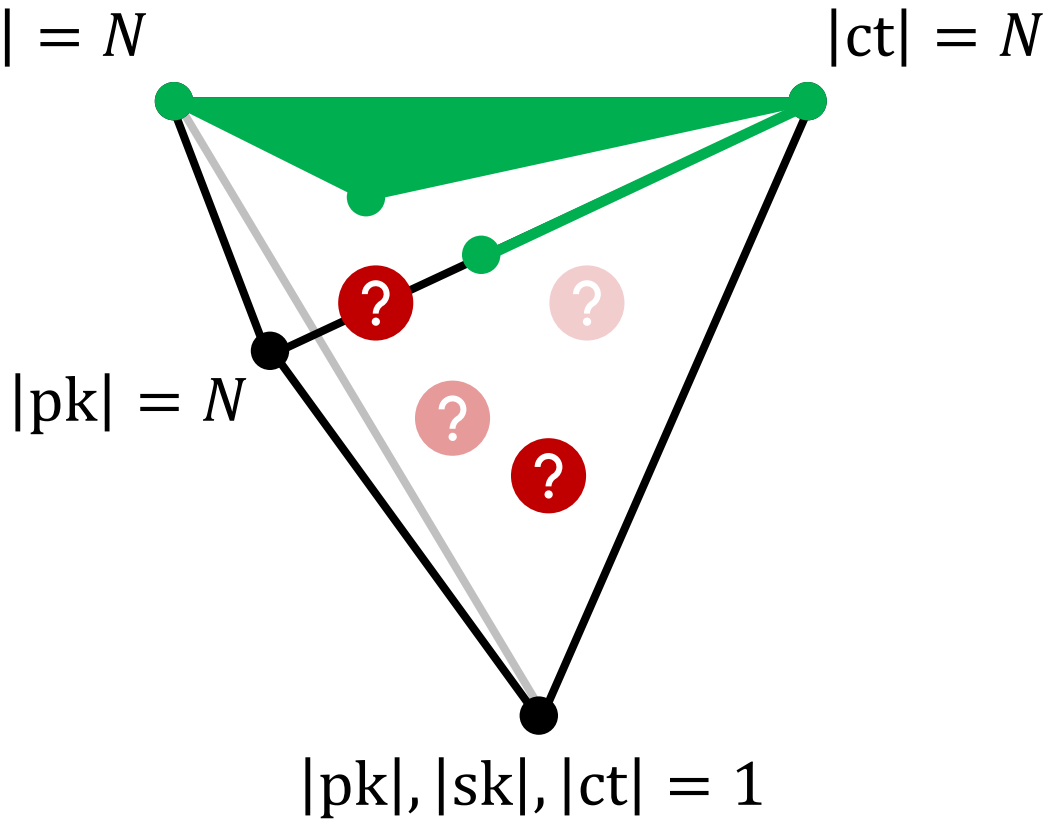

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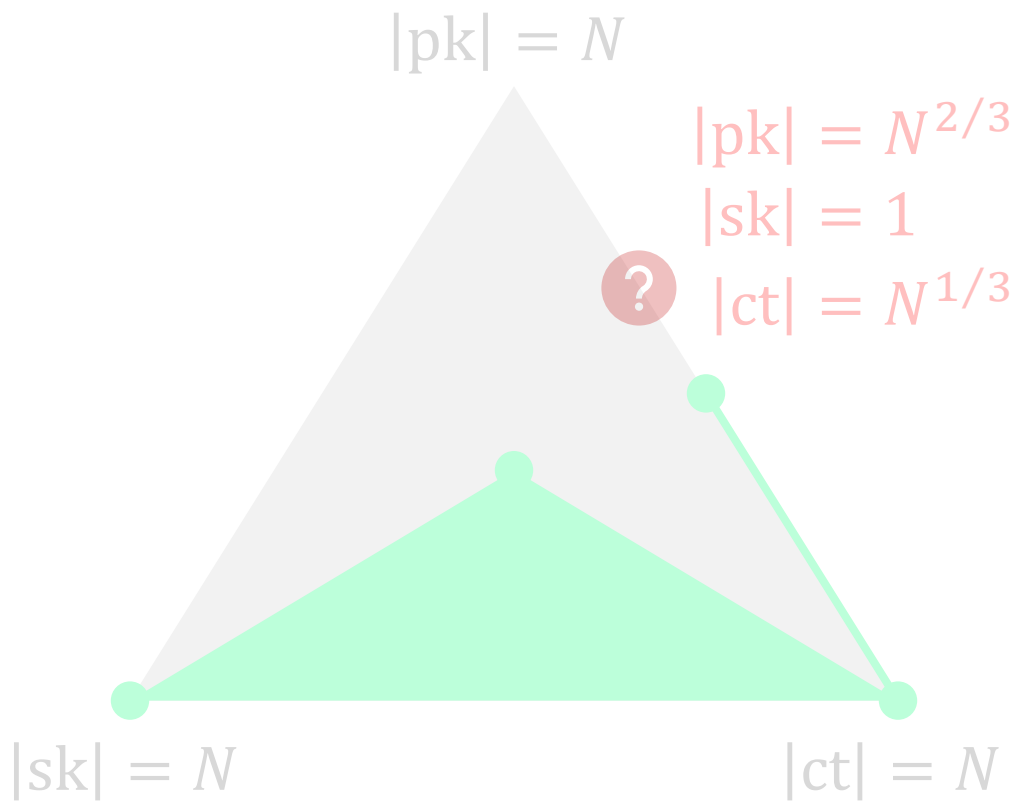

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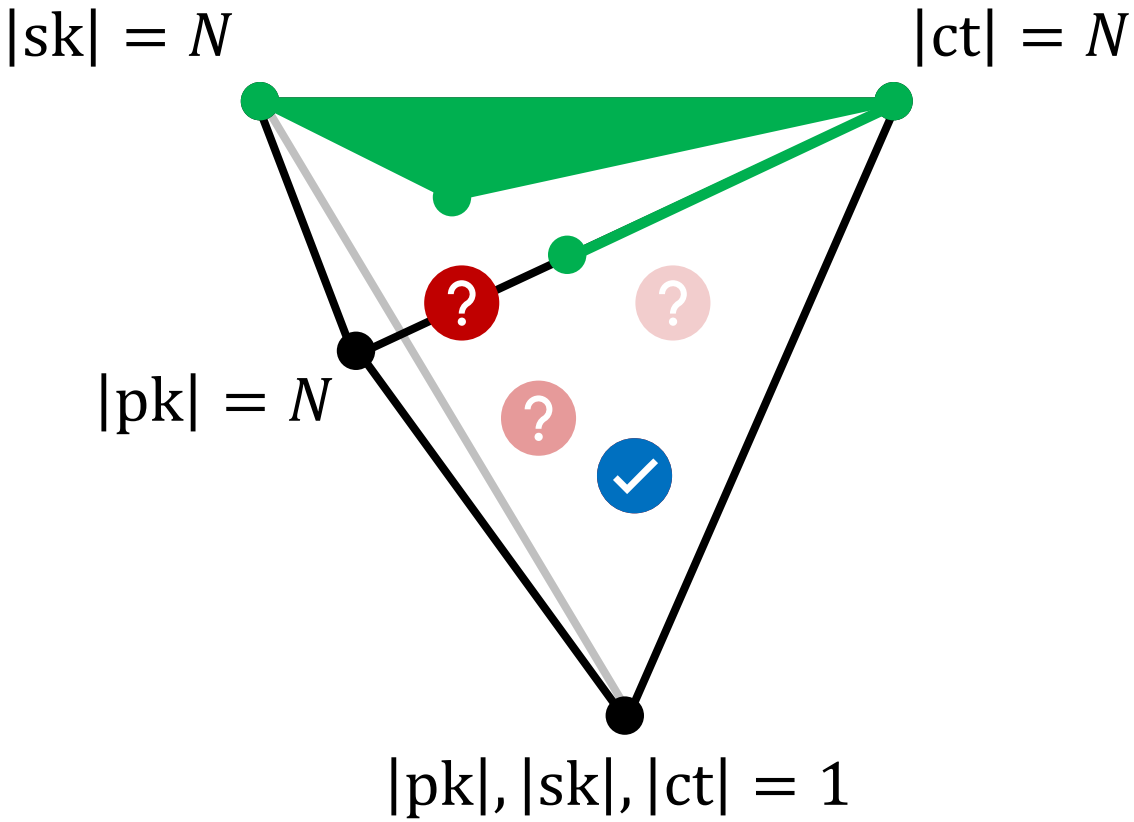

Our Results

	|pk|	|sk|	|ct|	**tracing**	**assumption**
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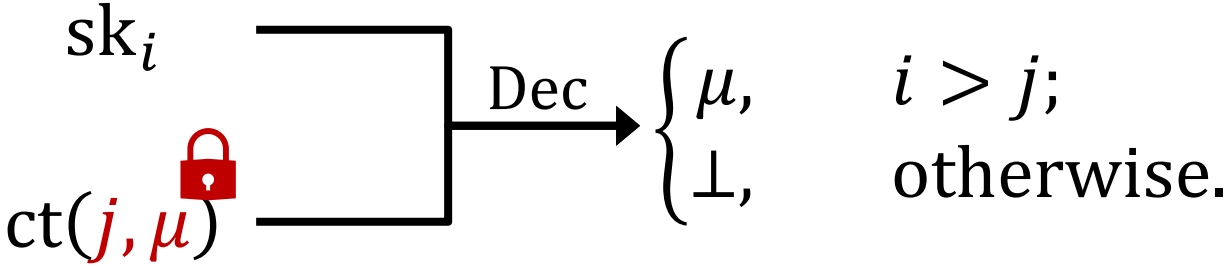
Quick Recap: Traitor Tracing from PLBE [BSW06]

Private Linear Broadcast Encryption



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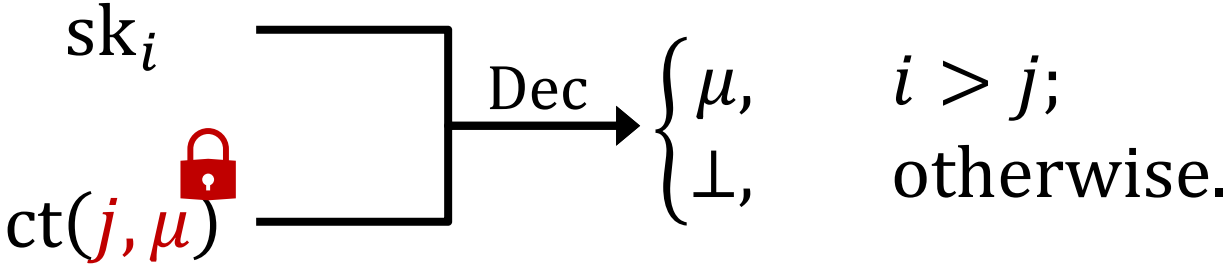

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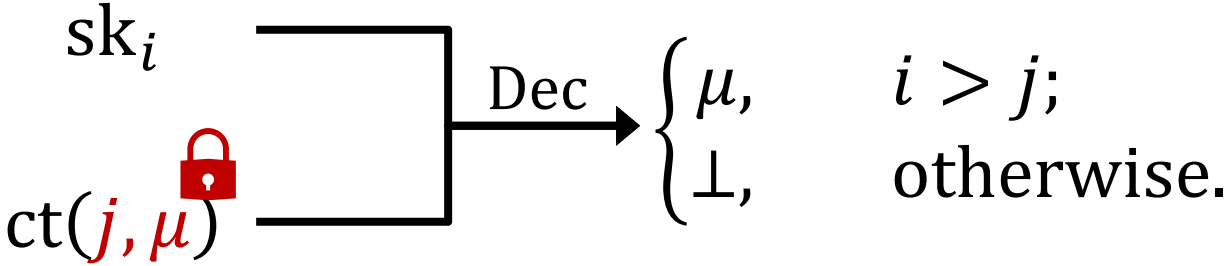

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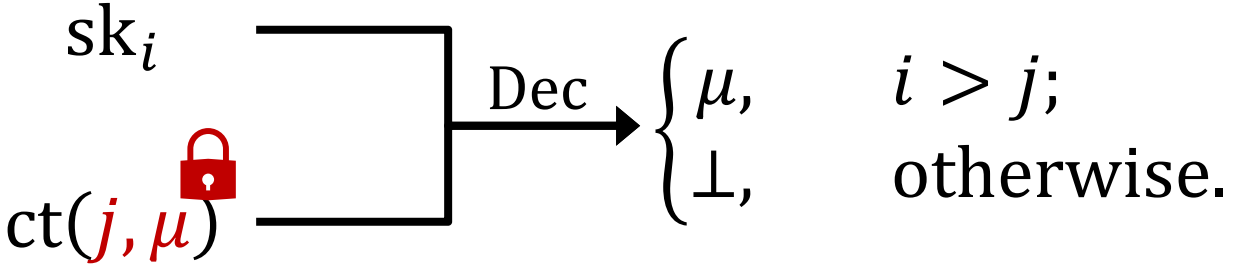

1

$$\mathrm{ct}(N, \mu) \approx \mathrm{ct}(N, 0) \text{ given } \mathrm{sk}_1, \ldots, \mathrm{sk}_N$$

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By (1), ≈ 0;
total gap of $\Omega(\varepsilon)$.

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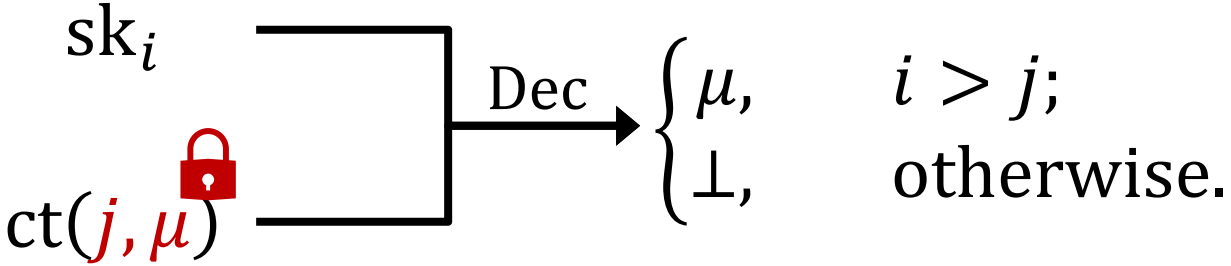

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By (2), if user i is honest, then $\Delta\varepsilon_i \approx 0$;

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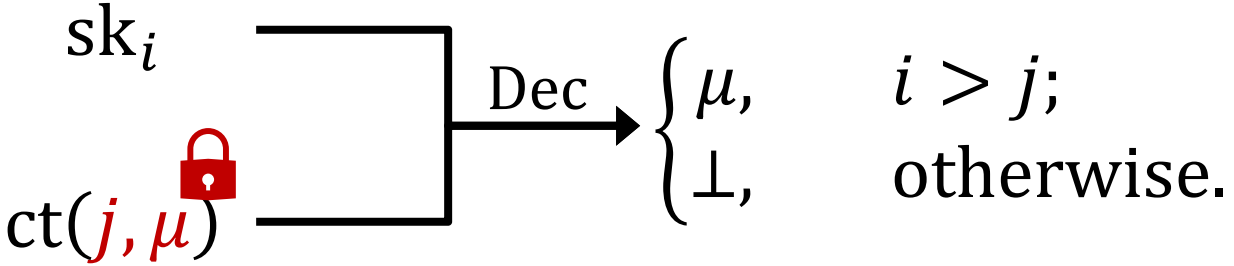

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By (2), if user i is honest, then $\Delta\varepsilon_i \approx 0$;
if $\Delta\varepsilon_i = \Omega(\varepsilon/N)$, then user i is a traitor.

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Revocable PLBE

Two Revocation Mechanisms: Index/Set Revocation

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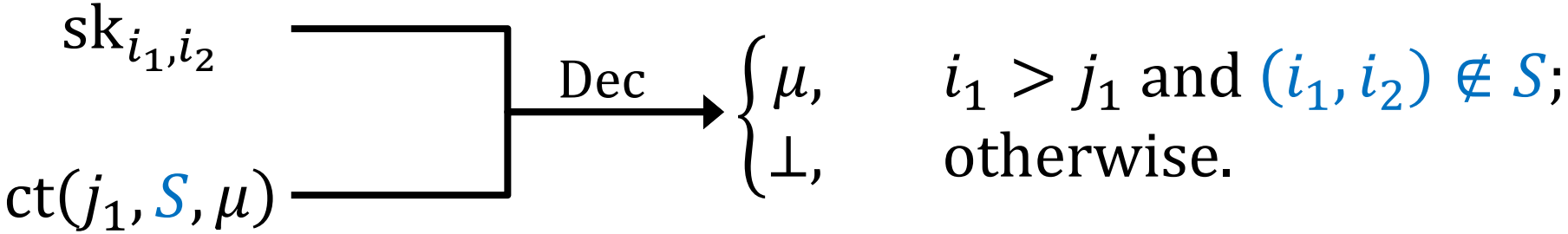
identity space $[N_1] \times [N_2]$



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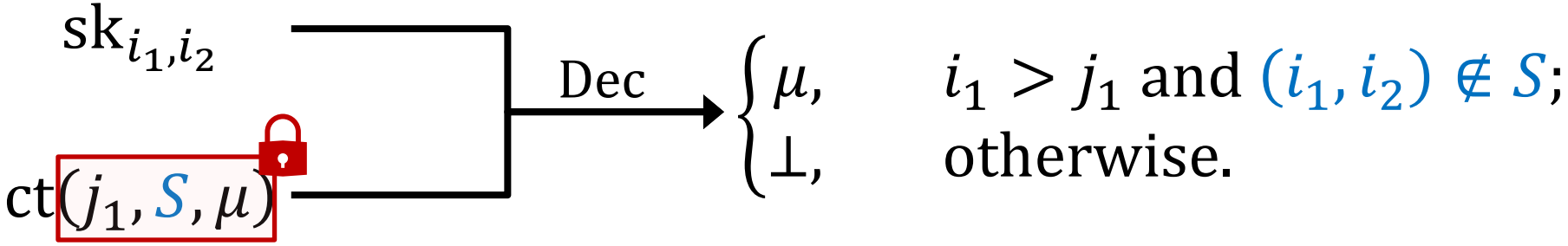

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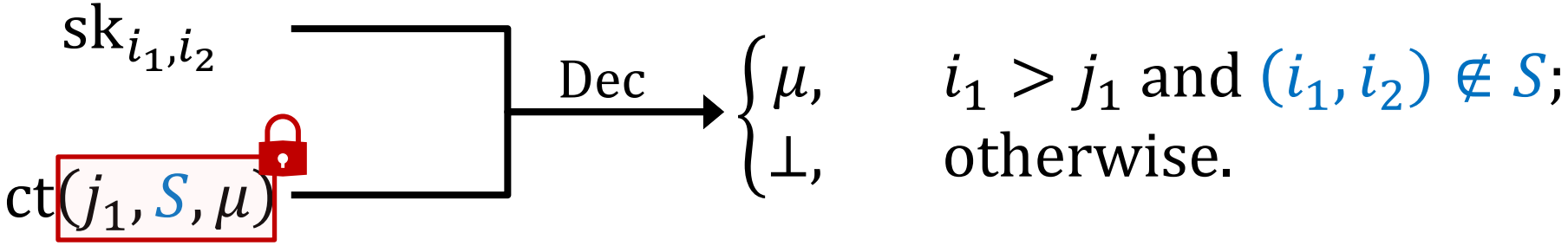

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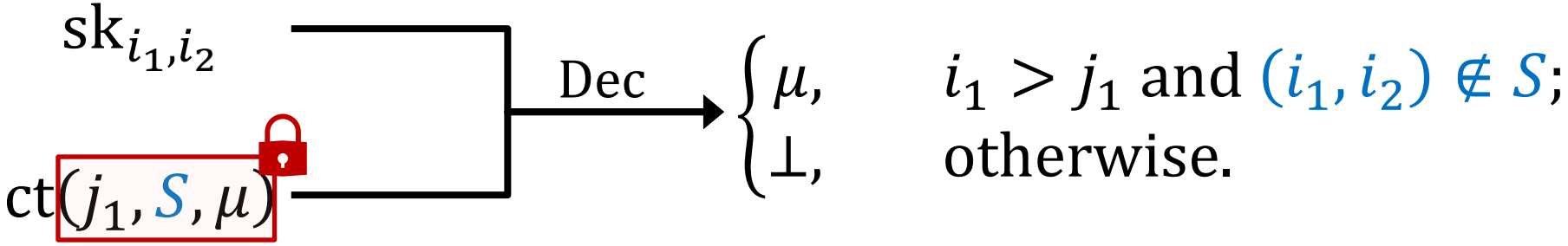

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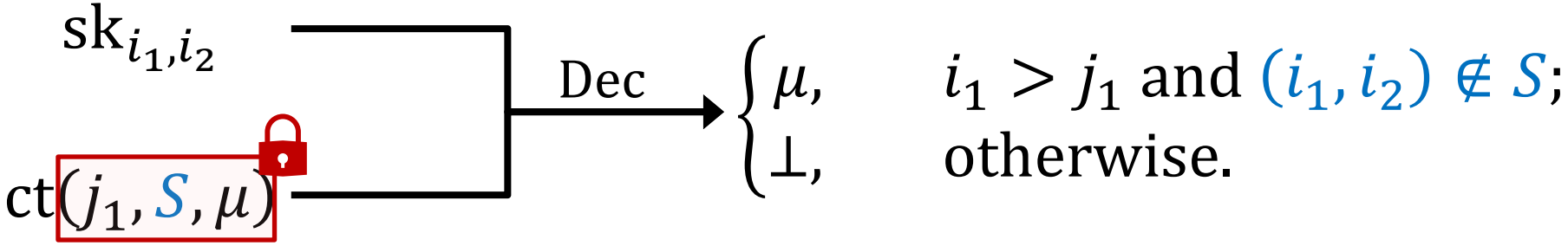

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Revocable PLBE and Traitor Tracing

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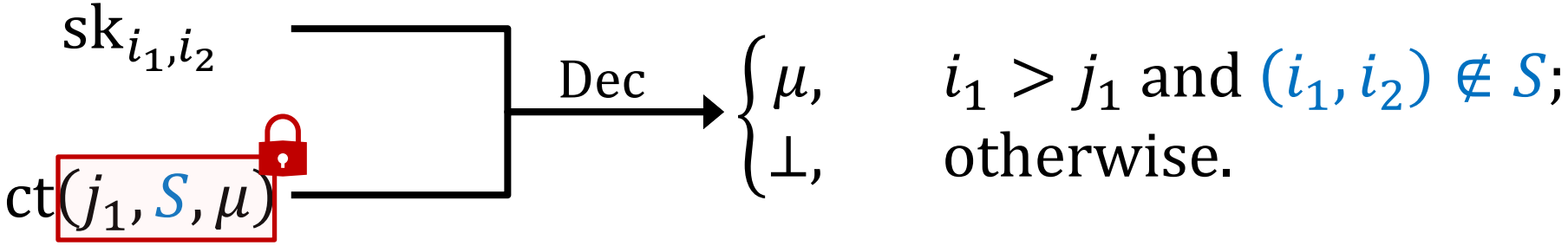

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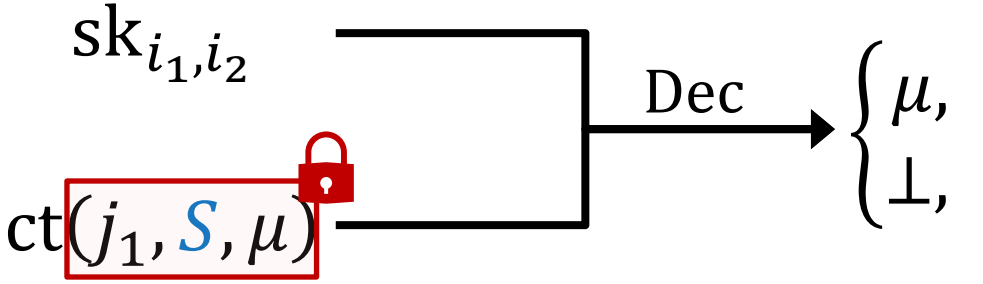

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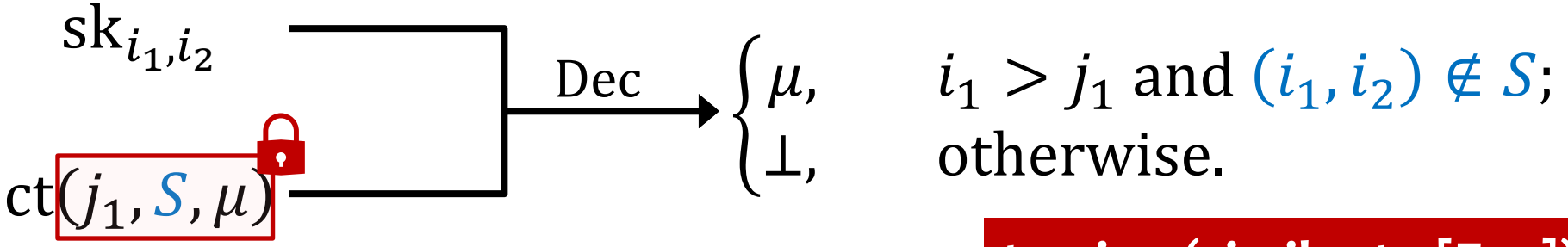

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Solution. if $f(z_0) = f(z_1)$, no move is needed;
if $f(z_0) \neq f(z_1)$, deduce $P(x) = 0$ and perform the move.

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Can we achieve such parameter sizes
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Thanks!

ePrint 2023/256

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$$\Delta\varepsilon_{i_1} = \Omega(\varepsilon / N_1)$$

$$\begin{array}{cc}
\mathrm{rPLBct}(i_1 - 1, \emptyset, \mu) & \mathrm{rPLBct}(i_1, \emptyset, \mu) \\
\mathrm{rPLBct}(i_1 - 1, S_1, \mu) & \mathrm{rPLBct}(i_1, S_1, \mu) \\
\vdots & \vdots \\
\mathrm{rPLBct}(i_1 - 1, S_{i_2 - 1}, \mu) & \mathrm{rPLBct}(i_1, S_{i_2 - 1}, \mu) \\
\mathrm{rPLBct}(i_1 - 1, S_{i_2}, \mu) & \mathrm{rPLBct}(i_1, S_{i_2}, \mu) \\
\vdots & \vdots \\
\mathrm{rPLBct}(i_1 - 1, S_{N_2}, \mu) & \mathrm{rPLBct}(i_1, S_{N_2}, \mu)
\end{array}$$

$\Delta\varepsilon \approx 0$ by **index-hiding**

$\mathrm{ct}(i_1 - 1, S_{N_2}, \mu) \approx \mathrm{ct}(i_1, S_{N_2}, \mu)$
given $\mathrm{sk}_{\neq i_1, \star}$'s and $\mathrm{sk}_{i_1, \star}$'s

Stock Question: Traitor Tracing from rPLBE: Identifying i_2

$\mathrm{TTsk}_i = \mathrm{rPLBsk}_i$

$\mathrm{TTct}(\mu) = \mathrm{rPLBct}(0, \emptyset, \mu)$ $\cdots$ $\mathrm{rPLBct}(i_1 - 1, \emptyset, \mu)$ $\mathrm{rPLBct}(i_1, \emptyset, \mu)$ $\cdots$ $\mathrm{rPLBct}(N_1, \emptyset, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_1, \mu)$ $\mathrm{rPLBct}(i_1, S_1, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_{i_2-1}, \mu)$ $\mathrm{rPLBct}(i_1, S_{i_2-1}, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_{i_2}, \mu)$ $\mathrm{rPLBct}(i_1, S_{i_2}, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_{N_2}, \mu)$ $\mathrm{rPLBct}(i_1, S_{N_2}, \mu)$

$$\Delta\varepsilon_{i_1} = \Omega(\varepsilon/N_1)$$

$$S_{i_2} = \{(i_1, \leq i_2)\}$$

By **set-hiding**,
if (i_1, i_2) is honest,
then $\Delta\Delta\varepsilon \approx 0$;

$\Delta\varepsilon \approx 0$ by **index-hiding**

$\mathrm{ct}(i_1 - 1, S_{N_2}, \mu) \approx \mathrm{ct}(i_1, S_{N_2}, \mu)$
given $\mathrm{sk}_{\neq i_1,\star}$'s and $\mathrm{sk}_{i_1,\star}$'s

Stock Question: Traitor Tracing from rPLBE: Identifying i_2

$\mathrm{TTsk}_i = \mathrm{rPLBsk}_i$

$\mathrm{TTct}(\mu) =$

$\mathrm{rPLBct}(0, \emptyset, \mu) \quad \cdots \quad \mathrm{rPLBct}(i_1 - 1, \emptyset, \mu) \quad \mathrm{rPLBct}(i_1, \emptyset, \mu) \quad \cdots \quad \mathrm{rPLBct}(N_1, \emptyset, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_1, \mu) \quad \mathrm{rPLBct}(i_1, S_1, \mu)$

$\vdots \qquad \vdots$

$\mathrm{rPLBct}(i_1 - 1, S_{i_2-1}, \mu) \quad \mathrm{rPLBct}(i_1, S_{i_2-1}, \mu)$

$\mathrm{rPLBct}(i_1 - 1, S_{i_2}, \mu) \quad \mathrm{rPLBct}(i_1, S_{i_2}, \mu)$

$\vdots \qquad \vdots$

$\mathrm{rPLBct}(i_1 - 1, S_{N_2}, \mu) \quad \mathrm{rPLBct}(i_1, S_{N_2}, \mu)$

$$\Delta\varepsilon_{i_1} = \Omega(\varepsilon/N_1)$$

$$S_{i_2} = \{(i_1, \leq i_2)\}$$

By **set-hiding**,
if (i_1, i_2) is honest,
then $\Delta\Delta\varepsilon \approx 0$;
so
if $\Delta\Delta\varepsilon = \Omega(\varepsilon/N_1N_2)$,
then (i_1, i_2) is a traitor.

$\Delta\varepsilon \approx 0$ by **index-hiding**

$\mathrm{ct}(i_1 - 1, S_{N_2}, \mu) \approx \mathrm{ct}(i_1, S_{N_2}, \mu)$
given $\mathrm{sk}_{\neq i_1,\star}$'s and $\mathrm{sk}_{i_1,\star}$'s