Proof-Carrying Data From Arithmetized Random Oracles

Thank you for many of the slides!

Jack O'Connor
Joint work with Megan Chen, Alessandro Chiesa, Tom Gur and Nicholas Spooner
Eurocrypt 2023

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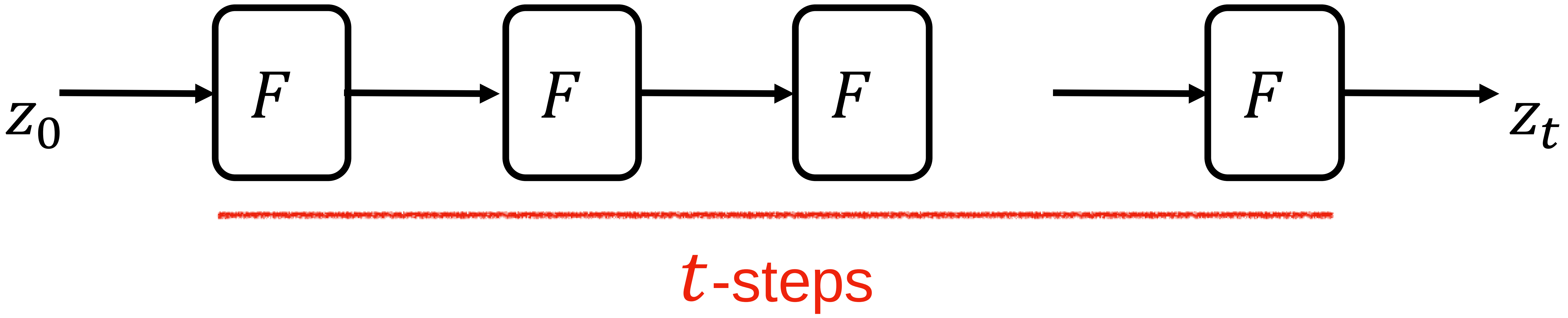

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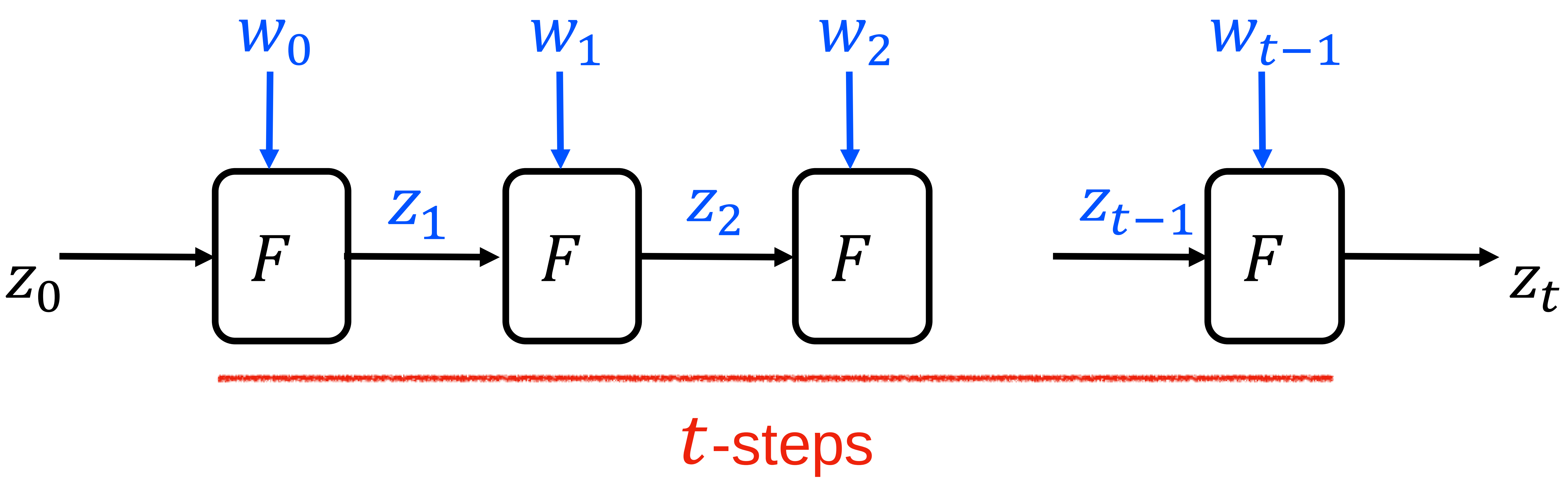

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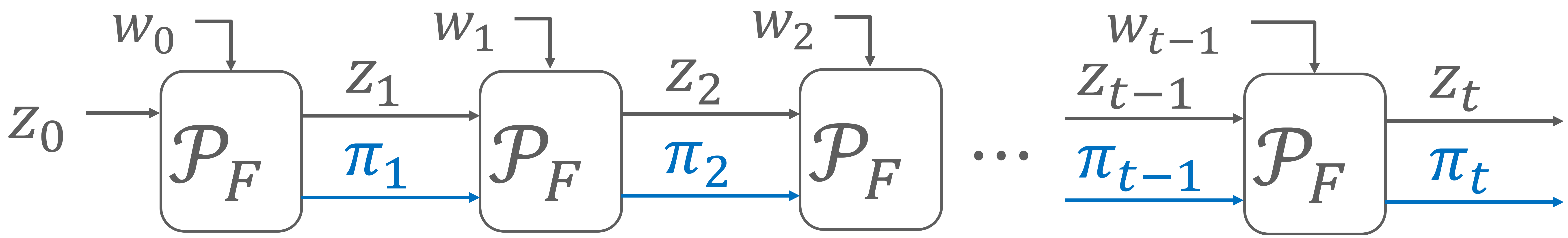
Incrementally verifiable computation (IVC) [Valiant08]



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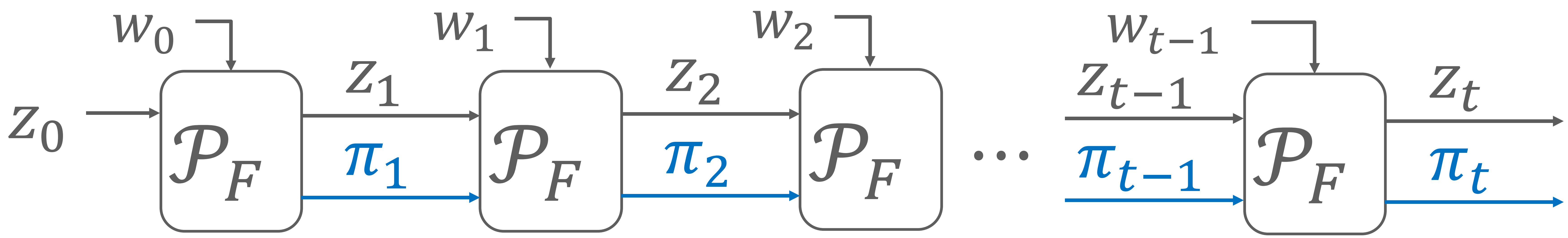

Proof-carrying Data (PCD) [CT10, BCCT13] generalizes path graph to DAG.

IVC from SNARK

SNARK = Succinct Non-interactive ARgument of Knowledge

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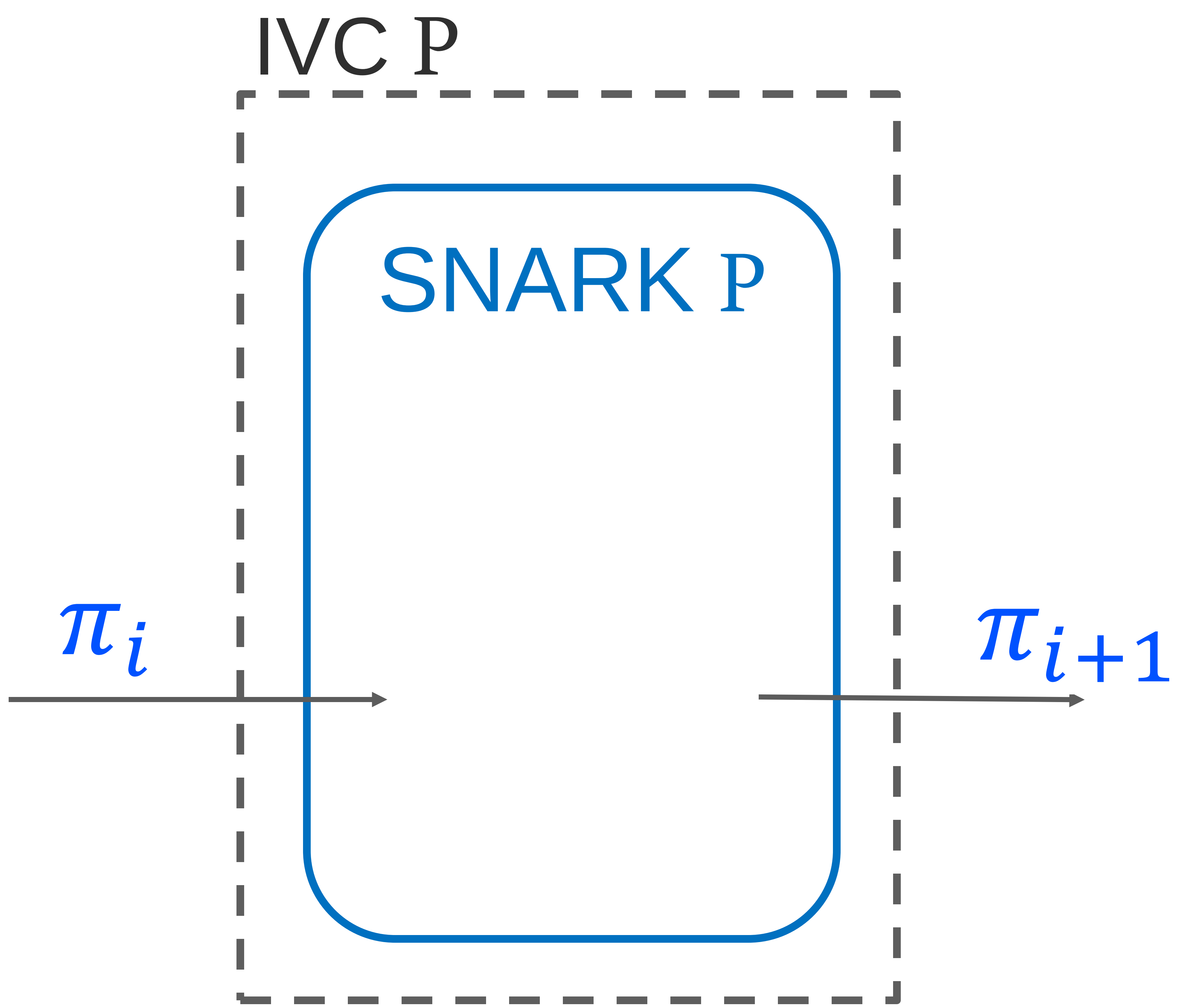

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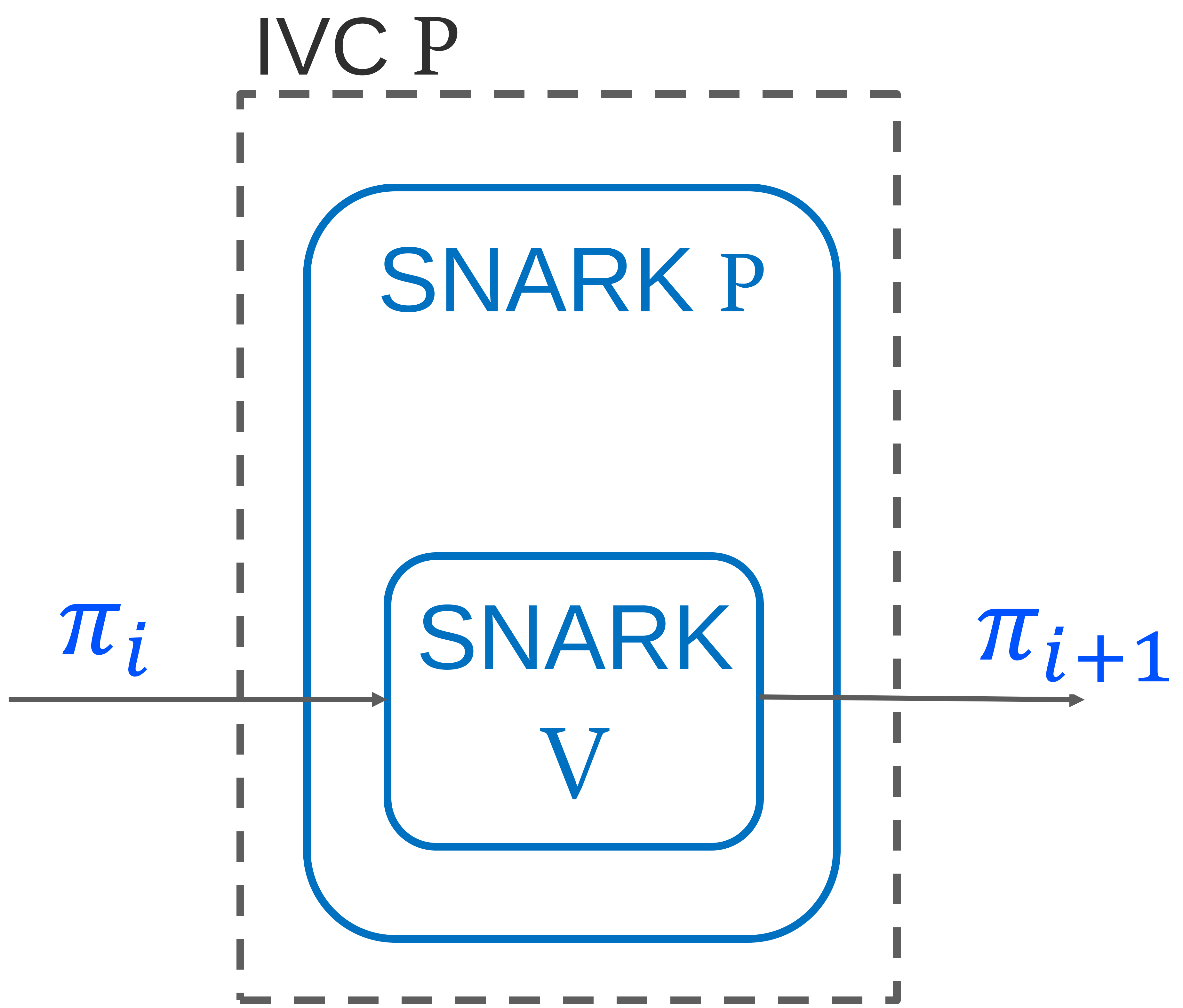
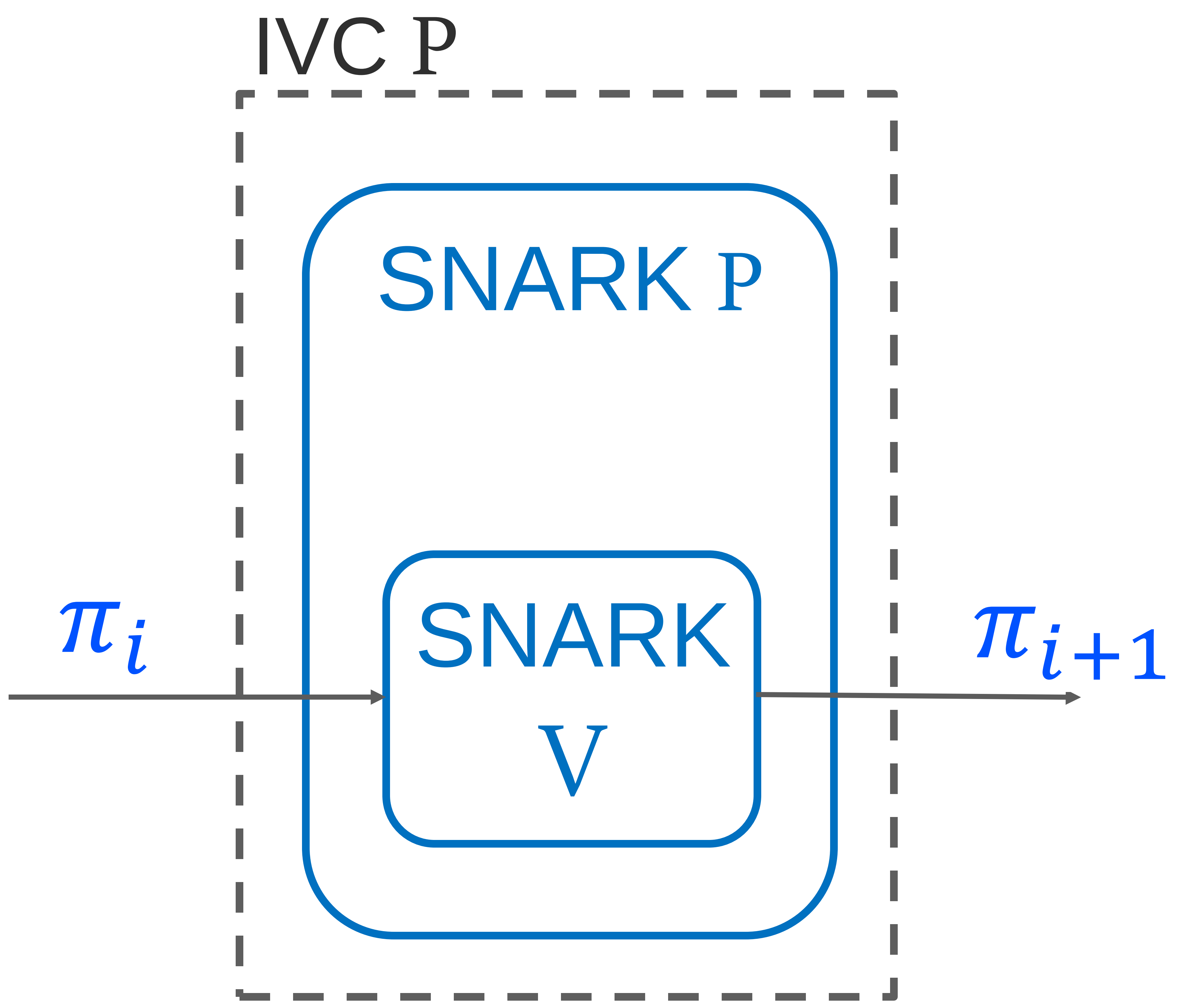


Prior works: SNARKs

SNARK = Succinct Non-interactive ARgument of Knowledge

Approach 1: CRS + knowledge (extraction) assumptions

[Groth10; GennaroGPR13; BitanskyCIOP13; Ben-SassonCTV14; BitanskyCCGLRT14; Groth16; GrothKMMM18]



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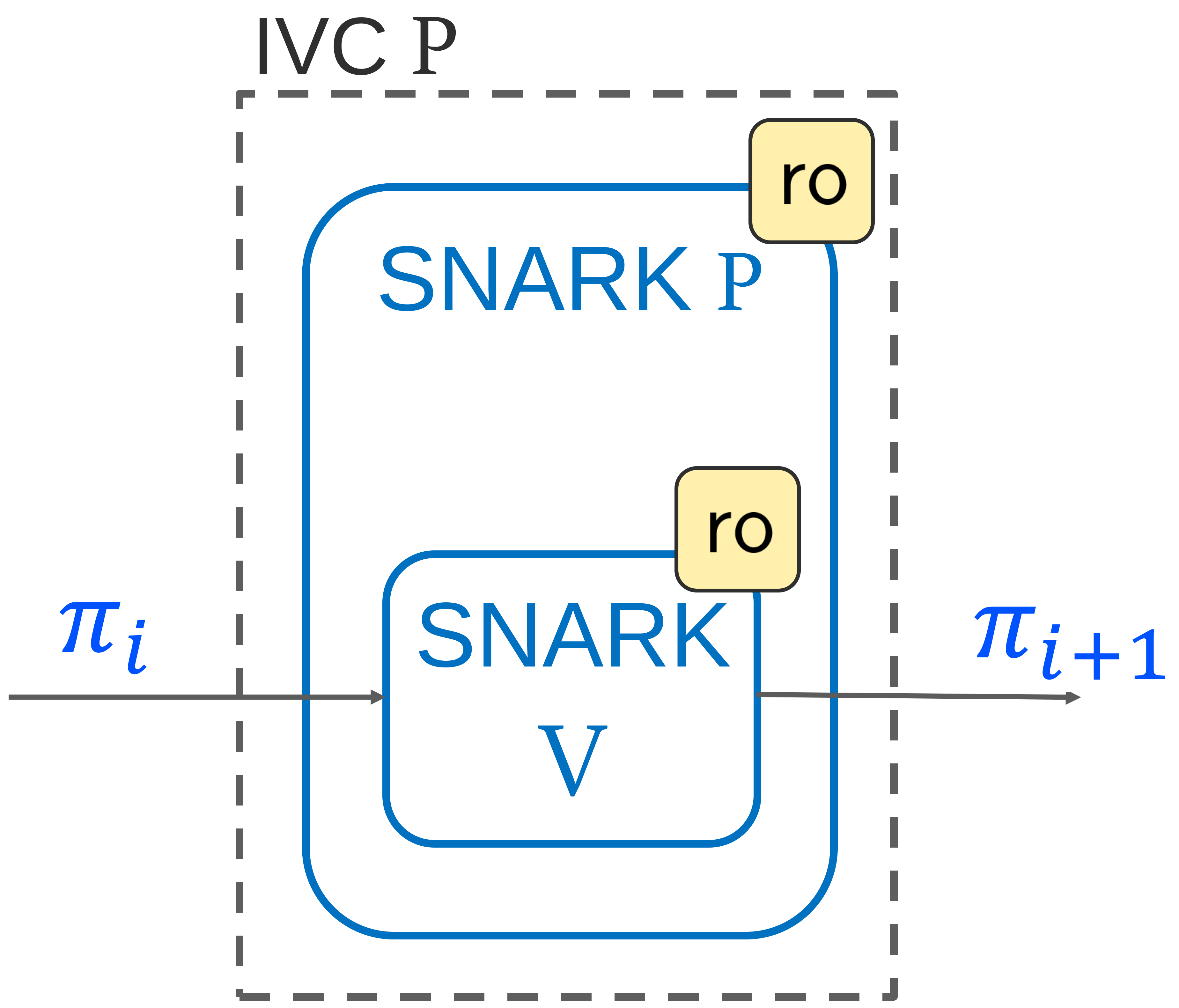
Approach 2: SNARKs in ROM

[Micali00; Ben-SassonCS16; ChiesaOS20; ChiesaHMMVW20]

Benefits:

- Transparent / universal setup
- Efficiency improvements



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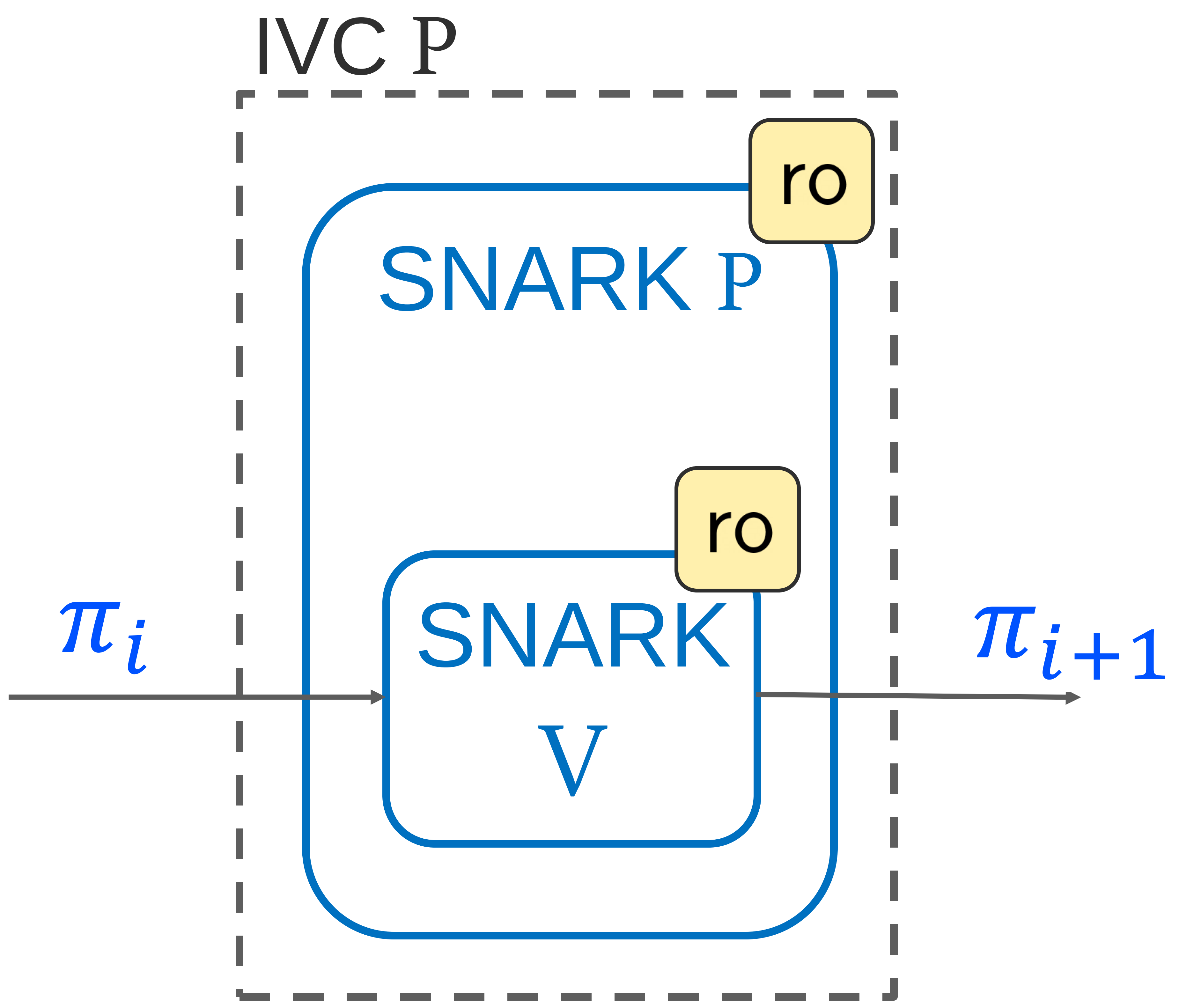

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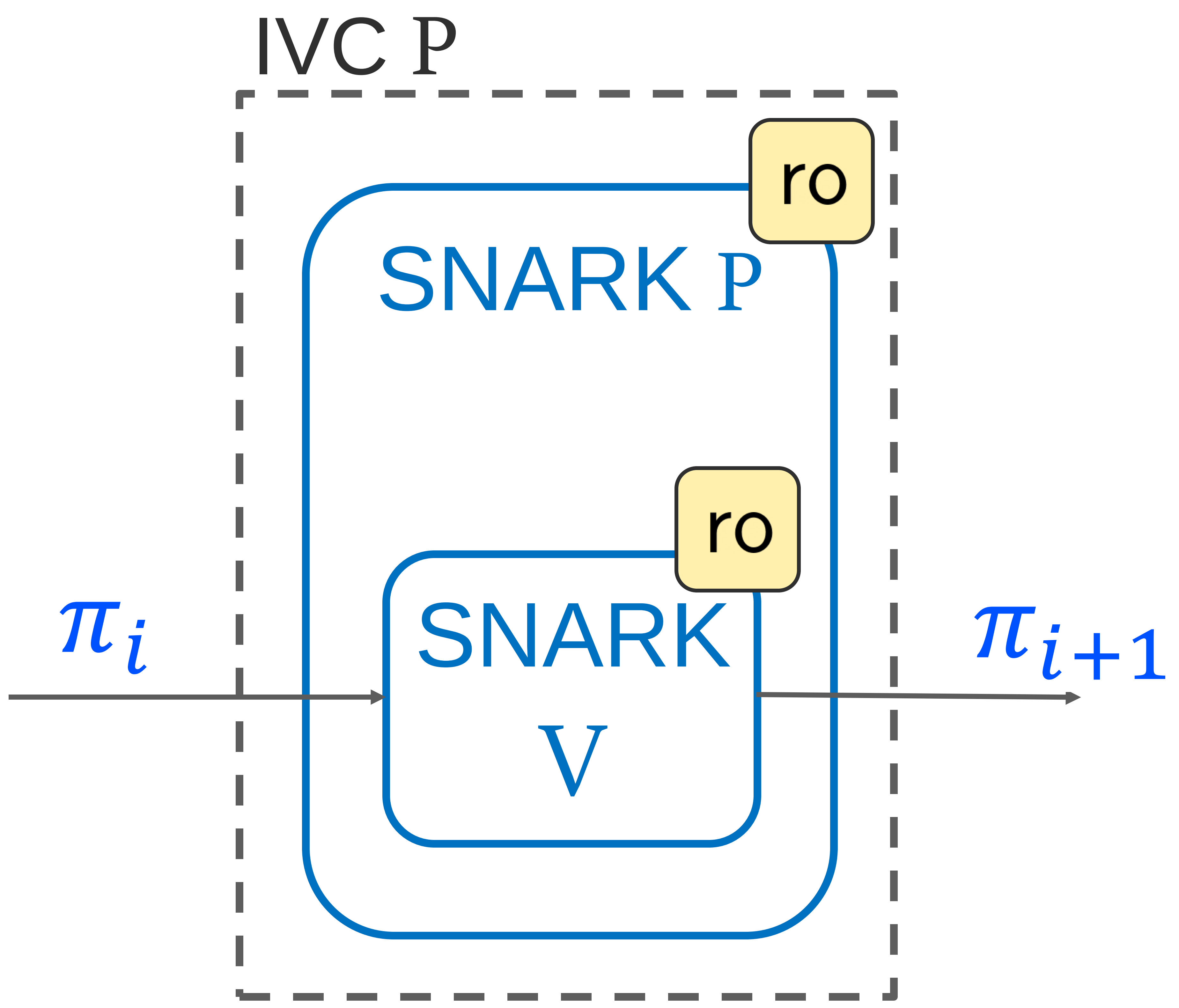

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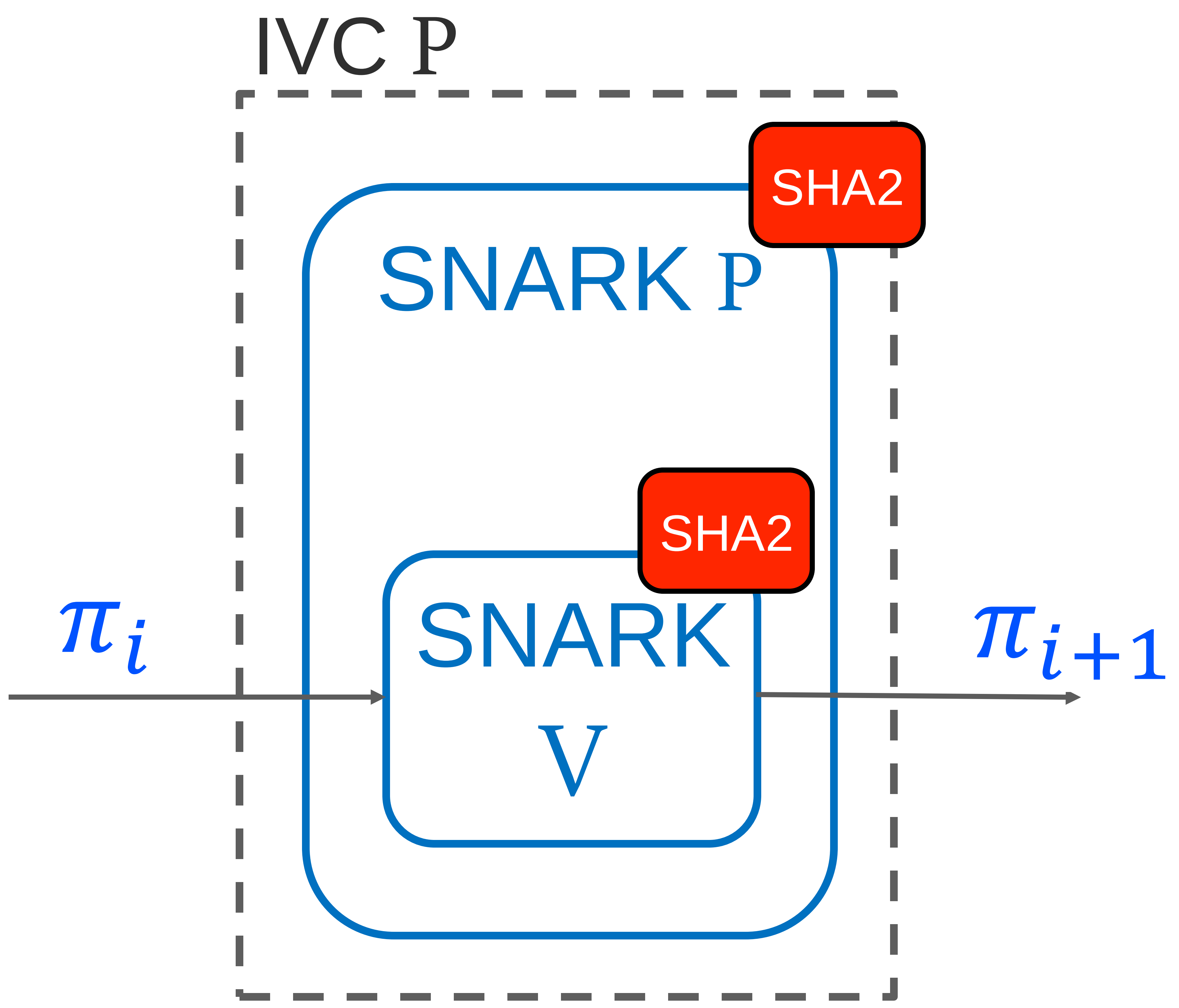
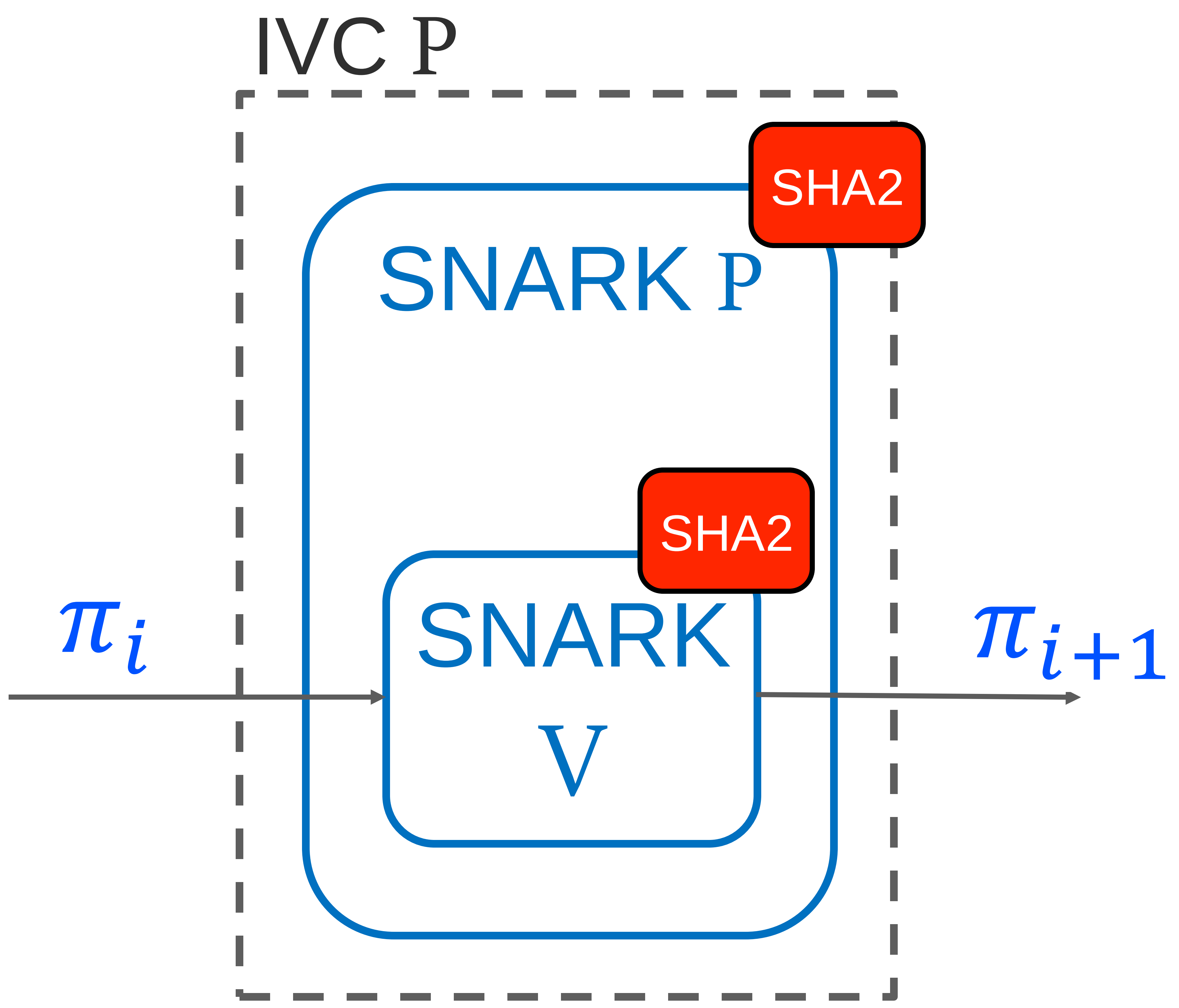
- [COS20;,…] Heuristically instantiate ro.



Issues with heuristic RO instantiation

Theoretical:



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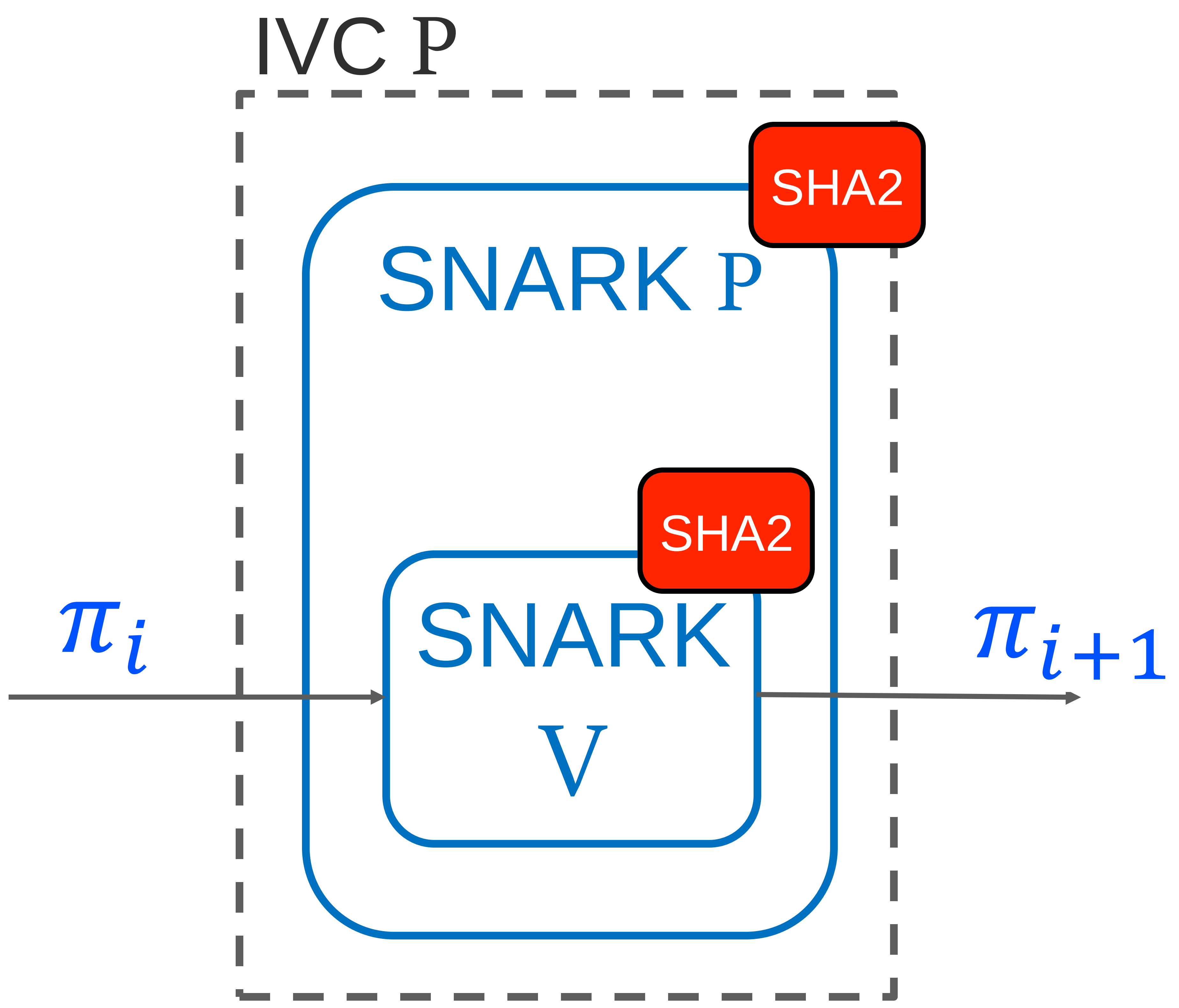

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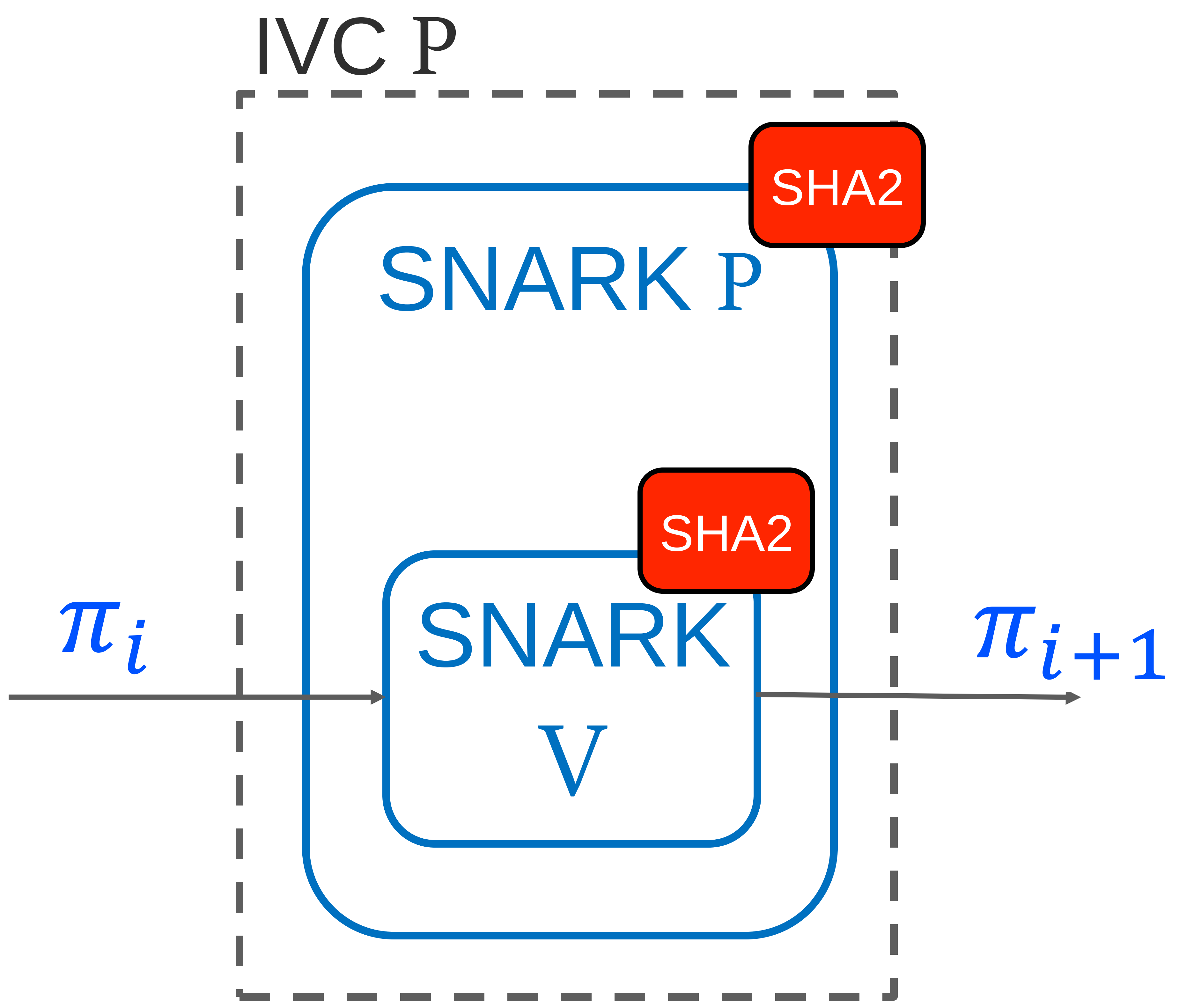

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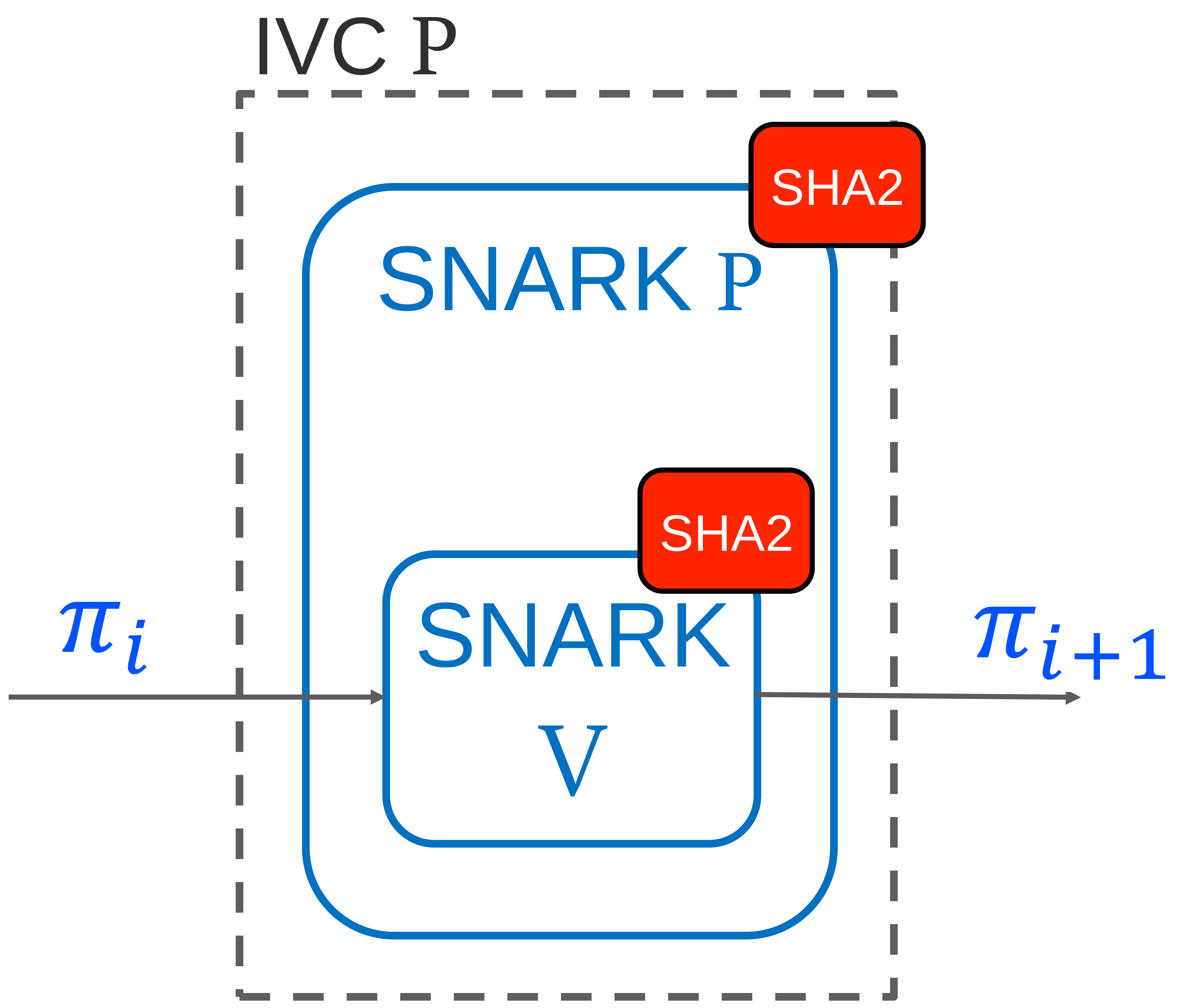

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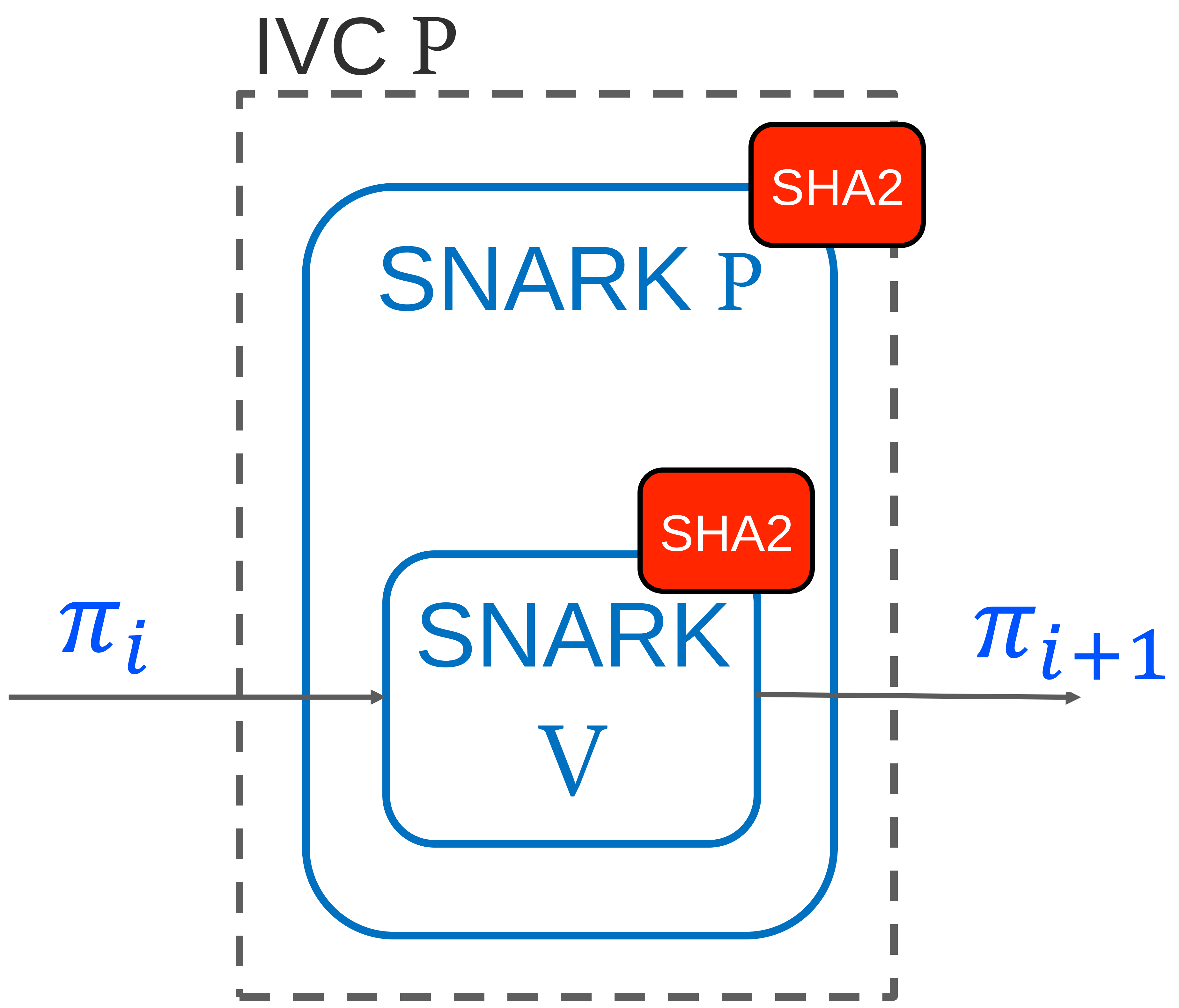

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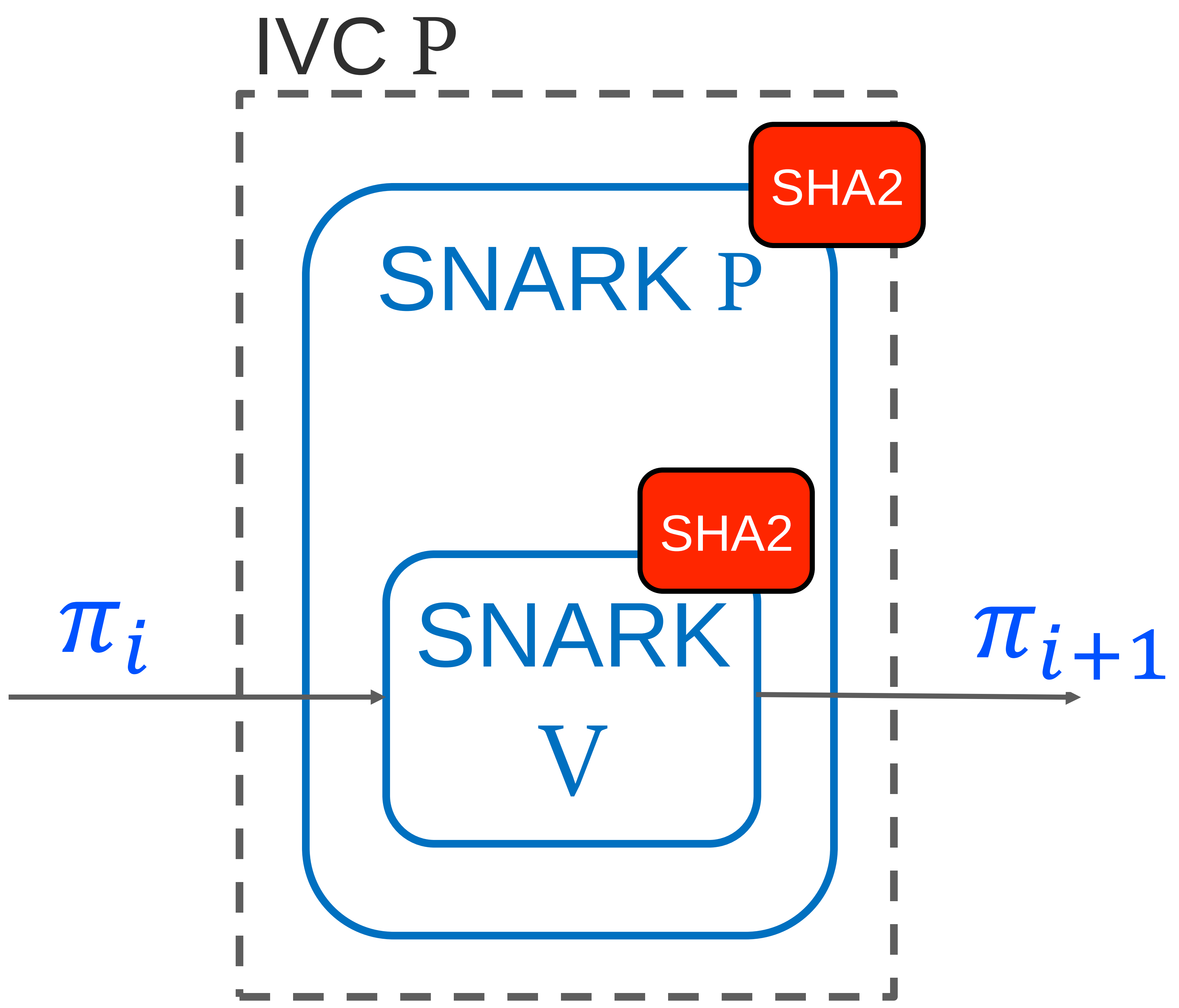
- No flexibility: Oracle must be instantiated as a circuit: can't use MPC, hardware token.
- Inefficient: SNARKs about SHA2, BLAKE are expensive!



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Is there an oracle model for which:

1. There exists a PCD scheme in this model under standard assumptions; and

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Our results

Yes!

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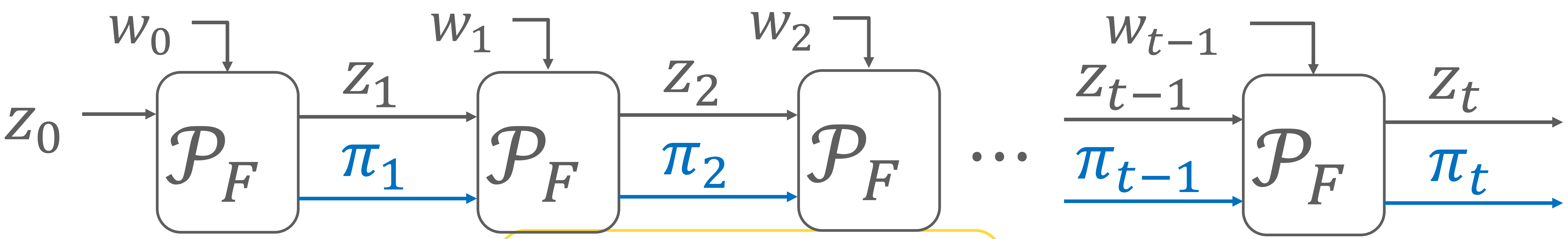

Secure to arbitrary depth t

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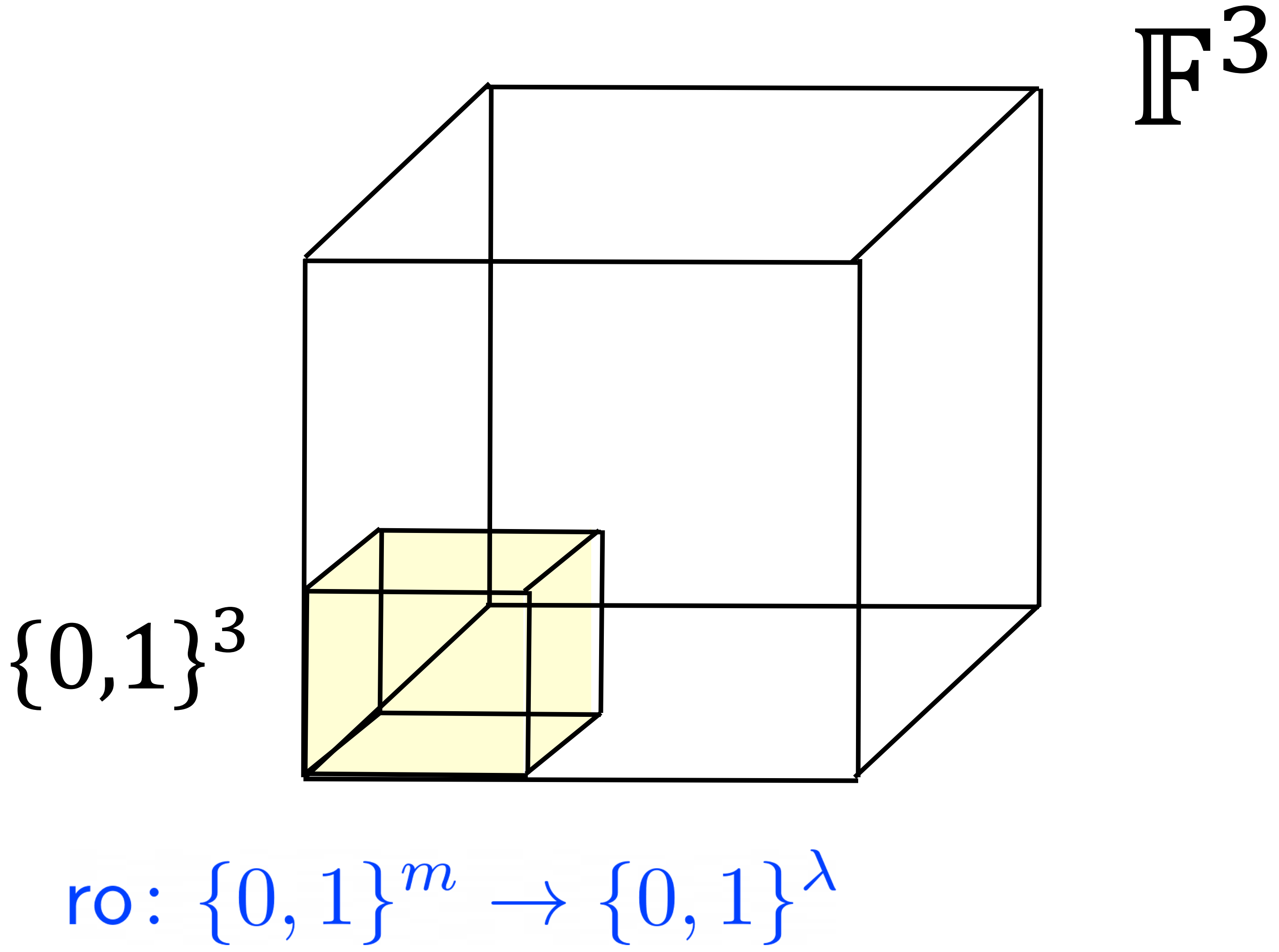

Random $\hat{\rho} \in \mathbb{F}^{\leq d}[X_1, \ldots, X_m]$ such that:

- Points in Boolean hypercube agree with the random oracle.
- Is low-degree (e.g. $d = O(1)$).
- Can query ANY point in $\mathbb{F}^m$.

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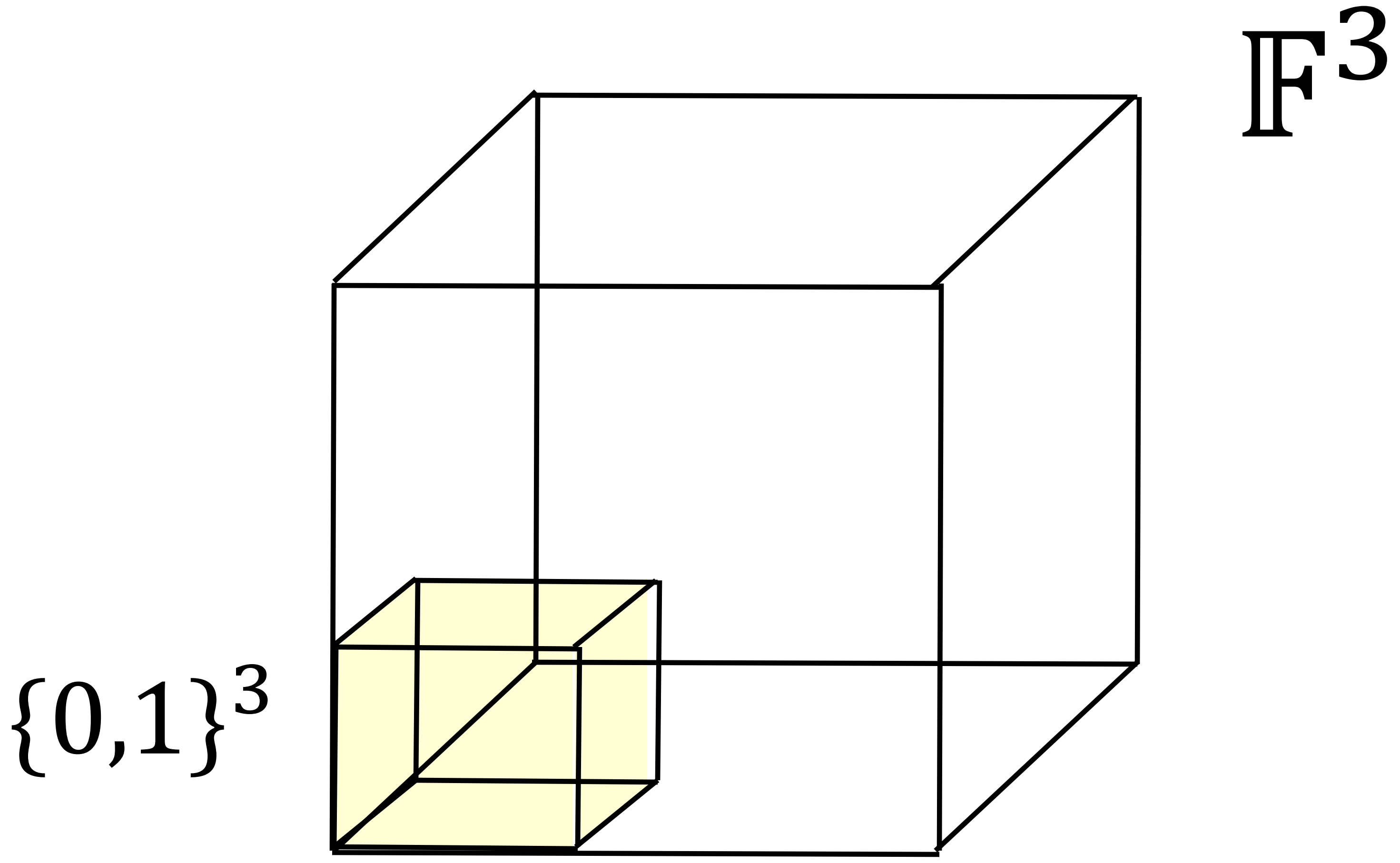

ro: $\{0,1\}^m \to \{0,1\}^\lambda$

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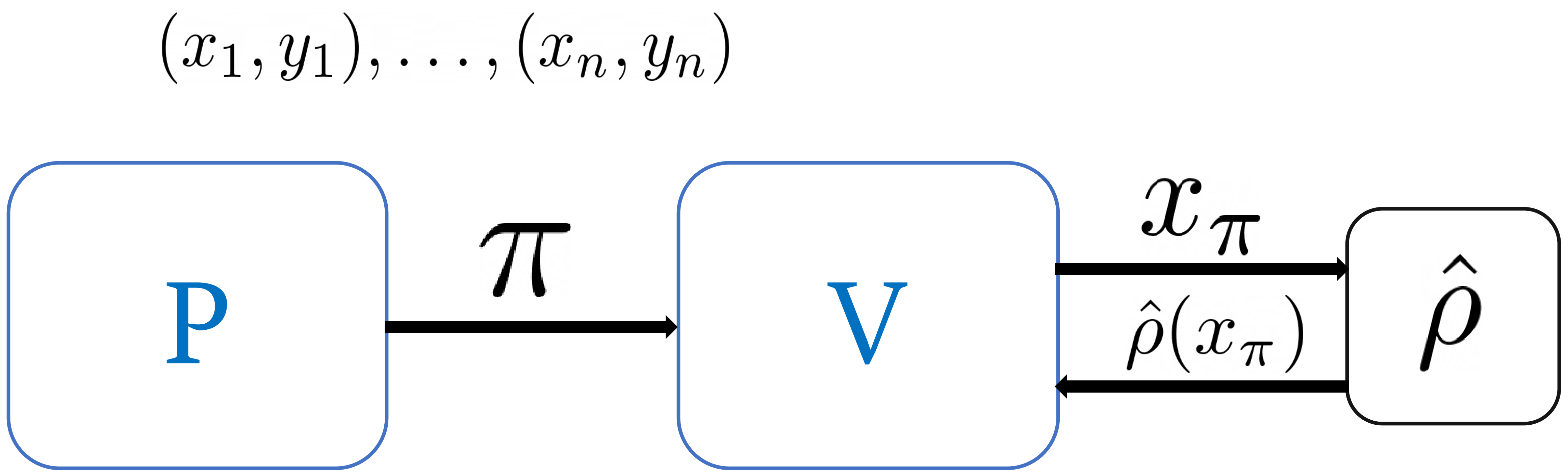
- [CCS22] give a non-interactive **query-reduction** protocol in the LDRO.
- This allows V to verify n queries to $\hat{\rho}$ with a single query to $\hat{\rho}$!



Arithmetization of hash functions

- Let's try to instantiate the low-degree random oracle...

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Hash function

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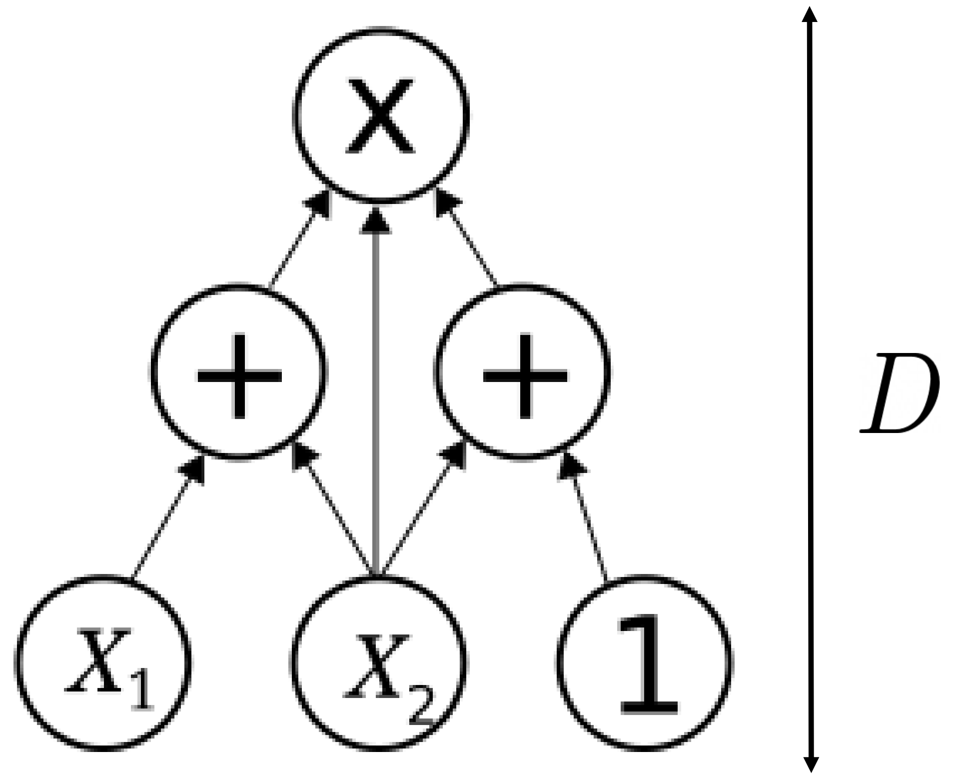

Arithmetic circuit

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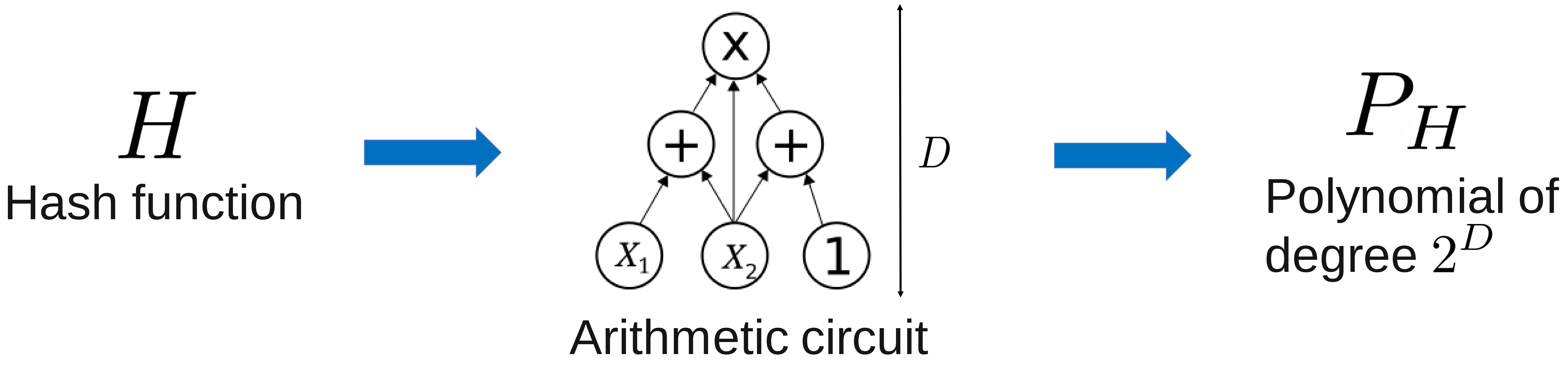

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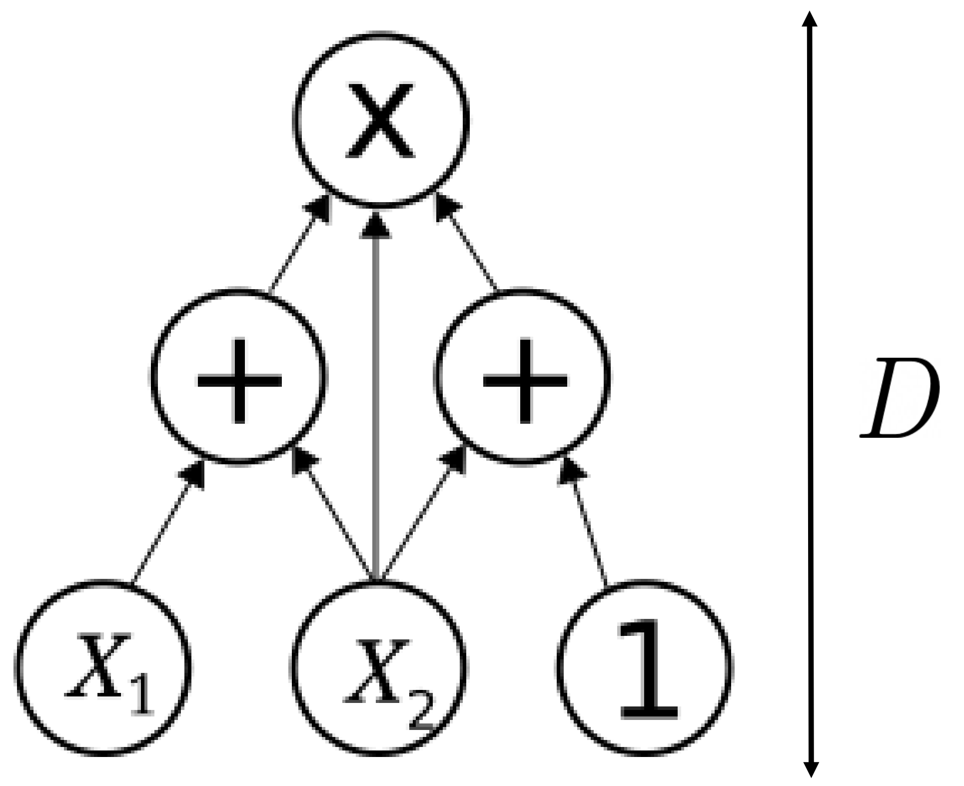

Arithmetic circuit

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Polynomial of degree 2^D

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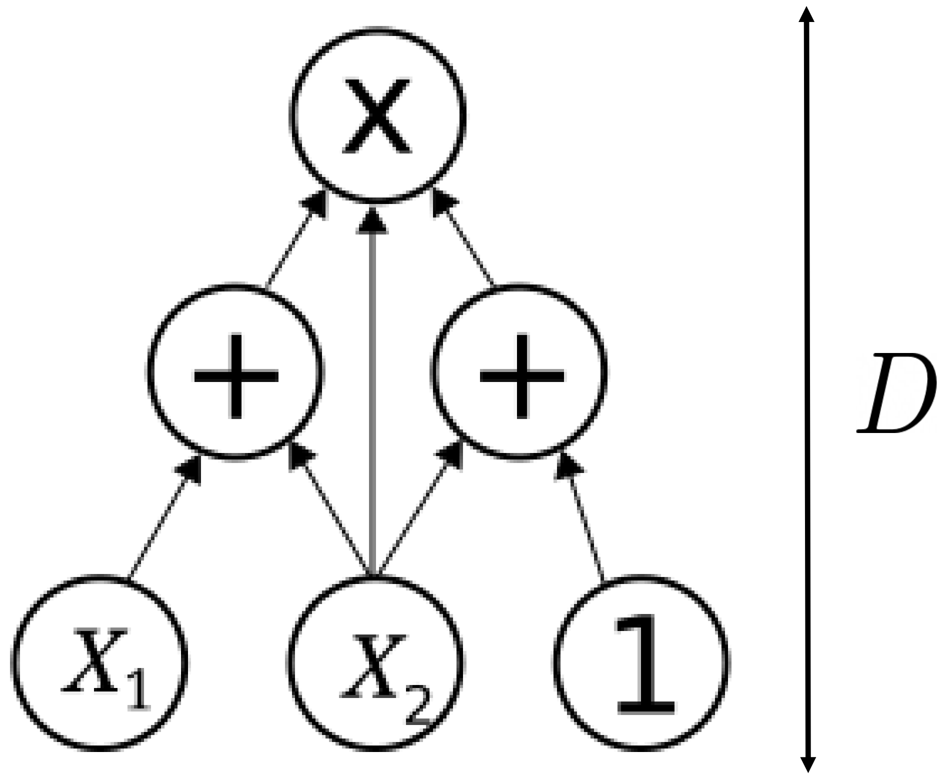

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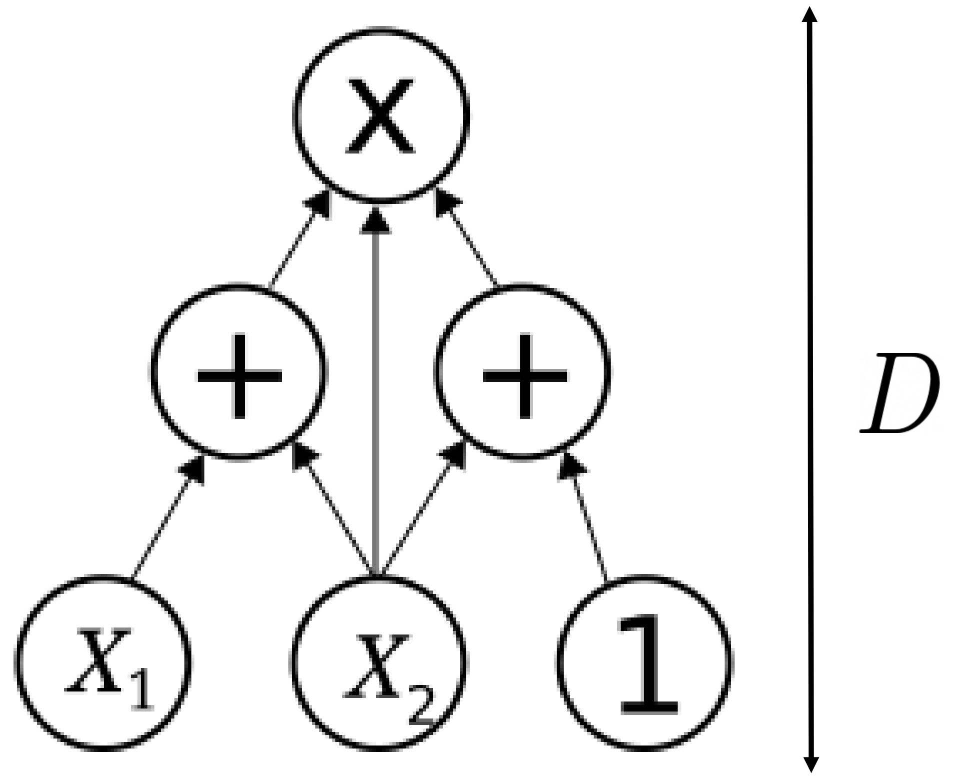

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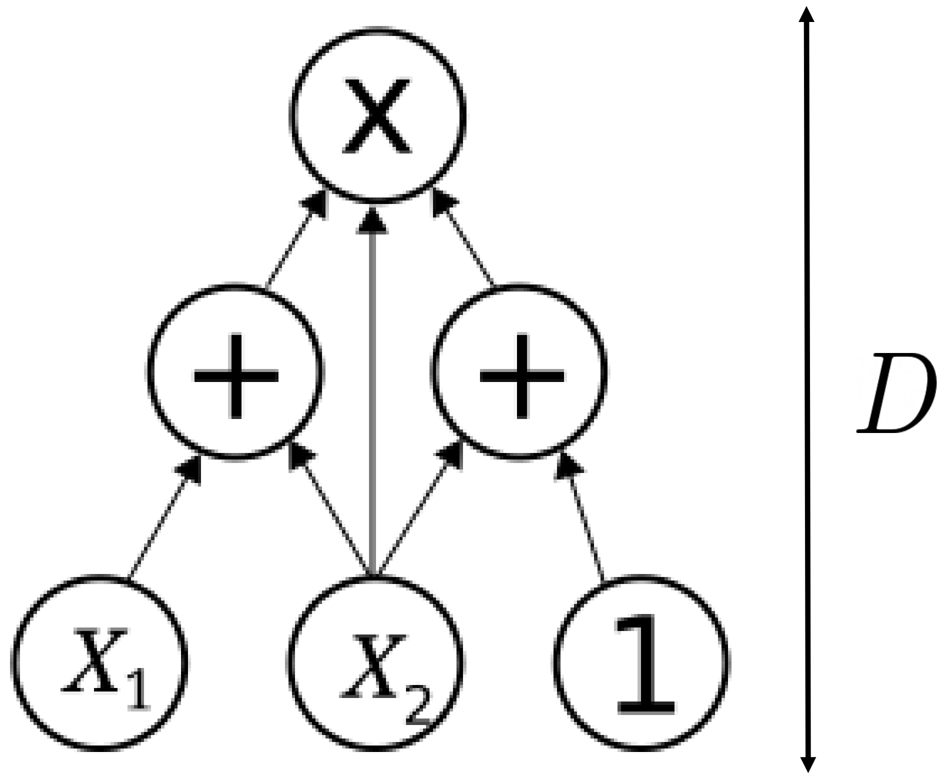

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- We need to reduce the depth of this circuit.

Degree reduction

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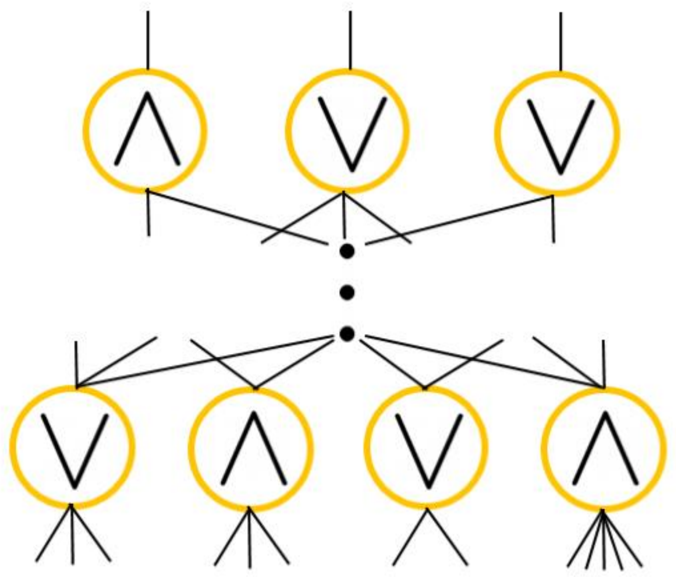

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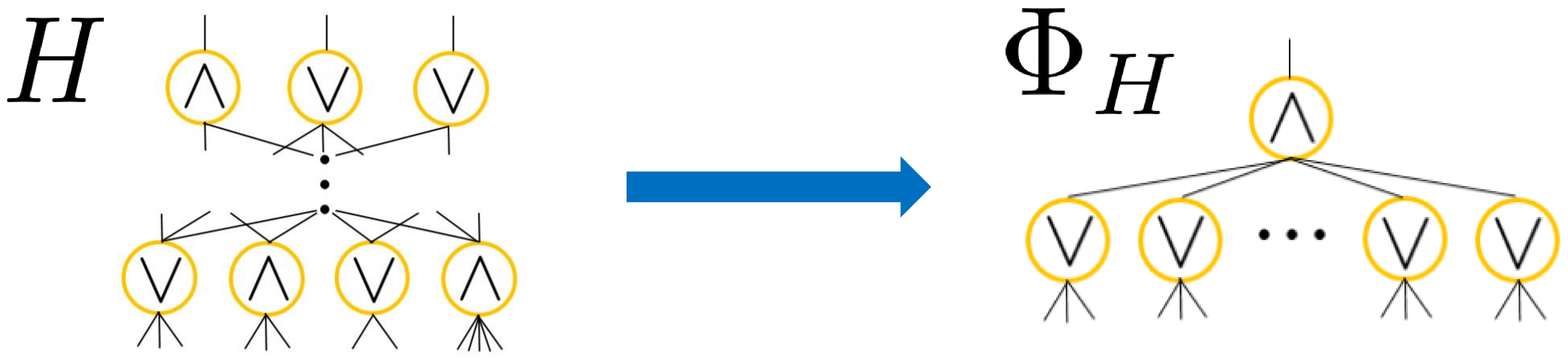

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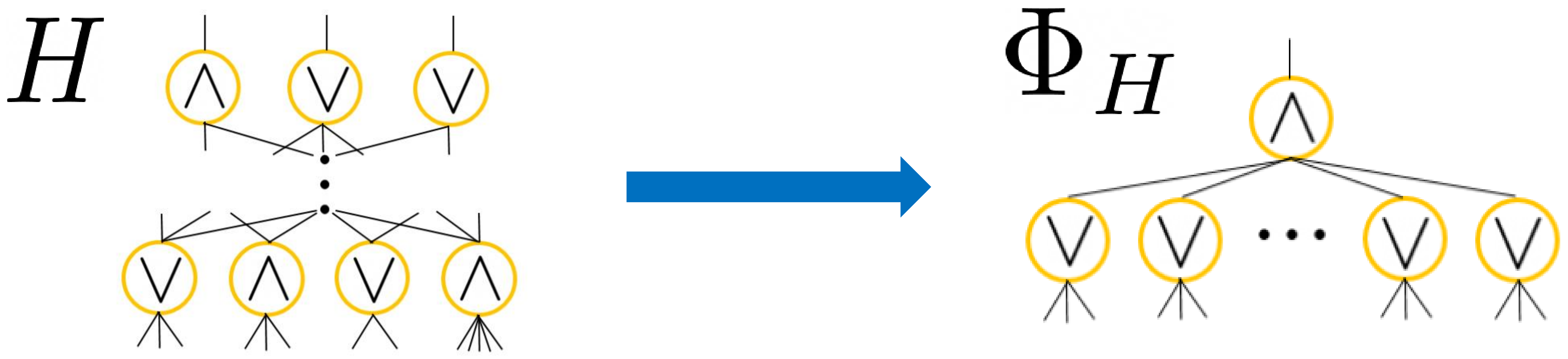
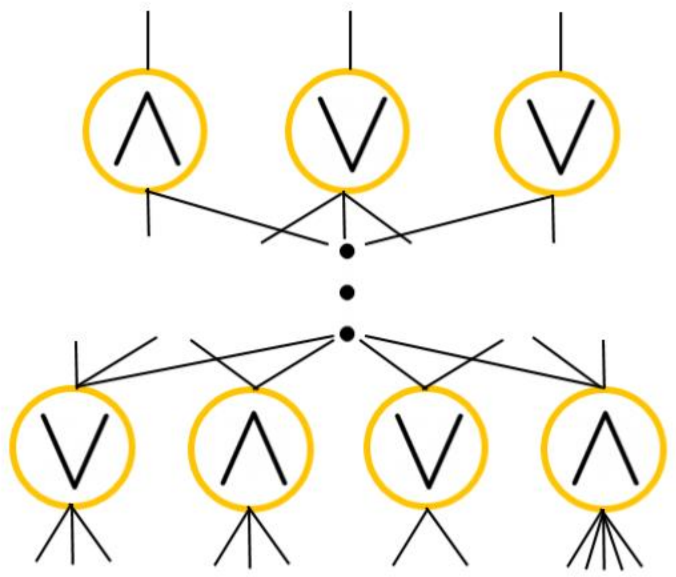


$$\Phi_H(x, y, z) = \begin{cases} 1 & \text{if } H(x) = y \text{ and } W_H(x) = z \\ 0 & \text{otherwise} \end{cases}$$

Efficiently computable

Degree reduction

H



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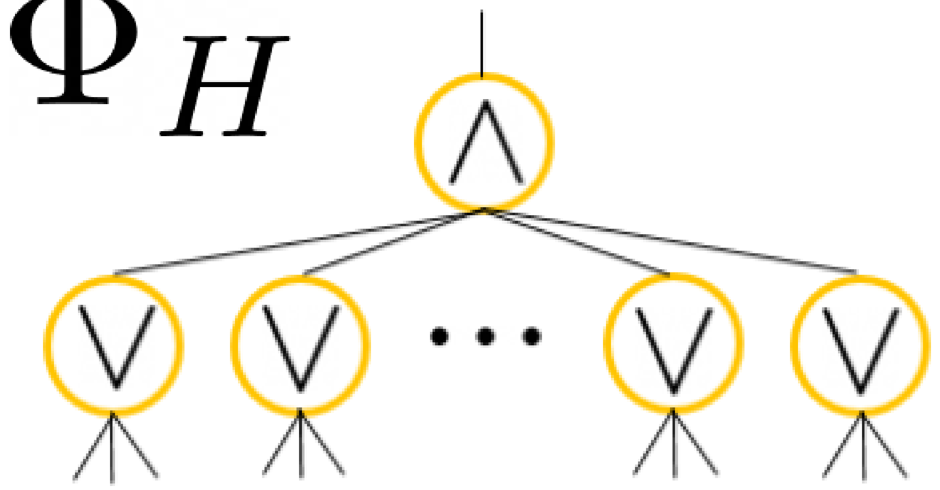

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Polynomial of degree $O(|H|)$

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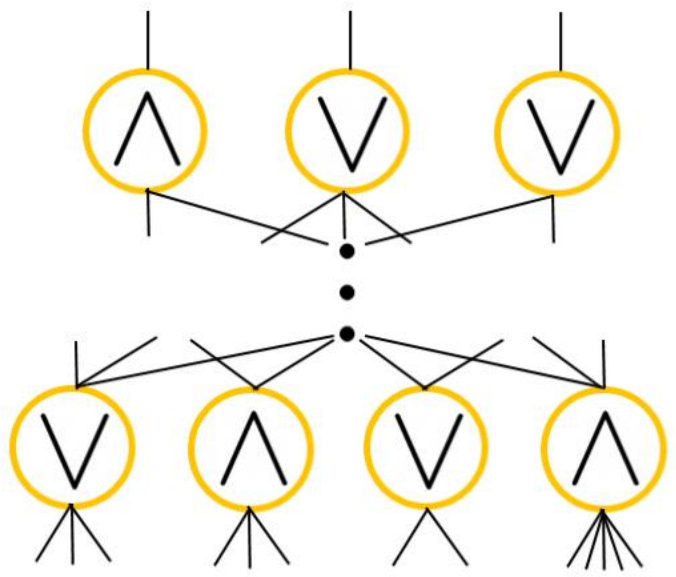

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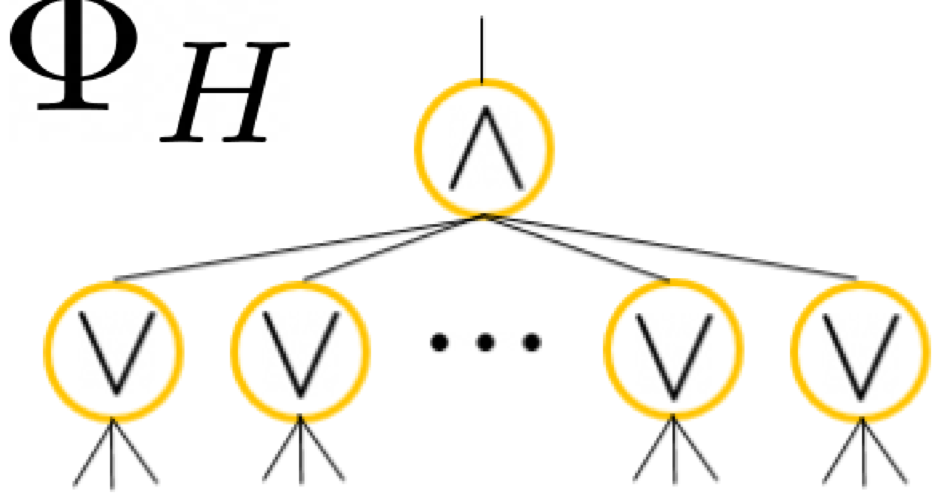

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Efficiently computable

- This is <u>not</u> a low-degree extension of H, so we can't instantiate the LDRO this way.

Arithmetized Random Oracle Model

Hash function H

Witness function W_H

Verification polynomial

$$P_H(x, y, z) = 1$$

$$\iff$$

$$H(x) = y \wedge W_H(x) = z$$

Arithmetized Random Oracle Model

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Random oracle
ro: $\{0,1\}^m \rightarrow \{0,1\}^\lambda$

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→

Extended verification oracle $\hat{\text{vo}}\colon \mathbb{F}^m \to \mathbb{F}$

low-degree extension of

$$\text{vo}(x,y,z) = 1$$
$$\iff$$
$$\text{ro}(x) = y \wedge \text{wo}(x) = z$$

Modelling Challenges

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$\hat{vo}$ is a uniformly random low-degree extension of vo with $d \geq 2$.

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- Weakening this choice is a question for future work.

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Security in ROM => Security in AROM.

Via the combinatorial nullstellensatz [Alo99] and results from algebraic query complexity [AW09].

PCD in the AROM.

Overview of construction

By constructing an accumulation scheme [BCMS20] for the arithmetized random oracle.

Security in ROM => Security in AROM.

+

Query reduction in the AROM.

=

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Thanks!

Emulation of the ARO

Lemma: There exists a probabilistic algorithm $\mathcal{M}$ such that for every security parameter $\lambda \in \mathbb{N}$, query bound $t \in \mathbb{N}$, and t-query adversary:

$$\left| \Pr_{(\mathsf{ro},\mathsf{wo},\hat{\mathsf{vo}}) \leftarrow \mathcal{O}(\lambda)} \left[\mathcal{A}^{(\mathsf{ro},\mathsf{wo},\hat{\mathsf{vo}})} = 1 \right] - \Pr_{(\mathsf{ro},\mathsf{wo},\hat{\mathsf{vo}}) \leftarrow \mathcal{O}(\lambda)} \left[\mathcal{A}^{\mathcal{M}^{(\mathsf{ro},\mathsf{wo})}} = 1 \right] \right| \leq \frac{t}{2^\lambda}$$