Quel est le prix du chiffrement fonctionnel ?

On the Optimal Succinctness and Efficiency of Functional Encryption and Attribute-Based Encryption

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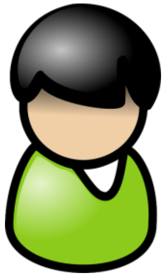
Partially Hiding Functional Encryption

$\mathsf{Setup} \rightarrow \mathsf{mpk}, \mathsf{msk}$

Partially Hiding Functional Encryption

Setup $\rightarrow$ mpk, msk

mpk



$\mathrm{Enc}(\mathrm{mpk}, x, y) \rightarrow \mathrm{ct}_x(y)$

public/private inputs

Partially Hiding Functional Encryption

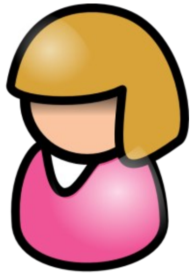
$\mathrm{Setup} \to \mathrm{mpk}, \mathrm{msk}$

$\mathrm{KeyGen}(\mathrm{msk}, f) \to \mathrm{sk}_f$

function

$\mathrm{mpk}, f, \mathrm{sk}_f$

mpk



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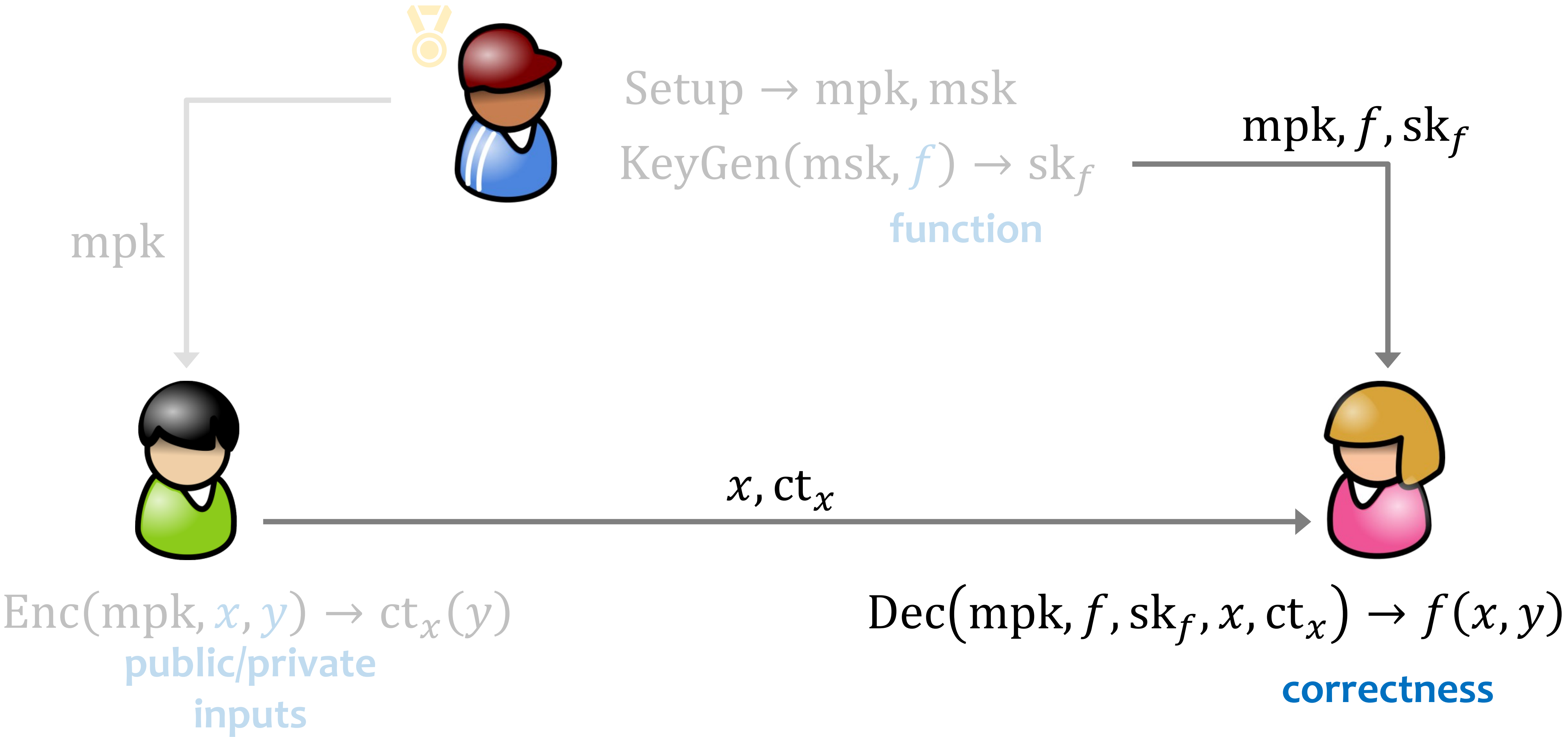


Partially Hiding Functional Encryption

Setup $\rightarrow$ mpk, msk

KeyGen(msk, f) $\rightarrow$ sk_f

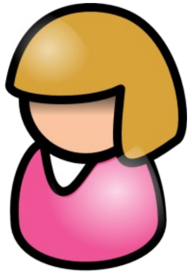
function

mpk

mpk, f, sk_f

f, x are always provided in the clear.

x, ct_x



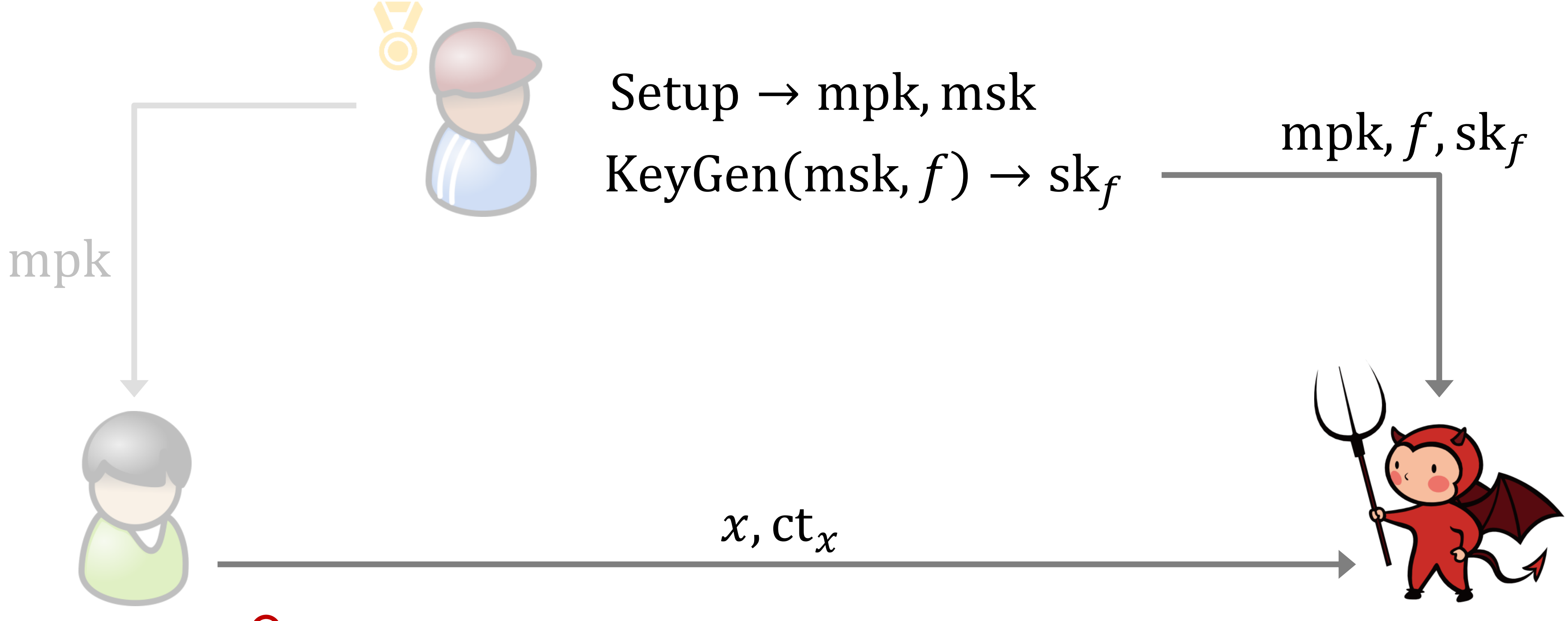
Enc(mpk, x, y) $\rightarrow$ $\mathrm{ct}_x(y)$

public/private inputs

$\mathrm{Dec}\big(\mathrm{mpk}, \boxed{f}, \mathrm{sk}_f, \boxed{x}, \mathrm{ct}_x\big) \rightarrow f(x, y)$

correctness

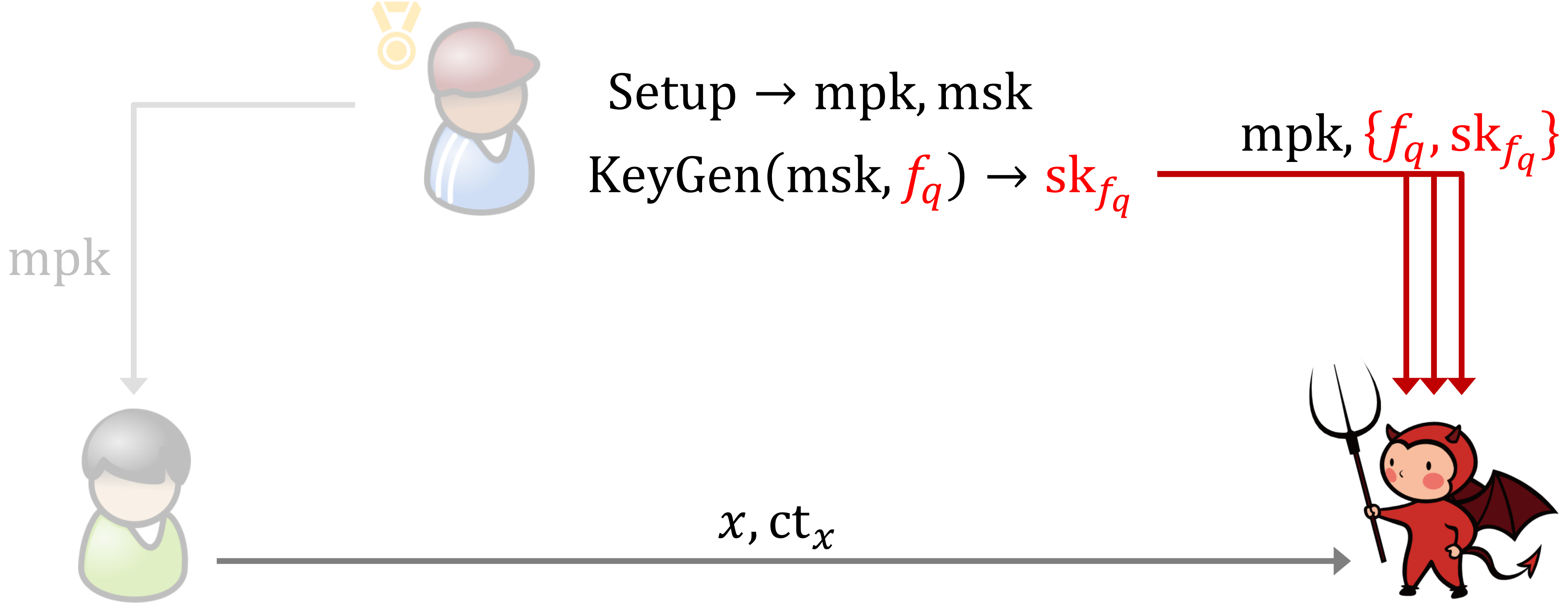
PHFE Security



$\mathrm{Enc}(\mathrm{mpk}, x, y) \to \mathrm{ct}_x(y)$

Security. Hides all information about y beyond $f(x, y)$.

PHFE Security: Collusion Resistance



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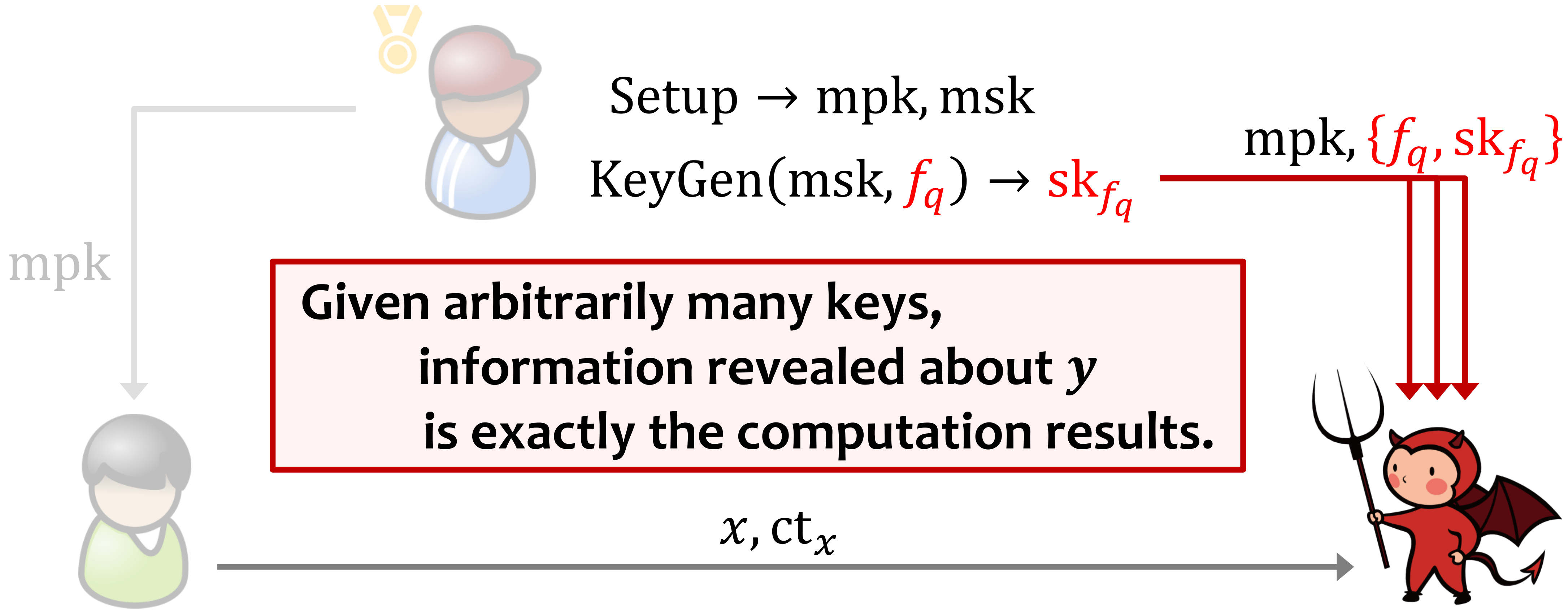

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PHFE Security: **IND-CPA**

$\mathrm{Exp}^{b}_{\mathrm{PHFE}}$

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$$\xrightarrow{\mathrm{mpk}}$$

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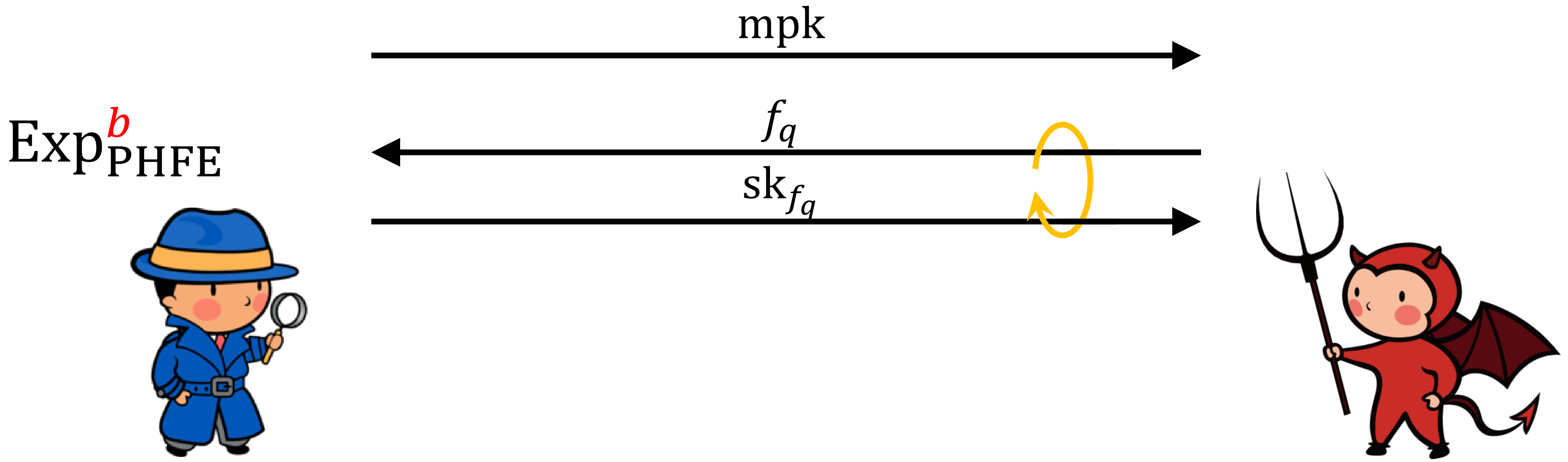
$\xleftarrow{f_q}$

$\xrightarrow{\mathrm{sk}_{f_q}}$

PHFE Security: **IND-CPA**



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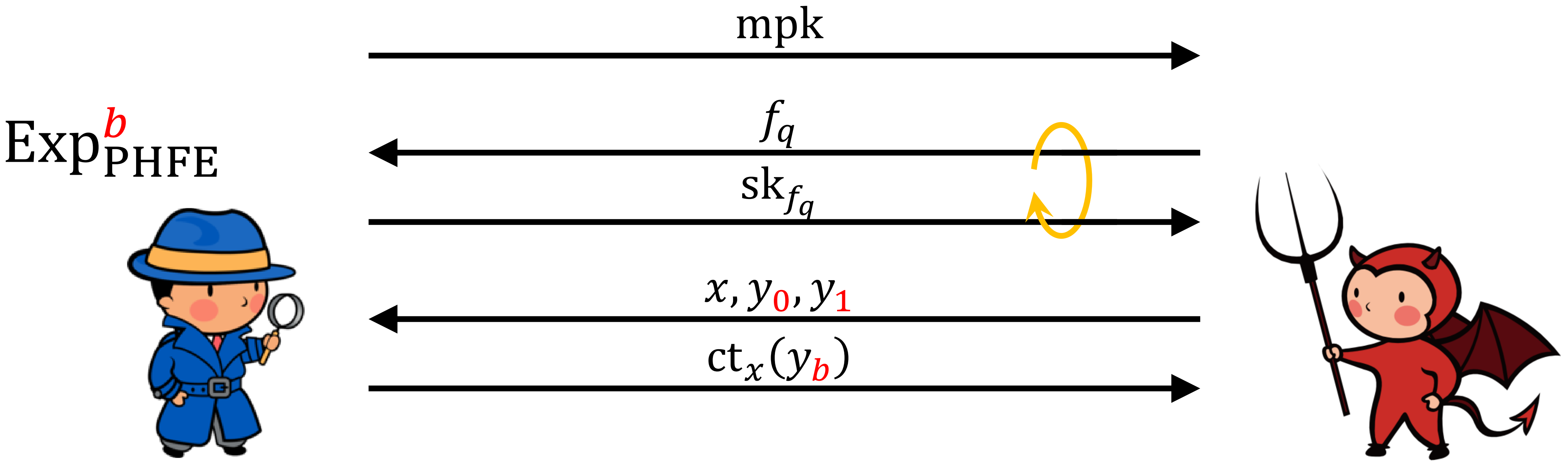

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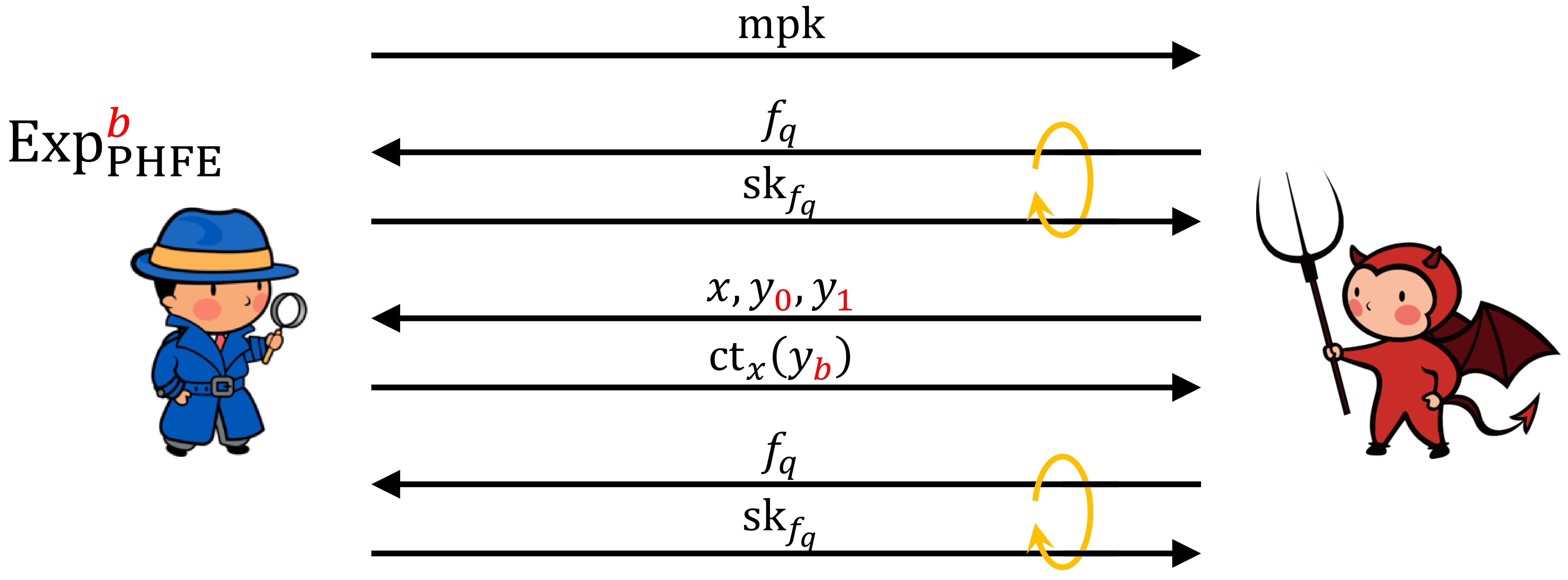

PHFE Security: IND-CPA

$\mathrm{Exp}^{b}_{\mathrm{PHFE}}$

$\xrightarrow{\mathrm{mpk}}$

$\xleftarrow{f_q}$

$\xrightarrow{\mathrm{sk}_{f_q}}$

$\xleftarrow{x, y_0, y_1}$

$\xrightarrow{\mathrm{ct}_x(y_b)}$

$\xleftarrow{f_q}$

$\xrightarrow{\mathrm{sk}_{f_q}}$

$\mathrm{Exp}^{0}_{\mathrm{PHFE}} \approx \mathrm{Exp}^{1}_{\mathrm{PHFE}}$ **if** $|y_0| = |y_1|$ **and** $\forall q\colon f_q(x, y_0) = f_q(x, y_1)$

What do we want for a PHFE?

★ expressive functionality

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$$T_{\mathrm{Dec}} = \mathrm{poly}(|f|, |x|, |y|, T)$$

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different aspects of efficiency?

(From what assumptions)
Can we **construct** optimally efficient PHFE?

Our Results: **Nearly Optimal PHFE for RAM**

polynomially secure FE for circuits

$\Longrightarrow$ **adaptively secure PHFE for RAM with**

$$|\mathrm{mpk}| = O(1), \quad |\mathrm{sk}_f| = O(1), \quad |\mathrm{ct}_x(y)| = 2|y| + O(1),$$

$$T_{\mathrm{KeyGen}} = O(|f|), \quad T_{\mathrm{Enc}} = O(|x| + |y|),$$

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?

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?

necessary / barrier

Select Related Works on FE

for	adaptive	$\lvert \mathrm{sk}_f \rvert$	$\lvert \mathrm{ct}_x(y) \rvert$	T_{Dec}	assumptions
RAM long output	this ✓	$O(1)$	$2\lvert y\rvert + O(1)$	$O(T + \lvert f\rvert + \lvert x\rvert + \lvert y\rvert)$	FE

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RAM	ACFQ	$\mathrm{poly}(\lvert f \rvert)$	$\mathrm{poly}(\lvert y \rvert)$	$T\,\mathrm{poly}(\lvert f \rvert)$	PK-DE-PIR + FE

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TM	AS	✓	$\mathrm{poly}(\lvert f\rvert)$	$\mathrm{poly}(\lvert y\rvert)$	$T\,\mathrm{poly}(\lvert f\rvert, \lvert y\rvert)$	$i\mathcal{O}$
	AJS	✓	$c\lvert f\rvert + O(1)$	$c\lvert y\rvert + O(1)$	$T\,\mathrm{poly}(\lvert f\rvert, \lvert y\rvert)$	subexp $i\mathcal{O}$
	AM	✓	$\mathrm{poly}(\lvert f\rvert)$	$O(\lvert y\rvert)$	$T\,\mathrm{poly}(\lvert f\rvert, \lvert y\rvert)$	dist. ind. FE
	KNTY		$\mathrm{poly}(\lvert f\rvert)$	$\mathrm{poly}(\lvert y\rvert)$	$T\,\mathrm{poly}(\lvert f\rvert, \lvert y\rvert)$	1-key sel. FE

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circuit	GGHRSW		$\mathrm{poly}(\lvert C\rvert)$	$\mathrm{poly}(\lvert y\rvert)$	$\mathrm{poly}(\lvert C\rvert)$	$i\mathcal{O}$
circuit	KNTY	✓	$\mathrm{poly}(\lvert C\rvert)$	$\mathrm{poly}(\lvert y\rvert)$	$\mathrm{poly}(\lvert C\rvert)$	1-key sel. FE
circuit	GWZ		$\mathrm{poly}(\lvert C\rvert)$	$\lvert y\rvert + O(1)$	$\mathrm{poly}(\lvert C\rvert)$	$i\mathcal{O}$

Significant Improvement and "Two Clouds"

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improvement from polynomial to *nearly optimal* efficiency

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first space-time efficiency trade-offs for (PH-)FE

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y? Linear-size components? Optimal decryption time? Connections to DE-PIR.

Doubly Efficient Private Information Retrieval

DE-PIR D

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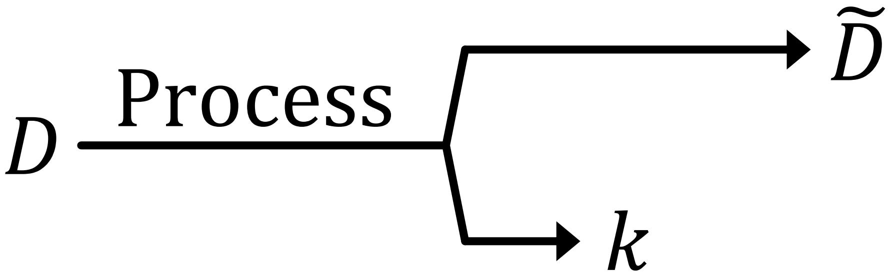


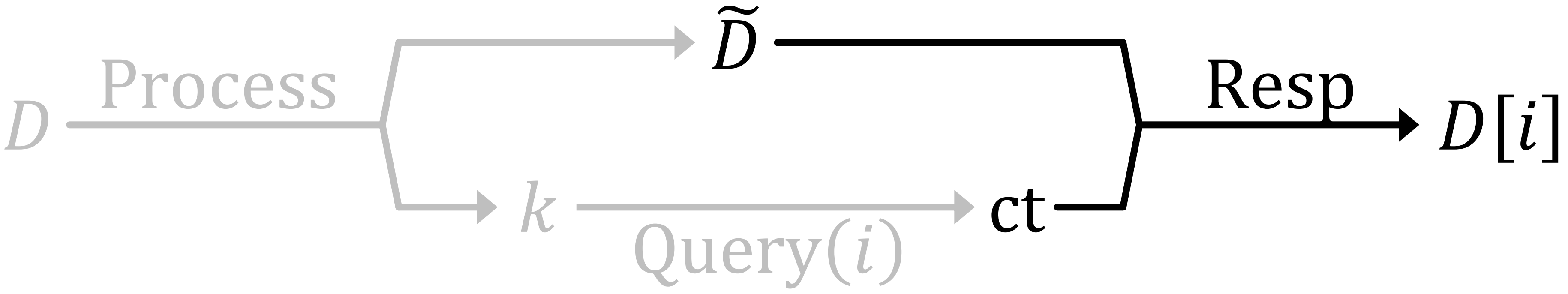
Doubly Efficient Private Information Retrieval

DE-PIR

$$D \xrightarrow{\text{Process}} \begin{cases} \widetilde{D} \\ k \xrightarrow[\text{Query}(i)]{} \text{ct} \end{cases}$$

Doubly Efficient Private Information Retrieval

DE-PIR



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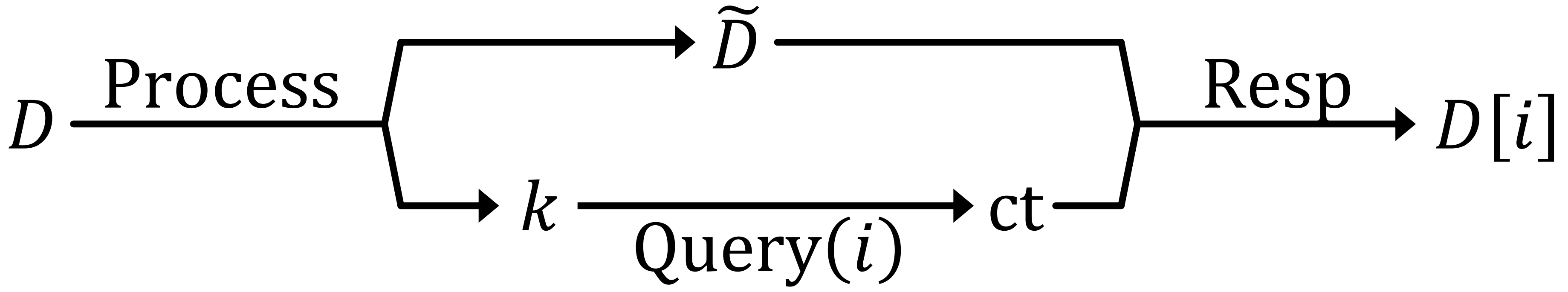

client efficiency $|k| = O(1)$ **and** $T_{\text{Query}} = O(|D|^{1-\varepsilon})$

server efficiency $T_{\text{Resp}} = O(|D|^{1-\varepsilon})$

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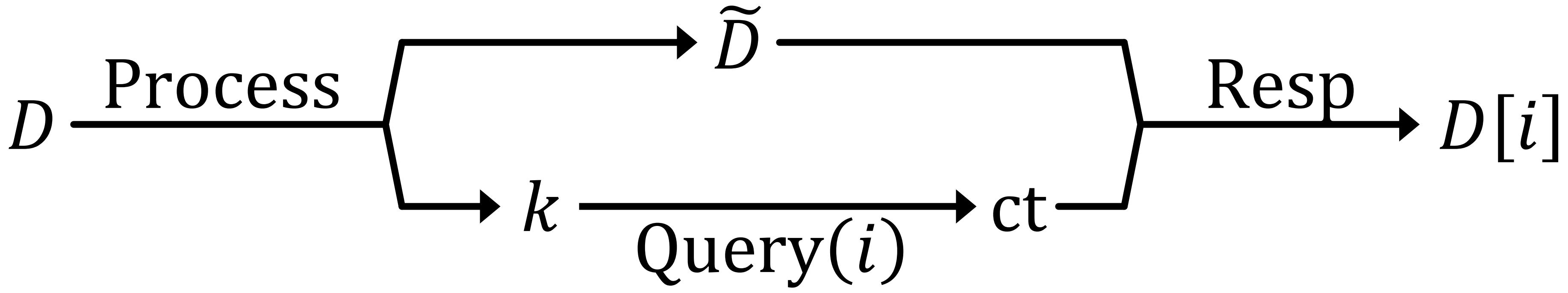

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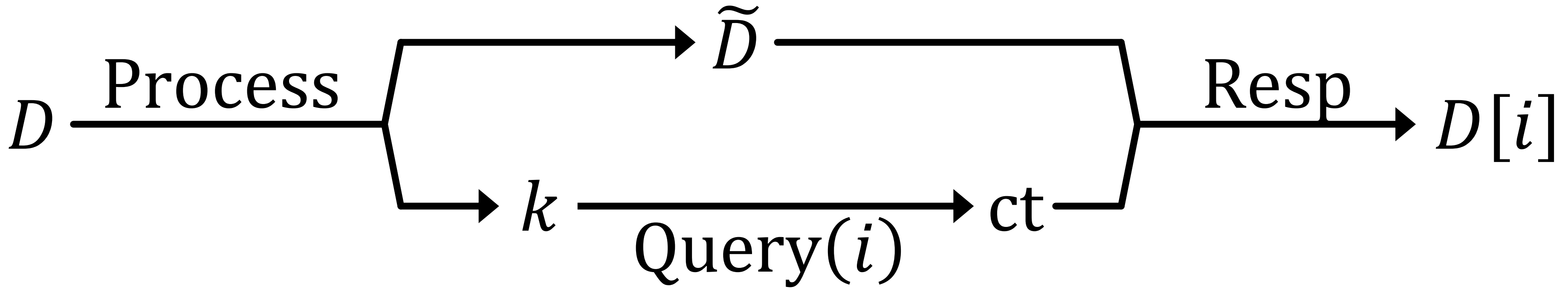

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dream efficiency $|\widetilde{D}| = O(|D|)$ **and** $T_{\text{Query}}, T_{\text{Resp}} = O(1)$

Doubly Efficient Private Information Retrieval

DE-PIR

$$D \xrightarrow{\text{Process}} \widetilde{D},\ k \qquad k \xrightarrow{\text{Query}(i)} \text{ct} \qquad \widetilde{D},\ \text{ct} \xrightarrow{\text{Resp}} D[i]$$

client efficiency $|k| = O(1)$ **and** $T_{\text{Query}} = O(|D|^{1-\varepsilon})$

server efficiency $T_{\text{Resp}} = O(|D|^{1-\varepsilon})$

security $\widetilde{D}, \{\text{ct}(i_q)\}$ **hides** $\{i_q\}$

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Is it known?

Results: Optimal Decryption Time Implies **DE-PIR**

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Assuming **sufficiently** expressive secure PHFE with

$$|\mathrm{ct}_x(y)| = |x|^A \,\mathrm{poly}(|y|)\,, \quad T_{\mathrm{Dec}} = |x|^B \,\mathrm{poly}(T, |f|, |y|)\,,$$

or

$$|\mathrm{ct}_x(y)| = |y|^A \,\mathrm{poly}(|x|)\,, \quad T_{\mathrm{Dec}} = |y|^B \,\mathrm{poly}(T, |f|, |x|)$$

for $B < 1$, then there exists **secret-key** DE-PIR with

$$|\widetilde{D}| = |D| + O(|D|^A)\,, \quad T_{\mathrm{Query}} = O(1)\,, \quad T_{\mathrm{Resp}} = O(|D|^B)\,.$$

essentially also proven in ACFQ

Results: Optimal Decryption Time Implies **DE-PIR**

Assuming **mildly** expressive secure PHFE with

new in this work

$$|\mathsf{sk}_f| = O(|f|^A), \quad T_{\mathsf{Dec}} = |f|^B \operatorname{poly}(T, |x|, |y|)$$

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Results: Constant-Overhead $i\mathcal{O}$ & ABE for RAM

subexp. secure FE for circuits

$\Longrightarrow$ subexp. secure $i\mathcal{O}$ for RAM with

$$|\widetilde{M}| = 2|M| + \mathrm{poly}(|D|)$$

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from PHFE	$O(1)$	$O(1)$	$O(T + \lvert f\rvert + \lvert x\rvert)$

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	$\lvert f\rvert + O(1)$	$\lvert x\rvert + O(1)$	$O(T)$

can move between size and time

(adaptive, based on FE for circuits)

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can move between size and time

Luo22

(adaptive, based on FE for circuits)

Where we stand now: **(PH-)FE**

$$y\text{-independent Dec}$$

$$\Uparrow$$

morally

$$\Downarrow$$

DE-PIR

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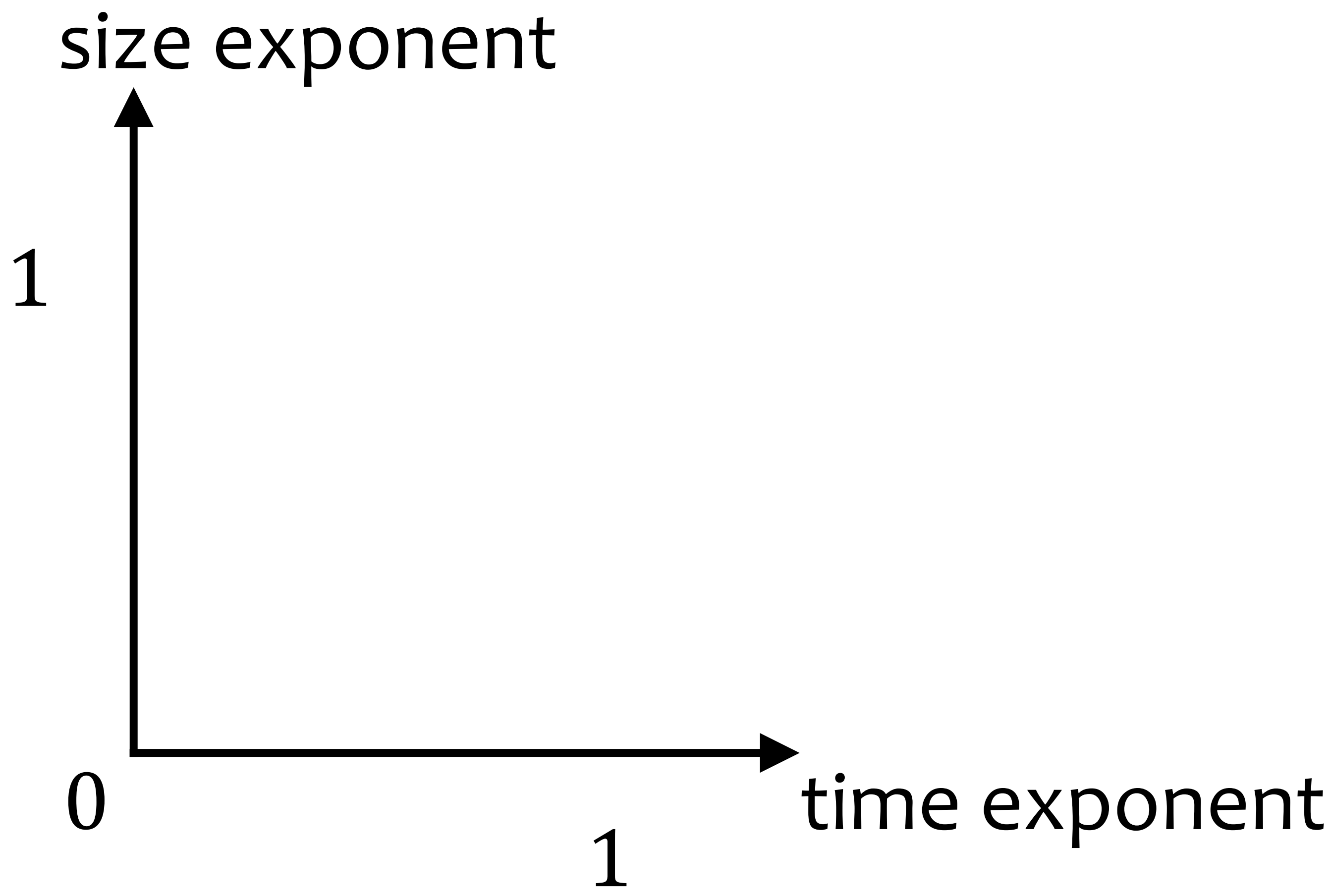

dependency on f or x
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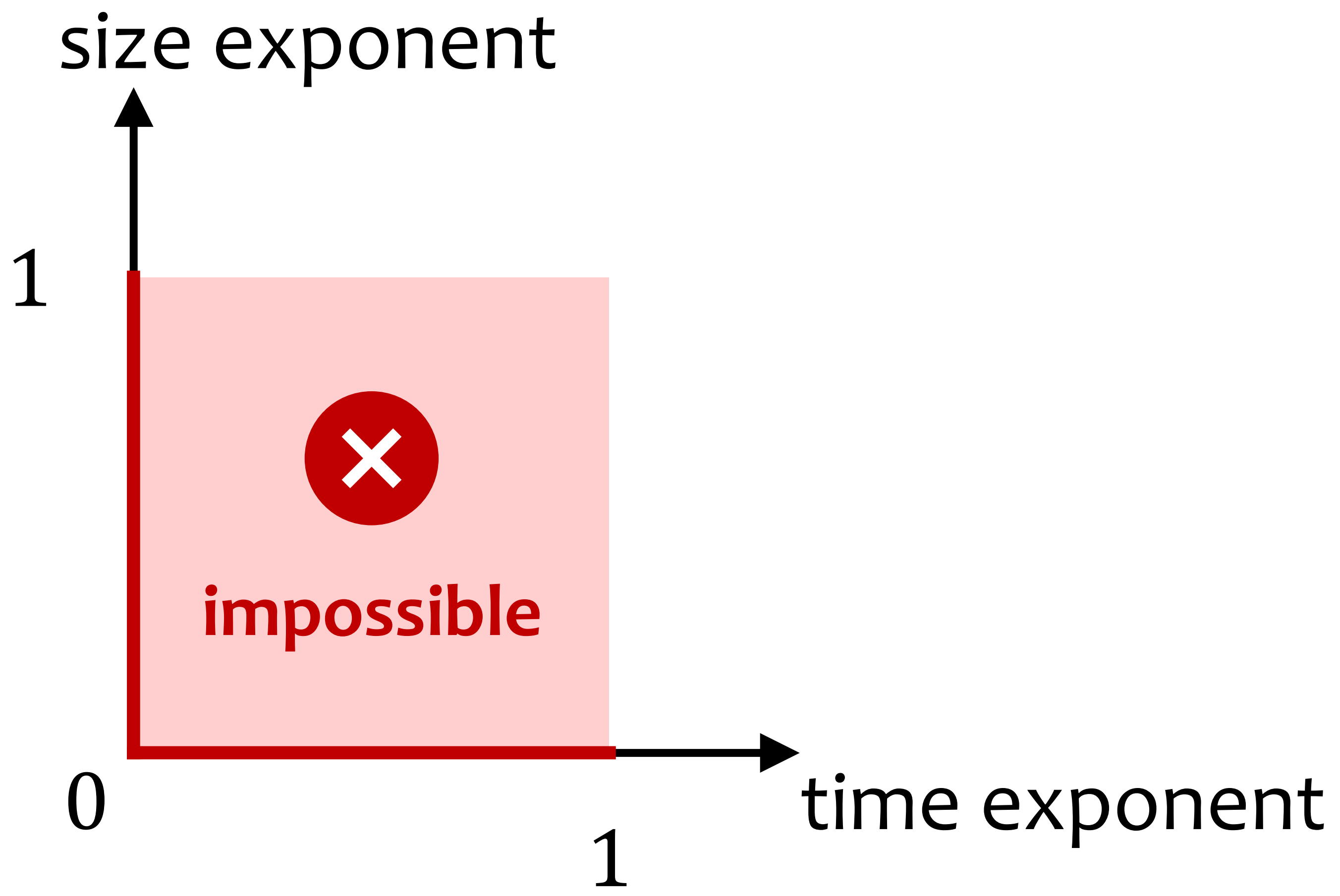
DE-PIR



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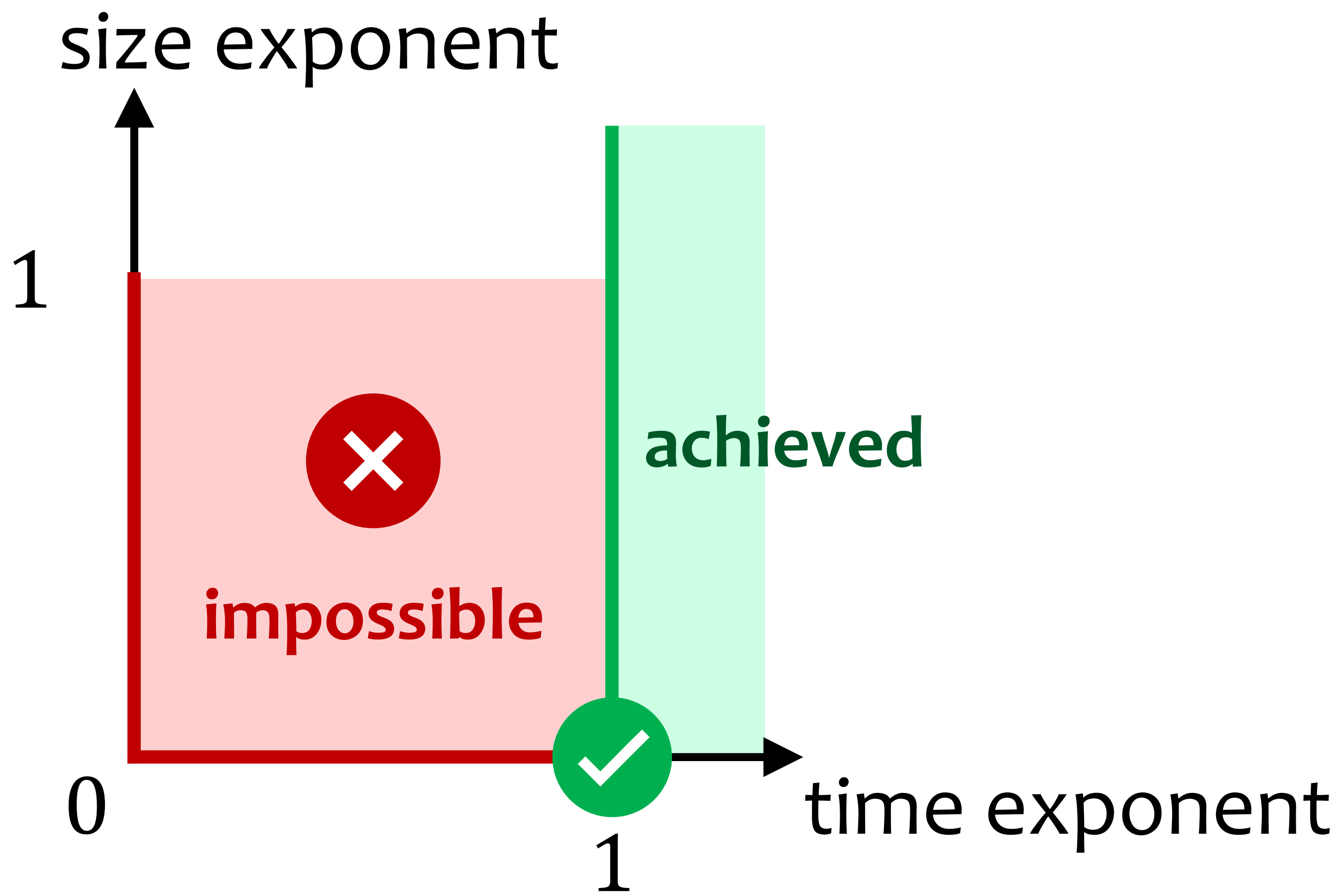
$\Updownarrow$ **morally**

DE-PIR



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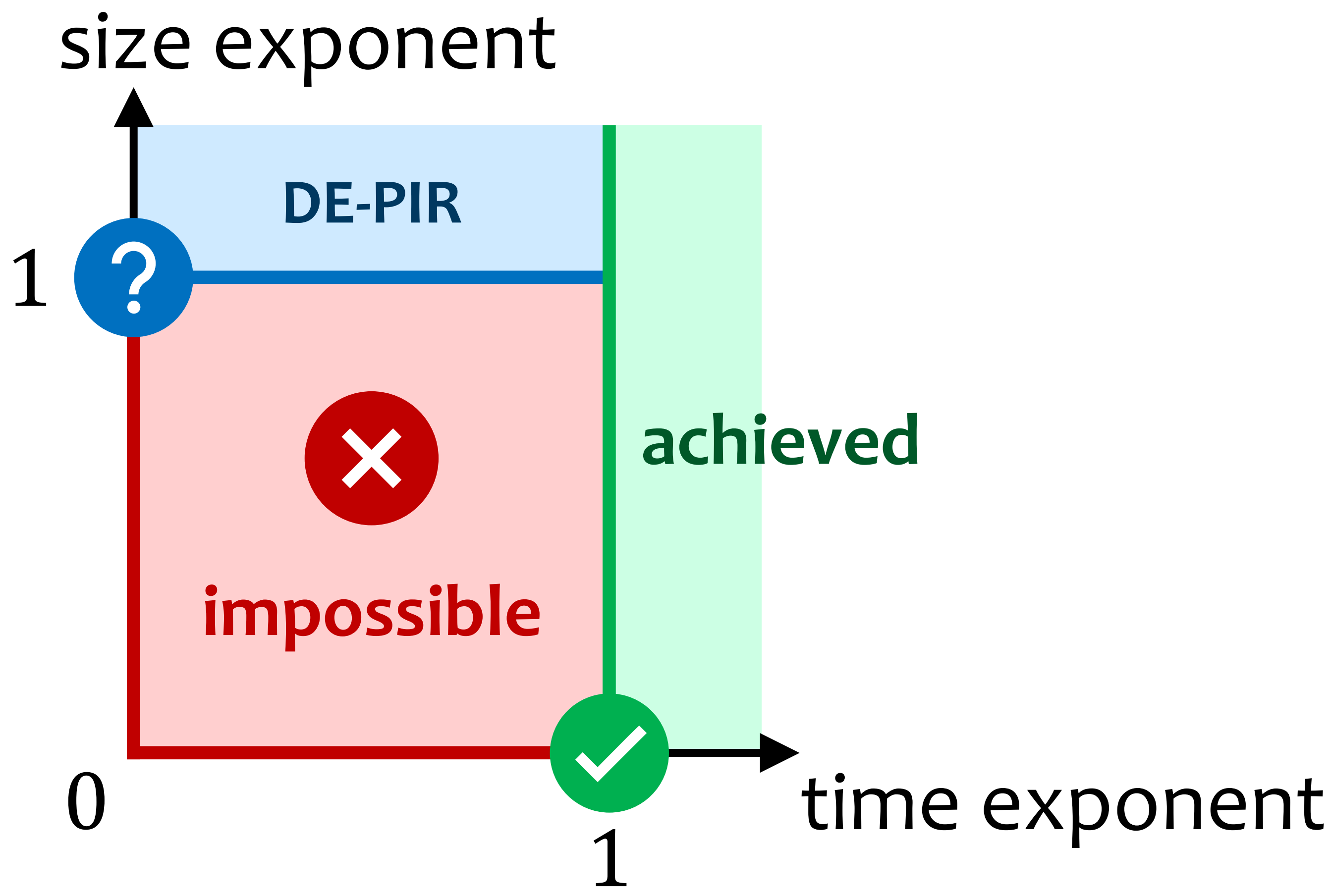

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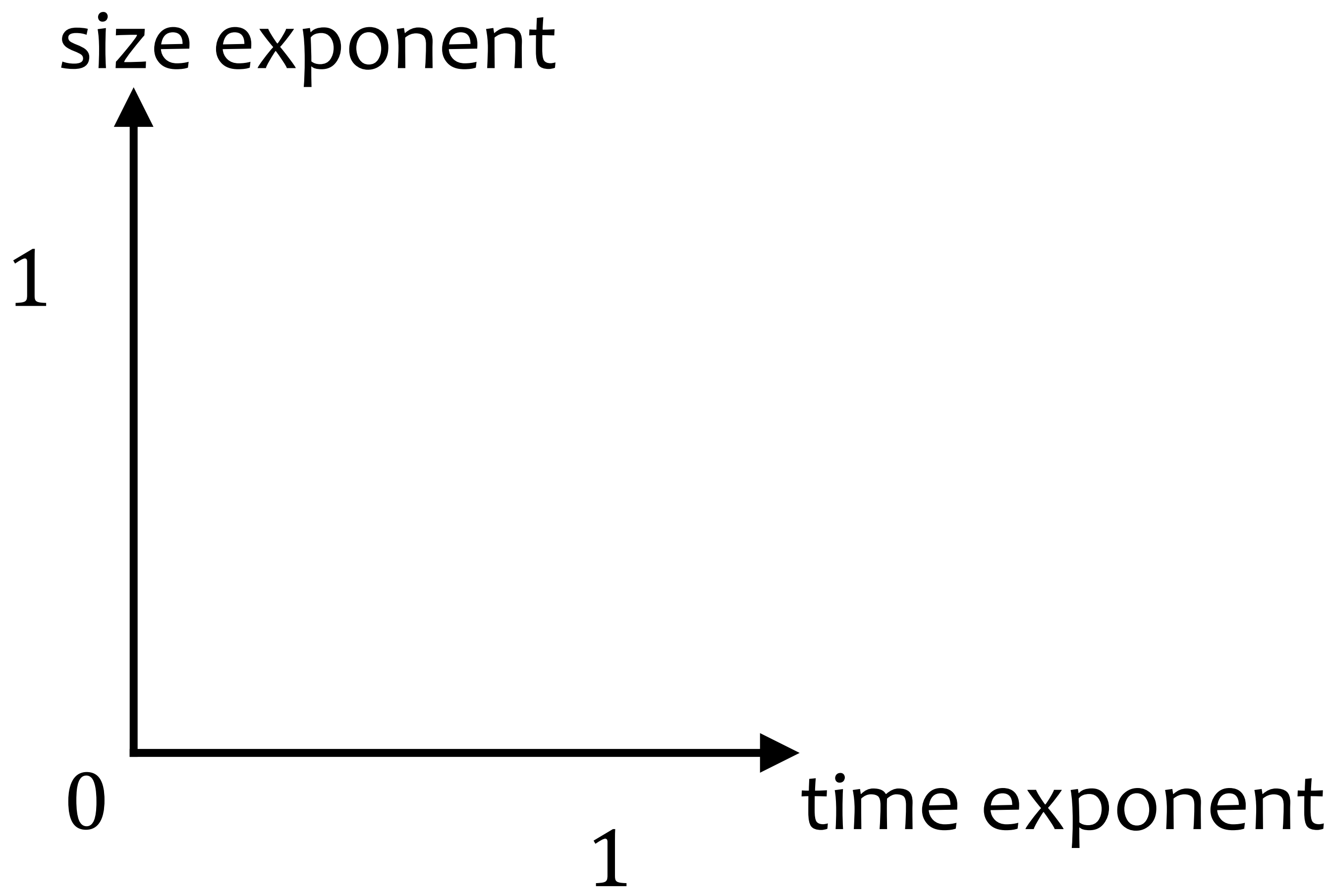

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$T_{\mathrm{Dec}} = O(T + \cdots)$

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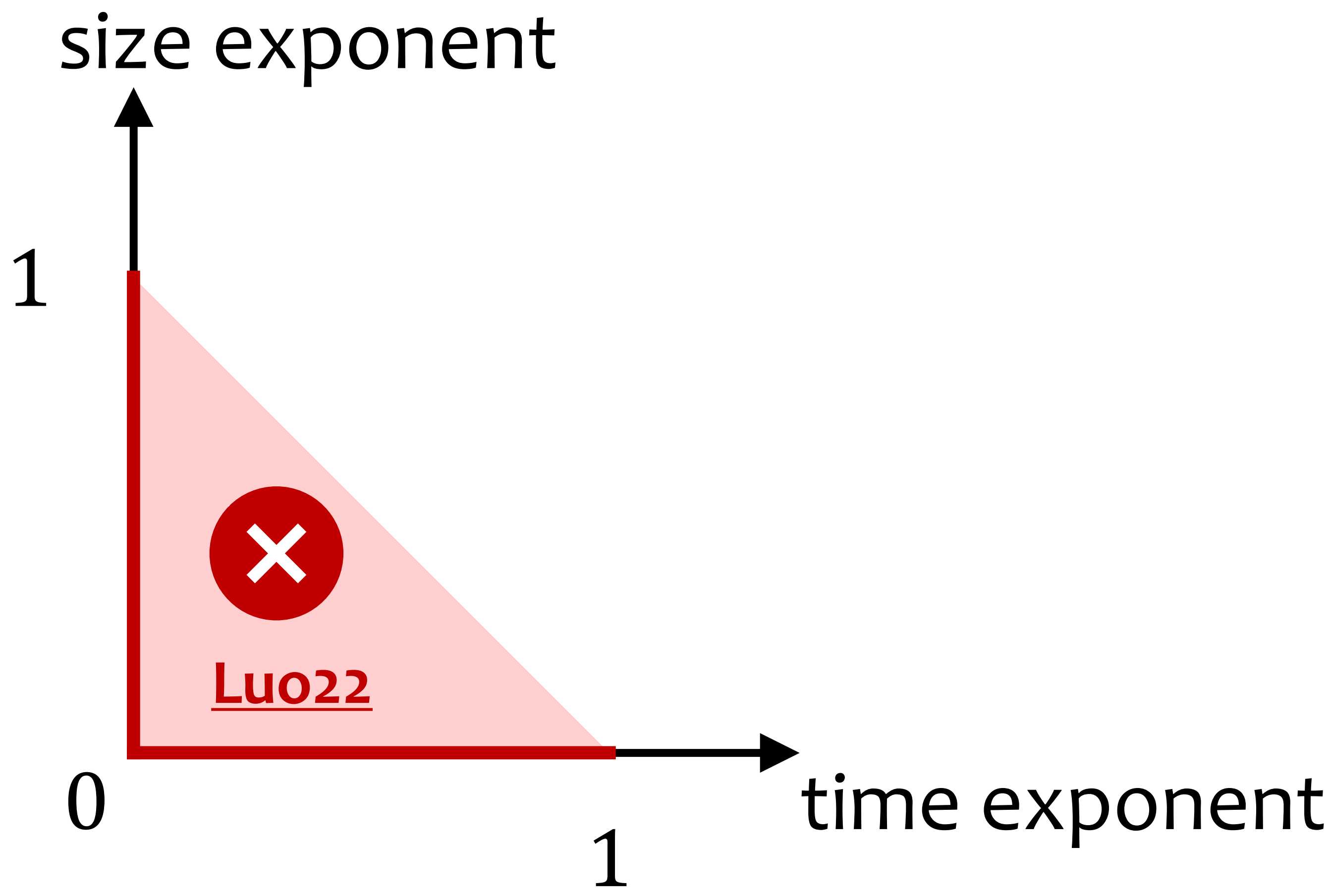

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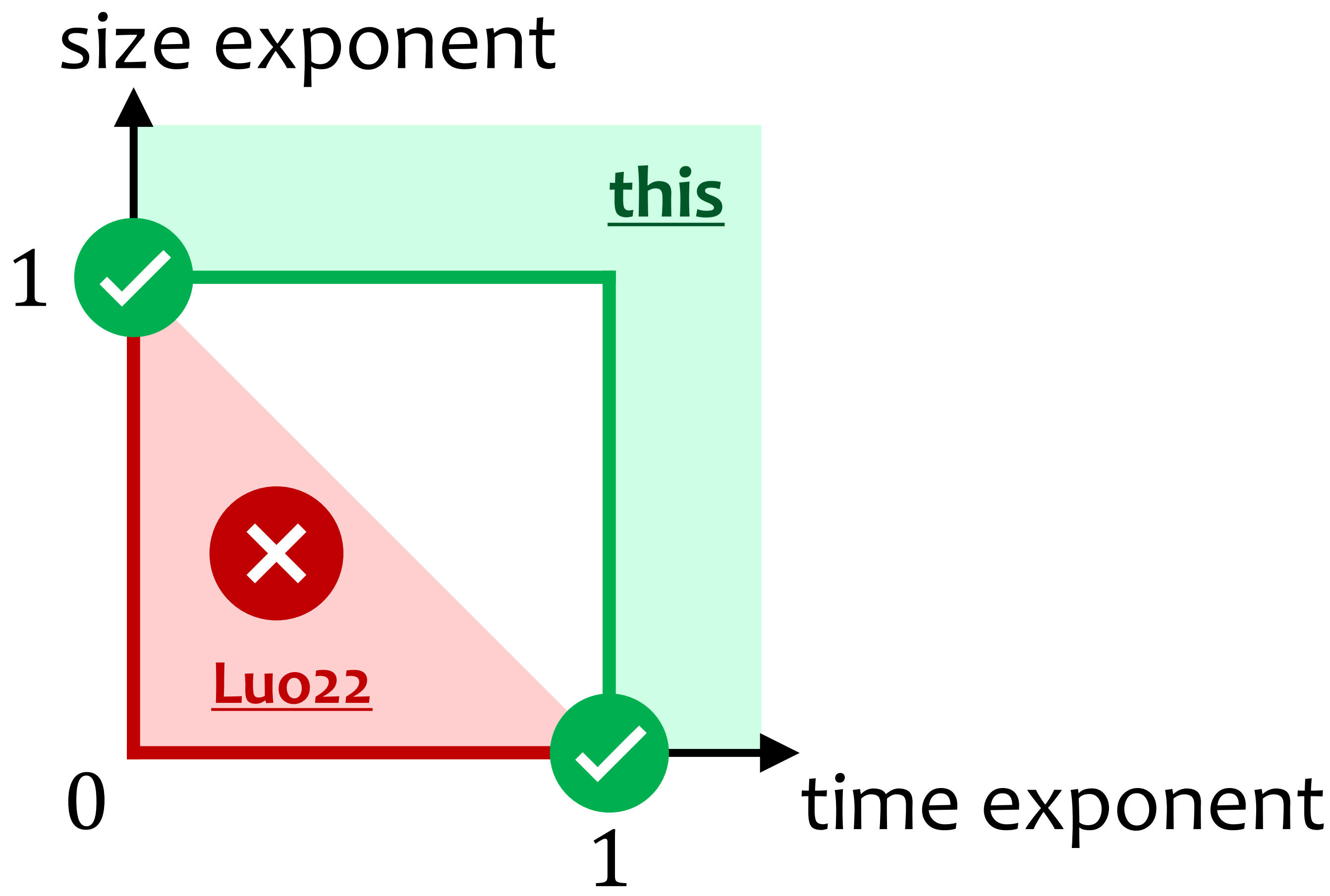

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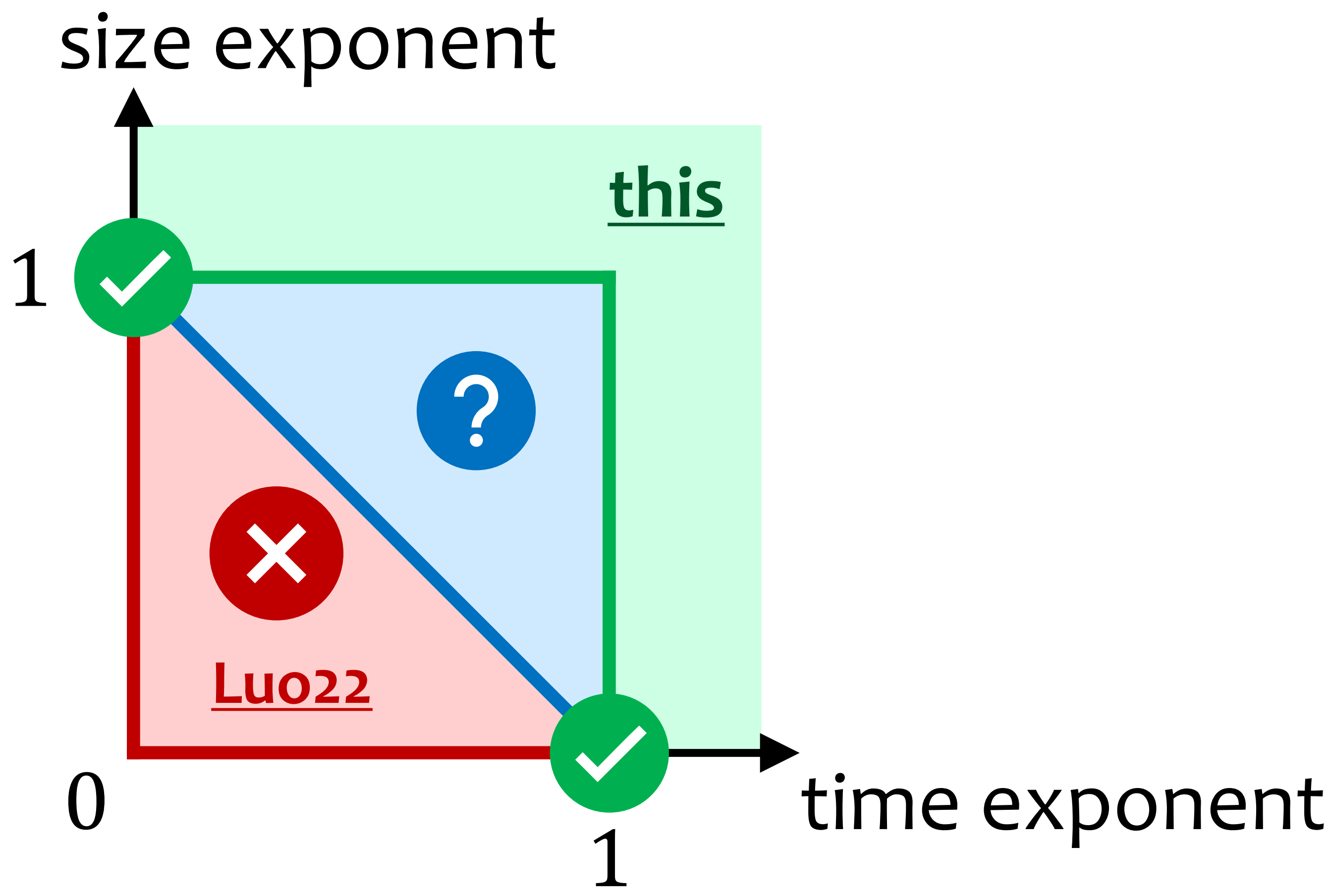

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Proof Sketch of **Unconditional** Lower Bound

$|\mathrm{sk}_f| = |f|^{A}$, $\quad T_{\mathrm{Dec}} = T + |f|^{B} + |x| + |y|$, $\quad A, B < 1.$

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$$f\ (x, y) = \begin{cases} R[I] \oplus w, & y = (I, w); \\ z, & y = z. \end{cases}$$

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$|\mathrm{sk}_f| = |f|^A$, $\quad T_{\mathrm{Dec}} = T + |f|^B + |x| + |y|$, $\quad A, B < 1.$

$= N^A \ll n \quad = n + N^B + 0 + n \approx n \ll N$

make $N^A, N^B \ll n \ll N$

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Unconditional Lower Bound (continued)

$|\mathrm{sk}_f| \ll n,$ $T_{\mathrm{Dec}} \ll N.$

$$f_R(x, y) = \begin{cases} R[I] \oplus w, & y = y_0 = (I, w); \\ z, & y = y_1 = z. \end{cases}$$

Unconditional Lower Bound (continued)

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How much of $R[I]$ does $\mathrm{Dec}^R(\mathrm{sk}_f, \mathrm{ct})$ read?

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choose random I, w and $z = R[I] \oplus w$

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$(\mathrm{sk}_f, \mathrm{ct})$ contains $\ll n$ bits of $R[I]$ (n bits)

must read **almost all** of $R[I]$ **(incompressibility argument)**

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When $y = y_1 = z$:

behavior of $\mathrm{Dec}^R(\mathrm{sk}_f, \mathrm{ct})$ **independent** of I

can **only** read $|I| \cdot \frac{T_{\mathrm{Dec}}}{N} \ll n$ bits from $R[I]$ **(hypergeometric distribution)**

$$f_R(x, y) = \begin{cases} R[I] \oplus w, & y = y_0 = (I, w); \\ z, & y = y_1 = z. \end{cases}$$

choose random I, w and $z = R[I] \oplus w$

Proof Sketch of Technical Barrier of **DE-PIR**

$|\mathrm{sk}_f| = |f|^{A}, \qquad T_{\mathrm{Dec}} = |f|^{B}\,\mathrm{poly}(T, |x|, |y|)\,, \qquad B < 1.$

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⚠ **IND-secure, selective, non-output-hiding, (if SK) non-database-hiding.**

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⚠ **IND-secure, selective, non-output-hiding, (if SK) non-database-hiding.**

Generic efficiency-preserving transformation for SIM-secure, adaptive, output-hiding, (if SK) database-hiding. ✓

Core of PHFE: **Laconic** Garbled **(Multi-Tape)** RAM

Step 1: **Formulate the right definition.**

Step 2: **Achieve it.**

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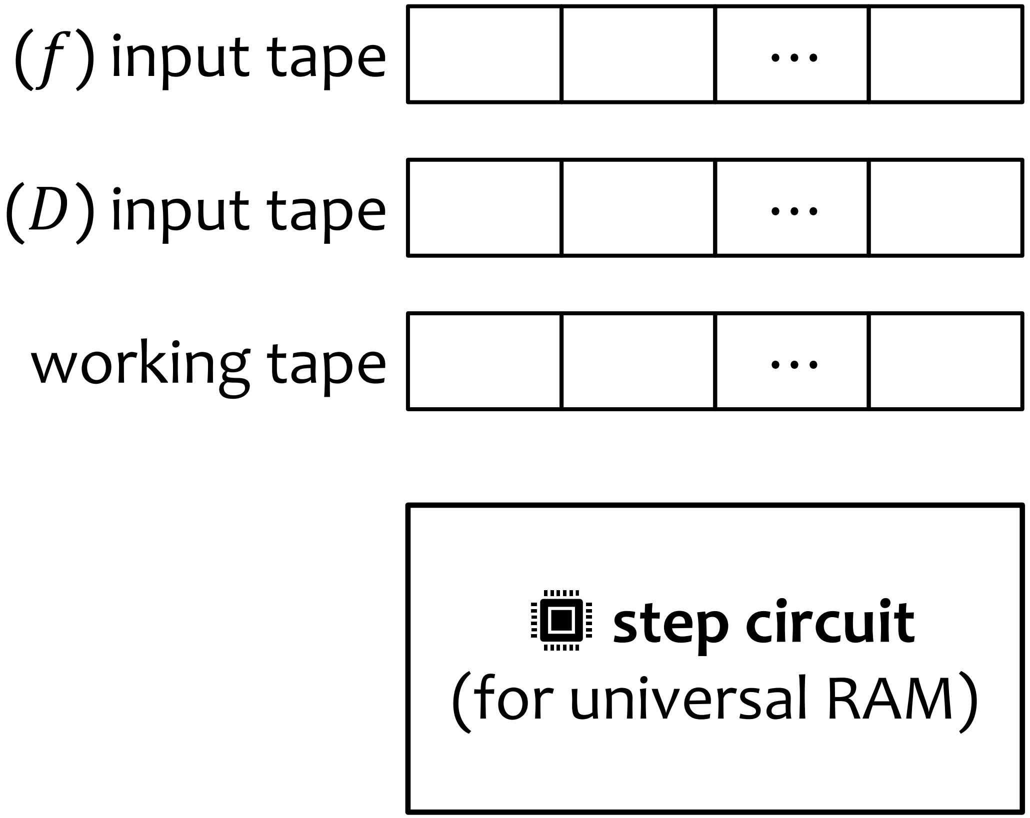

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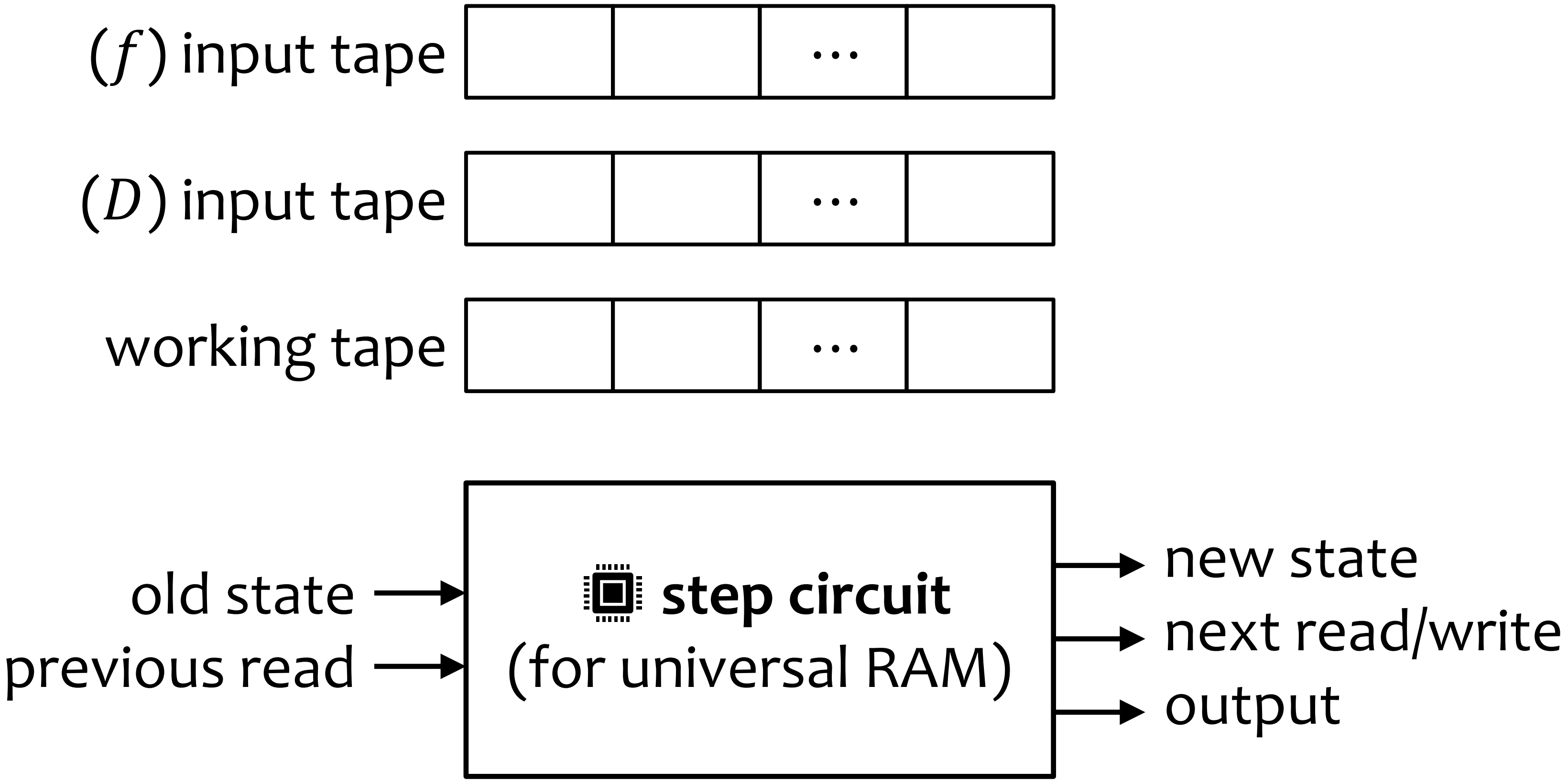

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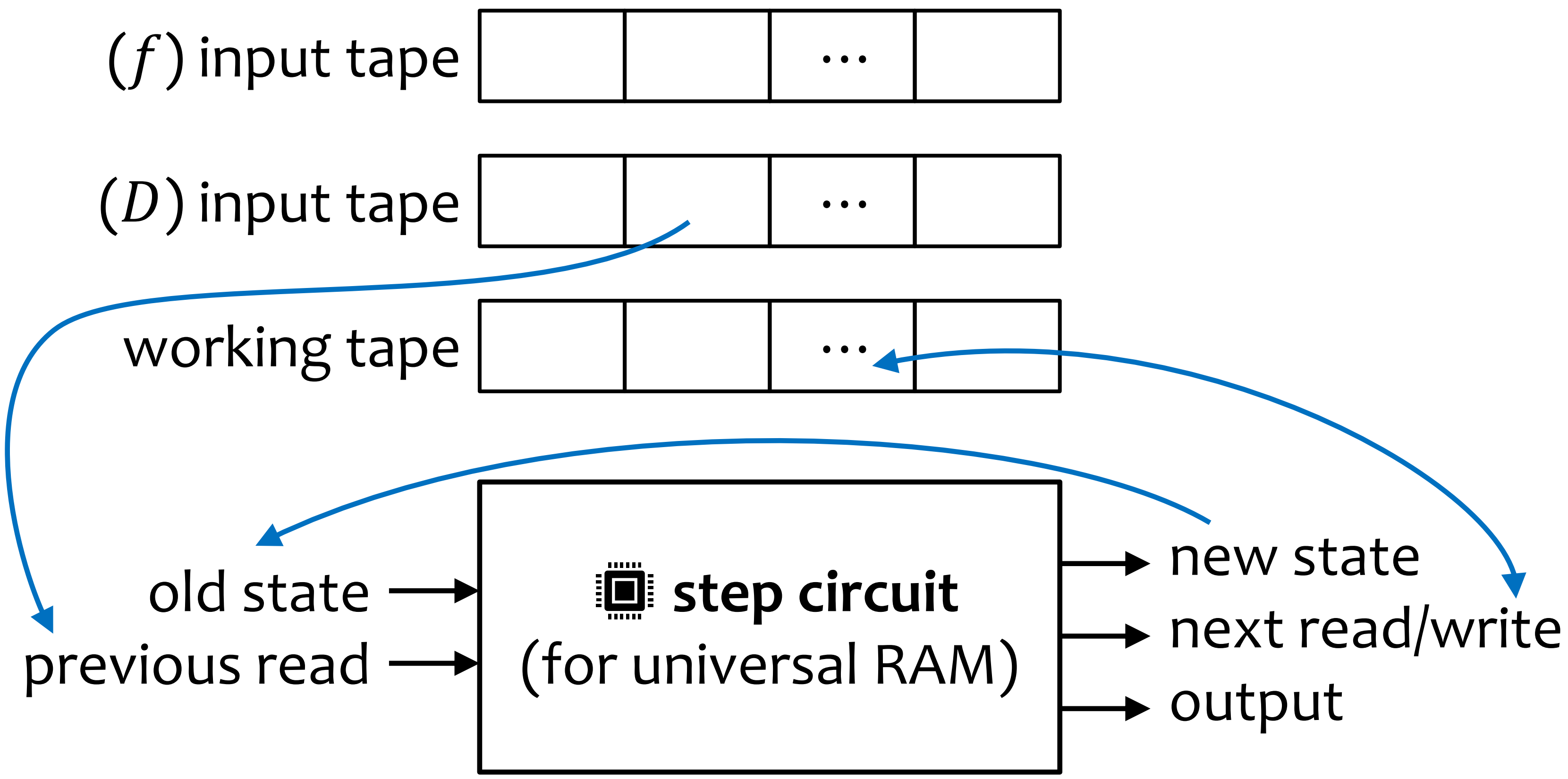

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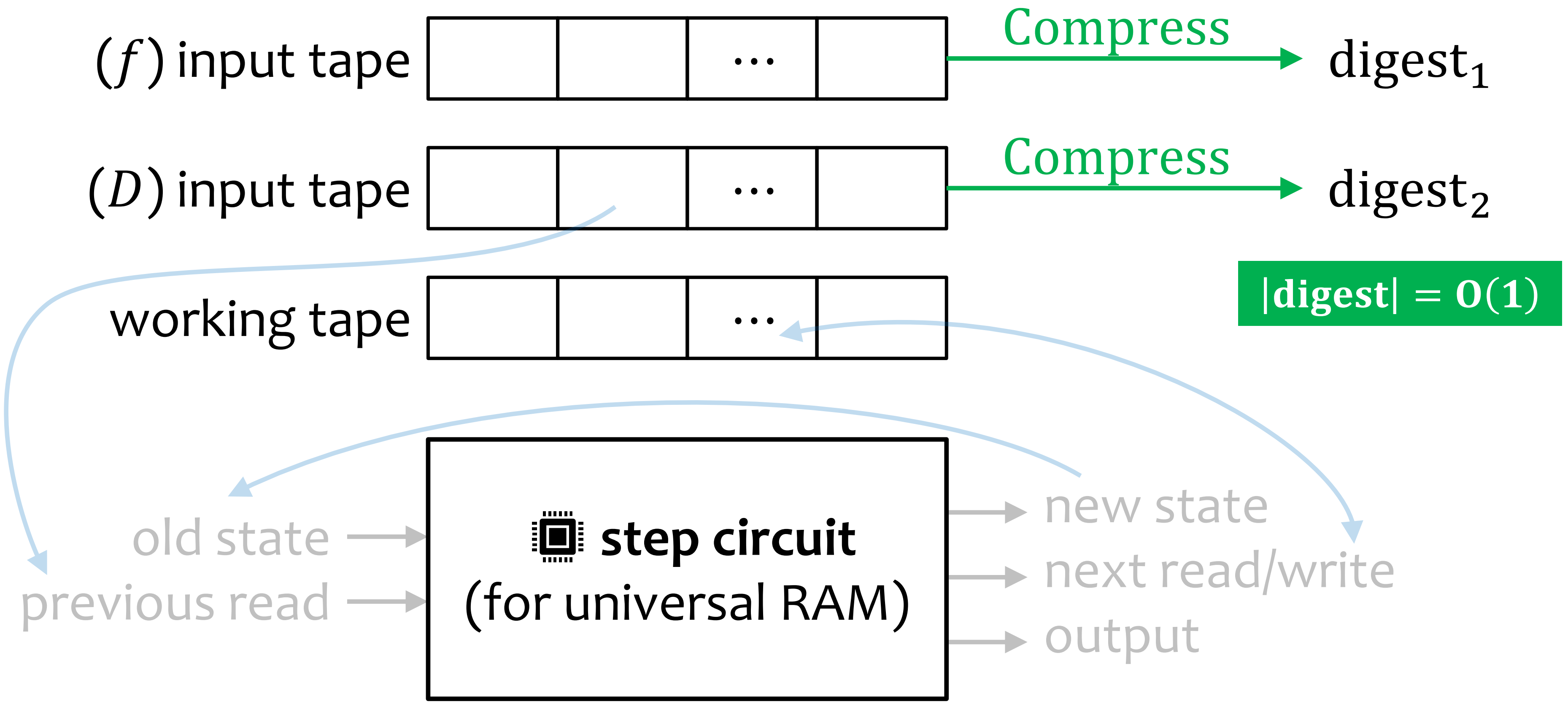

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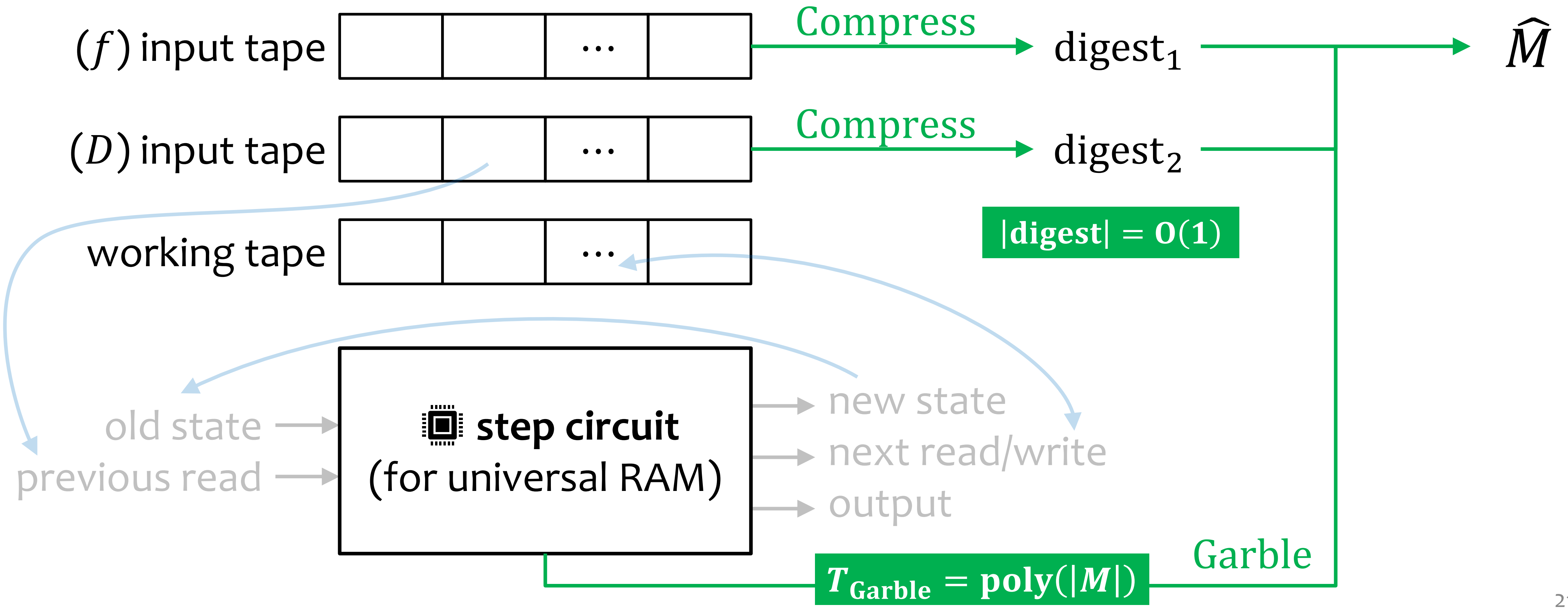

Core of PHFE: **Laconic** Garbled **(Multi-Tape)** RAM

$T_{\mathrm{Eval}} = (T + |f| + |D|)\,\mathrm{poly}(|M|)$

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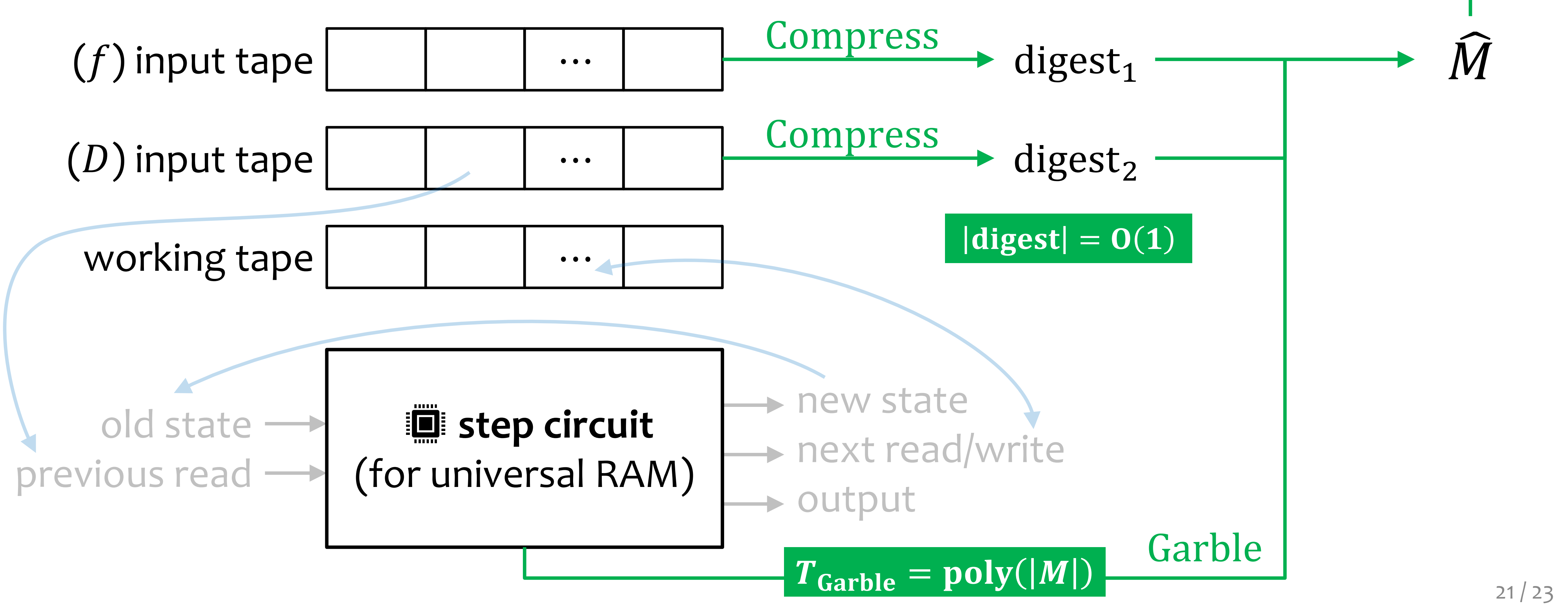
$M^{f,D}()$

$\mathrm{Eval}^{f,D}$



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Security is IND-based, not SIM-based.

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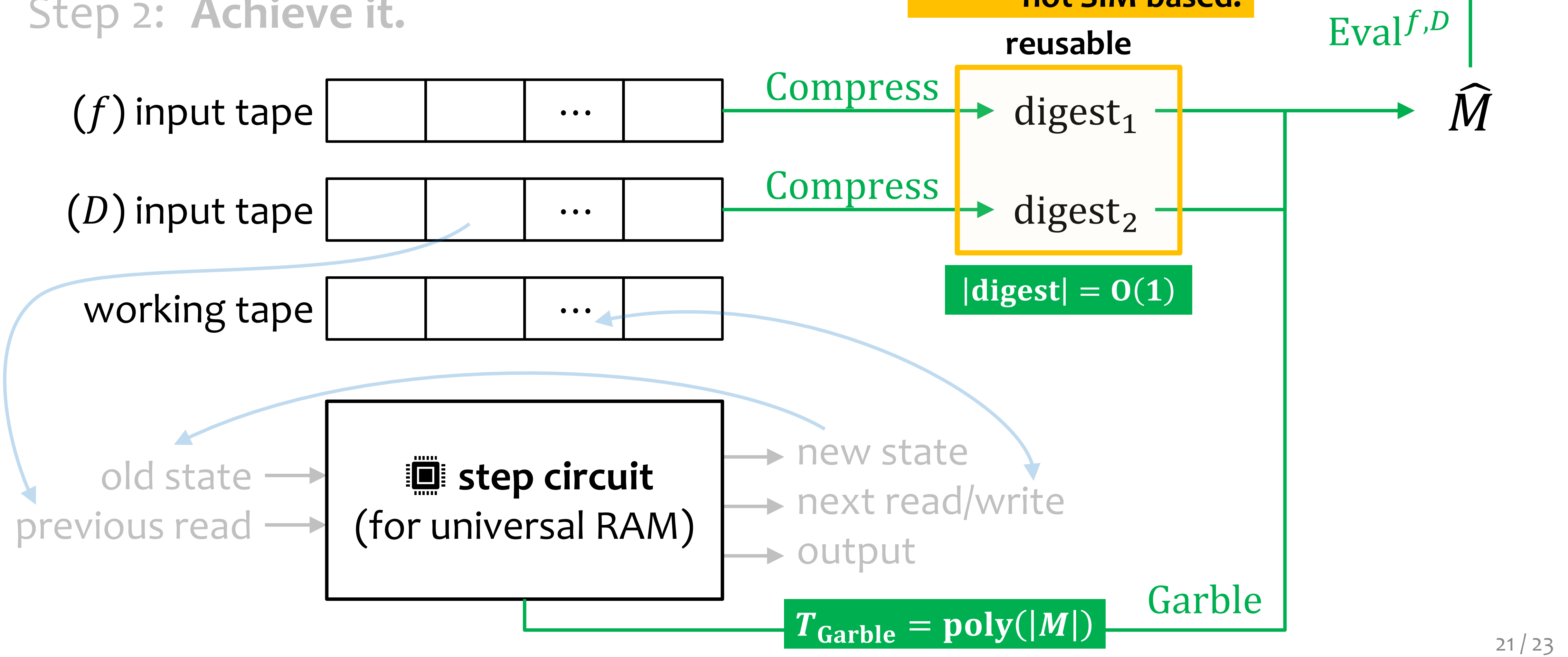

Core of PHFE: **Laconic** Garbled **(Multi-Tape)** RAM

important for nearly optimal efficiency

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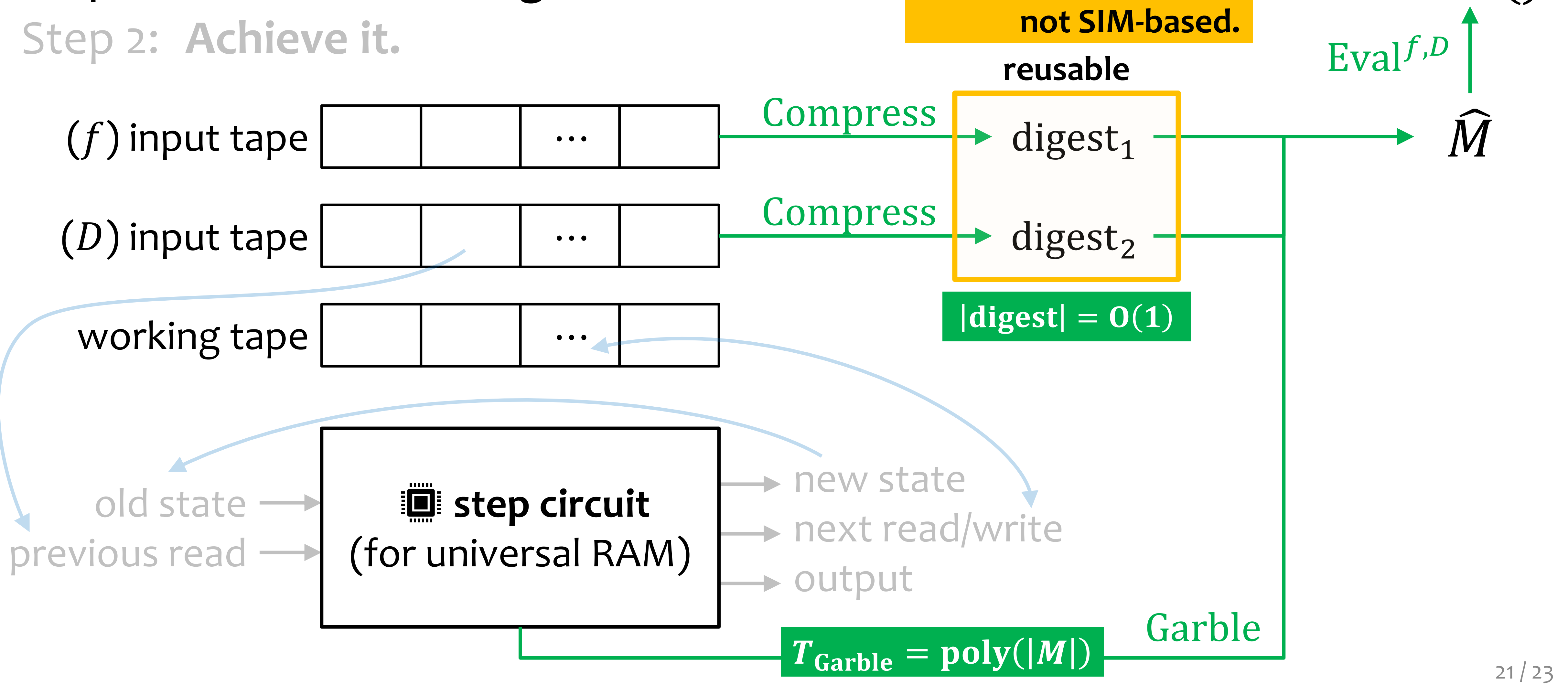

Open Questions: **What's next for (PH-)FE/ABE?**

1. **Construct** PHFE with optimal T_{Dec} from/and dream DE-PIR.
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3. **Tight relation** between optimal T_{Dec} and DE-PIR.

   FE for circuits + **PK**-DE-PIR $\Longrightarrow$ (x, y)-optimal Dec time $\Longrightarrow$ **SK**-DE-PIR.
   FE for circuits + **SK**-DE-PIR $\Longrightarrow$ … ?

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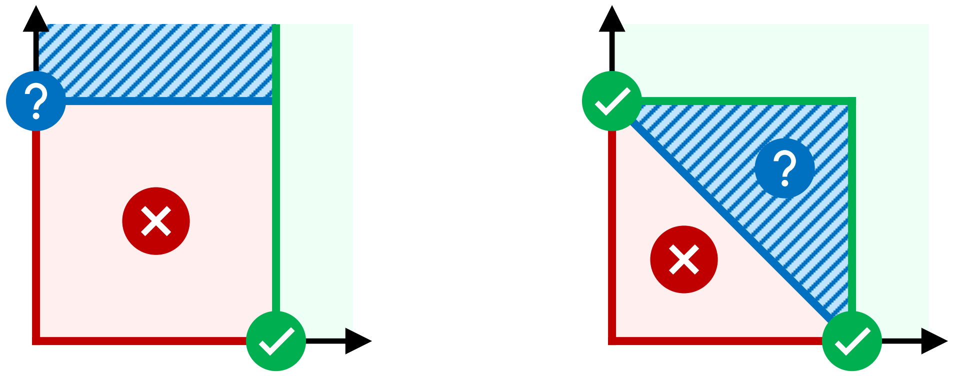
   FE for circuits + **SK**-DE-PIR $\Longrightarrow$ $\ldots$?

4. **Pin down** the exact Pareto frontier of efficiency.

   Demystify the **stripe area**.



Thanks!

ePrint 2022/1317 (revision coming soon)