Disorientation faults in CSIDH

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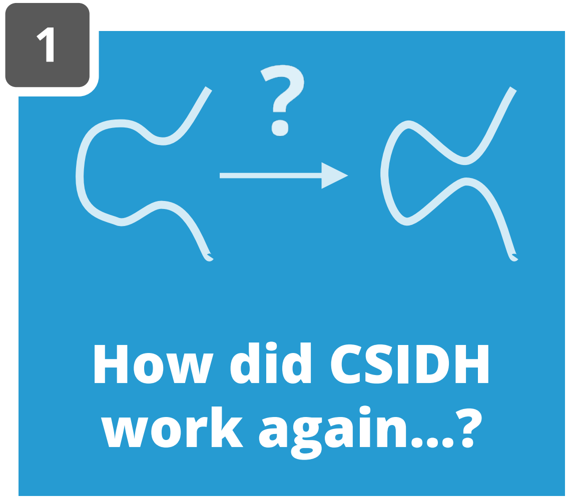
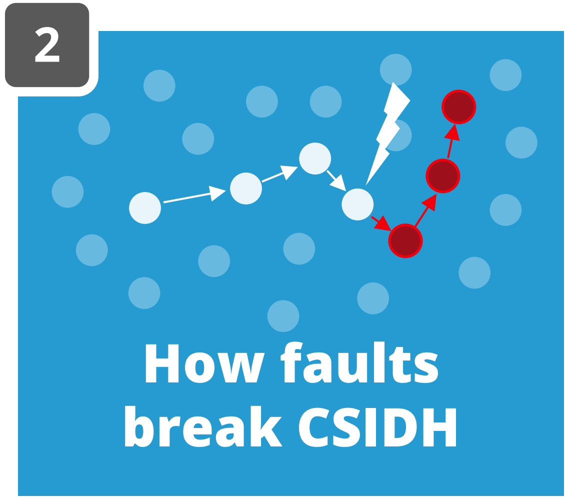
Monika Trimoska
Radboud University

Radboud University

26 April 2023
Eurocrypt '23

From disorientation attacks to key recovery





CSIDH FOR BEGINNERS

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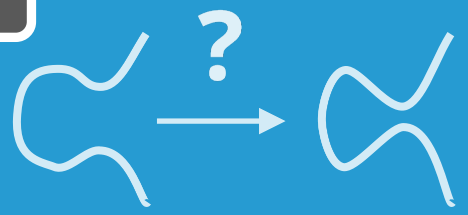

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$p = 419 = 4 \cdot 3 \cdot 5 \cdot 7 - 1$

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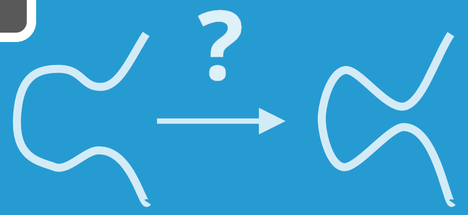

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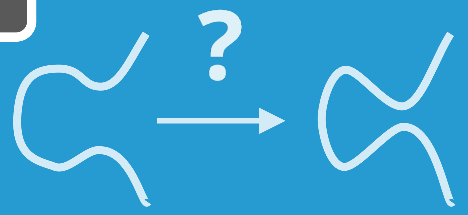

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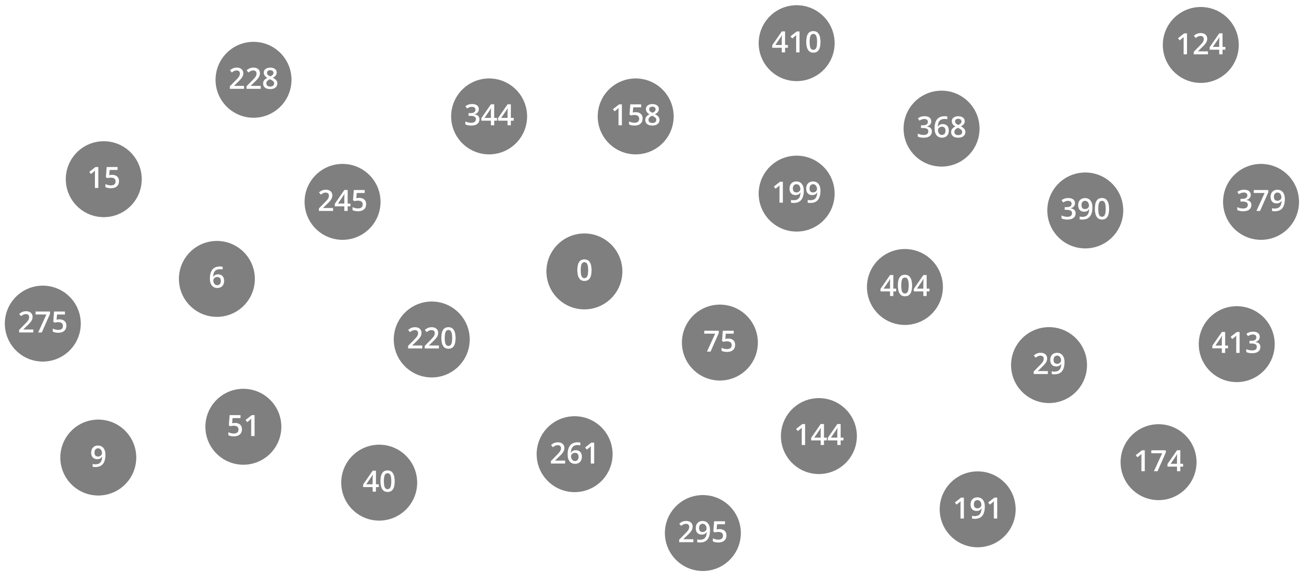

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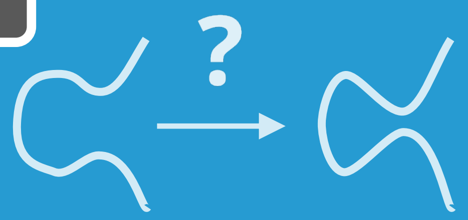

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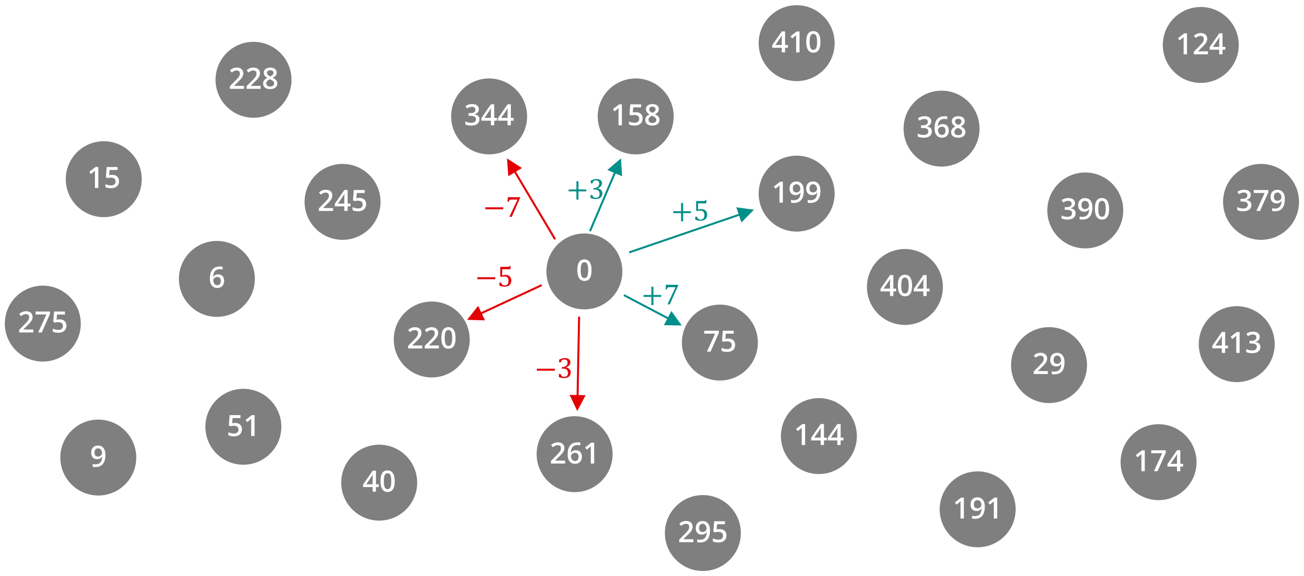

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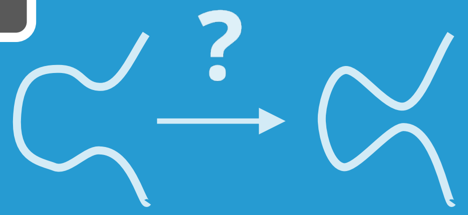

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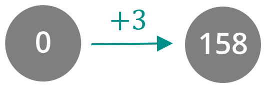

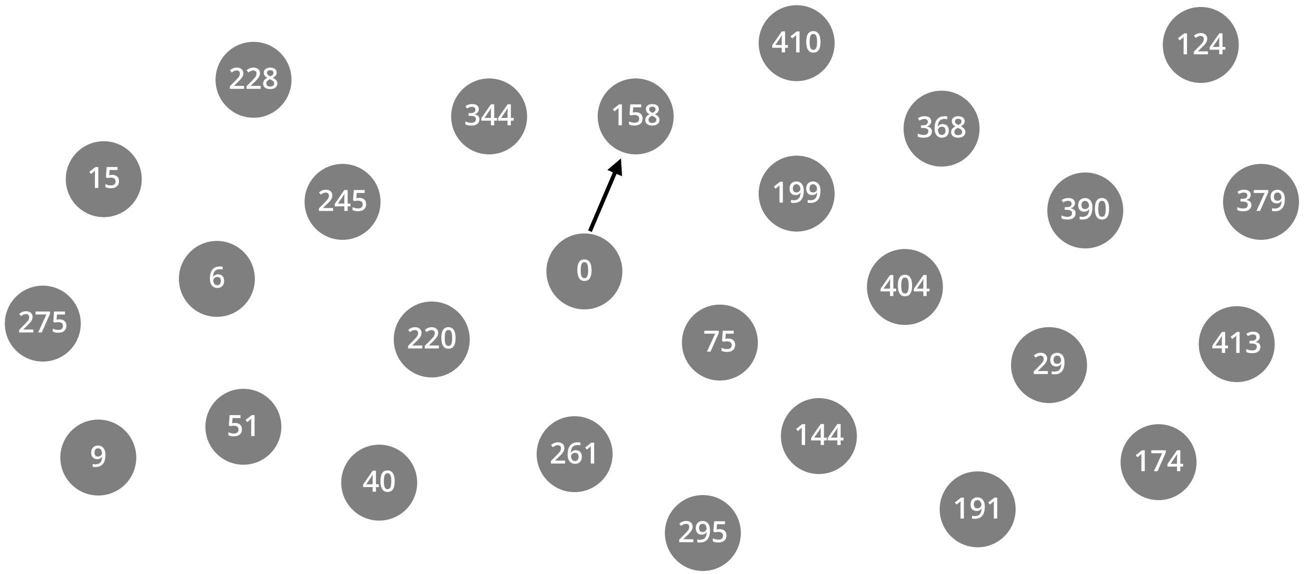

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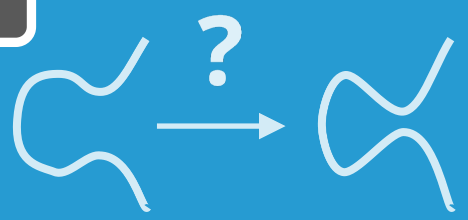

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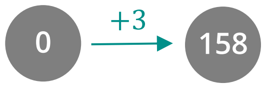

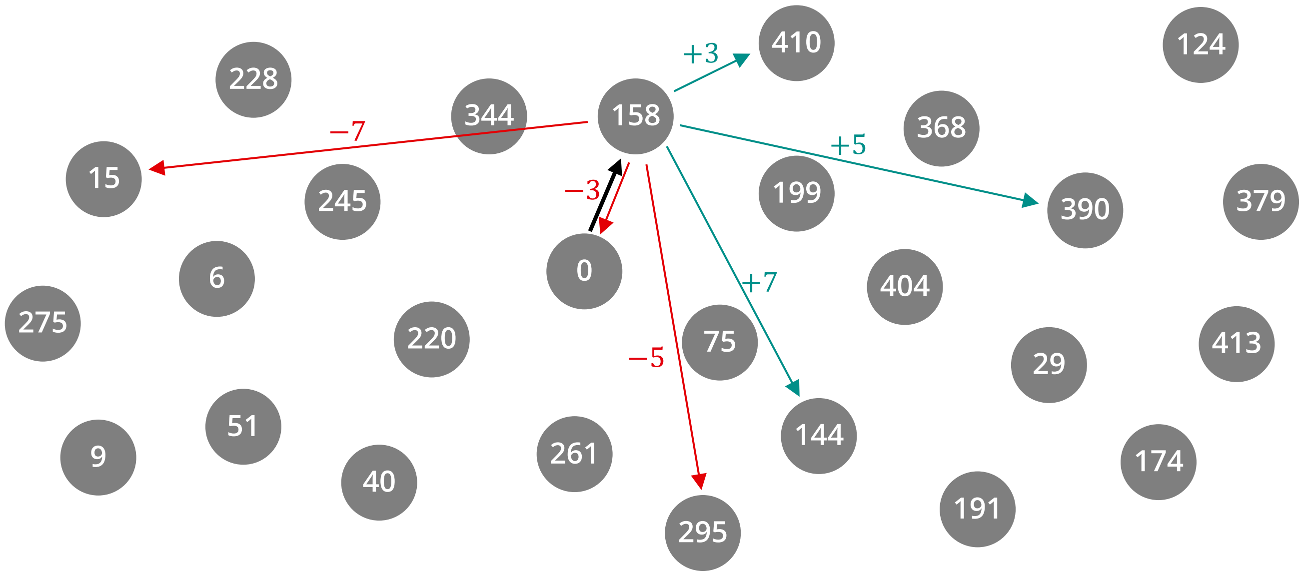

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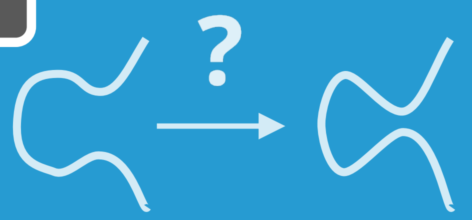

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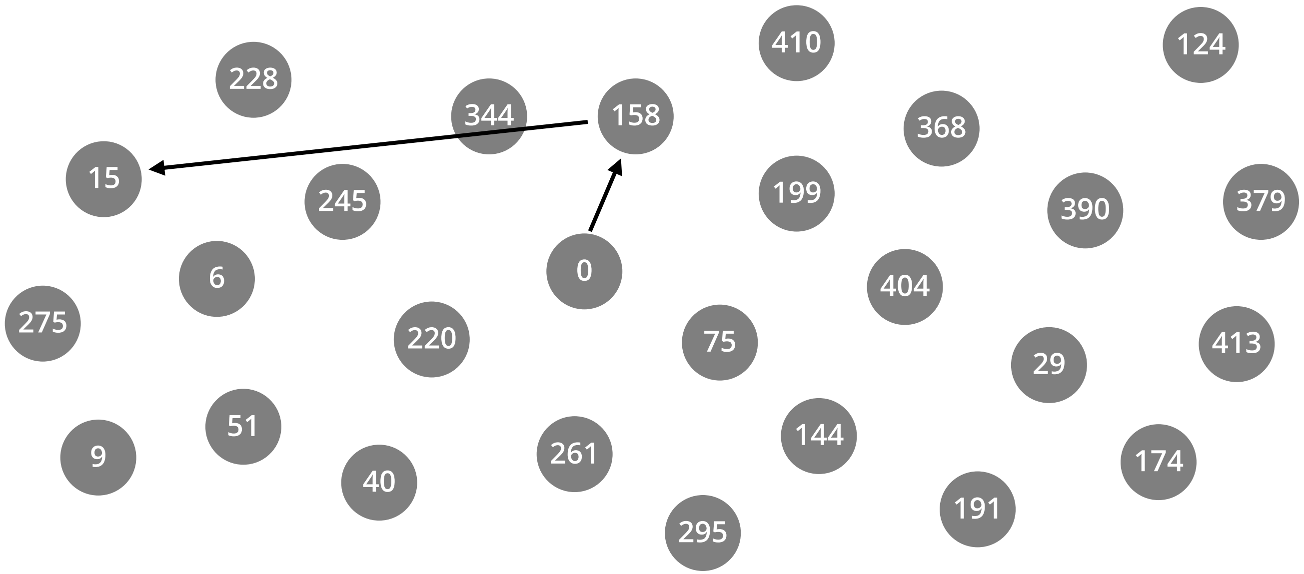

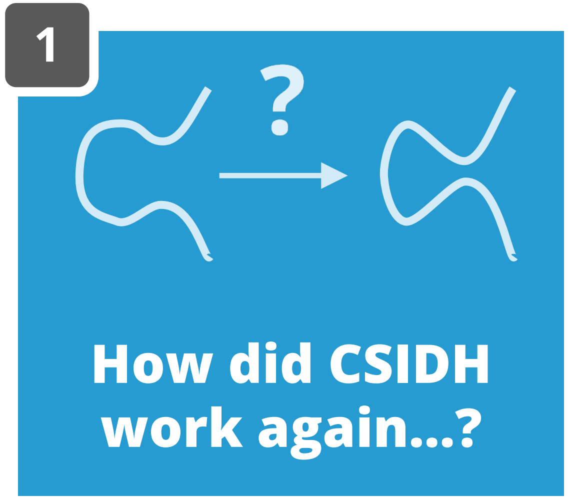

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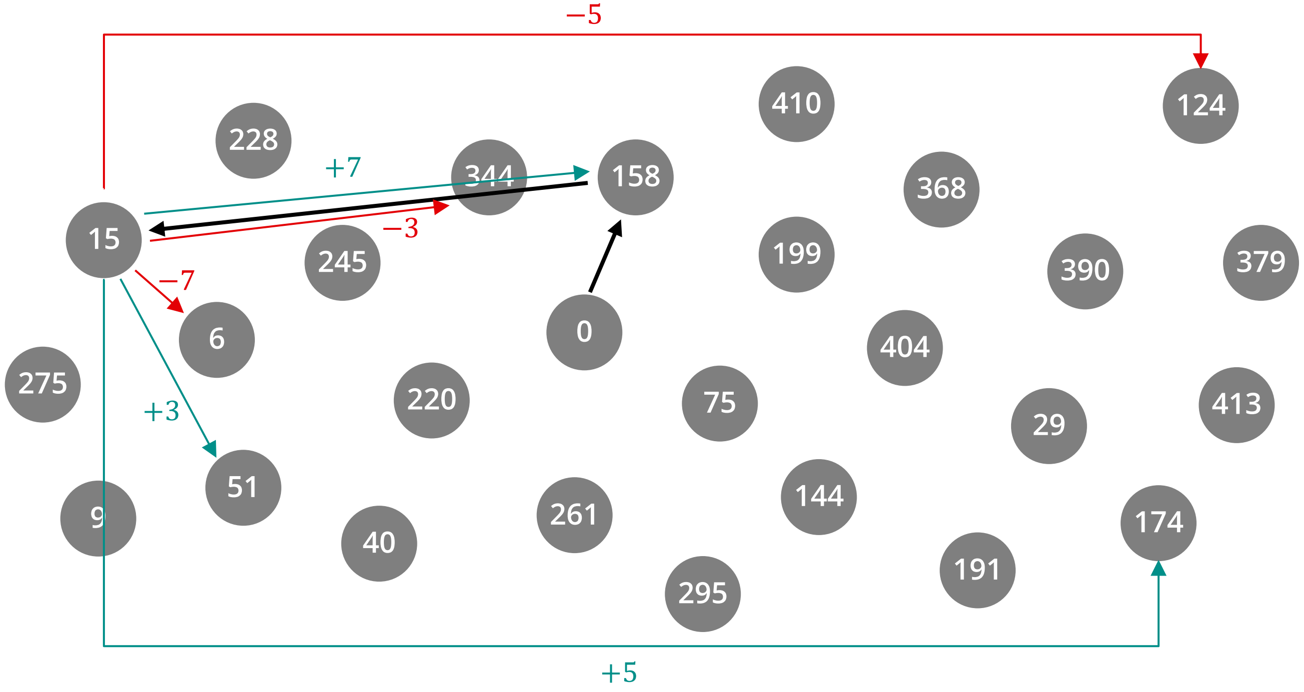

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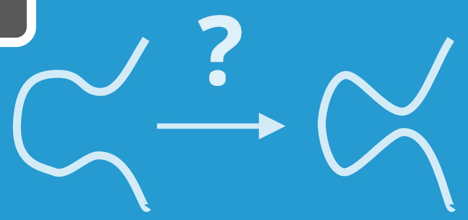

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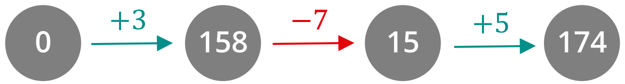

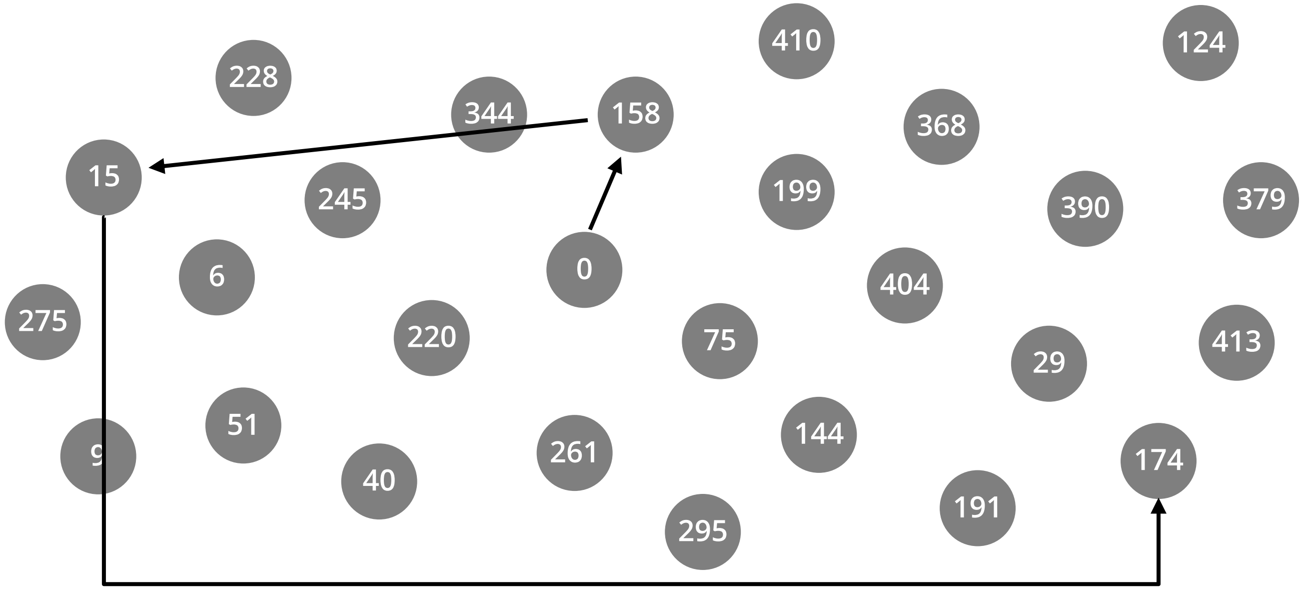

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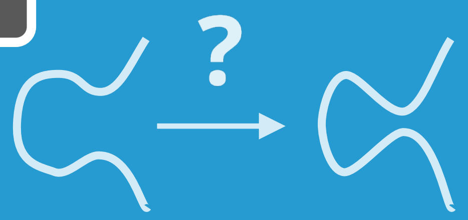

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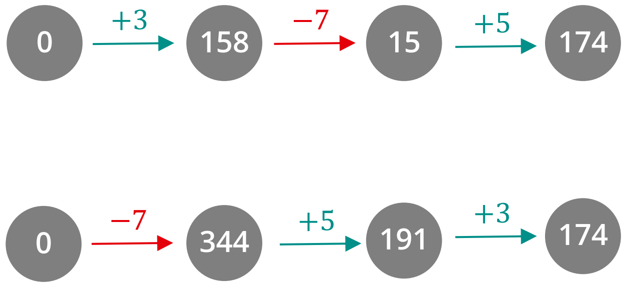

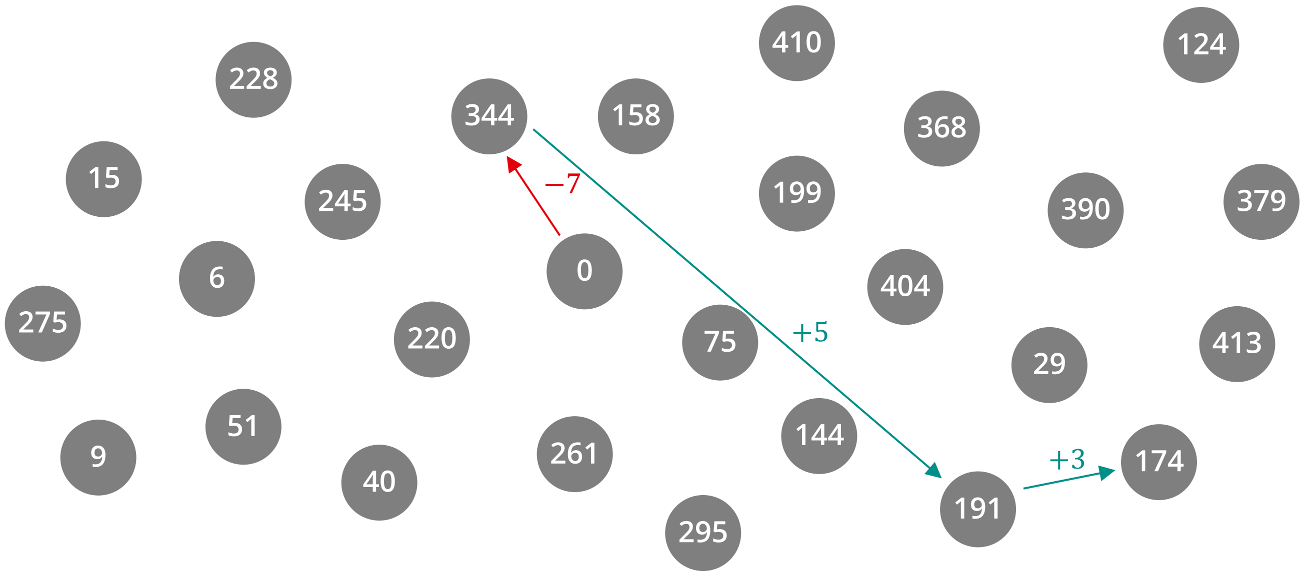

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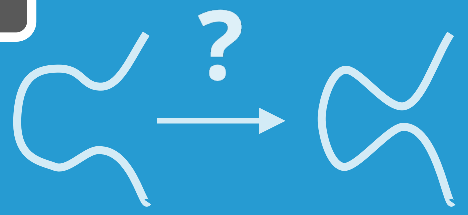

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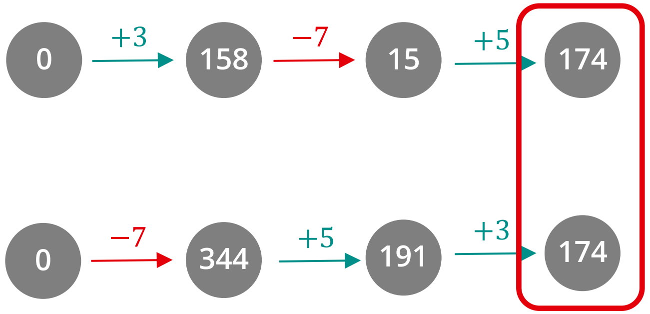

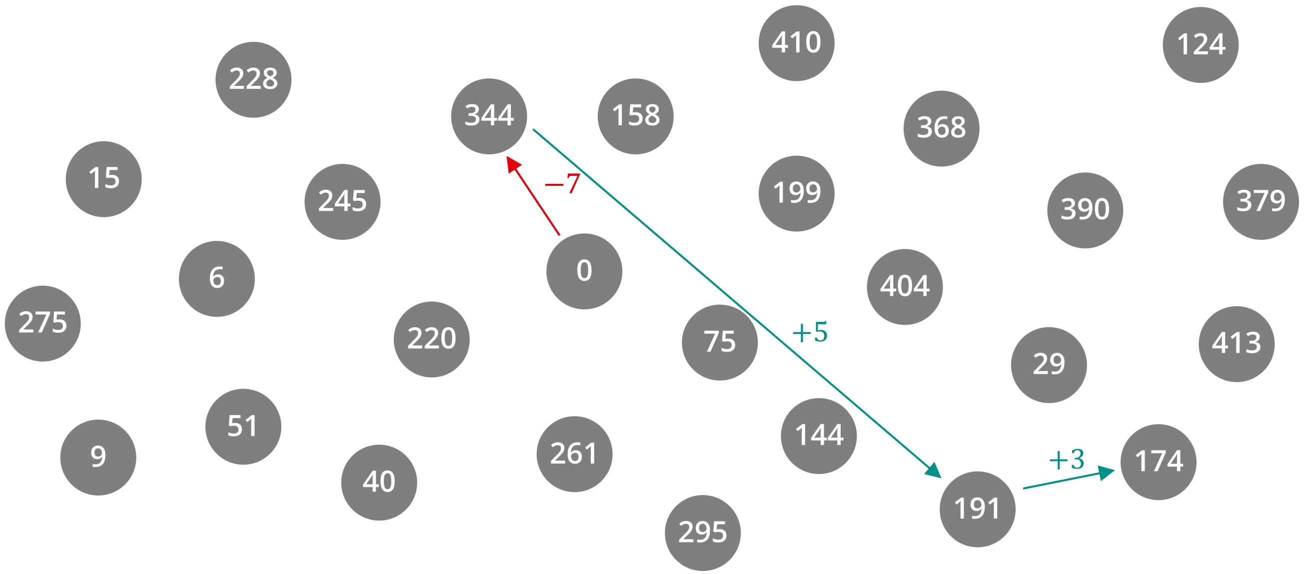

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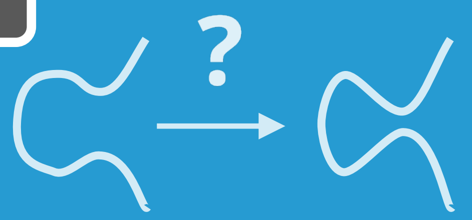

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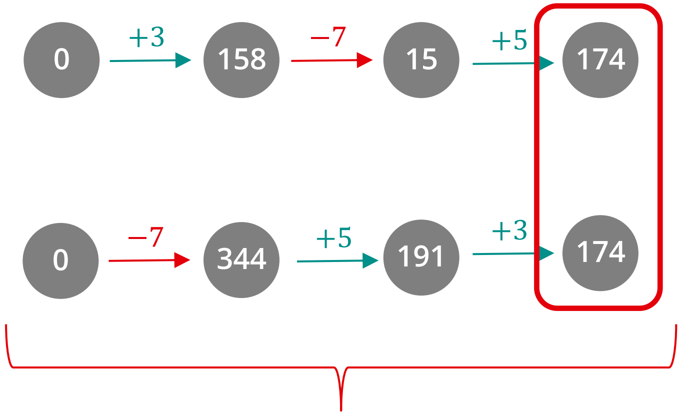
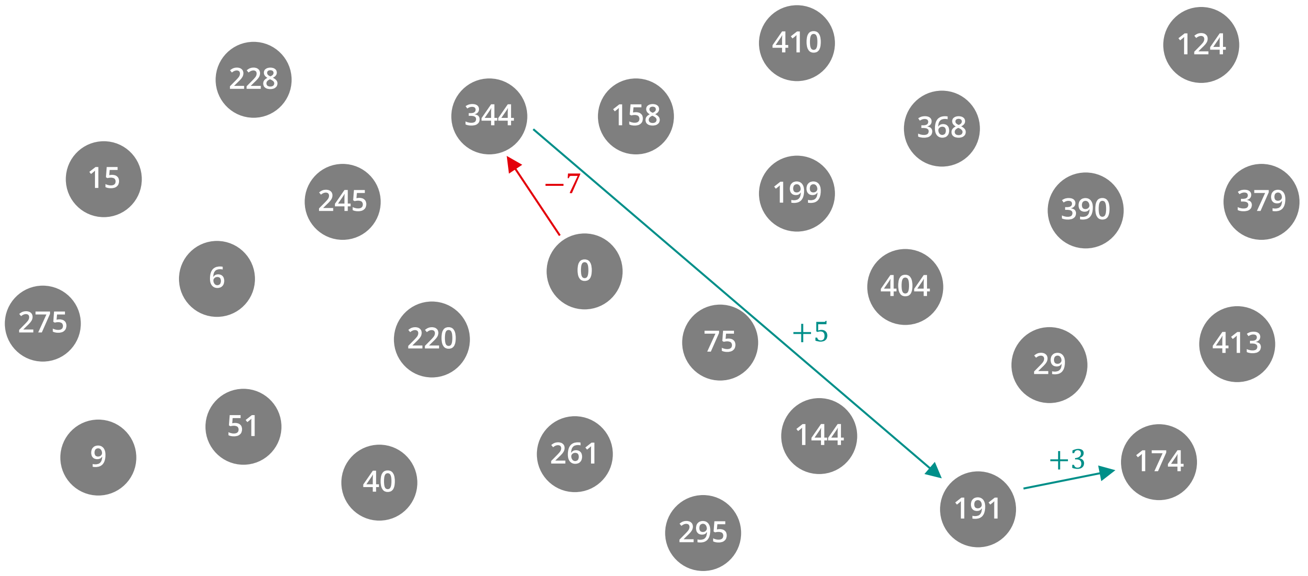
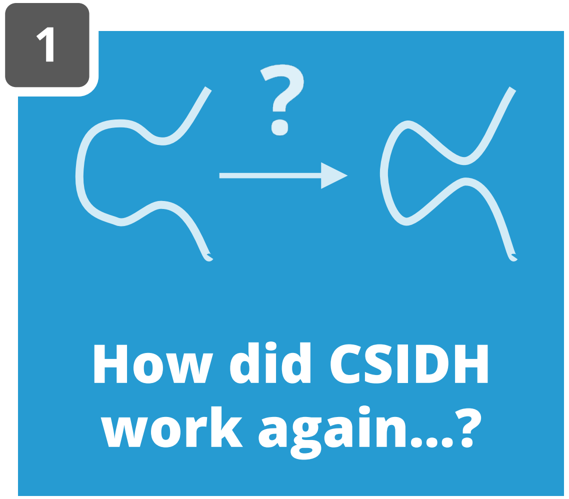


path is $(+1, +1, -1)$





CSIDH key exchange

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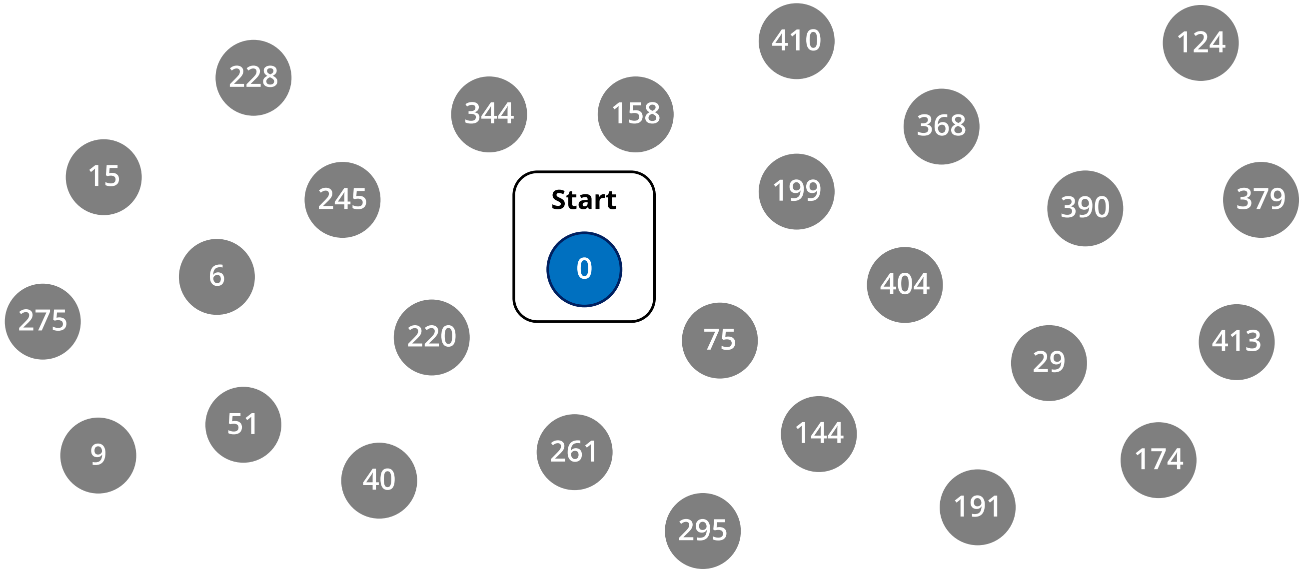

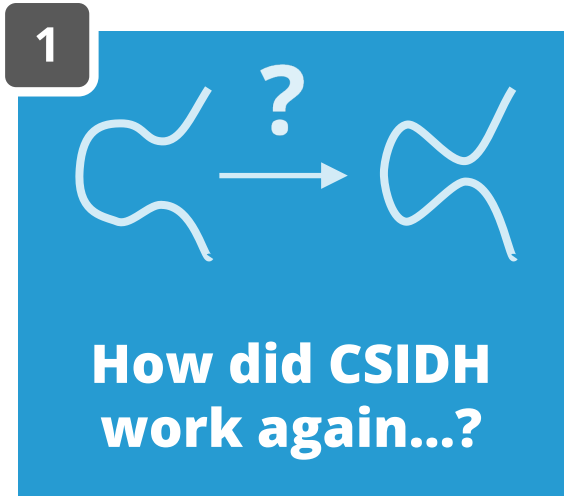

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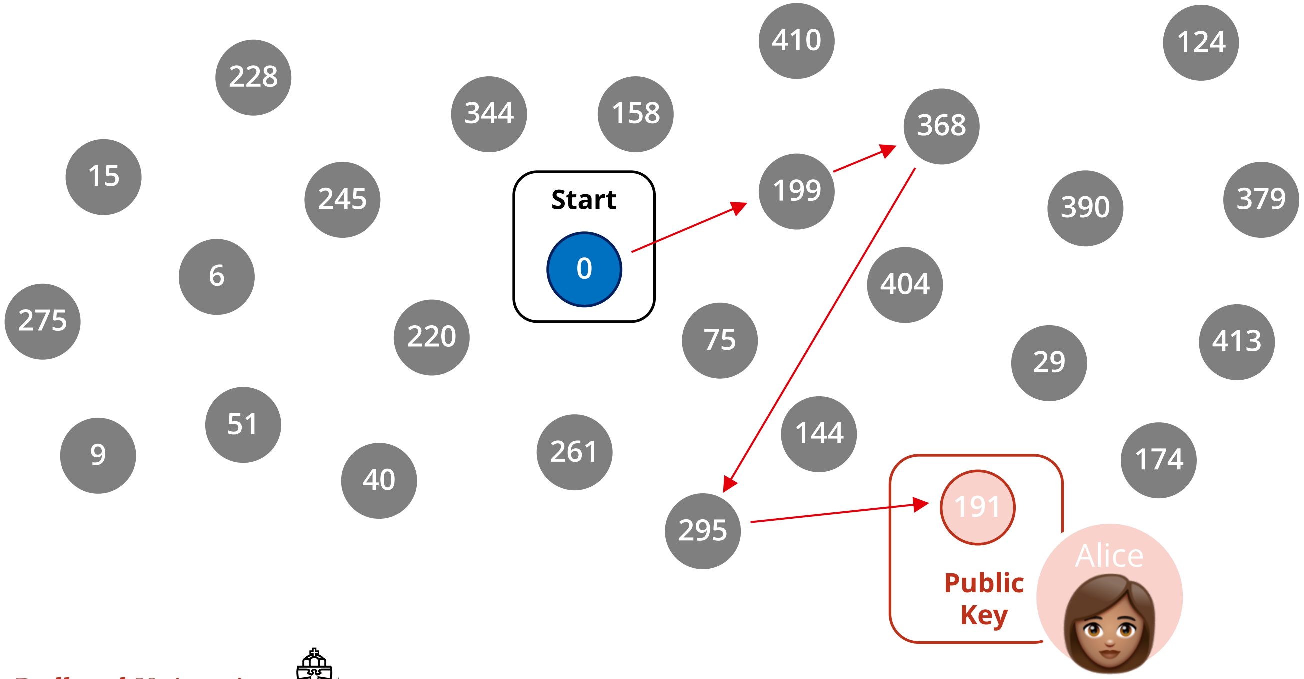

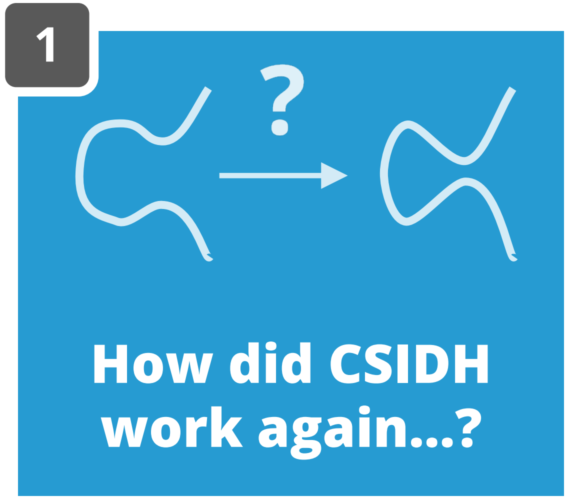

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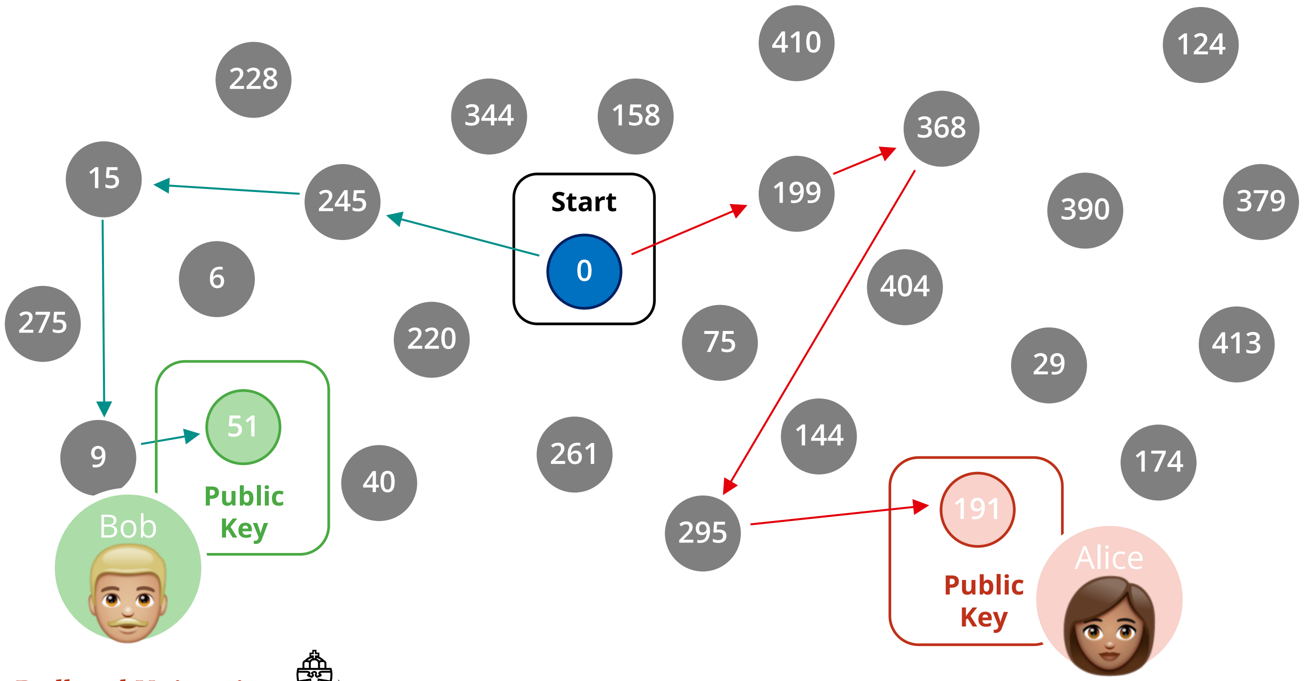

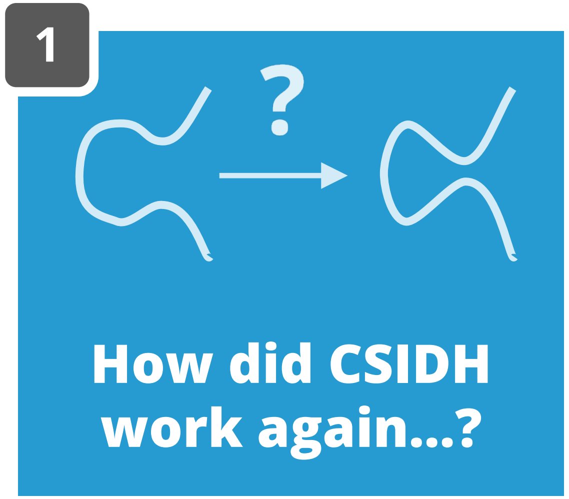

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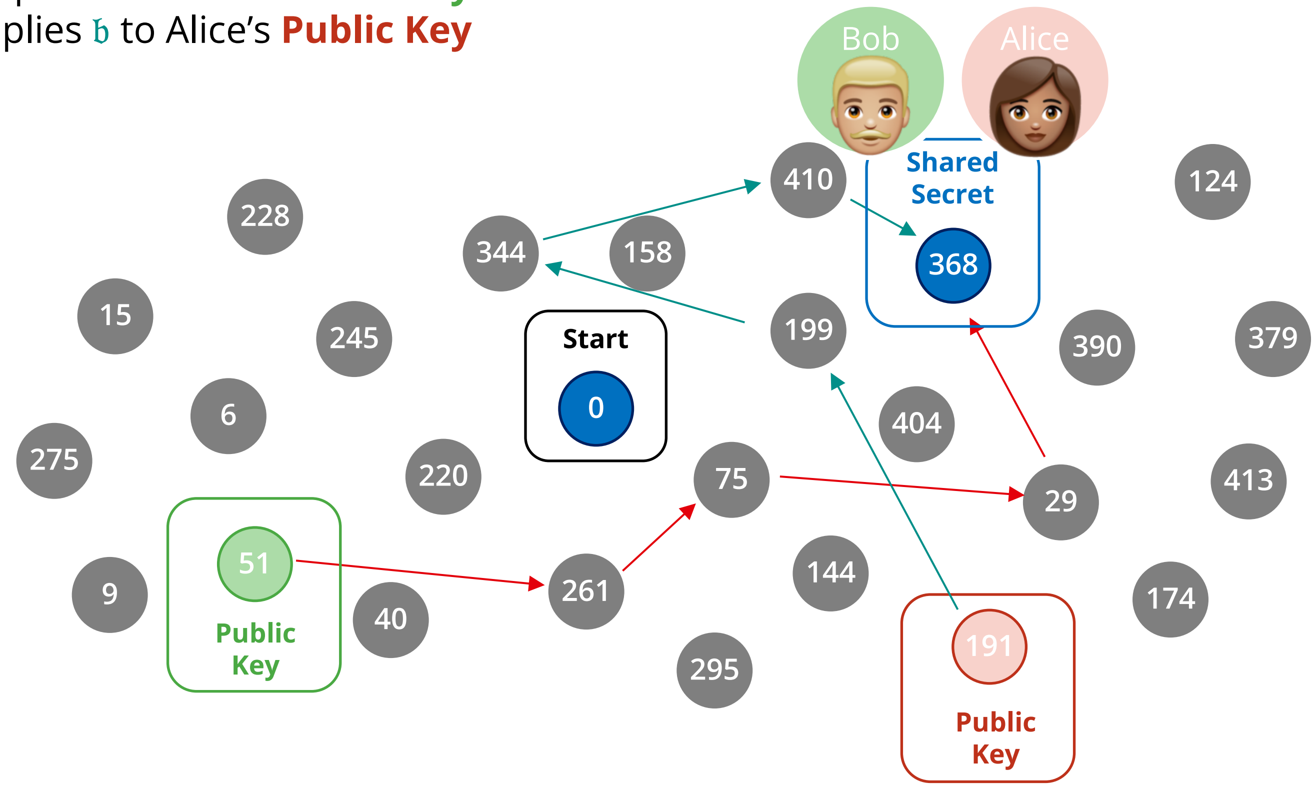

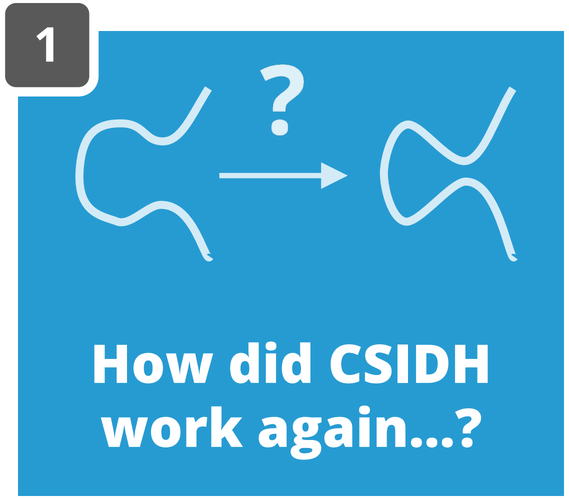

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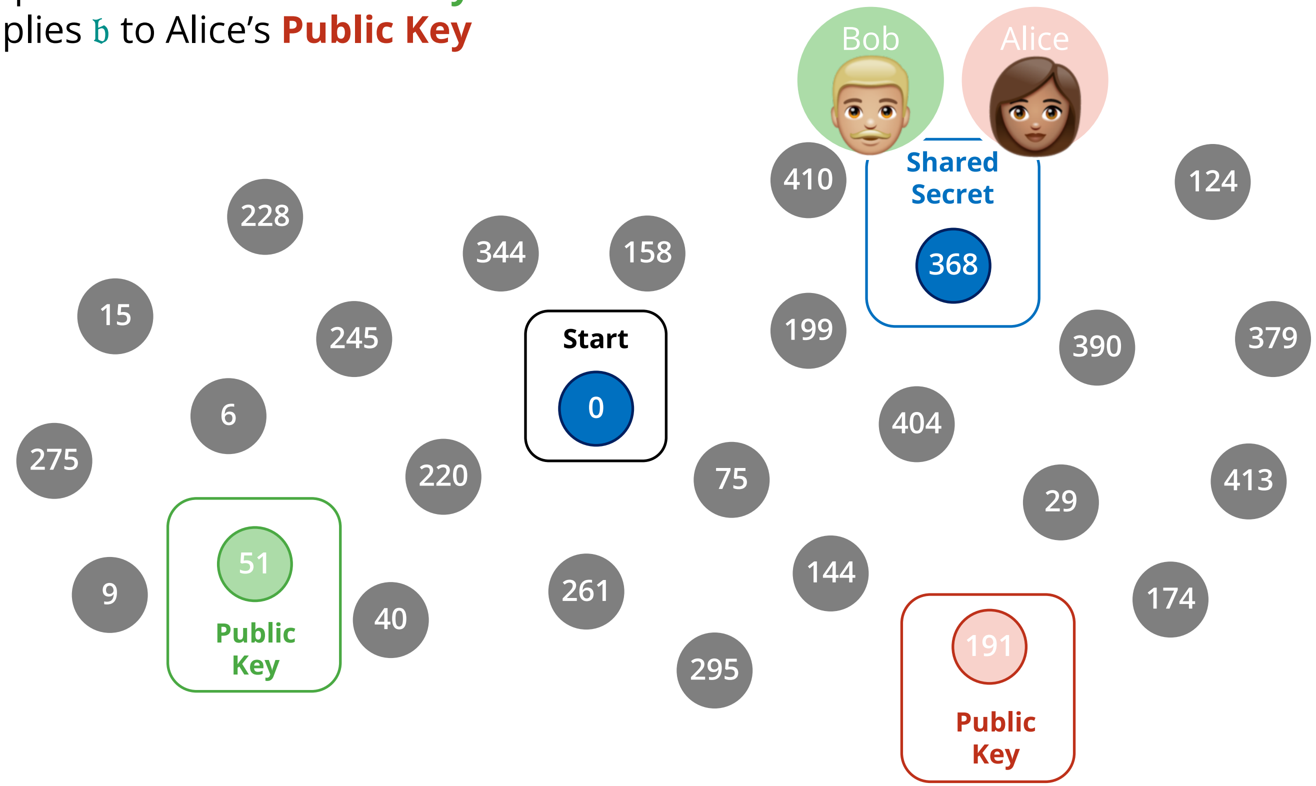

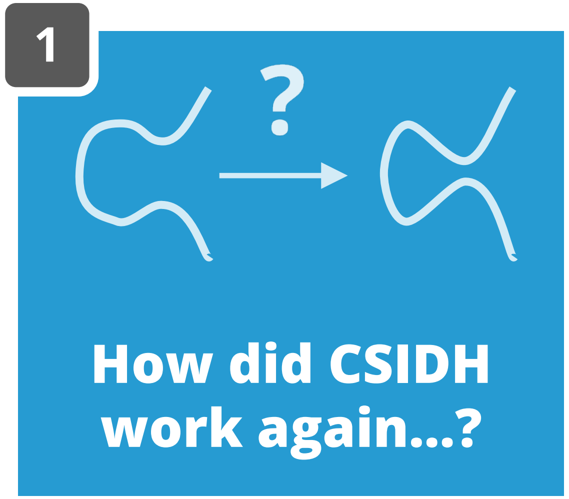

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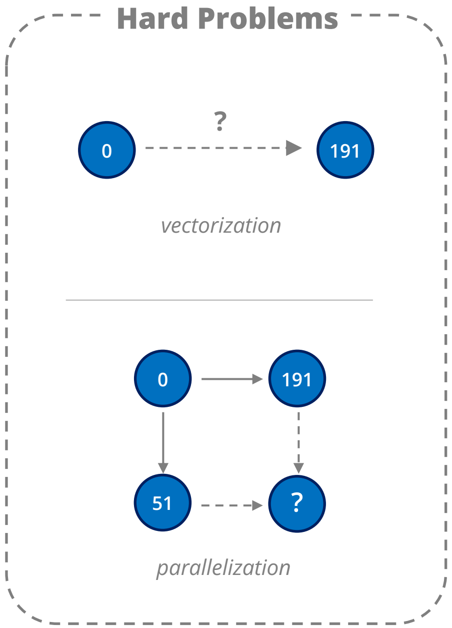
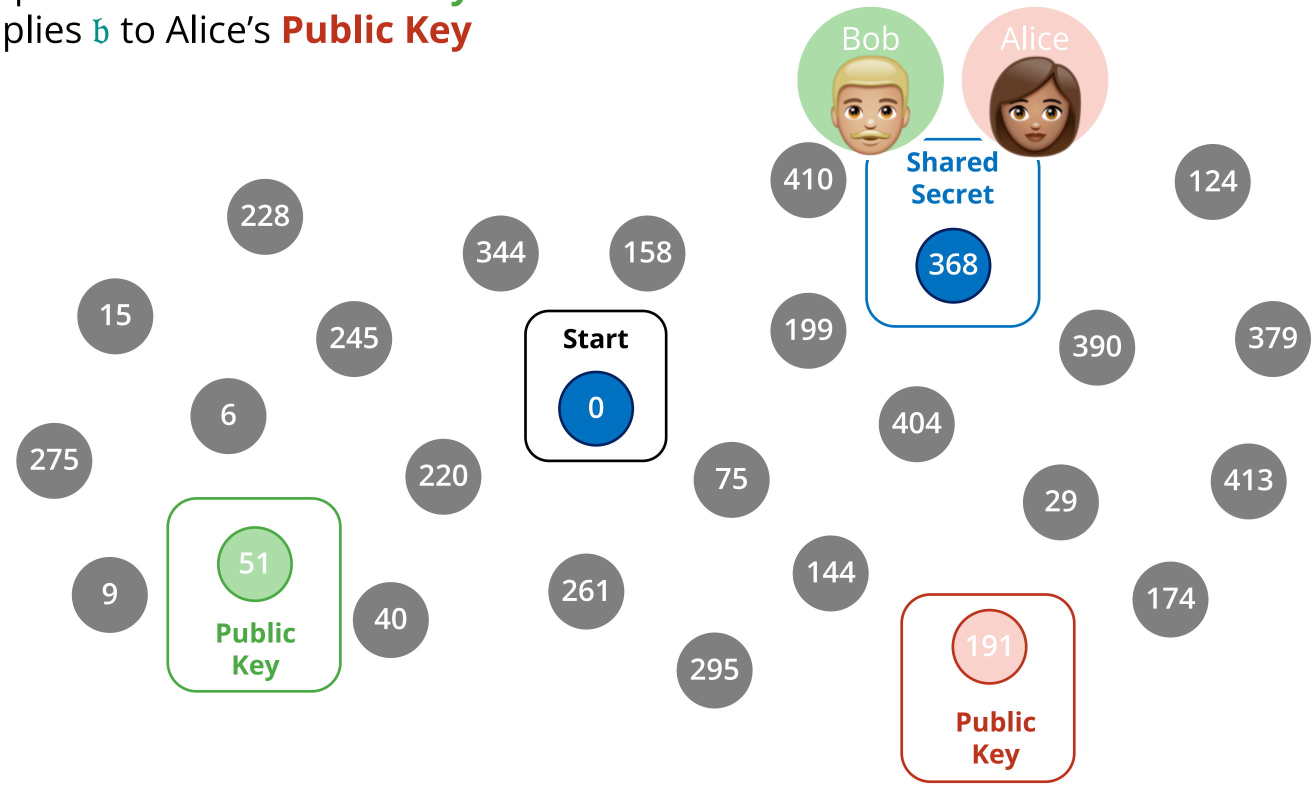
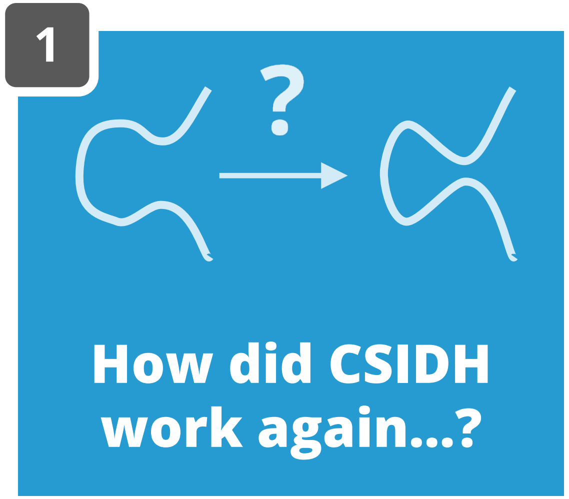
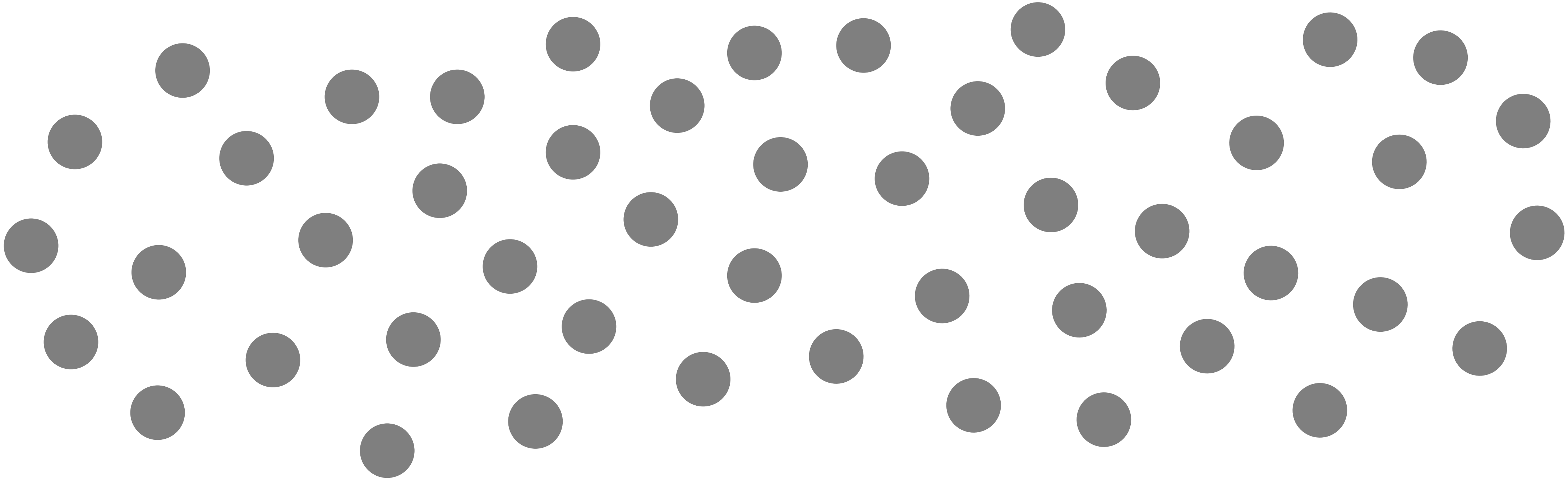


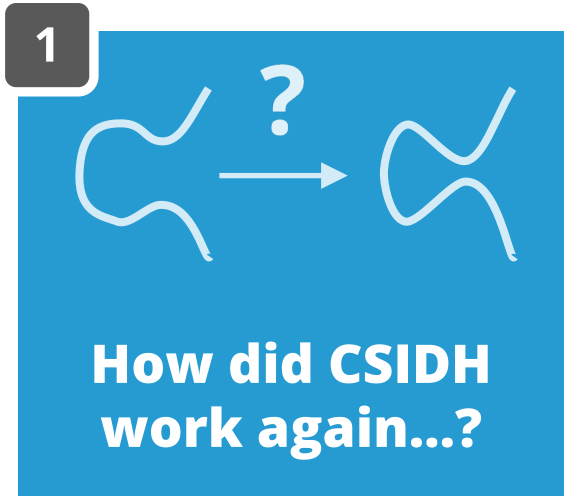


HOW TO WALK



How to compute walk





How to compute walk

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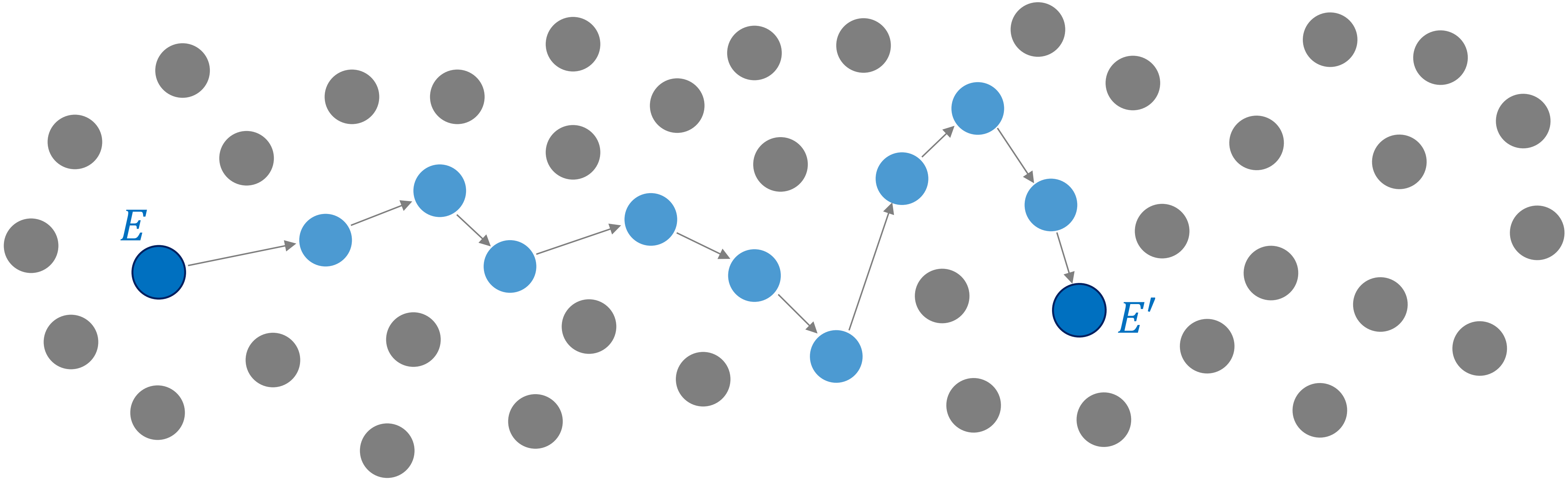

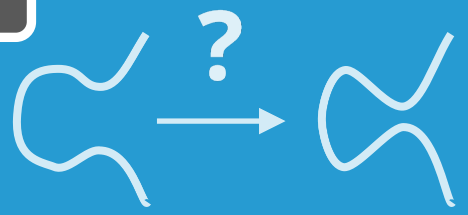

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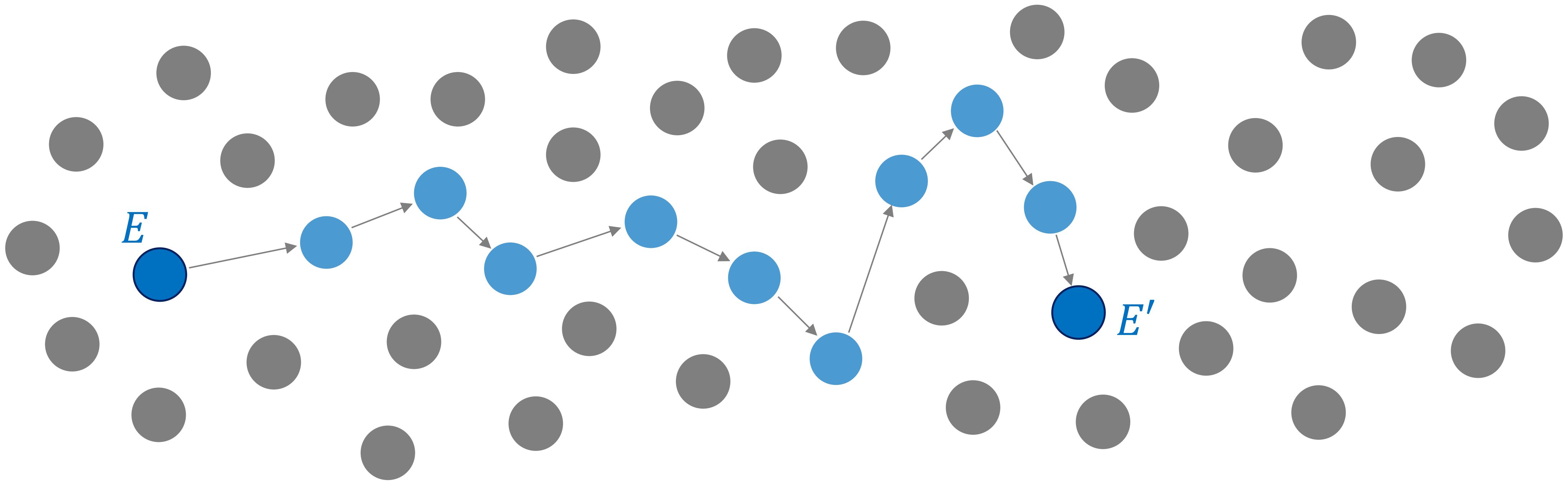
e.g. take two negative steps for third ℓ that divides $p + 1$



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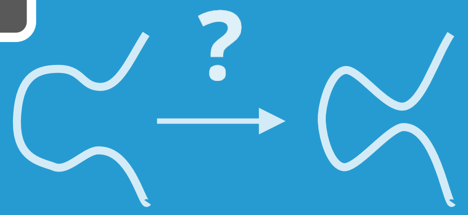
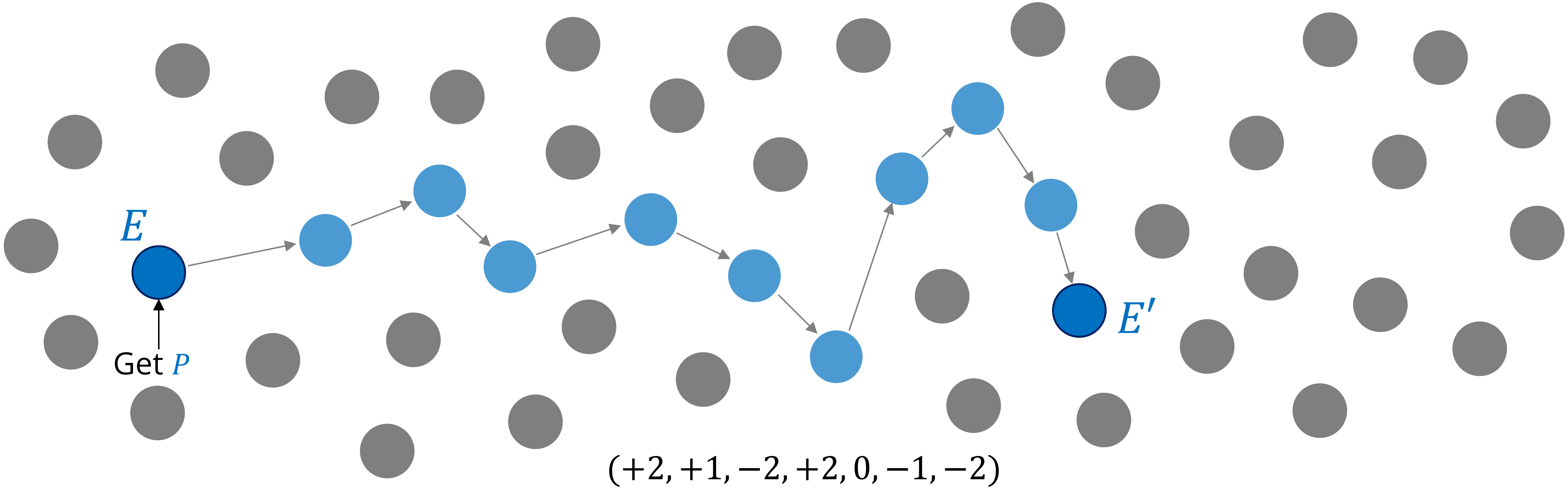


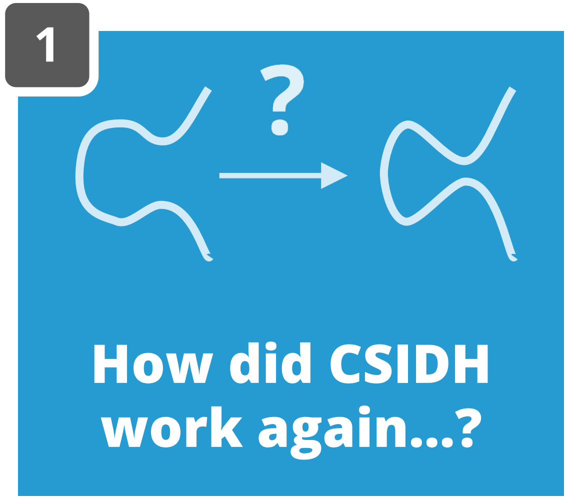
How did CSIDH work again...?

How to compute walk

Let's say $E \rightarrow E'$ is path $(+2, +1, -2, +2, 0, -1, -2)$

1. Sample point P, check if $+$ or $-$
2. Can use P to perform one step of each ℓ_i
3. Repeat until path is performed





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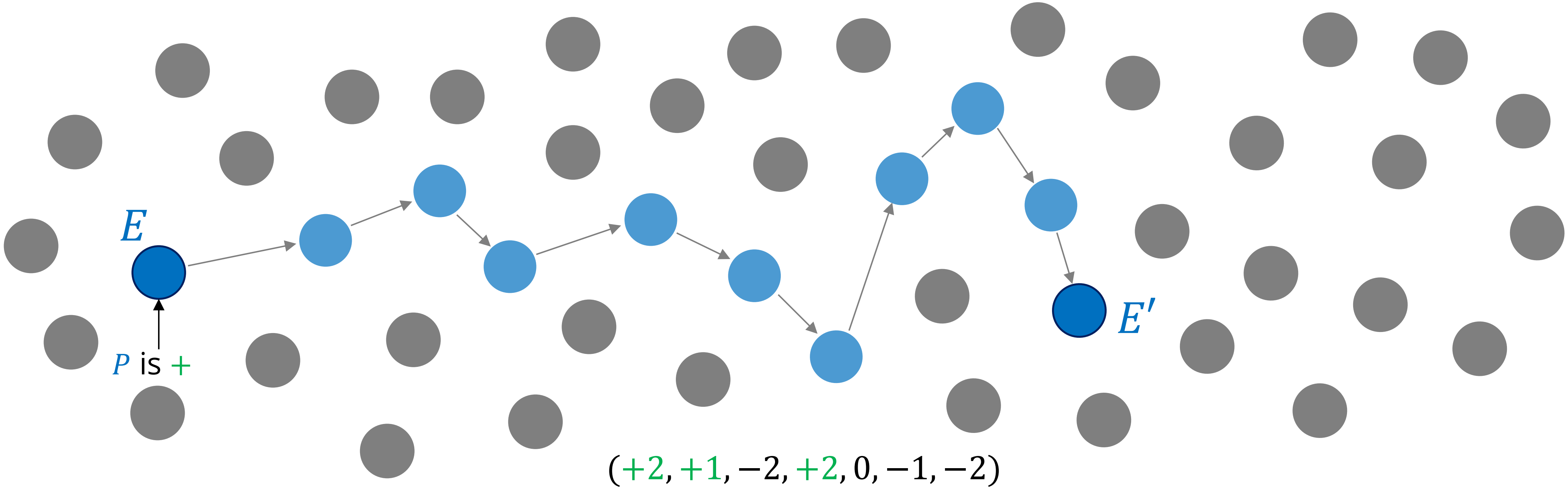
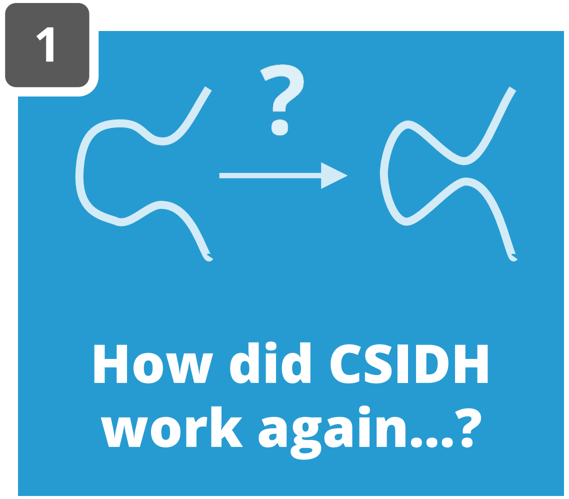


$(+2, +1, -2, +2, 0, -1, -2)$



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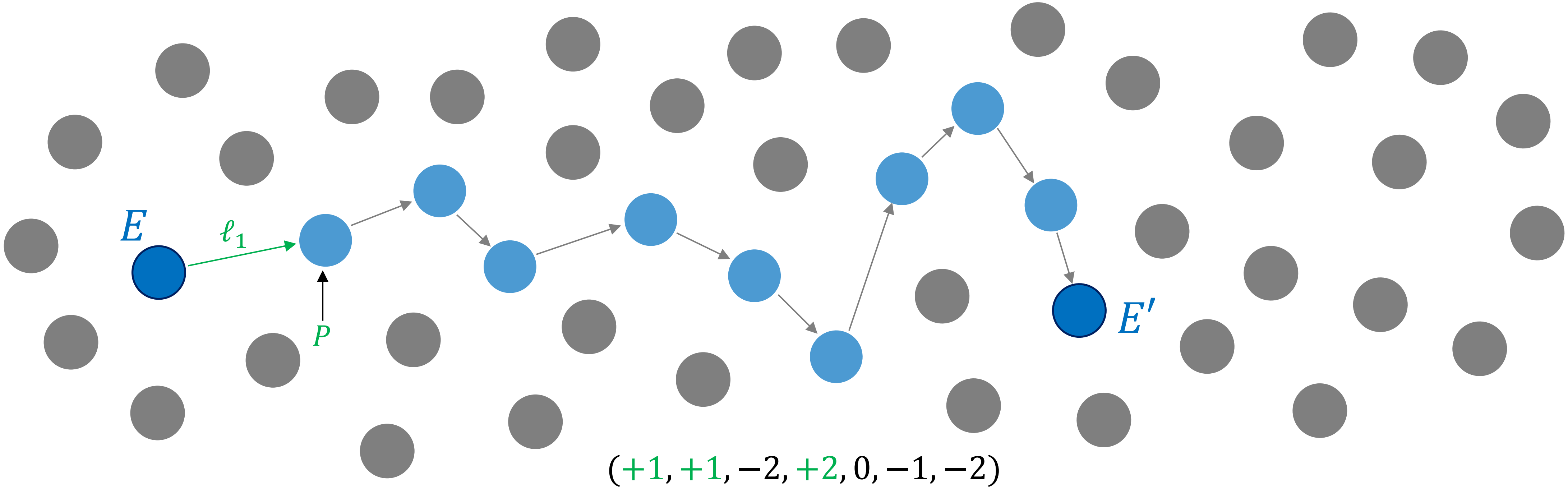

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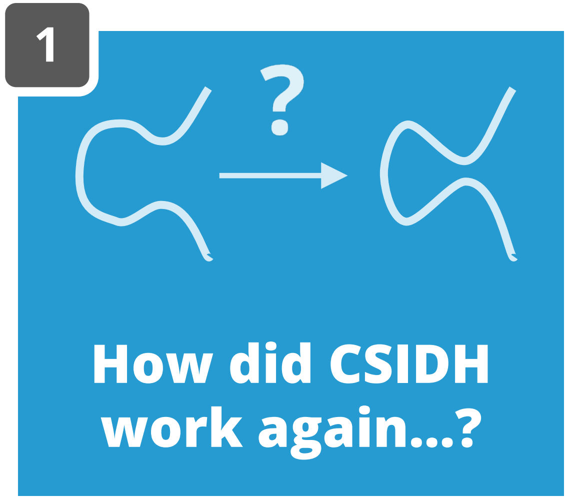

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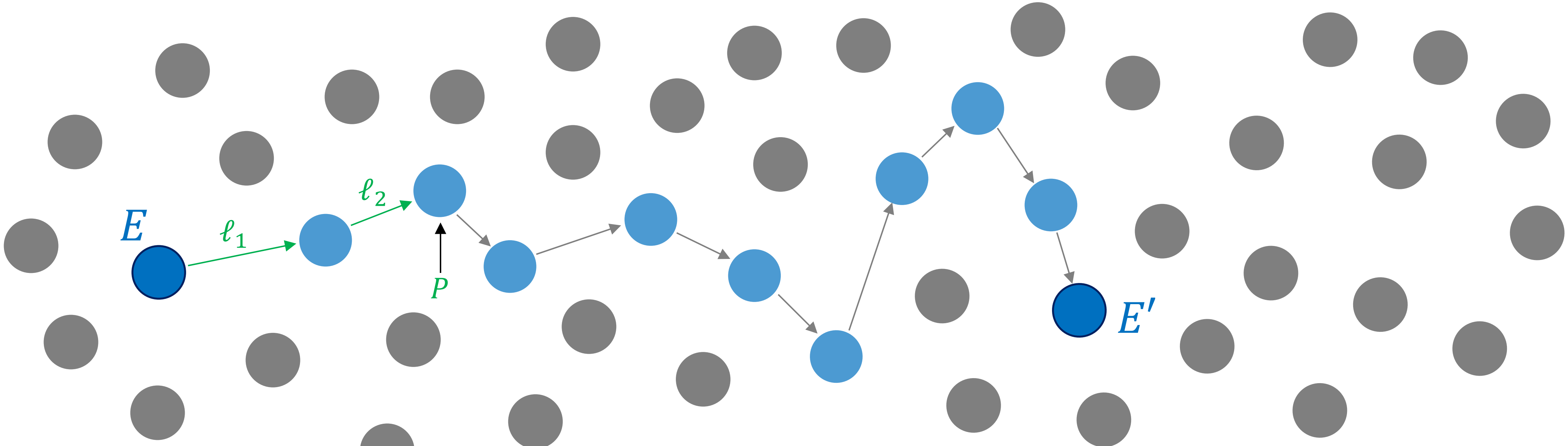
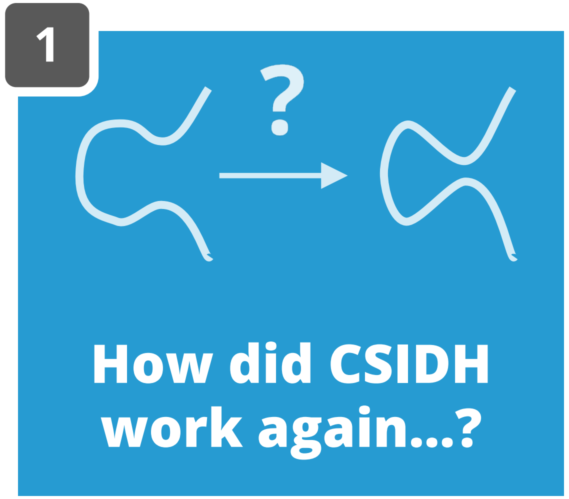


$(+1, +0, -2, +2, 0, -1, -2)$



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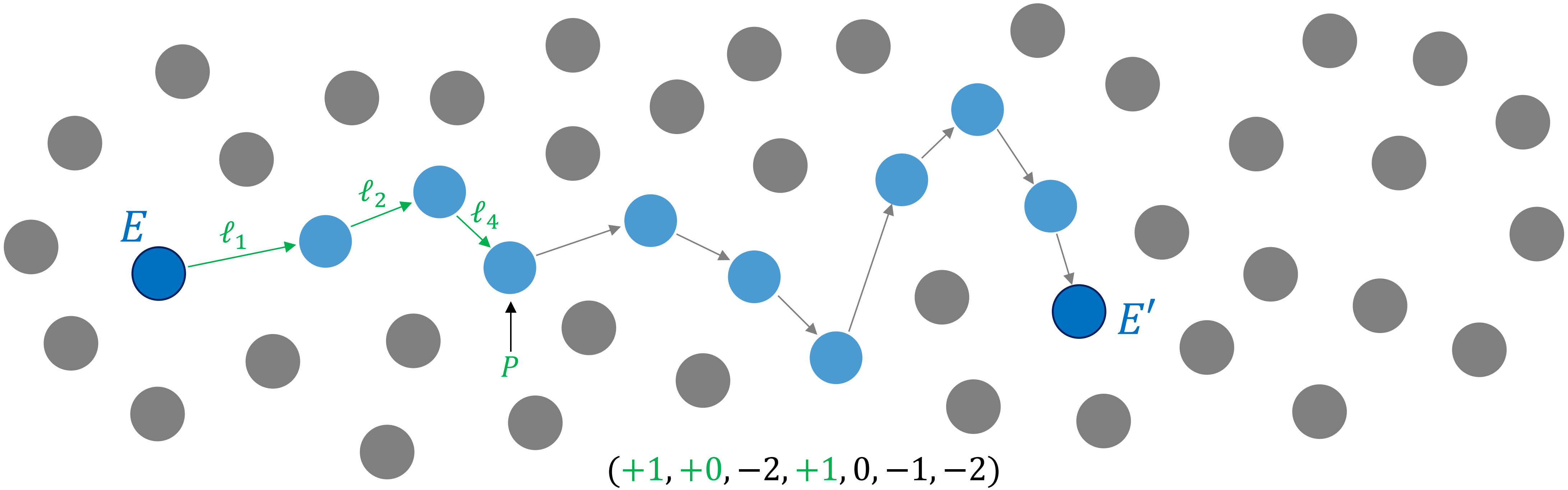
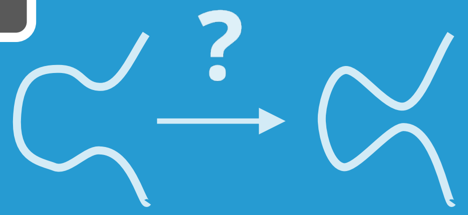


$(+1, +0, -2, +1, 0, -1, -2)$

1



How did CSIDH work again...?

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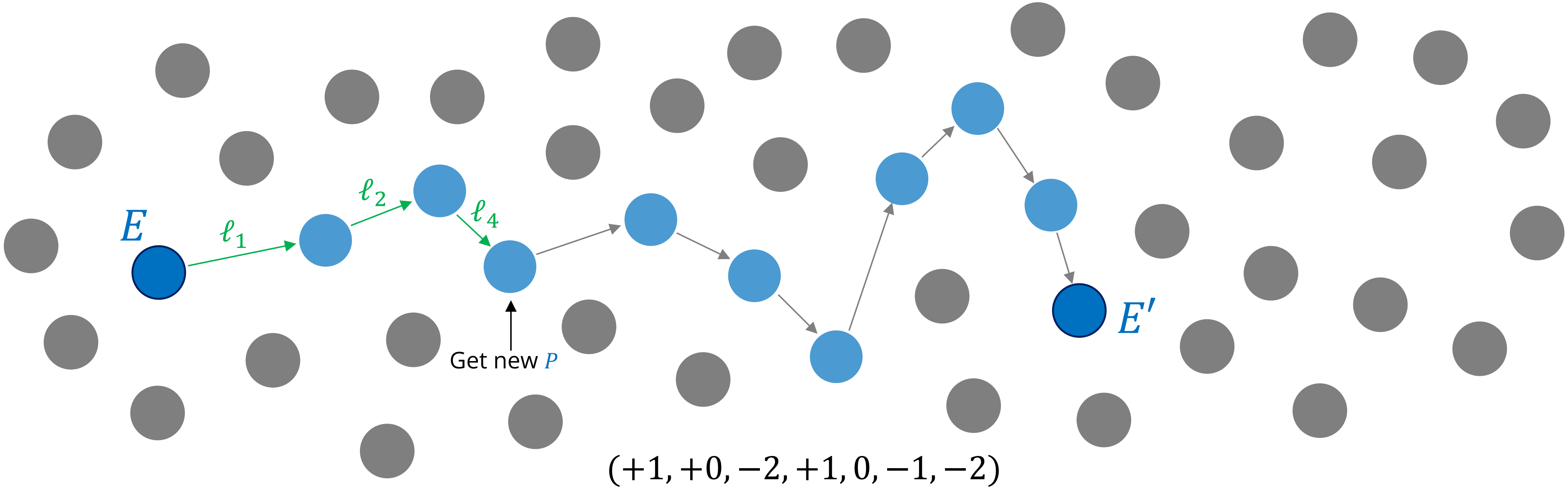
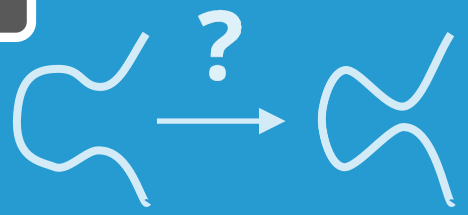


$(+1, +0, -2, +1, 0, -1, -2)$

1



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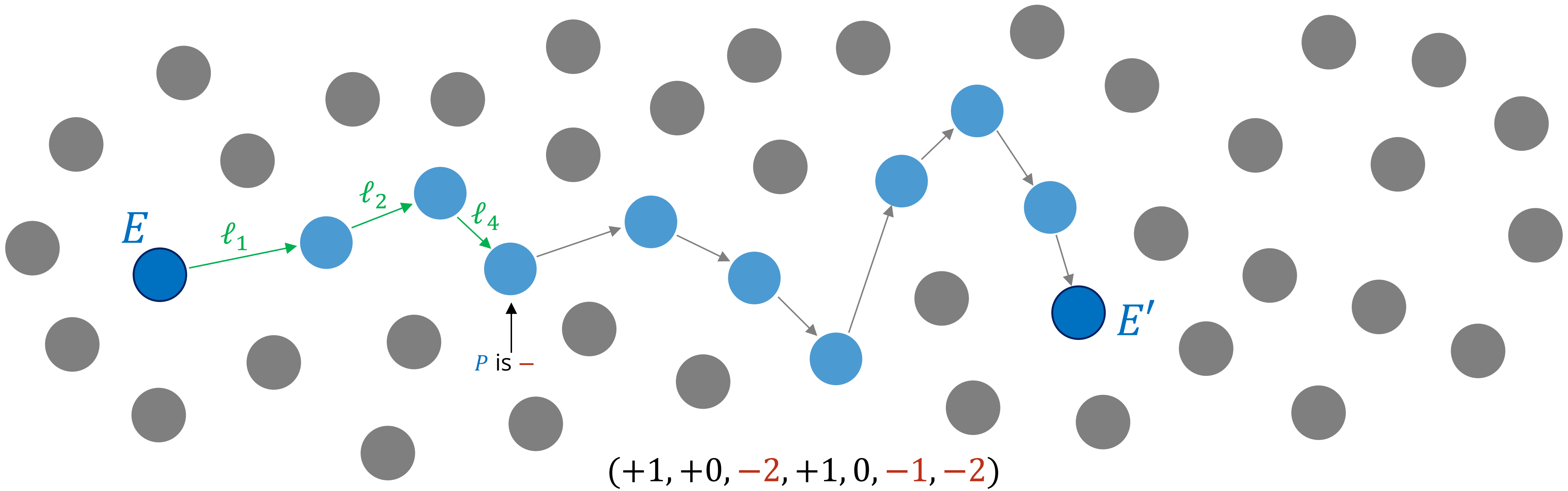
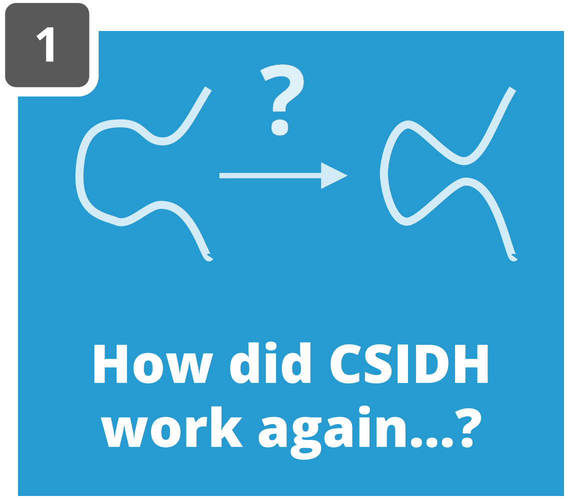


$$(+1, +0, -2, +1, 0, -1, -2)$$



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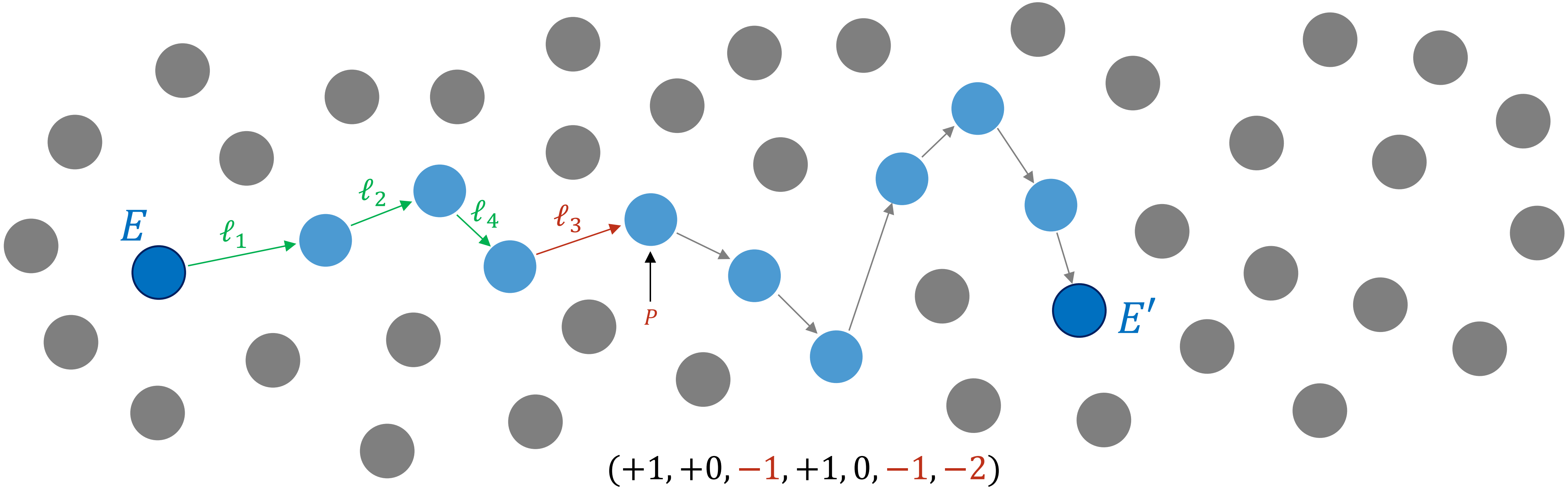

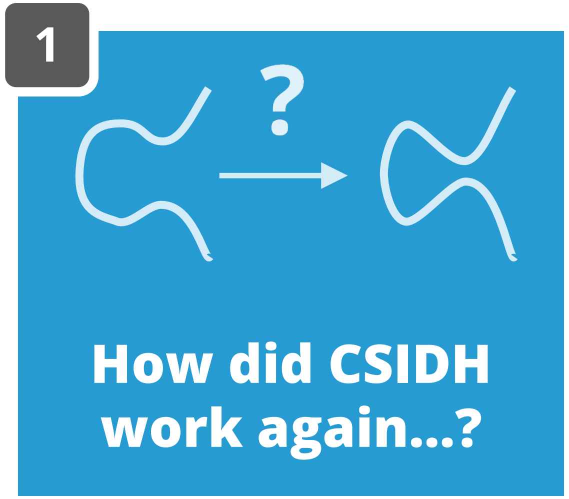

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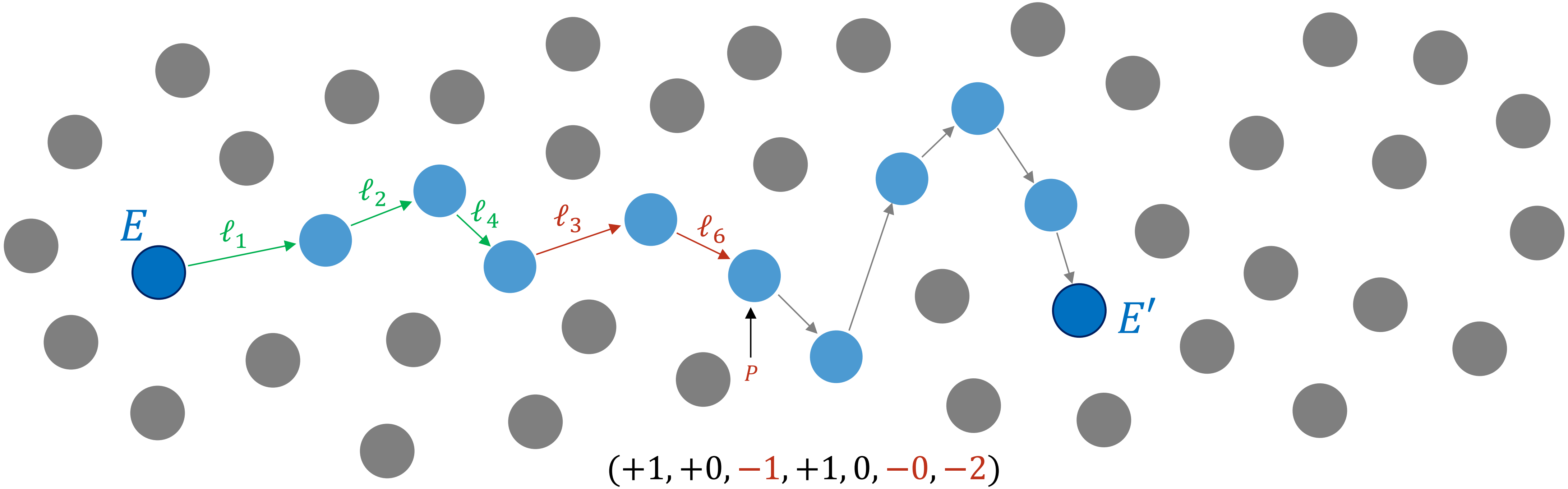
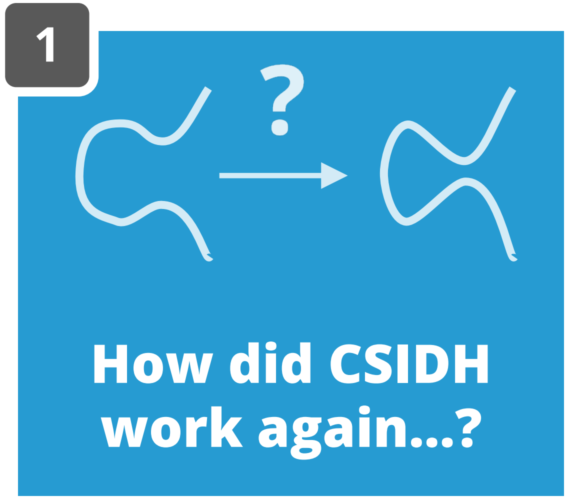


$(+1, +0, -1, +1, 0, -0, -2)$



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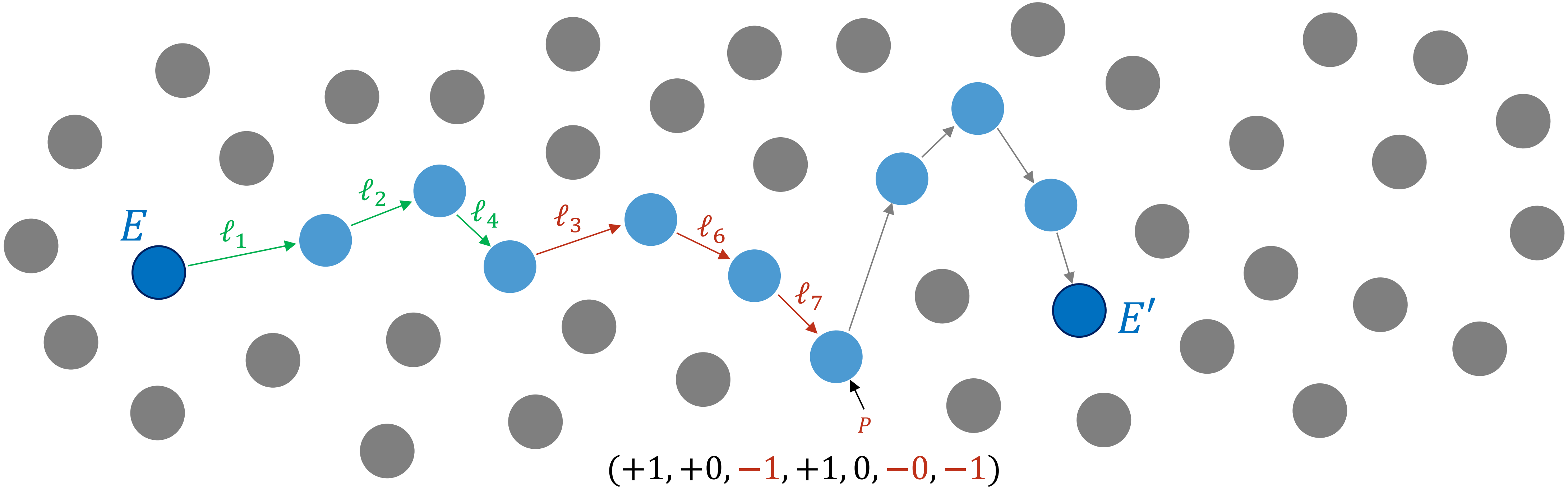
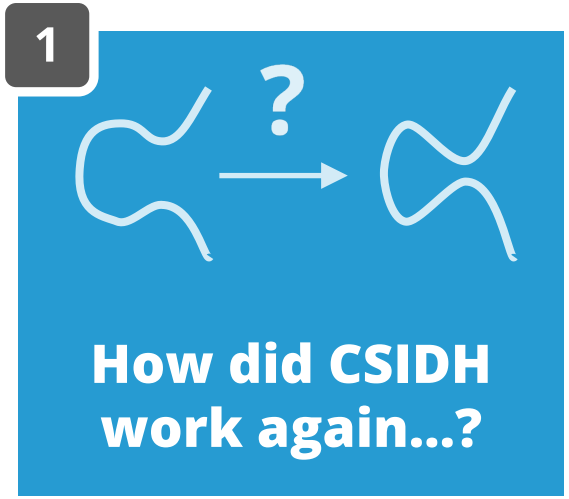


$(+1, +0, -1, +1, 0, -0, -1)$



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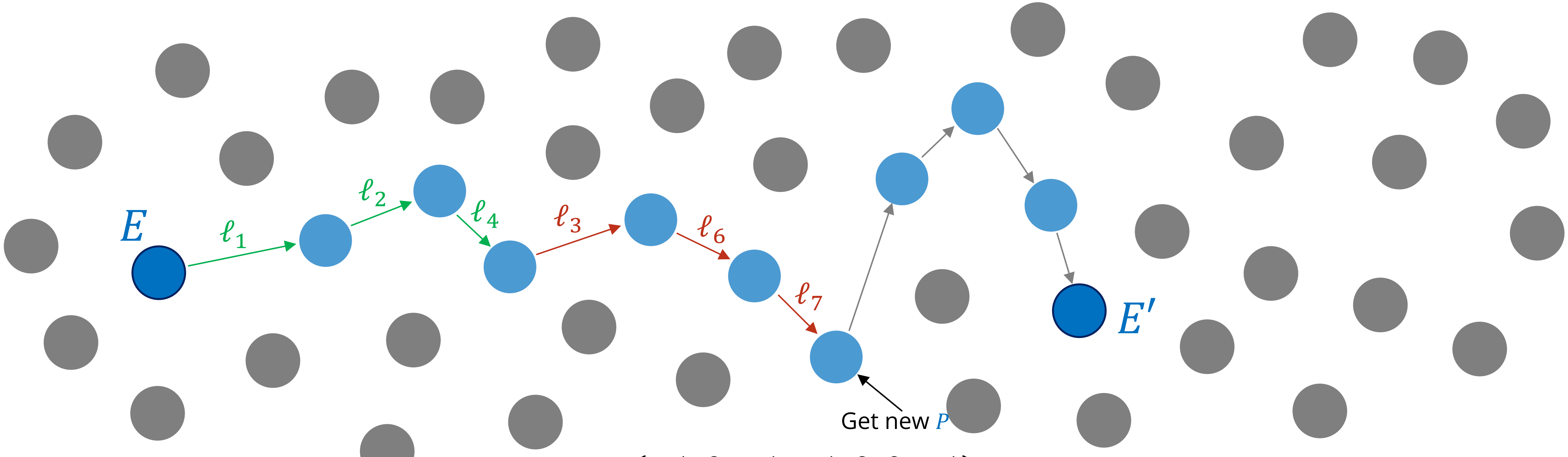
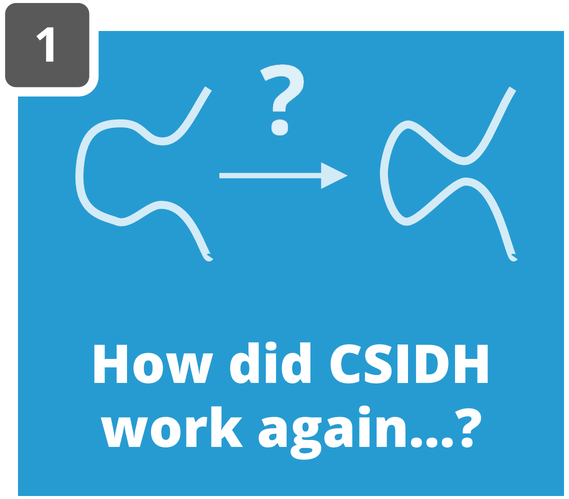


$(+1, 0, -1, +1, 0, 0, -1)$



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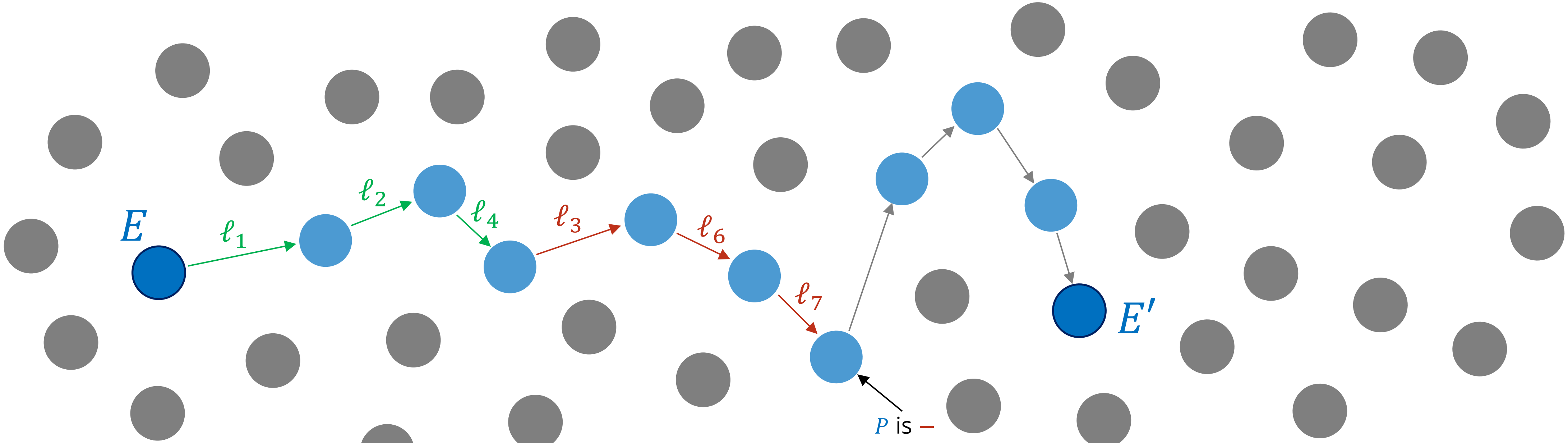
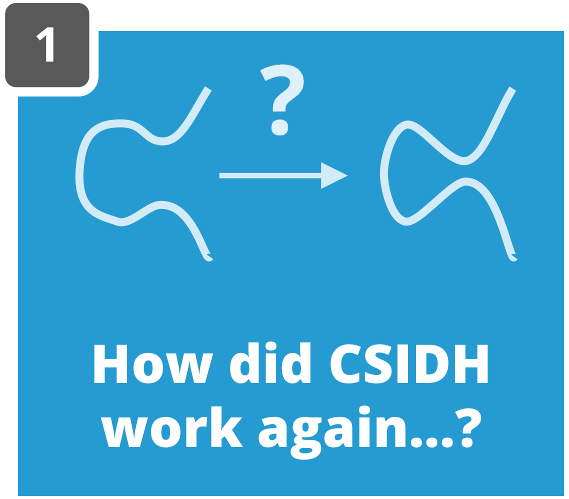


$(+1, 0, -1, +1, 0, 0, -1)$



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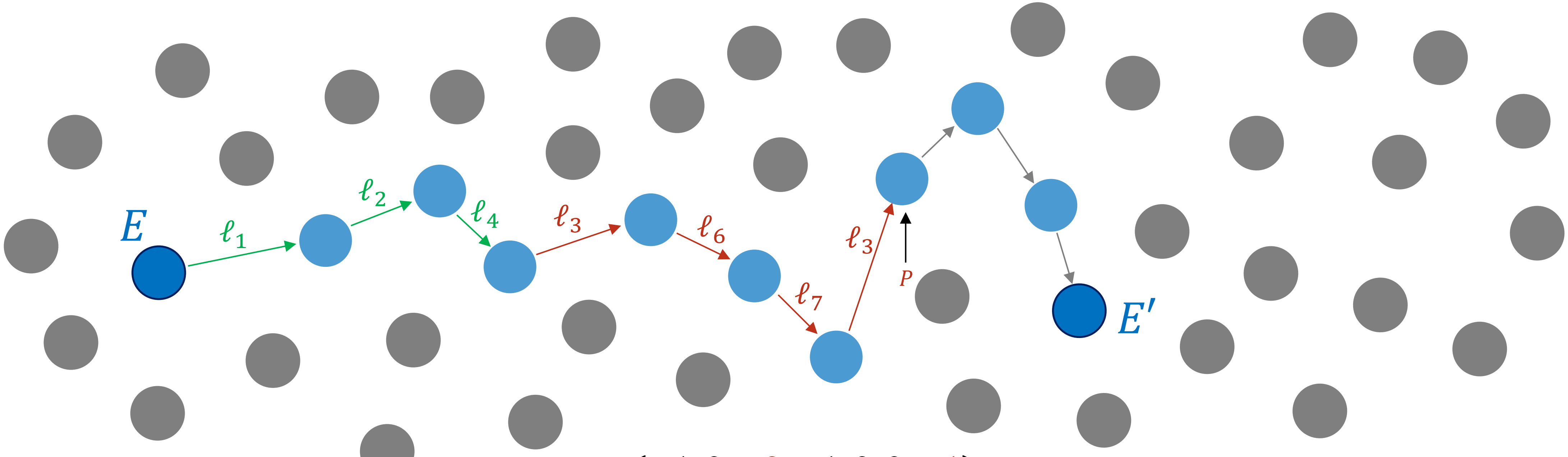
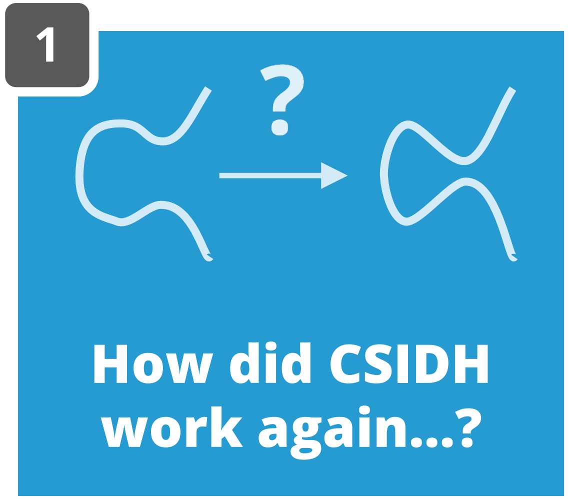


$(+1, 0, -0, +1, 0, 0, -1)$



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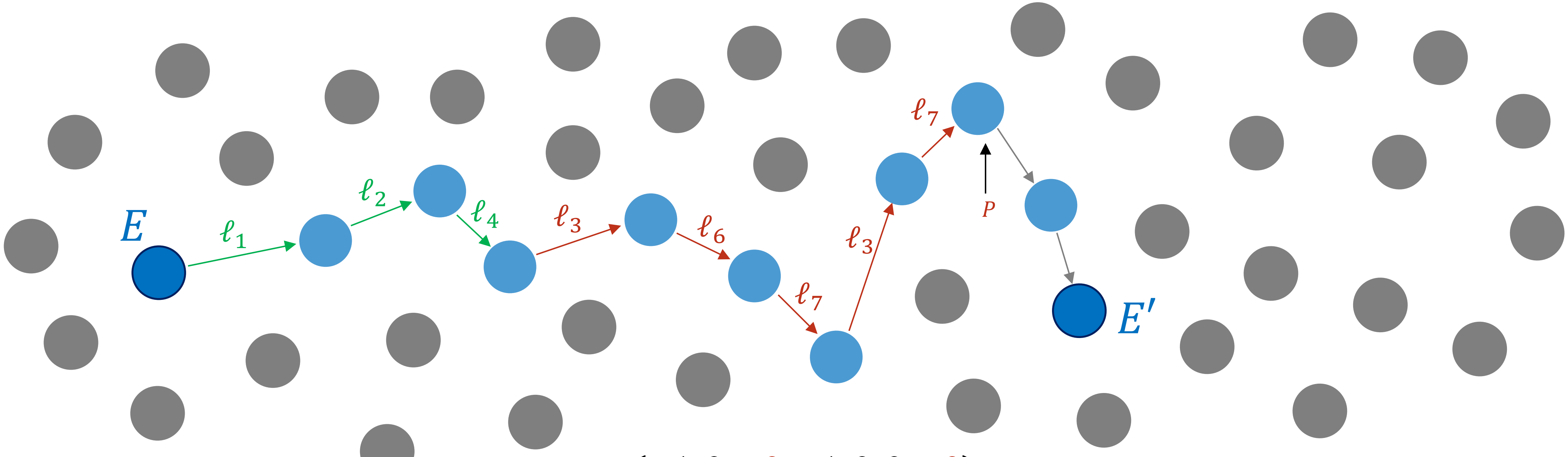
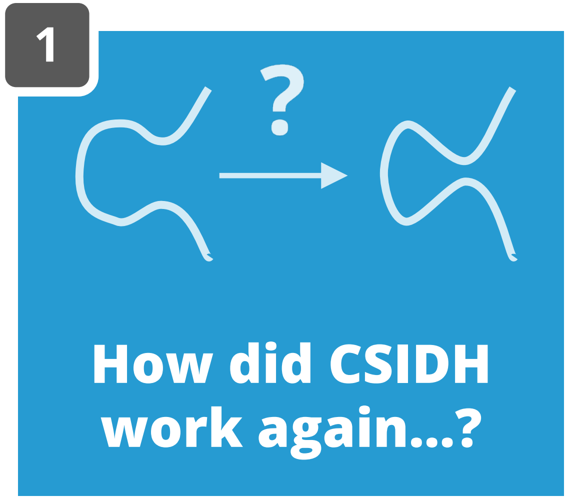


$(+1, 0, -0, +1, 0, 0, -0)$



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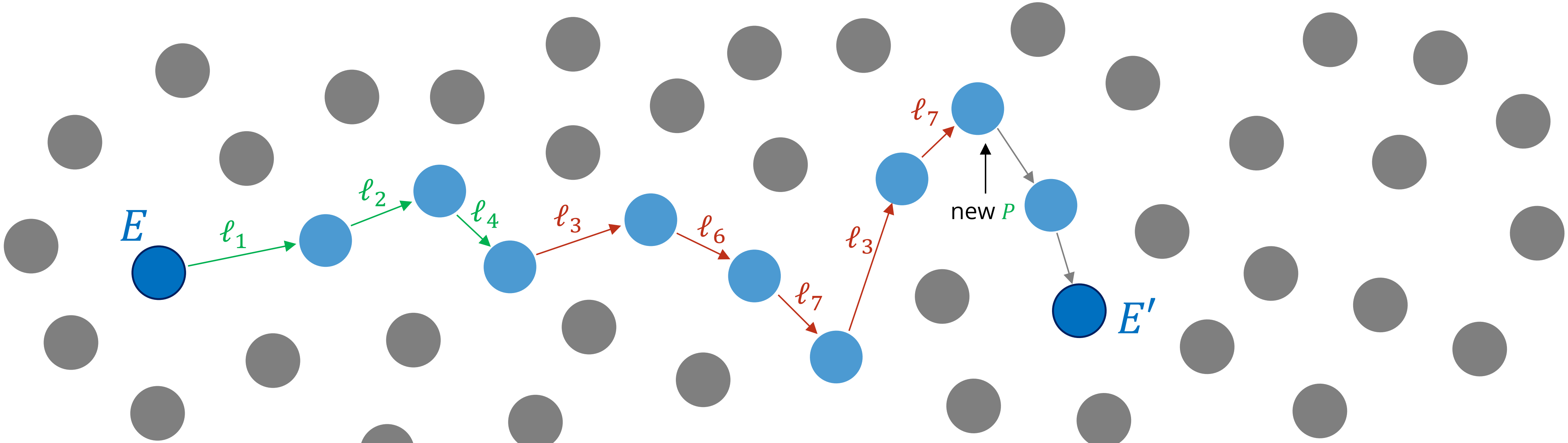
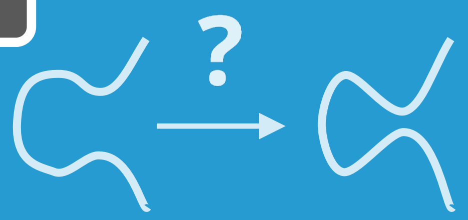


$(+1, 0, 0, +1, 0, 0, 0)$

1



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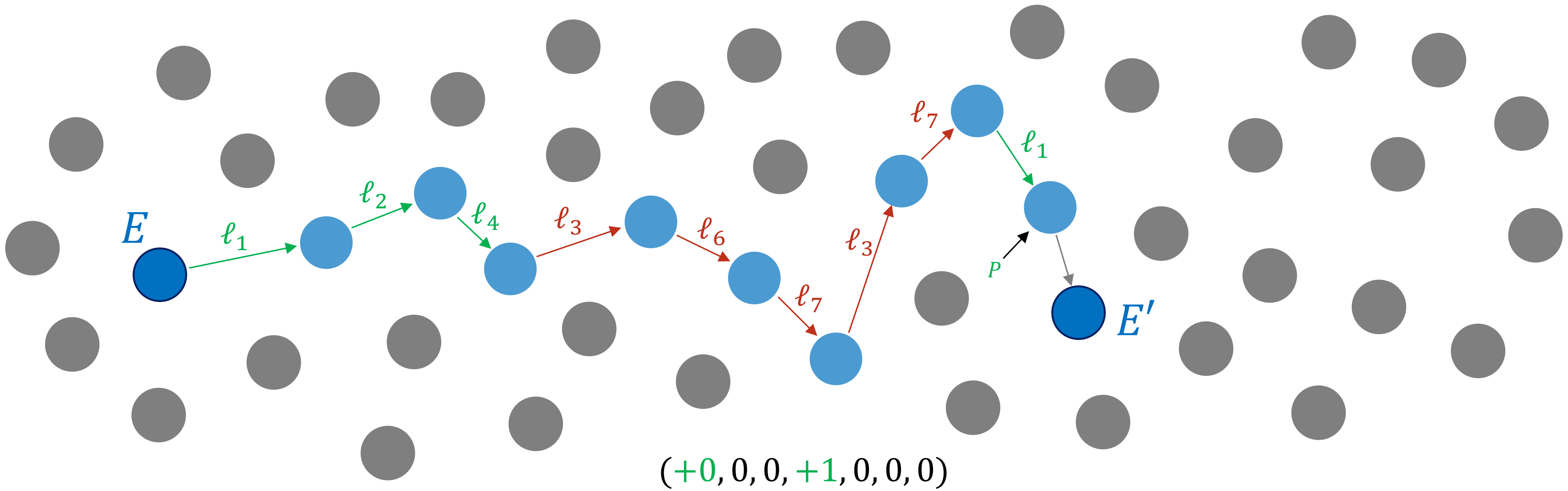
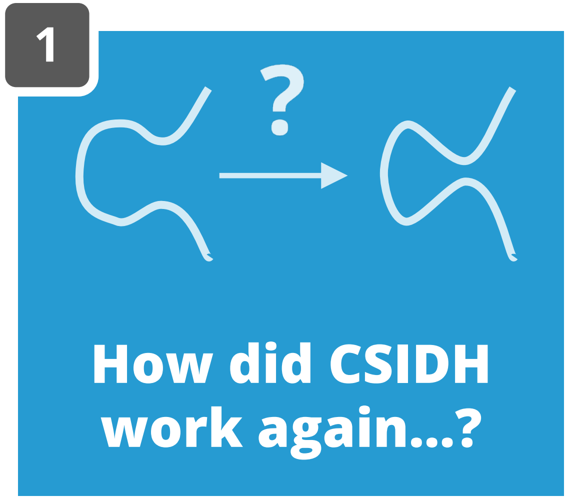


$(+0, 0, 0, +1, 0, 0, 0)$



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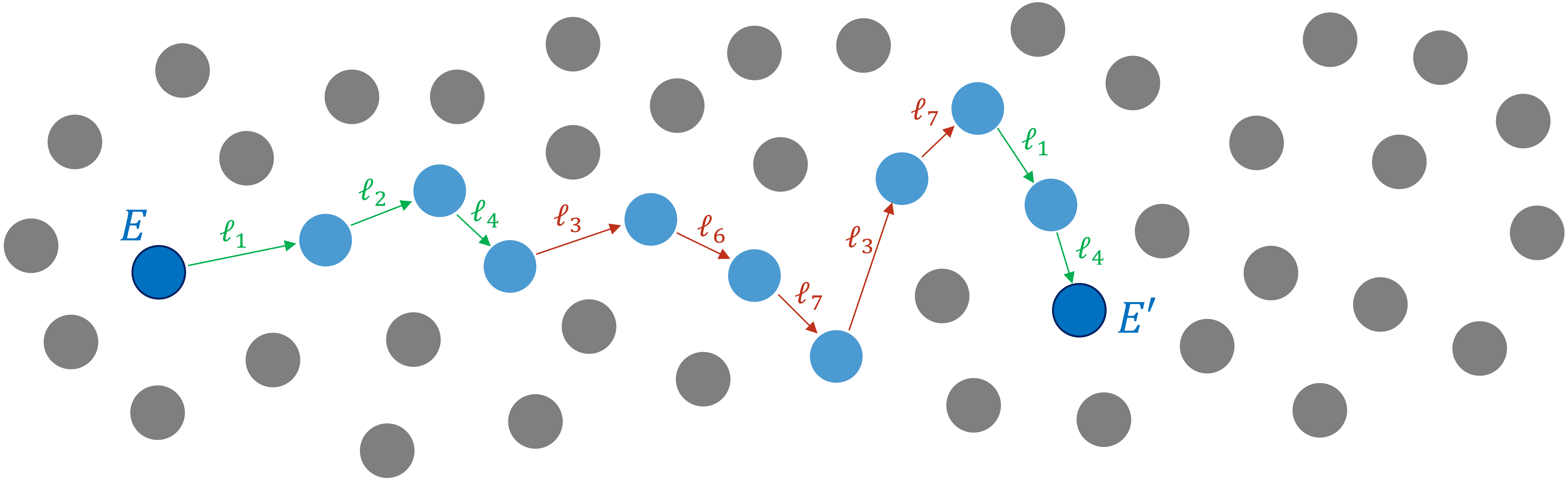

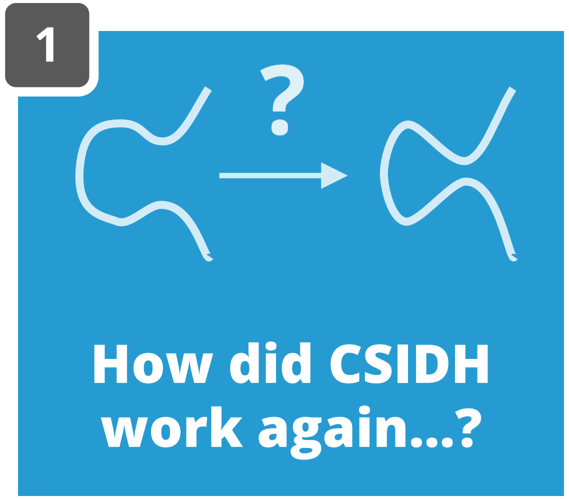

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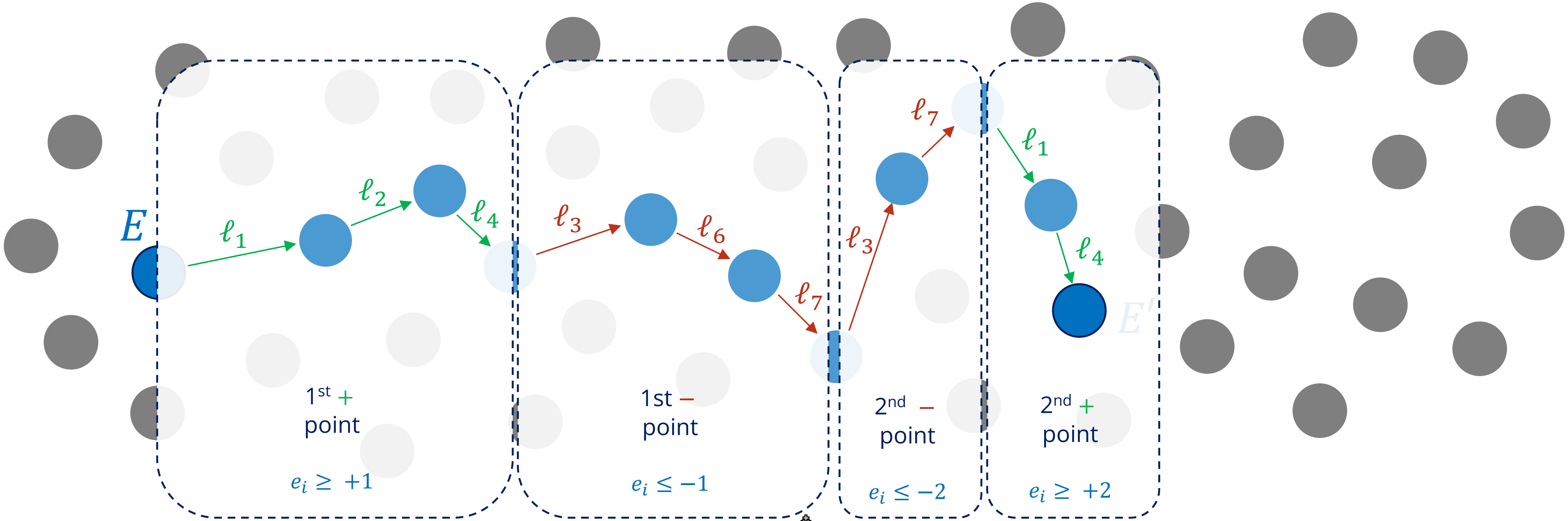
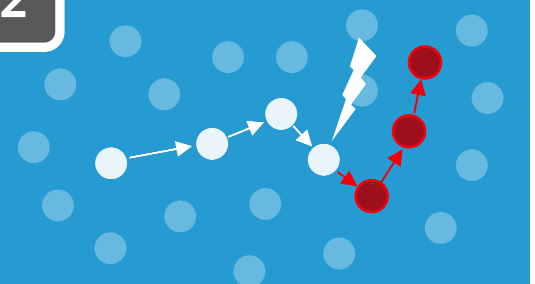


FAULT INJECTIONS

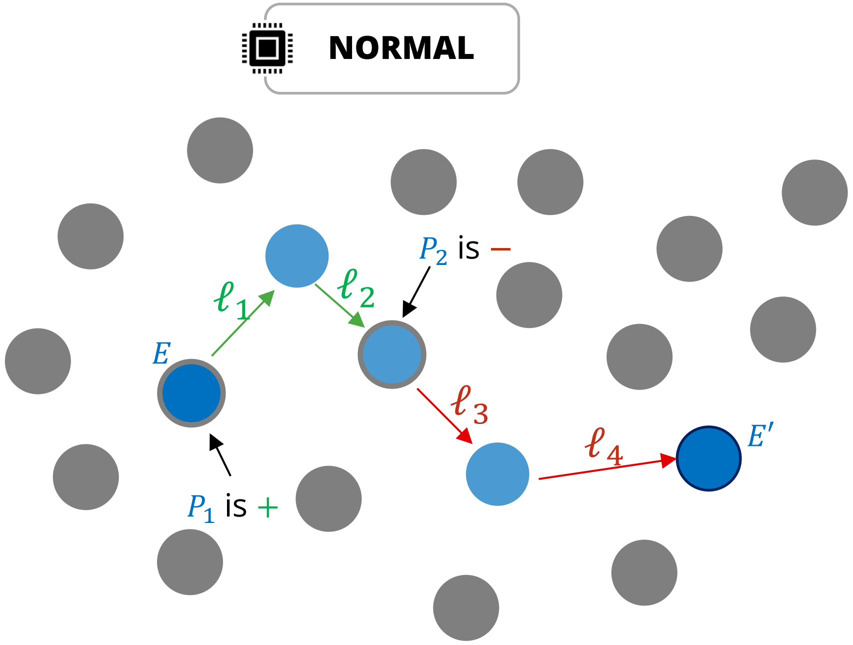
Or: How I Learned to Stop Worrying and Love the Laser

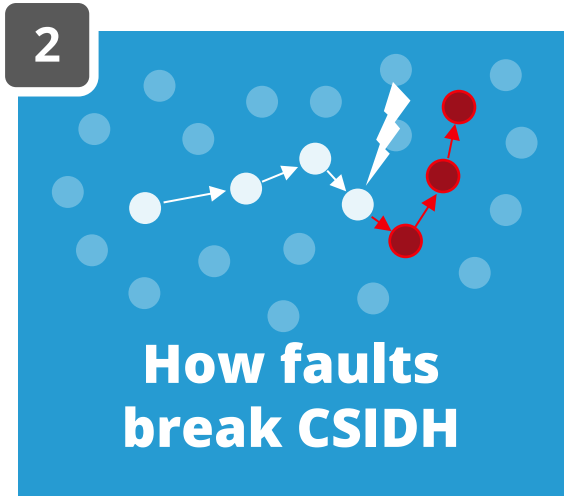


How faults break CSIDH

Toy example

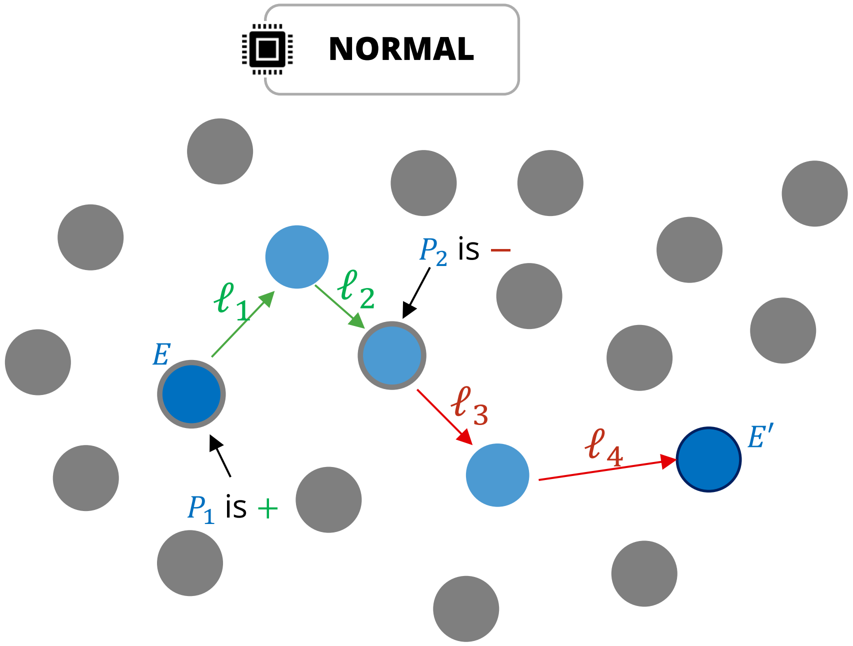
- Let's say $E \rightarrow E'$ is path $(+1, +1, -1, -1, 0, 0, 0)$

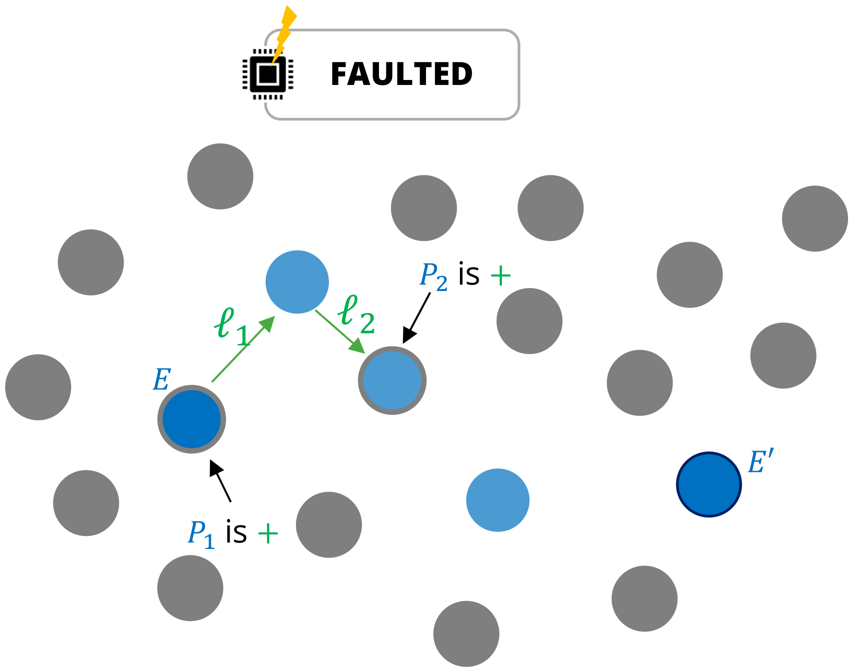




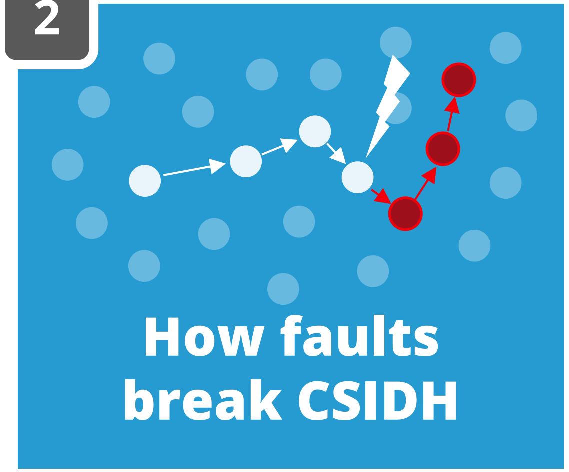
Toy example

- Let's say $E \rightarrow E'$ is path $(+1, +1, -1, -1, 0, 0, 0)$
- we sample a second positive point
- but fault inject so device thinks its negative





2



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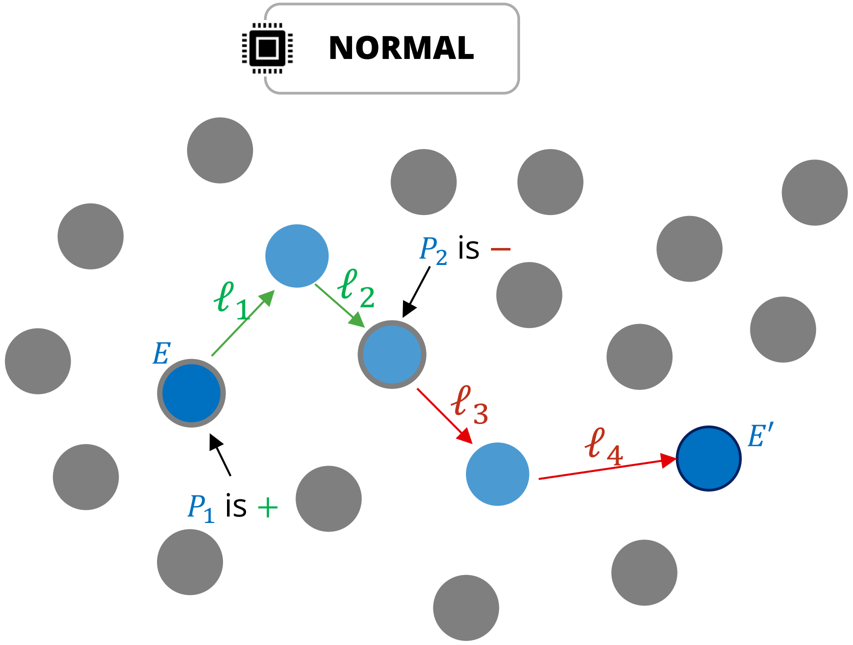

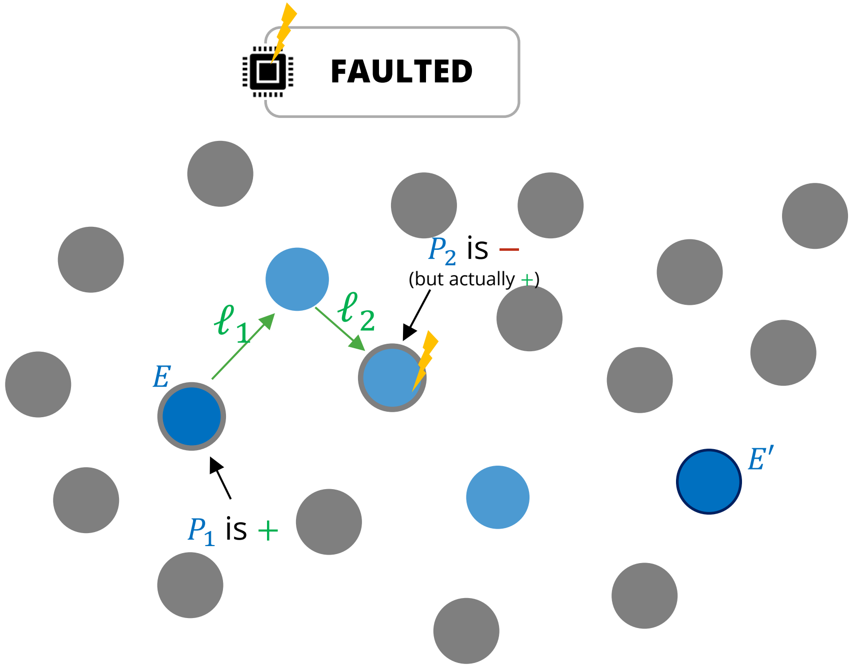

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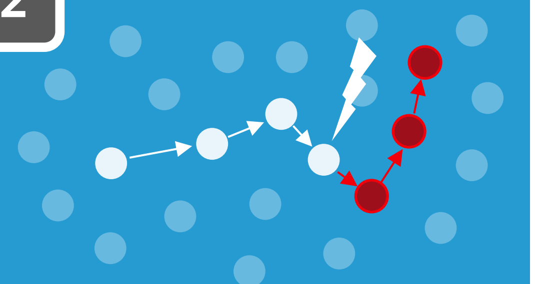

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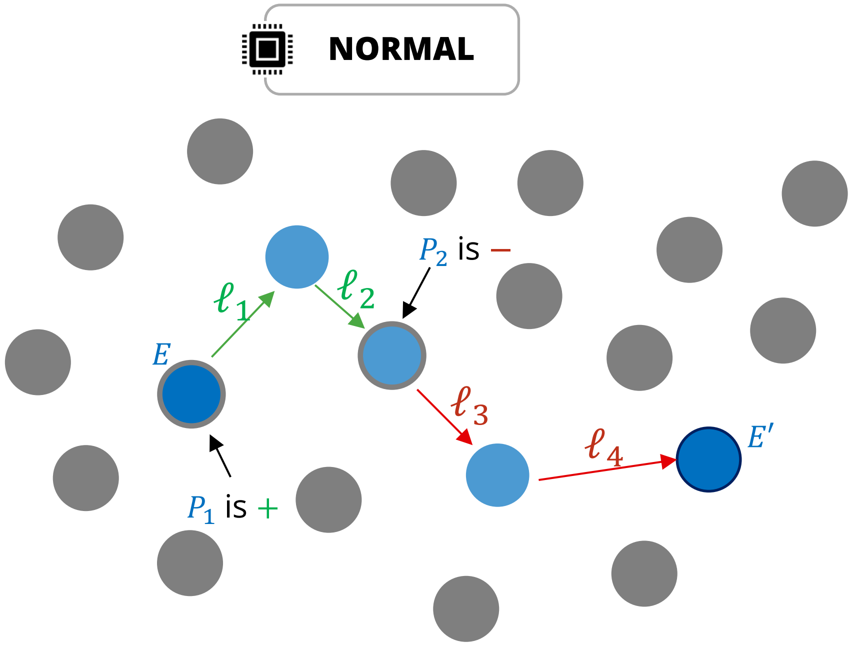

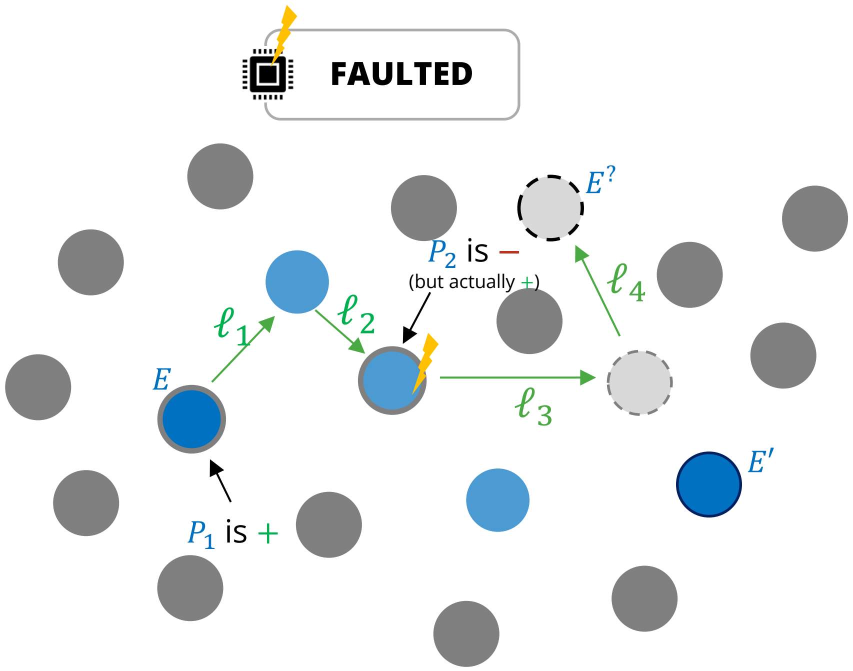

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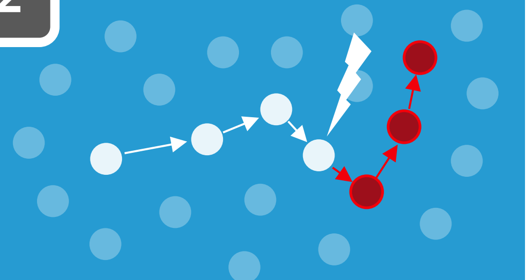

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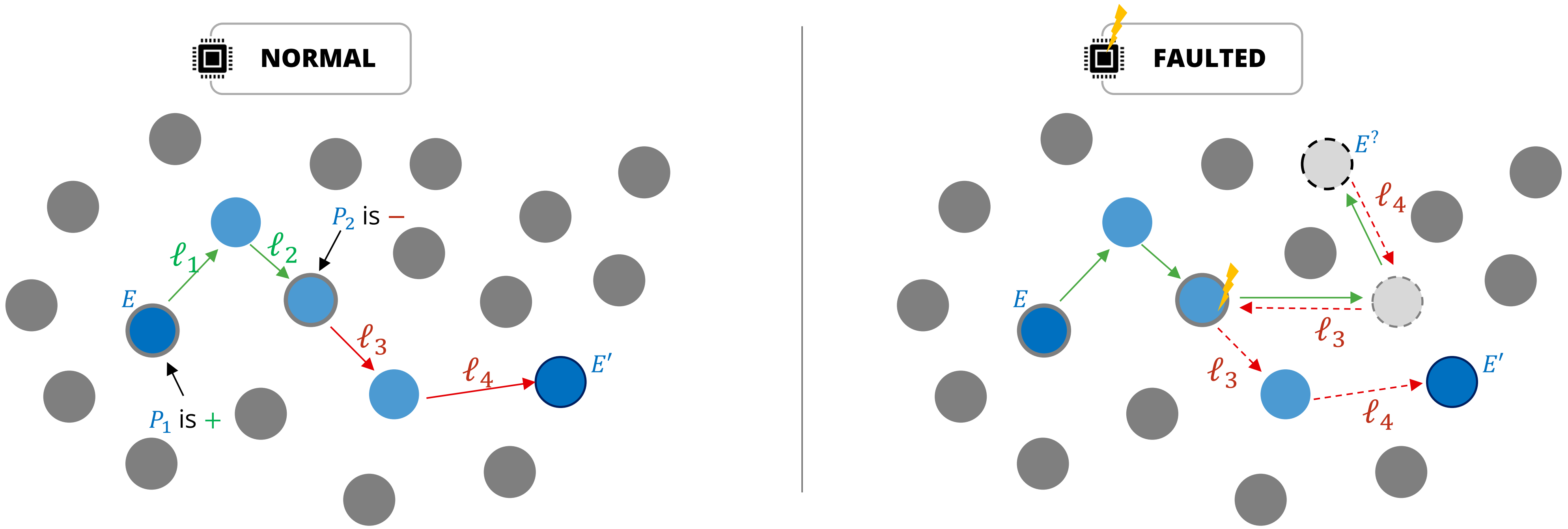

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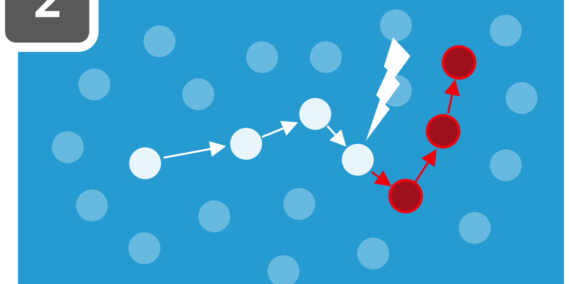

How faults break CSIDH

Toy example

- Let's say $E \rightarrow E'$ is path $(+1,+1,-1,-1,0,0,0)$
- we sample a second positive point
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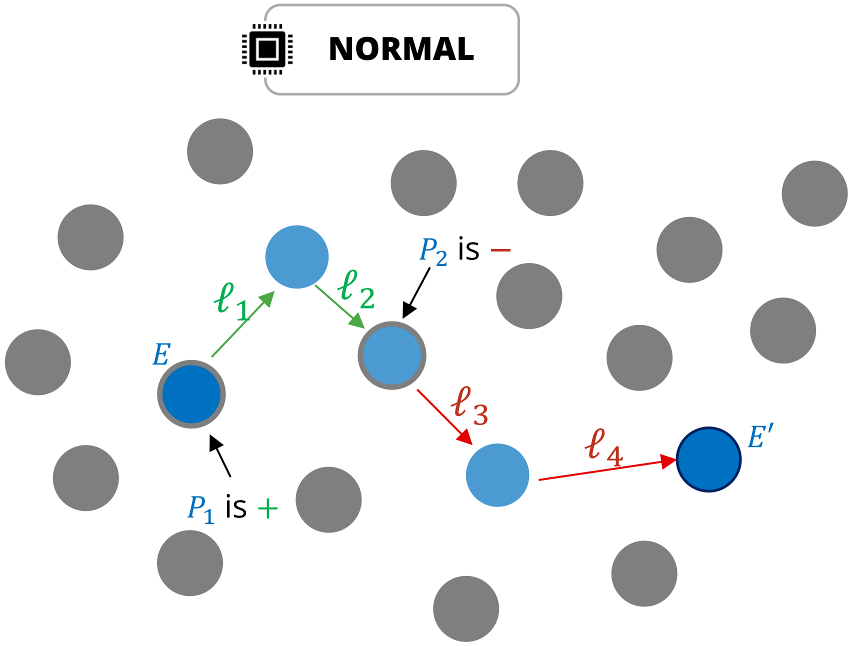
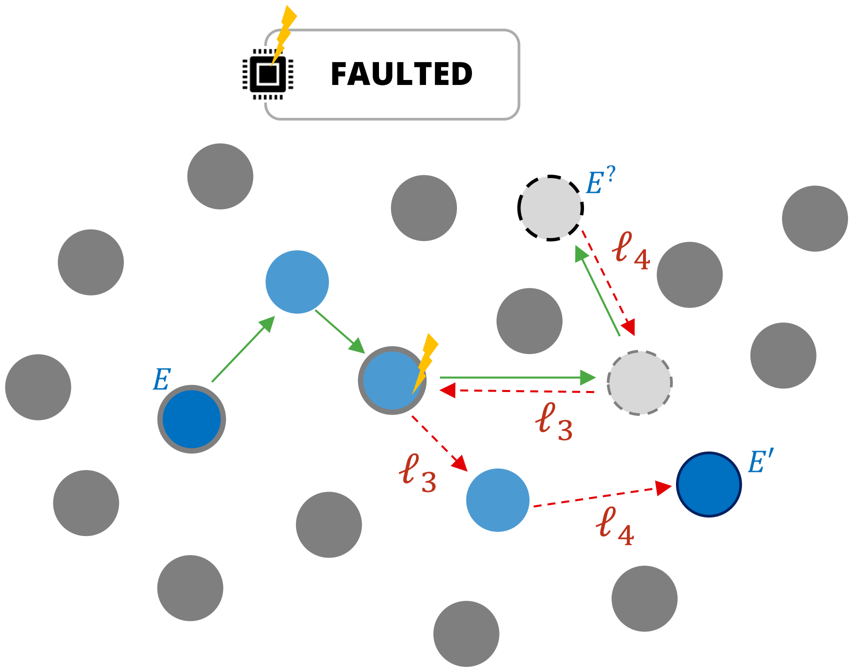
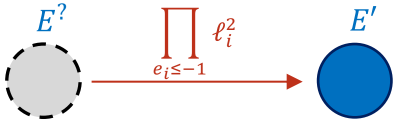
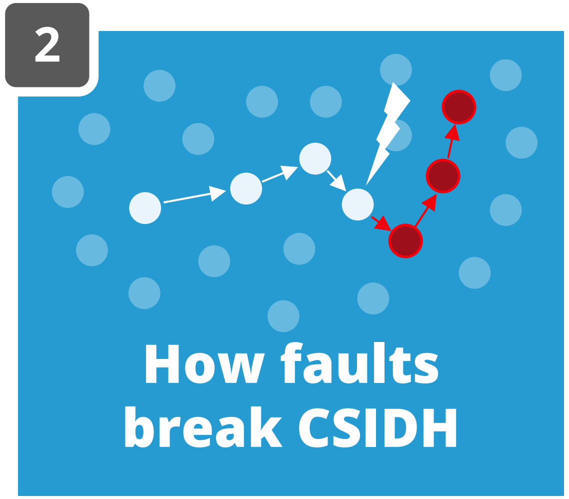




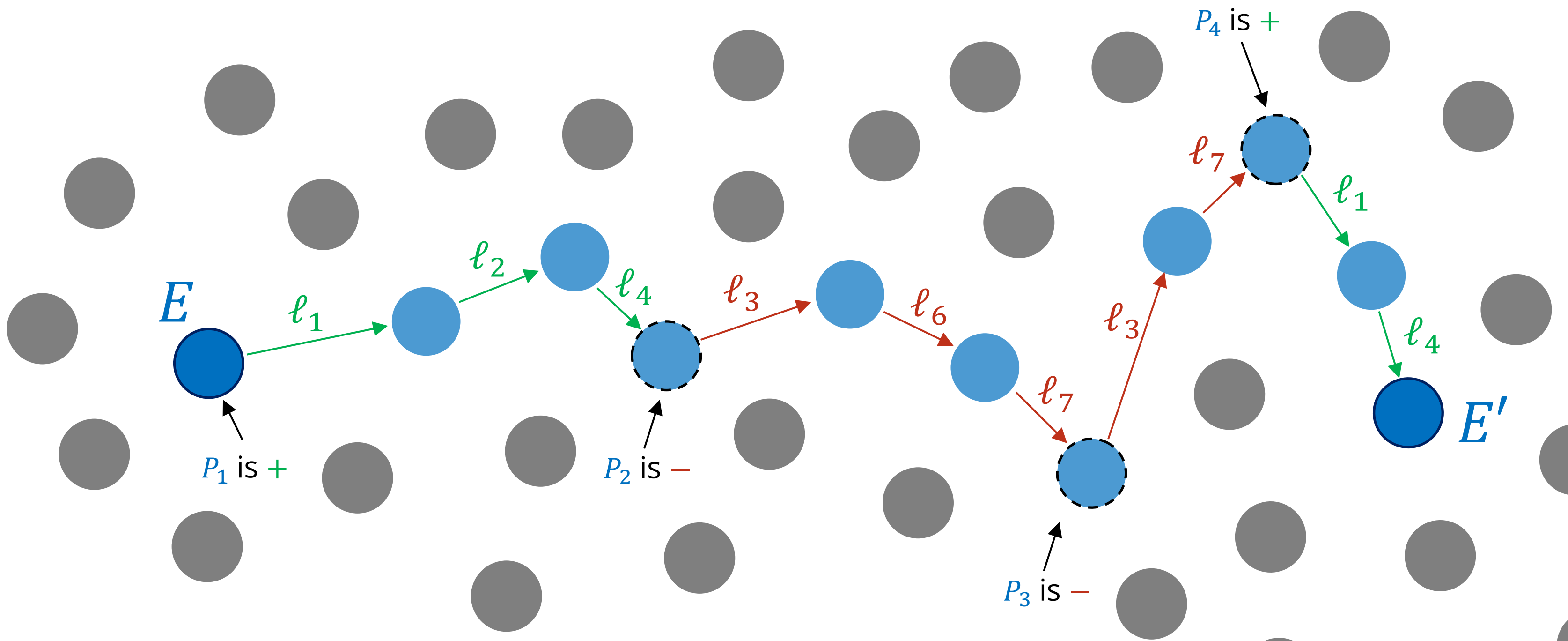
off by



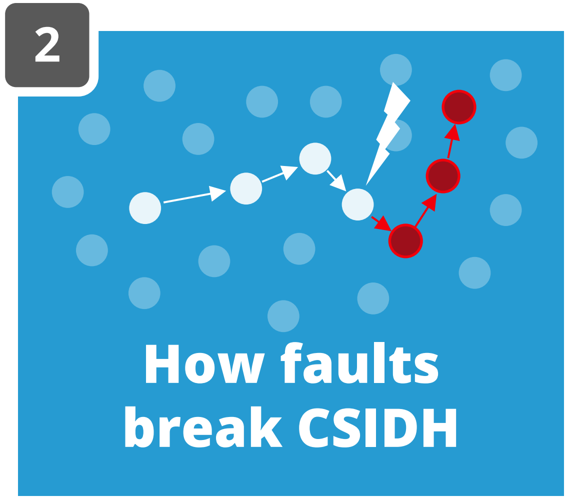


Back to example

- Path $E \rightarrow E'$ is $(+2, +1, -2, +2, 0, -1, -2)$
- What happens when we inject these points?

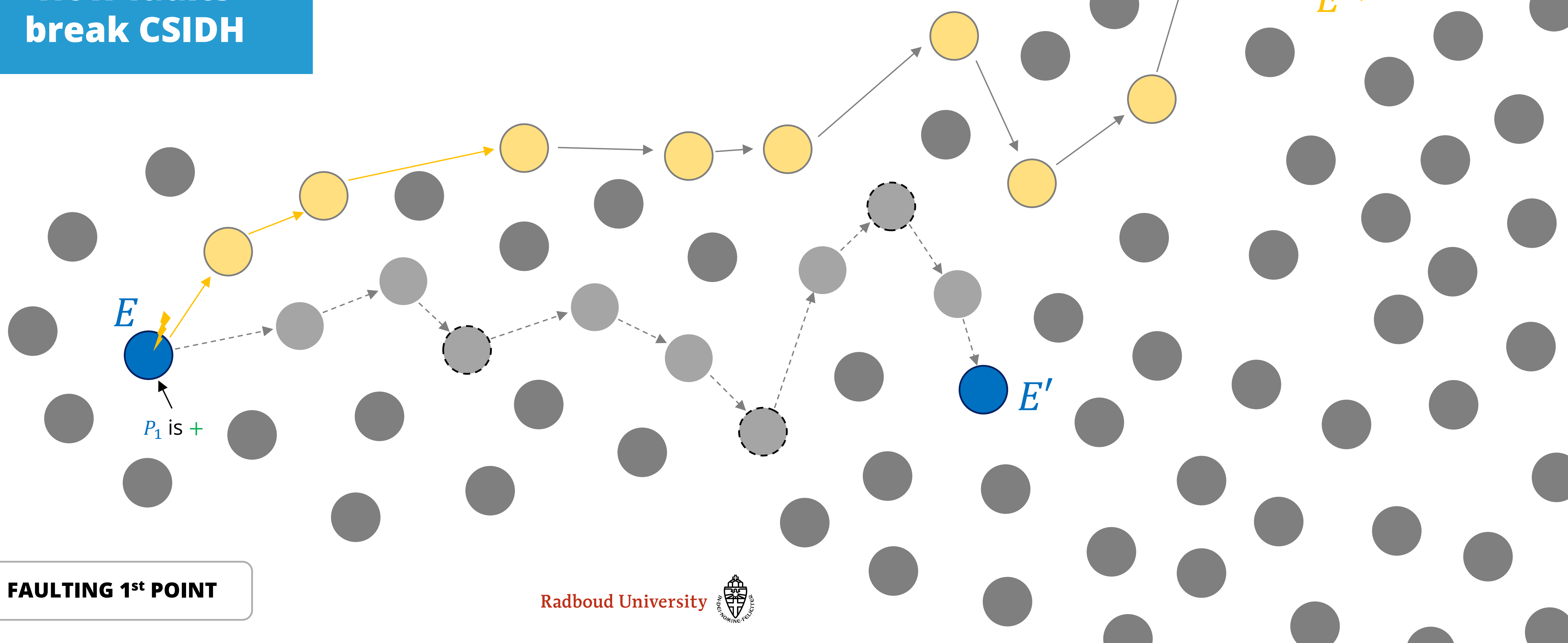


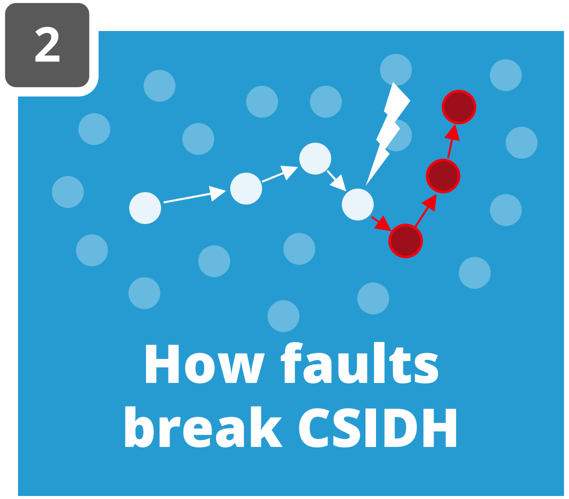
NORMAL



Back to example

- Path $E \rightarrow E'$ is $(+2, +1, -2, +2, 0, -1, -2)$
- What happens when we inject these points?





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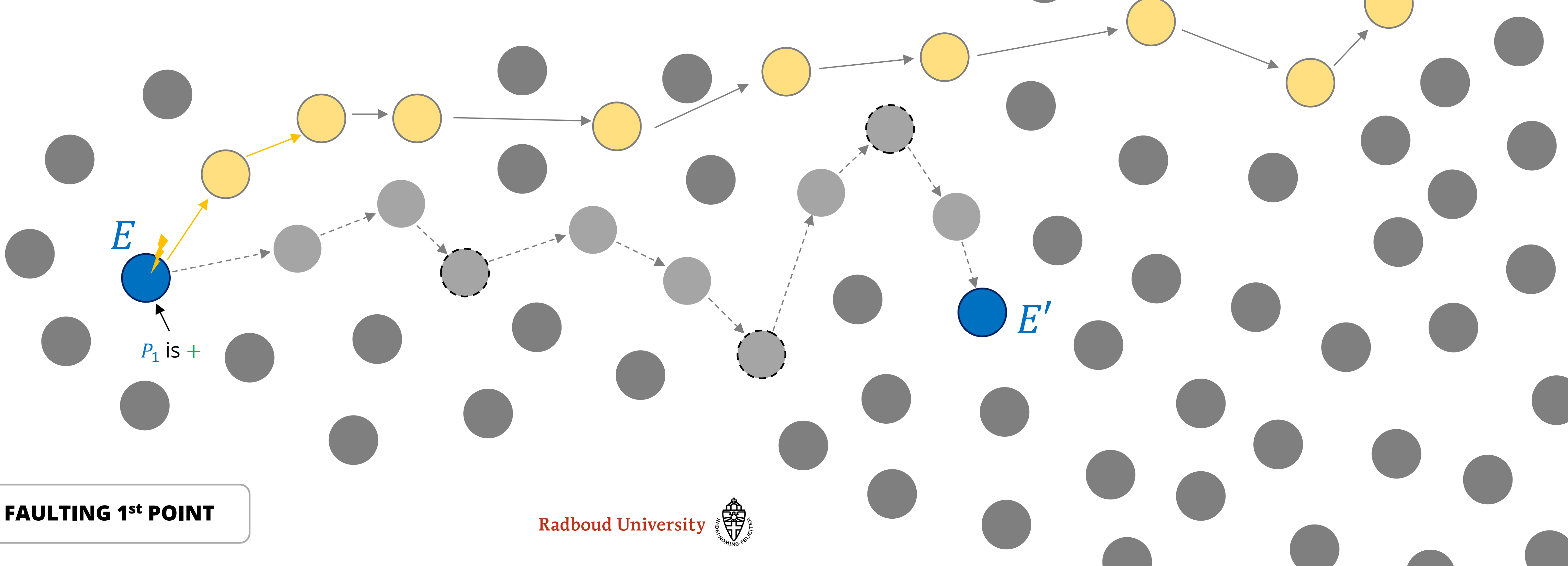

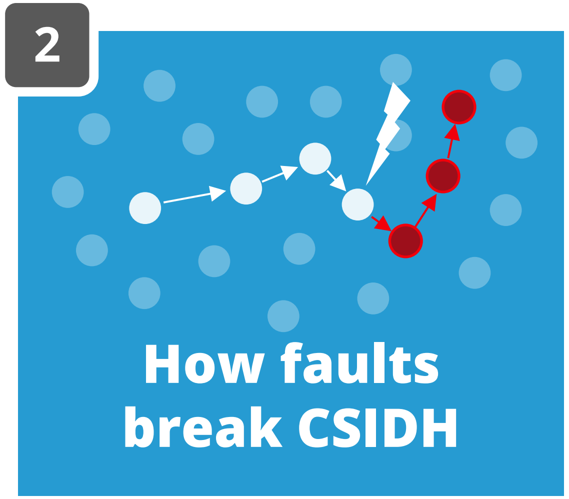

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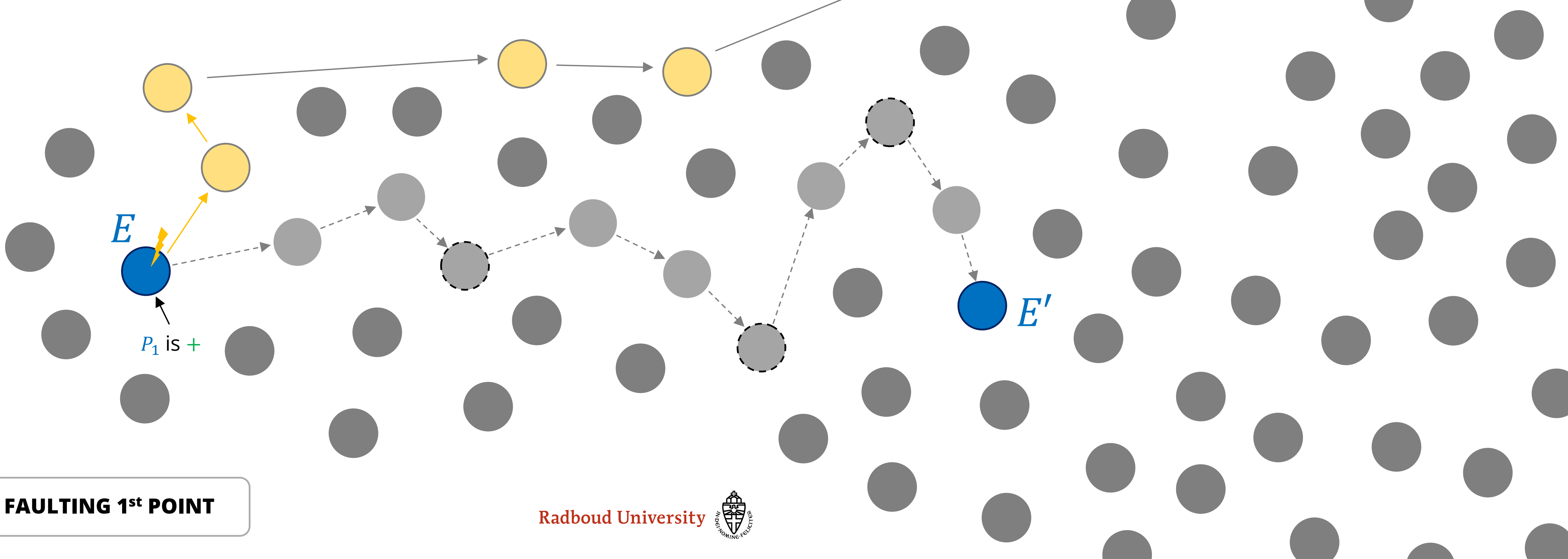

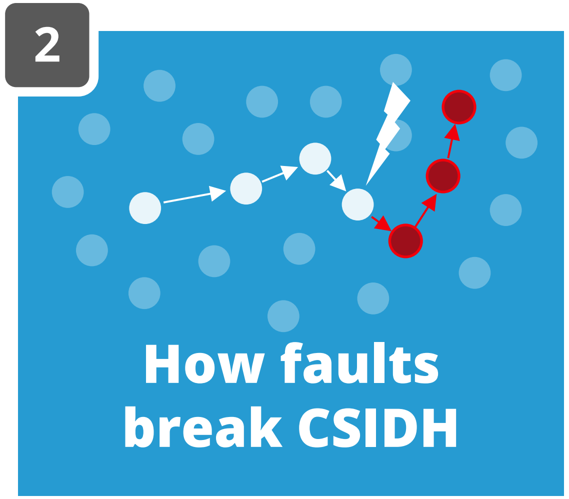

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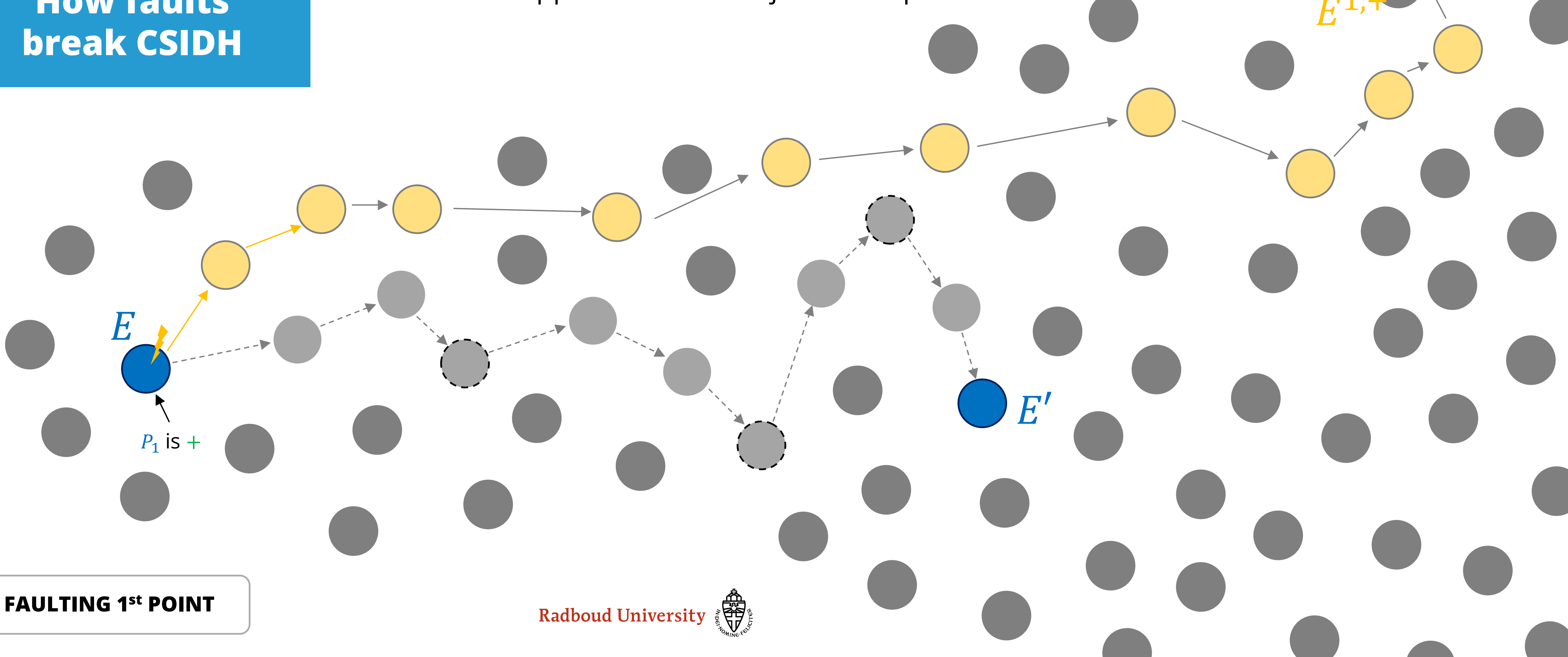
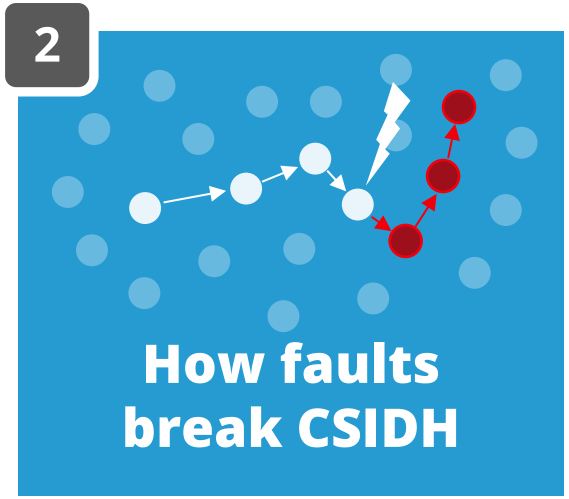


FAULTING 1st POINT



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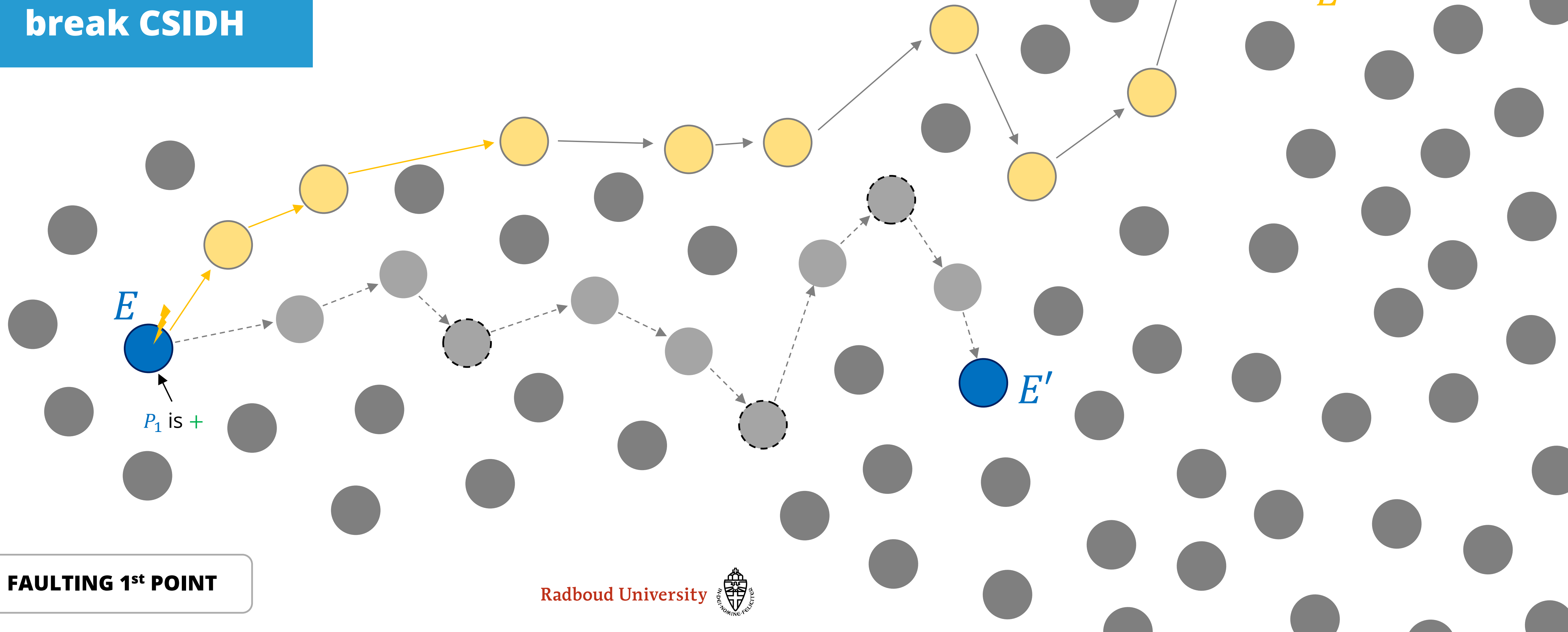

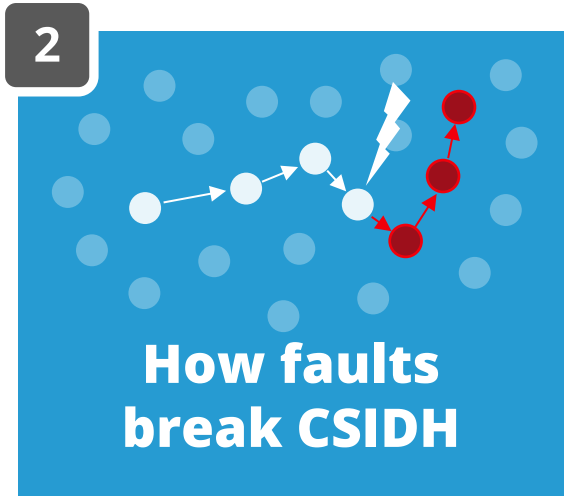

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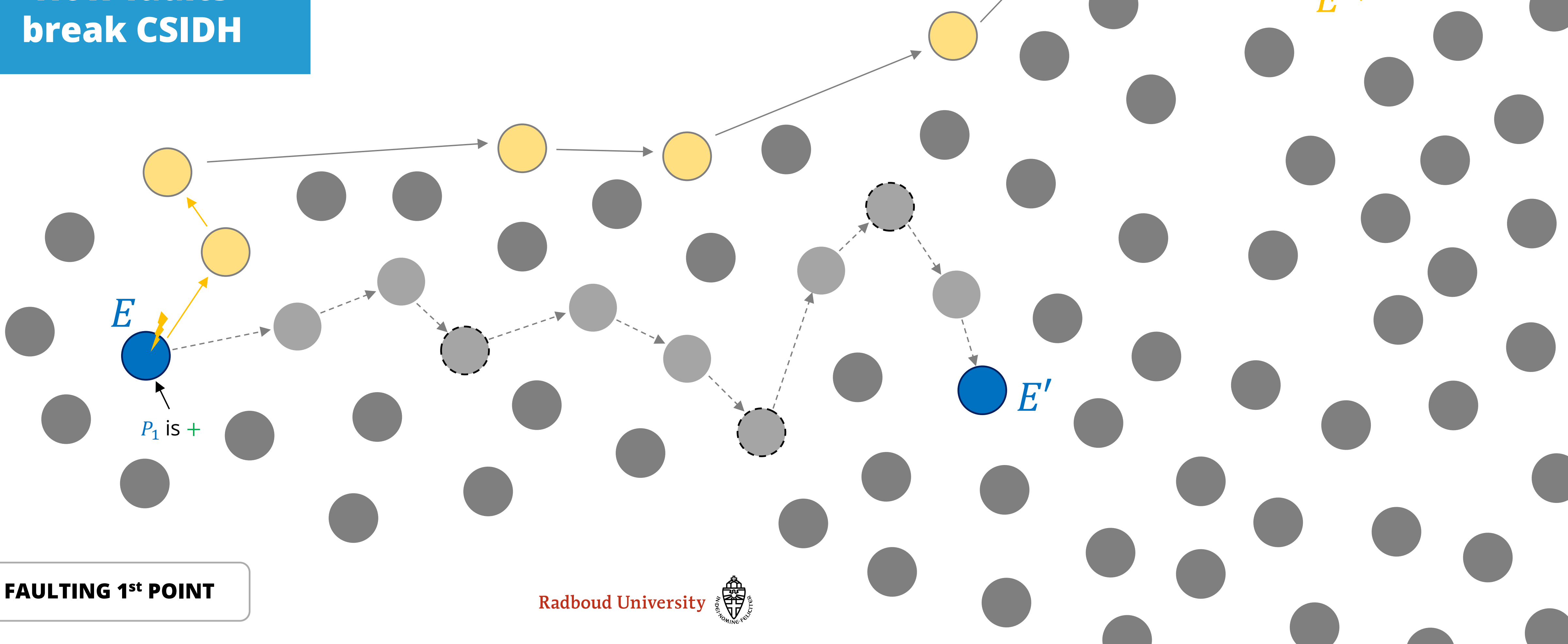

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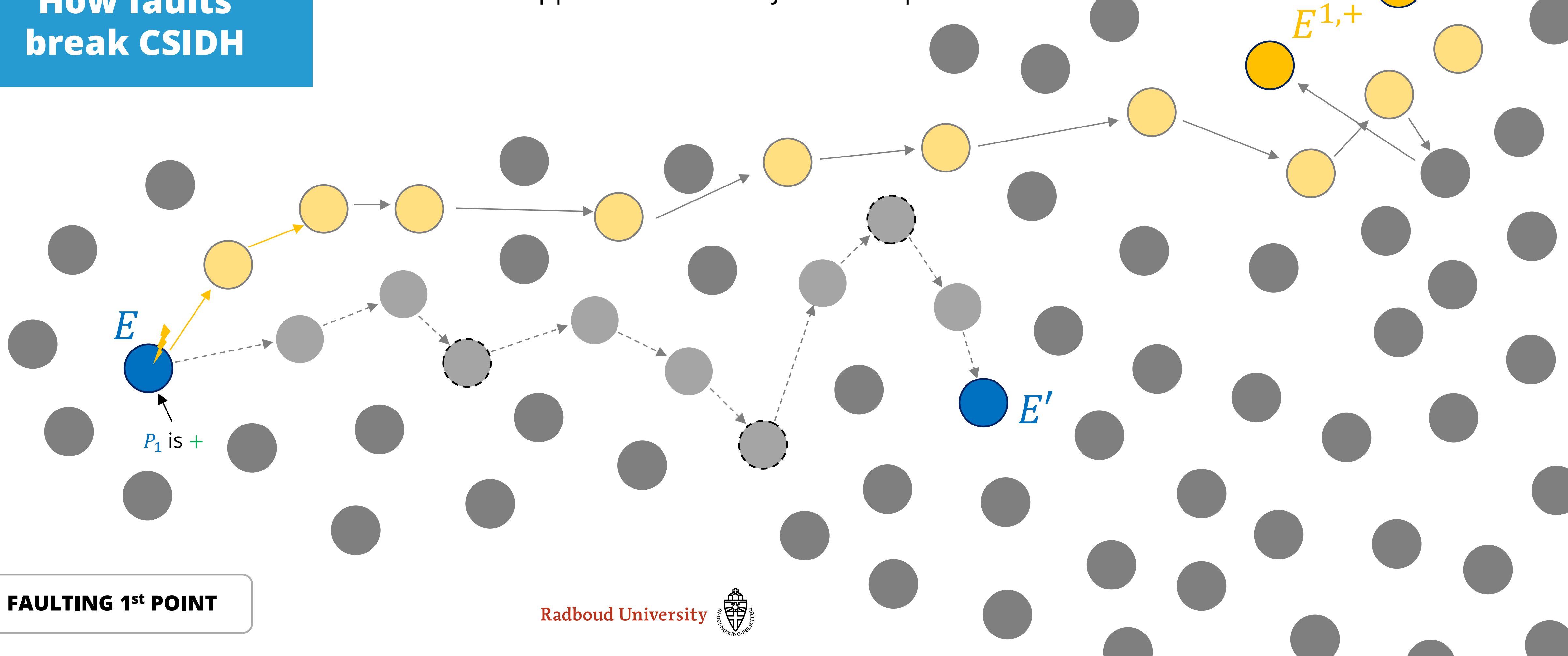

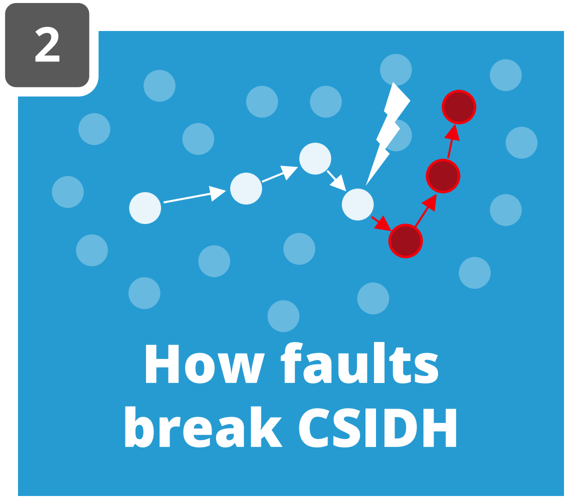

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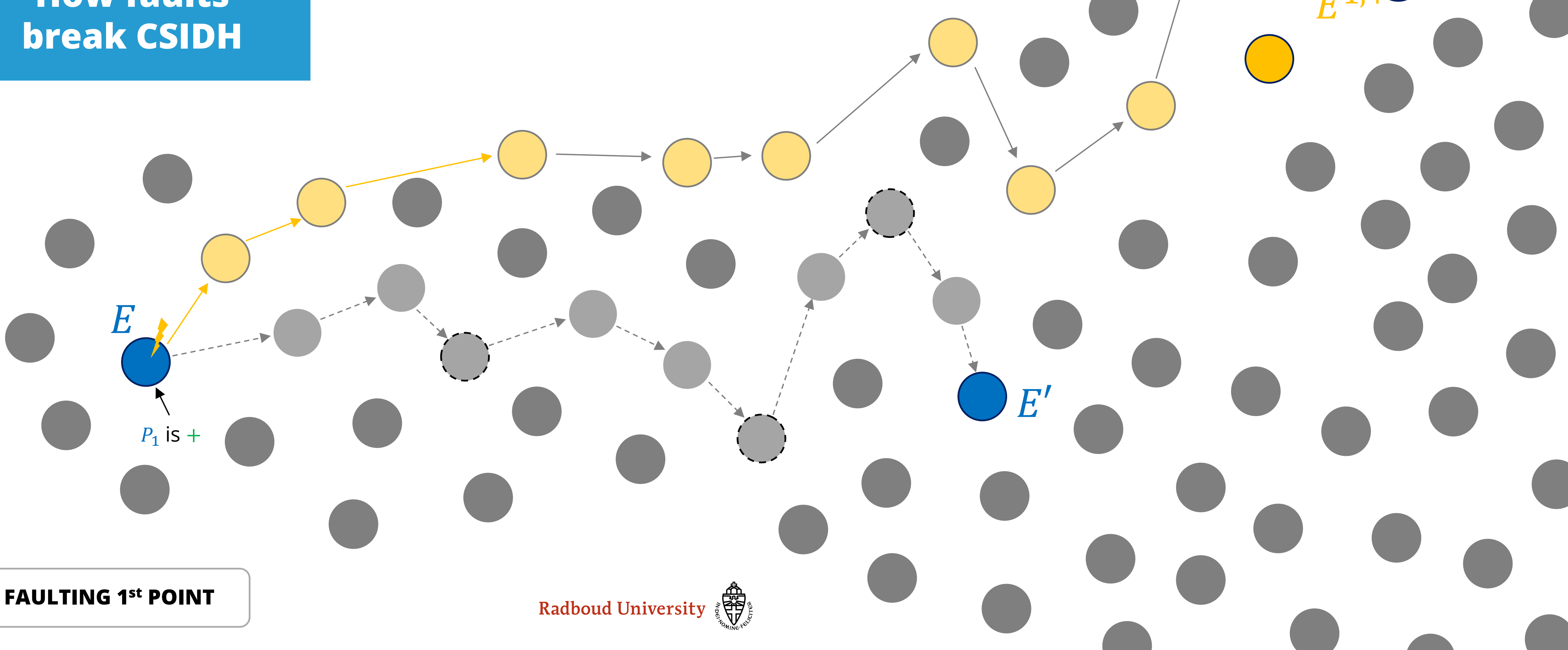

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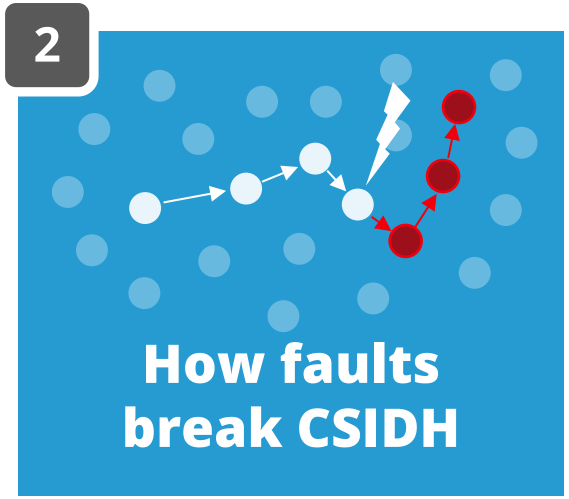

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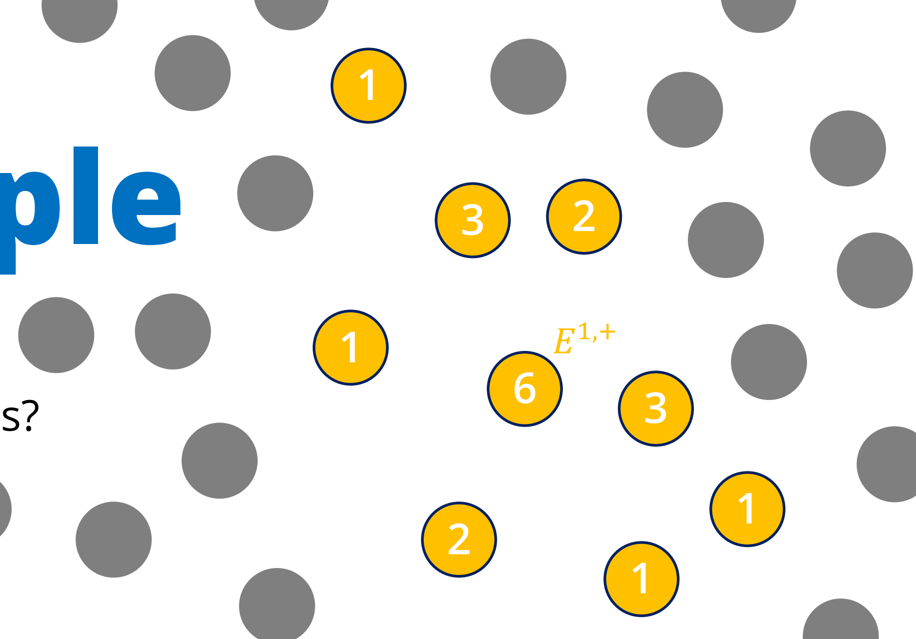

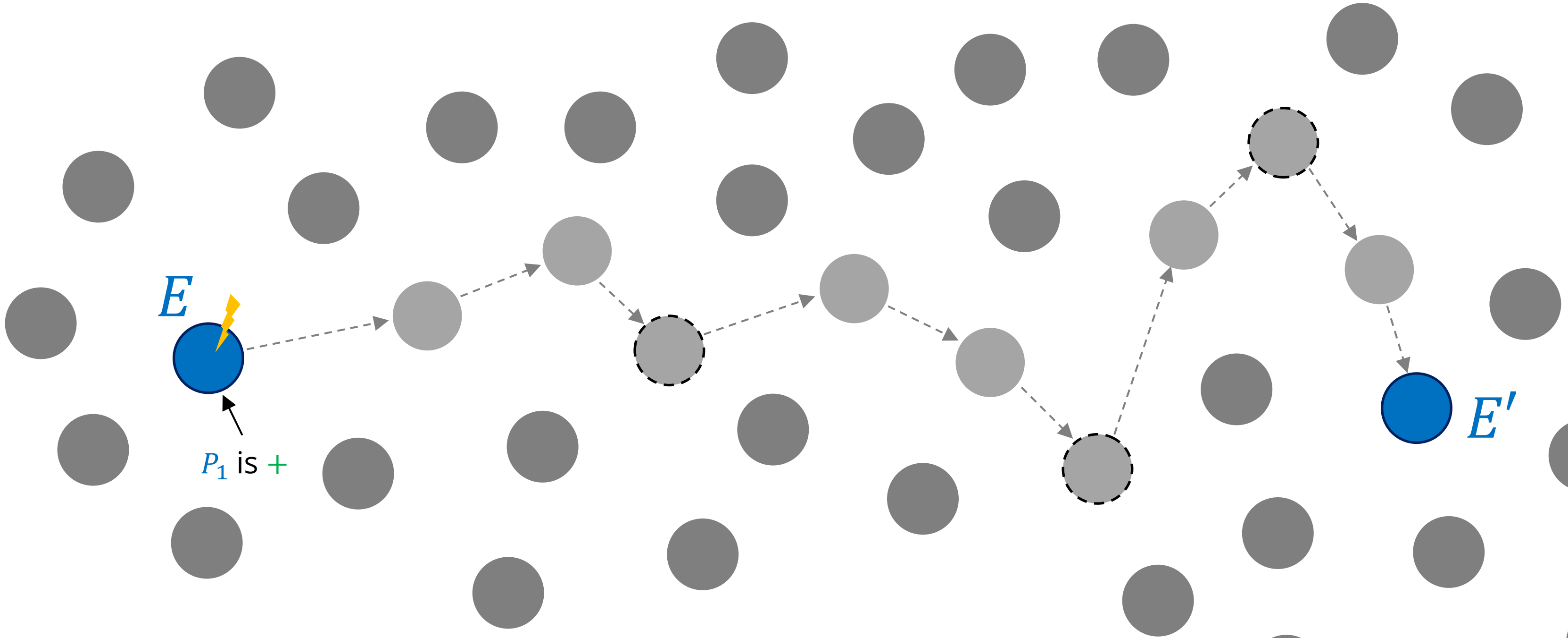

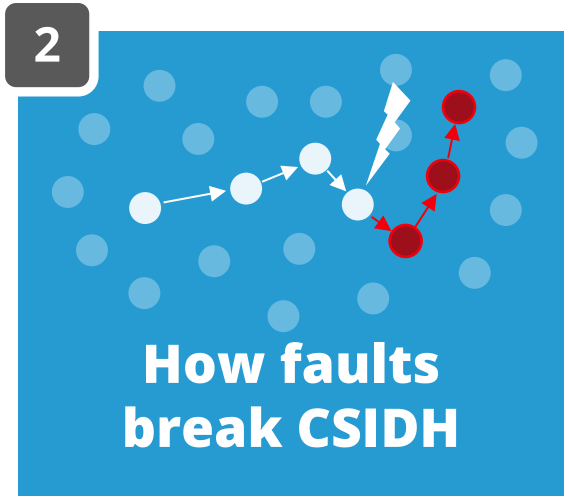

Back to example

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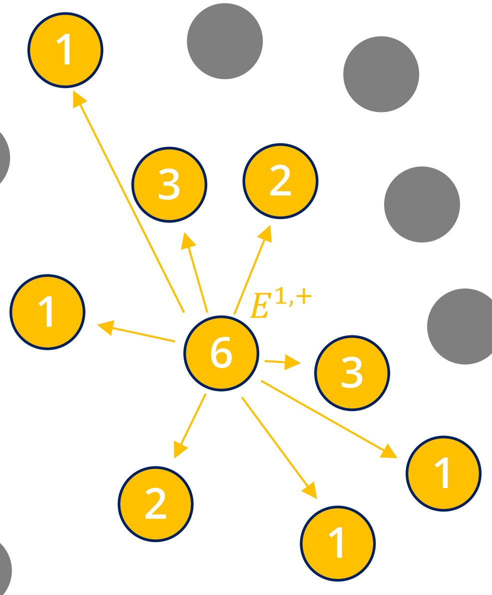
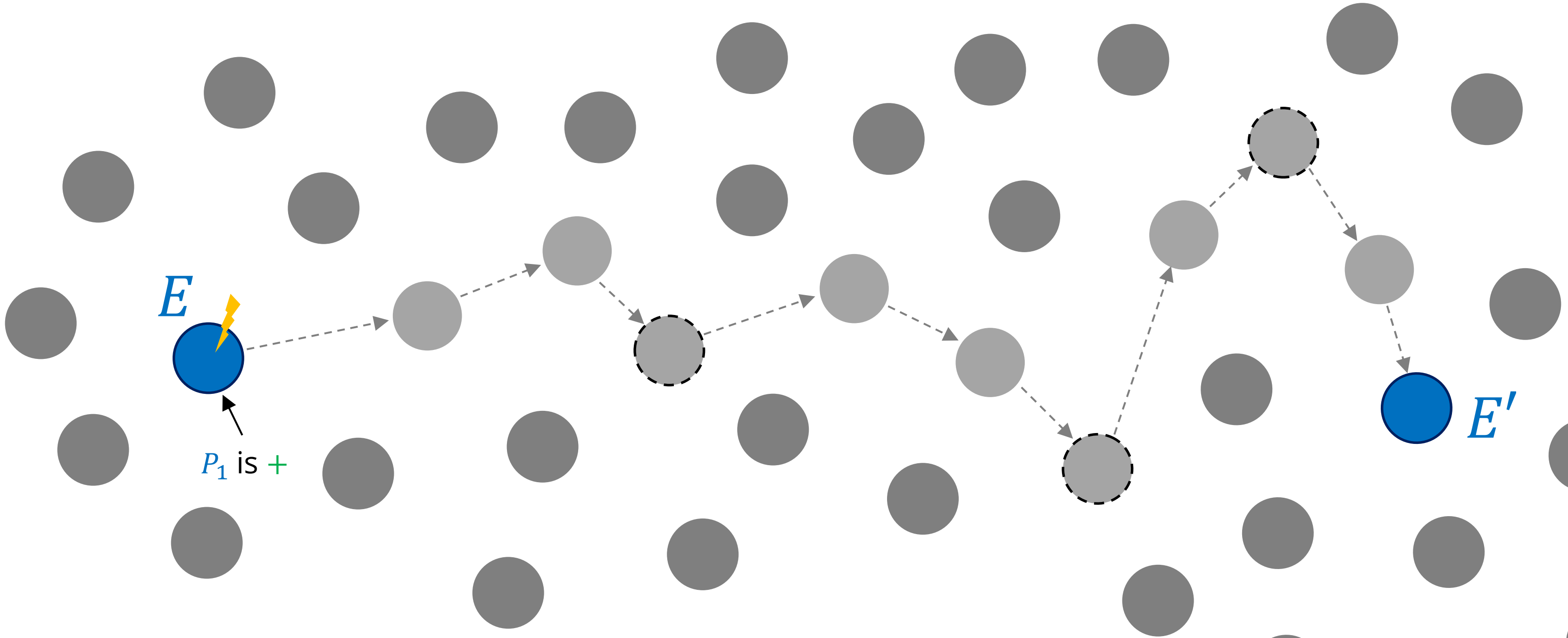
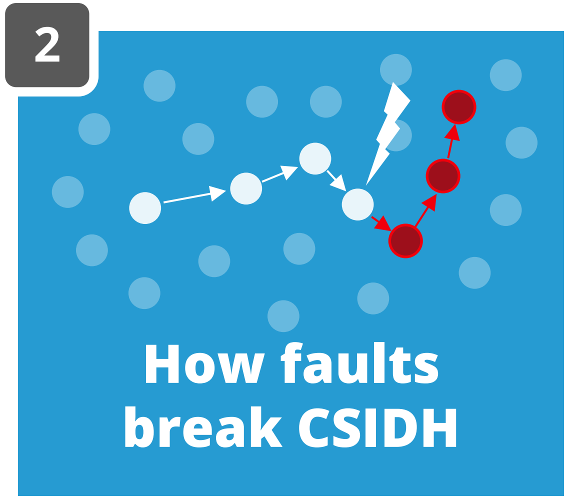




FAULTING 1st POINT



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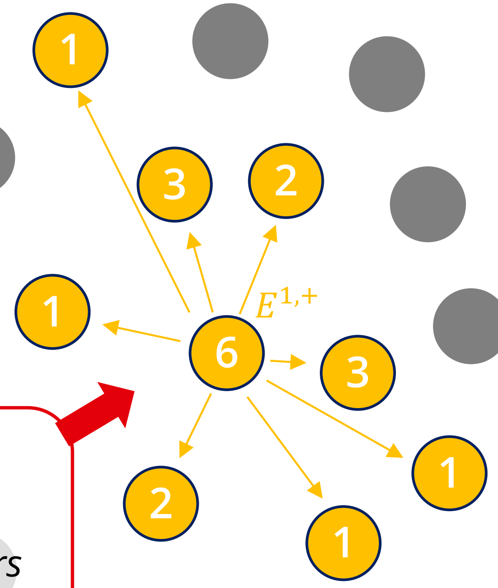
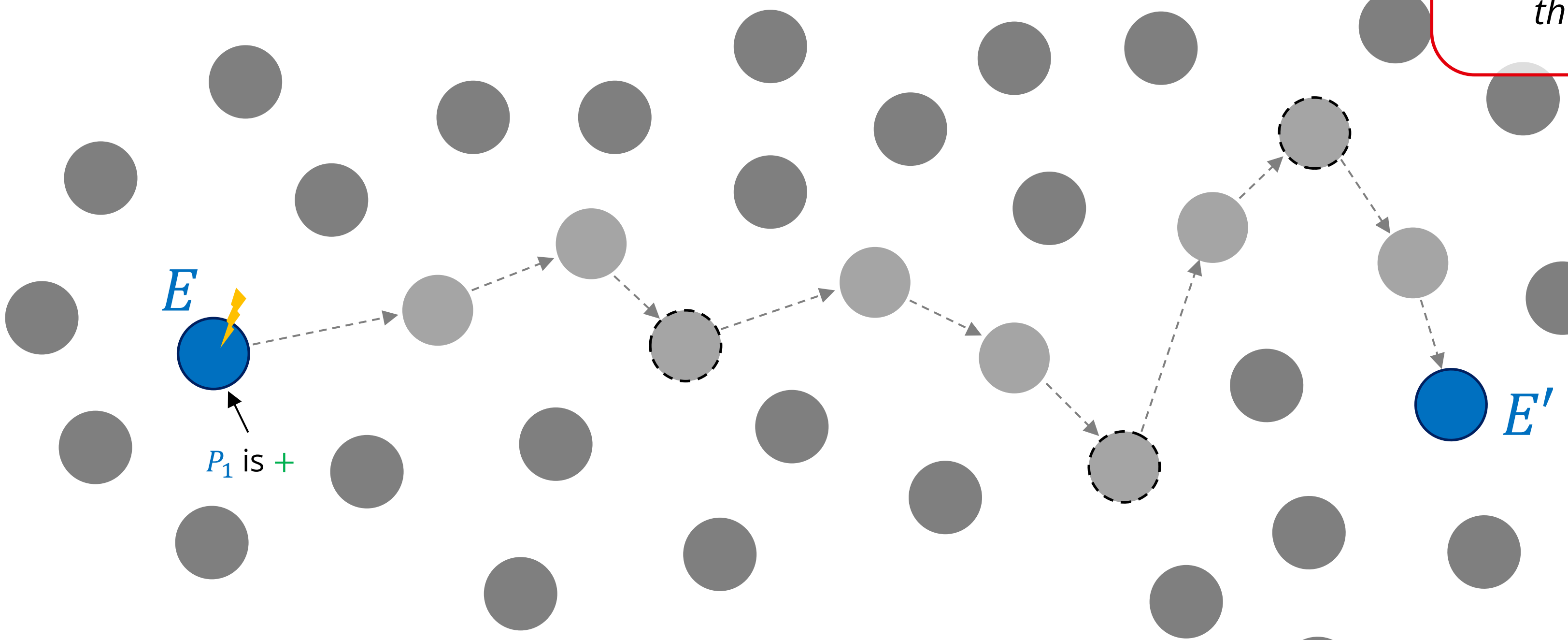


- *easy to find*
- *if* ℓ_i *appears then* $e_i \geq +1$



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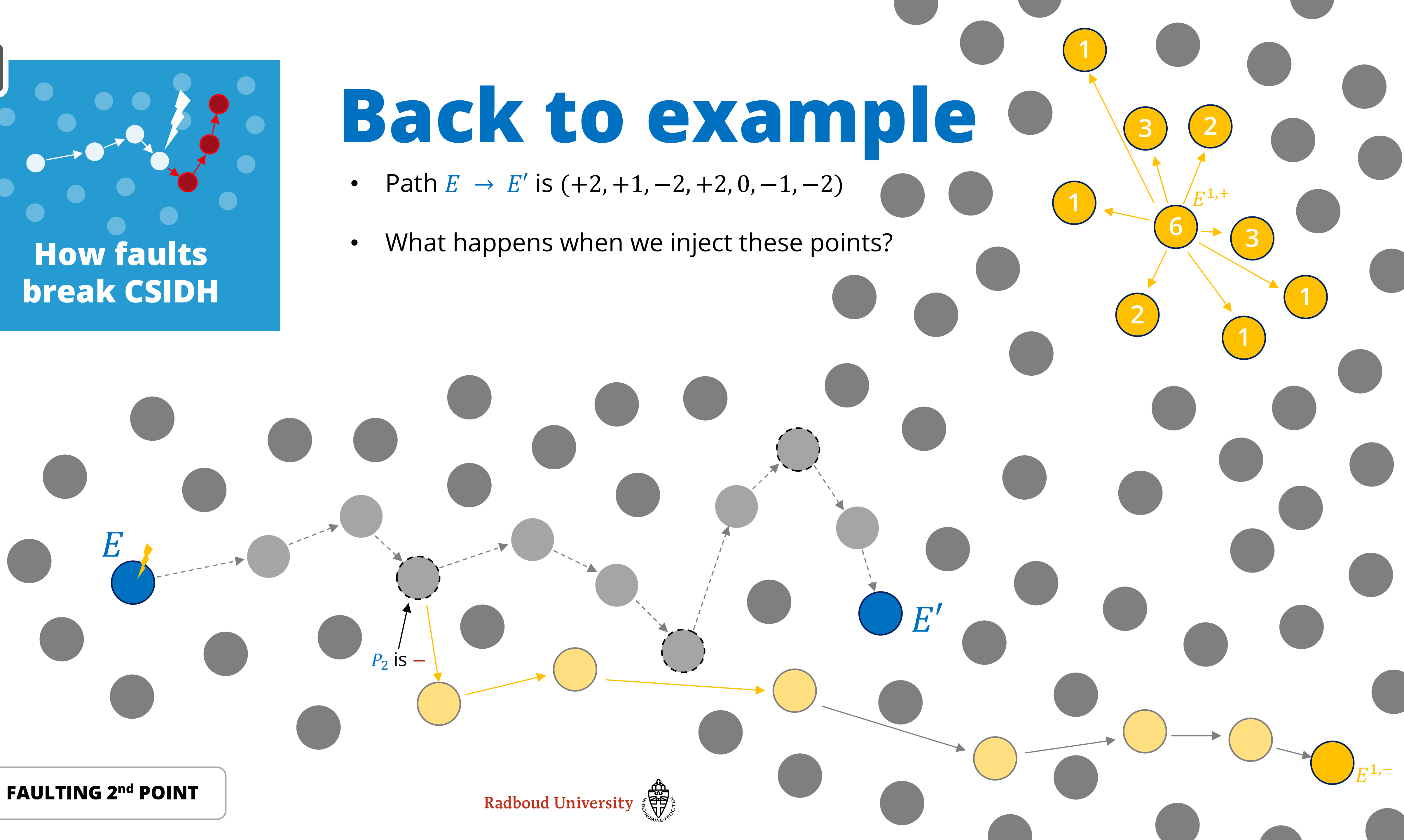

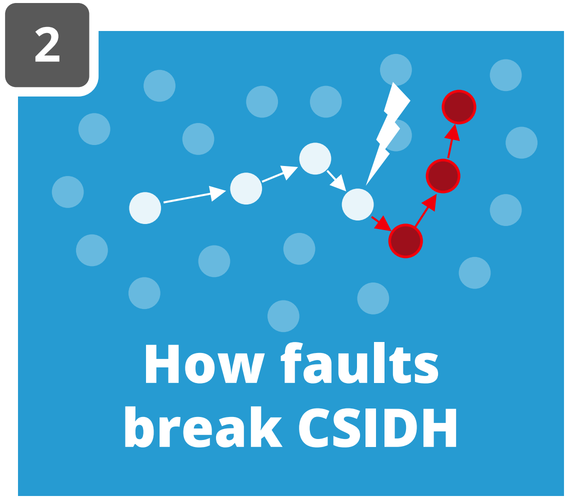

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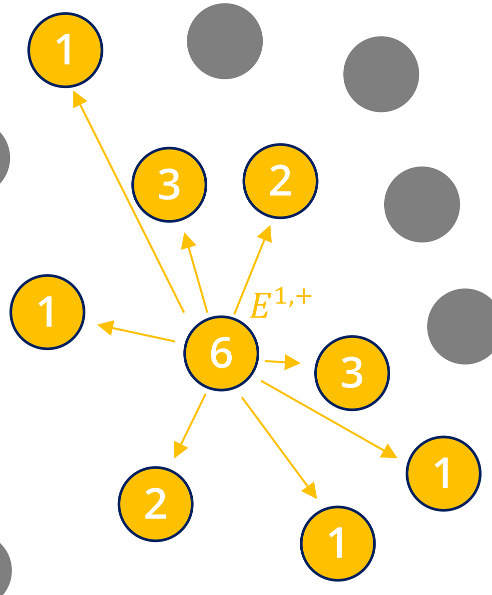

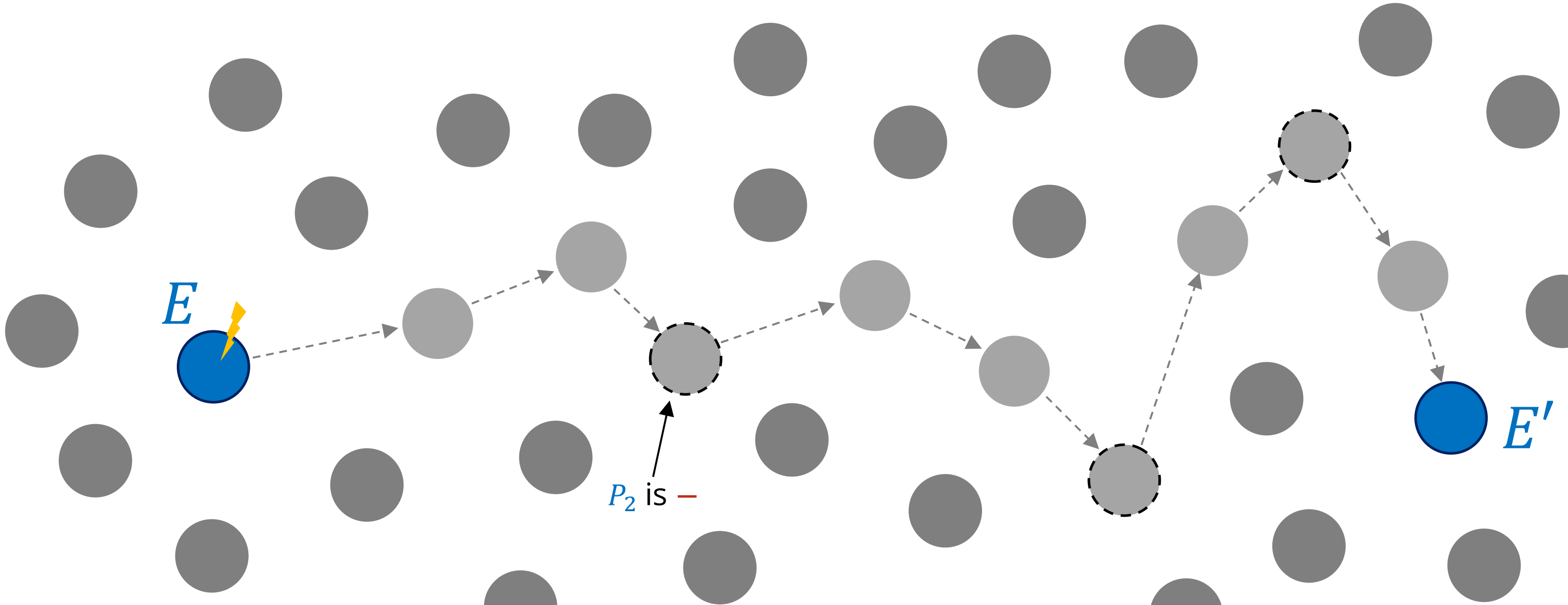

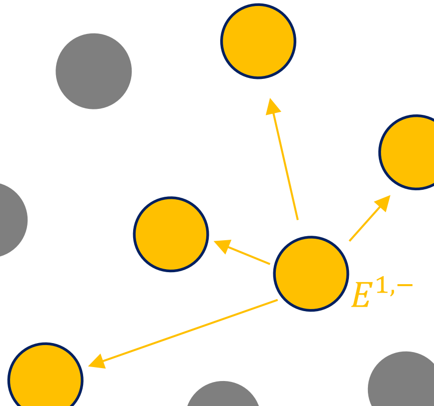

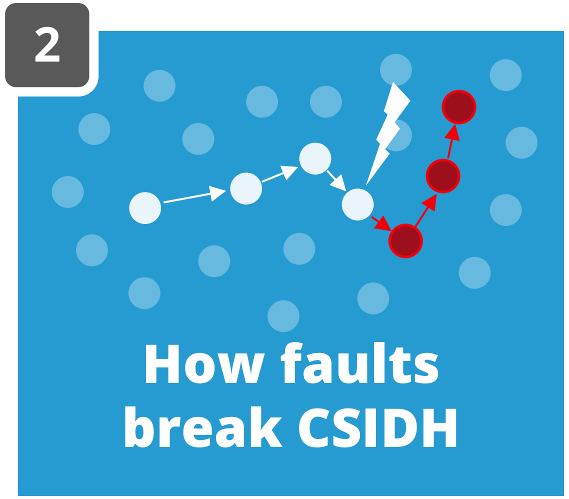

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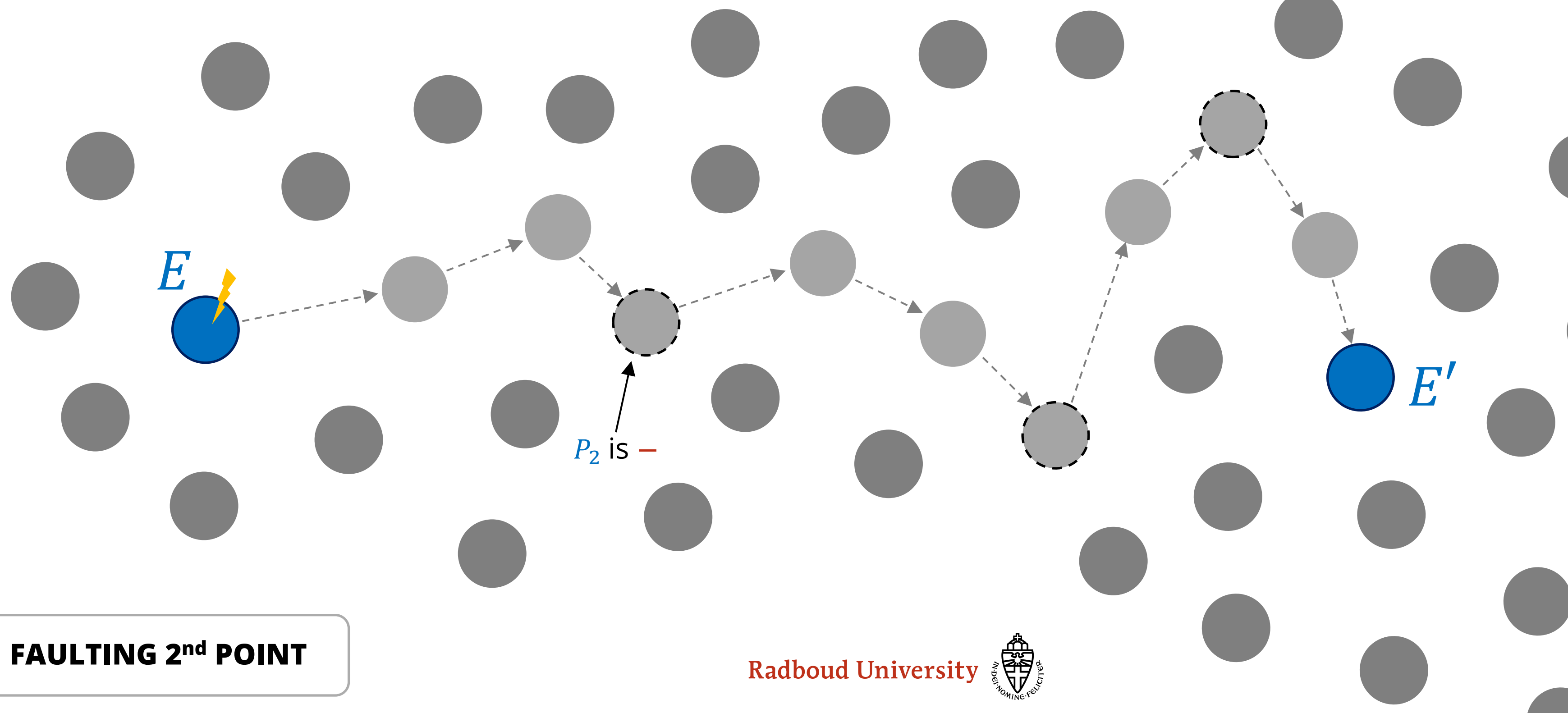

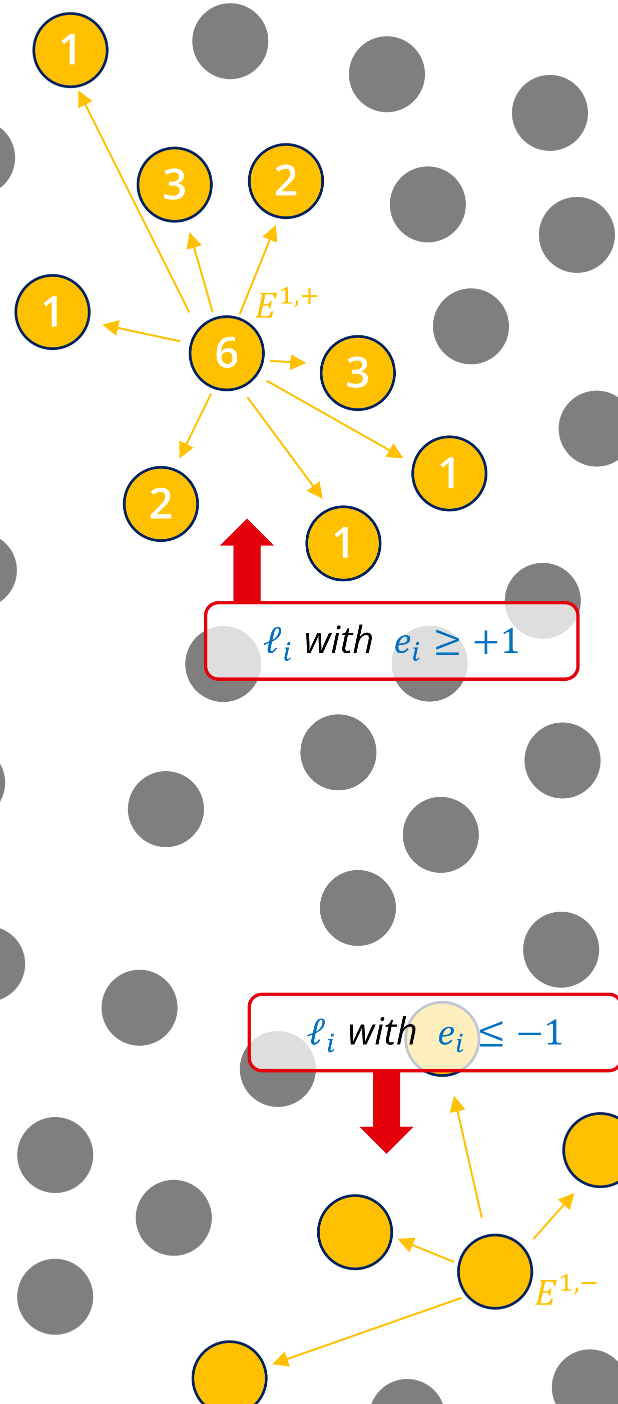

Back to example

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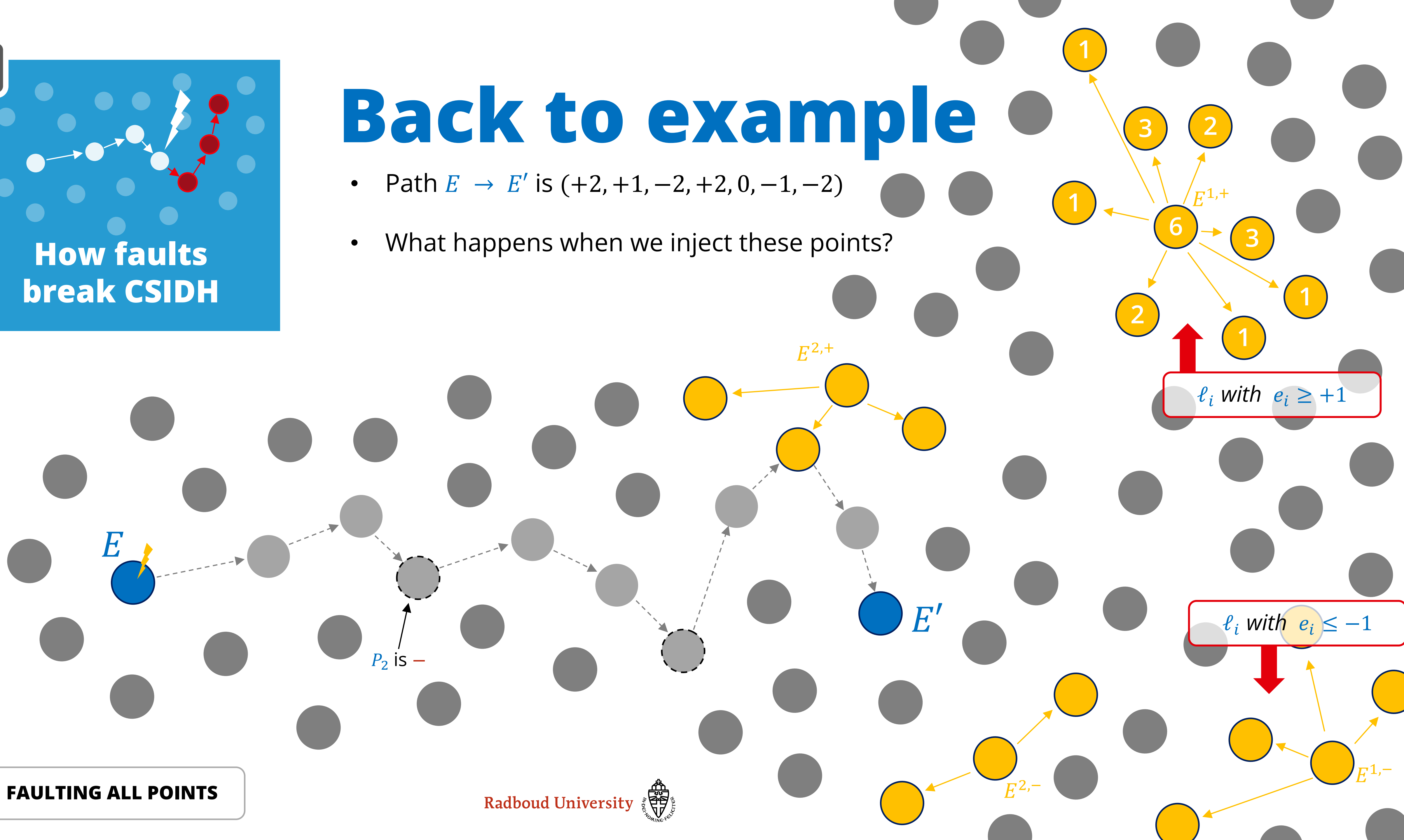

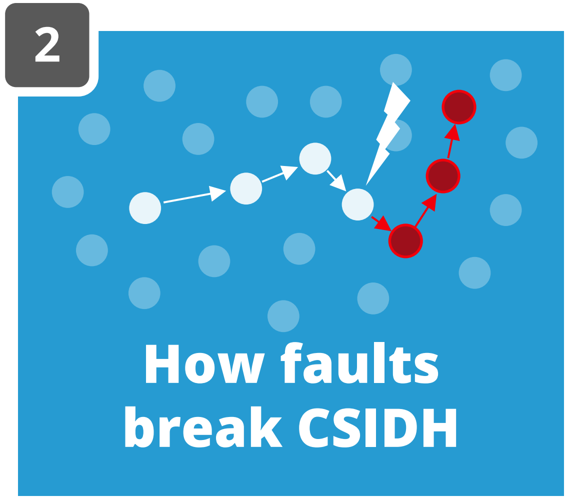

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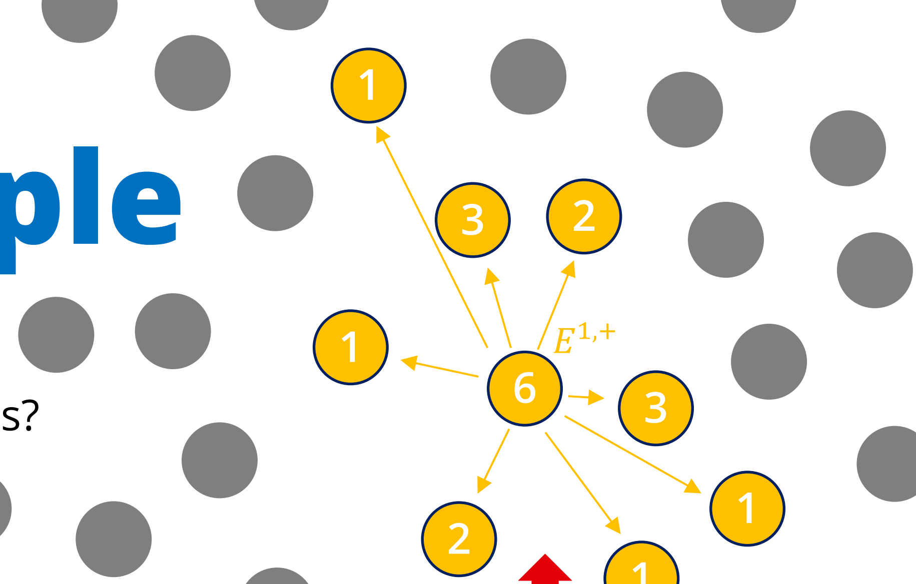
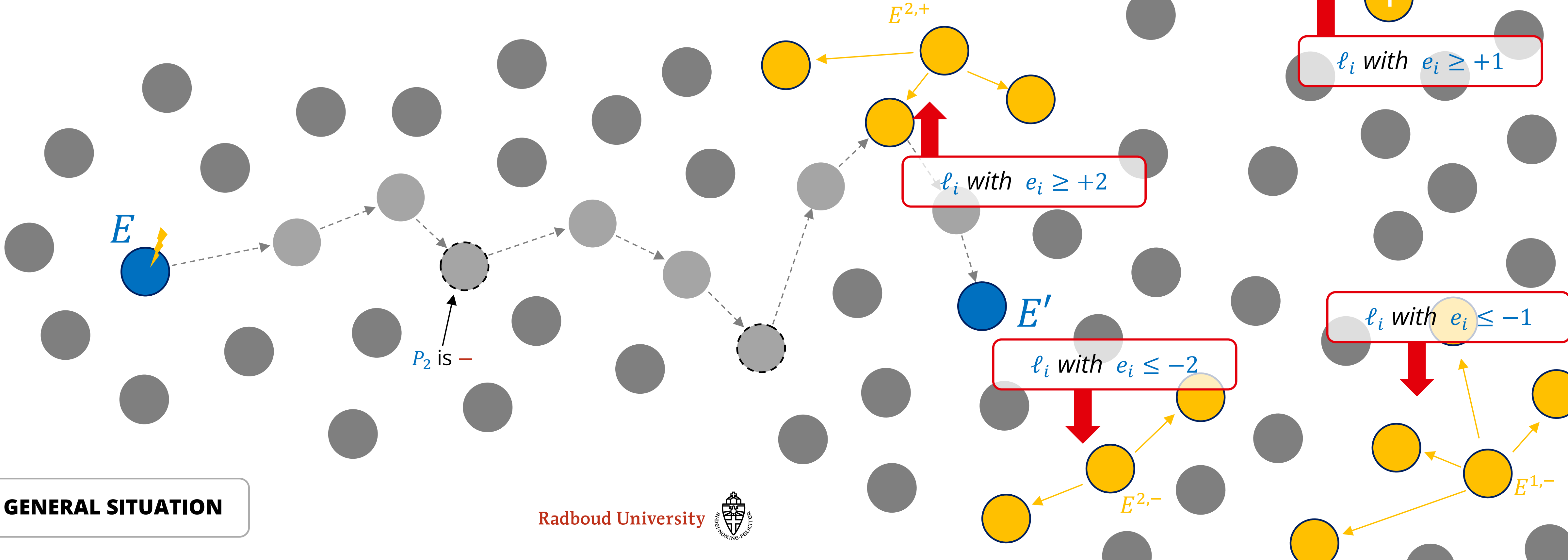
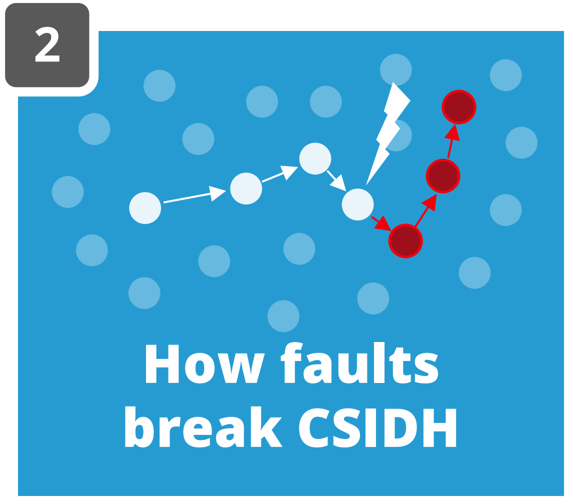



GENERAL SITUATION



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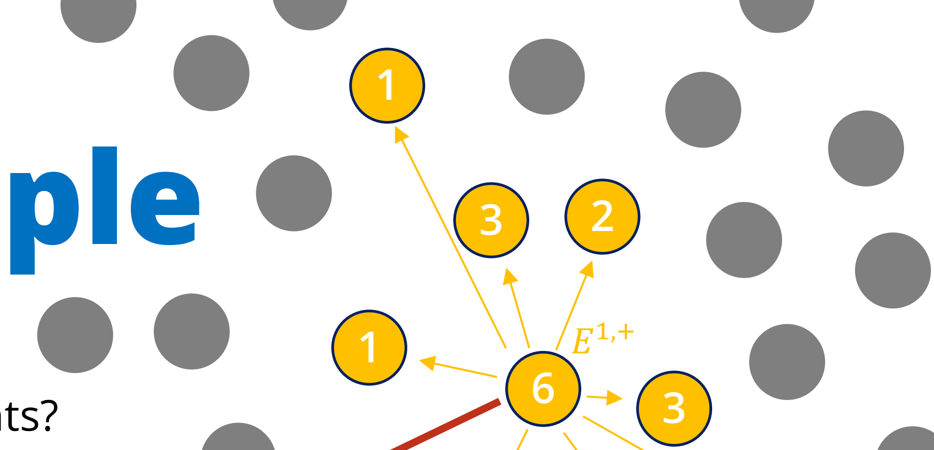

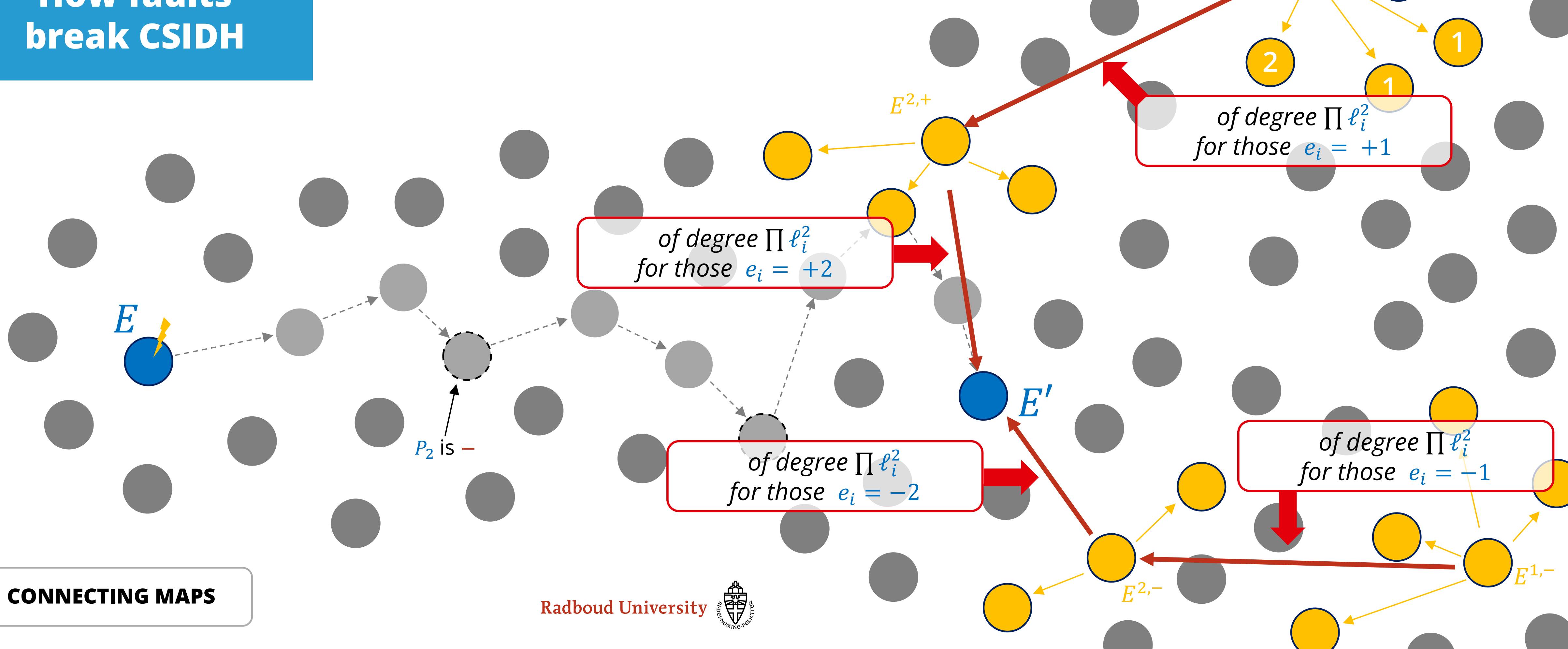

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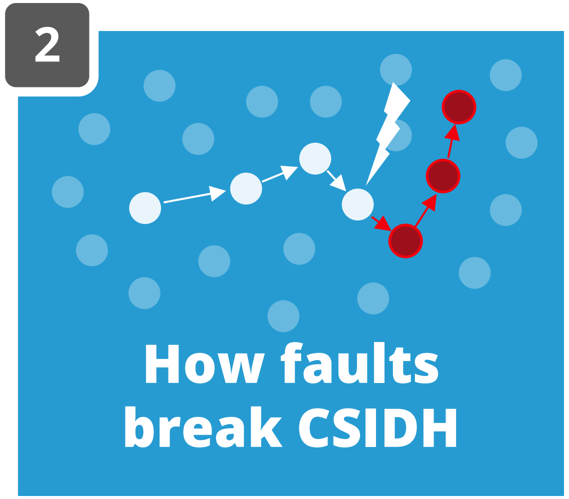

Back to example

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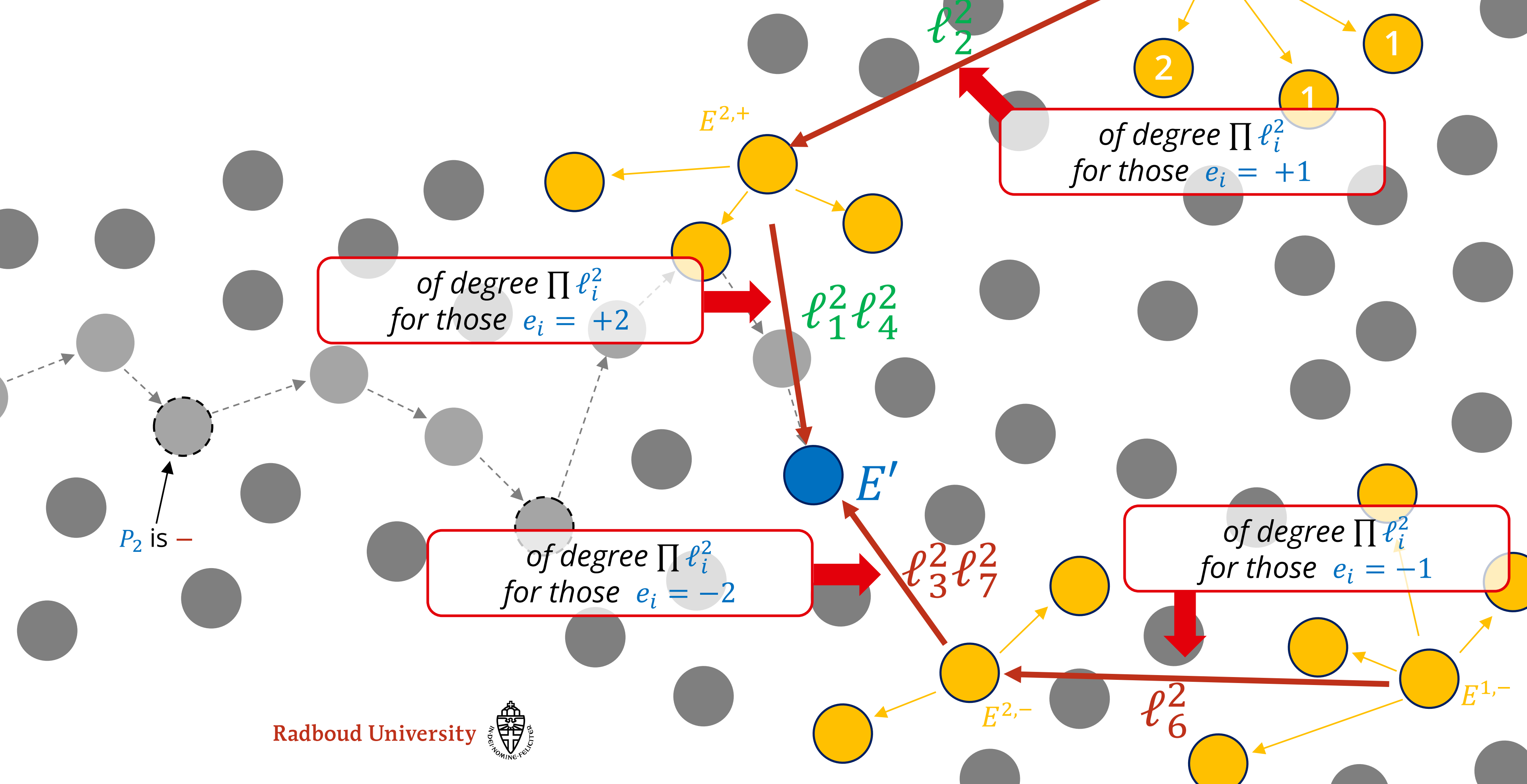
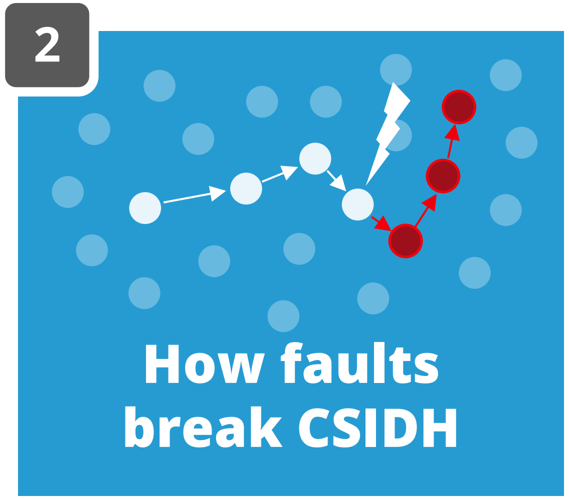
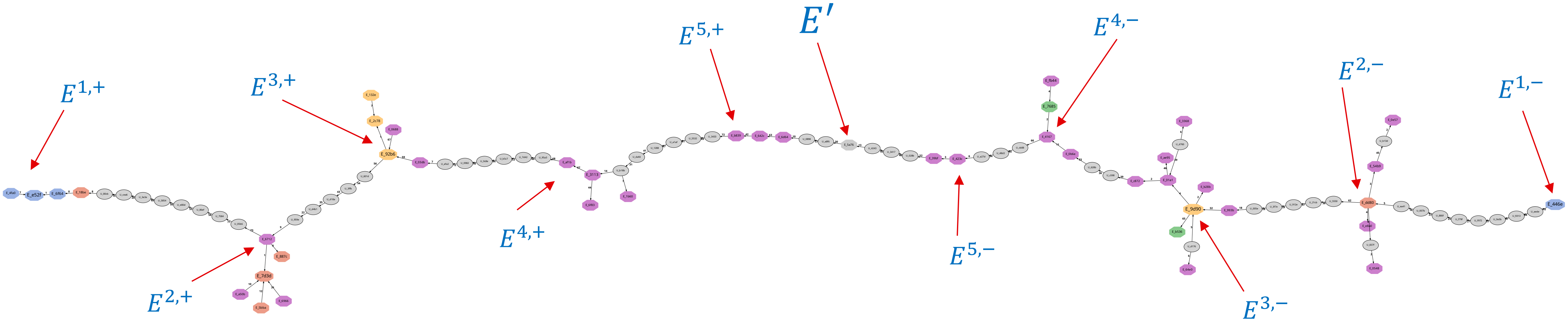


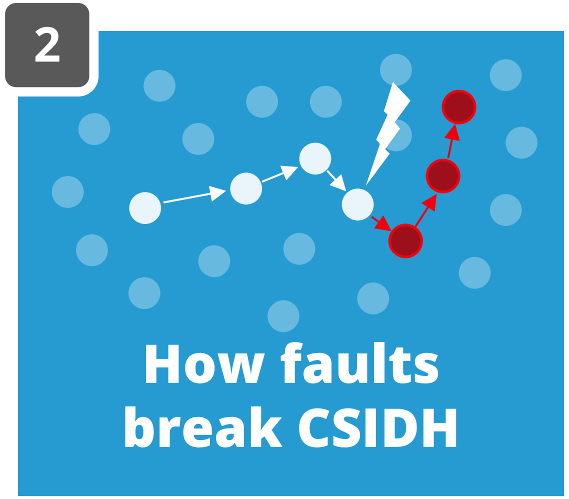
RECOVERING KEY



Real world: CSIDH-512

- uses 74 ℓ_i with $\mathrm{e_i} \in [-5, \dots, 5]$ for secret $(e_1, \dots, e_{74})$
- hence, need 10 points to perform computation so we get $E^{1,\pm}, \dots, E^{5,\pm}$ and a much larger graph
- overall strategy is exactly the same as before





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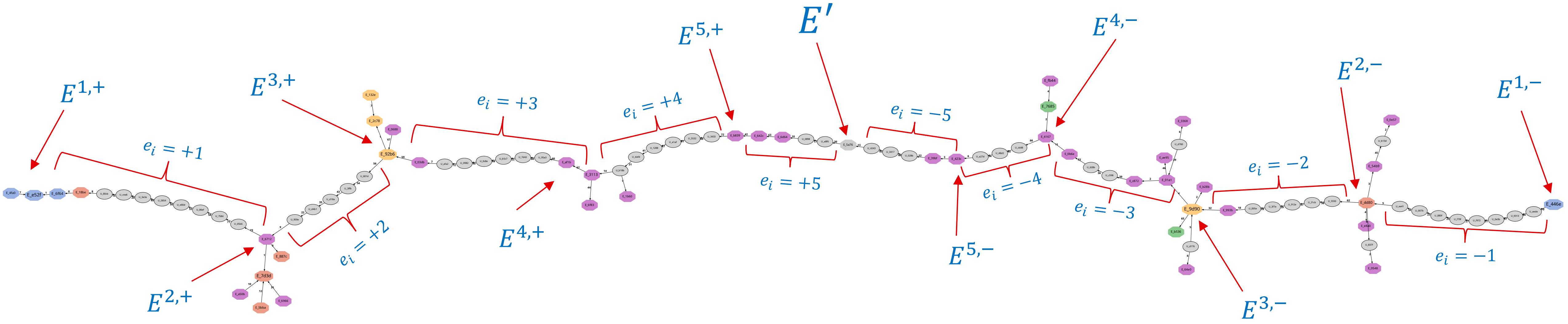
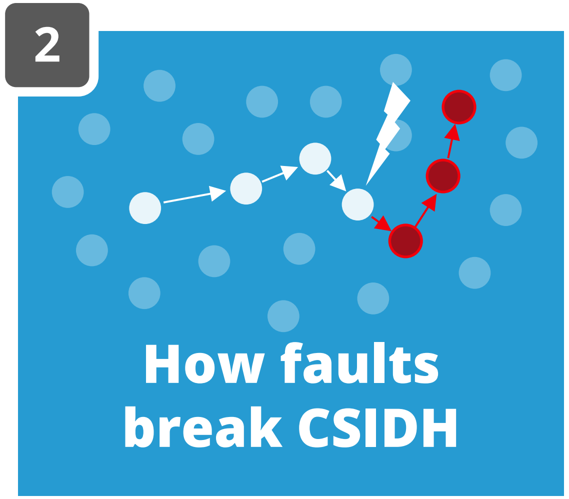
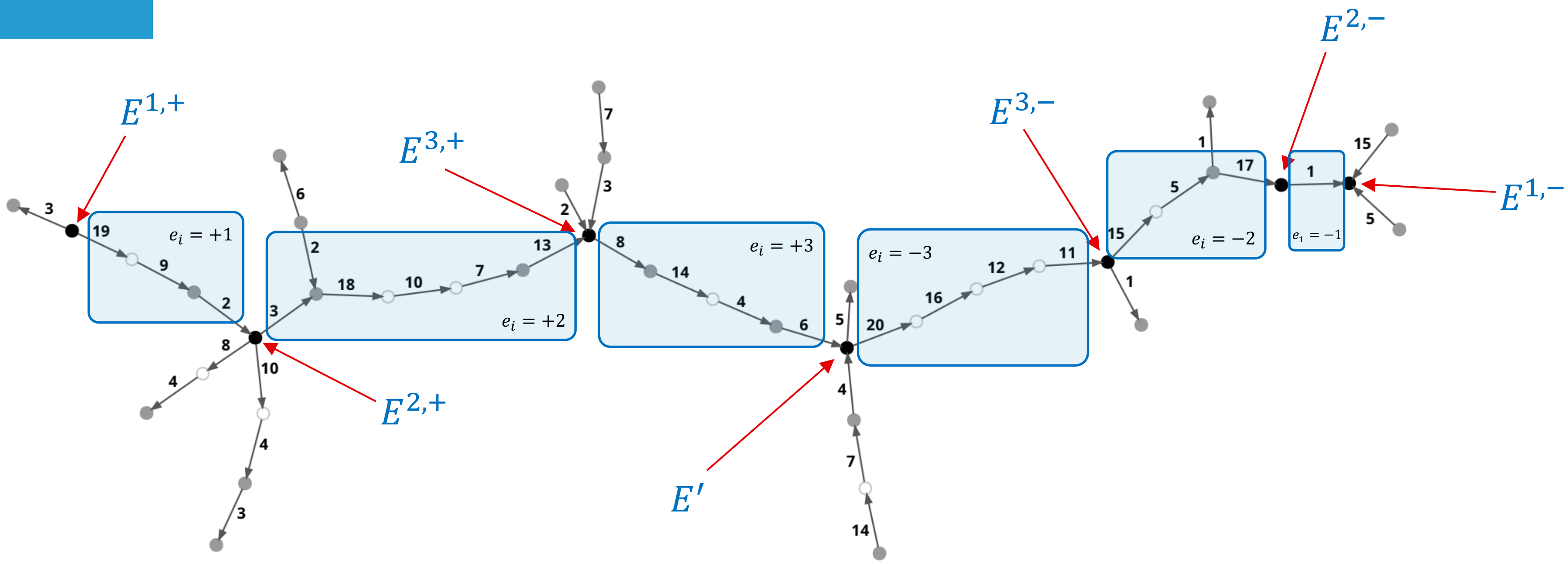




MORE READABLE: CSIDH-103

- uses 21 ℓ_i with $e_i \in [-3, \ldots, 3]$ for secret $(e_1, \ldots, e_{21})$
- hence, need 6 points to perform computation so we get $E^{1,\pm}, \ldots, E^{3,\pm}$ and a much larger graph

CSIDH-103



$[\mathfrak{a}] \sim (-1, +1, +2, +3, -2, +3, +2, +3, +1, +2, -3, -3, +2, +3, -2, -3, -2, +2, +1, -3, 0)$

IN SUMMARY

- fault injections allow us to break CSIDH-512 in about **100 samples**
  (one sample is a computation of group action with a single fault injection)
- similar strategy applied to CTIDH-512 needs only **40 samples**
- more advanced tricks (using the twist) moves most of computational effort to break CTIDH-512 to **one-off precomputation**
- countermeasure: **Elligreator.** (about 5% extra cost)
- hashed version: requires more samples and computations, still feasible