Impossibility of Indifferentiable Iterated Blockciphers from 3 or Less Primitive Calls

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Section 1

Background

Iterated blockciphers

- Virtually all modern blockciphers are iterated blockciphers

Iterated blockciphers

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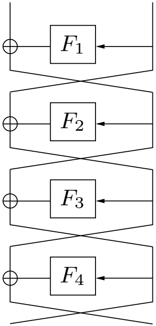
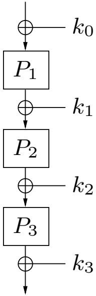
- Compositions of "rounds"/simpler blockciphers



Feistel network: DES, SIMON, etc.
Provable security up to $2^{n/2}$ queries.



Iterated Even-Mansour (IEM): AES, Skinny, etc.
Provable security up to $2^{3n/4}$ queries.

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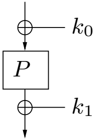

Even-Mansour cipher [EM97]: provable security up to $2^{n/2}$ queries

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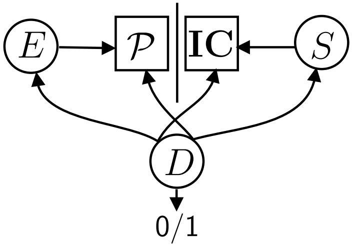
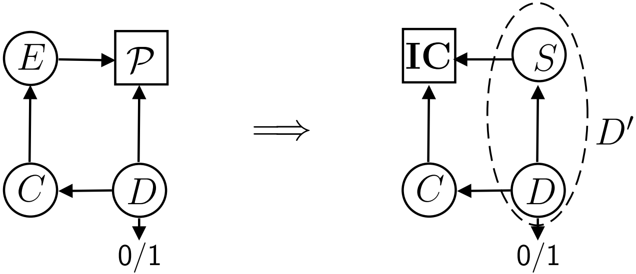
- How many random function/permutation-calls are necessary for a "non-trivial" iterated blockcipher?
- **Indistinguishability** from a random permutation: 1 call to a public permutation
- **Indifferentiability** from an ideal cipher?



Indifferentiability of blockciphers

Why indifferentiability?

- Because of the composition theorem: $C^{E^{\mathcal{P}}}$ using an indifferentiable blockcipher is as secure as $C^{\mathbf{IC}}$.
- Limitations: single-stage [RSS11, BBM13], complexity blow-up [DRST12, DGHM13].

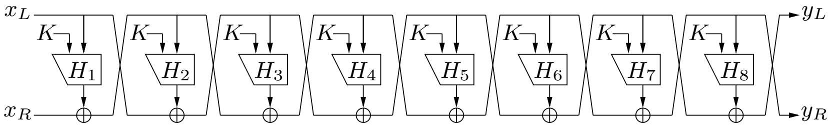


Adversary D against $C^{E^{\mathcal{P}}}$ $\Longrightarrow$ Adversary $D' = (D, S)$ constitutes an adversary against $C^{\mathbf{IC}}$

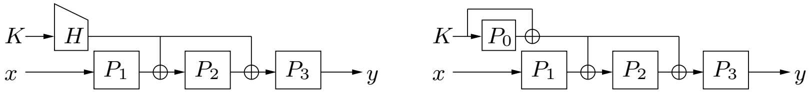
Indifferentiability of blockciphers

We had fruitful positive results:

- Key-prepended Feistel ciphers [CPS08, HKT11, DKT16, DS16]
- Iterated Even-Mansour ciphers [ABD$^+$13, LS13, DSST17, GL16]
- Confusion-diffusion networks [DSSL16]



8-round Key-prepended Feistel cipher [DS16]



3-round iterated Even-Mansour with an idealized key derivation [GL16]

Indifferentiability of blockciphers

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But no general lower bounds

- Only specific lower bounds for specific constructions
- Key-prepended Feistel: at least 6 rounds (6 calls to random functions) [CPS08]
- Iterated Even-Mansour with key derivation: at least 3 rounds (4 calls to random permutations or functions) [ABD+13, GL16]
- Iterated Even-Mansour without key derivation: 5 rounds (5 calls to random permutations) necessary and sufficient [LS13, DSST17]

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- Oracle $\mathcal{P} = (\mathbf{P}_1, \mathbf{P}_2, ..., \mathbf{P}_{|\mathcal{I}|})$
  - $\mathbf{P}_i : \{0,1\}^{(m(i))} \to \{0,1\}^{(m(i))}$ is a random permutation of width $m(i) = \text{poly}(n)$
  - $\mathcal{I} = \mathcal{I}_{\le n} \sqcup \mathcal{I}_{>n}$, where $m(i) \le n$ iff. $i \in \mathcal{I}_{\le n}$

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    - *BTW: when $|\mathcal{I}_{\le n}| = 2^{poly(n)}$, it seems our impossibility result on 1-call blockciphers $E1^{\mathcal{P}}$ still holds, but our differentiators on $E2^{\mathcal{P}}$ and $E3^{\mathcal{P}}$ don't work. This matches existing indifferentiable key-length extension results [CDMS10, GLL16].*

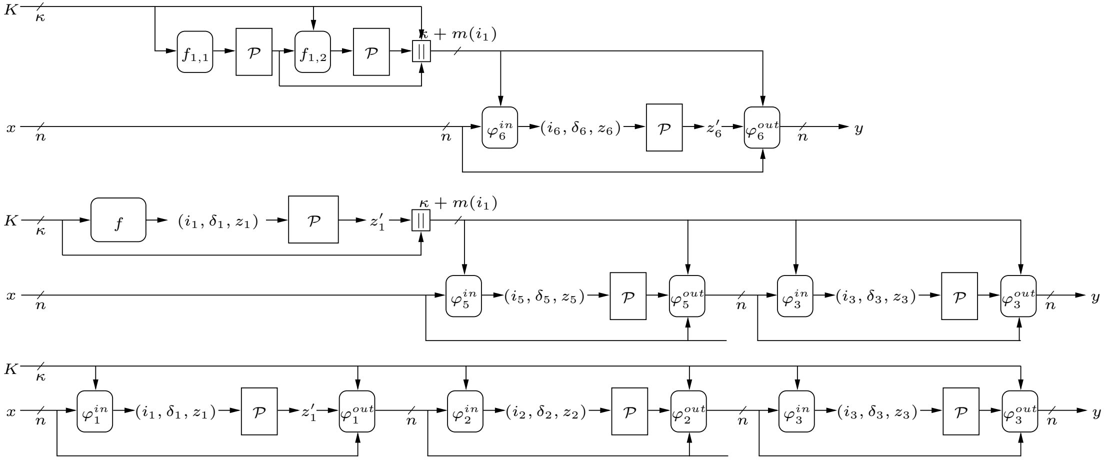
Section 2

Contributions

Main results: first general lower bound

- (Informal) No iterated blockcipher making 3 or less calls to the oracle $\mathcal{P}$ is statistically indifferentiable from ideal ciphers.
  - The 4-call positive result [GL16] is thus optimal.

3-call iterated blockcipher $E3^{\mathcal{P}} : \{0,1\}^{\kappa} \times \{0,1\}^n \to \{0,1\}^n$



Section 3

Results in detail

Four fundamental properties of a blockcipher oracle procedure $E^{\mathcal{P}}$

- By the definition of the notion of blockciphers:
  1. **Efficient invertibility**: there is a corresponding oracle procedure $(E^{-1})^{\mathcal{P}}$ computing its inverse;
  2. **Deterministic**: evaluating $E^{\mathcal{P}}(K,x) \to y$ and $(E^{-1})^{\mathcal{P}}(K,y)$ always yield the same transcript of $\mathcal{P}$-queries and responses.

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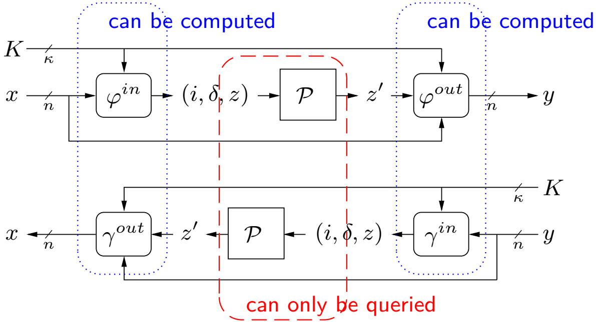
- By fixed descriptions of oracle procedures: sub-procedures in $E^{\mathcal{P}}$ are **oracle-independent**.
- **Non-degeneracy**: no encipherment $E^{\mathcal{P}}(K,x)$ can be approximately computed using less $\mathcal{P}$ calls than $E^{\mathcal{P}}$.

Model for 1-call cipher $E1$

- 1-call blockcipher/round $E1^{\mathcal{P}}(K,x) := \varphi^{out}\big(K, \mathcal{P}(\varphi^{in}(K,x)), x\big)$: $K \in \mathcal{K}$, $x \in \{0,1\}^n$
- Efficient inversion within 1 $\mathcal{P}$-call: $(E1^{-1})^{\mathcal{P}}(K,y) := \gamma^{out}\big(K, \mathcal{P}(\gamma^{in}(K,y)), y\big)$ for two other input and output functions γ^{in} and γ^{out}



Full characterization of 1-call cipher $E1$

The *Fundamental Properties* already ensure a number of non-trivial properties (on oracle procedures of blockciphers):

1. Inv-freeness and its oracle-independence.
2. Properties of inv-free $E1^{\mathcal{P}}(K, x)$.
3. Properties of non-inv-free $E1^{\mathcal{P}}(K, x)$.

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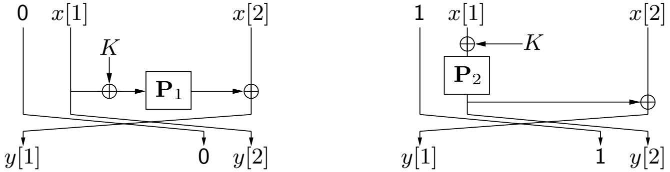
   - Inverse-free encipherments: $E1^{\mathcal{P}}(K,x) \to y$ and $(E1^{-1})^{\mathcal{P}}(K,y) \to x$ call $\mathcal{P}(i,\delta,\star)$ on the same direction δ.
   - Otherwise $E1^{\mathcal{P}}(K,x)$ is non-inverse-free



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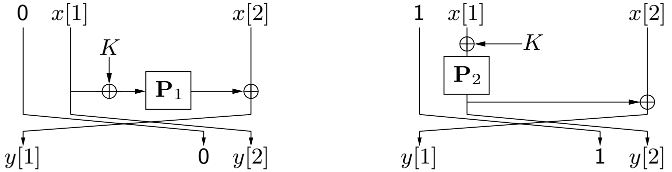

   - Observation: *in $E1^{\mathcal{P}}$, inv-freeness cannot depend on the oracle $\mathcal{P}$*, i.e., one can decide if an encipherment $E1^{\mathcal{P}}(K,x)$ is inv-free without querying $\mathcal{P}$.
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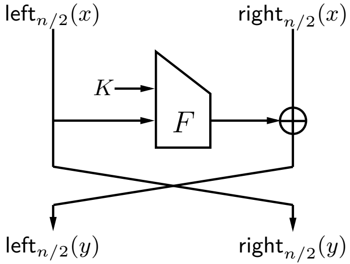
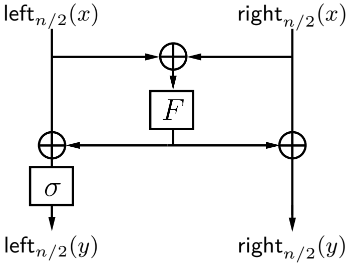


Feistel:
$\text{left}(x) = \text{right}(y)$,
$2^{n/2}$ distinct (K, x) call same $F\Big(K \| \text{left}(x)\Big)$



Lai-Massey:
$\text{left}(x) \oplus \text{right}(x) = \sigma^{-1}(\text{left}(y)) \oplus \text{right}(y)$,
$2^{n/2}$ distinct (K, x) call same $F\Big(\text{left}(x) \oplus \text{right}(x)$

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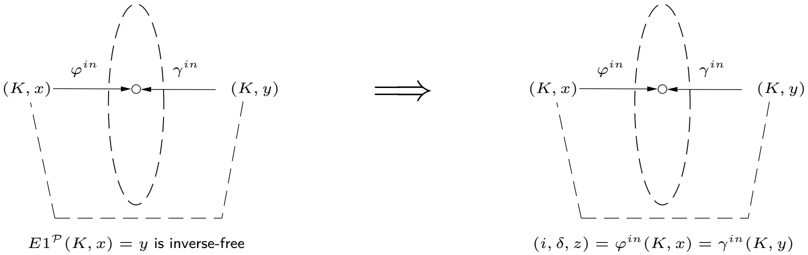

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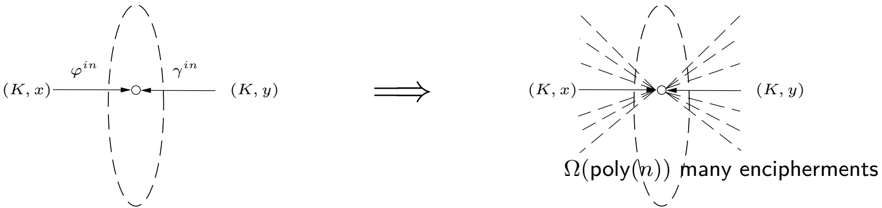

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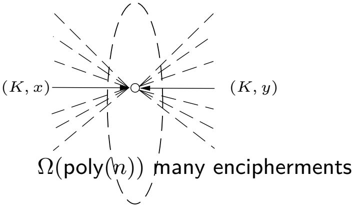
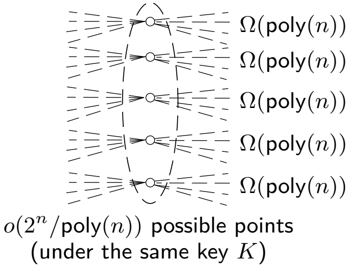


$\Longrightarrow$



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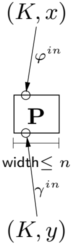
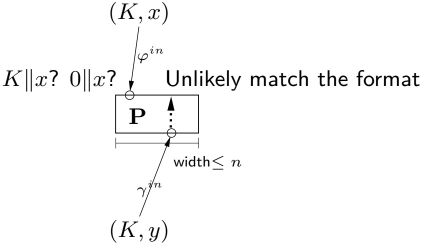


$(i, \delta, z) \in \{+, -\} \times \mathcal{I}_{\leq n} \times \{0,1\}^{m(i)}$:
At most $2^{n+1}|\mathcal{I}_{\leq n}|$ possibilities



Attack $E1^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

With the above properties, we are able to bump into our differentiator $D1$ on $E1^{\mathcal{P}}$. In detail, the cipher $E1^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ may fall into two cases.

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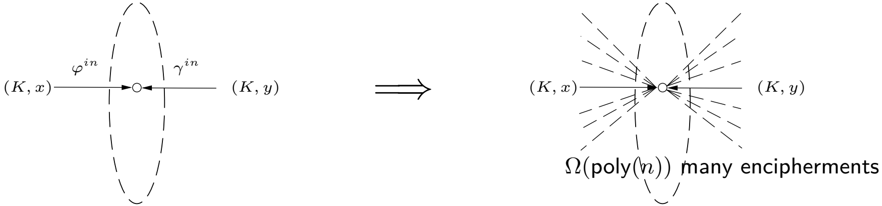
- Case 1: there exists at least 1 inv-free encipherment $E1^{\mathcal{P}}(K, x)$.
  As discussed, there are $t = \Omega(\text{poly}(n))$ distinct inv-free $E1^{\mathcal{P}}(K, x_1), ..., E1^{\mathcal{P}}(K, x_t)$ with $\varphi^{in}(K, x_1) = ... = \varphi^{in}(K, x_t) = (i, \delta, z)$. Thus, the restriction of $E1^{\mathcal{P}}(K, \cdot)$ to $\{x_1, ..., x_t\}$ is a bijection defined upon a polynomial-length random string $z' = \mathcal{P}(i, \delta, z)$.



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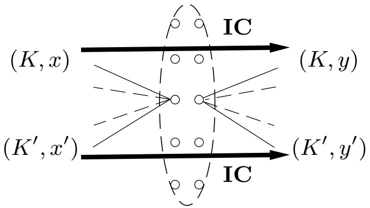
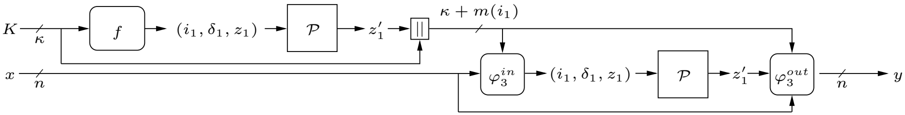
Condition: $|\mathcal{K}| \geq 2|\mathcal{I}_{\leq n}| + 1 = O(\text{poly}(n))$.

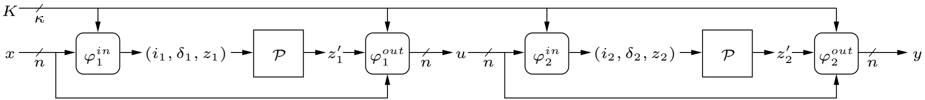
- Case 2: $E1^{\mathcal{P}}(K,x)$ is non-inv-free for all $(K,x) \in \mathcal{K} \times \{0,1\}^n$.
  The pigeonhole principle guarantees $\exists (K,x),(K',x') \in \mathcal{K} \times \{0,1\}^n$ with collision $\varphi^{in}(K,x) = \varphi^{in}(K',x')$ for attack.



width$\leq n$
$\Rightarrow \sharp$ points $\leq |\mathcal{I}_{\leq n}| 2^{n+1}$

2-call iterated blockcipher $E2^{\mathcal{P}}(K, x)$: $K \in \mathcal{K}$, $x \in \{0,1\}^n$





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- For all $K \in \mathcal{K}^{(1)}$: $E2^{\mathcal{P}}(K,x) = \Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big)$ invokes a 1-call key derivation function $\mathsf{kd}^{\mathcal{P}}$
  - $E2^{\mathcal{P}}(K,x) = \Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big)$ for a 1-call function $\mathsf{kd}^{\mathcal{P}} : \{0,1\}^{\kappa} \to \{0,1\}^{m_{max}}$ and a 1-call cipher $\Pi_3^{\mathcal{P}} : \{0,1\}^{\kappa+m_{max}} \times \{0,1\}^n \to \{0,1\}^n$;
  - $\mathsf{kd}^{\mathcal{P}}(K) = \mathcal{P}\big(f(K)\big)$ for another oracle-independent function f
  - $\Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big) = \varphi_3^{out}\big(K, \mathcal{P}(\varphi_3^{in}(K,x)), x\big)$

2-call iterated blockcipher $E2^{\mathcal{P}}(K,x)$: $K \in \mathcal{K}$, $x \in \{0,1\}^n$

- Key space can be partitioned: $\mathcal{K} = \mathcal{K}^{(0)} \sqcup \mathcal{K}^{(1)}$
- For all $K \in \mathcal{K}^{(1)}$: $E2^{\mathcal{P}}(K,x) = \Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big)$ invokes a 1-call key derivation function $\mathsf{kd}^{\mathcal{P}}$
  - $E2^{\mathcal{P}}(K,x) = \Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big)$ for a 1-call function $\mathsf{kd}^{\mathcal{P}} : \{0,1\}^{\kappa} \to \{0,1\}^{m_{max}}$ and a 1-call cipher $\Pi_3^{\mathcal{P}} : \{0,1\}^{\kappa+m_{max}} \times \{0,1\}^n \to \{0,1\}^n$;
  - $\mathsf{kd}^{\mathcal{P}}(K) = \mathcal{P}\big(f(K)\big)$ for another oracle-independent function f
  - $\Pi_3^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big) = \varphi_3^{out}\big(K, \mathcal{P}(\varphi_3^{in}(K,x)), x\big)$
- For all $K \in \mathcal{K}^{(0)}$: $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$, there is no key derivation in the form of oracle procedures
  - $\Pi_1^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big) = \varphi_1^{out}\big(K, \mathcal{P}(\varphi_1^{in}(K,x)), x\big)$ and $\Pi_2^{\mathcal{P}}\big(K \| \mathsf{kd}^{\mathcal{P}}(K), x\big) = \varphi_2^{out}\big(K, \mathcal{P}(\varphi_2^{in}(K,x)), x\big)$ are two 1-call ciphers/rounds.

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

- Case 1: $E2^{\mathcal{P}}$ invokes kd for sufficiently many keys. (Formally, $|\mathcal{K}^{(1)}| \geq 2|\mathcal{I}_{\leq n}| + 1$.)
  1. We simply pick $\lambda = 2|\mathcal{I}_{\leq n}| + 1$ keys $K_1, ..., K_\lambda \in \mathcal{K}^{(1)}$ and derive subkeys $s_1 = \mathsf{kd}^{\mathcal{P}}(K_1), ..., s_\lambda = \mathsf{kd}^{\mathcal{P}}(K_\lambda)$. This consumes at most $\lambda = O(\mathsf{poly}(n))$ P-queries.
  2. We then view the round $\Pi_3^{\mathcal{P}}$ as a 1-call cipher with keyspace $\{K_1 \| s_1, ..., K_\lambda \| s_\lambda\}$ and apply our differentiator $D1$.
  3. It is thus crucial that $D1$ can break $E1$ with polynomial-keyspace.

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

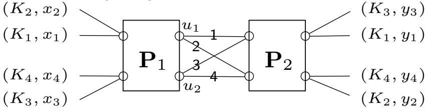
- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
  3. From boomerang to yoyo
  4. Non-degenerate input functions

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
     - In the 2-round Even-Mansour cipher $y = K \oplus \mathbf{P}_2(K \oplus \mathbf{P}_1(K \oplus x))$
     - Computing four distinct pairs $(K_1,u_1),(K_2,u_2),(K_3,u_3),(K_4,u_4)$ inducing two collided inputs to $\mathbf{P}_1^{-1}$ and two collide inputs to $\mathbf{P}_2$.
     - Can compute a 4-tuple of cipher inputs/outputs $\big((K_1,x_1,y_1),\ldots,(K_4,x_4,y_4)\big)$ that has $K_1 \oplus x_1 = K_2 \oplus x_2$, $K_3 \oplus x_3 = K_4 \oplus x_4$; $K_1 \oplus y_1 = K_3 \oplus y_3$, $K_2 \oplus y_2 = K_4 \oplus y_4$.

$u_1 = u_2$, $u_3 = u_4$;
$K_1 \oplus u_1 = K_3 \oplus u_3$, $K_2 \oplus u_2 = K_4 \oplus u_4$

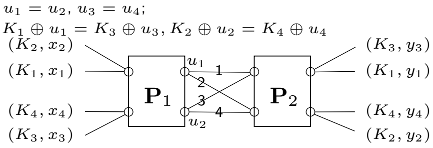


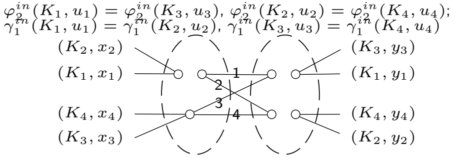
  2. Using graph theory on girth
  3. From boomerang to yoyo

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
     - Generalized boomerang: find pairs $(K_1,u_1),(K_2,u_2),(K_3,u_3),(K_4,u_4) \in \mathcal{K}^{(0)} \times \{0,1\}^n$ that induce similar collided $\mathcal{P}$-calls.
     - Can computes a 4-tuple of cipher inputs/outputs $\big((K_1,x_1,y_1),...,(K_4,x_4,y_4)\big)$ that has $\varphi_1^{in}(K_1,u_1) = \varphi_1^{in}(K_2,u_2)$, $\varphi_1^{in}(K_3,u_3) = \varphi_1^{in}(K_4,u_4)$, $\gamma_2^{in}(K_1,u_1) = \gamma_2^{in}(K_3,u_3)$ and $\gamma_2^{in}(K_2,u_2) = \gamma_2^{in}(K_4,u_4)$.



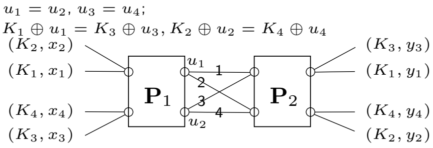


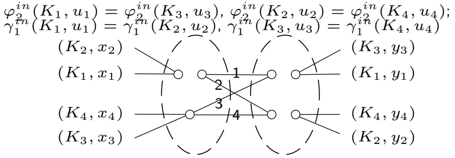
  2. Using graph theory on girth
  3. From boomerang to yoyo

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\left(K, \Pi_1^{\mathcal{P}}(K,x)\right)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
     - Existence of $(K_1,u_1),(K_2,u_2),(K_3,u_3),(K_4,u_4) \in \mathcal{K}^{(0)} \times \{0,1\}^n$?
     - A 4-cycle C_4 in a bipartite graph with left and right shore size$\leq |\mathcal{I}_{\leq n}|2^{n+1}$ and $|\mathcal{K}^{(0)}|2^n$ edges
     - Hoory [Hoo02]: when $|\mathcal{K}^{(0)}| = \Theta(2^n)$ (key-length$\approx n$), cycles of length≤ 4 must exist



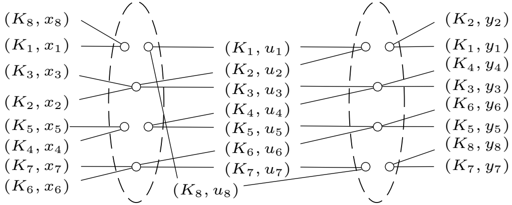


  3. From boomerang to yoyo

Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

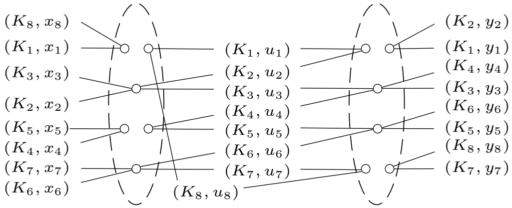
- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\left(K, \Pi_1^{\mathcal{P}}(K,x)\right)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
  3. From boomerang to yoyo
     - But with $|\mathcal{K}^{(0)}| = \Theta(2^n)$, we cannot invoke the attack for $E3^{\mathcal{P}}$ with KDFs!
     - A general yoyo distinguisher [RBH17]: consider longer cycles $C_{2\lambda}$, $\lambda \le n+1$. I.e., find 2λ-tuple $\left((K_1,u_1)\ ,\ldots,(K_{2\lambda},u_{2\lambda})\right)$ with $\varphi_2^{in}(K_1,u_1) = \varphi_2^{in}(K_2,u_2), \gamma_1^{in}(K_2,u_2) = \gamma_1^{in}(K_3,u_3), \varphi_2^{in}(K_3,u_3) = \varphi_2^{in}(K_4,u_4), \gamma_1^{in}(K_4,u_4) = \gamma_1^{in}(K_5,u_5), \ldots, \varphi_2^{in}(K_{2\lambda-1},u_{2\lambda-1}) = \varphi_2^{in}(K_{2\lambda},u_{2\lambda}), \gamma_1^{in}(K_{2\lambda},u_{2\lambda}) = \gamma_1^{in}(K_1,u_1)$.



Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

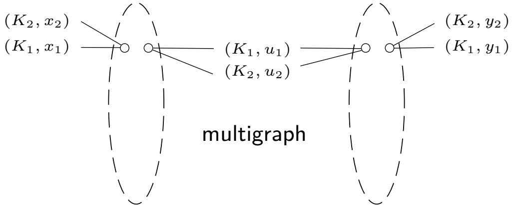
- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
  3. From boomerang to yoyo
     - Can compute a 2λ-tuple of $E2$ inputs/outputs $\big((K_1,x_1,y_1),...,(K_{2\lambda},x_{2\lambda},y_{2\lambda})\big)$ that has a "cycle of collisions". I.e., $\gamma_2^{in}(K_1,y_1) = \gamma_2^{in}(K_2,y_2), \varphi_1^{in}(K_2,x_2) = \varphi_1^{in}(K_3,x_3),...,\gamma_2^{in}(K_{2\lambda-1},y_{2\lambda-1}) = \gamma_2^{in}(K_{2\lambda},y_{2\lambda}), \varphi_1^{in}(K_{2\lambda},x_{2\lambda}) = \varphi_1^{in}(K_1,x_1)$.
     - By Hoory [Hoo02], $|\mathcal{K}^{(0)}| \geq \Big(6\big(3|\mathcal{I}_{\leq n}|\big)^{\frac{1}{n}} + 3\Big)|\mathcal{I}_{\leq n}| = O(\mathsf{poly}(n))$ suffices!



Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

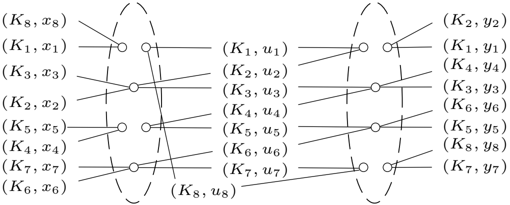
- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\left(K, \Pi_1^{\mathcal{P}}(K,x)\right)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
  3. From boomerang to yoyo
     - Hoory [Hoo02] does not apply when $\mathcal{G}$ is a multigraph, but this implies existence of C_2, which is an even weaker case.



Attack $E2^{\mathcal{P}} : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$

Built upon our above results on $E1^{\mathcal{P}}$, we further consider our 2-call model $E2^{\mathcal{P}}$.

- Case 2 (less obvious): $E2^{\mathcal{P}}(K,x) = \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)$ is 2-iteration for most keys.
  1. Starting point: boomerang property
  2. Using graph theory on girth
  3. From boomerang to yoyo
  + Crucial to restrict our discussion to iterated blockciphers with a clear valid intermediate value set $\{0,1\}^n$: an attacker can pick such a u and compute forward or backward.



3-call iterated blockcipher $E3^{\mathcal{P}} : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$

- Partition the key space: $\mathcal{K} = \mathcal{K}^{(0)} \sqcup \mathcal{K}^{(1)} \sqcup \mathcal{K}^{(2)}$
- For all $K \in \mathcal{K}^{(2)}$: $E3^{\mathcal{P}}(K,\cdot)$ invokes a 2-call key derivation function
  - $E3^{\mathcal{P}}(K,x) = \Pi_6^{\mathcal{P}}\big(K \| \mathsf{kd}_1^{\mathcal{P}}(K), x\big)$ for a 2-call KDF $\mathsf{kd}_1^{\mathcal{P}} : \{0,1\}^\kappa \to \{0,1\}^{2m_{max}}$ and a 1-call cipher $\Pi_6^{\mathcal{P}} : \{0,1\}^{\kappa+2m_{max}} \times \{0,1\}^n \to \{0,1\}^n$;
- For all $K \in \mathcal{K}^{(1)}$: $E3^{\mathcal{P}}(K,\cdot)$ invokes a 1-call key derivation function
  - $E3^{\mathcal{P}}(K,x) = \Pi_5^{\mathcal{P}}\big(K \| \mathsf{kd}_2^{\mathcal{P}}(K), \Pi_4^{\mathcal{P}}(K \| \mathsf{kd}_2^{\mathcal{P}}(K), x)\big)$ for a 1-call KDF $\mathsf{kd}_2^{\mathcal{P}} : \{0,1\}^\kappa \to \{0,1\}^{m_{max}}$ and two 1-call ciphers $\Pi_4^{\mathcal{P}}, \Pi_5^{\mathcal{P}} : \{0,1\}^{\kappa+m_{max}} \times \{0,1\}^n \to \{0,1\}^n$;
- For all $K \in \mathcal{K}^{(0)}$: there is no key derivation in the form of oracle procedures
  - $E3^{\mathcal{P}}(K,x) = \Pi_3^{\mathcal{P}}\big(K, \Pi_2^{\mathcal{P}}\big(K, \Pi_1^{\mathcal{P}}(K,x)\big)\big)$ for three 1-call ciphers $\Pi_1^{\mathcal{P}}, \Pi_2^{\mathcal{P}}, \Pi_3^{\mathcal{P}} : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$.

Section 4

Conclusion

Conclusion

The first general lower bounds on indifferentiable blockciphers.

1. (Informal) No iterated blockcipher making 3 or less calls to the oracle $\mathcal{P}$ is statistically indifferentiable from ideal ciphers.
   - The 4-call positive result [GL16] is thus optimal.
2. Model of blockciphers: oracle procedures built upon the oracle $\mathcal{P} = (\mathbf{P}_1, ..., \mathbf{P}_{|\mathcal{I}|})$
3. *Fundamental Properties* of blockciphers oracle procedures
4. Concrete models for 1-call blockciphers $E1^{\mathcal{P}}$ and 2- and 3-call iterated blockciphers $E2^{\mathcal{P}}$ and $E3^{\mathcal{P}}$
5. Attack ideas: using invertibility; using Extremal Graph Theory

Discussion: on blockcipher designs

1. *Expense of overcoming non-invertibility*: inverse-free rounds must admit severe weakness, regardless of its design.
2. *Unhelpfulness of wide permutations*: wide permutations with width$> n$ are not "more helpful" in constructing n-bit blockciphers.
3. *"Excluding-type" theoretical support for popular structures, e.g., the IEM ciphers [DSST17, GL16]*: *no other choice can be better*.
4. An anonymous reviewer: permutation-based cryptography are more efficient than ideal ciphers.

Discussion: on blockcipher designs

1. *Expense of overcoming non-invertibility*: inverse-free rounds must admit severe weakness, regardless of its design.
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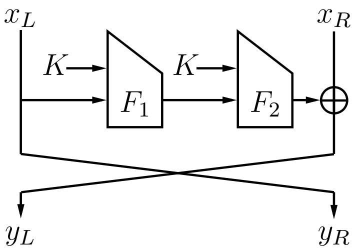
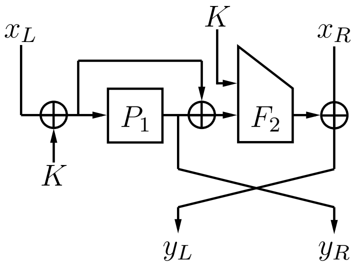
4. An anonymous reviewer: permutation-based cryptography are more efficient than ideal ciphers.

- Usual caveats: information-theoretic security upper bounds only

Possible future Directions

1. Extending our treatments to fully general 2- and 3-call blockciphers
   - Blockciphers are *not* necessarily iterated...





Possible future Directions

1. Extending our treatments to fully general 2- and 3-call blockciphers
2. Smart ideas to unify the complicated cases in $E3$ analysis
3. Fully concrete security characterizations of $E2$ and $E3$ (may need a new paradigm)
4. Other aspects: memory restrictions on adversaries/simulators, etc.
5. Achievability of computational indifferentiability with 3 calls
   - Hardness assumptions on graph problems or key derivation functions might be helpful.
6. Relaxing the condition "$Et^{\mathcal{P}}$ computes a blockcipher for all $\mathcal{P}$ and all n"?

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