An Incremental PoSW for General Weight Distributions

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Eurocrypt 2023 in Lyon, France

FWF Der Wissenschaftsfonds.

Outline

Proofs of Sequential Work, Standalone (PoSW) and Incremental (iPoSW)

The Skiplist PoSW

Make it Incremental (iPoSW)

Generalize it to *General Weight Distributions* (Motivated by Blockchain Applications)

All Constructions Are in the ROM

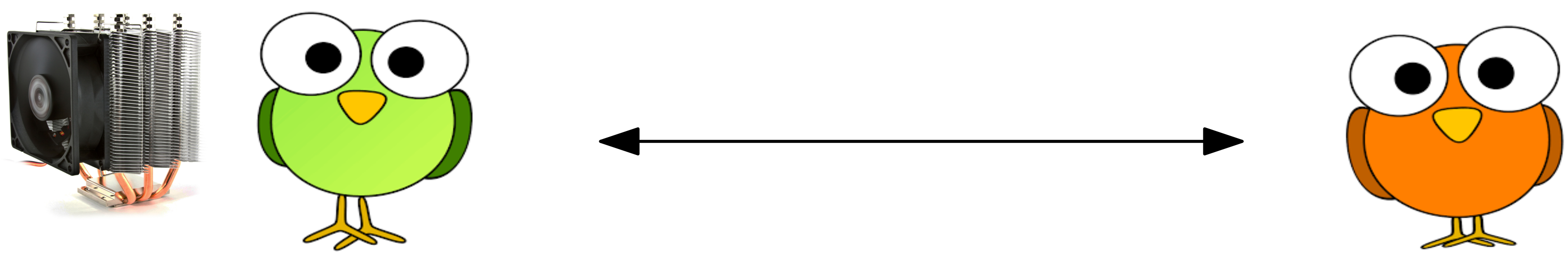
(We don't cover (continuous) verifiable delay functions)

PoSW

[Mahmoody-Moran-Vadhan'13]

Parameter: n

$\mathsf{P}^{\tau(\cdot)}$ $\mathsf{V}^{\tau(\cdot)}$



Completeness: Honest $\mathsf{P}^{\tau(\cdot)}$ making n **sequential** $\tau(\cdot)$ queries makes V accept w.p. 1

PoSW

[Mahmoody-Moran-Vadhan'13]

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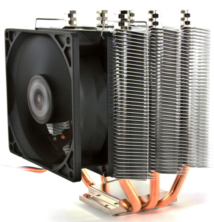
$\mathsf{P}^{\tau(\cdot)}$



$\mathsf{V}^{\tau(\cdot)}$

$\mathsf{poly}(\lambda, \log n)$ time

Completeness: Honest $\mathsf{P}^{\tau(\cdot)}$ making n **sequential** $\tau(\cdot)$ queries makes V accept w.p. 1

Succinctness: For every honest proof π :
$|\pi| \leq \mathsf{poly}(\lambda, \log n)$, $\mathsf{Time}(\mathsf{V}) \leq \mathsf{poly}(\lambda, \log n)$, and $\mathsf{Time}(\mathsf{P}) \leq \mathsf{poly}(\lambda, n)$

PoSW

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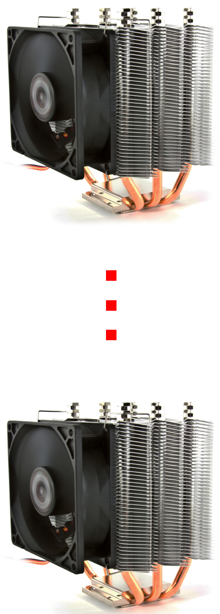


$\tilde{\mathsf{P}}^{\tau(\cdot)}$

$\mathsf{V}^{\tau(\cdot)}$

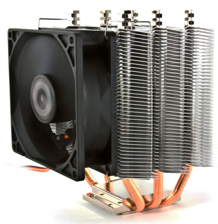
poly$(\lambda, \log n)$ time

(α, ϵ)-**Soundness**: A parallel $\tilde{\mathcal{P}}^{\tau(\cdot)}$ making $\leq \alpha \cdot n$ **sequential** queries to $\tau(\cdot)$ makes V accept with prob. $\leq \epsilon(\lambda)$

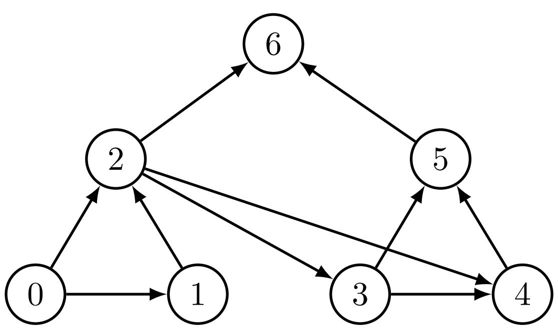
All PoSW Constructions Look Like

Parameters: $G_n, t, \cdots$

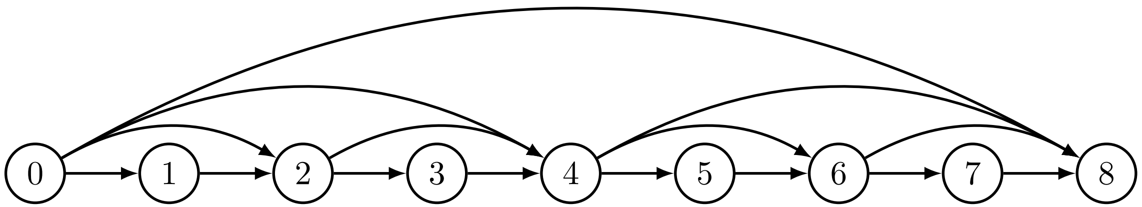
$\mathsf{P}^{\tau(\cdot)}$



$\mathsf{V}^{\tau(\cdot)}$



[Cohen-Pietrzak'18]



[Abusalah-Kamath-Klein-Walter-Pietrzak'19][Abusalah-Fuchsbauer-Gaži-Klein'22]

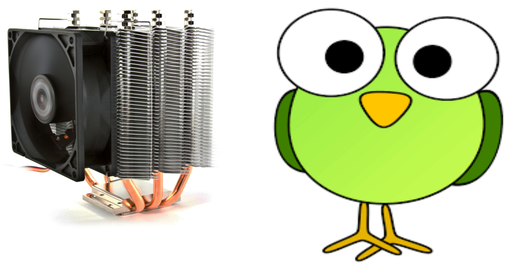
All PoSW Constructions Look Like

Parameters: $G_n, t, \cdots$

$\mathsf{P}^{\tau(\cdot)}$ $\qquad$ $\mathsf{V}^{\tau(\cdot)}$

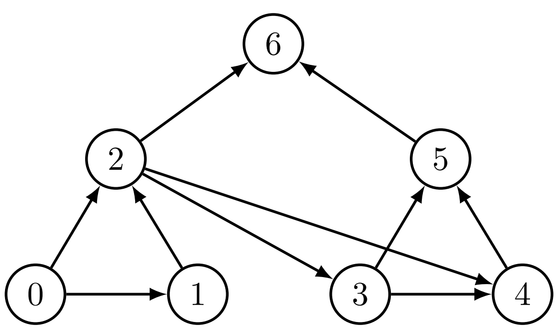
$$\mathsf{P} \xleftarrow{\ \chi\ } \mathsf{V}$$

$$\mathsf{P} \xrightarrow{\ L(n)\ } \mathsf{V}$$



Random oracle $\tau : \{0,1\}^* \to \{0,1\}^\lambda$ with $\tau := \tau(\chi, \cdot)$

$$L(i) := \begin{cases} \tau(i) & \text{if } \mathsf{parents}(i) = \emptyset, \\ \tau\big(i, L(\mathsf{parents}(i))\big) & \text{otherwise.} \end{cases}$$



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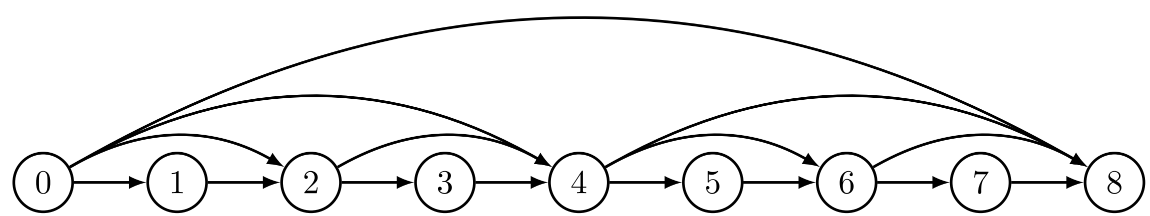

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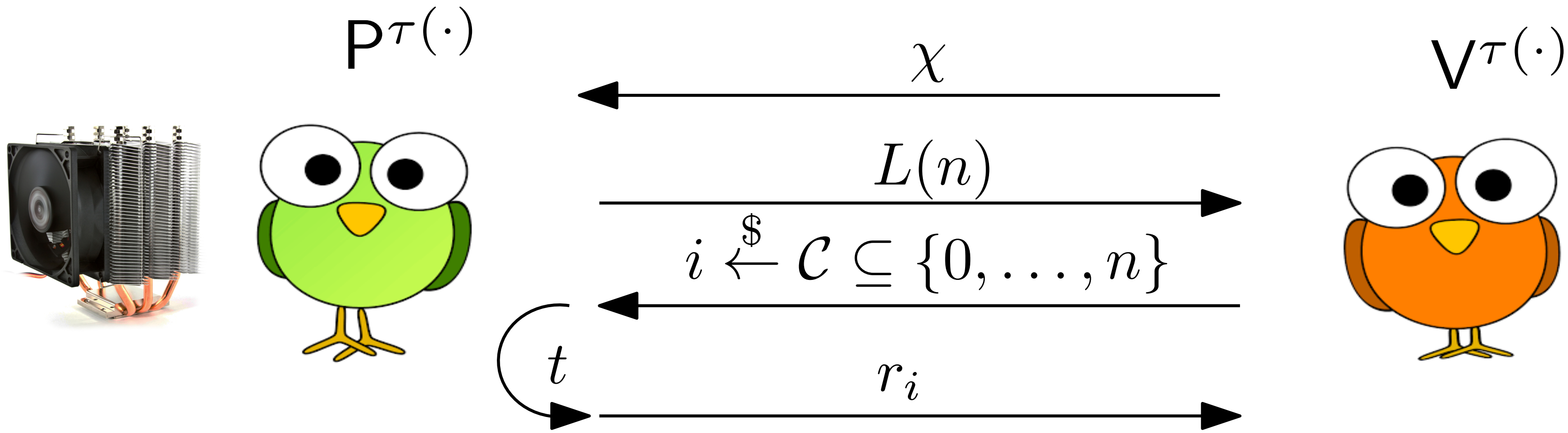

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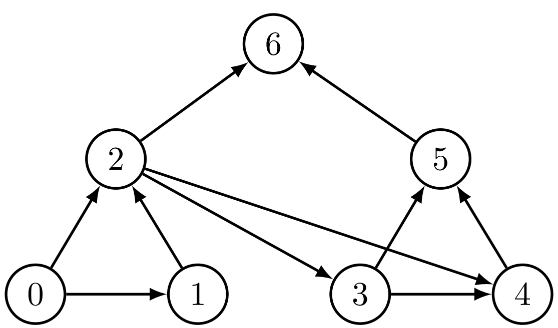

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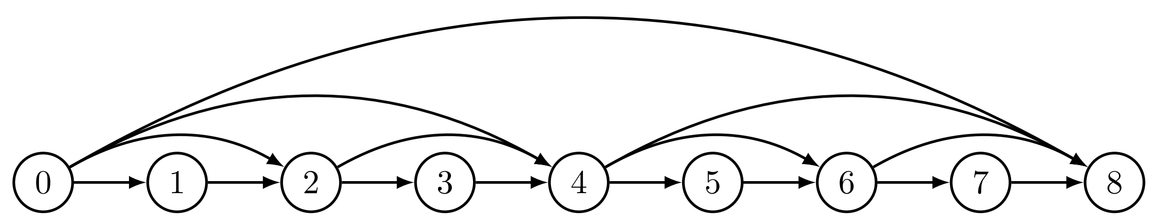

[Abusalah-Kamath-Klein-Walter-Pietrzak'19][Abusalah-Fuchsbauer-Gaži-Klein'22]

The Skiplist PoSW

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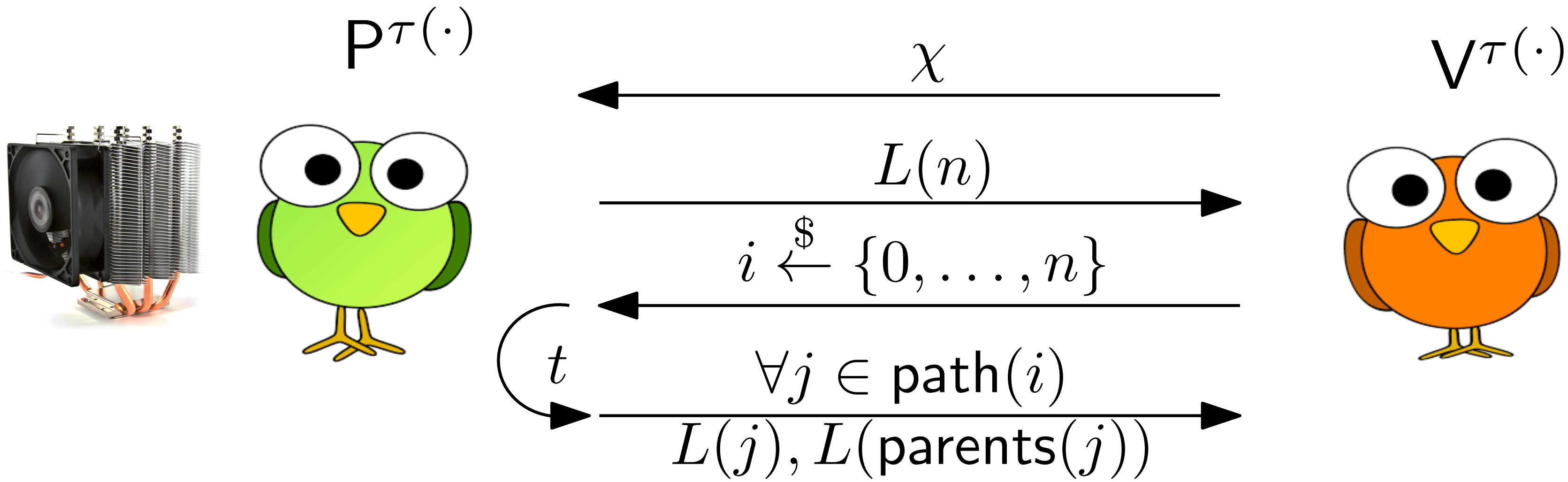

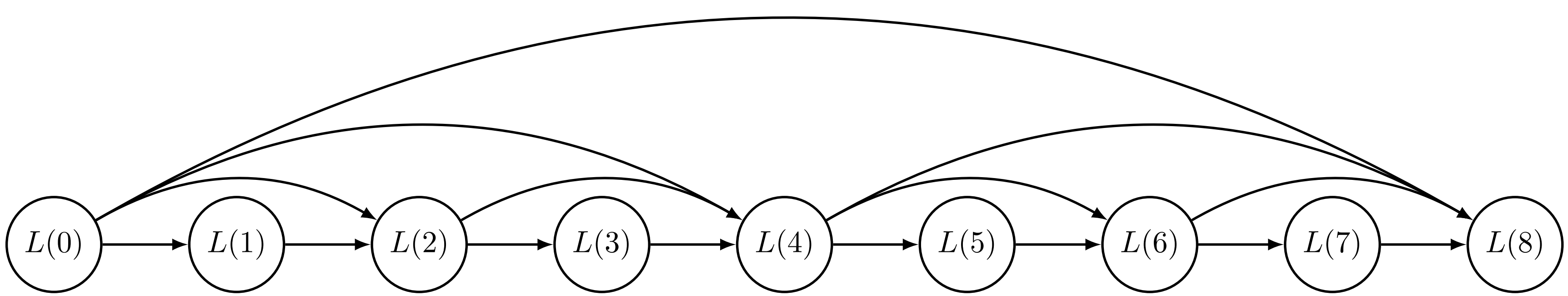

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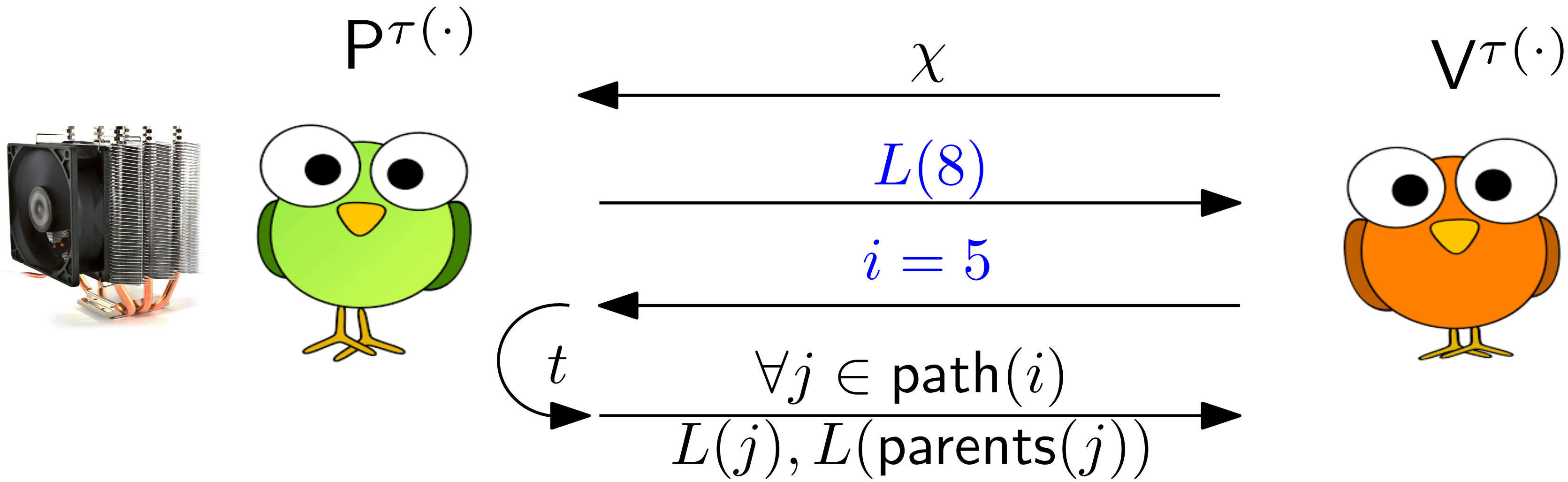

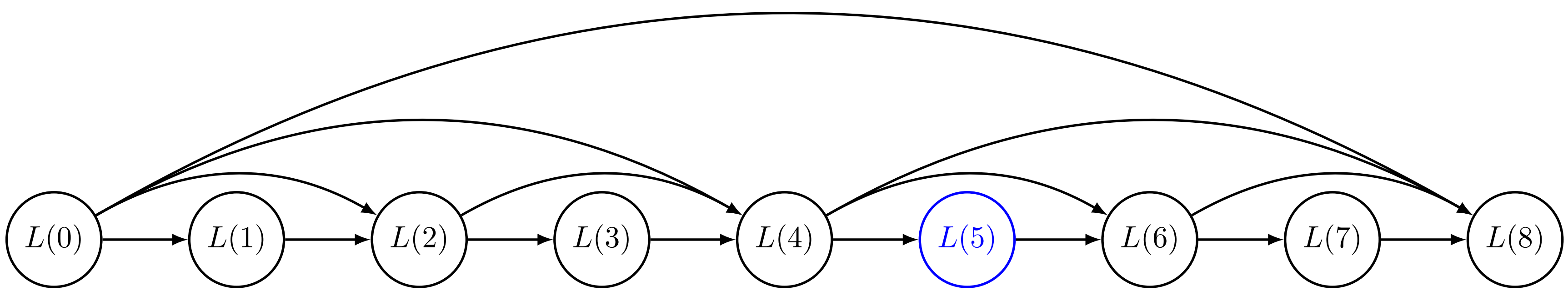

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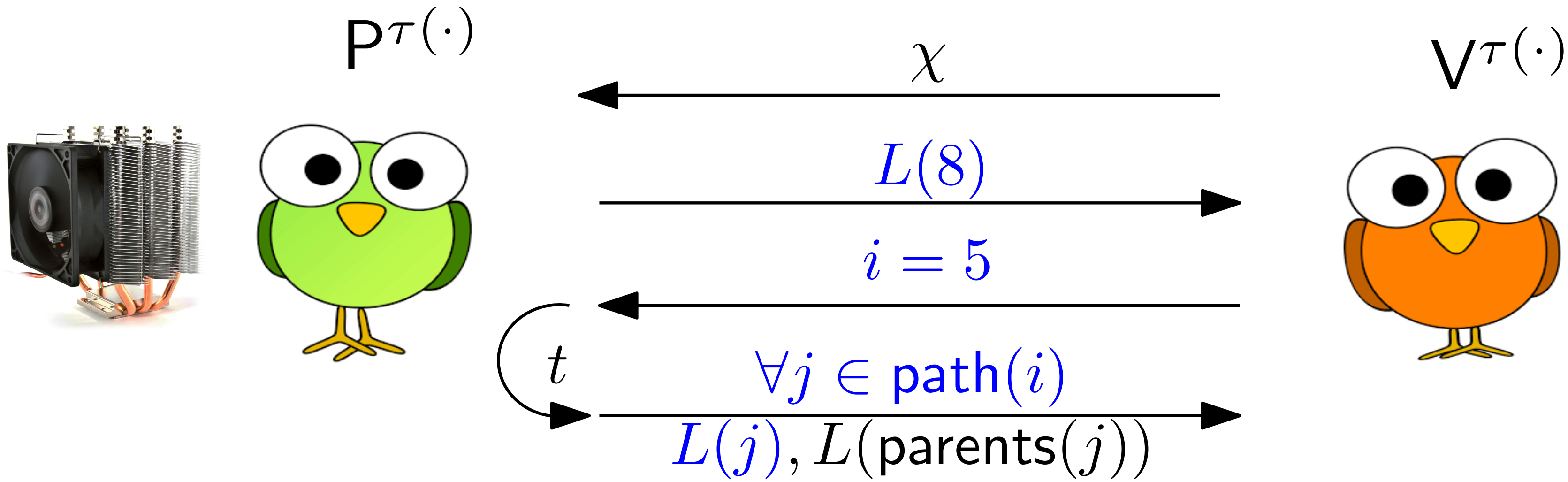

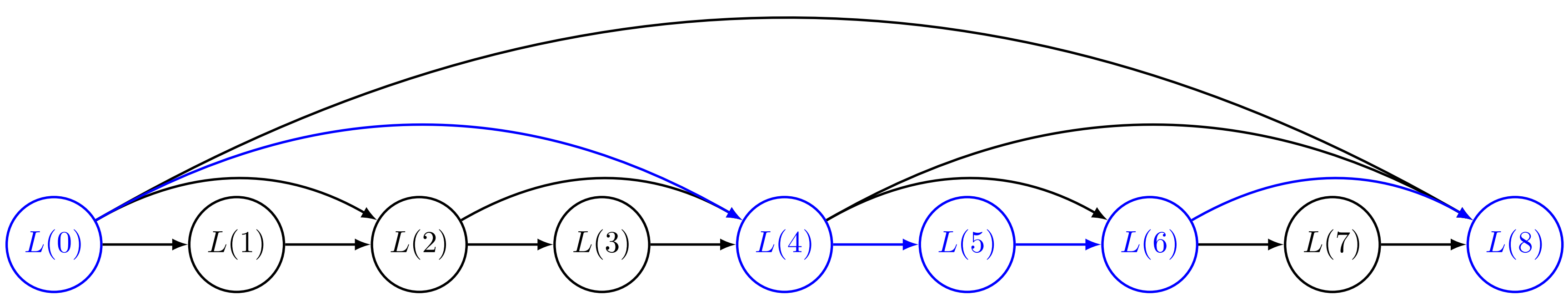

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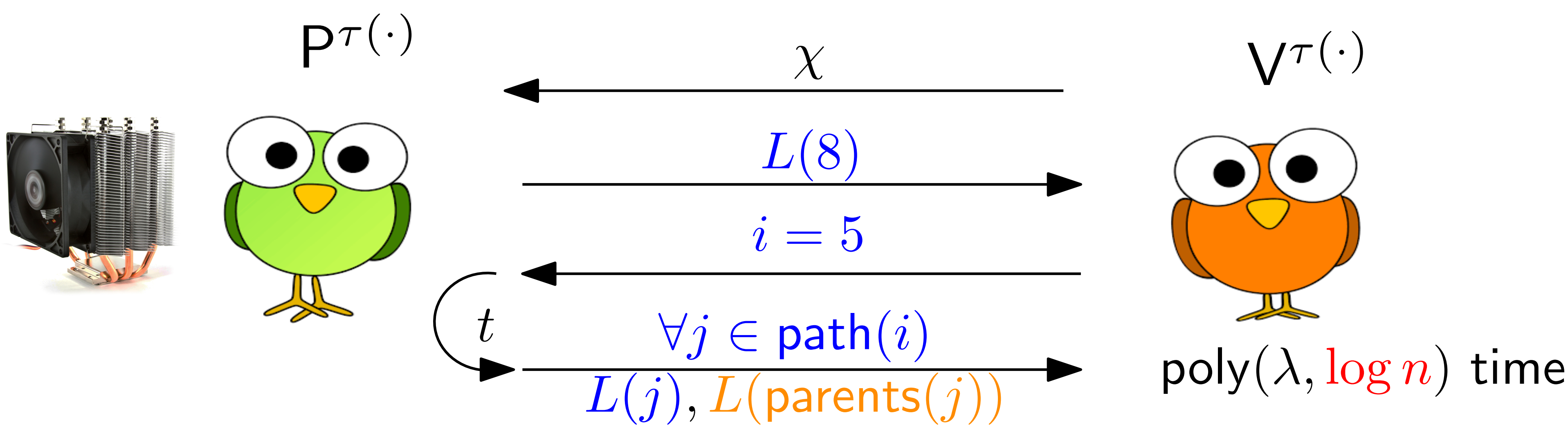

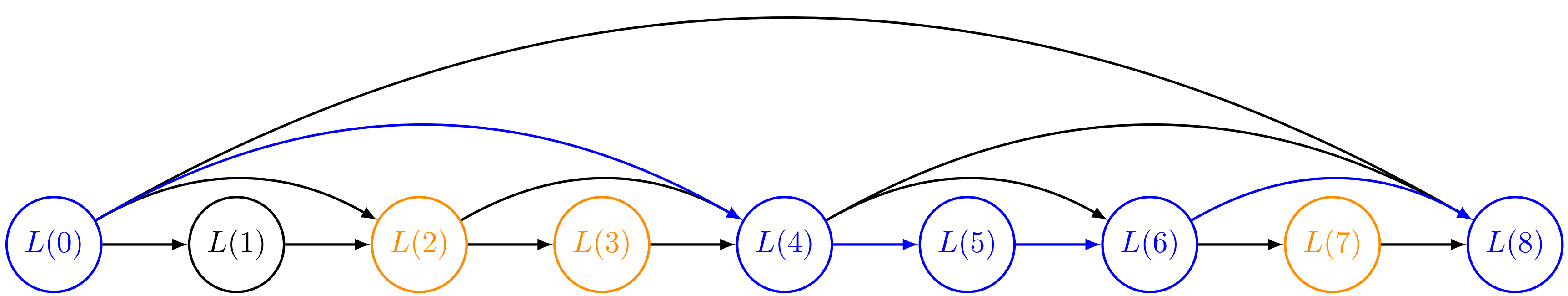

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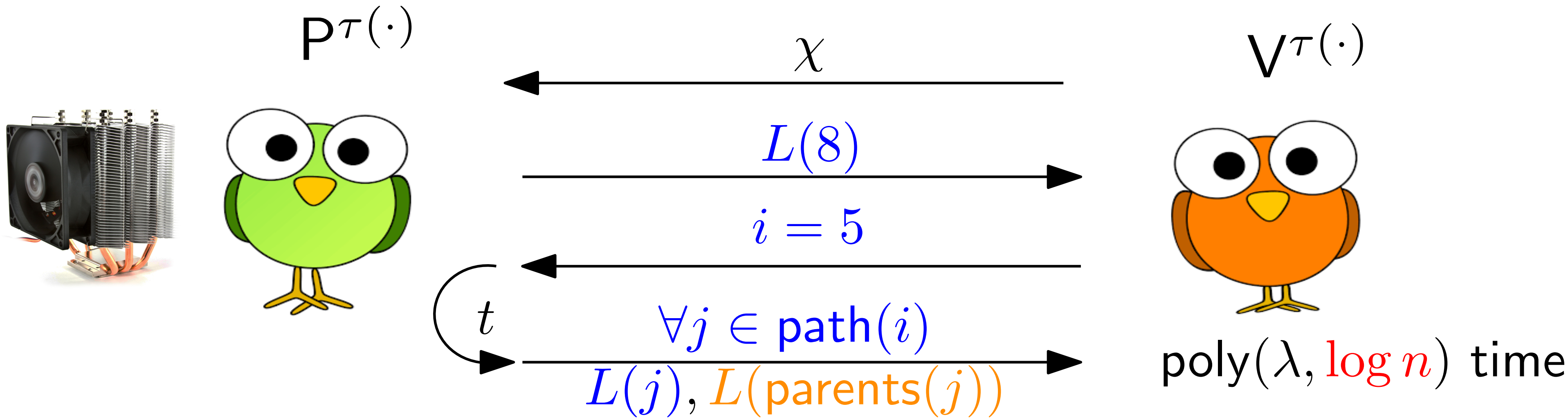
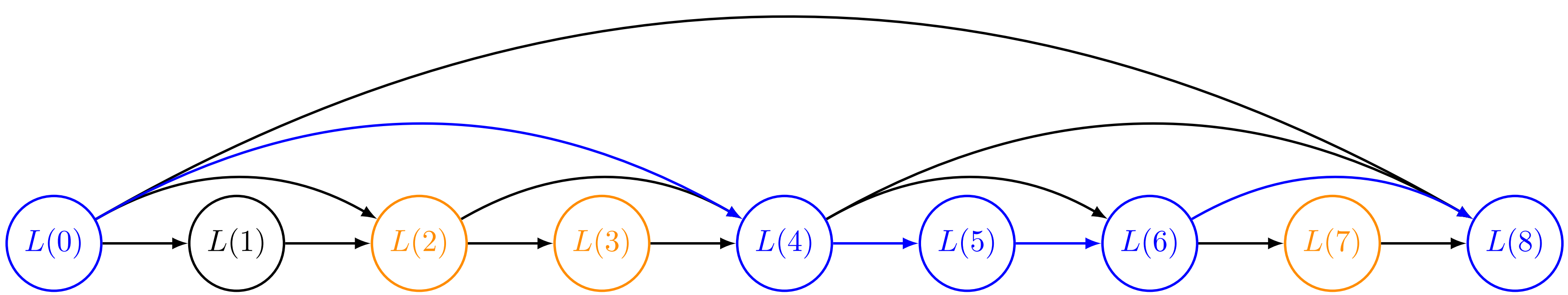


Thm: If 1. $\tilde{\mathsf{P}}_1$ made $\leq \alpha \cdot n$ sequential queries to $\tau(\cdot)$ before sending $L(n)$

2. $\tilde{\mathsf{P}} := (\tilde{\mathsf{P}}_1, \tilde{\tilde{\mathsf{P}}}_2)$ made a total of $\leq q$ queries to $\tau(\cdot)$

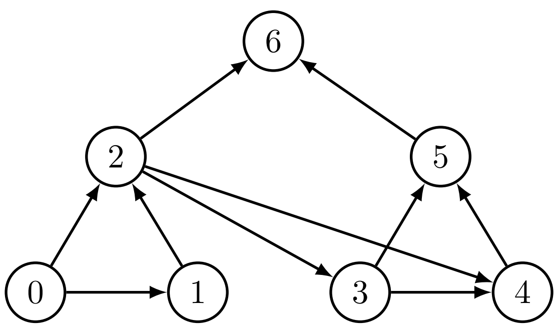
Then $\tilde{\mathsf{P}}$ makes V accept w.p. $\leq \alpha^t + 3 \cdot q^2/2^\lambda$

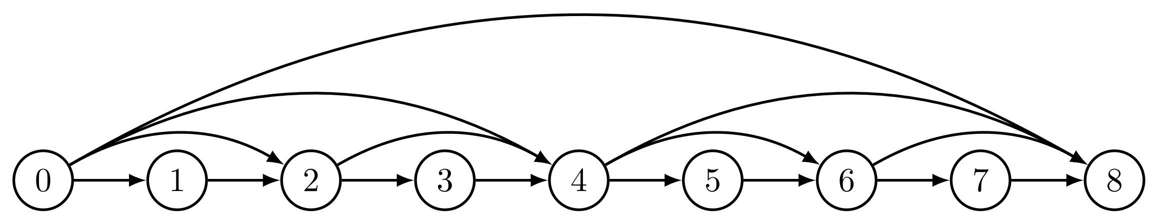


On Our Way to iPoSW

To answer challenges, P has two extremes

- **store all labels** $L(0), \ldots, L(n)$: answering a challenge is just a look-up
- store nothing and spend an extra n **sequential steps** to relabel and answer





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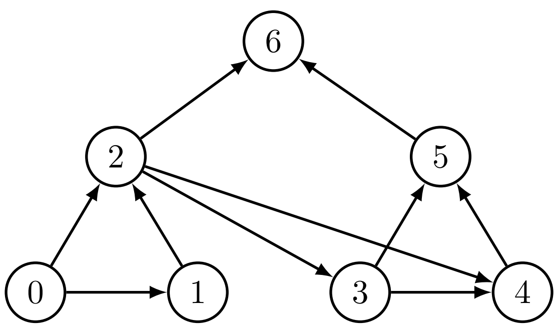

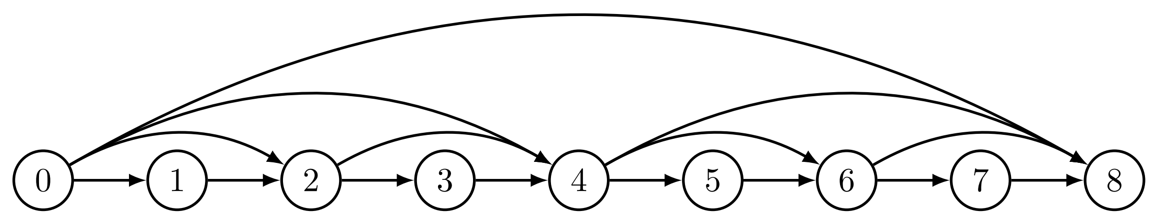

Space-time tradeoffs:

store $\sqrt{n}$ labels and spend an extra $\sqrt{n}$ sequential steps

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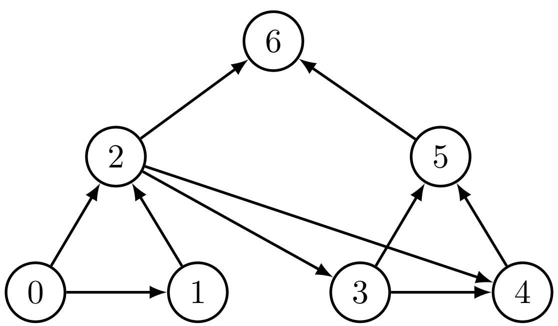

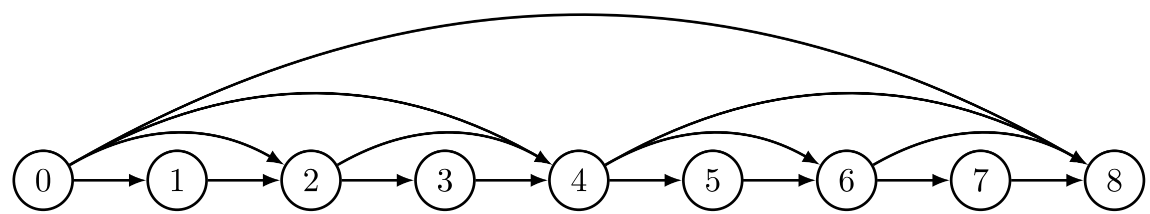

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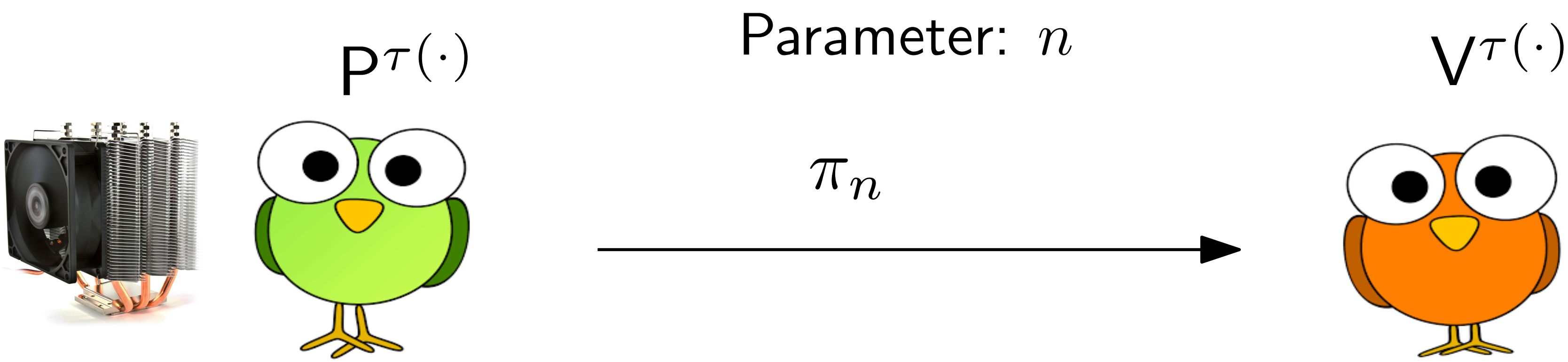
Question: Best of both worlds: can we store a succinct state and spend no extra time?

iPoSW

[Döttling-Lai-Malavolta'19]



An iPoSW is a *non-interactive* proof system $(\mathsf{P}, \mathsf{V}, \mathsf{Inc})$ where

- (P, V) is a PoSW: complete, sound, and succinct

iPoSW

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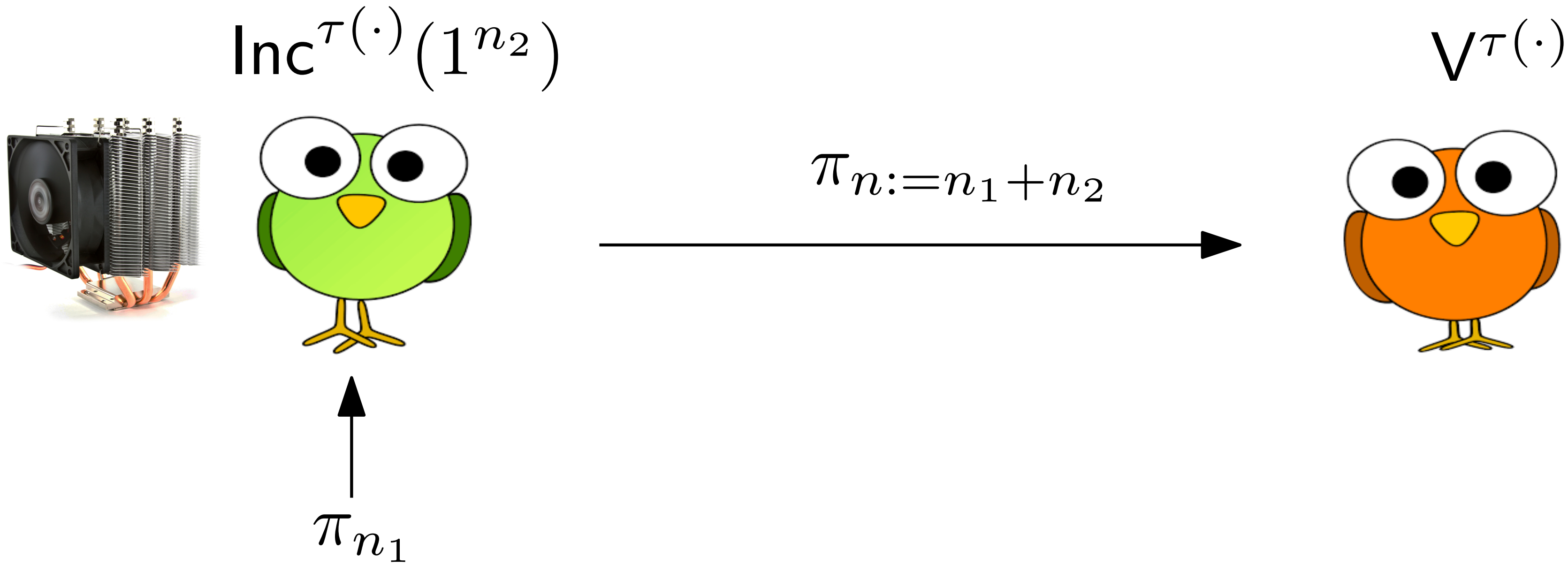

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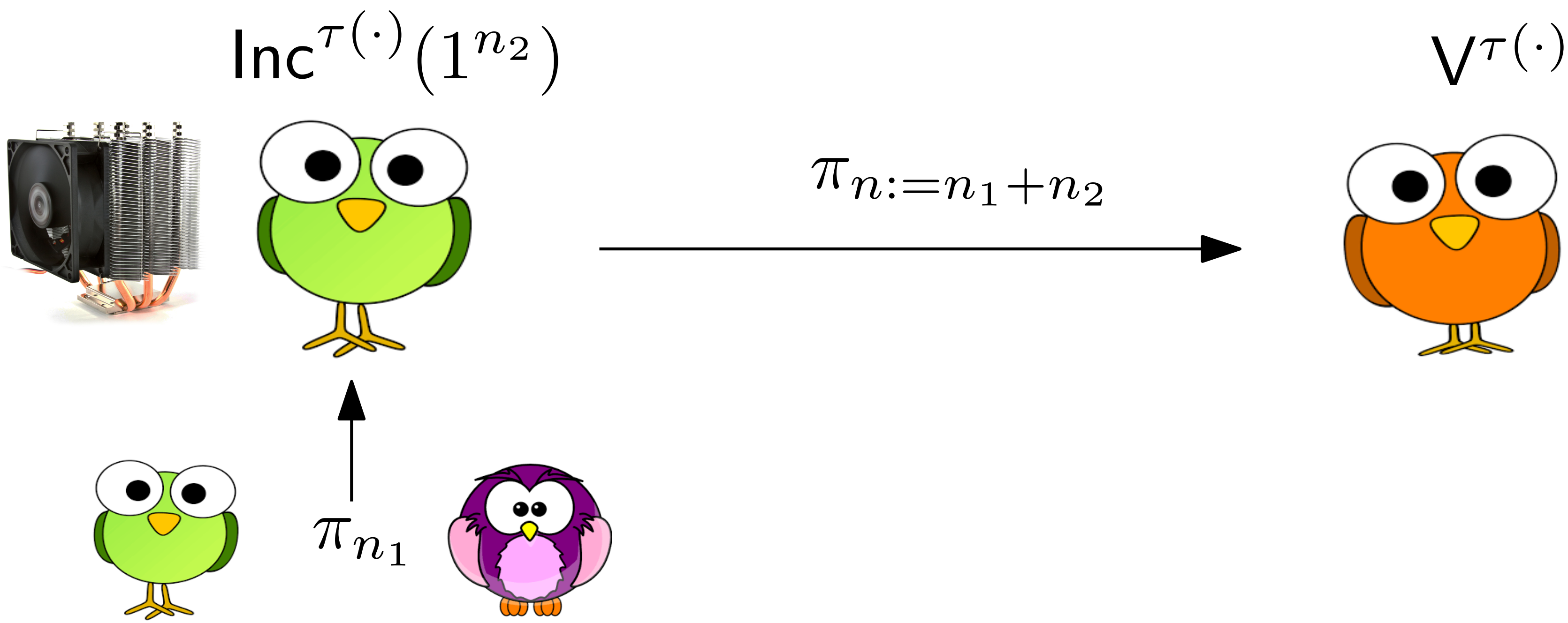

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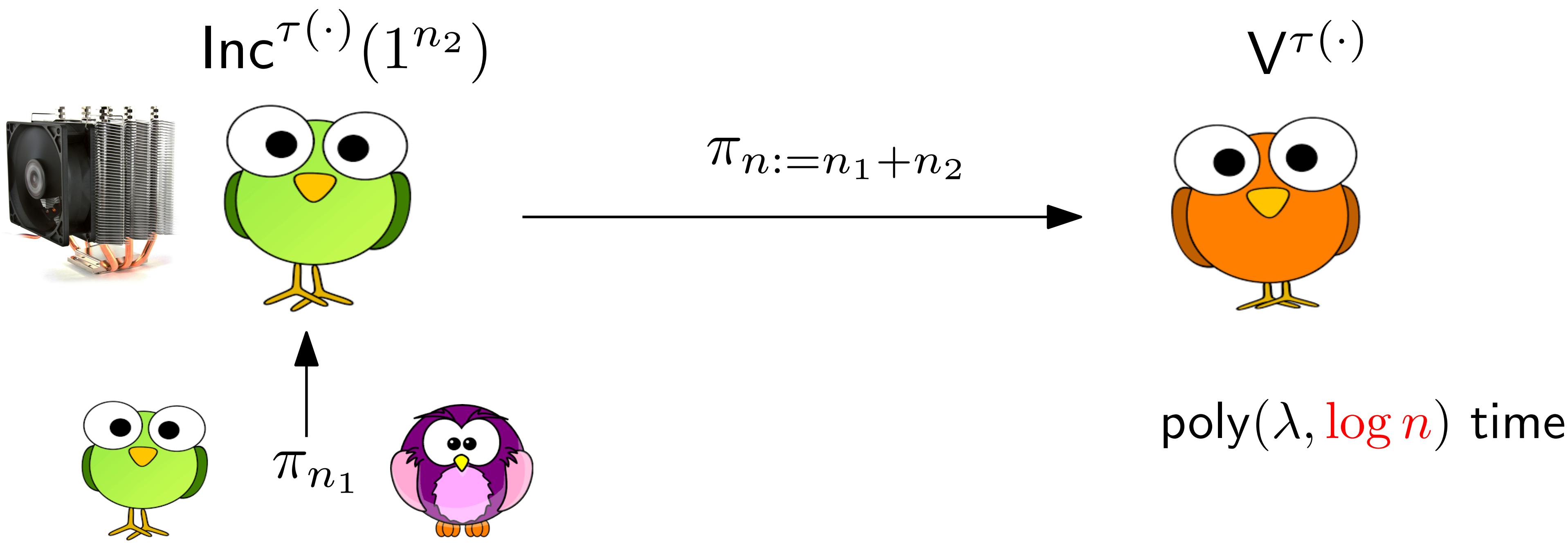

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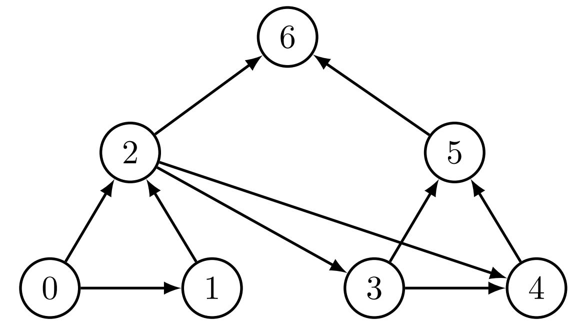
$$\pi_{n_1} \to \mathsf{Inc}(n_2) \to \pi_{n_1+n_2} \to \mathsf{Inc}(n_3) \to \pi_{n_1+n_2+n_3} \to \cdots \to \pi_{n=n_1+\cdots+n_k}$$

An iPoSW Construction

[Döttling-Lai-Malavolta'19]

- Döttling-Lai-Malavolta made Cohen-Pietrzak'18 incremental by *sampling challenges on the fly*
- Efficient, yet incurs an extra small security loss



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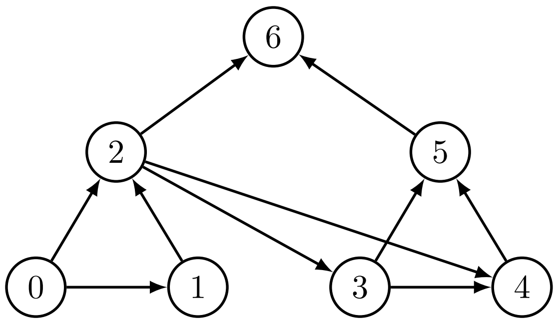

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In this work

1. We similarily make the skiplist PoSW incremental

   We apply the same on-the-fly sampling

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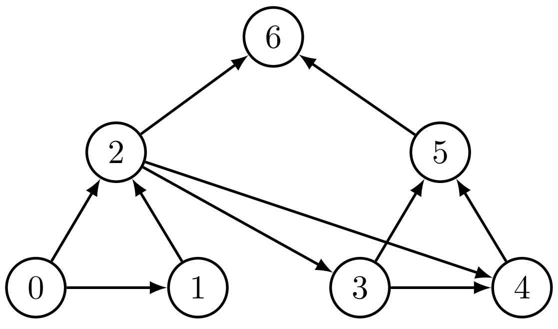

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In this work

1. We similarily make the skiplist PoSW incremental

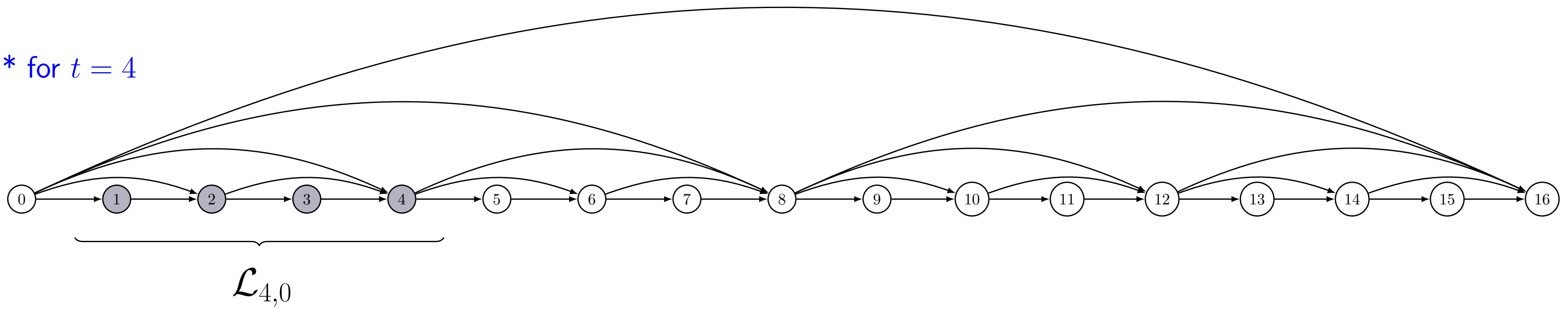
   We apply the same on-the-fly sampling

2. We generalize the skiplist iPoSW to general weight distributions

   We devise a new variant of the on-the-fly sampling technique

Our Standalone iPoSW: The Prover P

* for $t = 4$



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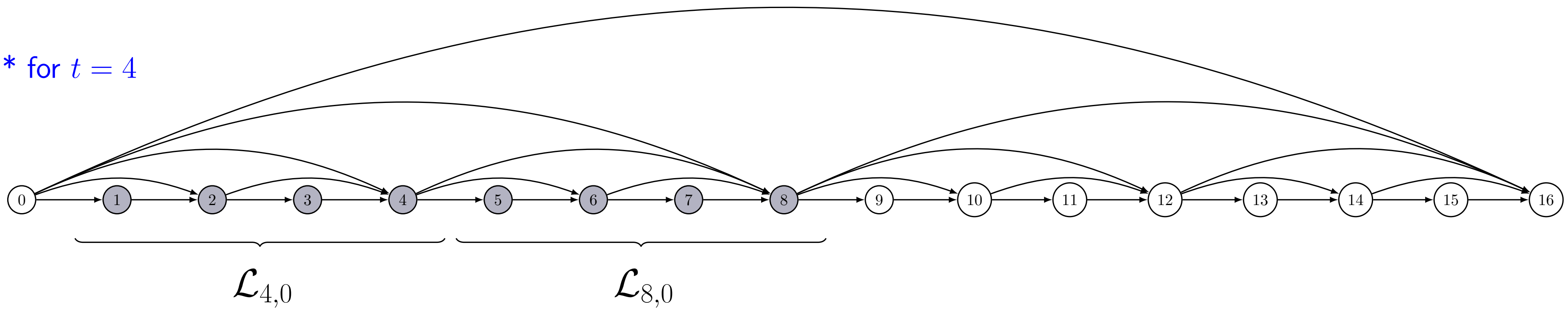

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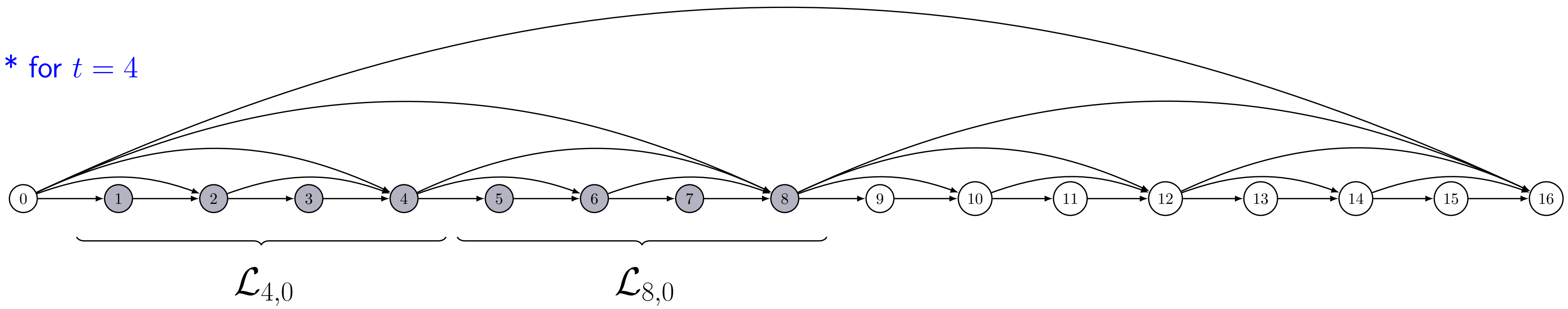
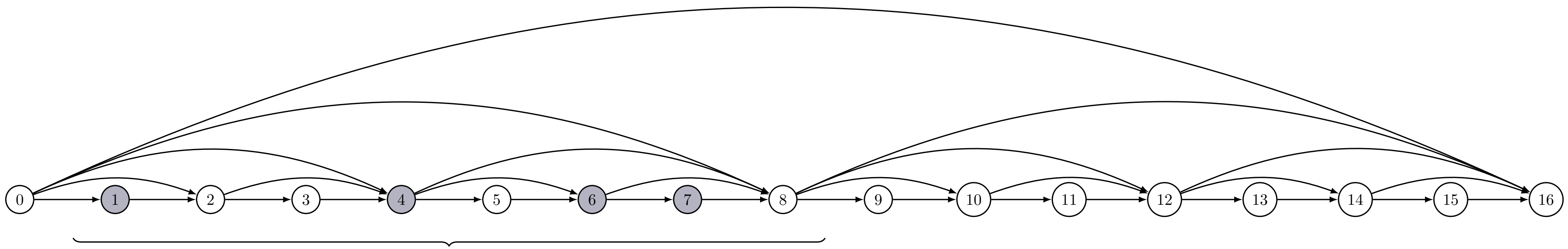


Use randomness from $L(8)$ to randomly sample a set of size 4 from $\{1, \ldots, 8\}$

Our Standalone iPoSW: The Prover P

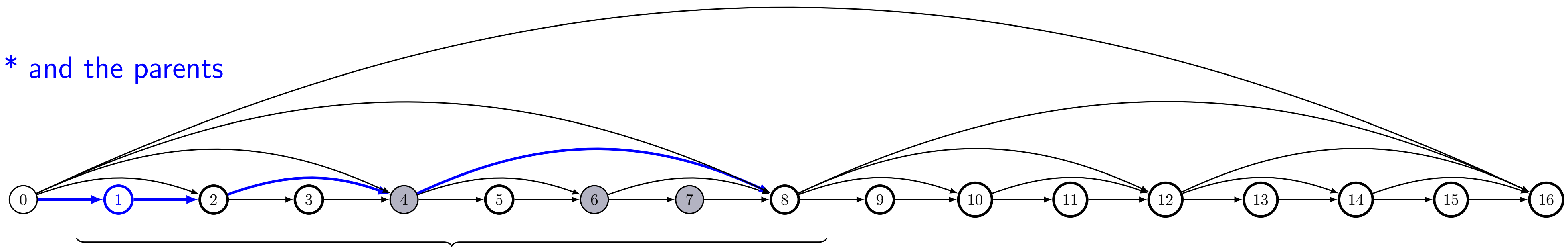


$$\mathcal{L}_{8,1} = \{\pi_1, \pi_4, \pi_6, \pi_7\}$$

$$\pi_i := L(j), L(\text{parents}(j)) \quad \forall j \in \text{path}(i) \text{ in } G_{[0:8]}$$

We have all labels to complile π_i

Our Standalone iPoSW: The Prover P



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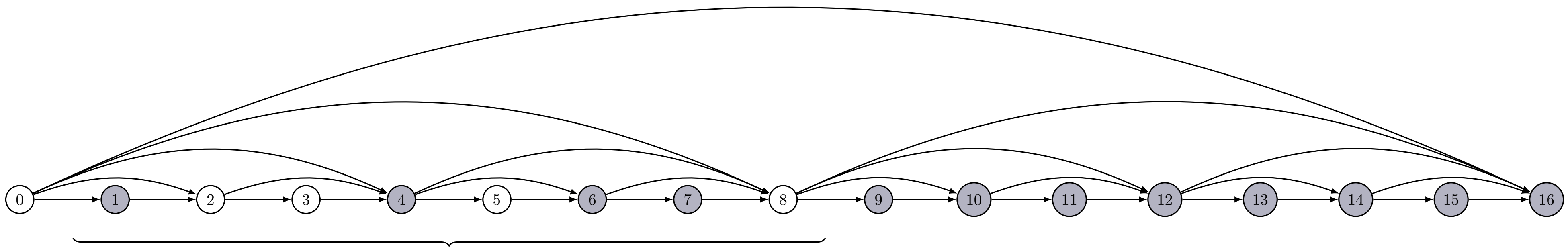

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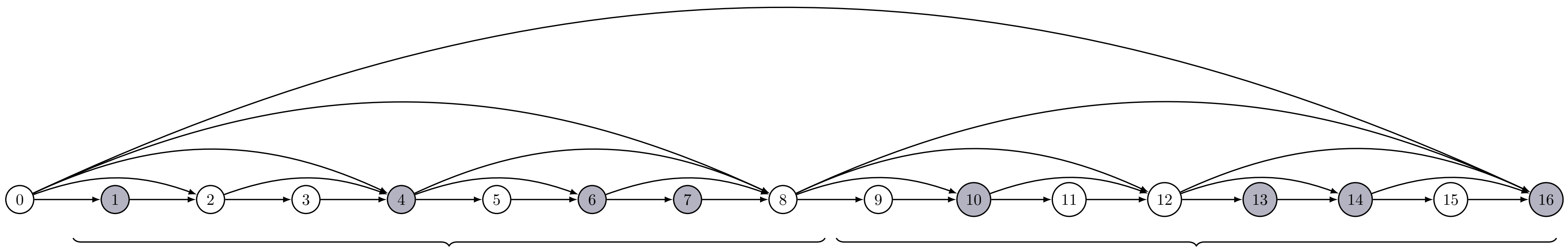

$\mathcal{L}_{8,1} = \{\pi_1, \pi_4, \pi_6, \pi_7\}$ $\mathcal{L}_{16,1} = \{\pi_{10}, \pi_{13}, \pi_{14}, \pi_{16}\}$

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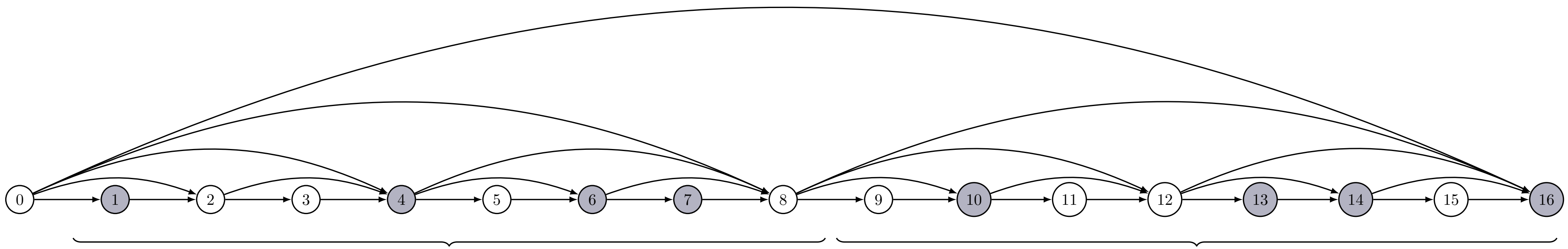

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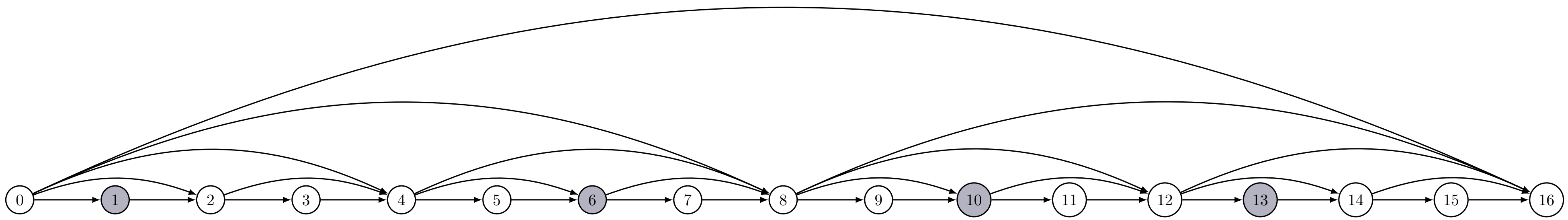
Use randomness from $L(16)$ to randomly sample a set of size 4 from $\{9, \ldots, 16\}$

Our Standalone iPoSW: The Prover P



$$\mathcal{L}_{8,1} = \{\pi_1, \pi_4, \pi_6, \pi_7\} \qquad \mathcal{L}_{16,1} = \{\pi_{10}, \pi_{13}, \pi_{14}, \pi_{16}\}$$

$$\mathcal{L}_{16,2} = \{\pi_1, \pi_6, \pi_{10}, \pi_{13}\}$$

Computing $\mathcal{L}_{16,2}$ from $\mathcal{L}_{8,1}$ and $\mathcal{L}_{16,1}$:

$$\pi_i := L(j), L(\text{parents}(j)) \quad \forall j \in \text{path}(i) \text{ in } G_{[0:8]}, G_{[8:18]} \text{respt.}$$

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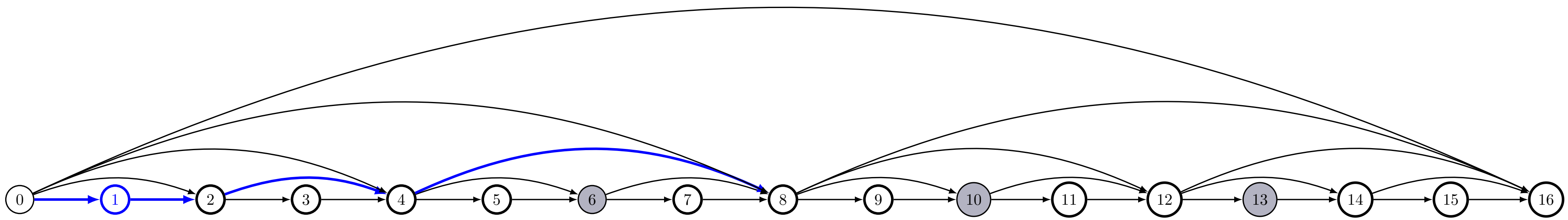

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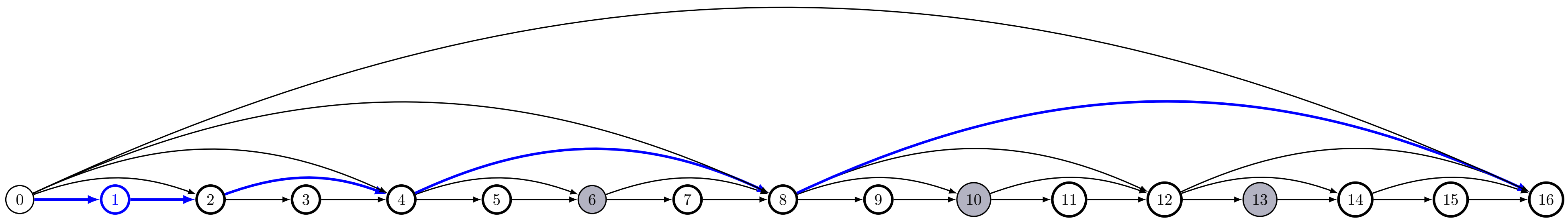
$$\underbrace{\pi_i \text{ in } G_{[0:8]}}_{\text{from } \mathcal{L}_{8,1}}$$

Our Standalone iPoSW: The Prover P



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Computing $\mathcal{L}_{16,2}$ from $\mathcal{L}_{8,1}$ and $\mathcal{L}_{16,1}$:

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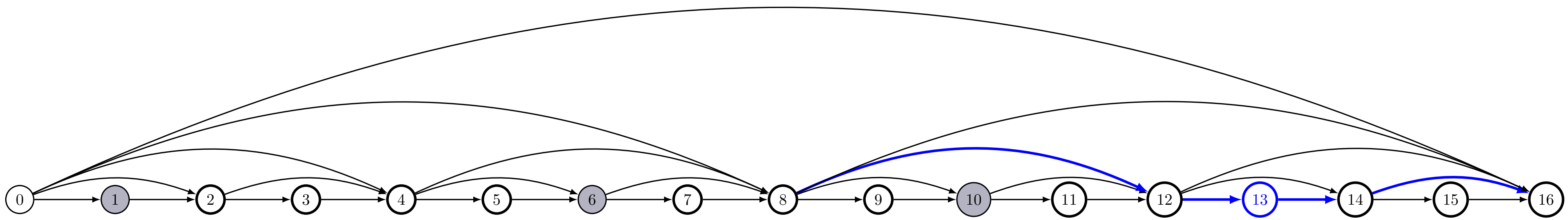

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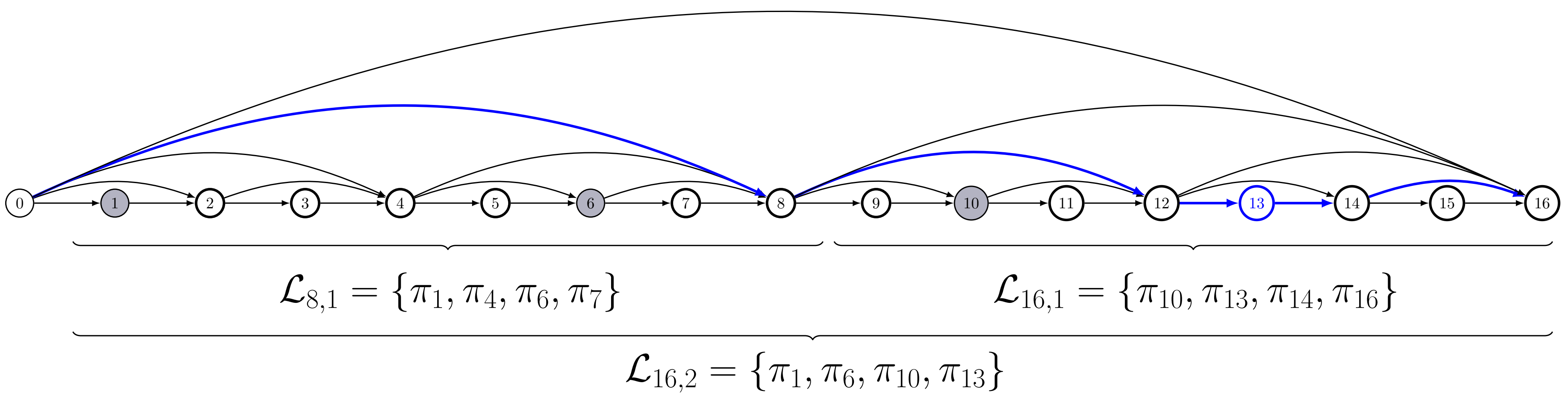

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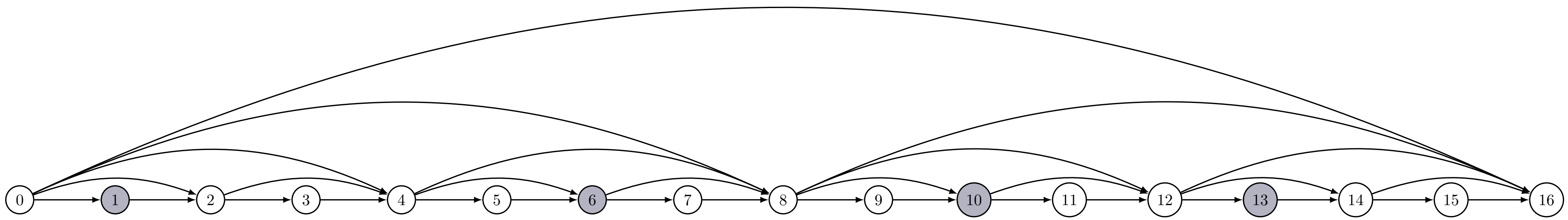
Our Standalone iPoSW: The Inc Algorithm



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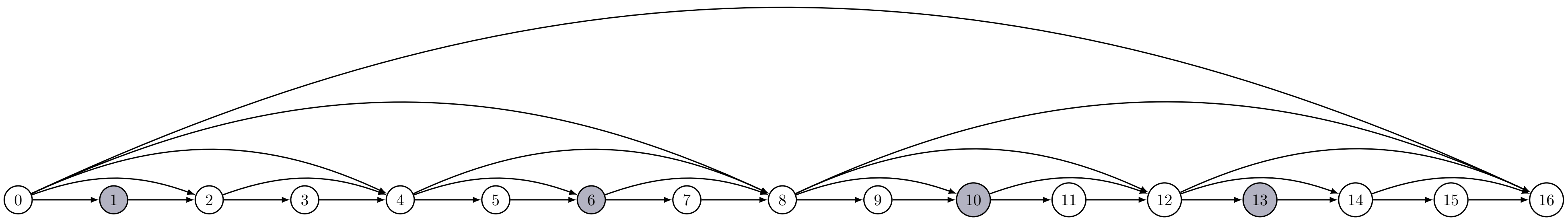
Inc **works exactly as** P: it picks up the computation where P leaves it and it constinues exactly as P would have continued

Standalone iPoSW Verifier V



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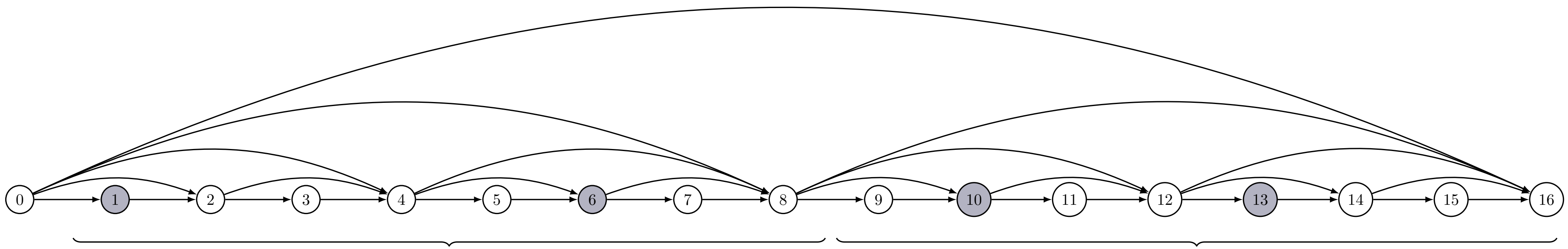

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- P provides V for every π_i with an index set $\mathcal{I}_i$
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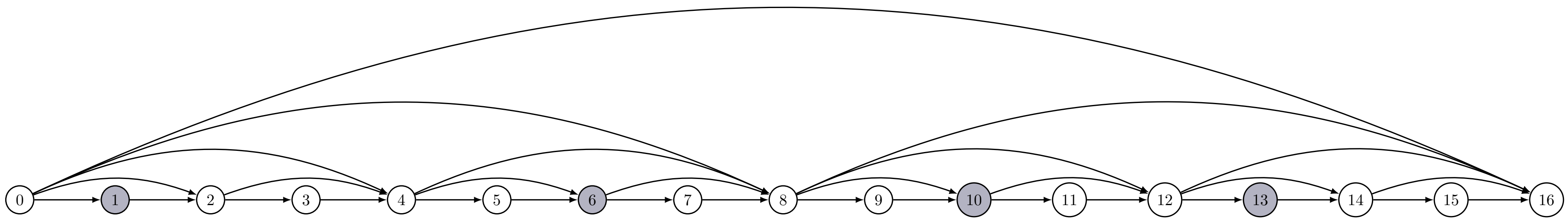

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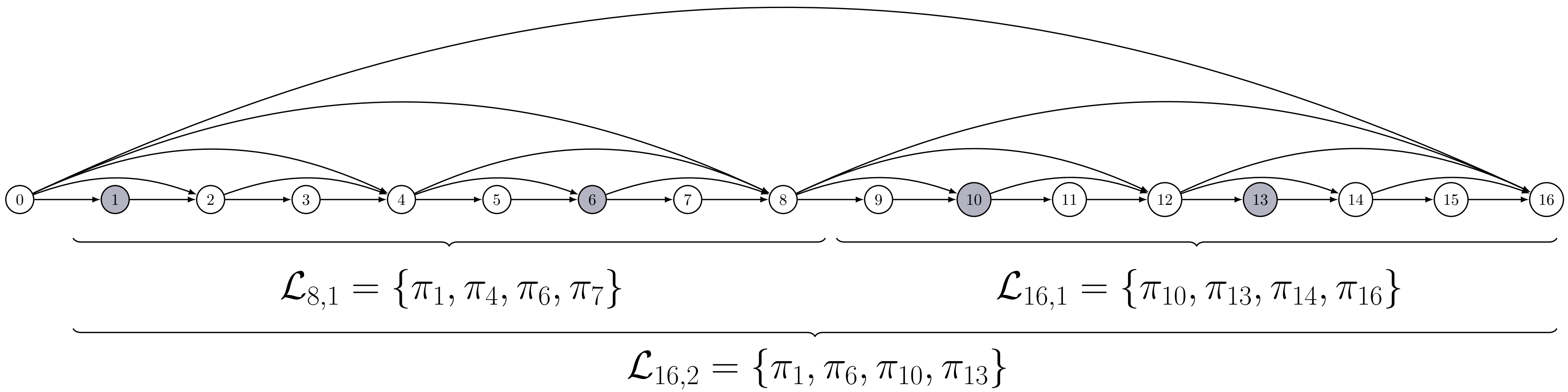

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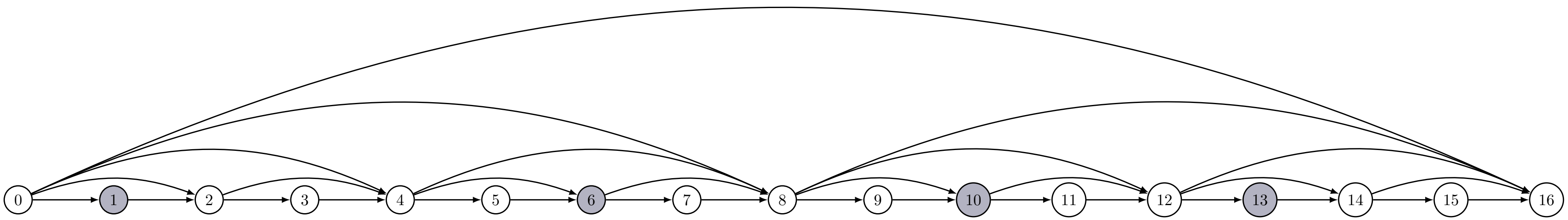
For general weight distributions: the sampling sets are of size t **on expectation**

Soundness



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1. Bound the advantage the on-the-fly sampling gives a malicous $\tilde{\mathsf{P}}$
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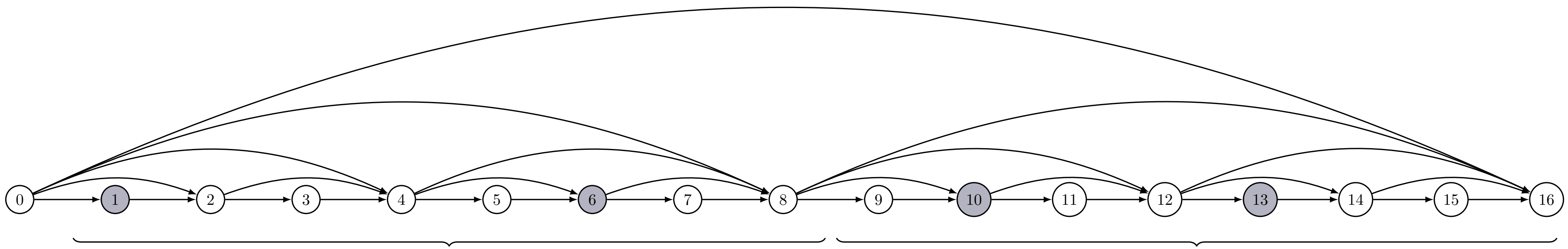

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The On-The-Fly Sampling Lemma:

1. $S := \mathcal{L}_{16,2}$ sampled from $S_0 := \mathcal{L}_{8,1} \cup S_1 := \mathcal{L}_{16,1}$ as above, or
2. S sampled directly from $\{1, \ldots, 16\}$

Show that the $\%$ of incosnsistent nodes in S in 1. and 2. are close

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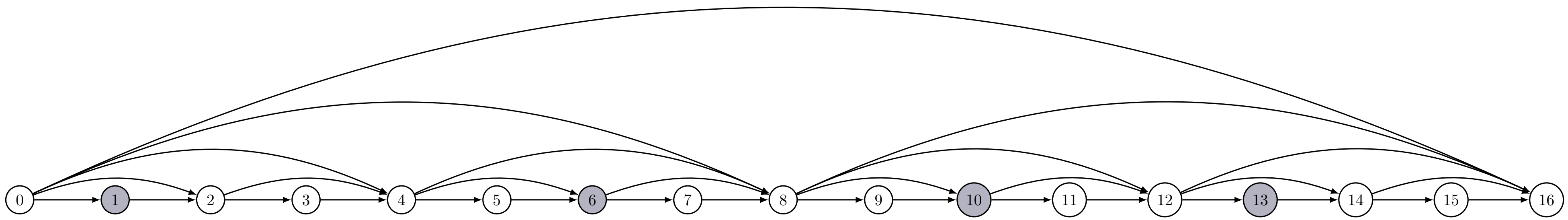

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Show that the % of incosnsistent nodes in S in 1. and 2. are close

This follows easily from a Hoeffding bound

Security Statement

Thm: $(\mathsf{P}, \mathsf{V}, \mathsf{Inc})$ an (α, ϵ)-sound **iPoSW** for $\alpha \in (0, 1]$ and

$$\epsilon = \frac{1 + q^2}{2^\lambda} + \frac{q(q-1)}{2^{\lambda+1}} + q \cdot e^{-2t \cdot \left(\frac{1-\alpha}{\log n}\right)^2} \quad .$$

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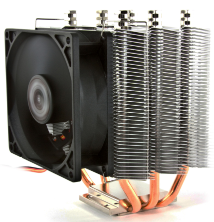
Thm (Standalone PoSW): (P, V) is an (α, ϵ)-sound **PoSW** for $\alpha \in (0, 1]$ and

$$\epsilon = \frac{3 \cdot q^2}{2^\lambda} + \alpha^t .$$

General Weight/Challenge Distributions

Parameters: $G_n, t, \cdots$

$\mathsf{P}^{\tau(\cdot)}$ $\qquad$ $\mathsf{V}^{\tau(\cdot)}$



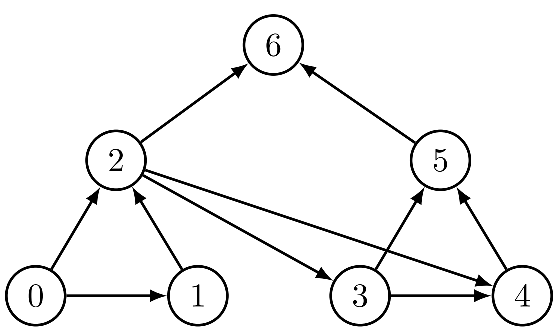
$\xleftarrow{\quad \chi \quad}$

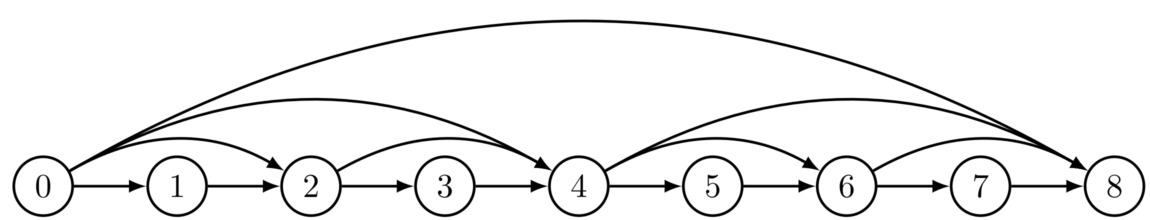
$\xrightarrow{\quad L(n) \quad}$

$\xleftarrow{\quad S \xleftarrow{\$} \mathcal{C} \subseteq \{0, \ldots, n\} \quad}$

$\xrightarrow{\quad r_S \quad}$

Standalone (i)PoSW: Sample a random S of size t from $\mathcal{C}$





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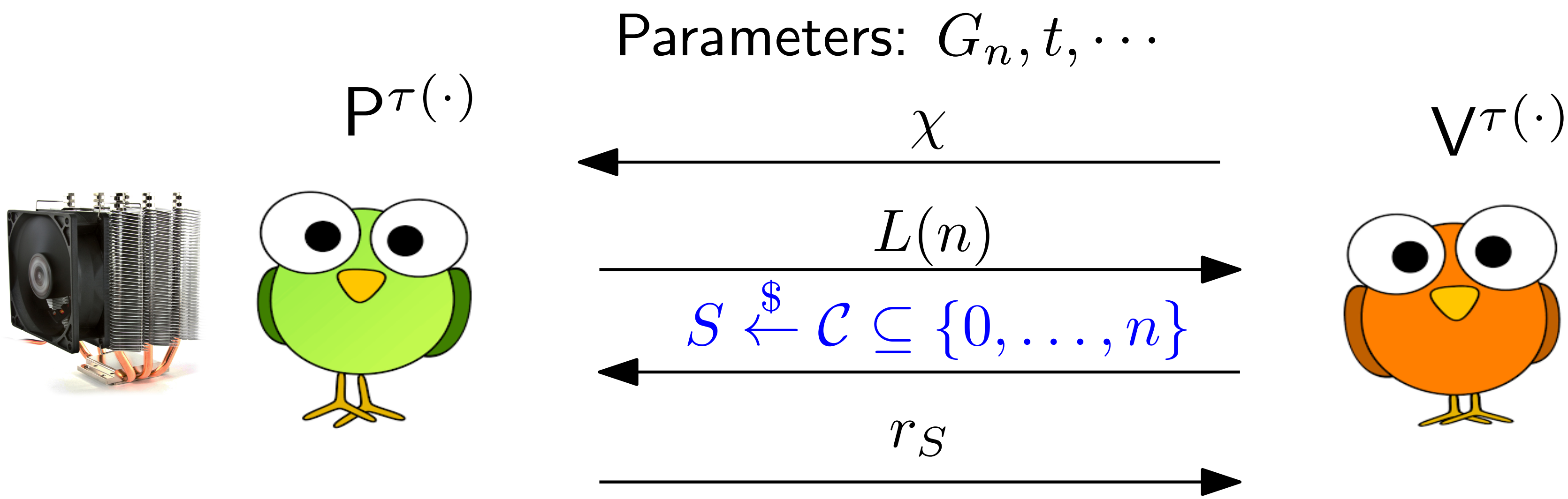
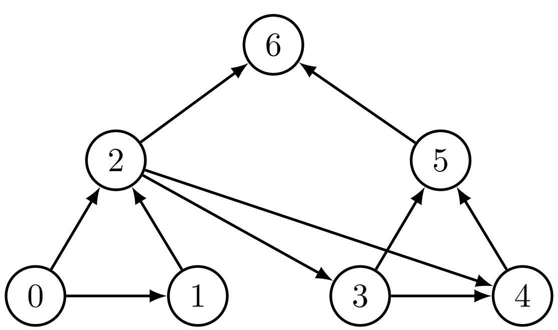
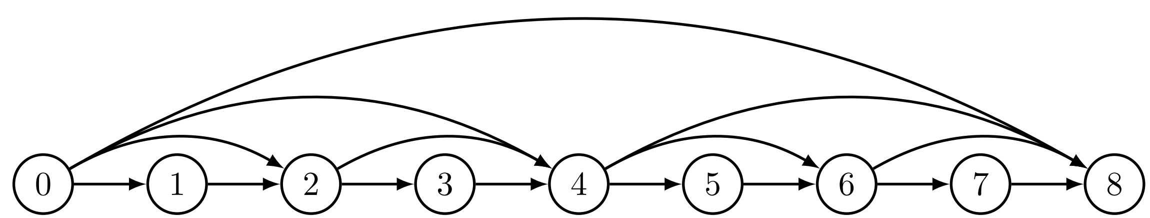


Standalone (i)PoSW: Sample a random S of size t from $\mathcal{C}$

For some applications, not all challenges in $\mathcal{C}$ are treated equally



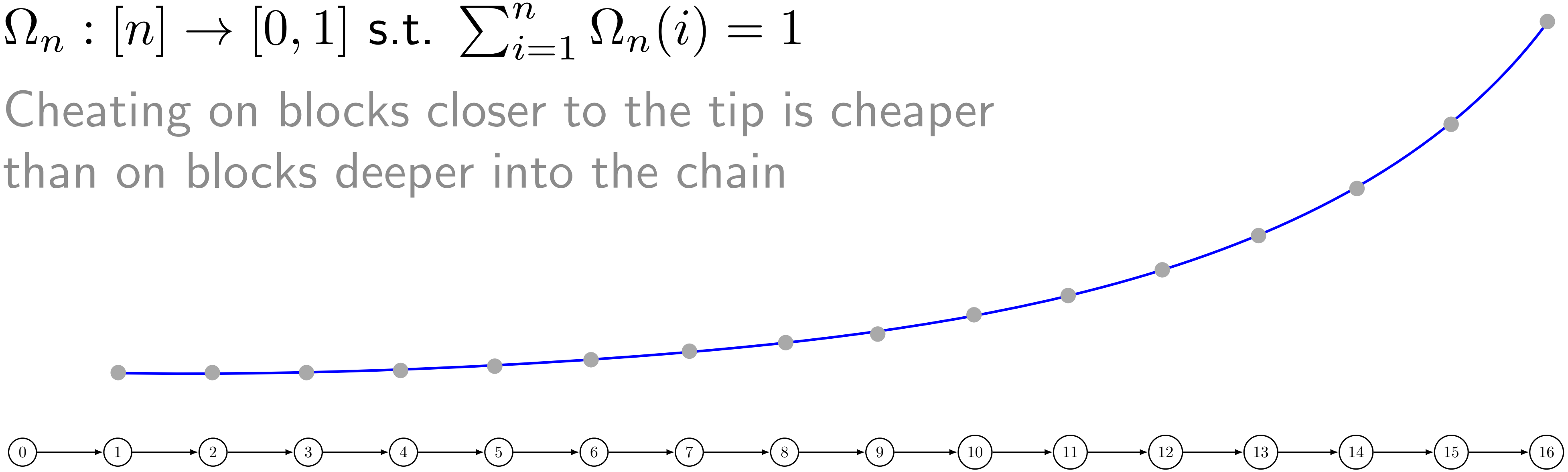


The SNACK Weight Distribution

The SNACK weight distribution $\Omega_n(i) \sim \frac{1}{n-i+c}$

$\Omega_n : [n] \to [0,1]$ s.t. $\sum_{i=1}^{n} \Omega_n(i) = 1$

Cheating on blocks closer to the tip is cheaper than on blocks deeper into the chain



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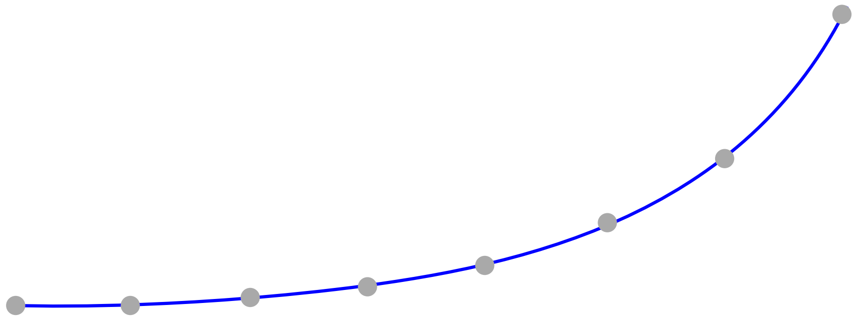

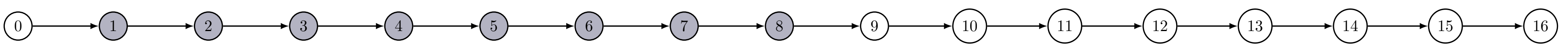

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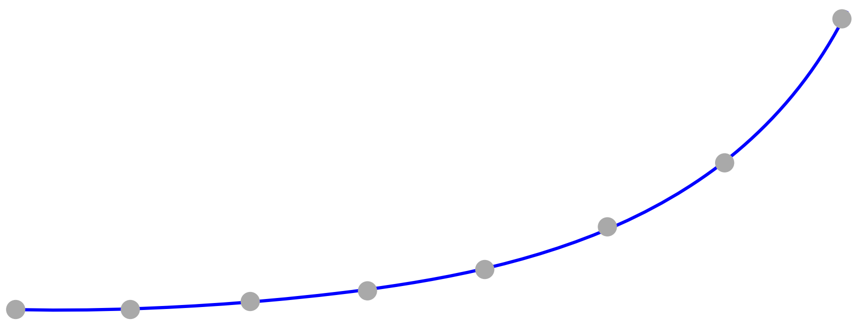

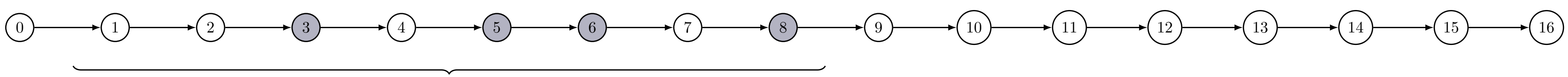

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Add π_i to $\mathcal{L}_{8,1}$ w.p. $t \cdot \Omega_n(i)$ $\Rightarrow$ $|\mathcal{L}_{8,1}| = t$ on expectation

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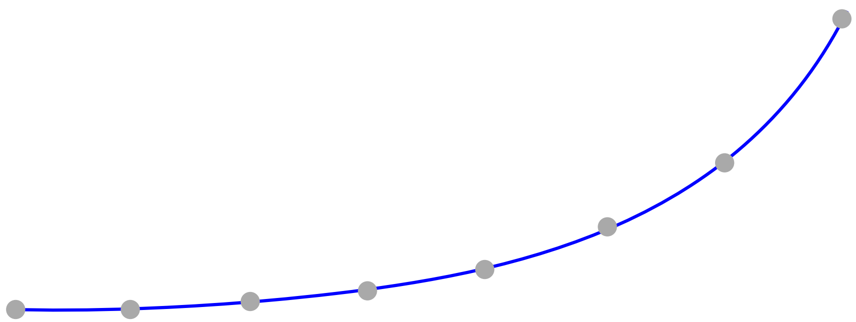

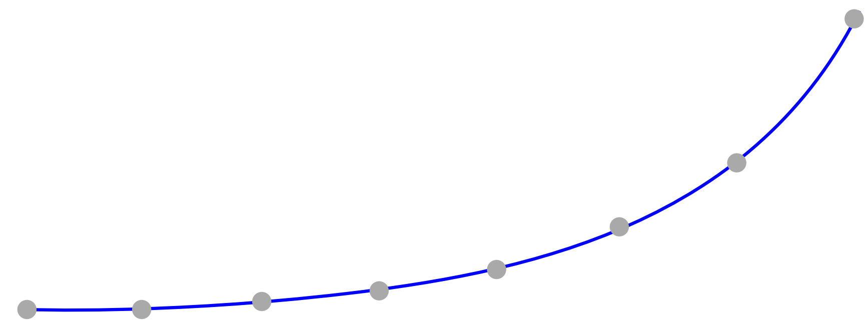

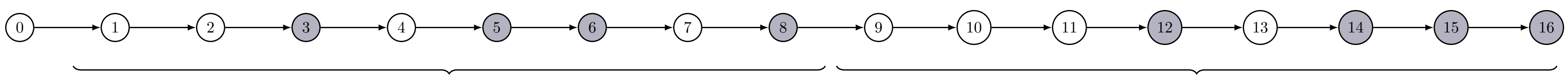

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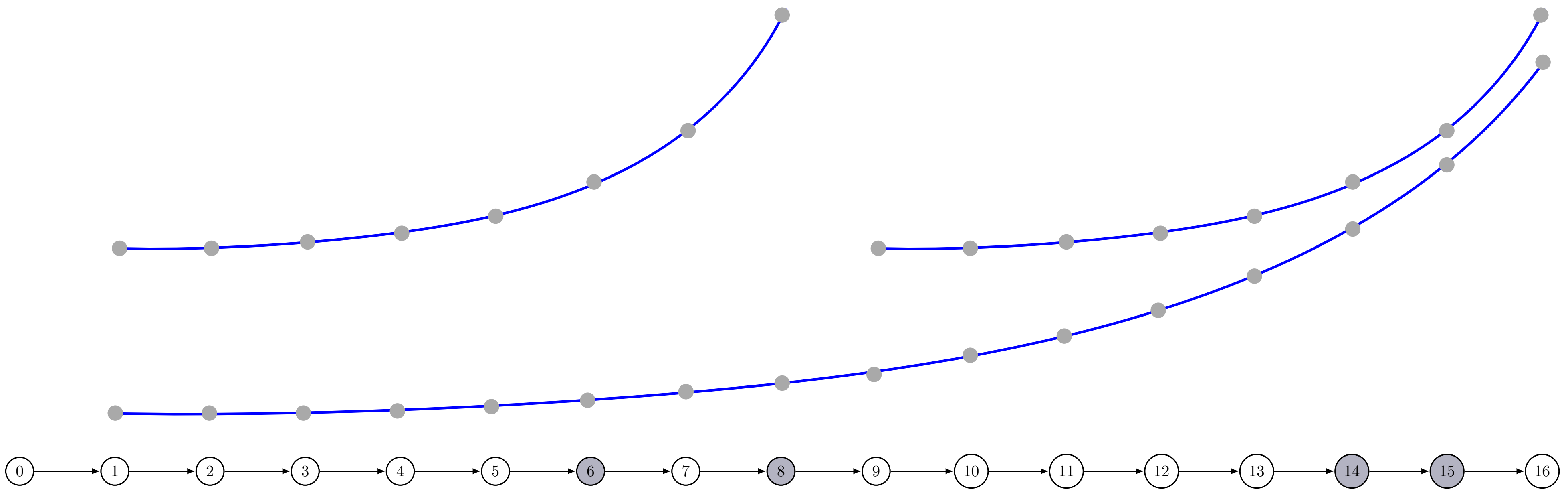

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Add π_i to $\mathcal{L}_{16,2}$ w.p. $t \cdot \Omega_n(i)$ $\Rightarrow$ $|\mathcal{L}_{16,2}| = t$ on expectation

Our iPoSW for General Weight Distributions

We characterize distributions that can be sampled incrementally:
t-**Incrementally Sampleable Distributions**

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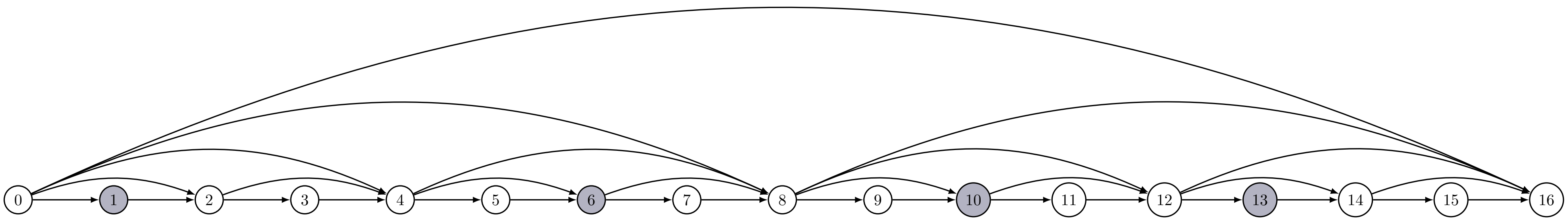
- P now provides the sampling sets explicitly as part of the proof
- V recursively checks the consistency of the samplings as before and that these sets are within their expected size

Soundness



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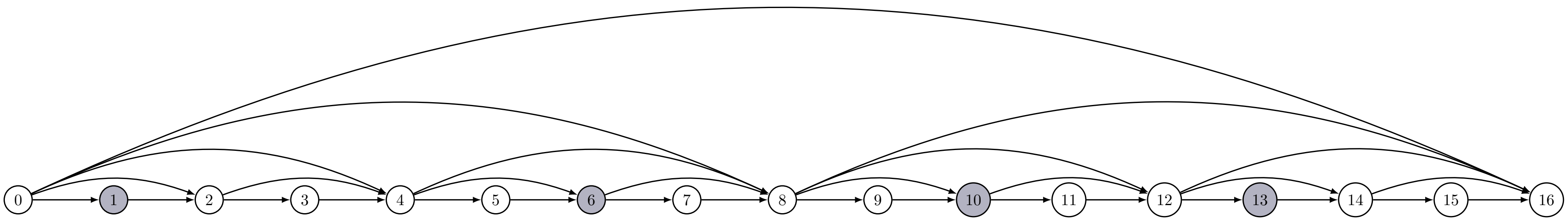

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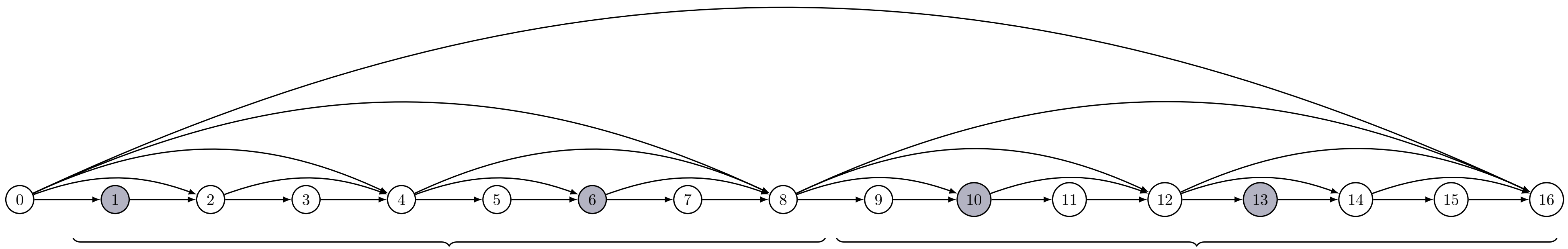
We give bounds for any t-incrementally sampleable weight distributions

We give concrete bounds for the uniform and SNACK distributions

Thank you

Additional Material

Efficiency Measures



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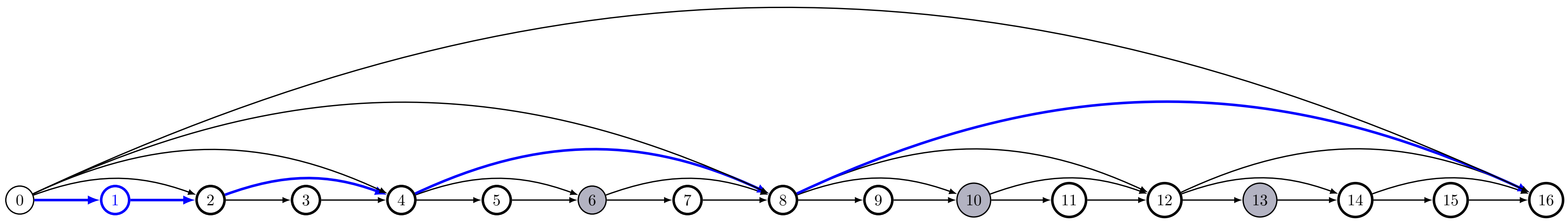
Prover space complexity:

1. The skiplist G_n can be topologically labeled with space $(\log n + 1)\lambda$ bits
2. At no time P or Inc keeps more than $\log n + 1$ lists $\mathcal{L}_{v,i}$, each of succinct size

$\Rightarrow$ proof size: $O(\lambda \cdot t \cdot \log^3 n)$

Our Standalone iPoSW: The Verifier V



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$\mathcal{I}_1 = \{1, 1\}$

$$S_{0,1} = \{1,2,3,4\} \cup S_{1,1} = \{5,6,7,8\}$$

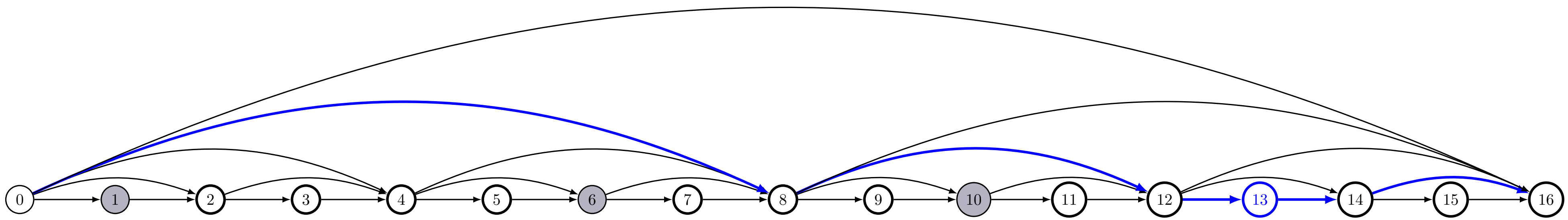
$$\rightarrow_{L(8)} S_1 = \{1,4,6,7\}$$

$$S_{0,2} = \{1,4,6,7\} \cup S_{1,2} = \{10,13,14,16\}$$

$$\rightarrow_{L(16)} S_2 = \{1,6,10,13\}$$

Note: $L(8), L(16) \in \pi_1$

Our Standalone iPoSW: The Verifier V



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$\mathcal{I}_{13} = \{1, 2\}$

$$S_{0,1} = \{9, 10, 11, 12\} \cup S_{1,1} = \{13, 14, 15, 16\}$$

$$\to_{L(16)} S_1 = \{10, 13, 14, 16\}$$

$$S_{0,2} = \{1, 4, 6, 7\} \cup S_{1,2} = \{10, 13, 14, 16\}$$

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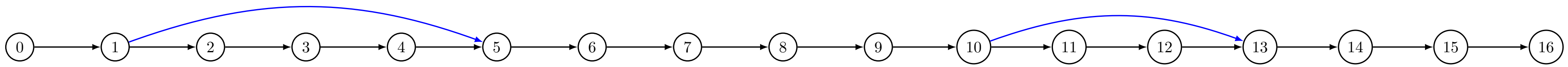
Note: $L(16) \in \pi_{13}$

Succinct Non-interactive Arguments of Chain Knowledge

[Abusalah-Fuchsbauer-Gaži-Klein'22]

Weighted Blockchain: $\Gamma_n = (H_n, \Omega_n)$ with weight function

$$\Omega_n : [n] \to [0,1] \text{ s.t. } \sum_{i=1}^{n} \Omega_n(i) = 1$$



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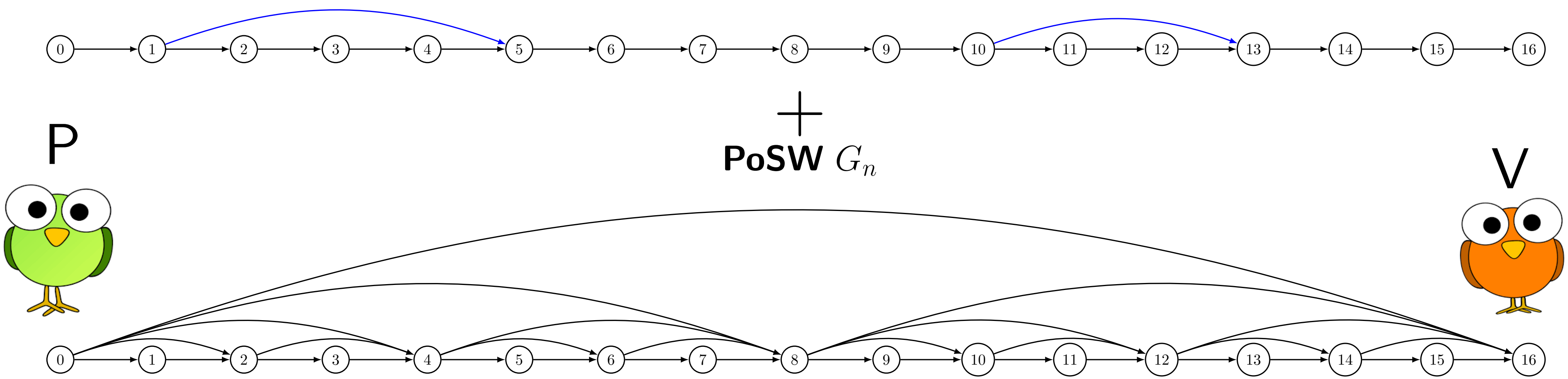

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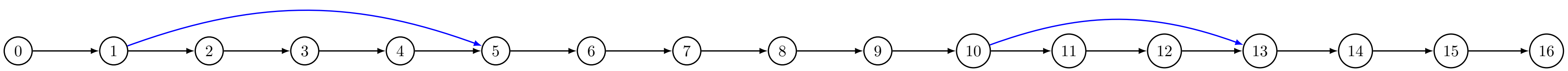

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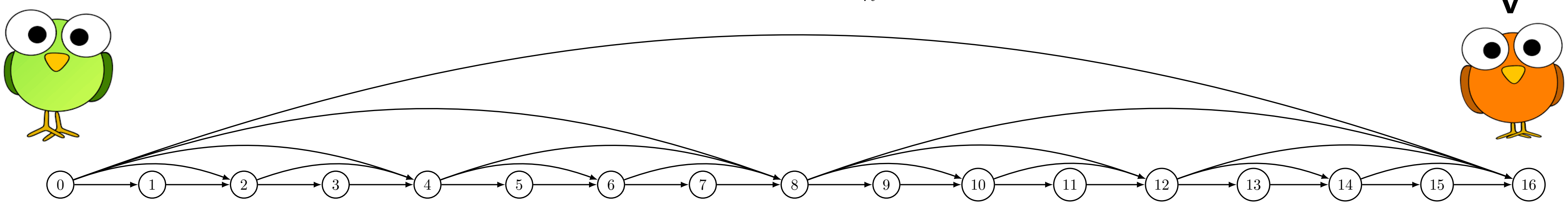
P

PoSW G_n

V



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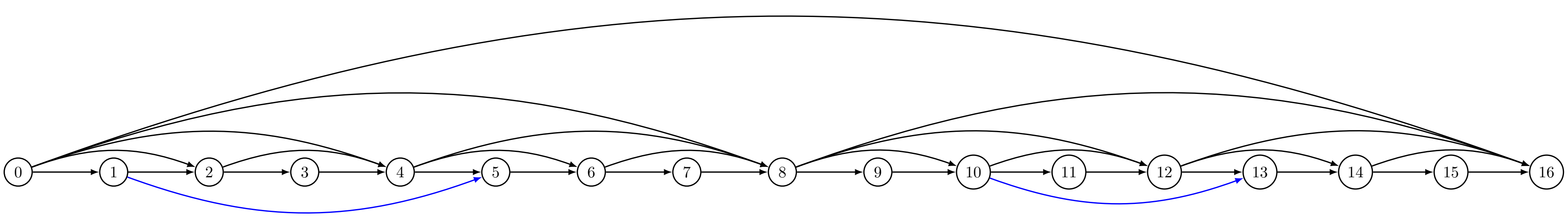
Augmented DAG $(K := ([n]_0, E_K := E_H \cup E_G), \Omega_n)$



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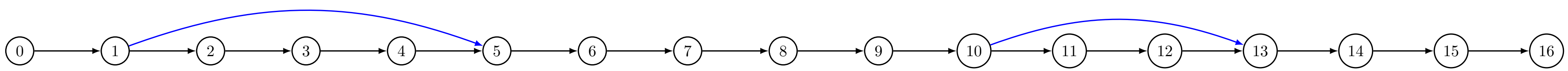

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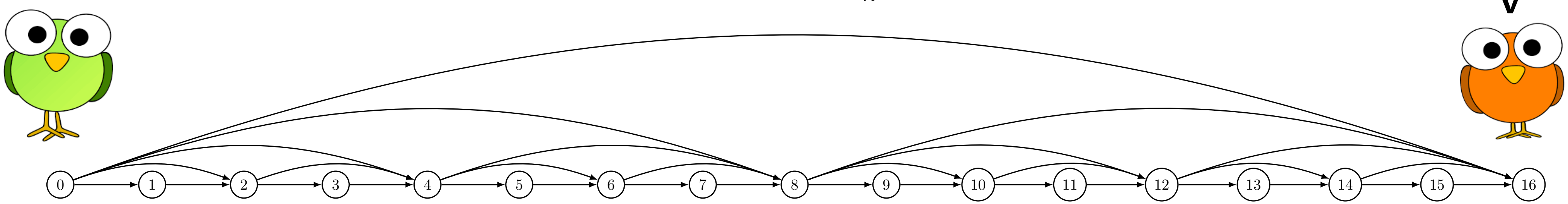

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Augmented DAG $(K := ([n]_0, E_K := E_H \cup E_G), \Omega_n)$

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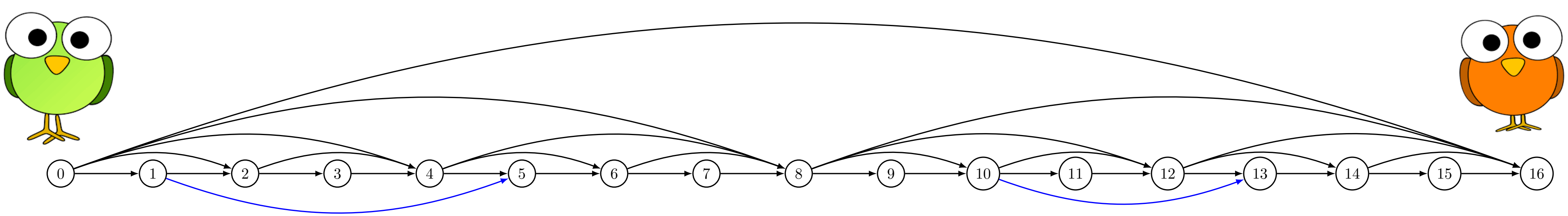
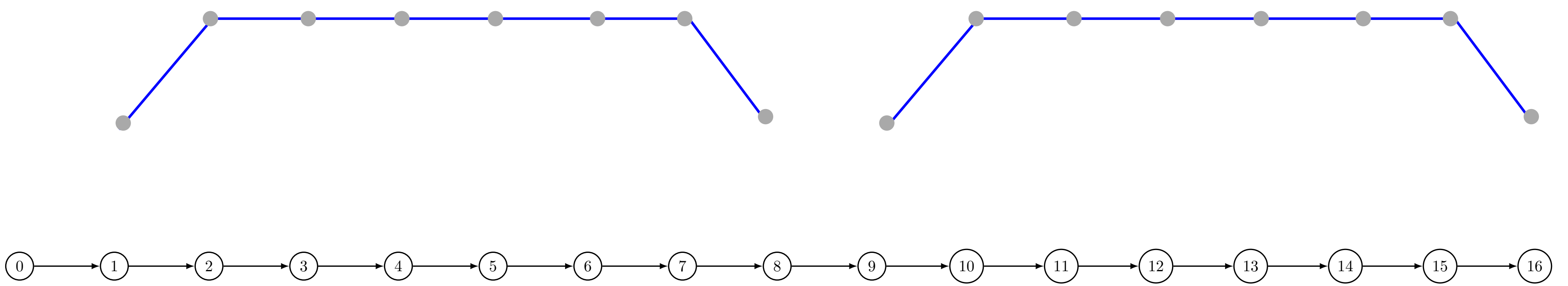


Not All Distributions Are Incrementable



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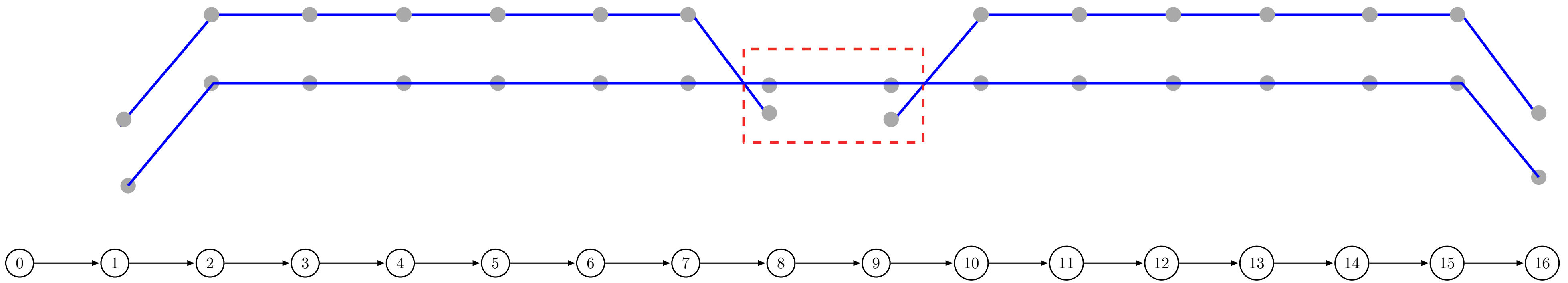

We characterize distributions that can be sampled incrementally:
t-incrementally Sampleable Distributions

Give a constrution for any t-sampleable distribution over the skiplist graph

Malicious Sampling Sets

$\tilde{\mathsf{P}}$ can maliciously choose the sampling sets

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We bound the advantage of $\tilde{\mathsf{P}}$ in these cases and give concrete bounds for the uniform and SNACK distributions

Security Statement

Thm (iPoSW for the uniform weight distribution): $(\mathsf{P}, \mathsf{V}, \mathsf{Inc})$ an (α, ϵ)-sound **iPoSW** for $\alpha \in (0, 1]$ and

$$\epsilon = \frac{1+q^2}{2^\lambda} + \frac{q(q-1)}{2^{\lambda+1}} + q \cdot e^{-2t \cdot \left(\frac{1-\alpha}{\log n}\right)^2} + q \cdot 2^{-\zeta \cdot t} \quad .$$

Thm (Standalone iPoSW): $(\mathsf{P}, \mathsf{V}, \mathsf{Inc})$ an (α, ϵ)-sound **iPoSW** for $\alpha \in (0, 1]$ and

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Thm (Standalone PoSW): (P, V) is an (α, ϵ)-sound **PoSW** for $\alpha \in (0, 1]$ and

$$\epsilon = \frac{3 \cdot q^2}{2^\lambda} + \alpha^t \quad .$$