

# Broadcast, Trace and Revoke with Optimal Parameters from Polynomial Hardness

Shweta Agrawal  
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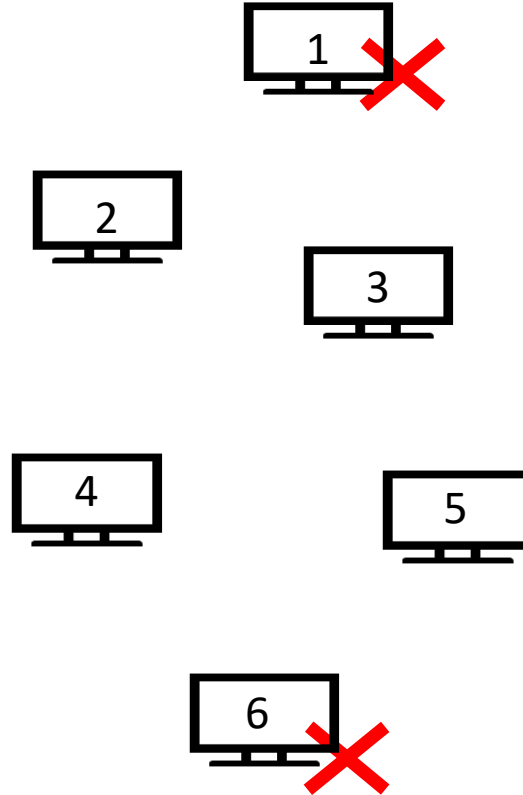
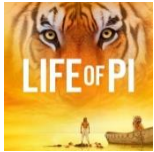
Simran Kumari  
IIT Madras, India

Anshu Yadav  
IIT Madras, India

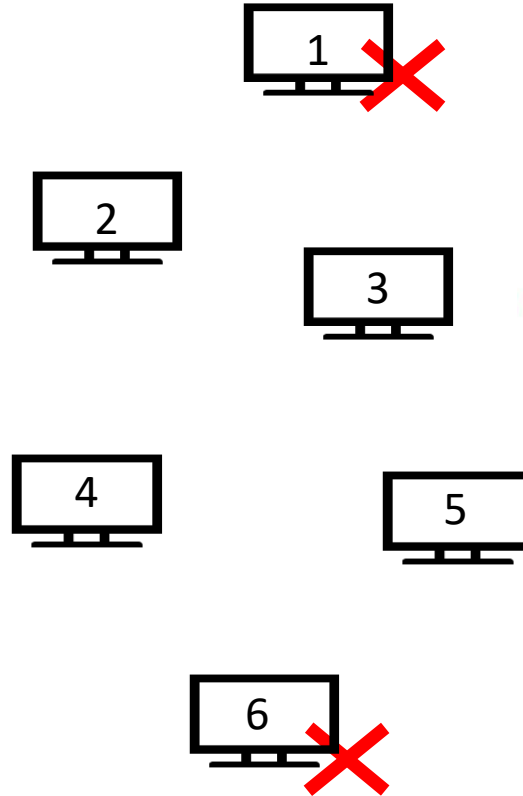
Shota Yamada  
AIST, Japan

**EUROCRYPT 2023**

# Motivation



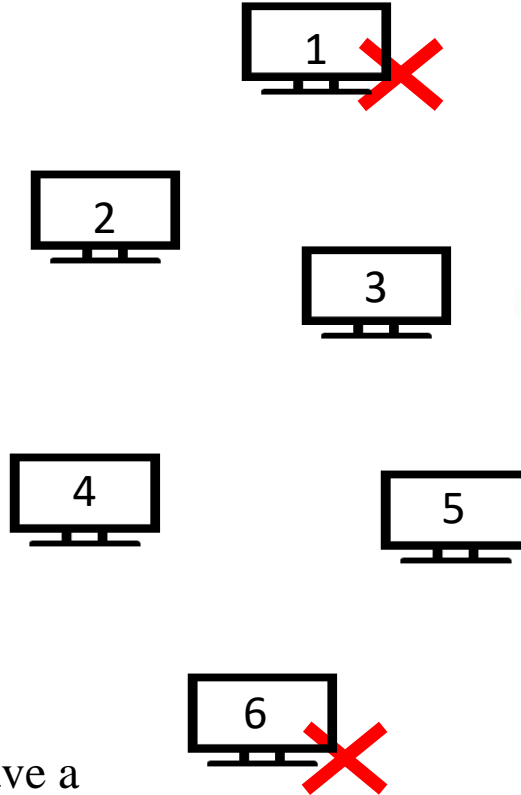
# Motivation



Trivial Approach : Using Public Key Encryption

!  $|ct|$  grows linearly with the no. of users

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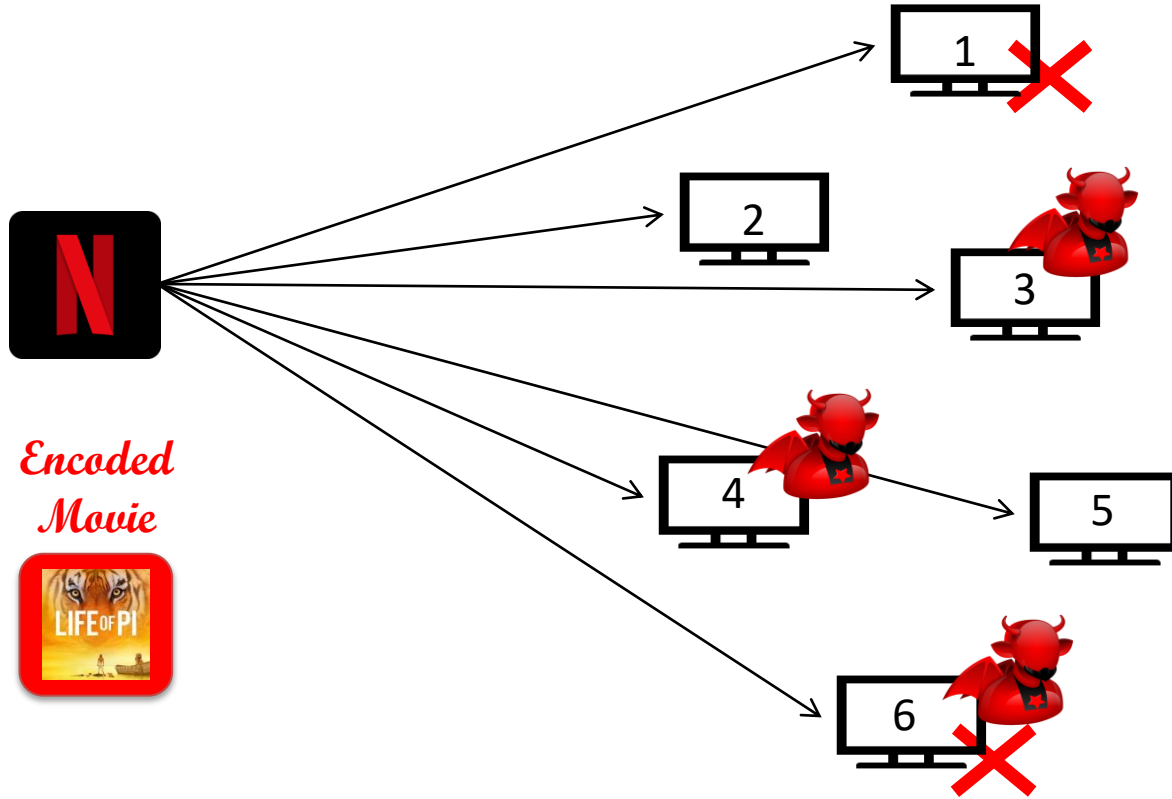


Trivial Approach : Using Public Key Encryption

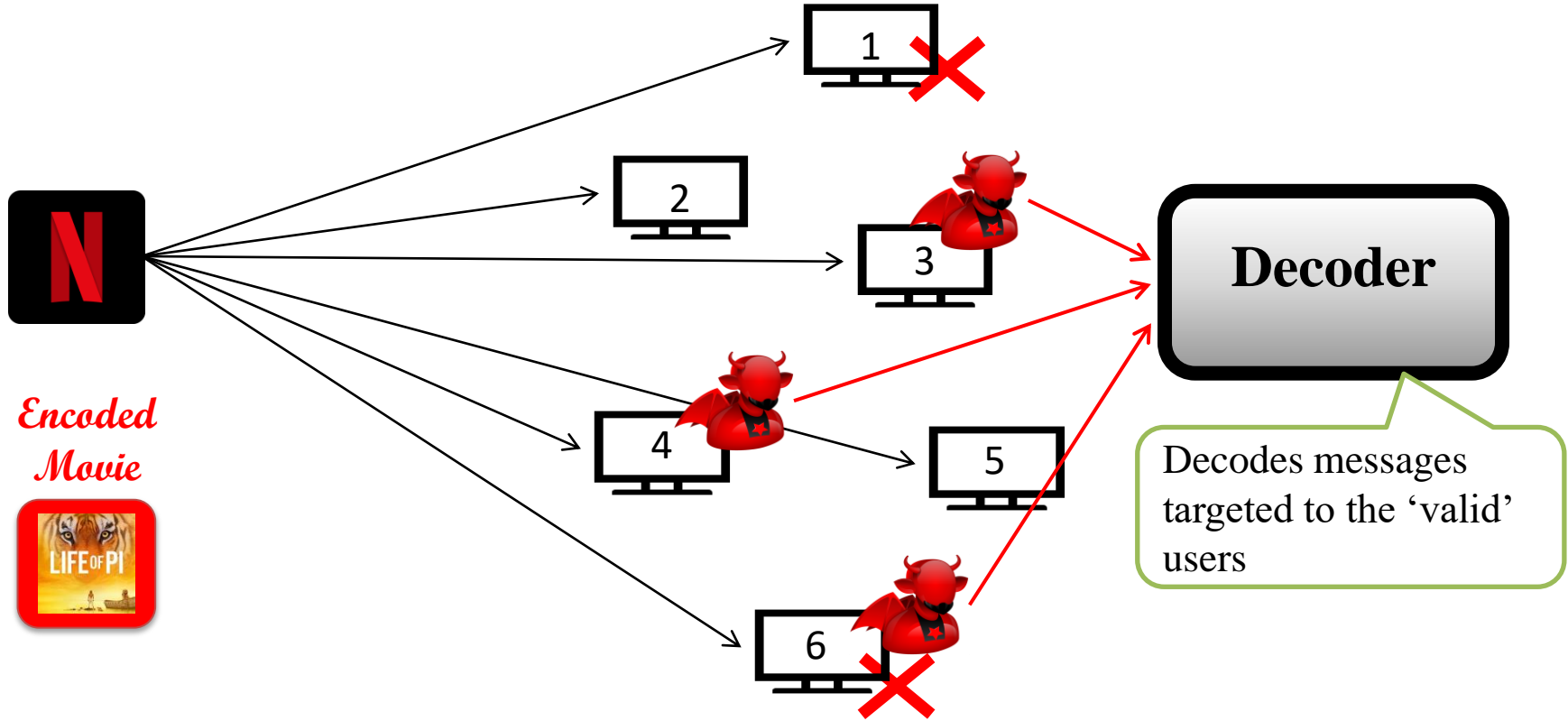
!  $|ct|$  grows linearly with the no. of users

A better approach? Can we have a **short common ct** for all the users?

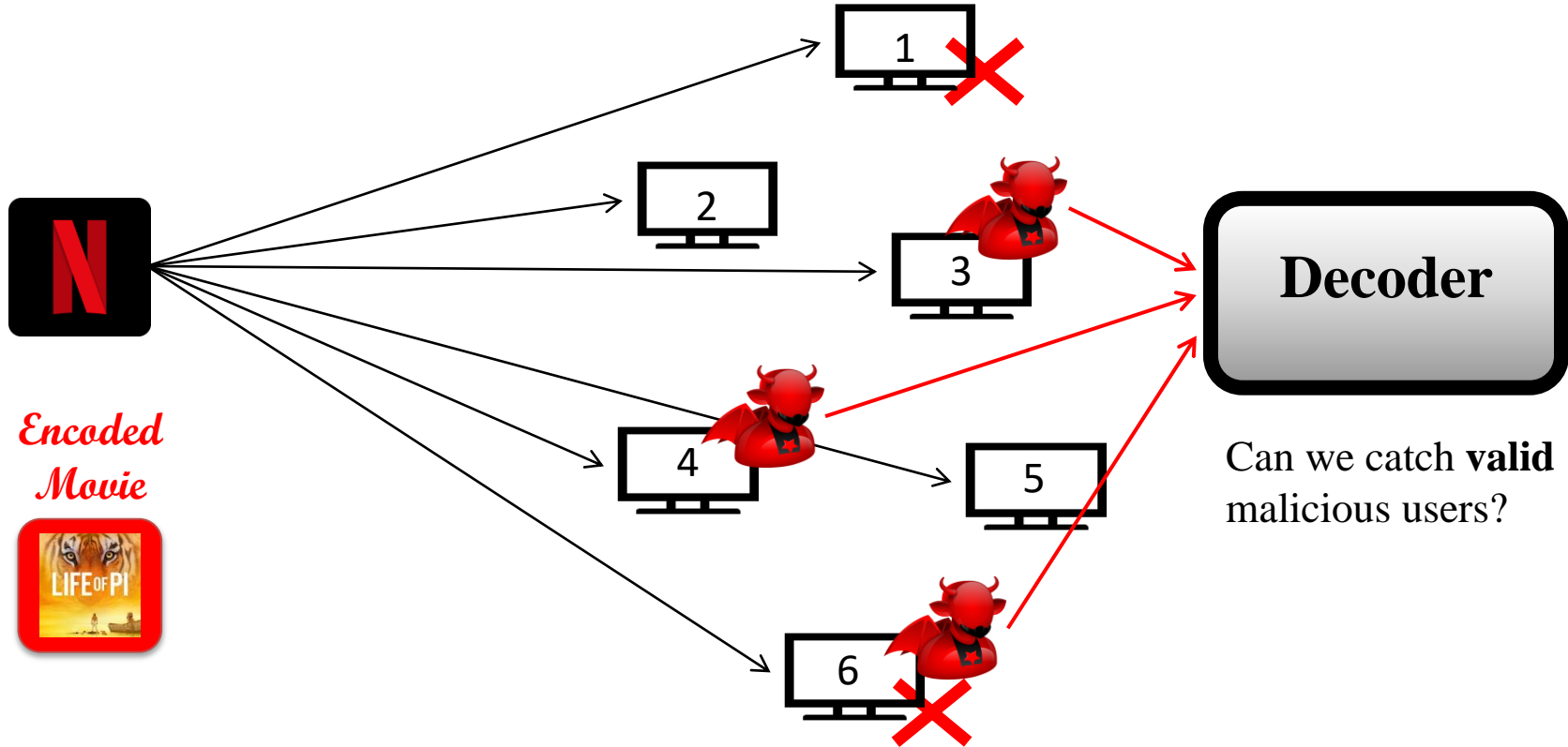
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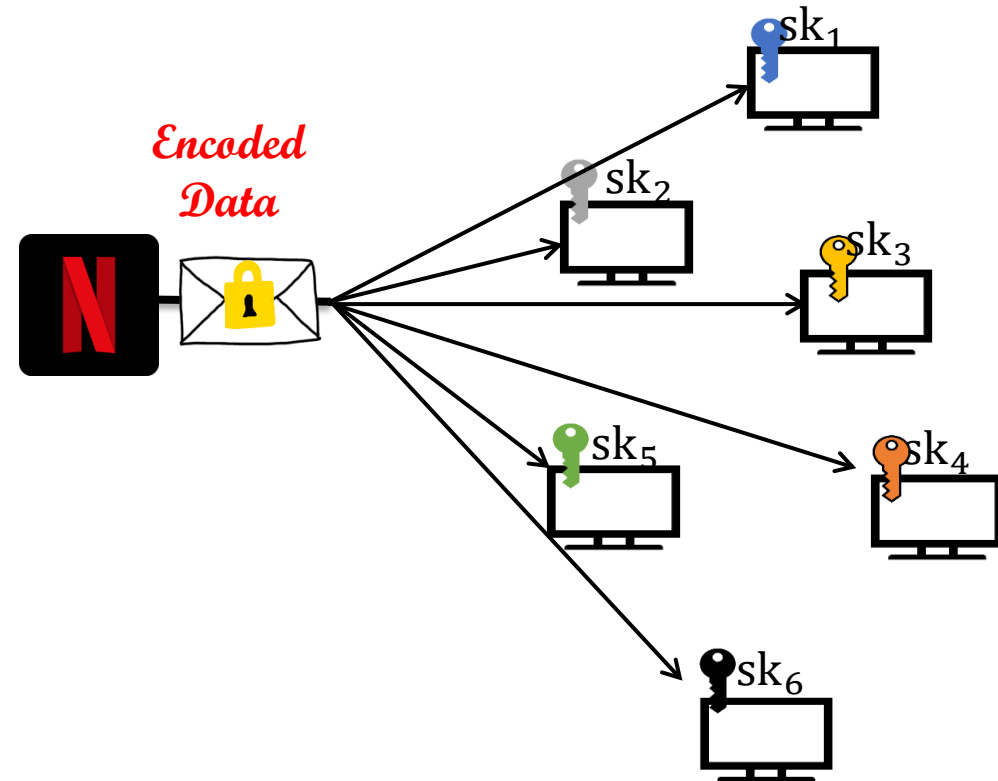
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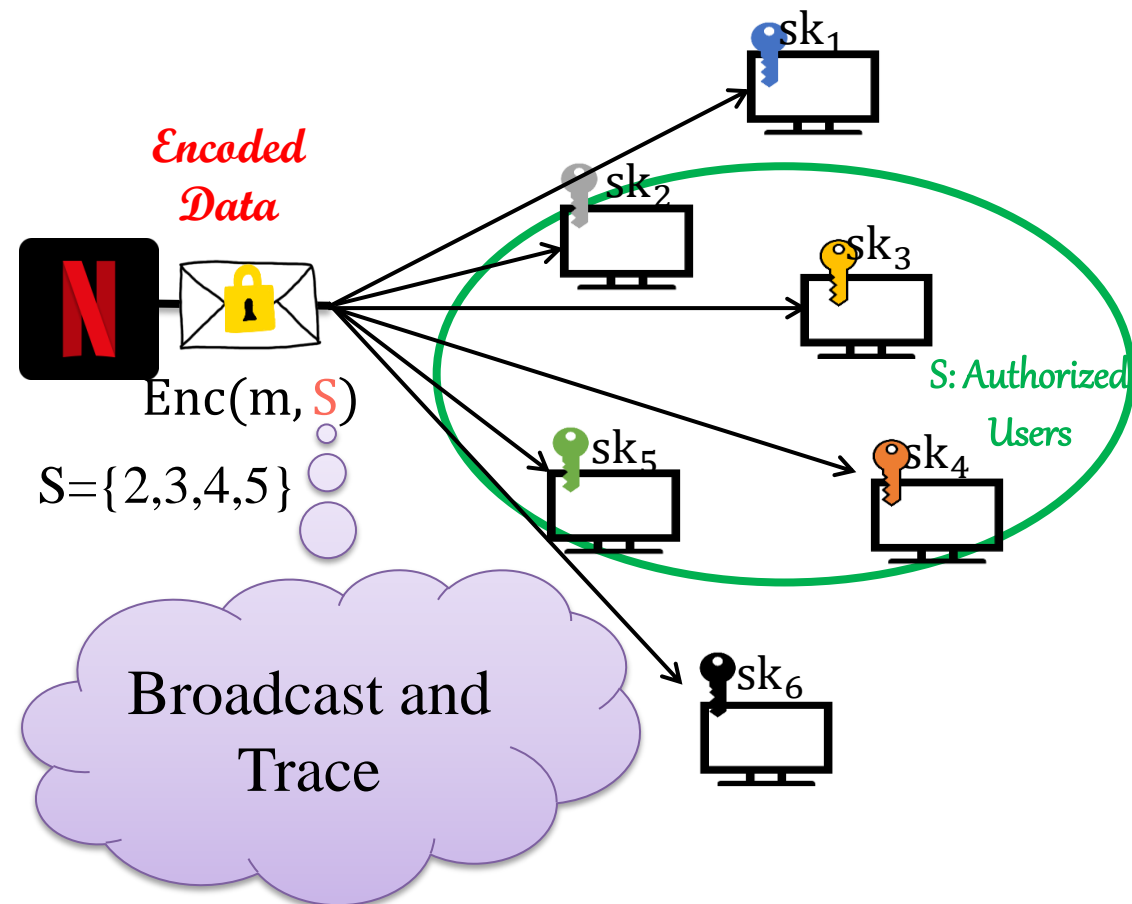
# Broadcast, Trace and Revoke<sub>[NP00]</sub>





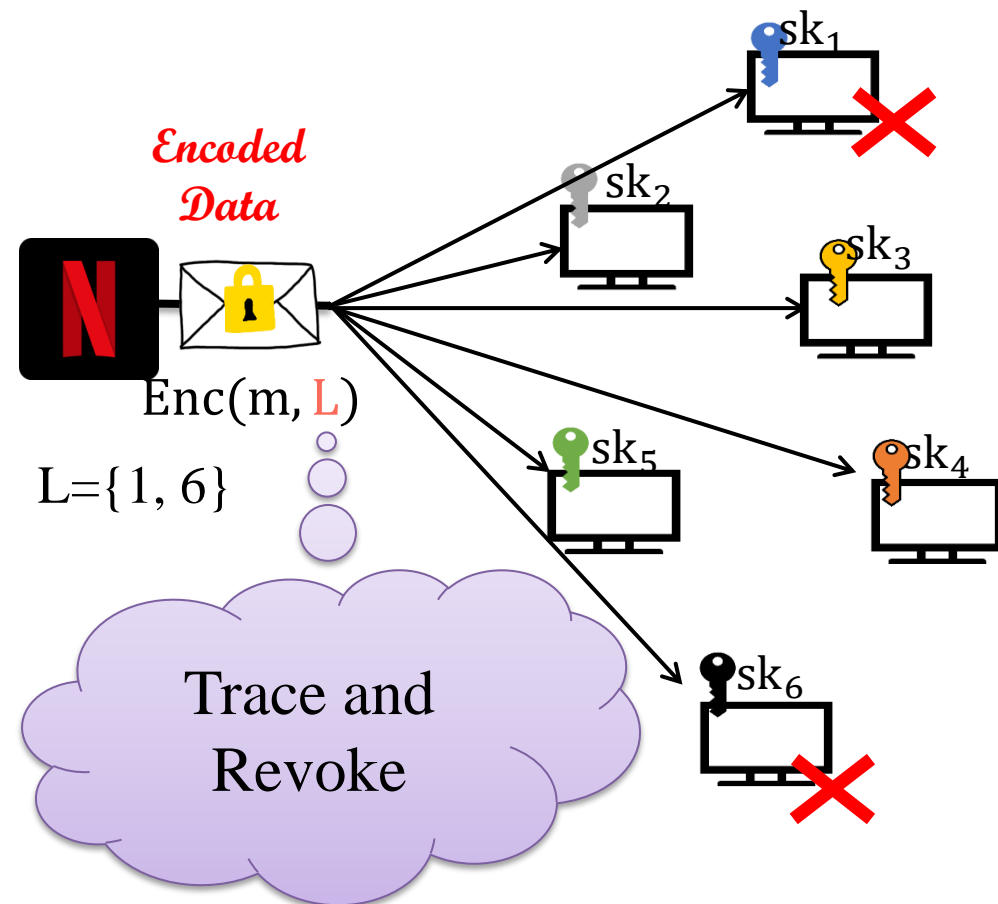
# Broadcast, Trace and Revoke<sub>[NP00]</sub>

- ❖ **Correctness:** Any user in  $S$  can decrypt



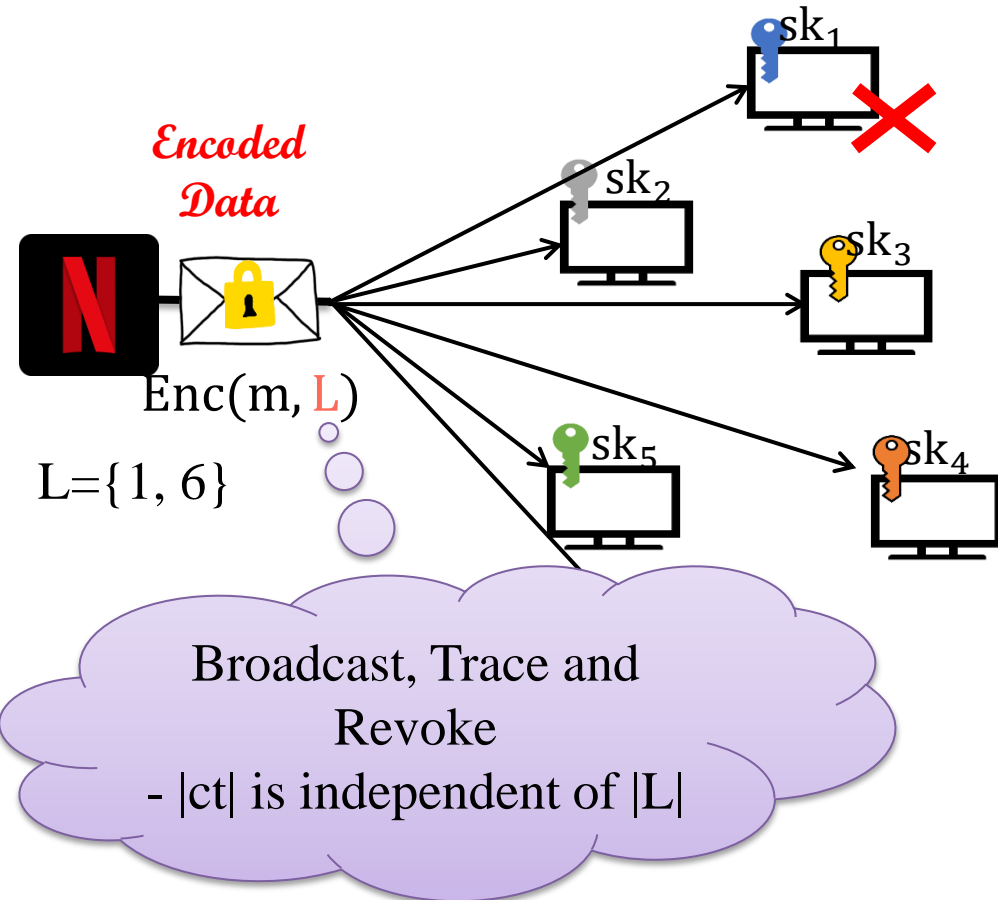
# Broadcast, Trace and Revoke<sub>[NP00]</sub>

- ❖ **Correctness:** Any user outside  $L$  can decrypt

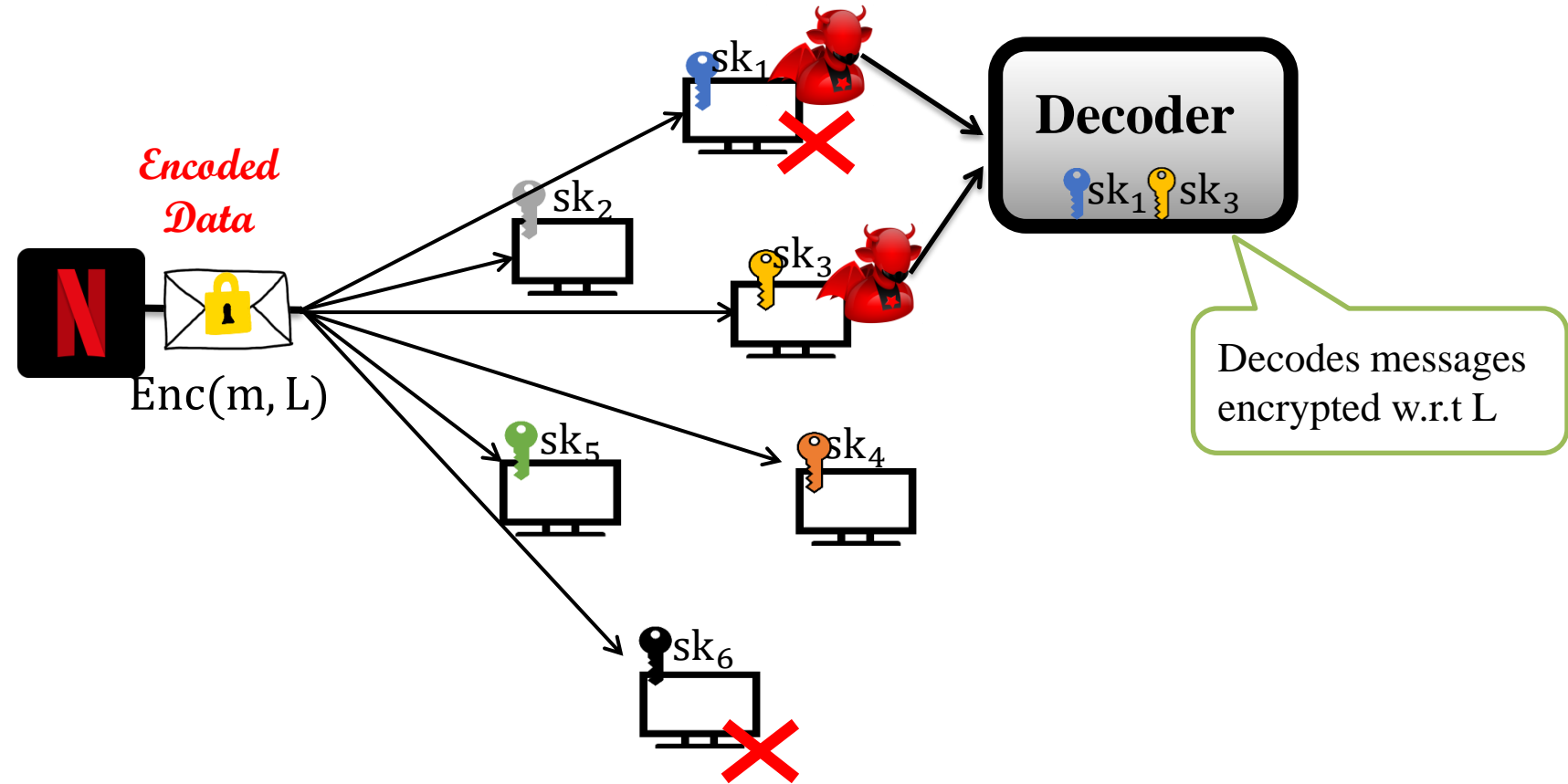


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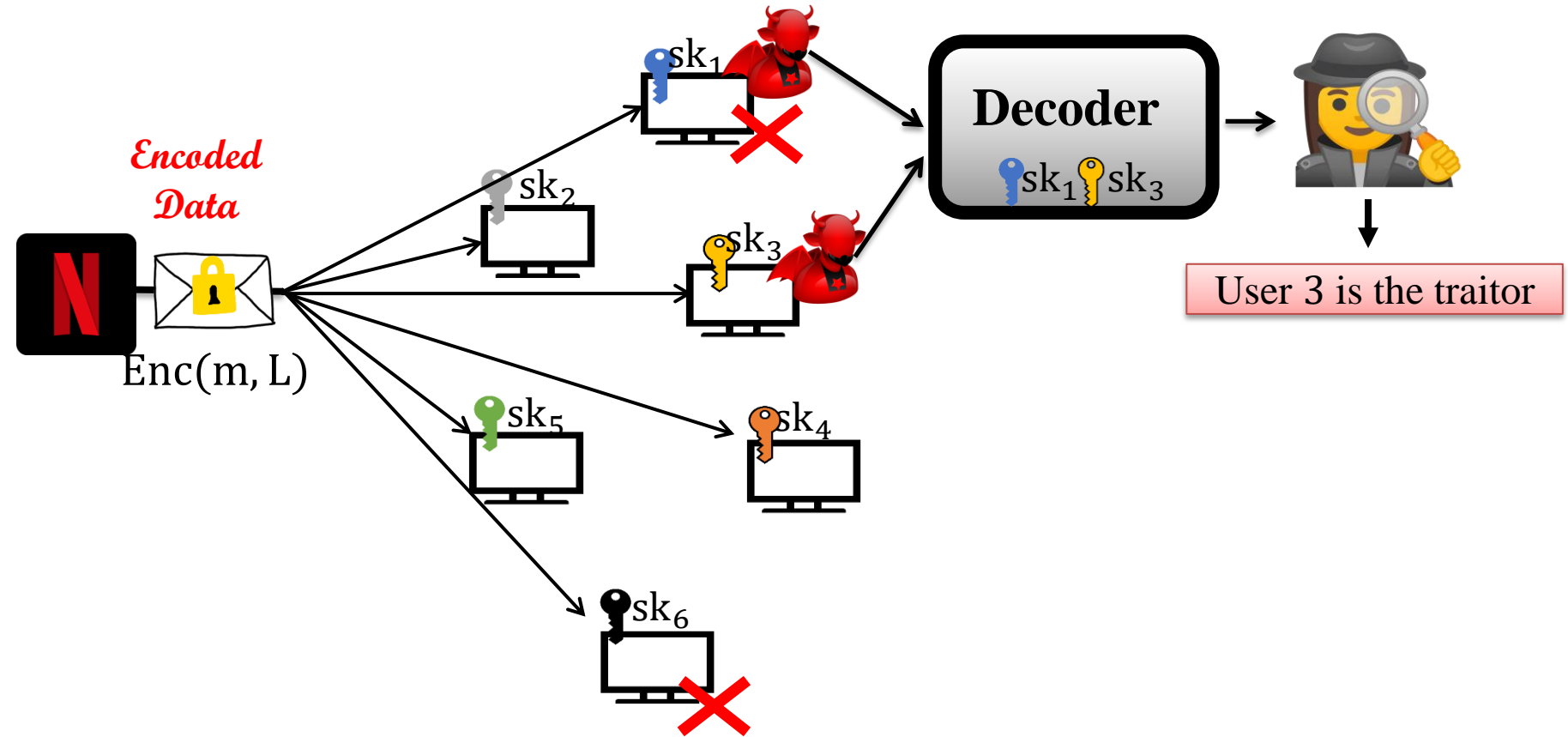
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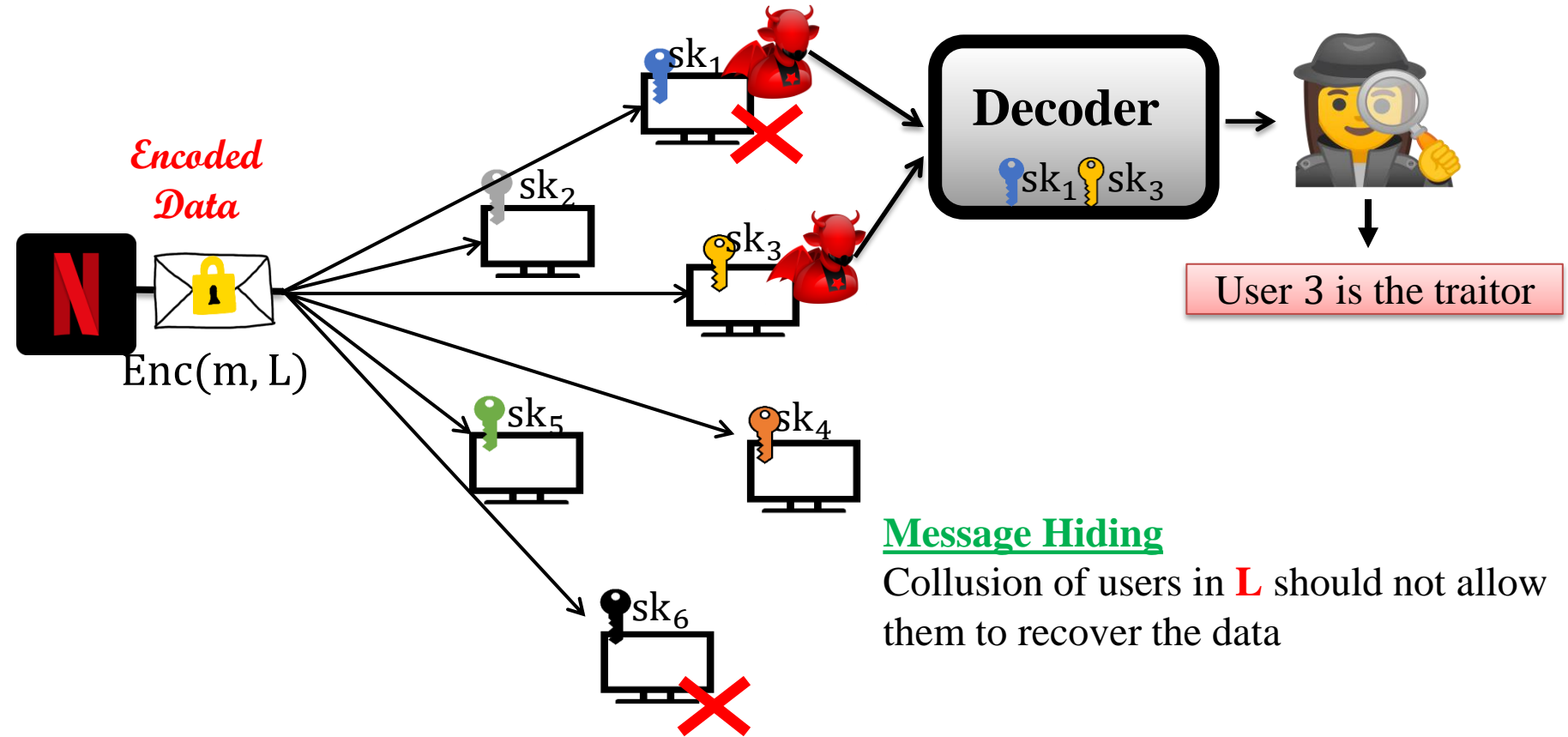
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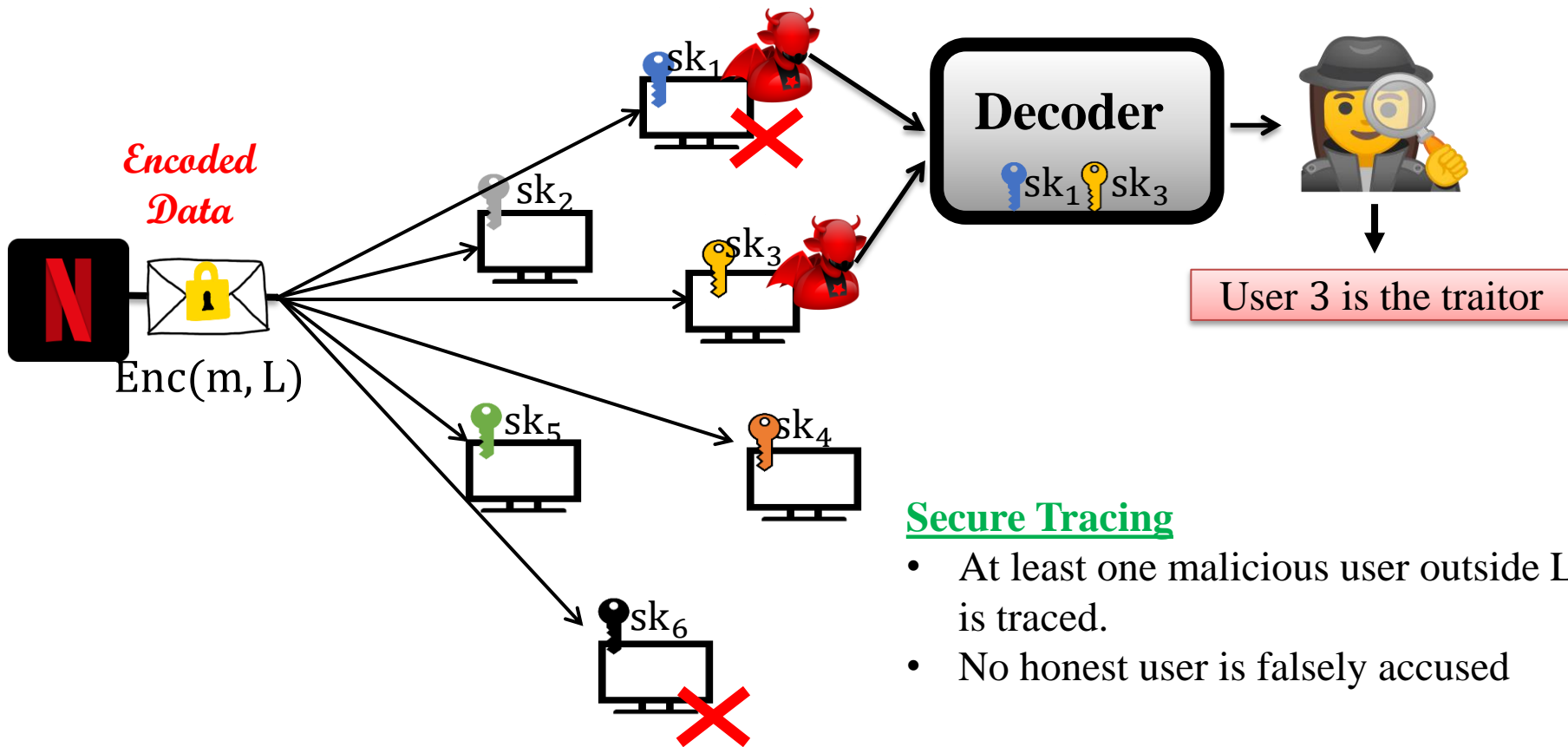
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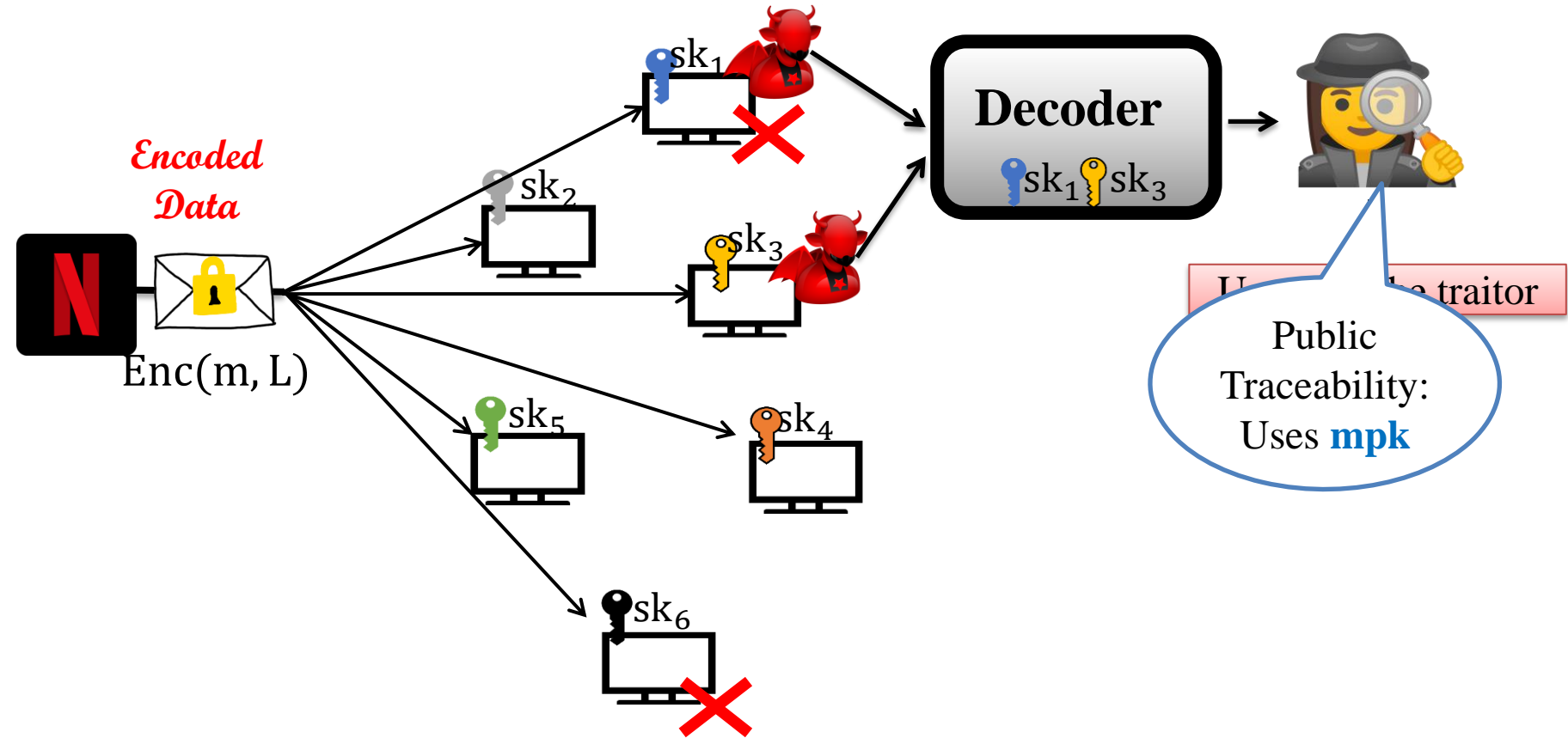
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## Secure Tracing

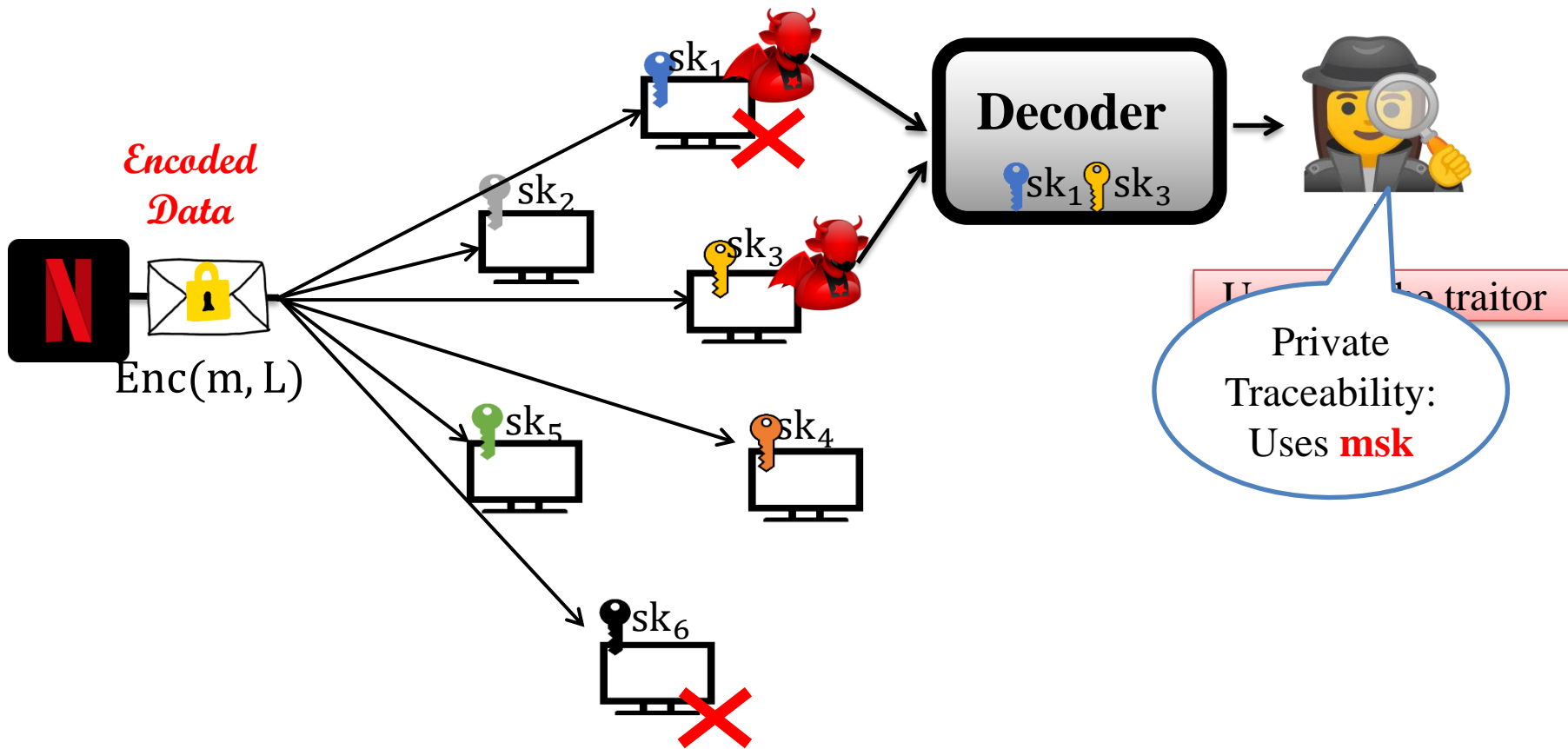
- At least one malicious user outside  $L$  is traced.
- No honest user is falsely accused

# Broadcast, Trace and Revoke<sub>[NP00]</sub>

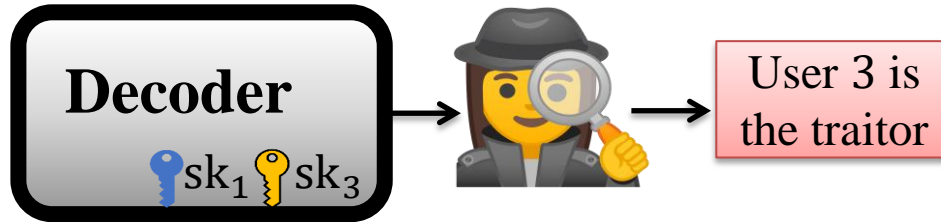




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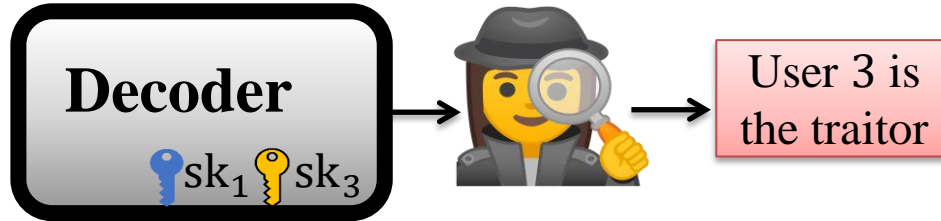
# Embedded Identities [NWZ16]



User Index	User Identity
1	Nisha
2	Sampan
3	Rohin
4	Anuja
5	Anshu

'Rohin' is the traitor.  
Revoke 'Rohin'

# Embedded Identities [NWZ16]

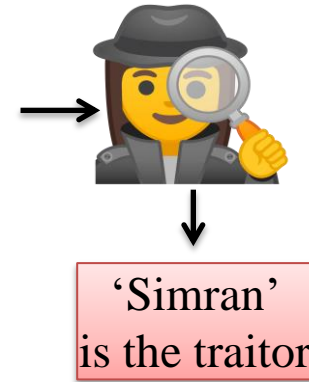
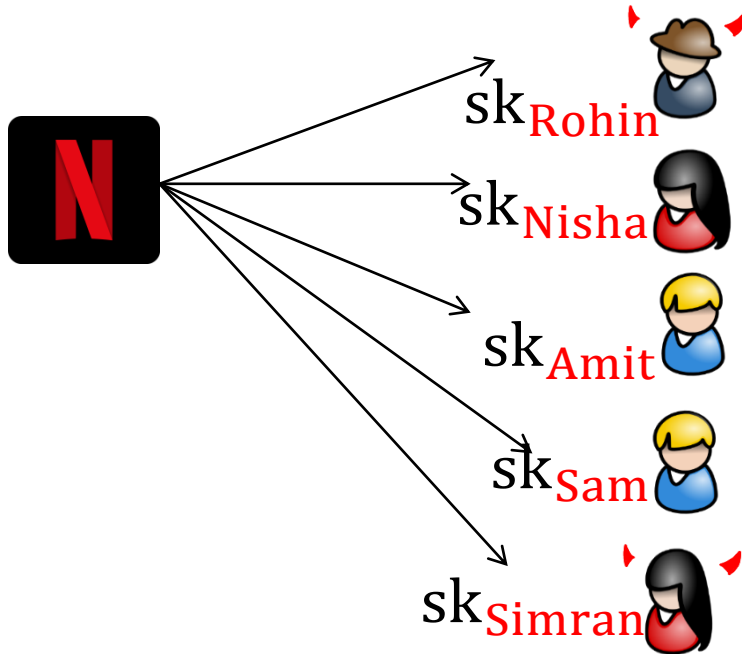


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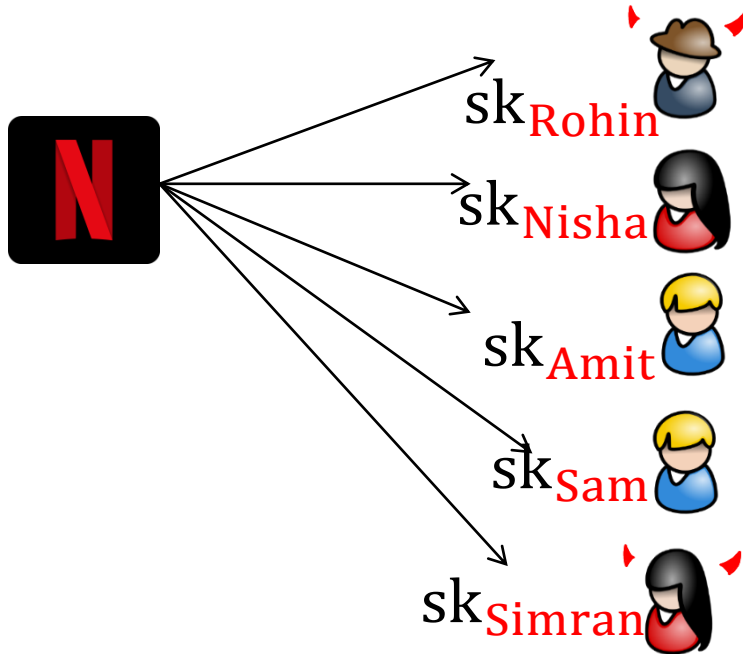
## Limitations

- The authority needs to maintain user index-identity mapping
- Violates user's identity privacy.

# Embedded Identities [NWZ16]



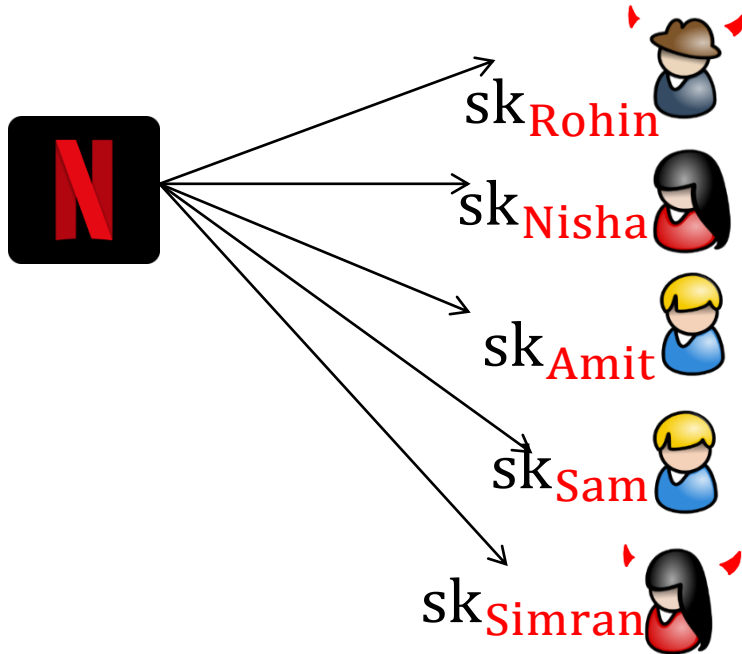
# Embedded Identities [NWZ16]



## Challenges:

- Traditional tracing method - *linear search* on the user space.
- Identity space can be exponential -- *exponential time* to output traitors

# Embedded Identities<sub>[NWZ16]</sub>



## Solutions:

- [\[Nishimaki-Wichs-Zhandry16\]](#) - Traitor Tracing & Trace and Revoke schemes with embedded identities.
- [\[Goyal-Koppula-Waters19\]](#) - Traitor tracing scheme with embedded identities

# Broadcast, Trace and Revoke : Goals



Optimal  
Parameters

$|ct|$ ,  $|pk|$  and  $|sk|$  independent  
of no. of users

# Broadcast, Trace and Revoke : Goals



Optimal  
Parameters



Adaptive Security  
w.r.t Revocation  
List L

$ct \leftarrow Enc(m, L)$

**Adaptive:** L can be chosen after the adversary gets the public parameters and user secret keys.



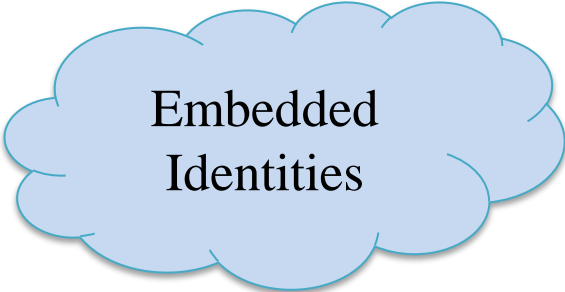
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Optimal  
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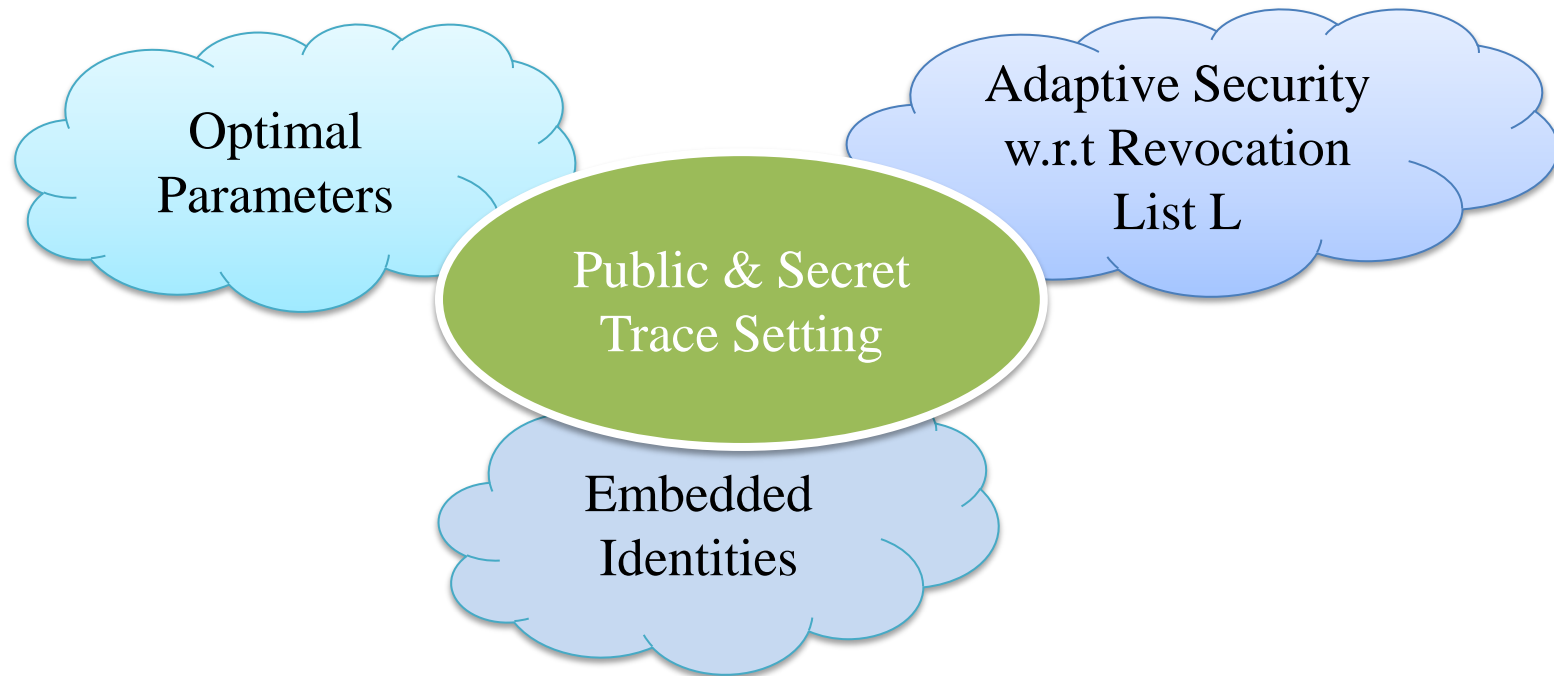


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# Broadcast, Trace and Revoke : Goals



# Prior Work : Public Traceability

	CT	SK	PK	Trace Space	Selective/ Adaptive	Asspn	Identities
[NWZ16]	1	1	1	Exp	<b>Selective</b>	<b>Subexp</b> (iO)	Yes

polylog(N, L) and poly(sec param) shown as 1.

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[GVW19]	1	1	1	<b>Poly</b>	Adaptive	<b>Subexp</b> (subexp Positional Witness Enc)	<b>No</b>

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<b>This Work</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>Exp</b>	<b>Adaptive</b>	<b>Poly</b> ( FE and Spcl ABE)	<b>Yes</b>

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# Public Traceability : Assumptions

[NWZ16]

**iO**

Inherently sub-exponential  
assumption

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[GVW19]

**Sub-exp  
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No construction from  
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**This  
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Spcl FE

Spcl ABE

# Building Blocks

## Functional Encryption (FE)

$$sk_C + Enc(m) \xrightarrow{\text{Decrypt}} C(m)$$

The decryptor learns **only**  $C(m)$  and nothing else.



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## Key Policy Attribute Based Encryption (KP-ABE)

$$sk_C + Enc(m, x) \xrightarrow{\text{Decrypt}} m \text{ iff } C(x) = 1$$

Restricted access using the secret key

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## CT Policy Attribute Based Encryption (CP-ABE)

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Restricted access using the secret key

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**This  
Work**

Spcl FE  
[JLS21,JLS22]

Spcl KP-ABE  
[BGG+14]

Known from polynomially  
hard assumptions

# Prior Work: Secret Traceability

	CT	SK	PK	Trace Space	Asspn	Identities
[GQWW19]	$N^a$	$N^c$	$N$	Poly	LWE and Pairings	No

$0 < a < 1$ ,  $c$  large constant.

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[KW20]	<b>L</b>	1	1	Exp	Subexp LWE	Yes

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<b>This Work</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>Exp</b>	<b>Poly (spcl CP-ABE &amp; KP-ABE)</b>	<b>Yes</b>

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# Secret Traceability : Assumptions

[GQWW19]

[Zha20]



**Pairings**

Post-Quantum insecure

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**Sub-exp  
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**This  
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**Spcl CP-ABE**

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**This  
Work**

Spcl CP-ABE

Evasive & Tensor  
LWE [Wee22]

Spcl KP-ABE

Std. LWE [BGG+14]

- Polynomially hard
- Lattice based – conjectured post quantum safe

# Public-Key vs Secret-Key BTR

## Public Traceability

- Succinct KP-ABE
- Compact FE

## Secret Traceability

- Succinct KP-ABE
- Compact CP-ABE

# Public-Key vs Secret-Key BTR

## Public Traceability

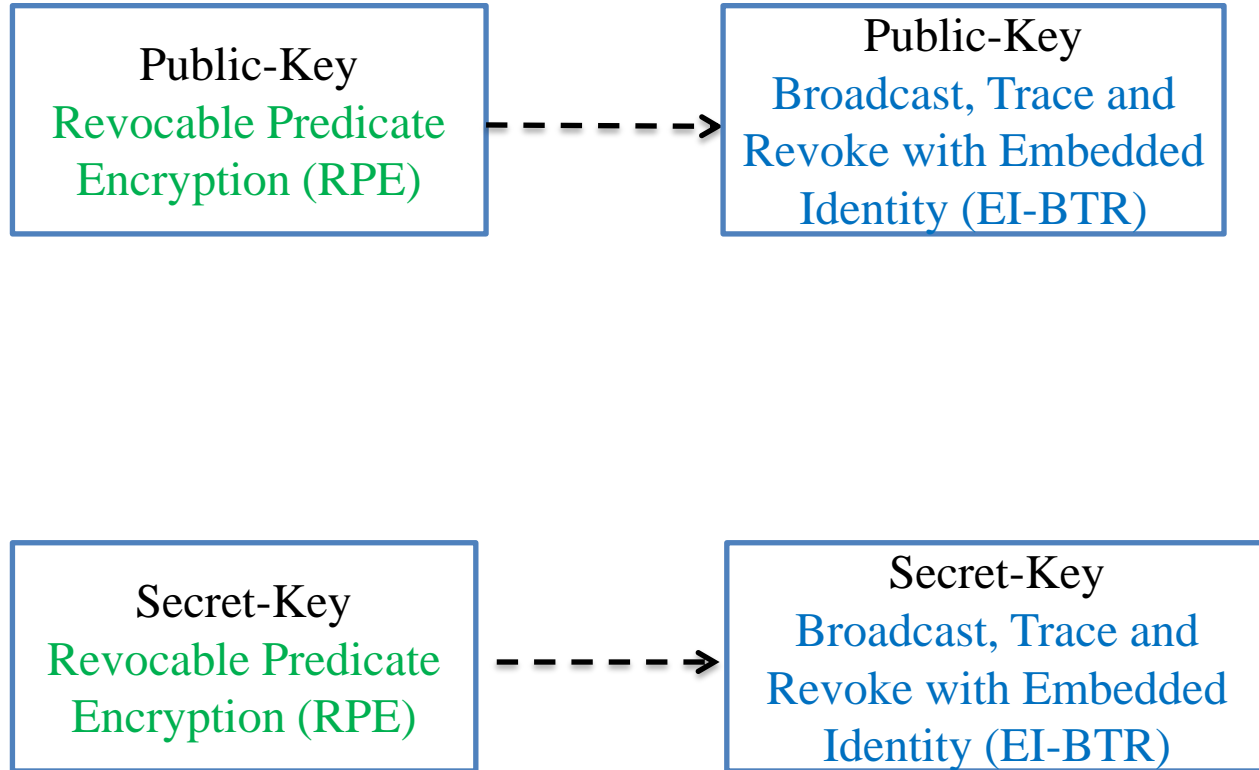
- Succinct KP-ABE
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## Secret Traceability

- Succinct KP-ABE
- Compact CP-ABE

ABE is weaker than FE since  
ABE is an all or nothing  
primitive in contrast to FE.

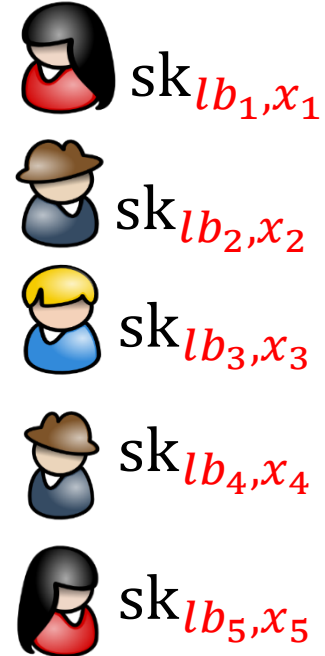
# Outline



# Revocable Predicate Encryption<sub>[KW19]</sub>

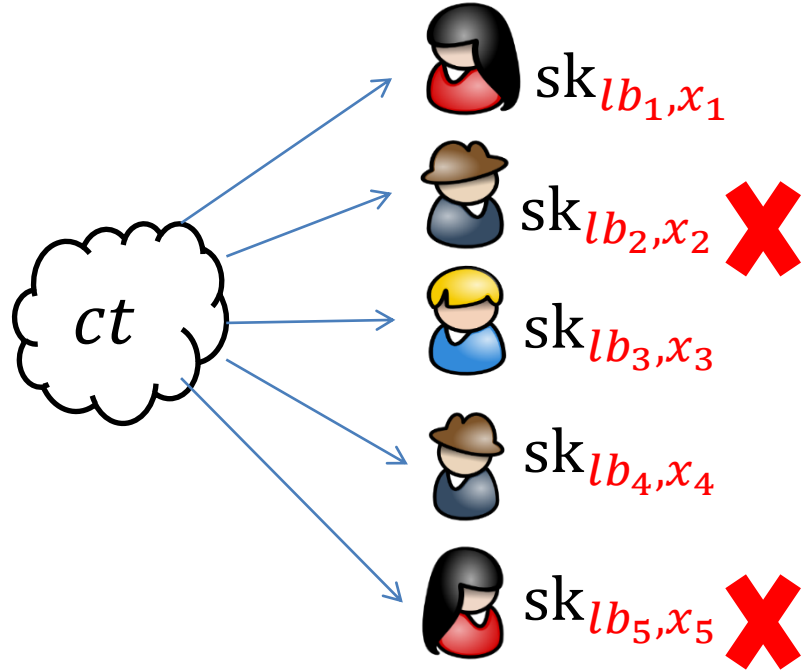
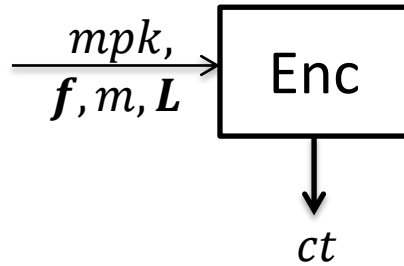


Revoked labels  
 $L = \{lb_2, lb_5\}$



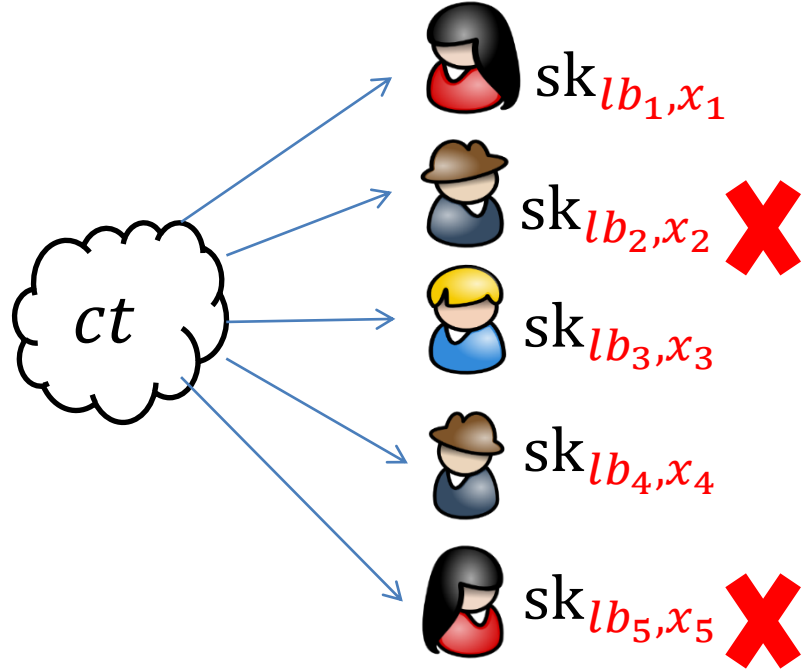
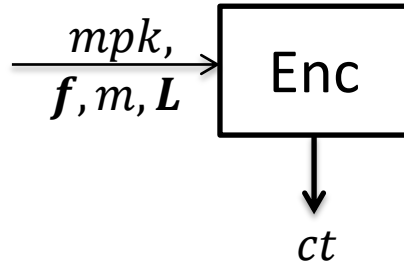


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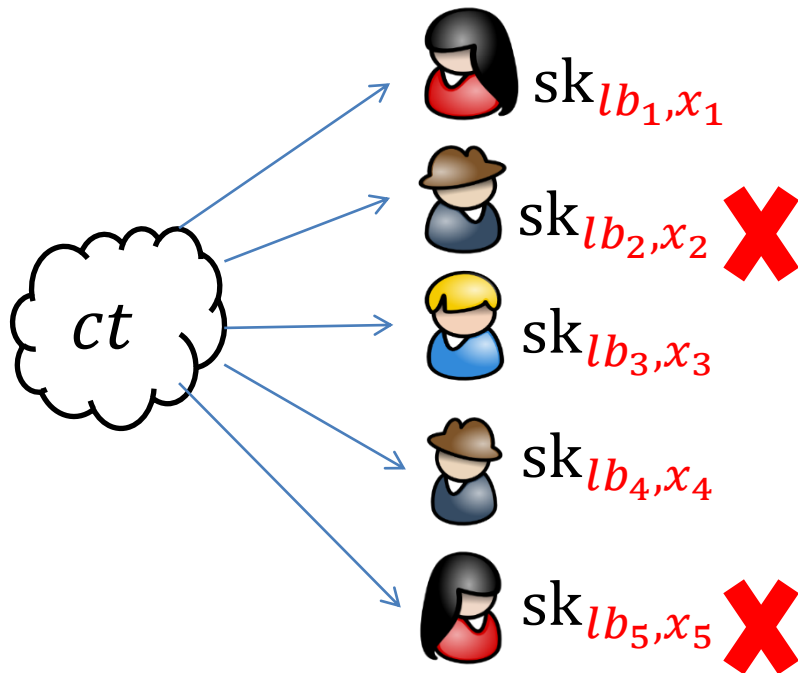
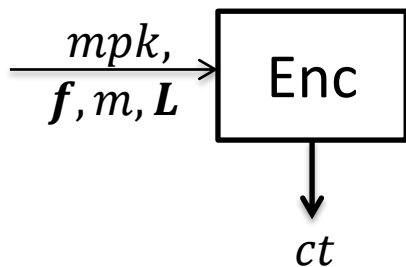
# Revocable Predicate Encryption<sub>[KW19]</sub>



Revoked labels  
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Correctness : A user with associated  $(lb_i, x_i)$  can recover message  $m$  if  $f(x_i) = 1$  and  $lb_i \notin L$

# Revocable Predicate Encryption<sub>[KW19]</sub>



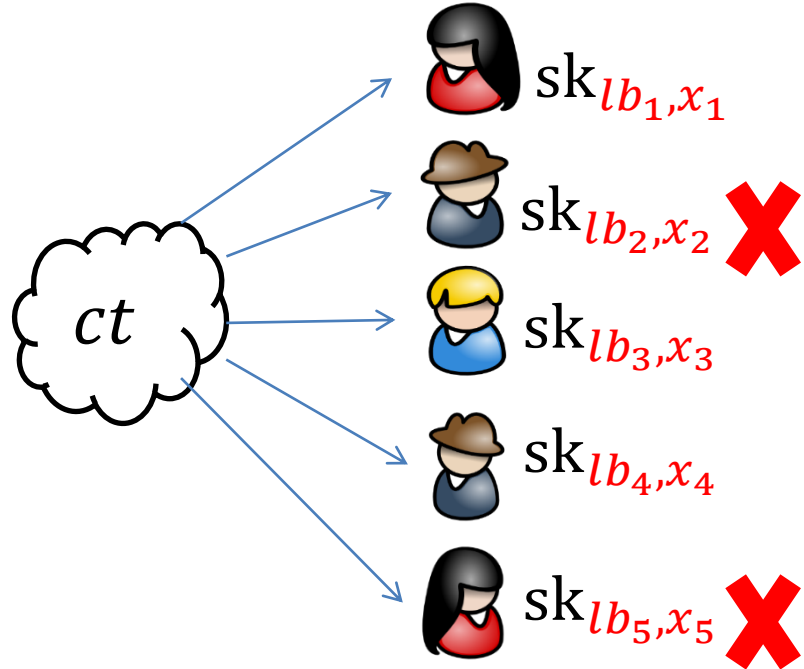
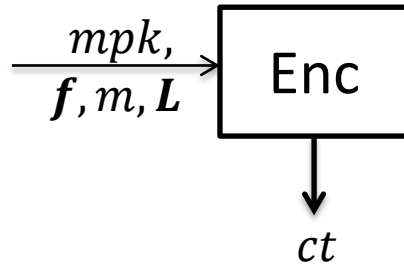
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## Message Hiding Security

$\text{Enc}(mpk, f, m_0, L) \approx \text{Enc}(mpk, f, m_1, L)$  ; if for all key queries  $(lb, x)$

$f(x) = 0$  or  $lb \in L$

# Revocable Predicate Encryption<sub>[KW19]</sub>



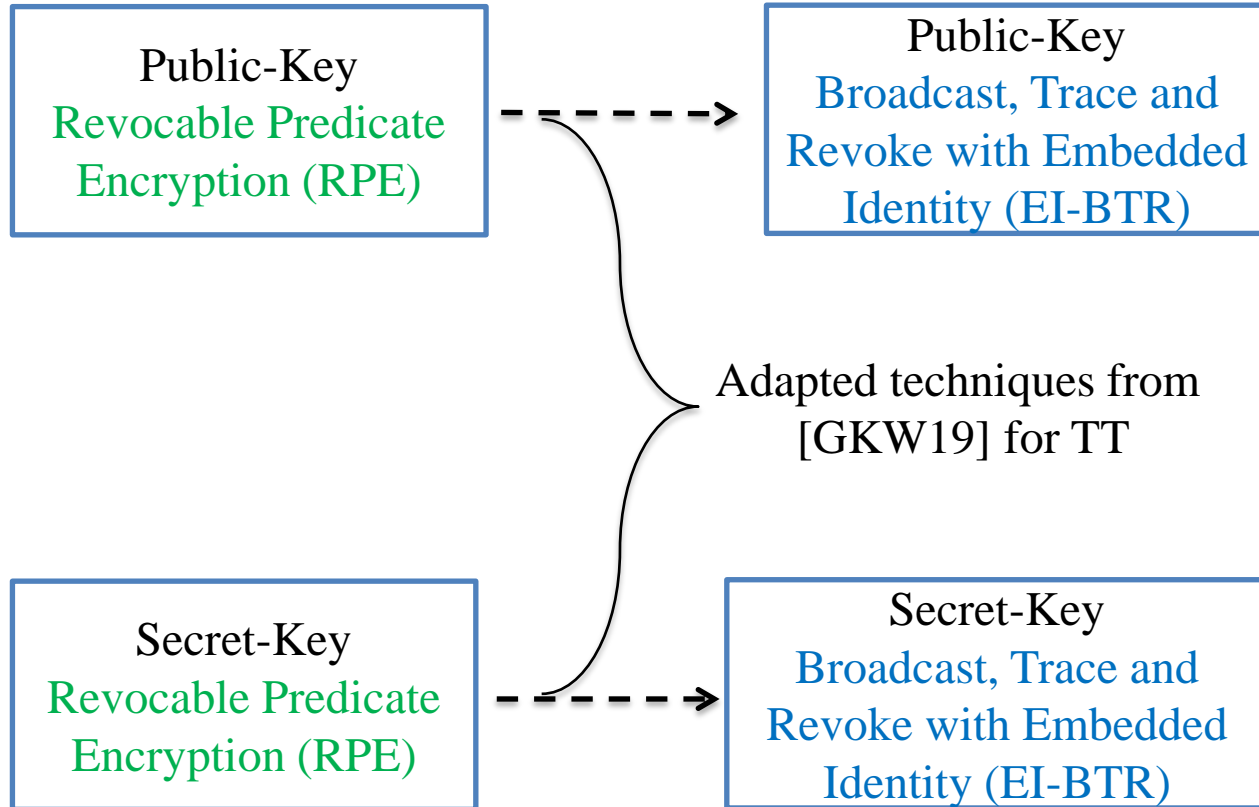
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## Function Hiding Security

$\text{Enc}(mpk, f_0, m, L) \approx \text{Enc}(mpk, f_1, m, L)$  ; if for all key queries  $(lb, x)$

$f_0(x) = f_1(x)$  or  $lb \in L$

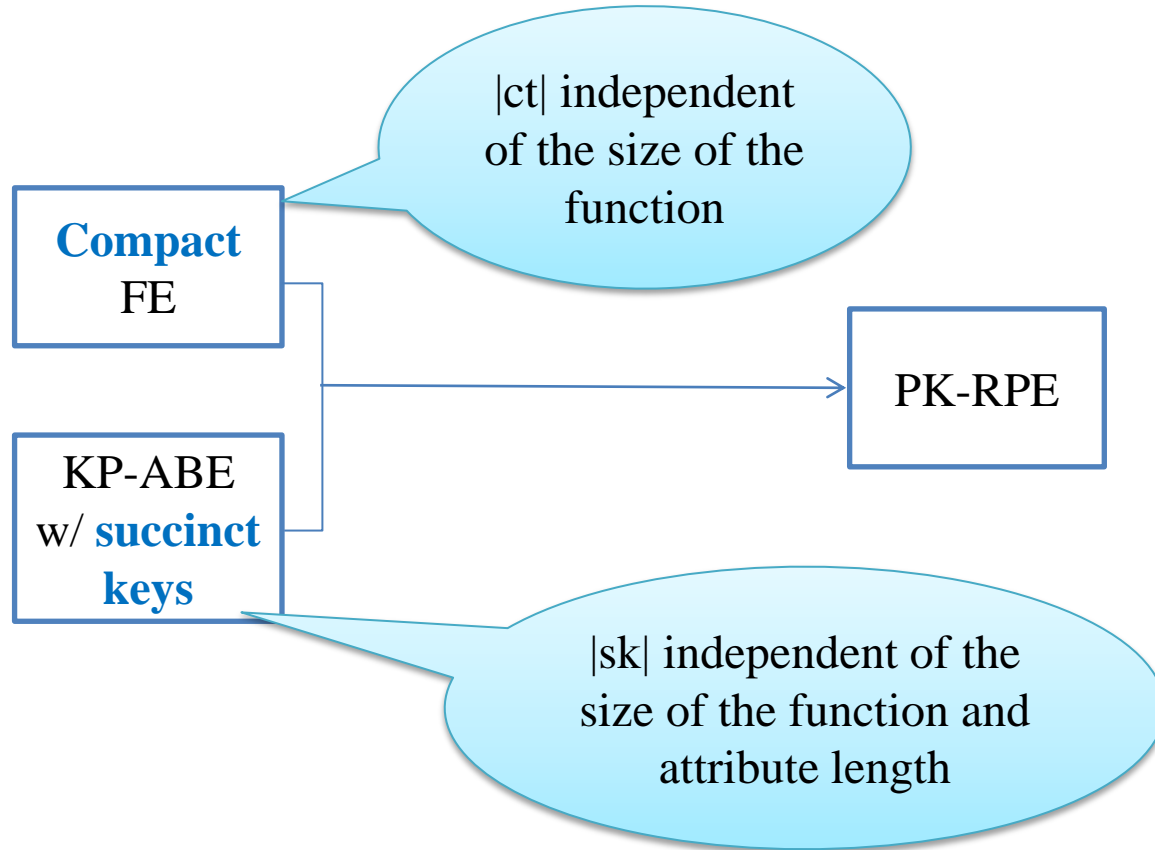
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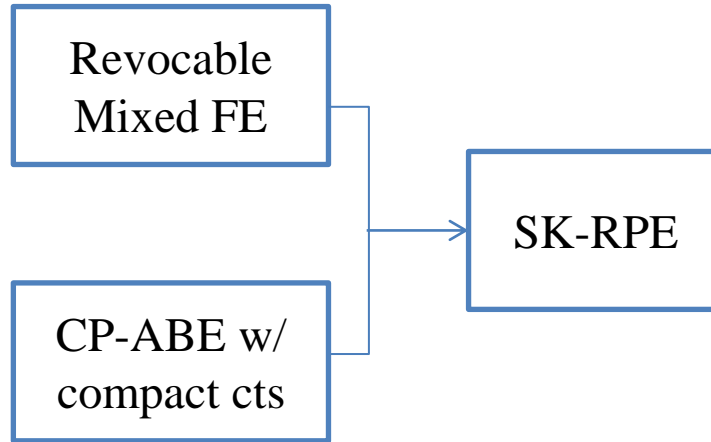
# Public Key RPE : Outline



# Public Key RPE : Optimal Parameters

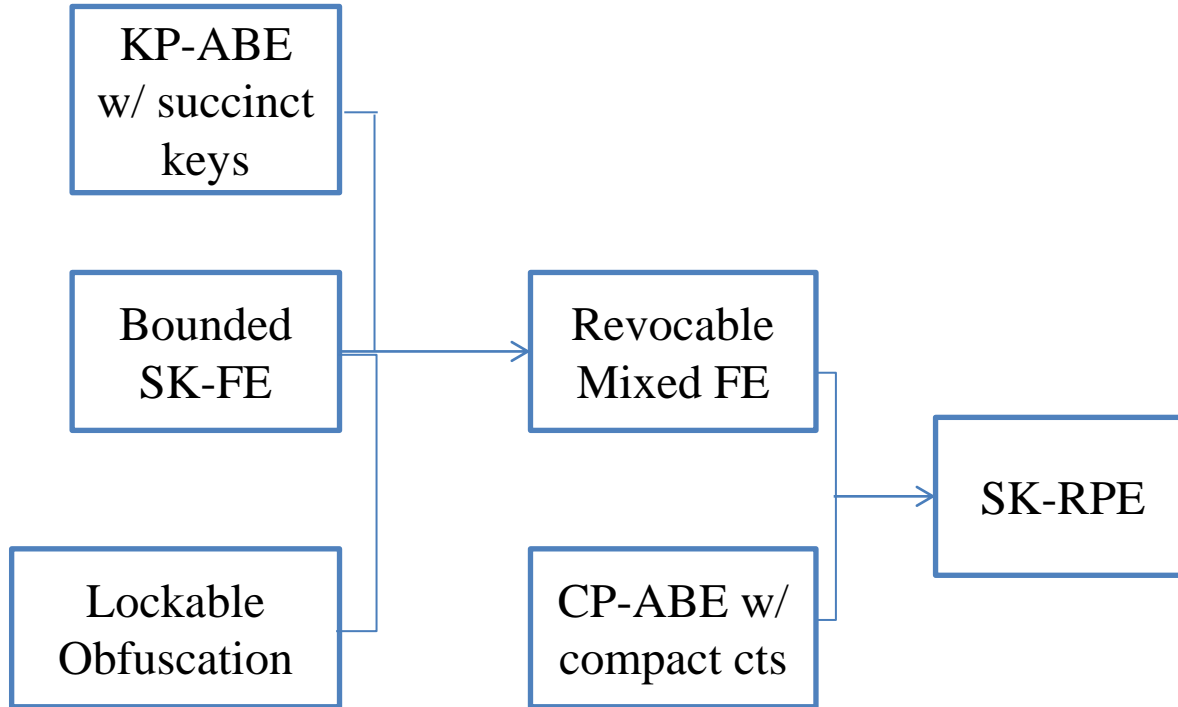


# Secret Key RPE : Outline

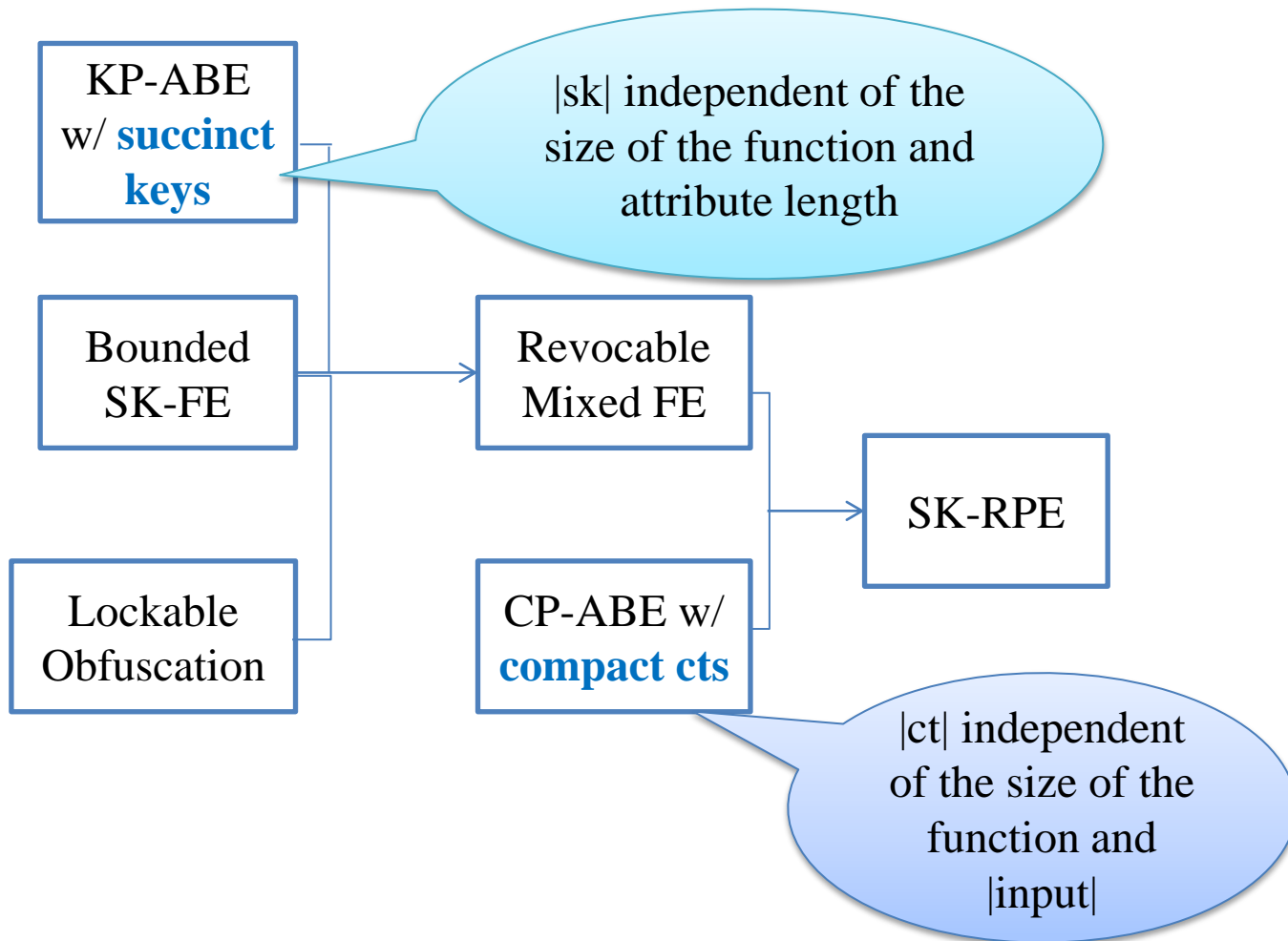




# Secret Key RPE : Outline



# Secret Key RPE: Optimal Parameters



# Summary

Unified framework for secret and public EI-BTR via RPE

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- Optimal size of CT, PK, SK
- Embedded Identities
- Adaptive security
- Poly hard assumptions: FE, ABE

**Open:** Weaker version of FE to allow post quantum scheme?

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First work to support superpoly revocation list size!  
(with efficient representation & membership testing)

Thank you!