

# Caveat Implementor!

## Key Recovery Attacks on MEGA

Martin R. Albrecht<sup>1</sup> Miro Haller<sup>2,3</sup> *Lenka Mareková*<sup>4</sup> Kenneth G. Paterson<sup>3</sup>

<sup>1</sup>King's College London  
martin.albrecht@kcl.ac.uk

<sup>2</sup>UC San Diego  
mhaller@ucsd.edu

<sup>3</sup>Applied Cryptography Group, ETH Zürich  
kenny.paterson@inf.ethz.ch

<sup>4</sup>Information Security Group, Royal Holloway, University of London  
lenka.marekova.2018@rhul.ac.uk

*24 April 2023*

# Introduction

- MEGA – E2EE cloud storage and communication platform with 280M registered users

# Introduction

- MEGA – E2EE cloud storage and communication platform with 280M registered users
- Previous work by Backendal, Haller and Paterson [BHP23] gave 5 attacks, the first two completely breaking confidentiality of user files

# Introduction

- MEGA – E2EE cloud storage and communication platform with 280M registered users
- Previous work by Backendal, Haller and Paterson [BHP23] gave 5 attacks, the first two completely breaking confidentiality of user files
- MEGA did not implement suggested countermeasures, instead relying on validation of plaintext payloads

# Introduction

- MEGA – E2EE cloud storage and communication platform with 280M registered users
- Previous work by Backendal, Haller and Paterson [BHP23] gave 5 attacks, the first two completely breaking confidentiality of user files
- MEGA did not implement suggested countermeasures, instead relying on validation of plaintext payloads
- These checks were sufficient to prevent the specific attacks, but (as we will show) not sufficient in general

# Background

Our new attacks are enabled by

# Background

Our new attacks are enabled by

- the continued lack of key separation and integrity protection

# Background

Our new attacks are enabled by

- the continued lack of key separation and integrity protection
- an ECB encryption oracle present in MEGAdrop, a feature unrelated to the protocol under attack



# Background

Our new attacks are enabled by

- the continued lack of key separation and integrity protection
- an ECB encryption oracle present in MEGAdrop, a feature unrelated to the protocol under attack
- detailed reporting of errors by the client to the server, added shortly after the patch for the attacks of [BHP23]

# Background

Our new attacks are enabled by

- the continued lack of key separation and integrity protection
- an ECB encryption oracle present in MEGAdrop, a feature unrelated to the protocol under attack
- detailed reporting of errors by the client to the server, added shortly after the patch for the attacks of [BHP23]

and inspired by

- the small-order subgroup attacks on DH [vW96, LL97]

# Background

Our new attacks are enabled by

- the continued lack of key separation and integrity protection
- an ECB encryption oracle present in MEGAdrop, a feature unrelated to the protocol under attack
- detailed reporting of errors by the client to the server, added shortly after the patch for the attacks of [BHP23]

and inspired by

- the small-order subgroup attacks on DH [vW96, LL97]
- the key overwriting attacks on OpenPGP [KR02, BPH22]

# Bird's-eye view of the MEGA key hierarchy

# Bird's-eye view of the MEGA key hierarchy

- Each user has

# Bird's-eye view of the MEGA key hierarchy

- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password

# Bird's-eye view of the MEGA key hierarchy

- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password
  - a 128-bit *master key*  $k_M$

# Bird's-eye view of the MEGA key hierarchy

- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password
  - a 128-bit *master key*  $k_M$
  - a 2048-bit RSA keypair  $(pk, sk)$

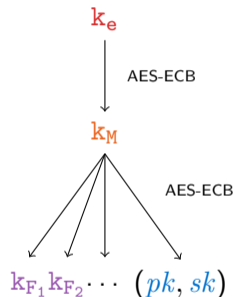


# Bird's-eye view of the MEGA key hierarchy

- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password
  - a 128-bit *master key*  $k_M$
  - a 2048-bit RSA keypair  $(pk, sk)$
  - *file encryption keys*  $k_{F_1}, k_{F_2}, \dots$

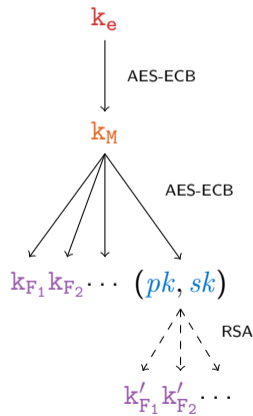
# Bird's-eye view of the MEGA key hierarchy

- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password
  - a 128-bit *master key*  $k_M$
  - a 2048-bit RSA keypair  $(pk, sk)$
  - *file encryption keys*  $k_{F_1}, k_{F_2}, \dots$
- The keys are encrypted using AES-ECB as
  - $[k_M]_{k_e}$
  - $[k_F]_{k_M}, [sk]_{k_M}$

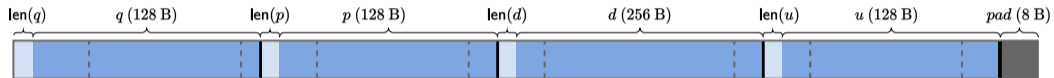


# Bird's-eye view of the MEGA key hierarchy

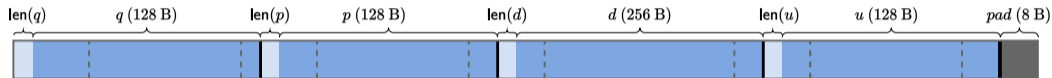
- Each user has
  - a 128-bit *encryption key*  $k_e$  derived from their password
  - a 128-bit *master key*  $k_M$
  - a 2048-bit RSA keypair  $(pk, sk)$
  - *file encryption keys*  $k_{F_1}, k_{F_2}, \dots$
- The keys are encrypted using AES-ECB as
  - $[k_M]_{k_e}$
  - $[k_F]_{k_M}, [sk]_{k_M}$
- *Shared-file encryption keys*  $k'_F$  are encrypted under  $pk$  using RSA



# Format of $sk$

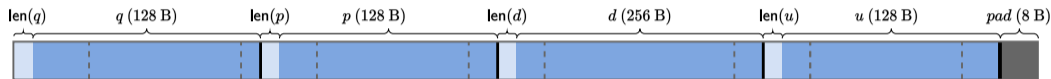


# Format of $sk$



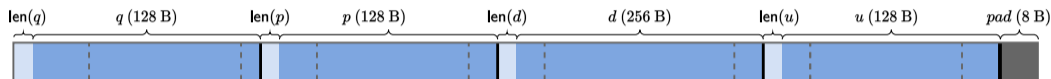
- Custom encoding of  $sk$  for RSA-CRT decryption, referred to as  $privk$

## Format of $sk$



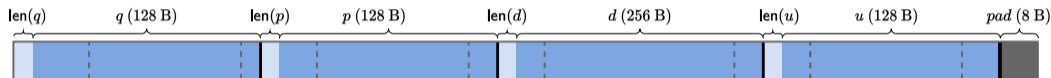
- Custom encoding of  $sk$  for RSA-CRT decryption, referred to as  $privk$ 
  - the prime factors  $p, q$  of the RSA modulus
  - the secret exponent  $d$
  - the value  $u = q^{-1} \bmod p$

## Format of $sk$



- Custom encoding of  $sk$  for RSA-CRT decryption, referred to as  $privk$ 
  - the prime factors  $p, q$  of the RSA modulus
  - the secret exponent  $d$
  - the value  $u = q^{-1} \bmod p$
- Each value is prefixed with a 2-byte length field

## Format of $sk$



- Custom encoding of  $sk$  for RSA-CRT decryption, referred to as  $privk$ 
  - the prime factors  $p, q$  of the RSA modulus
  - the secret exponent  $d$
  - the value  $u = q^{-1} \bmod p$
- Each value is prefixed with a 2-byte length field
- Split into 16-byte blocks for AES-ECB



# MEGA login procedure

User

MEGA

login request(*User*)



get ( $[k_M]_{k_e}, [privk]_{k_M}, uh$ ) of *User*

pick 43-byte *sid*

$[m]_{pk} \leftarrow \text{RSA.Enc}(pk, sid \parallel uh)$

$([k_M]_{k_e}, [privk]_{k_M}, [m]_{pk}, uh)$



$k_M \leftarrow \text{AES-ECB.Dec}(k_e, [k_M]_{k_e})$

$sid' \leftarrow \text{MegaDec}(k_M, [privk]_{k_M}, [m]_{pk}, uh)$

*sid'* or  $\perp$



$sid' \stackrel{?}{=} sid$

# Client decryption and parsing

**MegaDec**( $k_M$ ,  $[\text{privk}]_{k_M}$ ,  $[m]_{pk}$ ,  $uh$ ):

- 1  $sk \leftarrow \text{DecryptPrivk}(k_M, [\text{privk}]_{k_M})$  // AES-ECB
- 2  $sid' \leftarrow \text{DecryptSid}(sk, [m]_{pk})$  // RSA-CRT
- 3 Return  $sid'$

# Client decryption and parsing

**MegaDec**( $k_M$ ,  $[\text{privk}]_{k_M}$ ,  $[m]_{pk}$ ,  $uh$ ):

- 1  $sk \leftarrow \text{DecryptPrivk}(k_M, [\text{privk}]_{k_M})$  // AES-ECB
- 2  $sid' \leftarrow \text{DecryptSid}(sk, [m]_{pk})$  // RSA-CRT
- 3 Return  $sid'$

Both steps rely on validity checking of the decrypted values and return distinguishable errors to the server!

# Oracles from error reporting

# Oracles from error reporting

**Explicit** errors due to validity checking:

- In  $\text{DecryptSid}(sk, \cdot)$ , a length check on the plaintext together with a legacy padding check reveal if the second byte of  $m$  is 0  
⇒ attack based on small subgroups (#2)

# Oracles from error reporting

**Explicit** errors due to validity checking:

- In `DecryptSid( $sk$ , ·)`, a length check on the plaintext together with a legacy padding check reveal if the second byte of  $m$  is 0  
⇒ attack based on small subgroups (#2)

**Implicit** errors due to bugs in the low-level library:

- In `DecryptPrivk( $k_M$ , ·)`, failure in recomputing  $u' \leftarrow q^{-1} \bmod p$  reveals if  $\gcd(p, q) \neq 1$   
⇒ attack based on modular inverses (#1)

# ECB encryption oracle from MEGAdrop

# ECB encryption oracle from MEGAdrop

- **MEGAdrop** lets anyone upload files to a folder in the cloud storage of the recipient



## ECB encryption oracle from MEGAdrop

- **MEGAdrop** lets anyone upload files to a folder in the cloud storage of the recipient
- Clients automatically re-encrypt the received shared-file keys

# ECB encryption oracle from MEGAdrop

- **MEGAdrop** lets anyone upload files to a folder in the cloud storage of the recipient
- Clients automatically re-encrypt the received shared-file keys

MEGA

MEGAdrop.upload( $k_F$ )

$[F]_{k_F}, [k_F]_{pk}$



User

Webclient.update()

$[k_F]_{k_M}$



# ECB encryption oracle from MEGAdrop

- **MEGAdrop** lets anyone upload files to a folder in the cloud storage of the recipient
- Clients automatically re-encrypt the received shared-file keys

MEGA

MEGAdrop.upload( $k_F$ )

$[F]_{k_F}, [k_F]_{pk}$



User

Webclient.update()

$[k_F]_{k_M}$



A malicious provider can construct an ECB encryption oracle *without user interaction* and *without leaving traces*

# Attacks

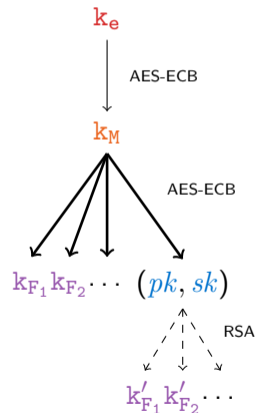
# Attacks

Setting: a malicious service provider

# Attacks

Setting: a malicious service provider

Goal: obtain ECB decryption ability under  $k_M$   
 $\implies$  recover  $sk$  (or any  $k_F$ )

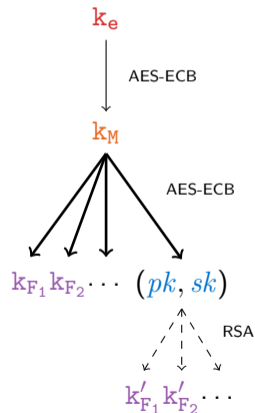


# Attacks

Setting: a malicious service provider

Goal: obtain ECB decryption ability under  $k_M$   
 $\implies$  recover  $sk$  (or any  $k_F$ )

Cost measured mainly in the number of login attempts



# Attack based on modular inverses



## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

- Construct  $[\text{priv}k^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

- Construct  $[\text{priv}k^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that
  - $p \bmod r = 0$  for small prime  $r$ , and

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

- Construct  $[\text{priv}k^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that
  - $p \bmod r = 0$  for small prime  $r$ , and
  - $q$  contains  $B$  in the least-significant position

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

- Construct  $[\text{privk}^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that
  - $p \bmod r = 0$  for small prime  $r$ , and
  - $q$  contains  $B$  in the least-significant position

so that  $\perp_{\text{inv}} \iff q \bmod r = 0$ , so by adjusting  $q$  we can learn  $B \bmod r$

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

- Construct  $[\text{privk}^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that
  - $p \bmod r = 0$  for small prime  $r$ , and
  - $q$  contains  $B$  in the least-significant positionso that  $\perp_{\text{inv}} \iff q \bmod r = 0$ , so by adjusting  $q$  we can learn  $B \bmod r$
- Repeat for a set of primes  $r_i$  such that their product  $R$  is 128 bits, so we can learn  $B \bmod R$  via CRT

## Attack based on modular inverses

Let  $[B]_{k_M}$  be the target ciphertext block,  $\text{OECB}_{k_M}$  be the ECB encryption oracle, and  $\perp_{\text{inv}}$  be the error output by **MegaDec** if  $\gcd(p, q) \neq 1$

Main idea is to

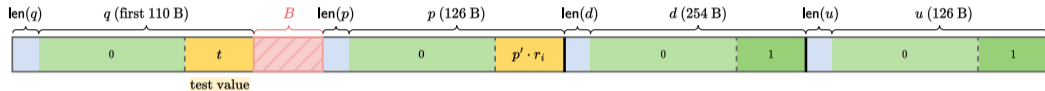
- Construct  $[\text{privk}^*]_{k_M}$  (using  $\text{OECB}_{k_M}$  and  $[B]_{k_M}$ ) such that
  - $p \bmod r = 0$  for small prime  $r$ , and
  - $q$  contains  $B$  in the least-significant positionso that  $\perp_{\text{inv}} \iff q \bmod r = 0$ , so by adjusting  $q$  we can learn  $B \bmod r$
- Repeat for a set of primes  $r_i$  such that their product  $R$  is 128 bits, so we can learn  $B \bmod R$  via CRT

Average cost: 627 login attempts and 66-91  $\text{OECB}_{k_M}$  queries

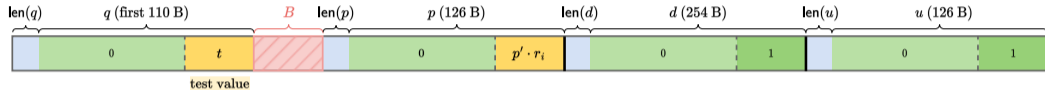
# Attack based on modular inverses (two versions)



# Attack based on modular inverses (two versions)



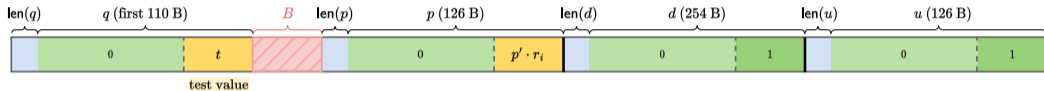
# Attack based on modular inverses (two versions)



Simple version could be prevented by extra validation checks

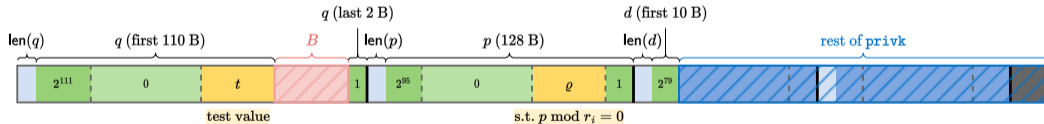
⇒ full version using more valid-looking values and parts of the original `[privk]km`

# Attack based on modular inverses (two versions)



Simple version could be prevented by extra validation checks

⇒ full version using more valid-looking values and parts of the original `[privk]2M`



# Attack based on small subgroups

# Attack based on small subgroups

Exploit the behaviour of  $\text{DecryptSid}(sk, [m]_{pk})$ :

# Attack based on small subgroups

Exploit the behaviour of  $\text{DecryptSid}(sk, [m]_{pk})$ :

```
...  
m ← RSA-CRT(sk, [m]_{pk})  
if m[1] ≠ 00 then m' ← 00 || m  
else m' ← m  
m' ← m'[2 : bytelen(m')]  
if bytelen(m') ≠ 255 then  
    return (⊥_{00}, bytelen(m'))  
...
```

# Attack based on small subgroups

Exploit the behaviour of  $\text{DecryptSid}(sk, [m]_{pk})$ :

```

...
m ← RSA-CRT(sk, [m]_{pk})
if m[1] ≠ 00 then m' ← 00 || m
else m' ← m
m' ← m'[2 : bytelen(m')]
if bytelen(m') ≠ 255 then
    return (⊥_{00}, bytelen(m'))
...

```

for  $\text{bytelen}(m) = 256$ ,  
 this means  $\perp_{00} \iff m[1] = 00$

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[privk^*]_{k_M}$  such that



## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[privk^*]_{k_M}$  such that
  - $d$  contains  $B$  in the least-significant position

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[privk^*]_{k_M}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[privk^*]_{k_M}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$
- Replace  $[m]_{pk}$  with  $c = g^x \bmod pq$  where

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[\text{privk}^*]_{\text{KM}}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$
- Replace  $[\text{m}]_{\text{pk}}$  with  $c = g^x \bmod pq$  where
  - $g$  has order  $r_i$  and  $\exists t \in \{1, \dots, r_i - 1\}: g^t \bmod pq$  has second byte 00

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[\text{privk}^*]_{\text{KM}}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$
- Replace  $[m]_{pk}$  with  $c = g^x \bmod pq$  where
  - $g$  has order  $r_i$  and  $\exists t \in \{1, \dots, r_i - 1\}: g^t \bmod pq$  has second byte 00
  - $x \in \{1, \dots, r_i - 1\}$  is a tested value so that  $\perp_{00}$  reveals  $x \cdot d \equiv t \pmod{r_i}$ , and we learn  $B \bmod r_i$

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[\text{privk}^*]_{k_M}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$
- Replace  $[m]_{pk}$  with  $c = g^x \bmod pq$  where
  - $g$  has order  $r_i$  and  $\exists t \in \{1, \dots, r_i - 1\}: g^t \bmod pq$  has second byte 00
  - $x \in \{1, \dots, r_i - 1\}$  is a tested value so that  $\perp_{00}$  reveals  $x \cdot d \equiv t \pmod{r_i}$ , and we learn  $B \bmod r_i$

Requires precomputation,  $\approx 3200$  logins and 15  $\text{OECB}_{k_M}$  calls

## Attack based on small subgroups (cont'd)

Main idea is to

- Construct  $[\text{privk}^*]_{k_M}$  such that
  - $d$  contains  $B$  in the least-significant position
  - $p, q$  are such that  $(p-1)(q-1)$  has small prime factors  $r_i$
- Replace  $[m]_{pk}$  with  $c = g^x \bmod pq$  where
  - $g$  has order  $r_i$  and  $\exists t \in \{1, \dots, r_i - 1\}$ :  $g^t \bmod pq$  has second byte 00
  - $x \in \{1, \dots, r_i - 1\}$  is a tested value so that  $\perp_{00}$  reveals  $x \cdot d \equiv t \pmod{r_i}$ , and we learn  $B \bmod r_i$

Requires precomputation,  $\approx 3200$  logins and 15  $\text{OECB}_{k_M}$  calls

The variety of errors used demonstrates the fragility of the system

# Recovering the full key

Naive use of previous attacks would require up to 9 blocks to recover  $sk$ , so instead



# Recovering the full key

Naive use of previous attacks would require up to 9 blocks to recover  $sk$ , so instead

- 1 Recover 4 blocks of  $q$

# Recovering the full key

Naive use of previous attacks would require up to 9 blocks to recover  $sk$ , so instead

- 1 Recover 4 blocks of  $q$
- 2 Run exhaustive search for the last 16 bits (non-aligned block)

# Recovering the full key

Naive use of previous attacks would require up to 9 blocks to recover  $sk$ , so instead

- 1 Recover 4 blocks of  $q$
- 2 Run exhaustive search for the last 16 bits (non-aligned block)
- 3 Efficiently recover the remainder using lattice reduction

# Attacks only get better

# Attacks only get better

- MEGA claimed the previous attacks of Backendal, Haller and Paterson [BHP23] were not practical due to the number of login attempts needed

# Attacks only get better

- MEGA claimed the previous attacks of Backedal, Haller and Paterson [BHP23] were not practical due to the number of login attempts needed
- ECB encryption oracle can be used to optimise one of the attacks on unpatched clients

# Attacks only get better

- MEGA claimed the previous attacks of Backendal, Haller and Paterson [BHP23] were not practical due to the number of login attempts needed
- ECB encryption oracle can be used to optimise one of the attacks on unpatched clients
- Recover [privk](#) using only 2 login attempts (vs. 512 of [BHP23] and 6 of [RH23])

# Responsible disclosure



# Responsible disclosure

- We disclosed to MEGA in September 2022, suggesting mitigations

# Responsible disclosure

- We disclosed to MEGA in September 2022, suggesting mitigations
- MEGA informed us that they were working on a larger redesign meant to

# Responsible disclosure

- We disclosed to MEGA in September 2022, suggesting mitigations
- MEGA informed us that they were working on a larger redesign meant to
  - Change how private keys are stored
  - Remove the ECB encryption oracle
  - Replace the low-level crypto/bigint library

# Responsible disclosure

- We disclosed to MEGA in September 2022, suggesting mitigations
- MEGA informed us that they were working on a larger redesign meant to
  - Change how private keys are stored
  - Remove the ECB encryption oracle
  - Replace the low-level crypto/bigint library
- Upgrade of clients in March 2023

# Responsible disclosure

- We disclosed to MEGA in September 2022, suggesting mitigations
- MEGA informed us that they were working on a larger redesign meant to
  - Change how private keys are stored
  - Remove the ECB encryption oracle
  - Replace the low-level crypto/bigint library
- Upgrade of clients in March 2023
- MEGA awarded a bug bounty

# Discussion

# Discussion

- Root causes highlighted in previous work [BHP23], but it took multiple series of attacks for MEGA to agree to some of the suggested mitigations

# Discussion

- Root causes highlighted in previous work [BHP23], but it took multiple series of attacks for MEGA to agree to some of the suggested mitigations
- ECB encryption oracle from an independent feature shows the fragility of the design



# Discussion

- Root causes highlighted in previous work [BHP23], but it took multiple series of attacks for MEGA to agree to some of the suggested mitigations
- ECB encryption oracle from an independent feature shows the fragility of the design
- Our attacks serve as an example of key overwriting attacks, which deserve more exploration

# Discussion

- Root causes highlighted in previous work [BHP23], but it took multiple series of attacks for MEGA to agree to some of the suggested mitigations
- ECB encryption oracle from an independent feature shows the fragility of the design
- Our attacks serve as an example of key overwriting attacks, which deserve more exploration

Cryptanalysis of protocols “in the wild” is needed to achieve the adoption of more secure and formally analysed cryptographic solutions in practice

# Discussion

See more details in [ia.cr/2023/329](https://ia.cr/2023/329)

# Discussion

See more details in [ia.cr/2023/329](https://ia.cr/2023/329)

Thank you for your attention. Any questions?

# References

- [BHP23] Matilda Backendal, Miro Haller, and Kenneth G. Paterson. MEGA: Malleable Encryption Goes Awry. In *IEEE S&P 2023, to appear*, 2023. <https://eprint.iacr.org/2022/959>.
- [BPH22] Lara Bruseghini, Kenneth G Paterson, and Daniel Huigens. Victory by KO: Attacking OpenPGP using key overwriting. In *ACM Conference on Computer and Communications Security (ACM CCS), to appear*, 2022.
- [KR02] Vlastimil Klima and Tomas Rosa. Attack on private signature keys of the OpenPGP format, PGP(TM) programs and other applications compatible with OpenPGP. Cryptology ePrint Archive, Report 2002/076, 2002. <https://eprint.iacr.org/2002/076>.
- [LL97] Chae Hoon Lim and Pil Joong Lee. A key recovery attack on discrete log-based schemes using a prime order subgroup. In Burton S. Kaliski Jr., editor, *CRYPTO'97*, volume 1294 of *LNCS*, pages 249–263. Springer, Heidelberg, August 1997.
- [RH23] Keegan Ryan and Nadia Heninger. The hidden number problem with small unknown multipliers: Cryptanalyzing MEGA in six queries and other applications. In *PKC 2023, to appear*, 2023. <https://eprint.iacr.org/2022/914>.
- [vW96] Paul C. van Oorschot and Michael J. Wiener. On Diffie-Hellman key agreement with short exponents. In Ueli M. Maurer, editor, *EUROCRYPT'96*, volume 1070 of *LNCS*, pages 332–343. Springer, Heidelberg, May 1996.