CASA
CYBER SECURITY IN THE AGE OF LARGE-SCALE ADVERSARIES

Pitfalls and Shortcomings for Decompositions and Alignment

Lyon, April 25

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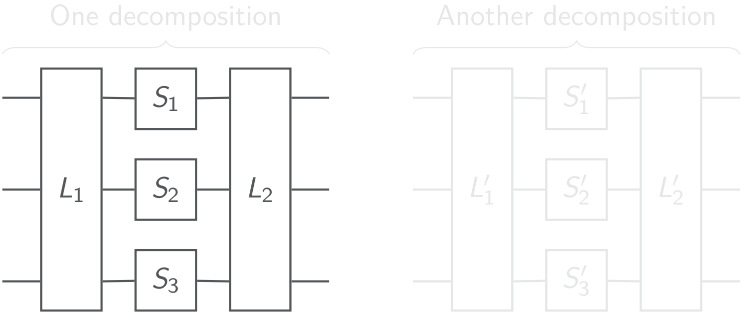
Decomposition of SPN Round Function



- Linear layers:
  L_1, L_2, L'_1, L'_2
- S-boxes:
  S_1, S_2, S_3, S'_1, S'_2, S'_3

- Security arguments based on decomposition
- Example: Bound probability of differential characteristic by counting active S-boxes
- Potential problem: Result may depend on the decomposition
- Question: When do multiple decompositions exist?

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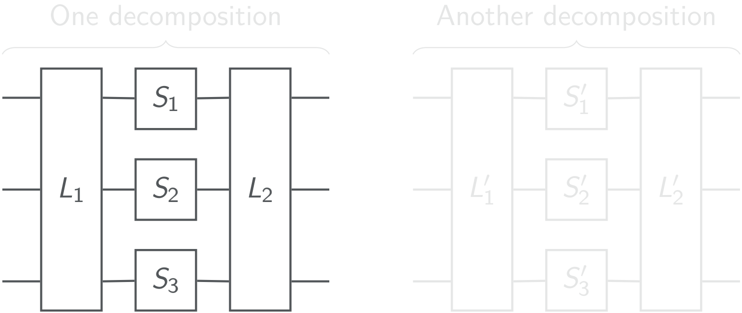

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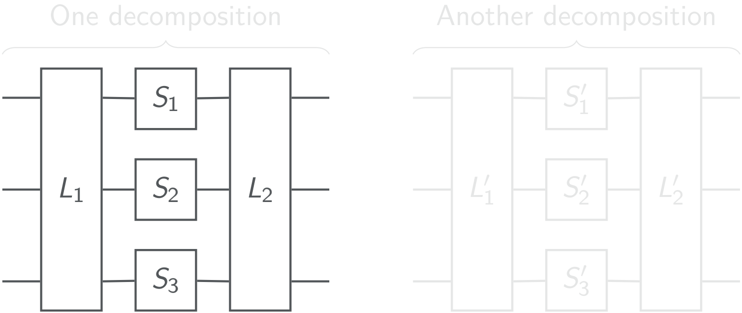

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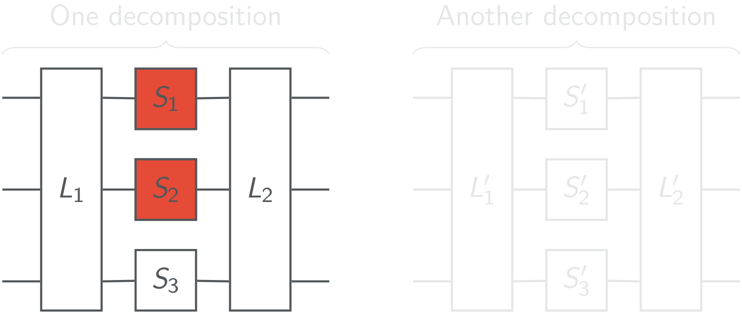

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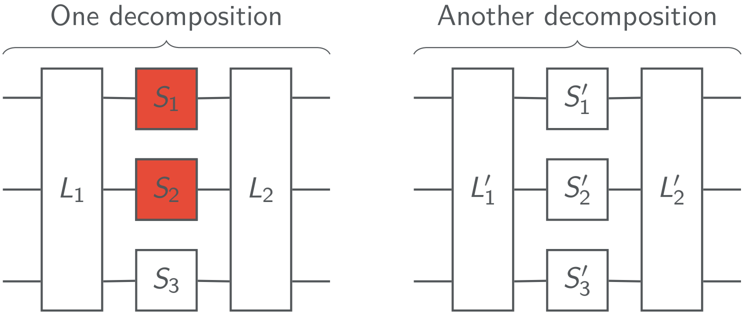

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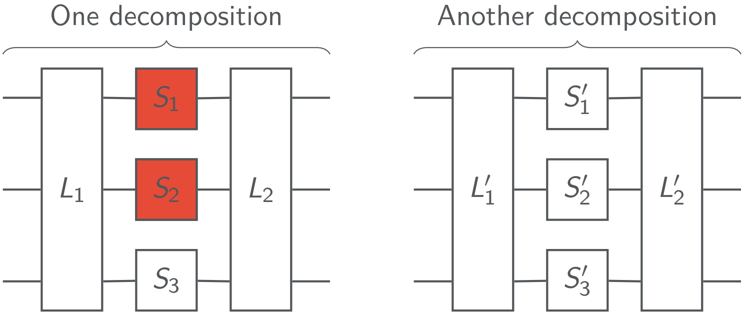

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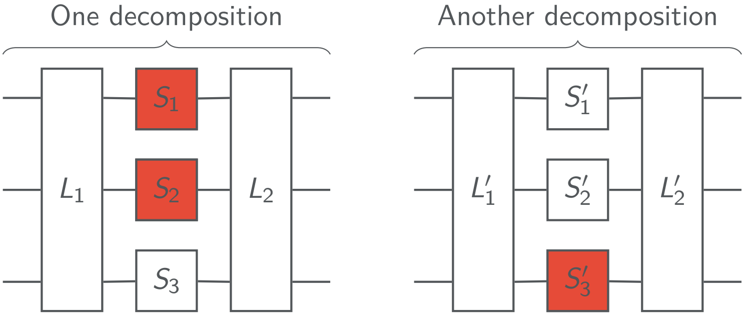

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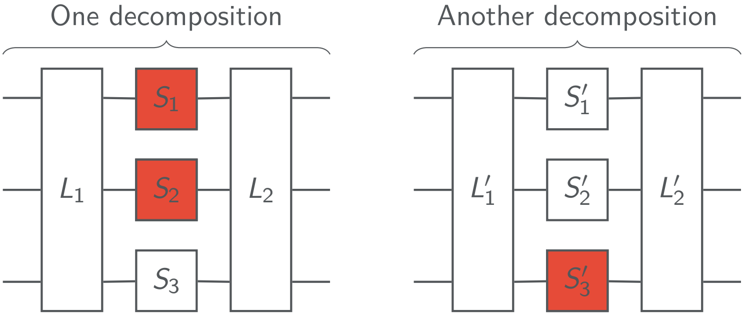

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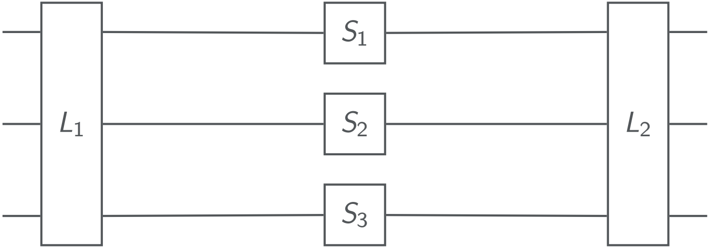
Uniqueness of Decompositions

- Well knows limitations to uniqueness
  - Reordering the S-boxes
  - Linear equivalence of the S-boxes
  - Combining S-boxes



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When is a decomposition unique (up to those limitations)?

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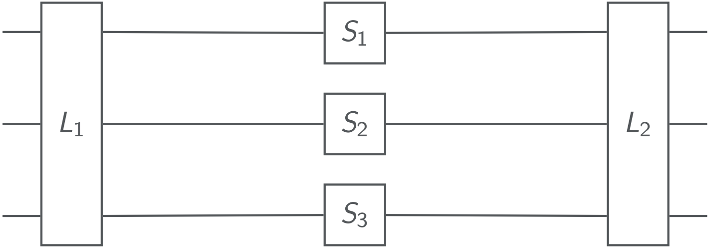

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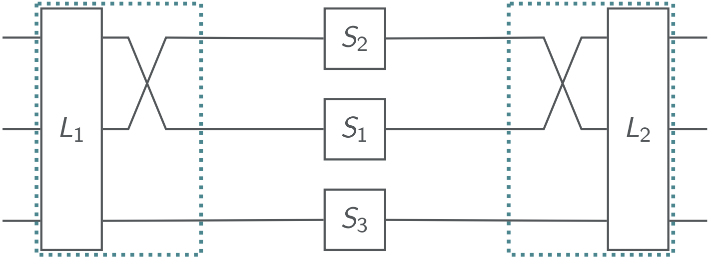

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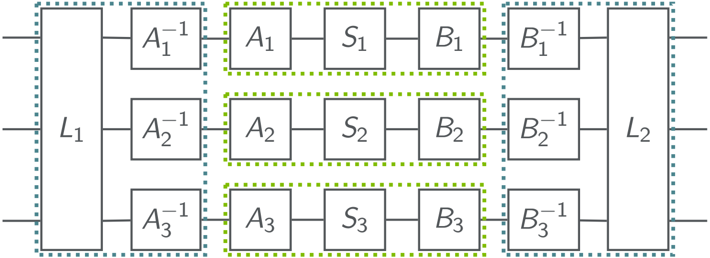

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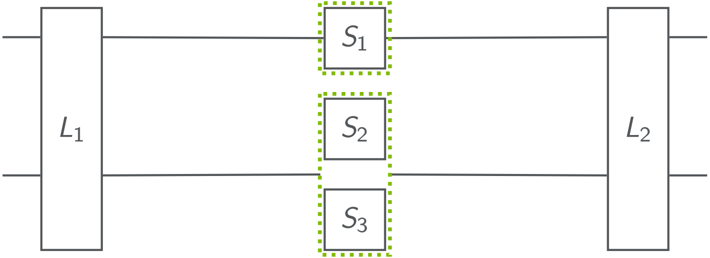

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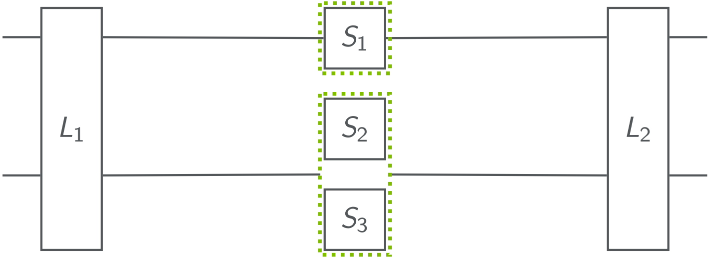

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Definition

A function F has maximal differential uniformity if $F(x) + F(x + \alpha) = \beta$ for some non-zero α, some β and all x.

Definition

A function F has maximal linearity if $\alpha^T \cdot F$ is affine for some non-zero α.

Main Theorem

A decomposition is not unique if and only if one S-box has maximal differential uniformity and another one has maximal linearity.

- Good, since decomposition is unique for all cryptographically strong S-boxes
- Easy/efficient to check (for all common S-box sizes)

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Sketch of Proof

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- Backward direction: If one S-box has maximal differential uniformity and another one has maximal linearity then there exist multiple decompositions

Lemma

Functions with maximal differential uniformity are exactly those that are linear equivalent to functions of the form

$$G\begin{pmatrix} x \\ y \end{pmatrix} = G\begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ y \end{pmatrix}.$$

Idea of Proof:

- F has maximal differential uniformity $\implies F(x') + F(x' + \alpha) = \beta$ for some $\alpha \neq 0, \beta$
- Linearly transform F to G such that α and β correspond to last bit y

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$$H\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\h\begin{pmatrix}x\\y\end{pmatrix}\end{pmatrix}.$$

Idea of Proof:

- F has maximal linearity $\implies$ $\alpha^T \cdot F(x') = \beta^T \cdot x' + c$ for some $\alpha \neq 0$, β and c
- Add $F(0)$ to F, as $\alpha^T \cdot (F + F(0))$ is linear
- Linearly transform $F + F(0)$ to H such that α and β both correspond to $(1, 0, \ldots, 0)^T$

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Corollary

Functions <u>without</u> unique decomposition are exactly those affine equivalent to ones of the form

$$R\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix} G\begin{pmatrix}x_1\\0\end{pmatrix} + \begin{pmatrix}0\\x_2\end{pmatrix} \\ x_3 \\ h\begin{pmatrix}x_3\\x_4\end{pmatrix}\end{pmatrix} \begin{matrix}\}\ \textit{S-box(es)} \\ \}\ \textit{S-box(es)}\end{matrix}.$$

Sketch of Proof

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$$= \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) R \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$$

- Decomposition not unique, as linear layers are different
- Other direction: See paper

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Uniqueness of Decompositions

- Example without unique decomposition: DEFAULT[1]
- Have to be careful if S-boxes with maximal differential uniformity and linearity are/can be used!
- One such case: Alignment
  - Misterious property:
    - Positive & negative correlations
    - For a long time: Not formally defined
  - First formal definition at CRYPTO'21[2]
  - Informally, alignment means that two rounds decompose with at least two S-boxes (referred to as super-boxes)
  - Investigate: Definition & impact of alignment

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Uniqueness of Decompositions

- Example without unique decomposition: DEFAULT[1]
- Have to be careful if S-boxes with maximal differential uniformity and linearity are/can be used!
- One such case: Alignment
  - Misterious property:
    - Positive & negative connotations
    - For a long time: Not formally defined
  - First formal definition at CRYPTO'21[2]
  - Informally, alignment means that two rounds decompose with at least two S-boxes (referred to as super-boxes)
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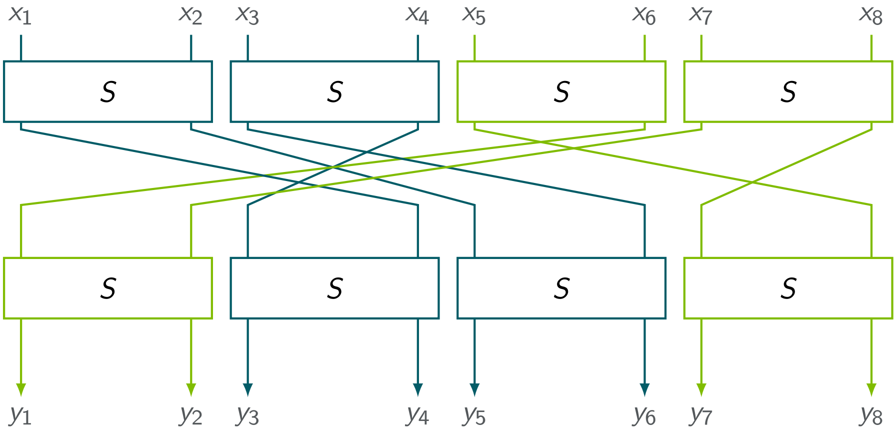
Alignment and Non-Unique Decompositions



- Function aligned

Alignment and Non-Unique Decompositions

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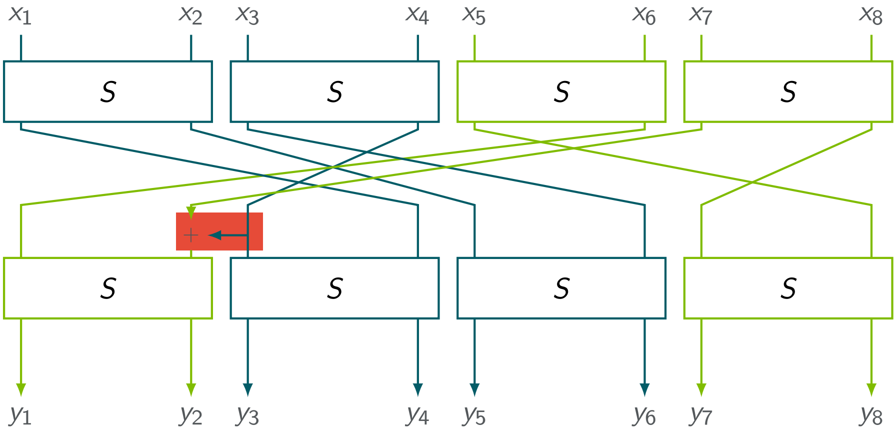

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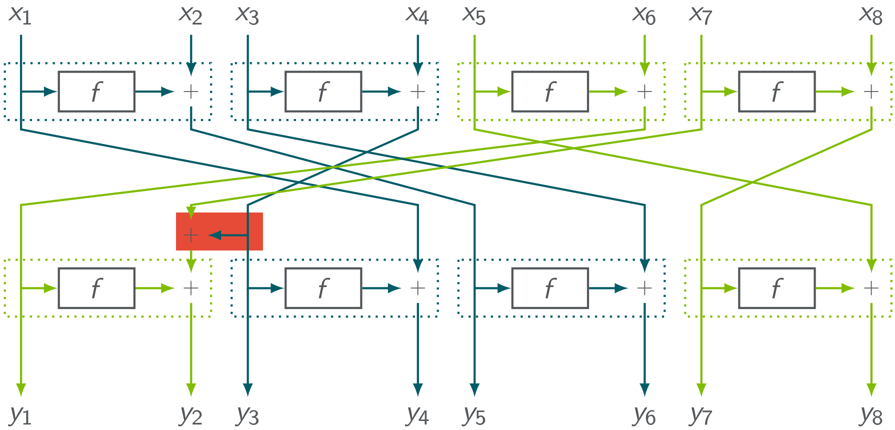

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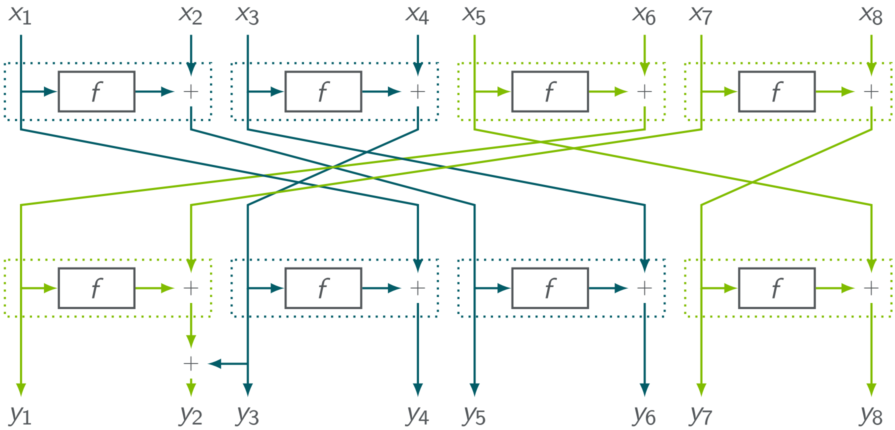

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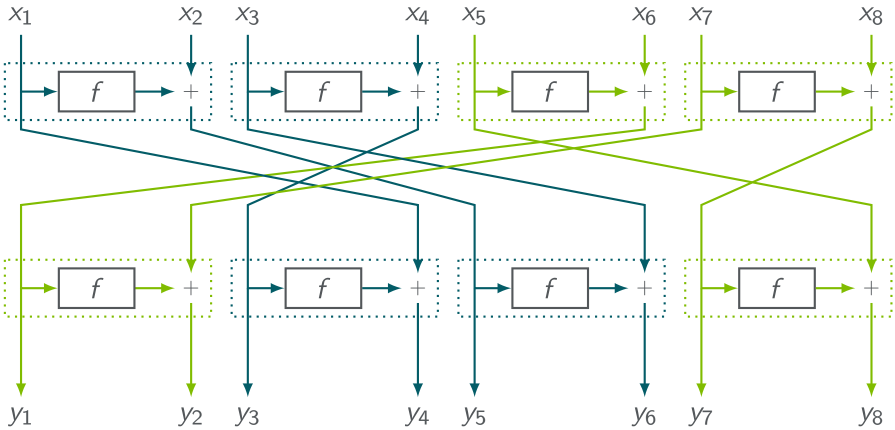

- Function unaligned, but also aligned?

Alignment – Impact

- (CRYPTO'21) compared aligned and unaligned ciphers
- Infer that alignment might lead to bigger clustering effects
- Question: Results due to alignment or due to other disparities?
- Idea: Change bit-permutation of PRESENT
  - Produce variants that are aligned and ones that are unaligned
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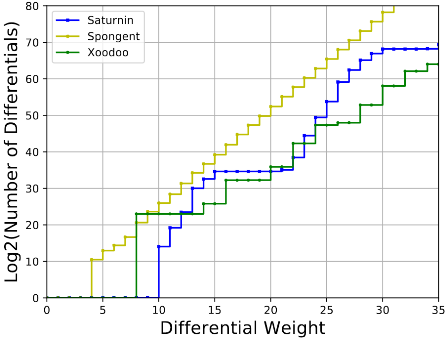
Cumulative histogram of the number of differentials of a given weight over 2 rounds



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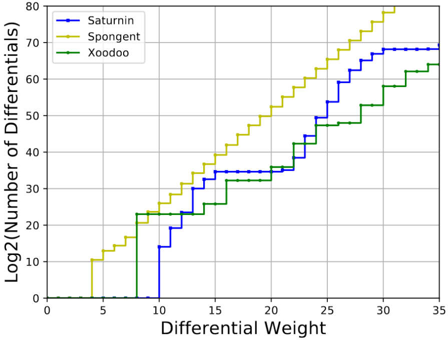

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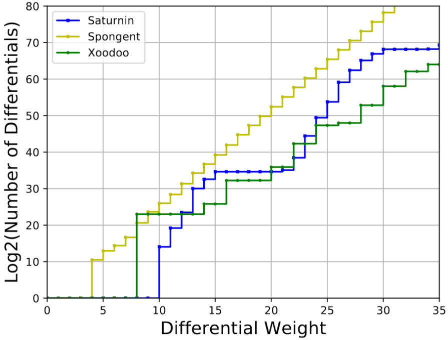

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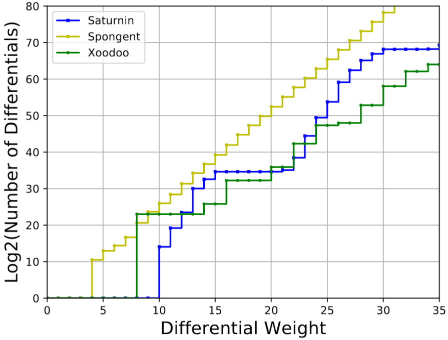

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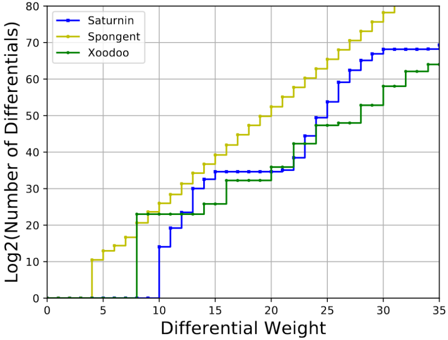

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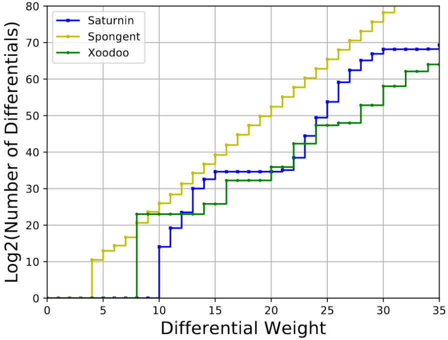

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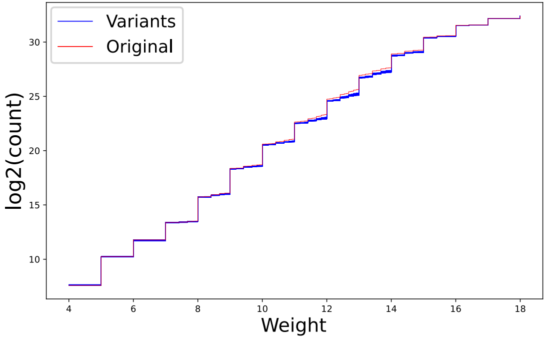

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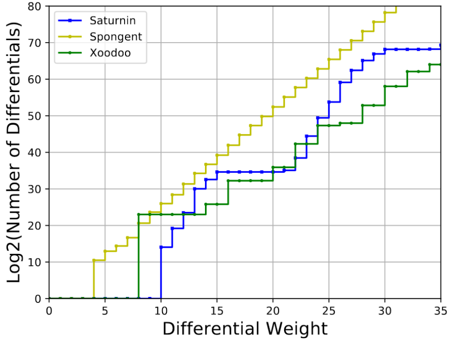

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- Overall result: PRESENT variants behave very similar in all aspects

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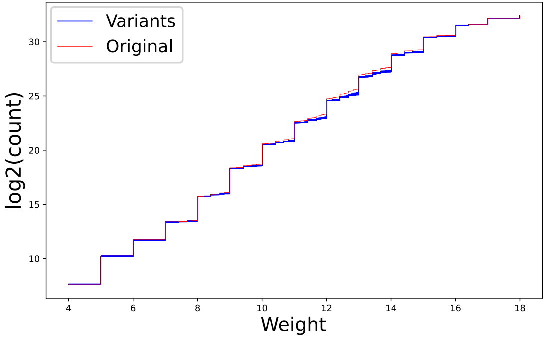

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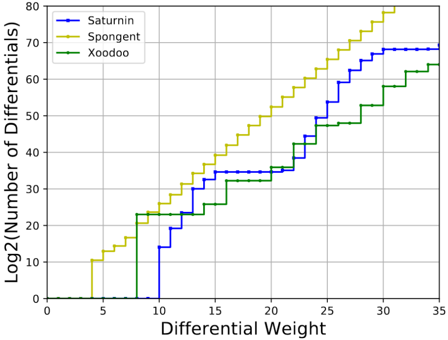

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- Under some mild conditions decomposition is unique
- Good, as it allows to base security arguments on the unique decomposition
- Still, have to be careful if conditions are not met

- Impact of alignment on clustering may be overestimated
- Benefits of alignment may outweigh this impact

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