Throwing Boomerangs into Feistel Structures

Application to CLEFIA, WARP, LBlock, LBlock-s and TWINE

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FSE 2023 - Kobe, Japan

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TU Graz

SCIENCE PASSION TECHNOLOGY

Research Gap and Our Contributions

Motivation and Our Contributions

Research gap:

- The lack of a tool to automatically find boomerang distinguishers for Feistel cipher

Contributions:

- Providing an easy to use and fast method to find boomerang distinguishers
- We applied our method to `CLEFIA`, `WARP`, `LBlock`, and `TWINE`
  - We improved the boomerang distinguisher of `WARP` by 2 rounds
  - We improved the boomerang distinguisher/attack of `CLEFIA` by 1 round
- Our method is applicable to any strongly aligned (Sbox-based) block cipher, e.g., SKINNY

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Outline

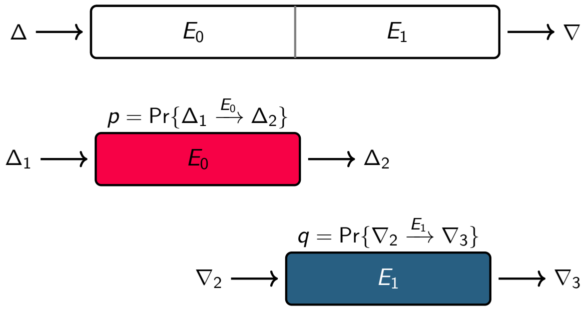
Effective Parameters in the Success Probability of Boomerang Distinguishers

Boomerang Distinguishers [Wag99]

$$\Delta \longrightarrow \boxed{E : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n} \longrightarrow \nabla$$

$$0 \lneq \Pr\{\Delta \xrightarrow{E} \nabla\} \lll 2^{-n}$$

Boomerang Distinguishers [Wag99]



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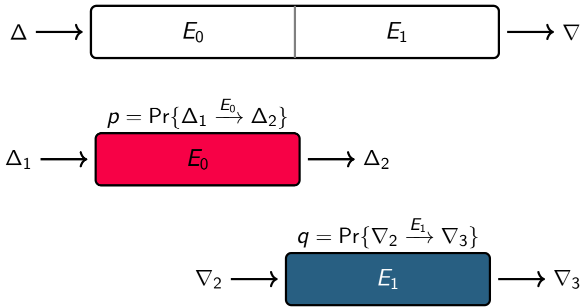

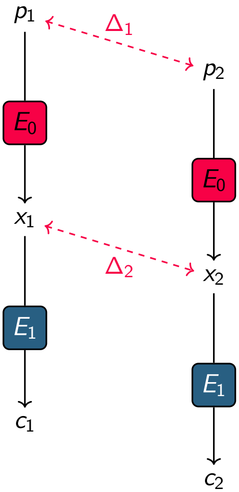

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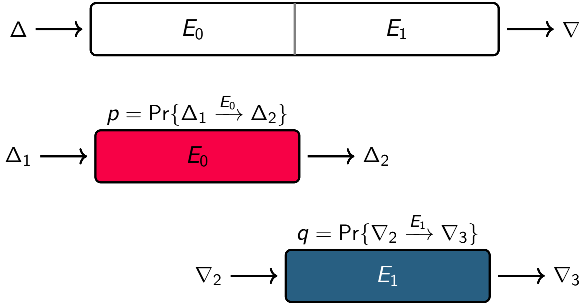

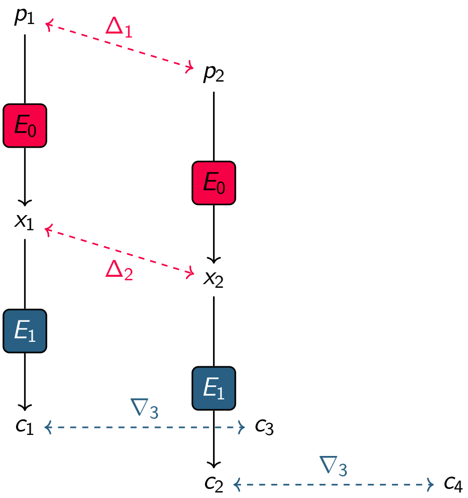

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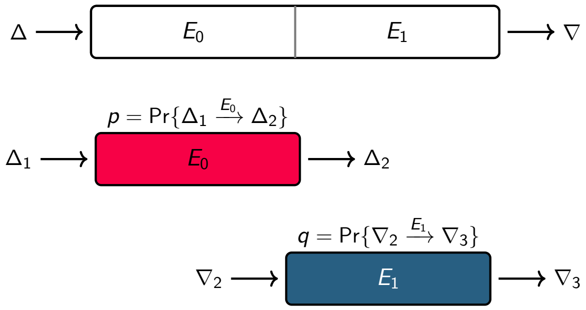

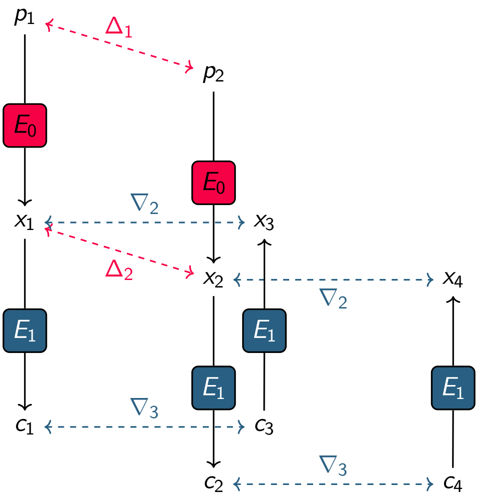

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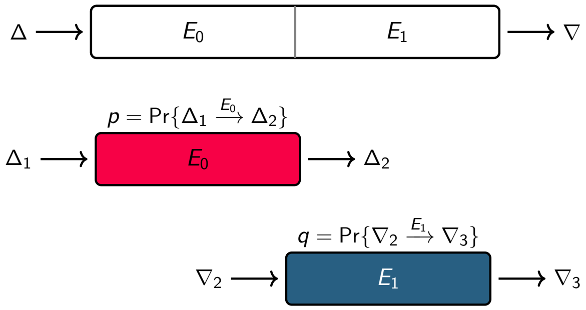
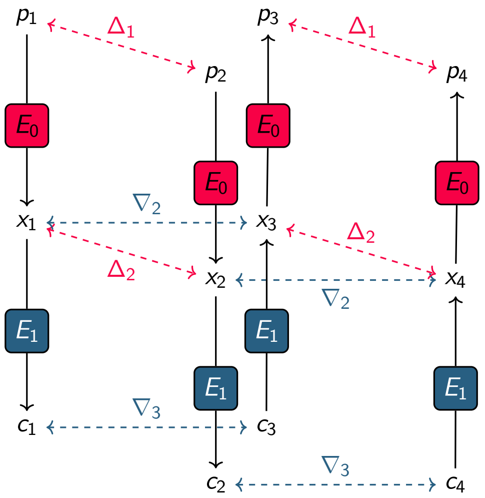


$$\Pr\{p_3 \oplus p_4 = \Delta_1\} = p^2 q^2$$



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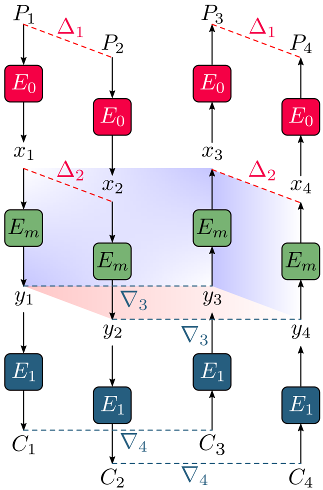
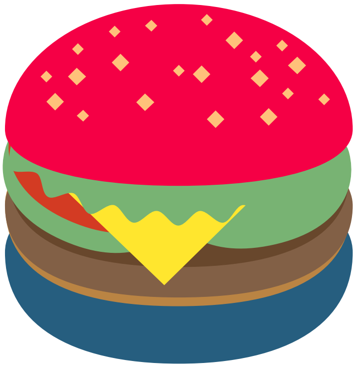
Sandwiching the Differentials! [DKS10]





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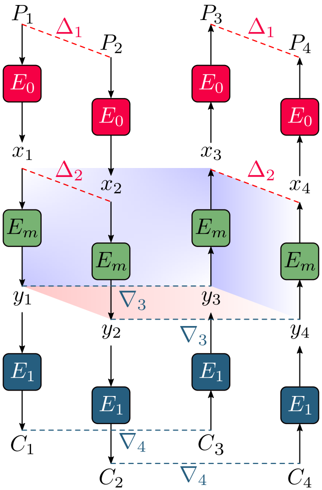

$$\Pr(P_3 \oplus P_4 = \Delta_1) \approx p^2 \times r \times q^2$$

$$r = \Pr(\Delta_2 \rightleftarrows \nabla_3)$$

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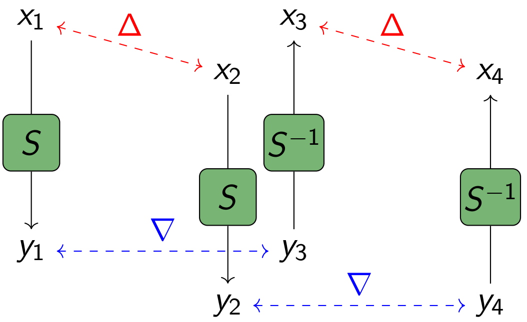
Boomerang Switch For SPN Block Ciphers



$$\mathrm{BCT}(\Delta, \nabla) := \#\{x \in \mathbb{F}_2^n \mid S^{-1}(S(x) \oplus \nabla) \oplus S^{-1}(S(x \oplus \Delta) \oplus \nabla) = \Delta\}$$

$$\mathrm{BCT}(0, \nabla) = \mathrm{BCT}(\Delta, 0) = 2^n$$

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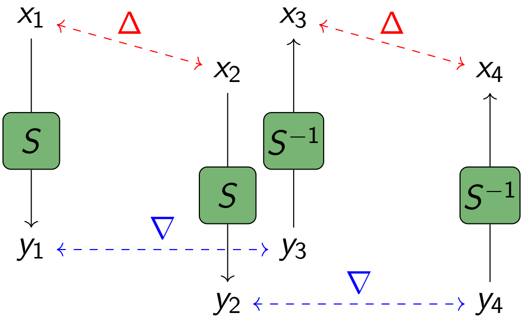

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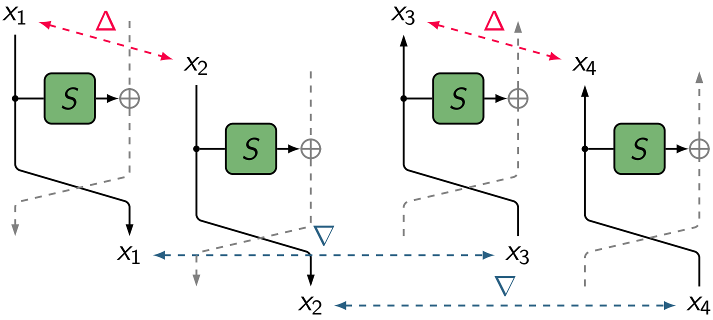

$$\mathtt{FBCT}(\Delta, \nabla) := \#\{x \in \mathbb{F}_2^n : S(x) \oplus S(x \oplus \Delta) \oplus S(x \oplus \nabla) \oplus S(x \oplus \Delta \oplus \nabla) = 0\}$$

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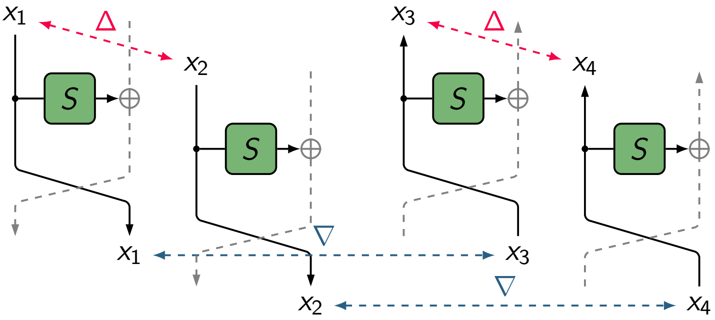
Boomerang Switch For Feistel Ciphers



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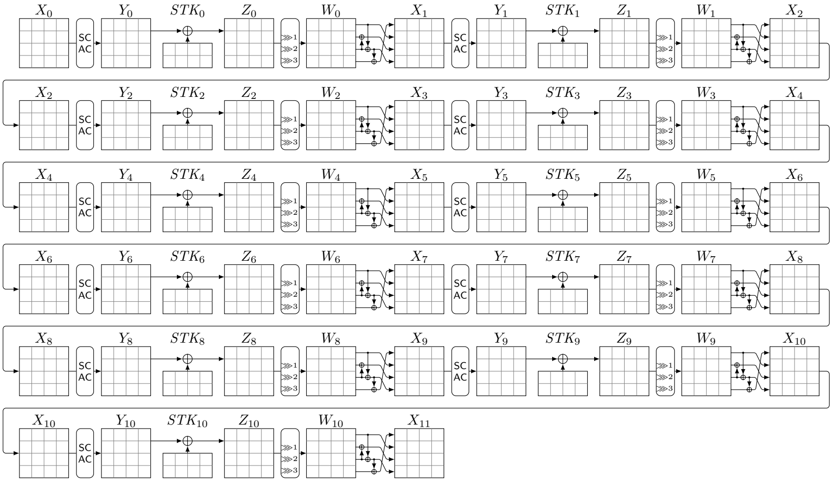
Building Deterministic Boomerang from Impossible Trails [HBS21]



- $p = 2^{-146}$ (impossible due to dependencies [PT22])
- $q = 2^{-179}$ (impossible due to dependencies [PT22])
- $\Pr_{\text{boom}} = 1$

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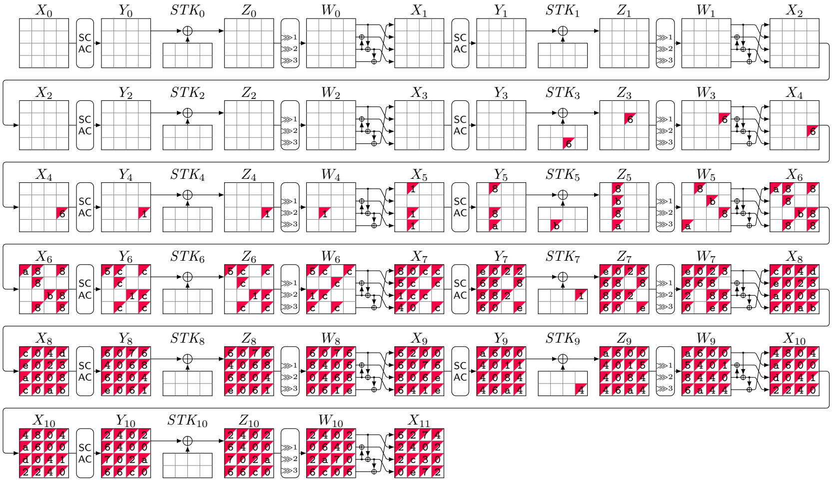

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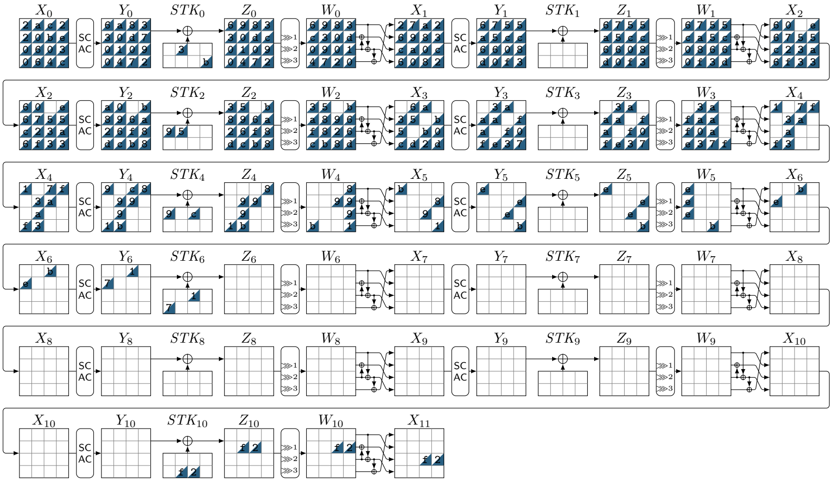

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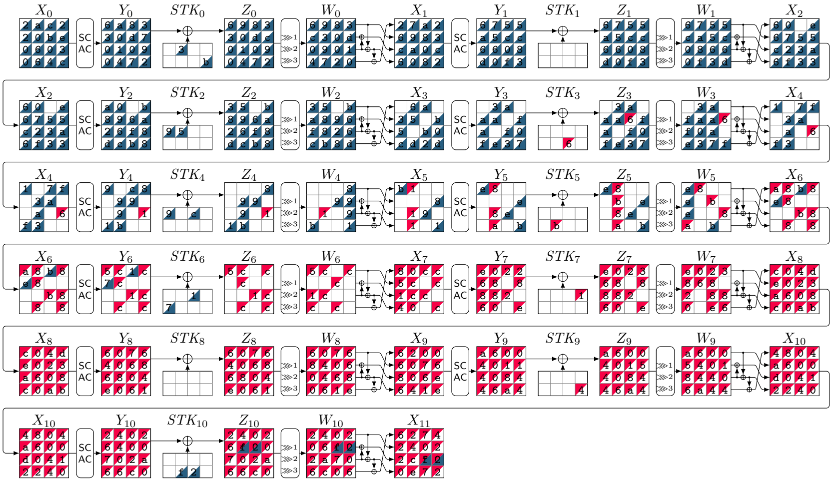

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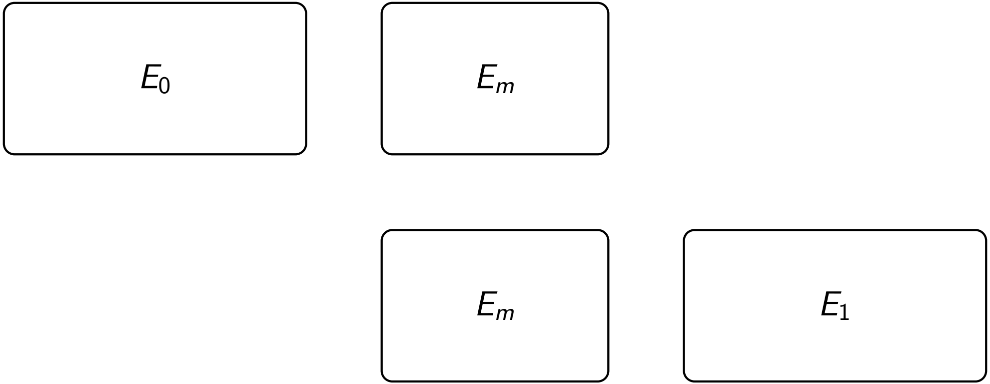
Effective Parameters in p^2q^2r Formula



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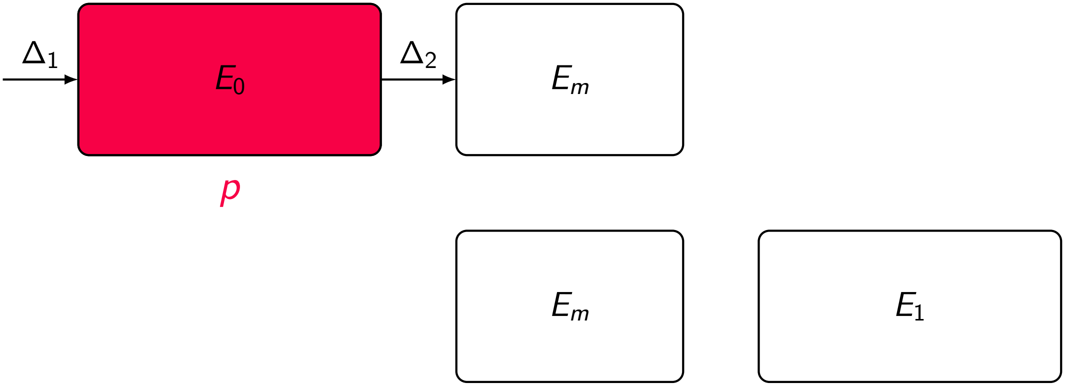

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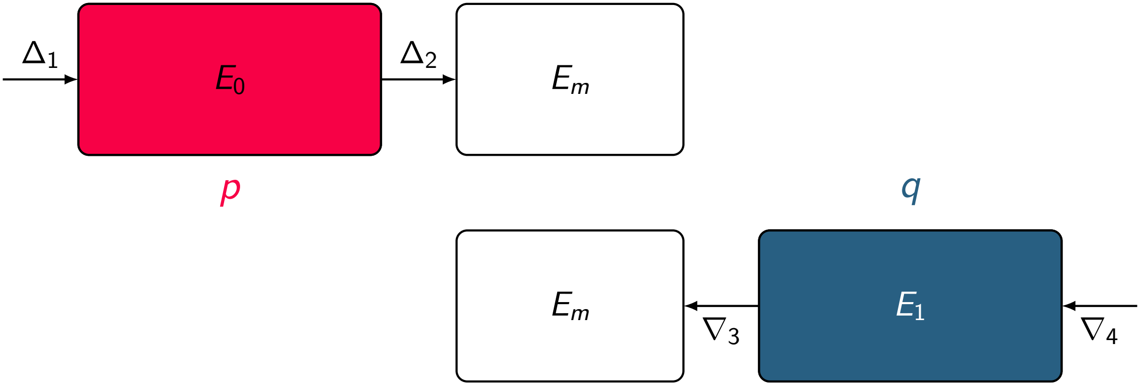

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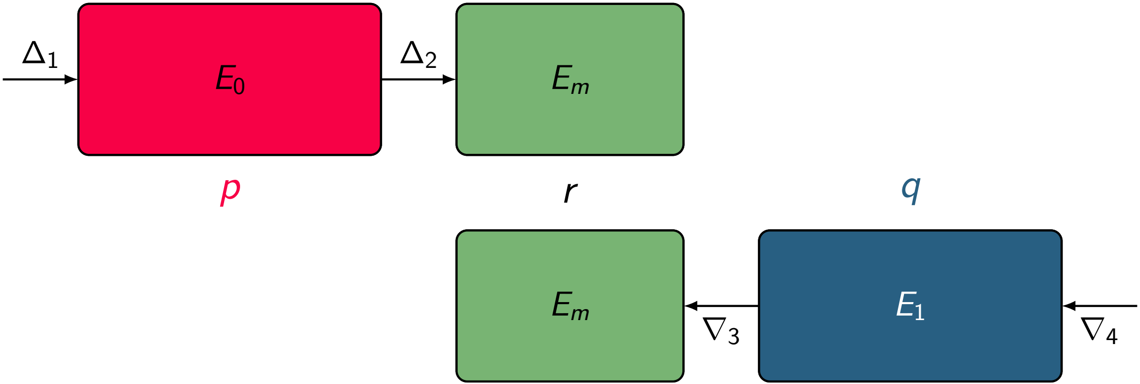

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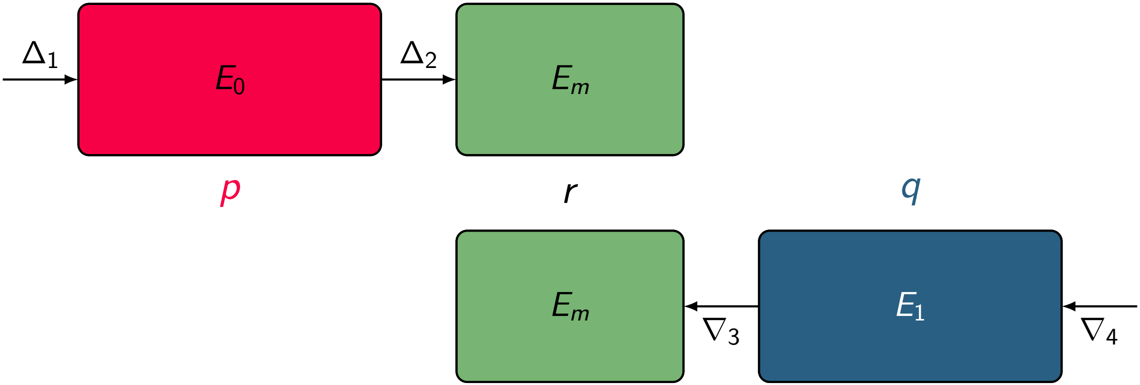

Active S-boxes in E_0, E_1 are more expensive than common active S-boxes in E_m

Our Method to Search for Boomerang Distinguishers

Our Method to Find Boomerang Distinguishers

Our method has three steps:

- Find good truncated upper and lower trails:
  - minimize number of active S-boxes in outer parts, i.e., E_0, and E_1
  - minimize number of common active S-boxes in the middle part, i.e., E_m
- Instantiate discovered truncated trails with concrete differential trails
- Compute p, q and r to derive the entire probability, i.e., p^2q^2r

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Find Good Truncated Upper and Lower Trails

E

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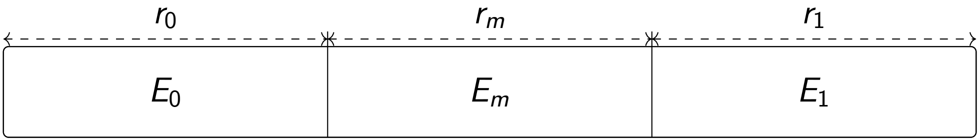

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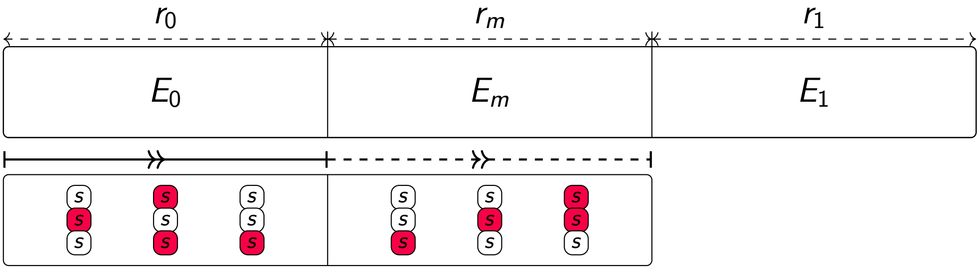

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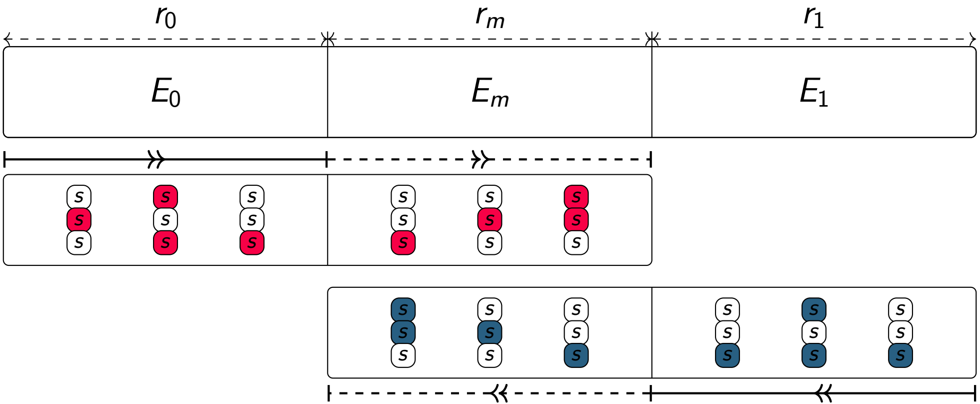

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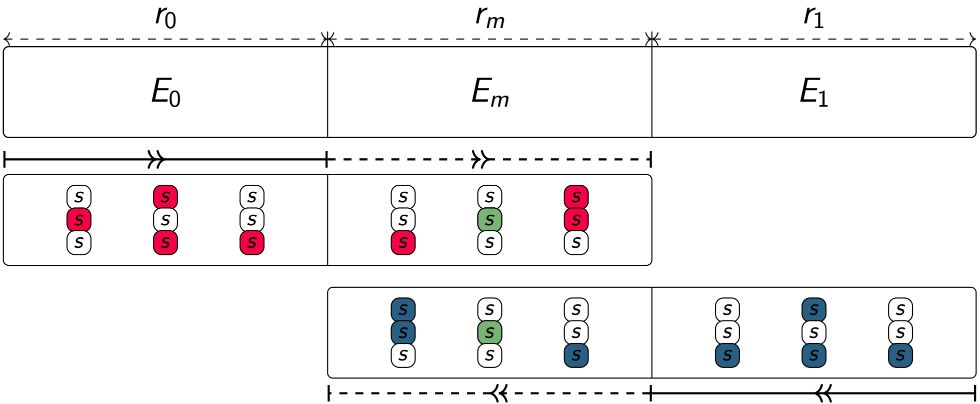

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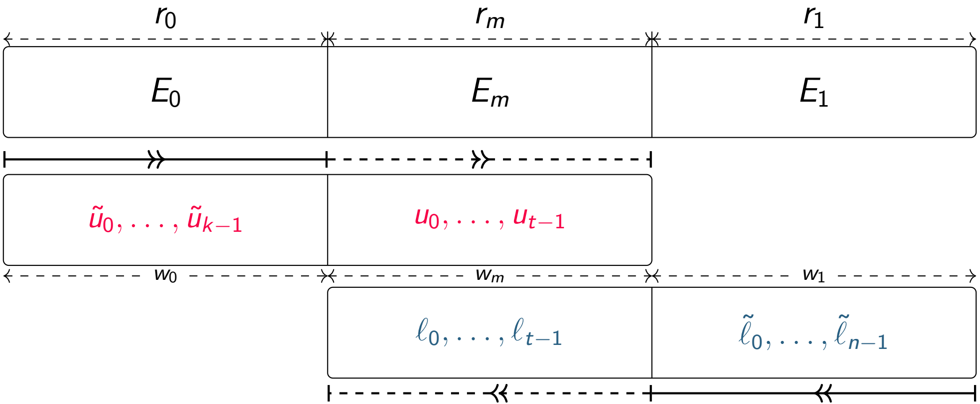

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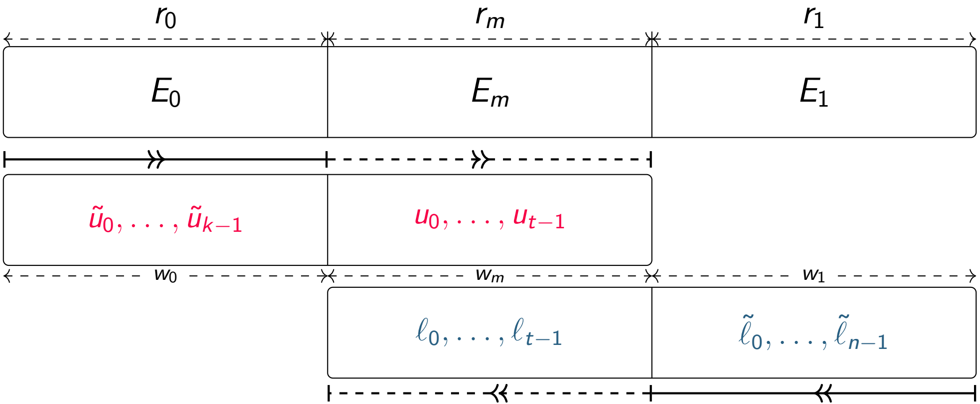

$$u_i - s_i \geq 0, \quad \ell_i - s_i \geq 0, \quad -u_i - \ell_i + s_i \geq -1$$

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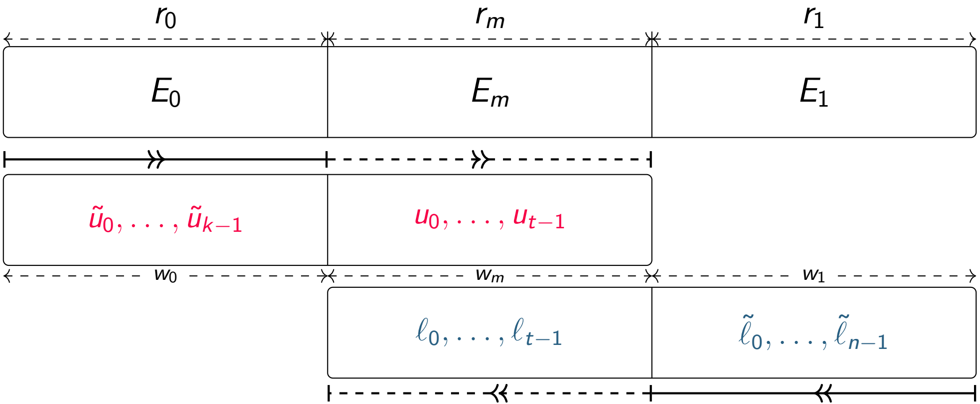

$$\min \sum_{i=0}^{k-1} w_0 \cdot \tilde{u}_i + \sum_{j=0}^{t-1} w_m \cdot s_j + \sum_{k=0}^{n-1} w_1 \cdot \tilde{\ell}_k$$

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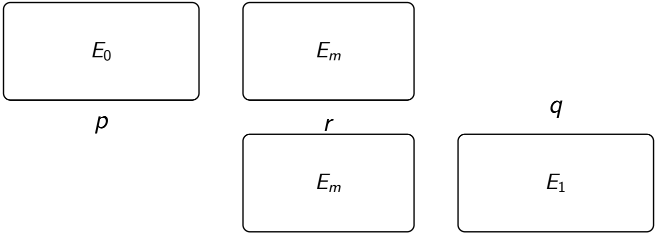
Instantiate Discovered Truncated Trails with Real Differentials

- We instantiate the truncated trails for E0 and E1 with bit-wise trails
- We only fix $\Delta_1, \Delta_2, \nabla_3$, and ∇_4 to compute p, and q
- We compute $r = \Pr\{\Delta_2 \rightleftarrows \nabla_3\}$ for E_m



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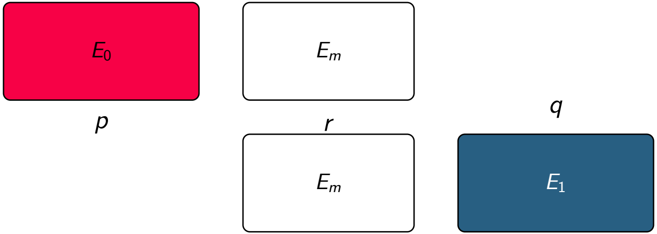

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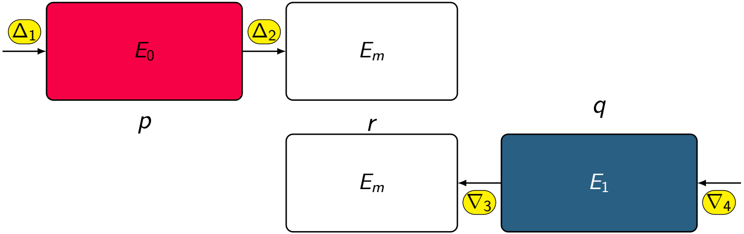

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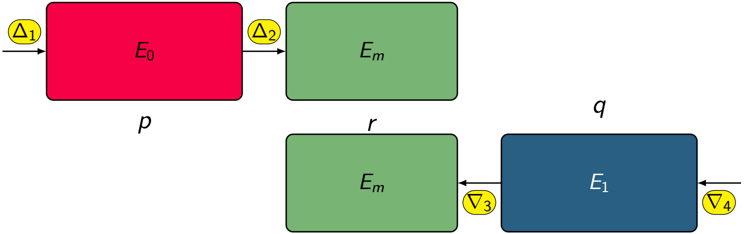

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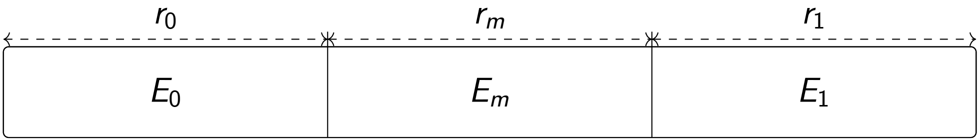
Applications of Our Method to CLEFIA, WARP, LBlock, and TWINE

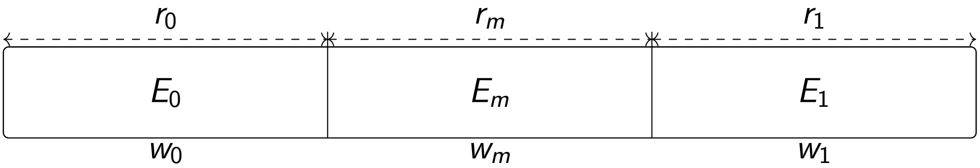
Usage of Our Tool

```
python3 boom.py -r0 6 -rm 10 -r1 7
```



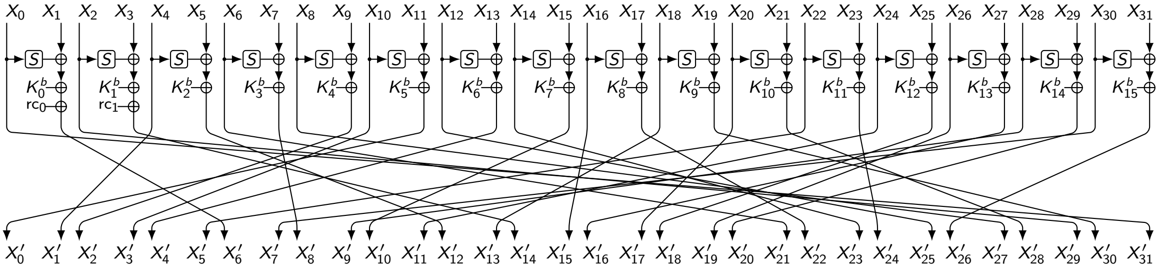
Usage of Our Tool

```
python3 boom.py -r0 6 -rm 10 -r1 7 -w0 2 -wm 1 -w1 2
```

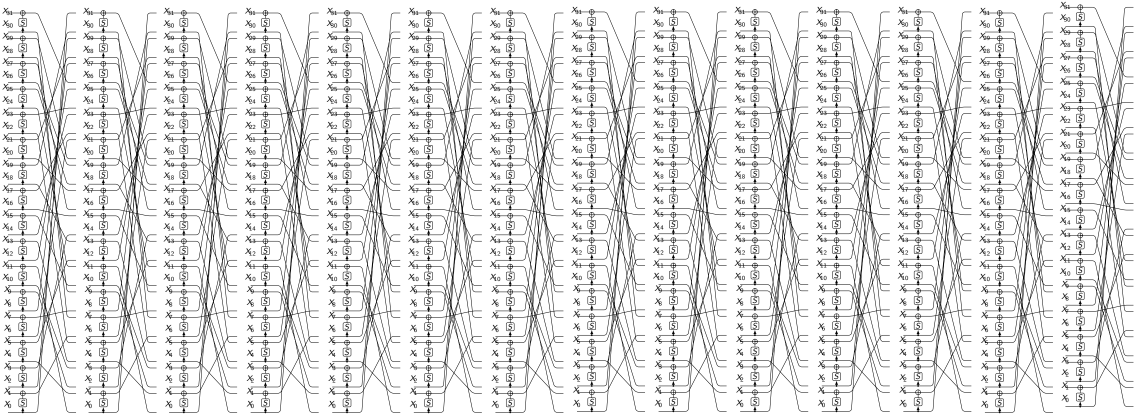


WARP

- Proposed in SAC 2020 [Ban+20] as the lightweight alternative of AES-128
- 128-bit block size, and 128-bit key size
- 41 rounds (40.5 rounds)



14-Round Boomerang Distinguisher for WARP



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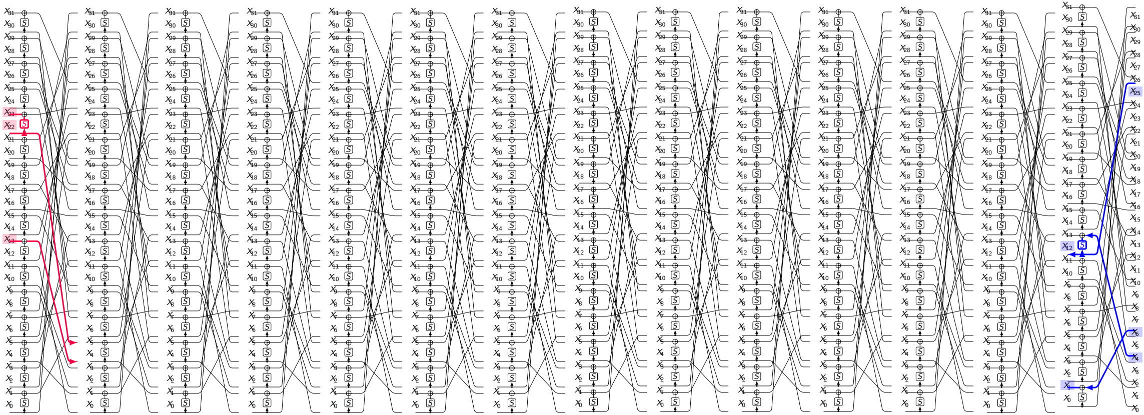

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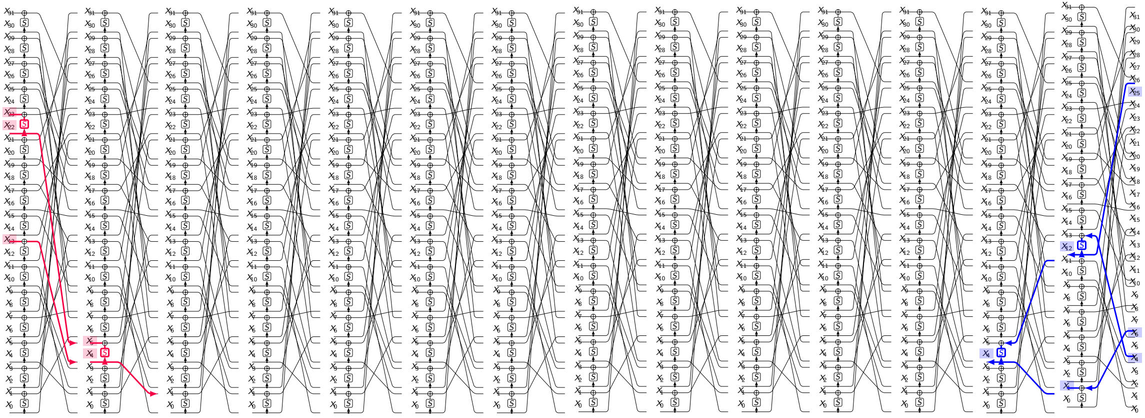
E_0 E_1



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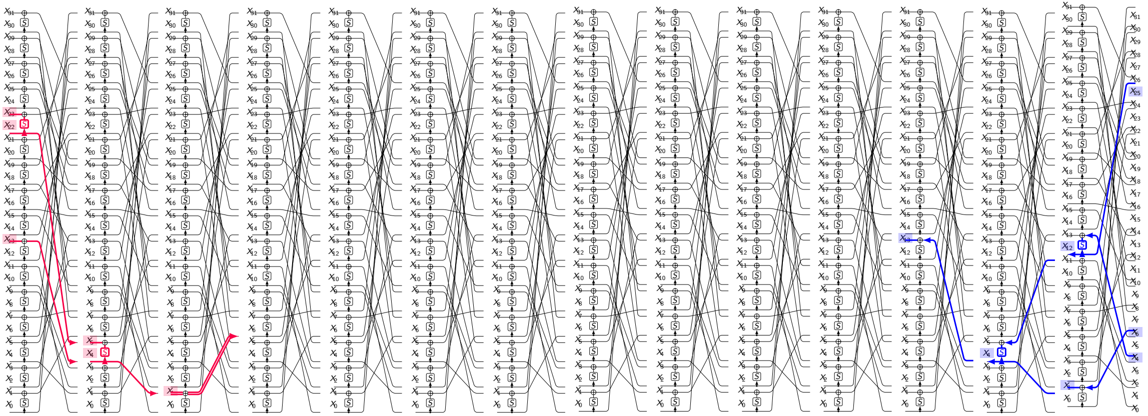
$p = 2^{-4}$ E_0

$q = 2^{-4}$ E_1



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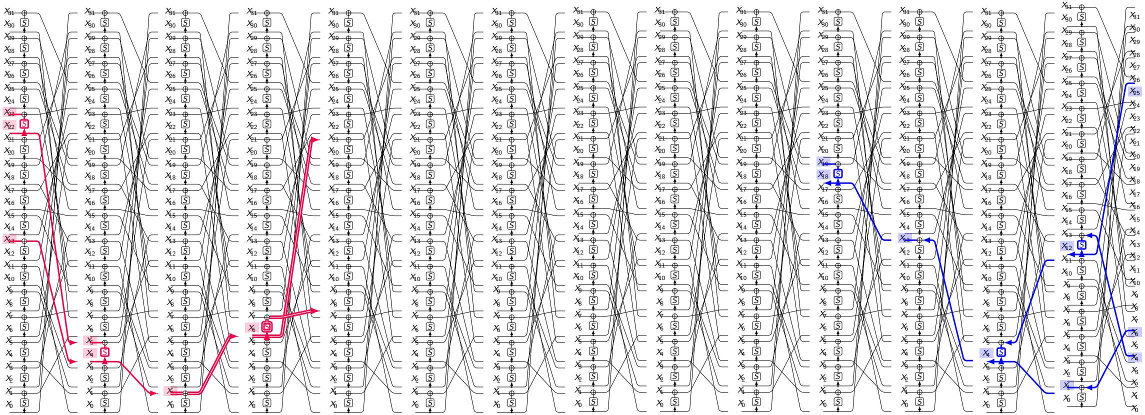
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$

E_0

$q = 2^{-4}$

E_1



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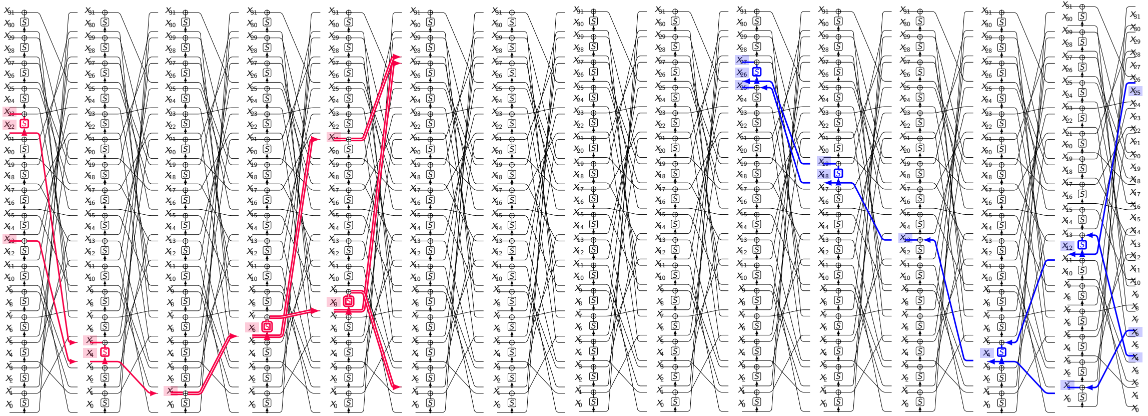
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$

E_0

$q = 2^{-4}$

E_1



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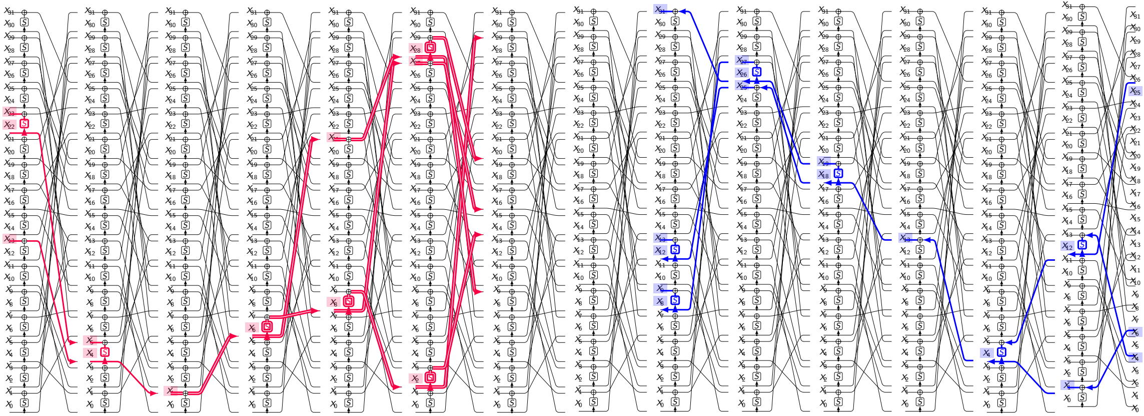
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$ E_0

$q = 2^{-4}$ E_1



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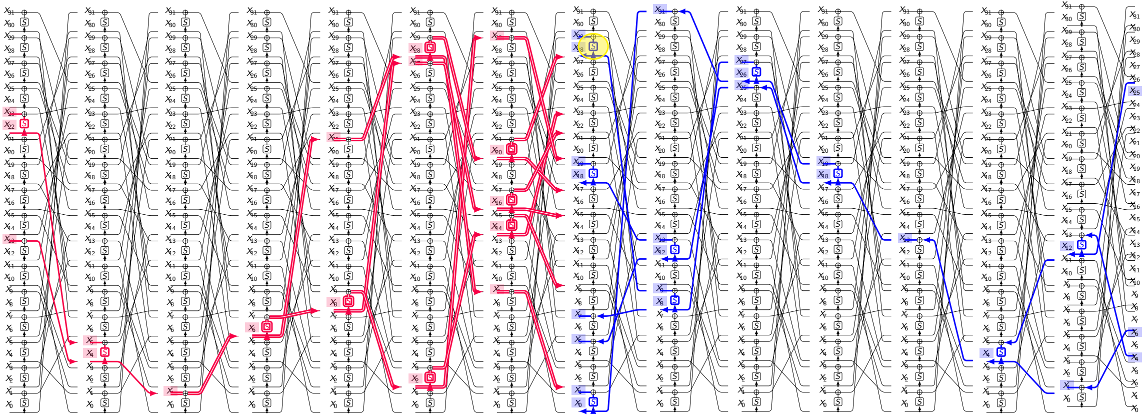
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$

E_0

$q = 2^{-4}$

E_1



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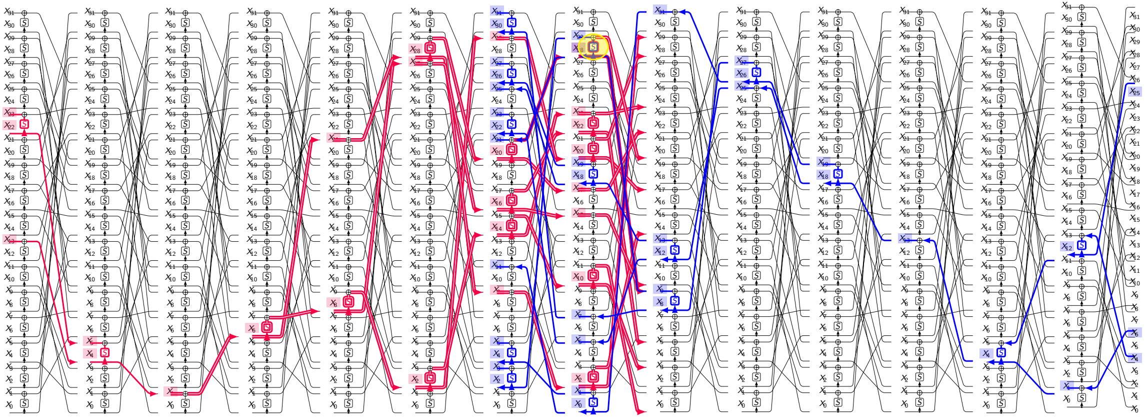
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$

E_0

$q = 2^{-4}$

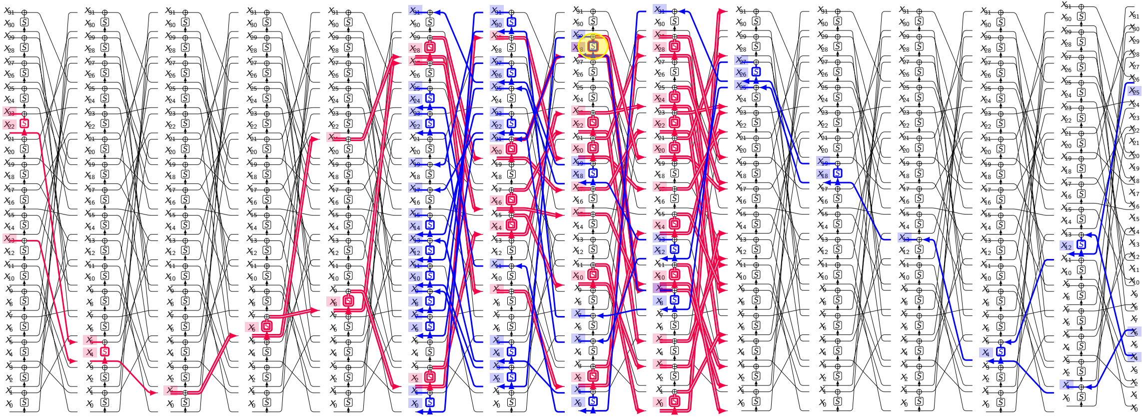
E_1



14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$ E_0

$q = 2^{-4}$ E_1



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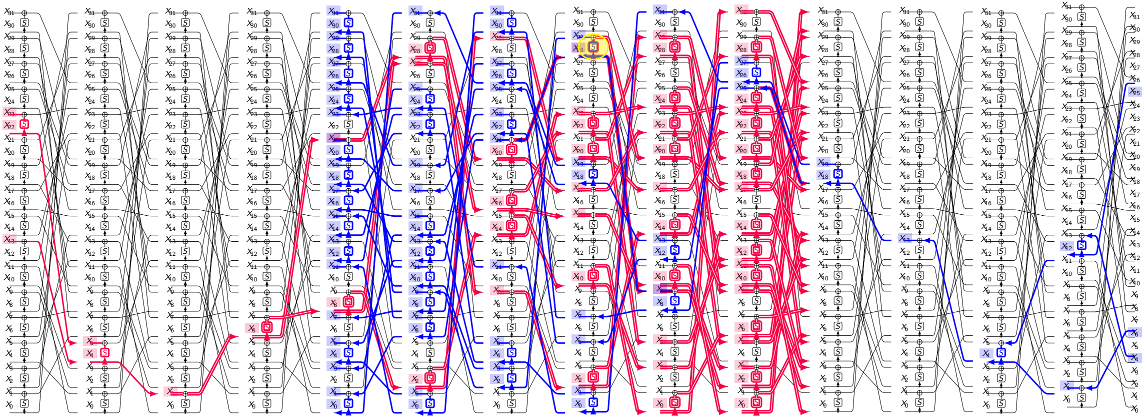
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$

E_0

$q = 2^{-4}$

E_1



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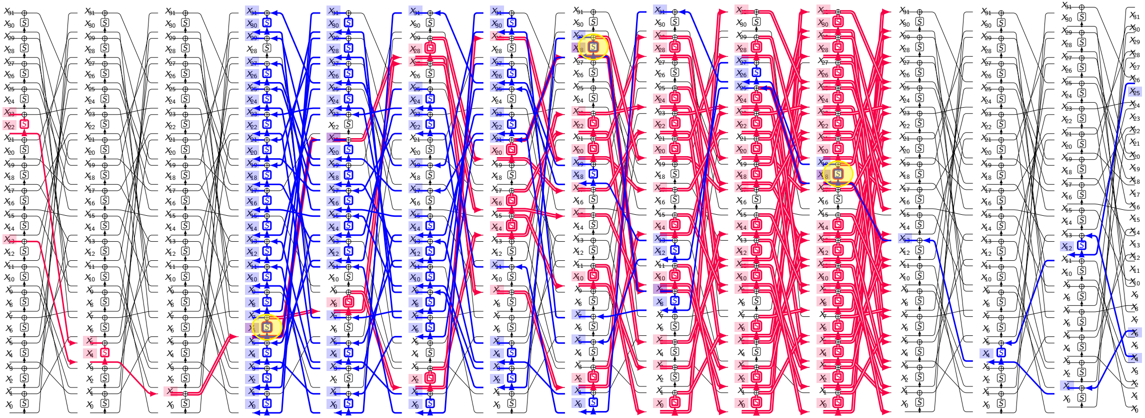
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$ E_0

$q = 2^{-4}$ E_1



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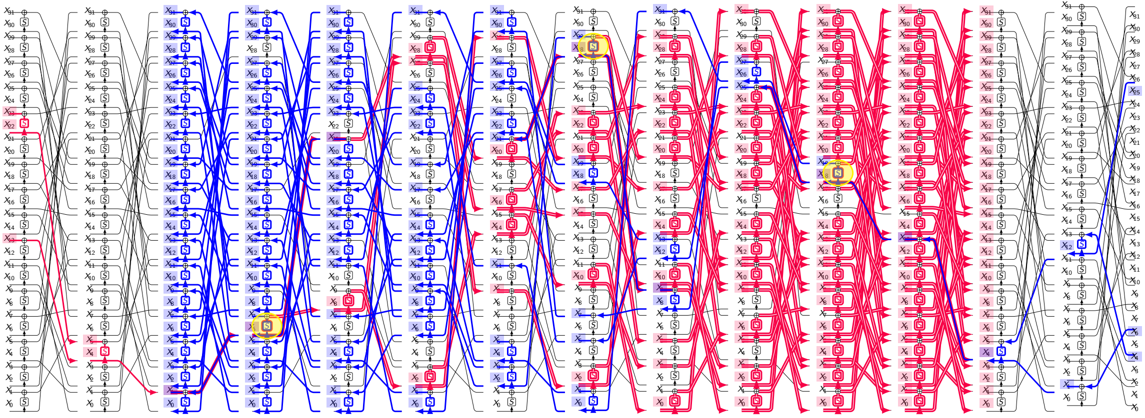
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$ E_0 $r = 2^{-4.58}$ E_m $q = 2^{-4}$ E_1



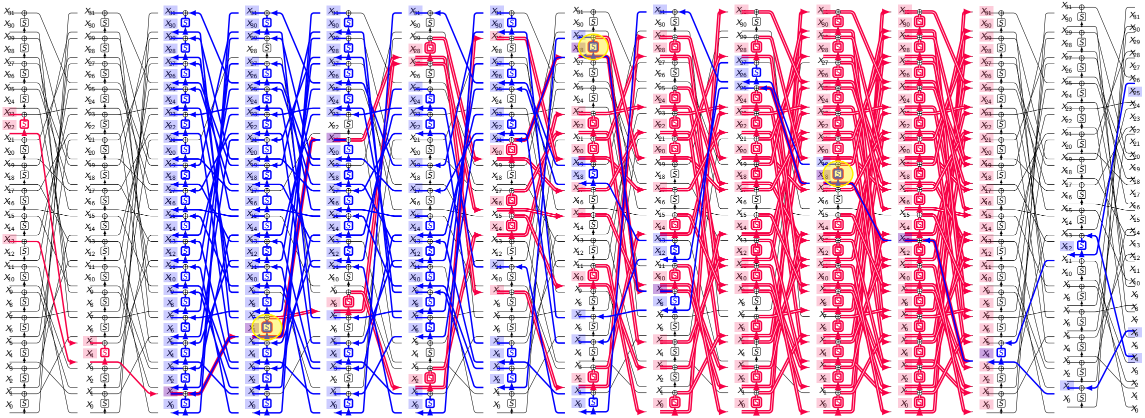
14-Round Boomerang Distinguisher for WARP

$p = 2^{-4}$ E_0

$r = 2^{-4.58}$ E_m

$p^2 q^2 r = 2^{-20.58}$

$q = 2^{-4}$ E_1



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Our Discoveries for WARP

Block cipher	#Rounds	Probability	Reference
WARP	20 / 40	$2^{-114.24}$	[TB22]
	20 / 40	$2^{-75.96}$	This paper
	21 / 40	$2^{-121.11}$	[TB22]
	21 / 40	$2^{-84.55}$	This paper
	22 / 40	$2^{-96.55}$	This paper
	23 / 40	$2^{-115.59}$	This paper

Conclusion

Our Main Contribution

- We provided an easy to use and fast method to find boomerang distinguishers
- We improved the boomerang distinguisher/attack of CLEFIA by 1 round
- We improved the boomerang distinguisher of WARP by 2 rounds
- Our method is applicable to any strongly aligned S-box based block cipher

Thanks for your attention!

: https://github.com/hadipourh/comeback

: https://github.com/hadipourh/sboxanalyzer

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FBCT of WARP

Δ \ ∇	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
1	16	16	4	4	0	0	0	0	0	0	0	0	0	0	0	0
2	16	4	16	4	4	0	4	0	0	4	0	4	4	0	4	0
3	16	4	4	16	0	0	0	0	0	0	0	0	0	0	0	0
4	16	0	4	0	16	0	4	0	0	0	0	0	0	0	0	0
5	16	0	0	0	0	16	0	0	0	0	8	0	0	0	0	8
6	16	0	4	0	4	0	16	0	0	0	0	0	0	0	0	0
7	16	0	0	0	0	0	0	16	0	0	8	0	0	8	0	0
8	16	0	0	0	0	0	0	0	16	0	0	0	0	0	0	0
9	16	0	4	0	0	0	0	0	0	16	0	4	0	0	0	0
a	16	0	0	0	0	8	0	8	0	0	16	0	0	8	0	8
b	16	0	4	0	0	0	0	0	0	4	0	16	0	0	0	0
c	16	0	4	0	0	0	0	0	0	0	0	0	16	0	4	0
d	16	0	0	0	0	0	0	8	0	0	8	0	0	16	0	0
e	16	0	4	0	0	0	0	0	0	0	0	0	4	0	16	0
f	16	0	0	0	0	8	0	0	0	0	8	0	0	0	0	16