A New Generic Transform from Multi-Round Interactive Proof to NIZK

Pierre-Alain Fouque, Adela Georgescu, Chen Qian, Adeline Roux-Langlois, Weiqiang Wen

Content

- Introduction
  - Muti-round PCIP and NIZK
  - Existing transforms: Fiat-Shamir and CPSV@TCC16
- CPSV@TCC16 transform and OR-proofs
- Our new OR-proof technique and new transform
- Conclusion

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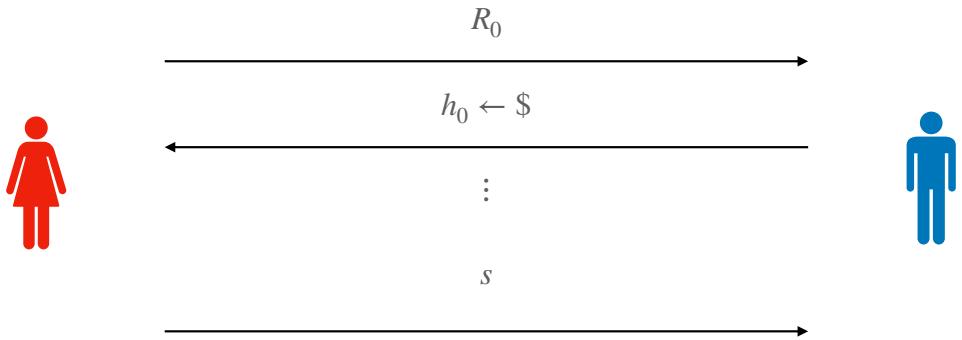
Multi-Round (Public-Coin) Interactive Proof



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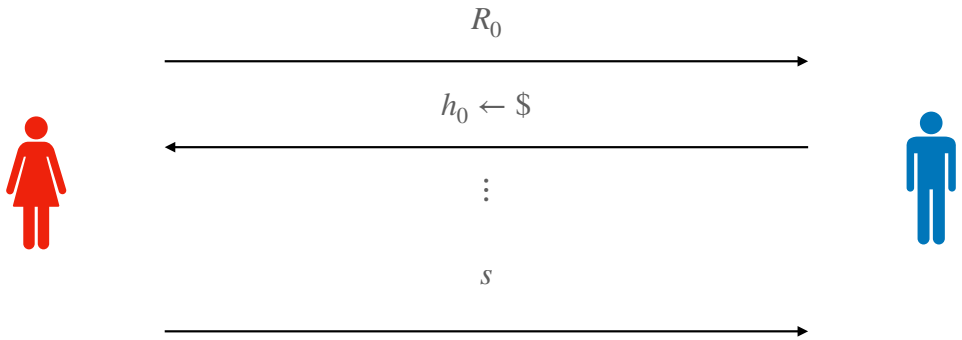
- Goal: Prove $x \in_w \mathscr{L}$ without leak w.



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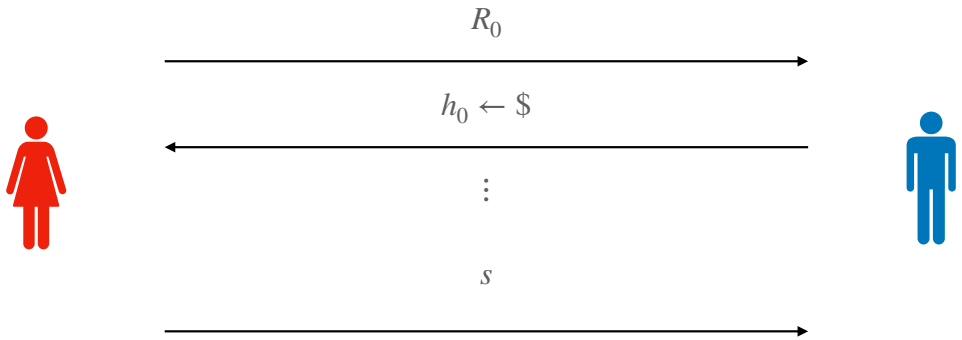

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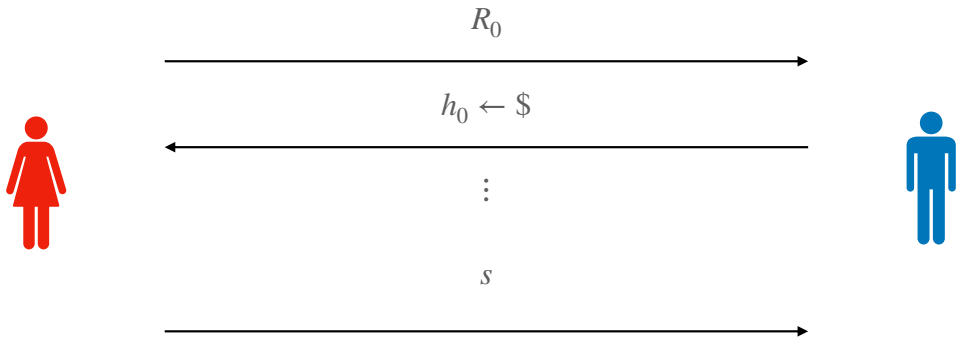

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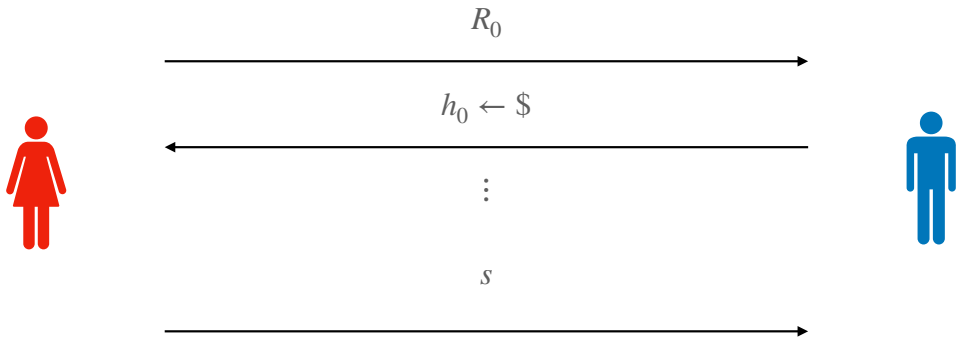

- Correctness: Honestly generated proof should be verifiable.
- Soundness: If $x \notin \mathscr{L}$, it is computationally hard to produce a valid proof.
- (Honest Verifier) Zero-knowledge: The transcript does not leak information more than $x \in \mathscr{L}$.

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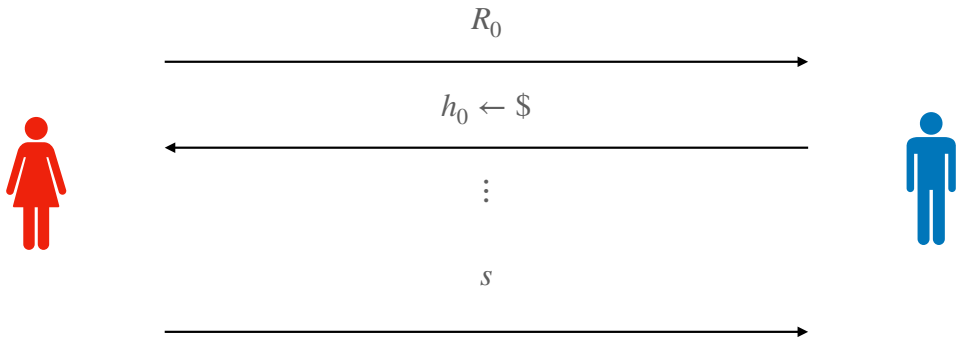

- Soundness: If $x \notin \mathscr{L}$, it is computationally hard to produce a valid proof.
  - Round-By-Round: Implied by negligible soundness.
  - Optimal: Generalization of special soundness.

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Fiat-Shamir	Yes	Random Oracle	Random Oracle
CPSV@TCC16	No	Non-programmable RO	standard model
This work	Yes	Non-programmable RO	standard model

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- multi-round: bulletproof, new lattice-based exact proofs…

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  - Soundness: $x' \notin \mathscr{L}'$
  - Zero-knowledge: switch CRS with $x' \in_{w'} \mathscr{L}'$, then the simulator prove with w'

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  - Parallel OR-proof: CDS@Crypto94
  - Sequential OR-proof: AOS@Asiacrypt02

Parallel OR-Proof

CDS@Crypto94

- Proof of the form: $\pi = (R_0, R_1, h_0, h_1, s_0, s_1)$
- $h_0 \oplus h_1 = H(R_0, R_1)$: The adversary can freely choose h_0 or h_1.

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- Remarks:
  - A natural extension of CDS@Crypto94 leads to $h_0 \oplus \hat{h}_1 = H(R_0, \hat{R}_1)$ and $\hat{h}_0 \oplus h_1 = H(R_0, \hat{R}_1, \hat{R}_0, R_1)$.

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  - The adversary can freely choose both h_0 and h_1.

Sequential OR-Proof

AOS@Asiacrypt02

- Proof of the form: $\pi = (R_0, R_1, h_0, h_1, s_0, s_1)$
- $h_0 = H(R_1)$ and $h_1 = H(R_0)$: The adversary can freely choose h_0 or h_1 depending on the order.

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Idea from parallel OR-proof

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- First attempt:
  - $h_{i,0} = H(\{R_{j,0}\}_{j=1}^{i}) \oplus a_{i,0}$, $h_{i,1} = H(\{R_{j,1}\}_{j=1}^{i}) \oplus a_{i,1}$

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- How to generate a proof with only w_b:
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  - Using HVZK to simulate $\pi_{1-b} = (R_{1,1-b}, h_{1,1-b}, \ldots, R_{n,1-b}, h_{n,1-b}, s_{1-b})$

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  - Let $\mathsf{A}_{1-b} = (a_{1,1-b}, \ldots, a_{n,1-b})$
  - Compute π_b using A_b and A_{1-b}

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- Quantum setting:
  - QROM based proof with security loss Q_H^4 instead of Q_H^{2n}.

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- Open problem:
  - The proof in the QROM require programmability, does there exist a transform in QROM without programmability?