Transparent Batchable Time-lock Puzzles and Applications to Byzantine Consensus













Shravan Srinivasan UMD

Julian Loss

Giulio Malavolta MPI-SP

Kartik Navak Duke

Charalampos Papamanthou

Sri AravindaKrishnan Thyagarajan NTT Research

PKC '23

• Encrypt a message "to the future"



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- Security against parallel adversaries
- Applications in auctions, blockchains, timed-commitments, and more



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Problem Statement

Scheme	Batchable	Transparent setup	No apriori bound	Compact puzzles
RSA-based [TBM ⁺ 20, MT19]	\checkmark	×	×	×
Class-groups based [TCLM21]	\checkmark	\checkmark	×	×

Problem Statement		Unbounded batching				
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	?	\checkmark	\checkmark	\checkmark	\checkmark	

Why do we care?

Useful in the decentralized setting [TBM⁺20, WXDS20, tez22]:

- Impractical to rely on trusted setup
- Unknown number of nodes
- Large puzzles increases communication overhead

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This work	\checkmark	\checkmark	\checkmark	\checkmark

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	–This work	\checkmark	\checkmark	\checkmark	\checkmark		
→ Uses indistinguishability obfuscation							



Applications in consensus:



Applications in consensus:

- First permissionless protocol in the mobile sluggish model
- First expected O(1)-round Byzantine broadcast under strongly adaptive and corrupt majority setting

Outline

- 1. Preliminaries
- 2. TLP construction
 - 2.1 Puzzle generation
 - 2.2 Batch Solving
- 3. Application: Permissionless Consensus
 - 3.1 Network model
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$$m_1^{(3)} + m_2^{(3)} = m_1 + m_2^{(3)} \xrightarrow{(5)} \rightarrow$$

$$m_1^{\textcircled{0}} + m_2^{\textcircled{0}} = m_1 + m_2^{\textcircled{0}} \xrightarrow{\textcircled{0}} m_1 + m_2$$

Homomorphic Time-Lock Puzzles (HTLP) [MT19, TCLM21]:

$$m_1^{(1)} + m_2^{(2)} = m_1 + m_2^{(2)} \xrightarrow{(1)} m_1 + m_2$$

Key-Homomorphic Pseudorandom Functions (KH-PRF) [BV15]:

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for all keys \mathbf{k}_1 , \mathbf{k}_2 and all message \mathbf{m} .

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$$\begin{bmatrix} F(k_i, 1) \\ \vdots \\ F(k_i, i) + m_i \\ \vdots \\ F(k_i, n) \end{bmatrix}$$



































$$\mathbf{k}^* = \sum_i \mathbf{k}_i^*$$

$$\sum_{i} F(\mathbf{k}_{i}, 1) + \mathbf{m}_{1}$$
$$\sum_{i} F(\mathbf{k}_{i}, 2) + \mathbf{m}_{2}$$
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Batch Solving (contd.)

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- Exact number of nodes unknown
- No authentication mechanism
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Nakamoto consensus:



Secure in the synchronous model [GKL15, PSS17, LG19]

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Honest msg. delay	Known Δ

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Permissionless	

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Permissionless		

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Permissionless	\bigcirc	8

Model	Synchronous	Sluggish	Partially/Asynchronous
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Permissionless	S		8

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Permissionless	\bigcirc			8

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 Δ upper bound on the honest message delay

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Permissionless			0	8

Is it possible to achieve consensus in the permissionless setting in the presence of mobile sluggish faults?

Overview

- Based on Nakamoto consensus
- All messages are time-lock encrypted
- Set hiding time, $\mathbf{T} = \Delta$

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- All messages are time-lock encrypted
- Set hiding time, $\mathbf{T} = \Delta$
- Non-block winners send decoys
- Decoys give "cover" for the block winner
- Adversary has to corrupt or deliver messages randomly

- Decoys: Dummy TLP messages
- Need to prevent Sybil attack

payload

0

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H(payload)

 2^{λ}

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- 2. Solve Phase:
 - Multicast the puzzle
 - Receive puzzles
 - Batch open puzzles
 - Extend the chain and go to step 1

X Miner 1







Miner 2









Miner 2

 \bigotimes Miner 4

Block

2

Miner 1

*

2



X Miner 3





Miner 2



Miner 1

*

2

Mine Phase

X Miner 3

\$

Block

Decoy









Block

\$

Decoy






X 2 Miner 1





Block

\$













Block

\$













Block

\$













Block

\$













Block







Decoy

2







Block

\$













Solve Phase







Solve Phase

 \geq



Protocol



\$

Decoy

Block







































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Solve Phase

Ы







Miner 4









Miner 4









► - - - - - - - - - - - - - - - - - × Miner 4







X Miner 4







X Miner 4



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@s_shravan



Broadcast under strongly adaptive and majority corruptions

- Byzantine broadcast is a classical problem in consensus
- Building block for other flavours of consensus
- Strongly adaptive adversary:
 - Can corrupt the victim on-the-fly
 - Can perform after-the-fact removal in the ongoing round
- Weakly adaptive adversary:
 - Can corrupt the victim on-the-fly
 - Cannot perform after-the-fact removal

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