QCCA-Secure Generic Transformations in the Quantum Random Oracle Model

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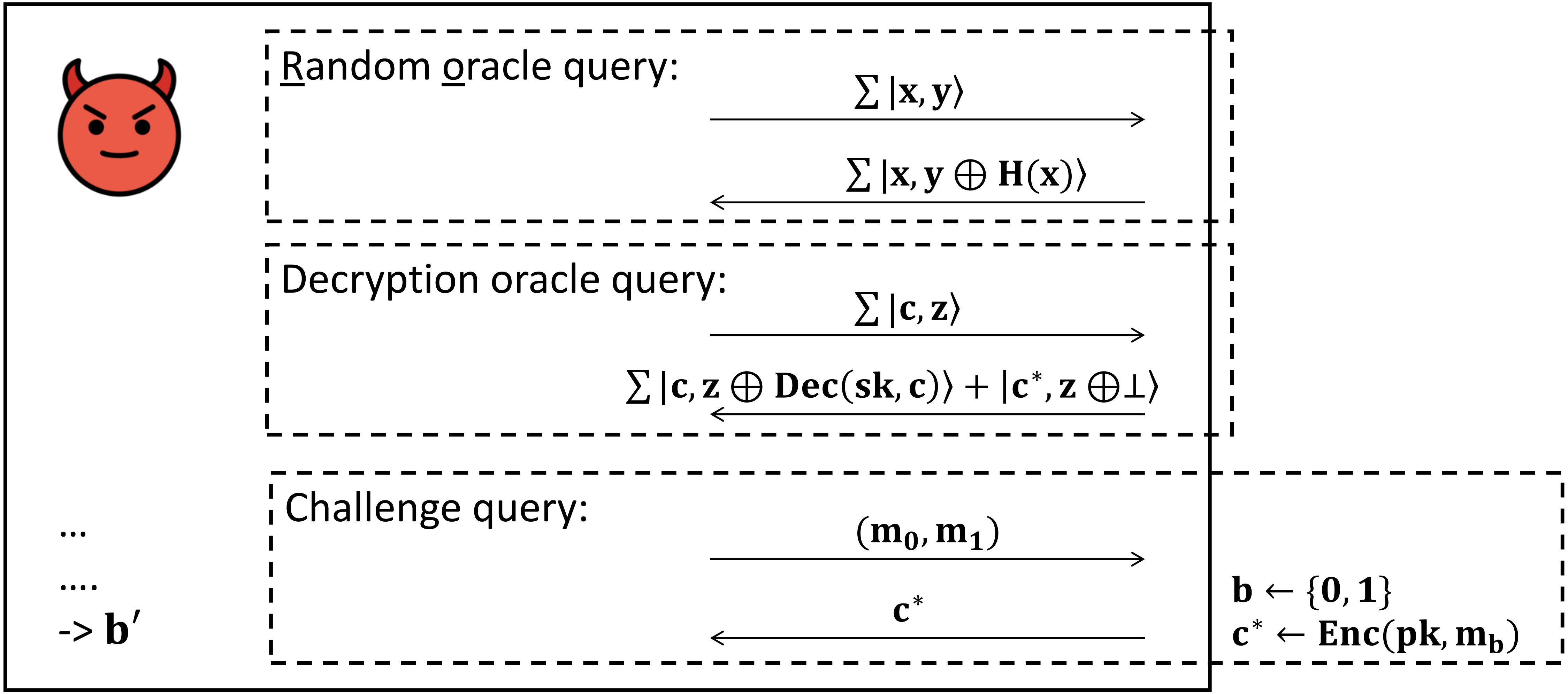
IND-qCCA security in the QROM



Compressed Standard Oracle[Z19]

QRO:

$\sum |x, y\rangle \rightarrow$

$H \leftarrow \Omega_H$

$|x, y\rangle \mapsto |x, y \oplus H(x)\rangle$

$\leftarrow \sum |x, y \oplus H(x)\rangle$

$=$

CStO:

$\sum |x, y\rangle \rightarrow$

1. $|x, y\rangle|D\rangle \mapsto |x, y\rangle \mathrm{StdDecomp}_x |D\rangle$
2. $|x, y\rangle|D\rangle \mapsto |x, y \oplus D(x)\rangle|D\rangle$
3. $|x, y\rangle|D\rangle \mapsto |x, y\rangle \mathrm{StdDecomp}_x |D\rangle$

$\leftarrow \sum |x, y \oplus D(x)\rangle$

Result

- **The definition of o-m schemes, including the resulting PKE of FO and REACT.**

Oracle-masked scheme

FO Transformation:

Enc(pk,m;δ)=(c,d)

d=E'(G(δ),m)

c=Enc'(pk,δ;H(δ,d))

Dec(sk,(c,d))=m

δ=Dec'(sk,c)

If c=Enc'(pk,δ;H(δ,d)),

m=D'(G(δ),d)

Otherwise,

m=⊥

δ: Randomness

E': Enc alg. of SKE

Enc': Enc alg. of PKE

H, G: Random oracle

D': Dec alg. of SKE

Dec': Dec alg. of PKE

REACT Transformation:

Enc(pk,m;(R,r))=(c_1, c_2, c_3)

c_1=Enc'(pk,R;r)

c_2=E'(G(R),m)

c_3=H(R,m,c_1, c_2)

Dec(sk,(c_1, c_2, c_3))=m

R=Dec'(sk,c_1)

m'=D'(G(R),c_2)

If c_3=H(R,m',c_1, c_2),

m=m'

Otherwise,

m=⊥

R, r: Randomness

E': Enc alg. of SKE

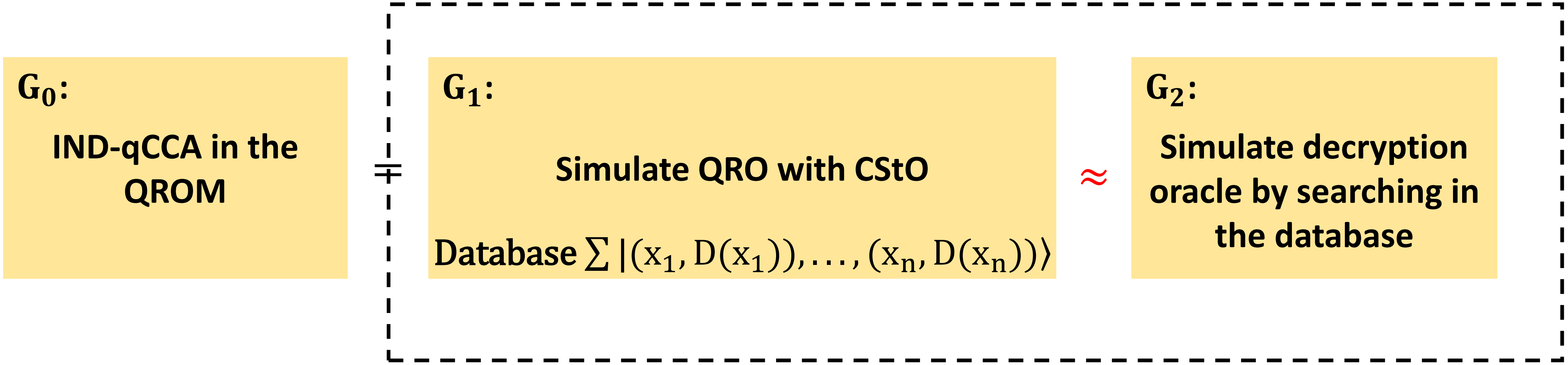
Enc': Enc alg. of PKE

H, G: Random oracle

D': Dec alg. of SKE

Dec': Dec alg. of PKE

- The definition of o-m schemes, including the resulting PKE of FO and REACT.
- An upper bound of the error caused by quantum-accessible decryption oracle simulation for o-m schemes.



- The definition of o-m schemes, including the resulting PKE of FO and REACT.
- An upper bound of the error caused by quantum-accessible decryption oracle simulation for o-m schemes.

G_0: IND-qCCA in the QROM

$=$

G_1: Simulate QRO with CStO

Database $\sum |(x_1, D(x_1)), \ldots, (x_n, D(x_n))\rangle$

$\approx$

G_2: Simulate decryption oracle by searching in the database

FO Transformation

γ-spreadness $\Longrightarrow$ $|\Pr[$ G_1: b'=b, Simulate QRO with CStO $] - \Pr[$ G_2: b'=b, Simulate decryption oracle without sk $]| \leq 5/\sqrt{2^{\gamma}}$

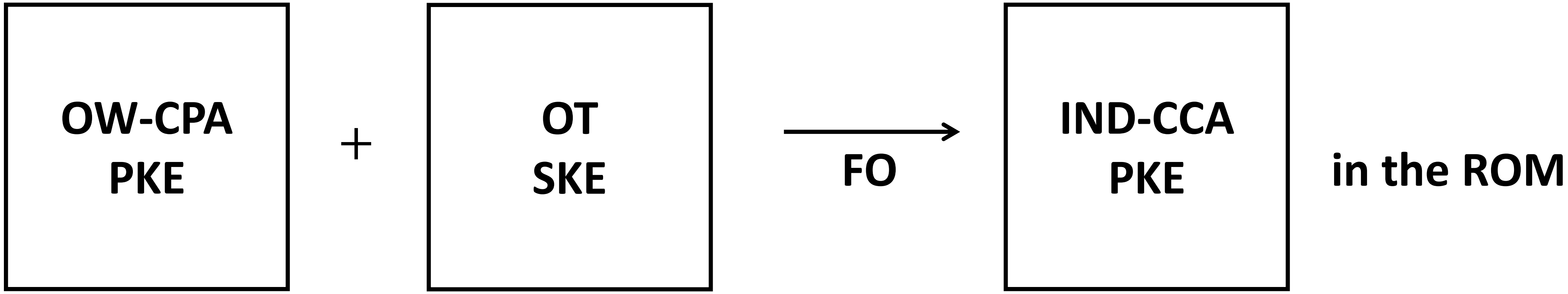
- **The definition of o-m schemes, including the resulting PKE of FO and REACT.**
- **An upper bound of the error caused by quantum-accessible decryption oracle simulation for o-m schemes.**
- **IND-qCCA security proof of FO, REACT and T_{CH} transformation, after revisiting the IND-qCCA security proof of FO by Zhandry [Z19].**

Transformation	Underlying Security	Achieved Security	Requirement
FO	OW-CPA	IND-qCCA	Well-spread
REACT	OW-qPCA	IND-qCCA	
T_{CH}	OW-qPCA	IND-qCCA	

Table. QCCA security of FO, REACT and T_{CH} transformation in the QROM.

Motivation

Classical security of FO[FO99]



Proof by games:

G_0: IND-CCA game in the ROM.

G_1: Simulate H and G on the fly.

G_2: Simulate decryption oracle without secret key.

G_3: Simulate (c^*, d^*) without $G(\delta^*)$, $H(\delta^*, d^*)$.

<u>Enc(pk,m;δ)=(c,d)</u>

d=E'(G(δ),m)

c=Enc'(pk,δ;H(δ,d))

δ: randomness

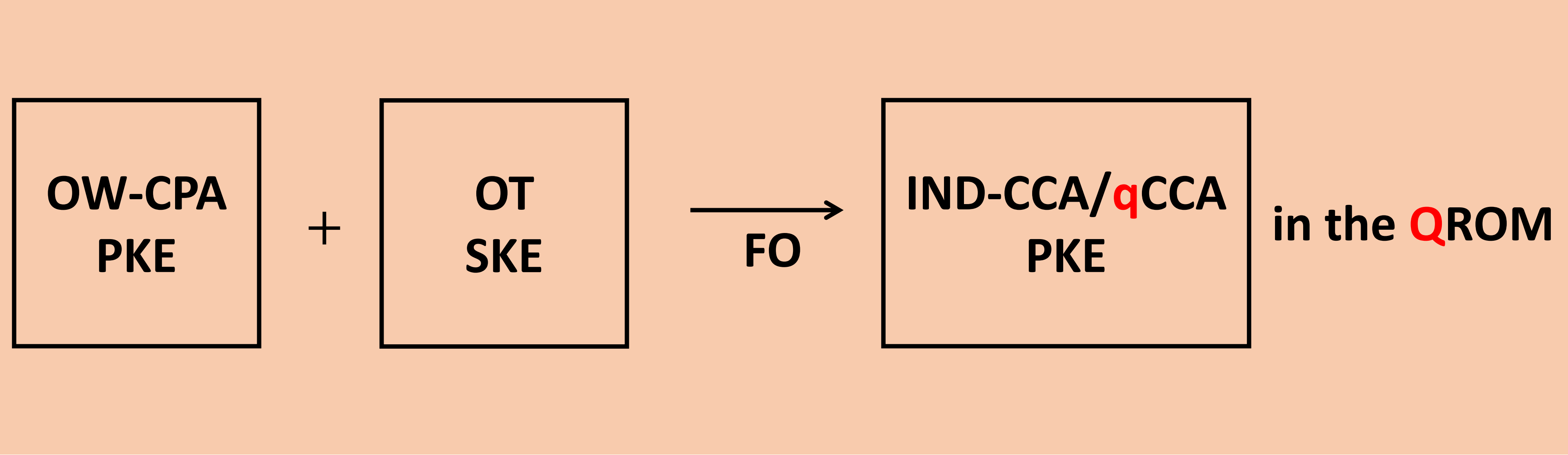
E': Enc alg. of SKE

Enc': Enc alg. of PKE

H, G: random oracle

What about the post-quantum security of FO?

Can we prove



?

Can we lift the ROM proof into the QROM proof?

- **Didn't know how to simulate quantum random oracles on the fly.**
- **Didn't know how to simulate decryption oracle without secret keys.**

We can modify FO to solve it!

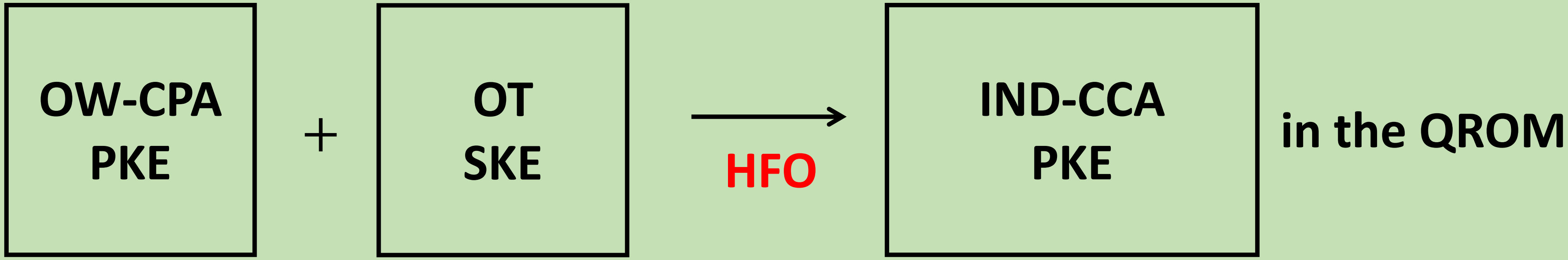
[TU16,AHU19]:

Enc(pk,m;δ)=(c,d)
d=E'(G(δ),m)
c=Enc'(pk,δ;H(δ,d))

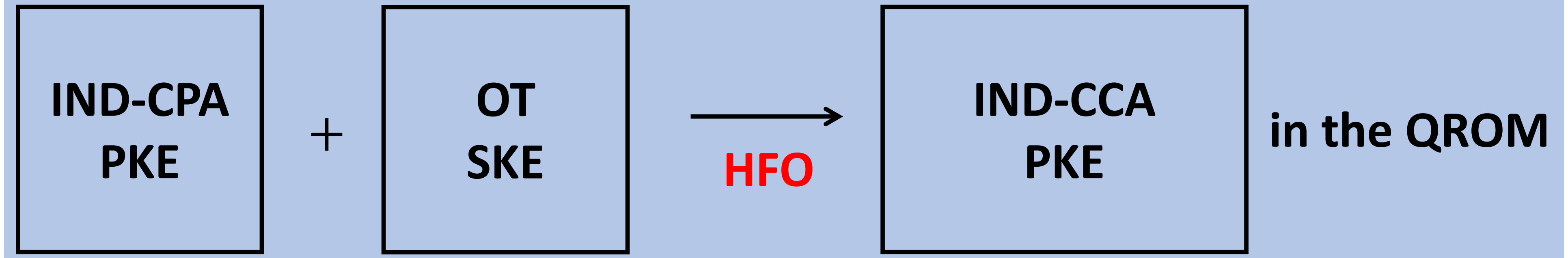
→ **HFO**

Enc(pk,m;δ)=(c,d,e)
e=H'(δ)
d=E'(G(δ),m)
c=Enc'(pk,δ;H(δ,d))

[TU16,AHU19]:



[AHU19]:



After the compressed oracle technique [Z19] being proposed...

Can we lift the ROM proof into the QROM proof?

- ~~Didn't know how to simulate quantum random oracles on the fly.~~
- ~~Didn't know how to simulate decryption oracle without secret keys.~~

Even simulate decryption oracle for quantum queries!

[Z19]:

OW-CPA PKE + **OT SKE** —FO→ **IND-qCCA PKE** **in the QROM**

But…

G_0: IND-qCCA game in the QROM.
G_1: Simulate H with compressed oracle technique.
G_2: …
G_3: …
G_4: …
G_5: … . In G_5, the secret key is not needed.

G_0: IND-CCA game in the ROM.
G_1: Simulate H and G on the fly.
G_2: Simulate decryption oracle without secret keys.

But...

G_0: IND-qCCA game in the QROM.

$\mathbf{G_1}$: Simulate H with compressed oracle technique.

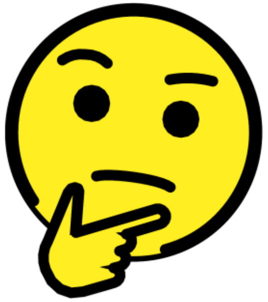
G_2: ...

G_3: ...

G_4: ...

$\mathbf{G_5}$: ... . In G_5, the secret key is not needed.

Emmm...can we give a simpler and tighter proof?



G_0: IND-CCA game in the ROM.

G_1: Simulate H and G on the fly.

G_2: Simulate decryption oracle without secret keys.

Two Points in the Proofs

Take FO as an example

Simulate decryption oracle without sk

G_0:

IND-qCCA game in the QROM

$=$

G_1:

Simulate H with CStO

well-spread

$\approx$

G_2:

Simulate decryption oracle by searching in the database:

U_{Ext}:

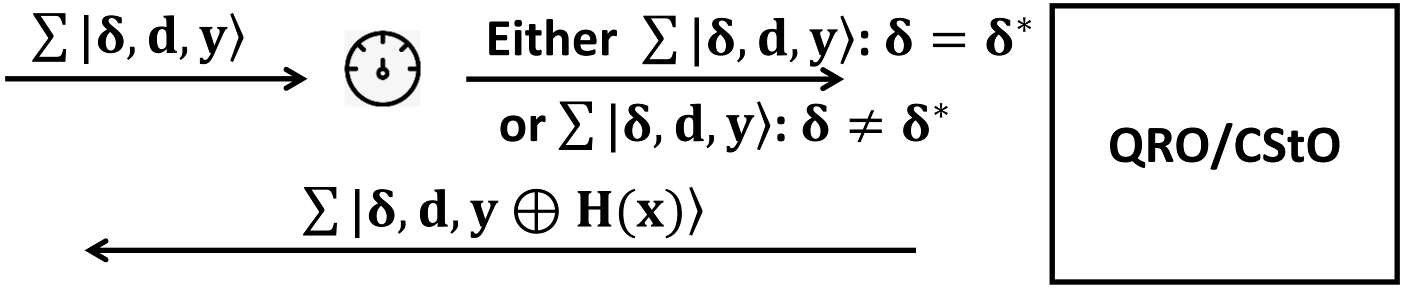
1. **If (c,d)=(c^*,d^*), output ⊥;**
2. **Else if ∄ (δ,d,D(δ,d)) s.t. Enc'(pk,δ;D(δ,d))=c, output ⊥;**
3. **Else, output D'(G(δ),d).**

Simulate (c^*, d^*) without $G(\delta^*), H(\delta^*, d^*)$

$$|\Pr[\ G_2: b'=b \quad (c^*, d^*) \quad d^* = E'(G(\delta^*), m_b) \quad c^* = Enc'(pk, \delta^*; H(\delta^*, d^*))\ ] - \Pr[\ G_3: b'=b \quad (c^*, d^*) \quad d^* = E'(k^*, m_b) \quad c^* = Enc'(pk, \delta^*; r^*)\ ]| \leq ?$$

$G(\delta^*)$, $H(\delta^*, d^*)$ are uniformly random if

- **A never queries δ^* to G;**
- **A never queries (δ^*, d^*) to H;**
- **More conditions?**



U_{Ext} disturbs the simulation of $H(\delta^*, d^*)$

G_2:

(m_0, m_1) →

← (c^*, d^*)

$\sum |c, d, z\rangle$ →

$b \leftarrow \{0,1\}, k^* \leftarrow G(\delta^*), r^* \leftarrow \text{CStO}(\delta^*, d^*)$

$d^* = E'(k^*, m_b), c^* = \text{Enc}'(pk, \delta^*; r^*)$

$\sum |c, d, z\rangle$: $c \neq c^*, d = d^*, \exists\, r$ s.t. $\text{Enc}'(pk, \delta^*; r) = c$

$$|c, d, z\rangle \otimes \text{StdDecomp}_{\delta^*, d^*} |(\delta^*, d^*, r^*)\rangle \xrightarrow{U_{\text{Ext}}} |c, d, z \oplus \bot\rangle |\psi_0\rangle + |c, d, z \oplus m_b\rangle |\psi_1\rangle$$

Patch: change the simulation of decryption oracle

After $(\mathbf{c}^*, \mathbf{d}^*)$ is defined,

$\mathbf{G_2}$:

$$\sum |\mathbf{c}, \mathbf{d}, \mathbf{z}\rangle|\mathbf{D}\rangle \xrightarrow{\mathbf{U_{Ext}}} \sum |\mathbf{c}, \mathbf{d}, \mathbf{z} \oplus \mathbf{m}\rangle|\mathbf{D}\rangle$$

$\mathbf{G_{2a}}$:

$$\sum |\mathbf{c}, \mathbf{d}, \mathbf{z}\rangle|\mathbf{D}\rangle \xrightarrow{\mathbf{StdDecomp}_{\delta^*, \mathbf{d}^*}} \xrightarrow{\mathbf{U_{Ext}}} \xrightarrow{\mathbf{StdDecomp}_{\delta^*, \mathbf{d}^*}} \sum |\mathbf{c}, \mathbf{d}, \mathbf{z} \oplus \mathbf{m}\rangle|\mathbf{D}\rangle$$

$\mathbf{G_2 \approx G_{2a}}$

$\mathbf{G_{2a}}$:

$$\sum|c,d,z\rangle|D\rangle \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \xrightarrow{U_{Ext}} \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \sum|c,d,z\oplus m\rangle|D\rangle$$

$$\approx \sum|c,d,z\rangle|D\rangle \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \xrightarrow{U_{Ext}} \sum|c,d,z\oplus m\rangle|D\rangle$$

$\mathbf{G_2 \approx G_{2a}}$

$\mathbf{G_{2a}}$:

$$\sum |\mathbf{c}, \mathbf{d}, \mathbf{z}\rangle|\mathbf{D}\rangle \xrightarrow{\mathbf{StdDecomp}_{\delta^*,\mathbf{d}^*}} \xrightarrow{\mathbf{U}_{\mathbf{Ext}}} \xrightarrow{\mathbf{StdDecomp}_{\delta^*,\mathbf{d}^*}} \sum |\mathbf{c}, \mathbf{d}, \mathbf{z} \oplus \mathbf{m}\rangle|\mathbf{D}\rangle$$

$$\approx \sum |\mathbf{c}, \mathbf{d}, \mathbf{z}\rangle|\mathbf{D}\rangle \xrightarrow{\mathbf{StdDecomp}_{\delta^*,\mathbf{d}^*}} \xrightarrow{\mathbf{StdDecomp}_{\delta^*,\mathbf{d}^*}} \xrightarrow{\mathbf{U}_{\mathbf{Ext}}} \sum |\mathbf{c}, \mathbf{d}, \mathbf{z} \oplus \mathbf{m}\rangle|\mathbf{D}\rangle$$

$G_2 \approx G_{2a}$

G_{2a}:

$$\sum |c,d,z\rangle|D\rangle \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \xrightarrow{U_{\text{Ext}}} \xrightarrow{\text{StdDecomp}_{\delta^*,d^*}} \sum |c,d,z\oplus m\rangle|D\rangle$$

$$\approx \sum |c,d,z\rangle|D\rangle \xrightarrow{\cancel{\text{StdDecomp}_{\delta^*,d^*}}} \xrightarrow{\cancel{\text{StdDecomp}_{\delta^*,d^*}}} \xrightarrow{U_{\text{Ext}}} \sum |c,d,z\oplus m\rangle|D\rangle$$

G_2:

$$= \sum |c,d,z\rangle|D\rangle \xrightarrow{U_{\text{Ext}}} \sum |c,d,z\oplus m\rangle|D\rangle$$

Thank you for listening