Decentralized Multi-Authority Attribute-Based Inner-Product FE: Large Universe and Unbounded

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NTT SIL

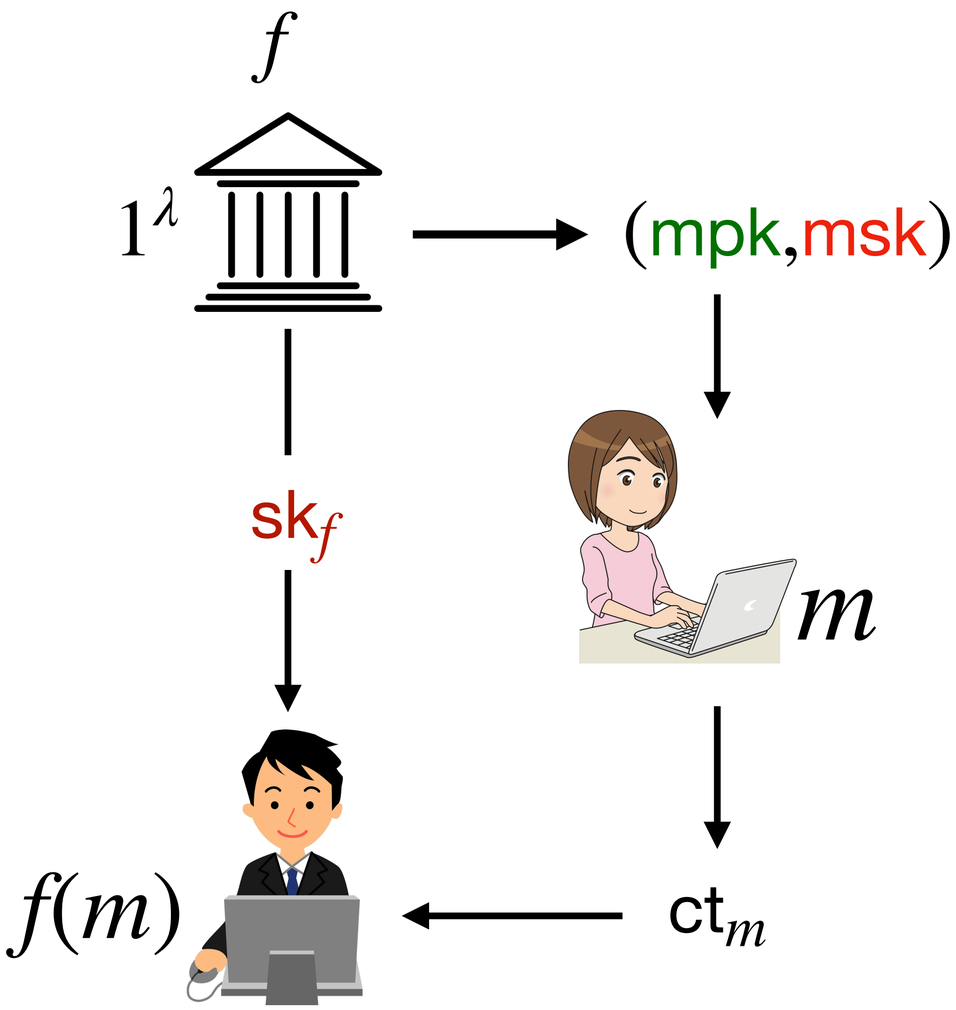
Functional Encryption [BSW11]

$\mathsf{Setup}(1^\lambda) \to (\mathsf{mpk}, \mathsf{msk})$

$\mathsf{KeyGen}(\mathsf{msk}, f) \to \mathsf{sk}_f$

$\mathsf{Enc}(\mathsf{mpk}, m) \to \mathsf{ct}_m$

$\mathsf{Dec}(\mathsf{sk}_f, \mathsf{ct}_m) \to f(m)$



Functional Encryption: Security [BSW11]

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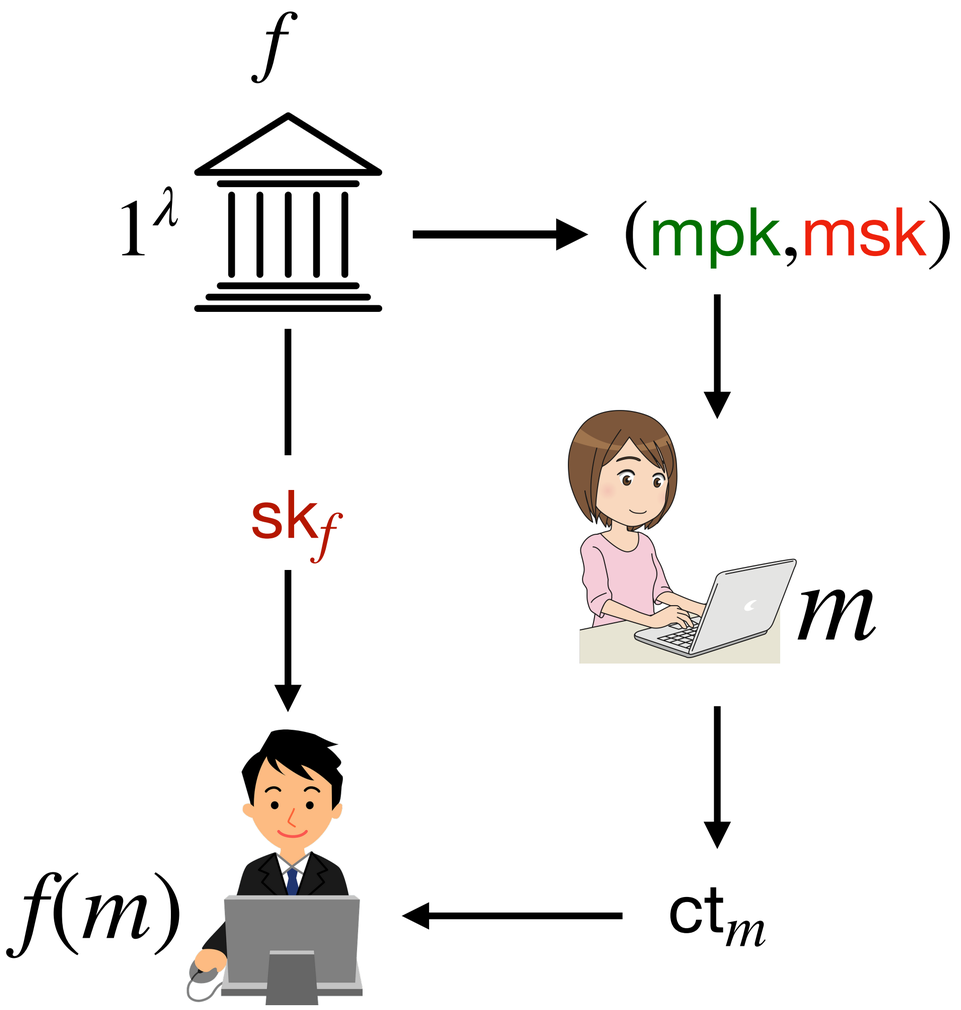


Security: users learn *only* $f(m)$

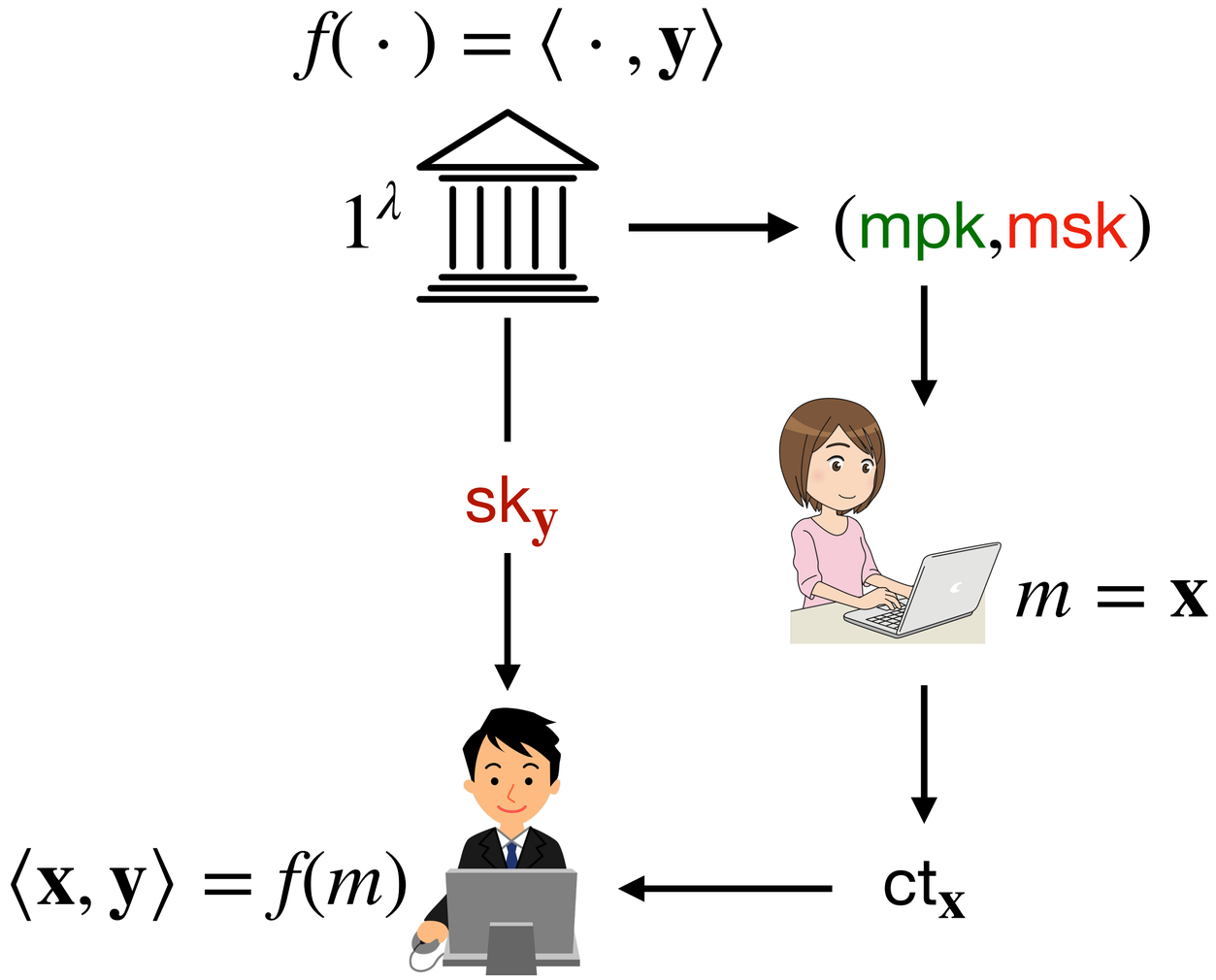
Inner Product Functional Encryption (IPFE) [ABDP15, ALS16]

Setup$(1^n) \to$ (mpk, msk)

KeyGen(msk, $\mathbf{y}$) $\to$ sk$_{\mathbf{y}}$

Enc(mpk, $\mathbf{x}$) $\to$ ct$_{\mathbf{x}}$

Dec(sk$_{\mathbf{y}}$, ct$_{\mathbf{x}}$) $\to \langle \mathbf{x}, \mathbf{y} \rangle$



Leakage of message in IPFE

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KeyGen(msk, $\mathbf{y}$) $\rightarrow$ sk$_{\mathbf{y}}$

Enc(mpk, $\mathbf{x}$) $\rightarrow$ ct$_{\mathbf{x}}$

Dec(sk$_{\mathbf{y}}$, ct$_{\mathbf{x}}$) $\rightarrow \langle \mathbf{x}, \mathbf{y} \rangle$

- sk$_{\mathbf{y}}$ reveals $\langle \mathbf{x}, \mathbf{y} \rangle$
- release sufficient # sk$_{\mathbf{y}_i}$

$$\text{solve} \begin{pmatrix} \cdots & \mathbf{y}_1 & \cdots \\ & \vdots & \\ \cdots & \mathbf{y}_n & \cdots \end{pmatrix} \cdot \mathbf{x} = \begin{pmatrix} \langle \mathbf{x}, \mathbf{y}_1 \rangle \\ \vdots \\ \langle \mathbf{x}, \mathbf{y}_n \rangle \end{pmatrix} \text{ for } \mathbf{x}$$

- system is completely broken

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- system is completely broken
- Cannot release sk$_{\mathbf{y}}$ for n L.I. $\mathbf{y}$ vectors

Leakage is high

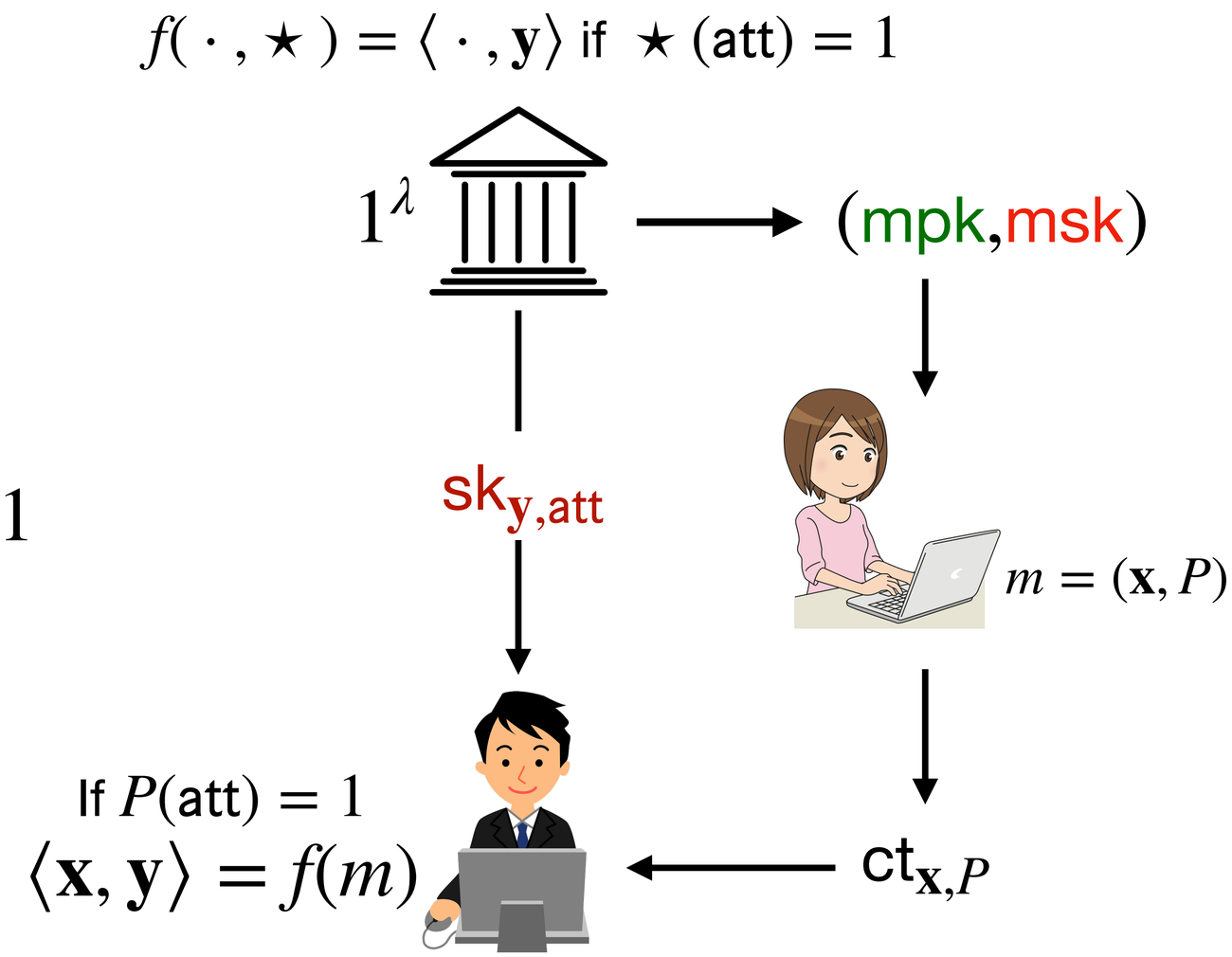
Attribute-Based IPFE (ABIPFE) [ACGU20]

Setup$(1^\lambda, 1^n) \to$ (mpk, msk)

KeyGen(msk, $\mathbf{y}$, att) $\to$ sk$_{\mathbf{y},\text{att}}$

Enc(mpk, $\mathbf{x}, P$) $\to$ ct$_{\mathbf{x},P}$

Dec(sk$_{\mathbf{y},\text{att}}$, ct$_{\mathbf{x},P}$) $\to \langle \mathbf{x}, \mathbf{y} \rangle$ if $P(\text{att}) = 1$



ABIPFE: IND-based Security [ACGU20]

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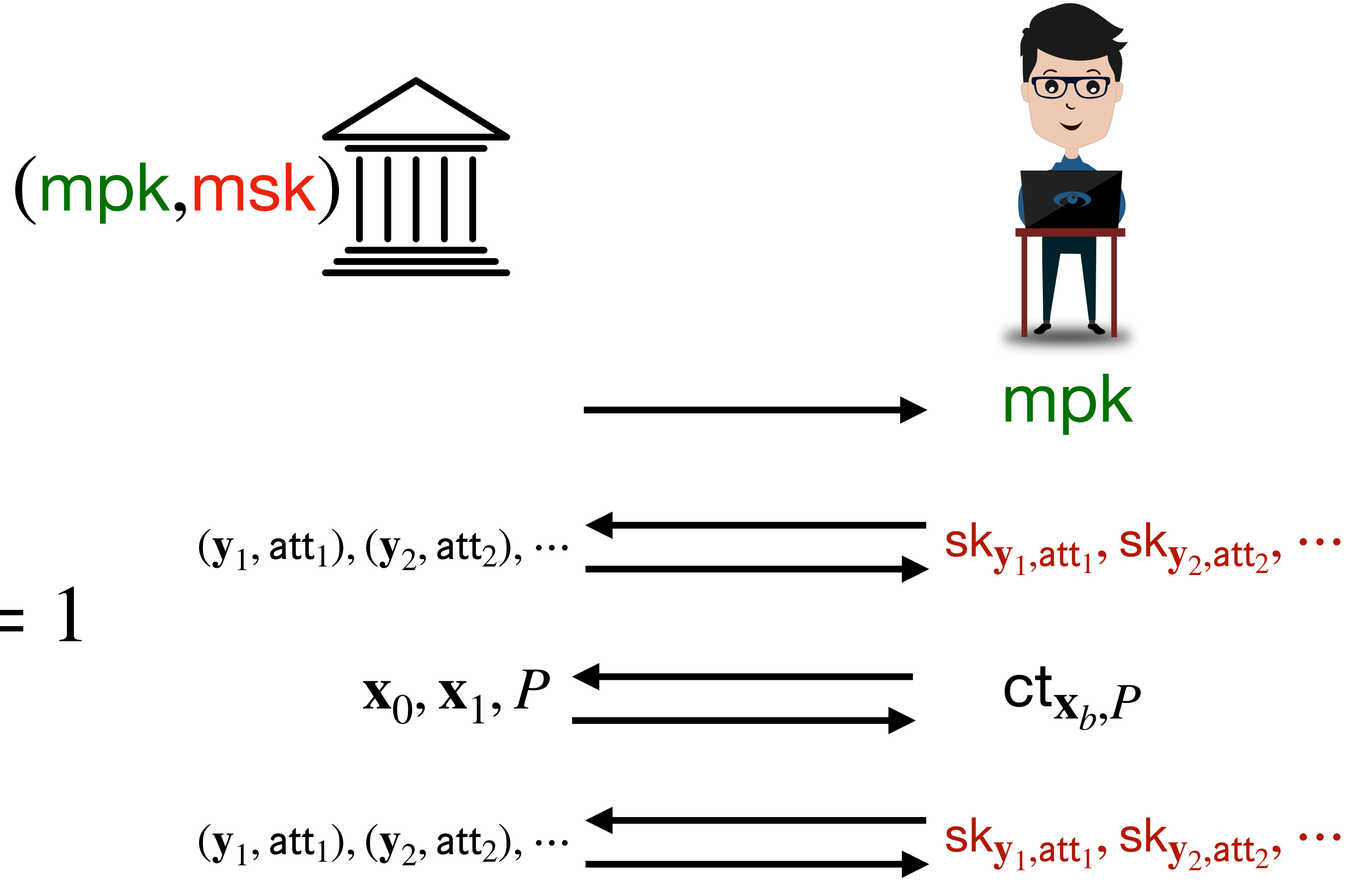

$\langle \mathbf{x}_0, \mathbf{y}_i \rangle = \langle \mathbf{x}_1, \mathbf{y}_i \rangle \ \forall i$ s.t. $P(\text{att}_i) = 1$

Security: cannot guess b

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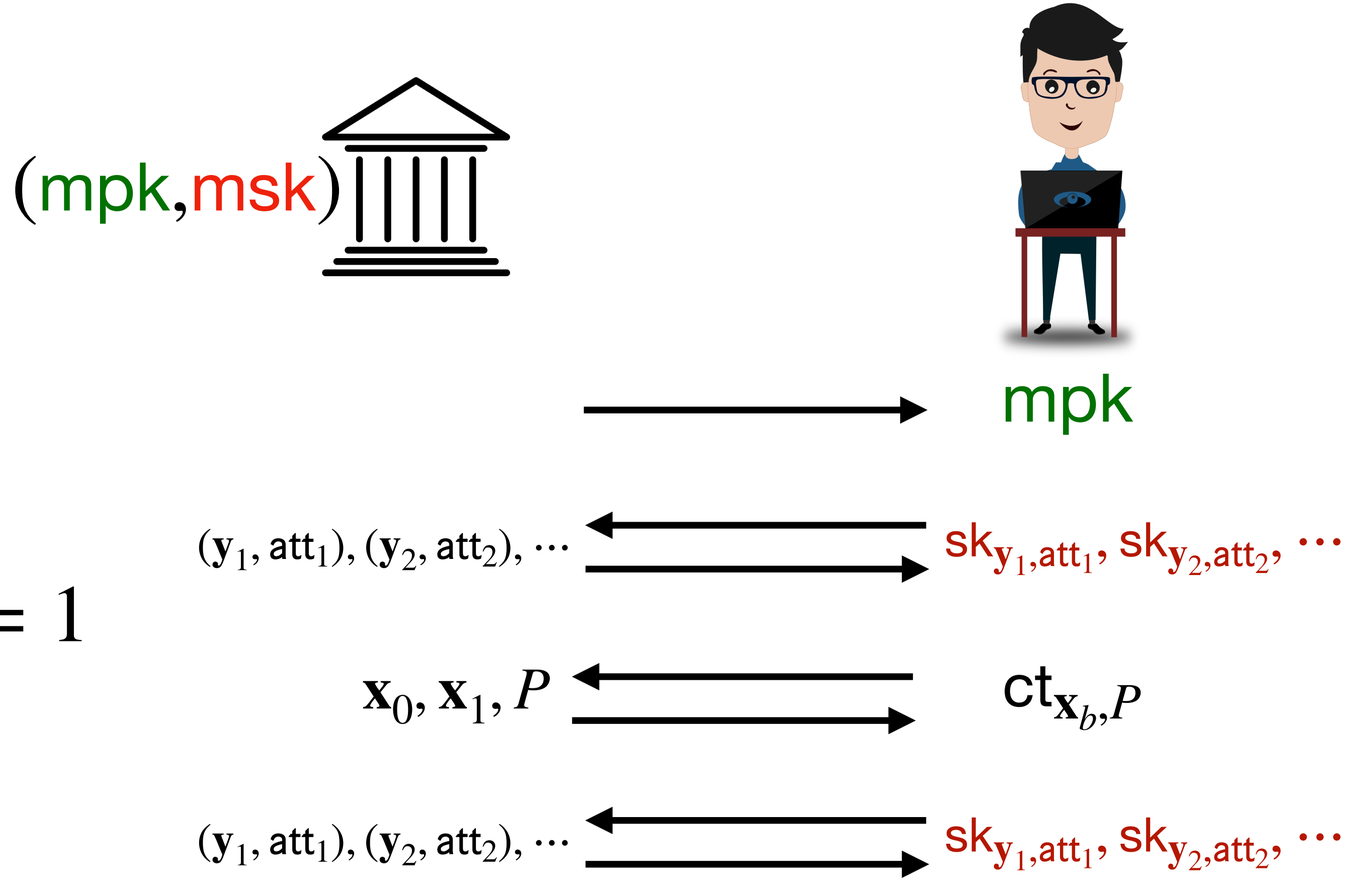
- Can release sk$_{\mathbf{y}}$ for n L.I. $\mathbf{y}$ vectors unless these are associated to satisfying attributes.

manage leakage by access control



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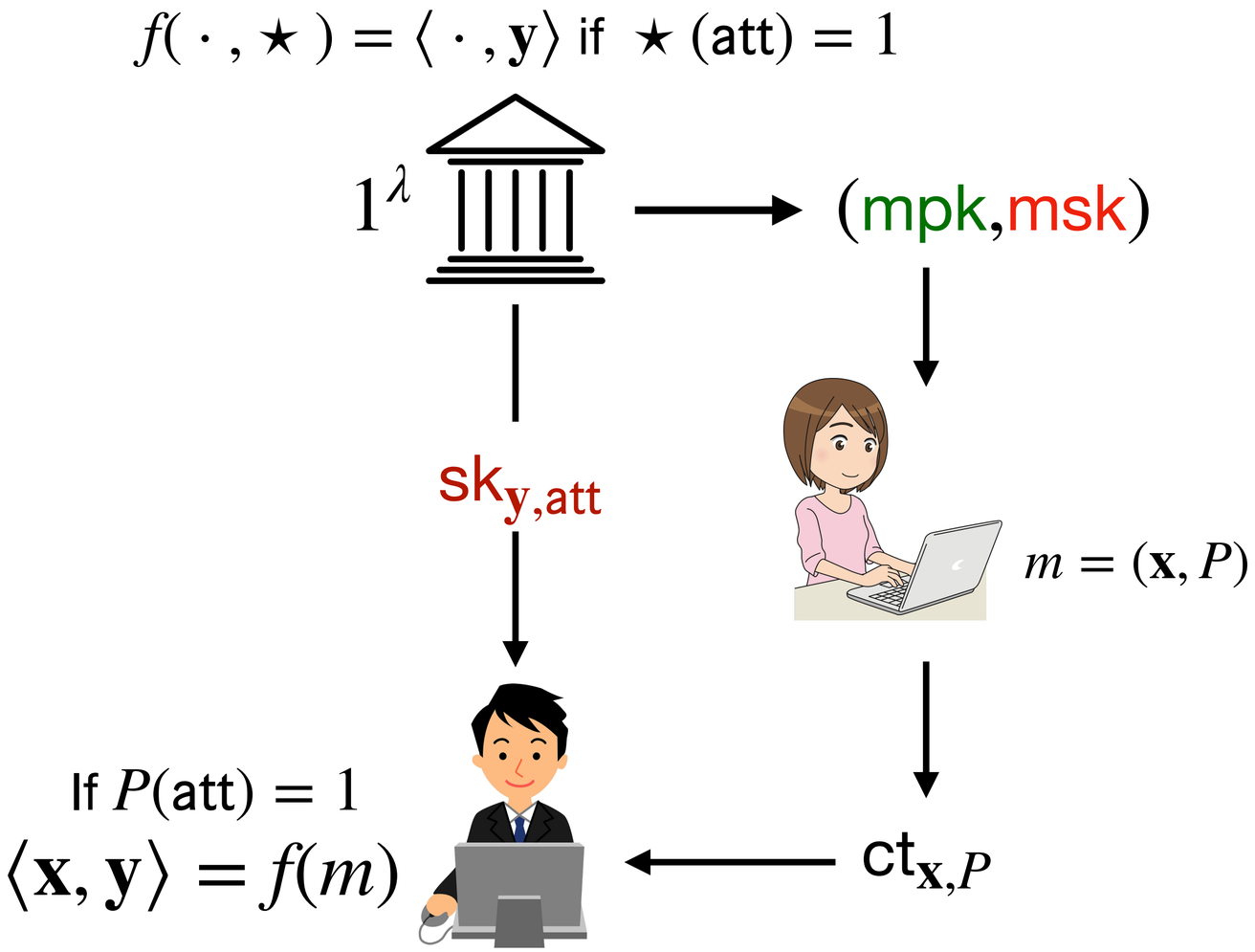
Attribute-Based IPFE (ABIPFE): Application in IoMT



Attribute-Based IPFE (ABIPFE): Application in IoMT

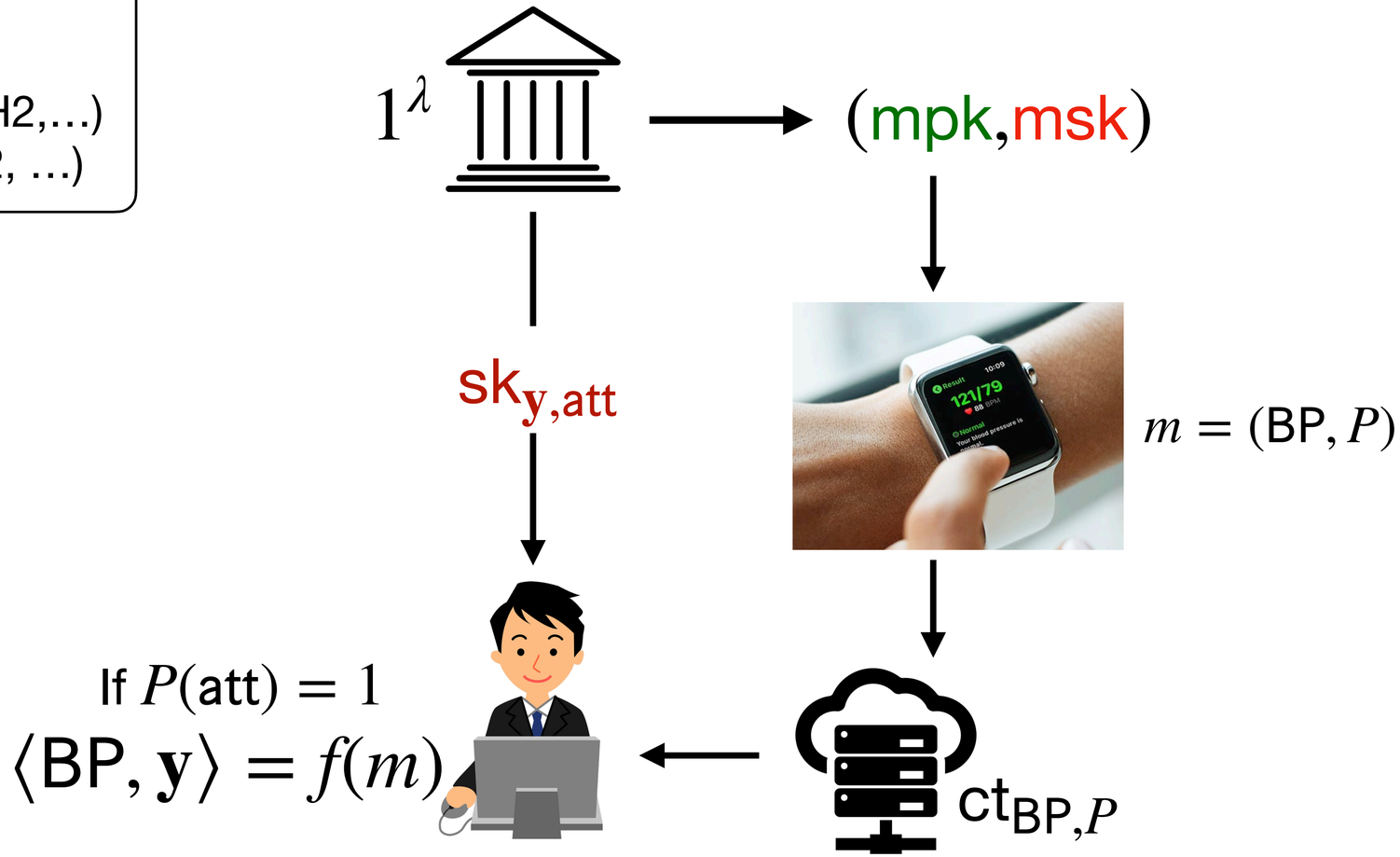
Data owner: IoMT device users

Data users:

- doctors of hospitals (H1, H2,…)
- researchers of labs (L1, L2, …)

att = {dept_H1/H2, section_L1/L2, …}

$f(\cdot, \star) = \langle \cdot, \mathbf{y} \rangle$ if $\star(\text{att}) = 1$



P: (doc@cardio_H1) ∧ (rs@drug_L2)

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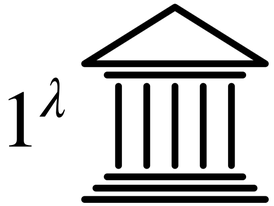
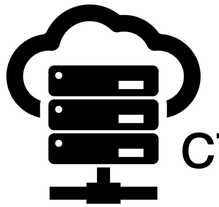


1^λ → (mpk,msk)

$\text{sk}_{\mathbf{y},\text{att}}$

$m = (\text{BP}, P)$



$\text{ct}_{\text{BP},P}$

If $P(\text{att}) = 1$

$\langle \text{BP}, \mathbf{y} \rangle = f(m)$

- all attributes are controlled by a *single* authority
- the security of the system depends on the *single* authority

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Multi-Authority ABIPFE (MA-ABIPFE) [AGT21]

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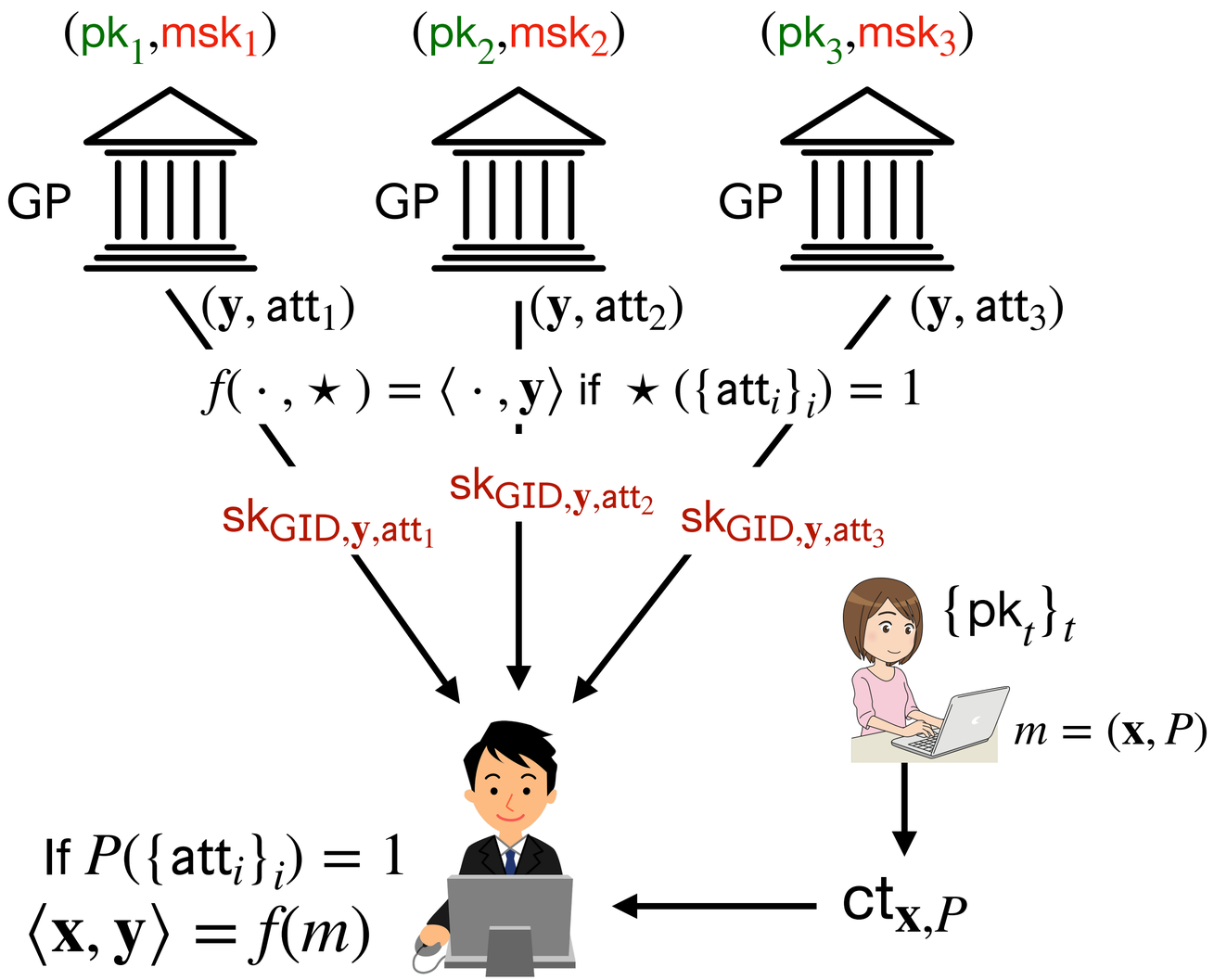

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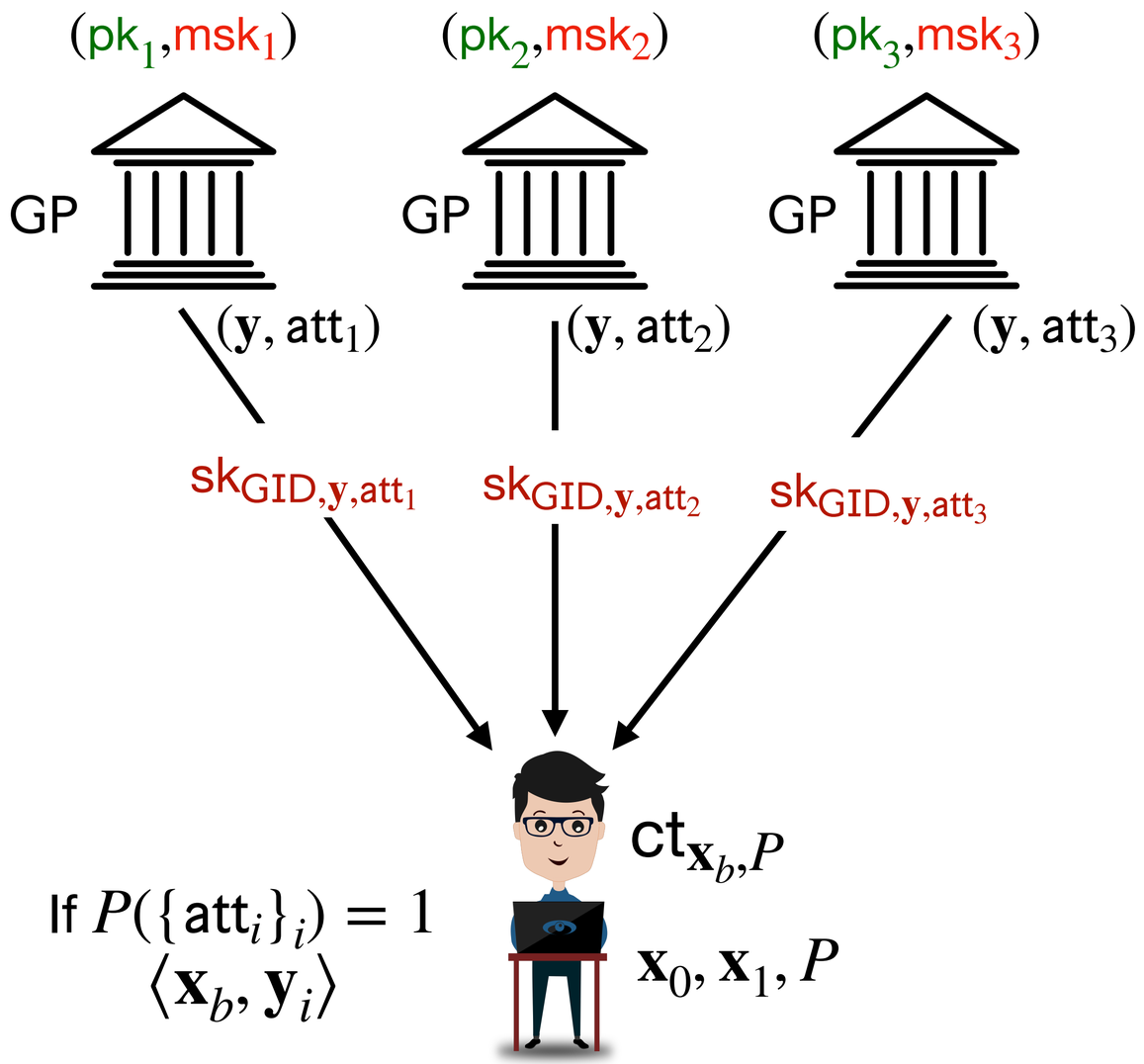

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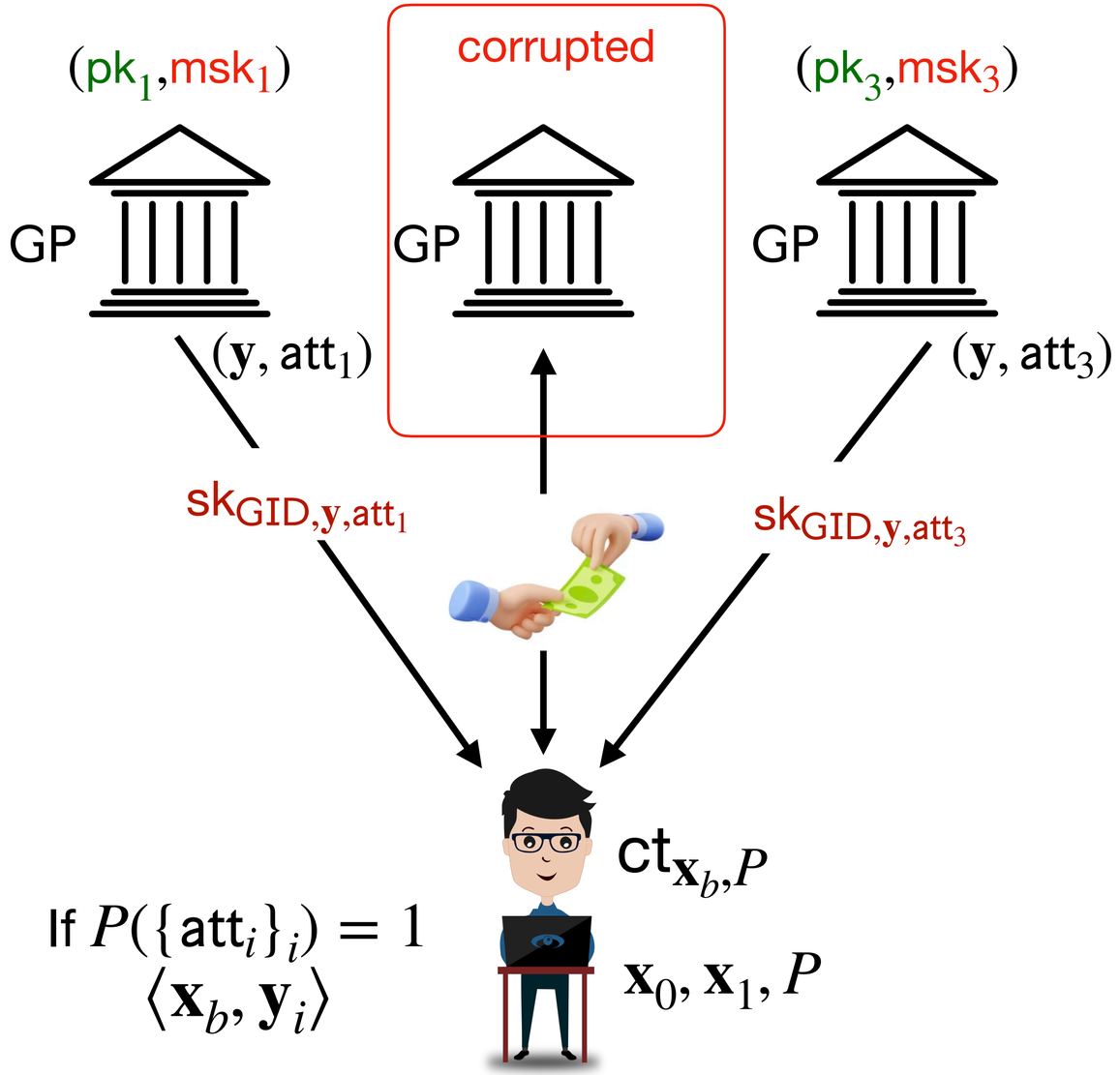
- authorities can be corrupted



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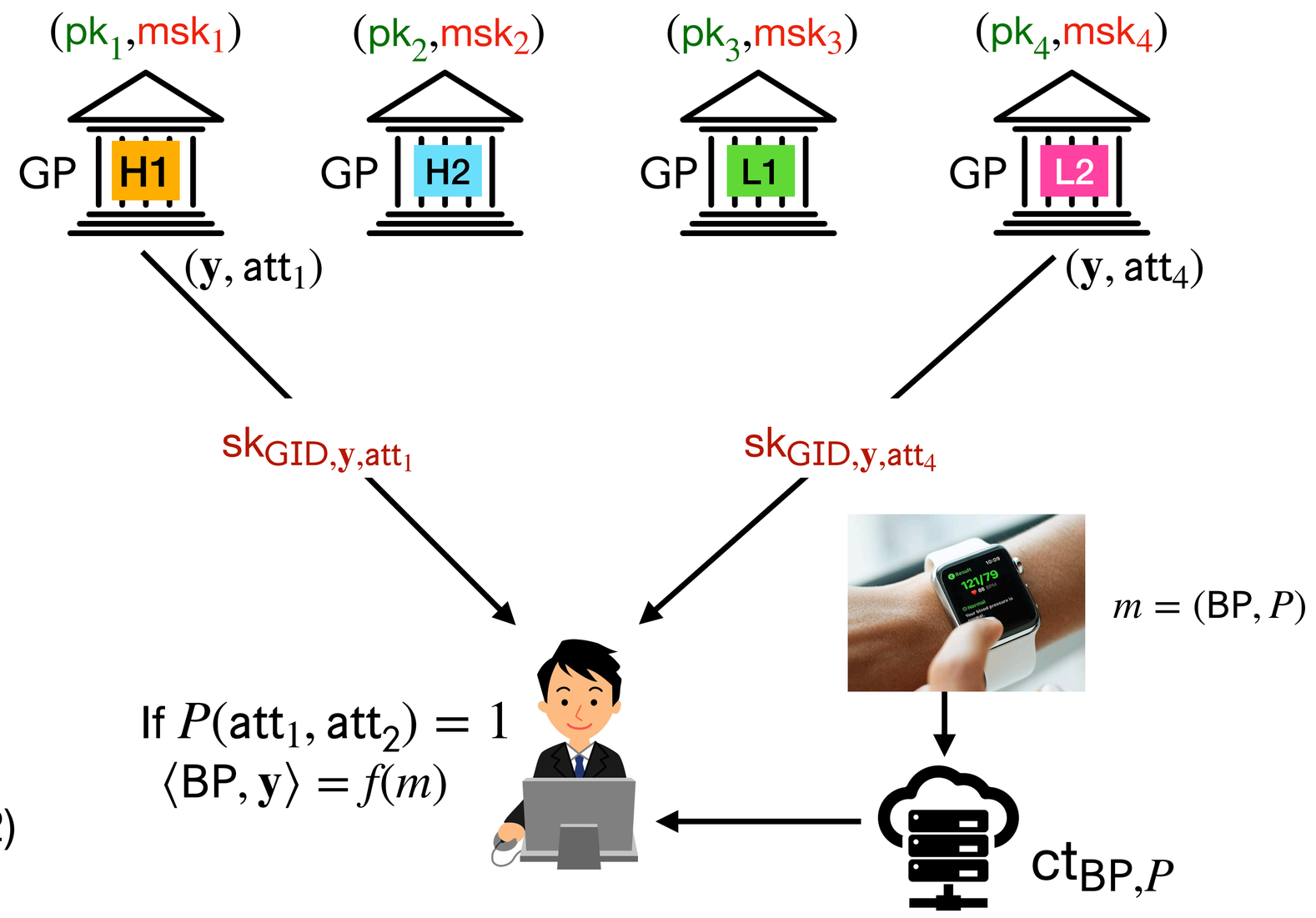

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att_1 att_2

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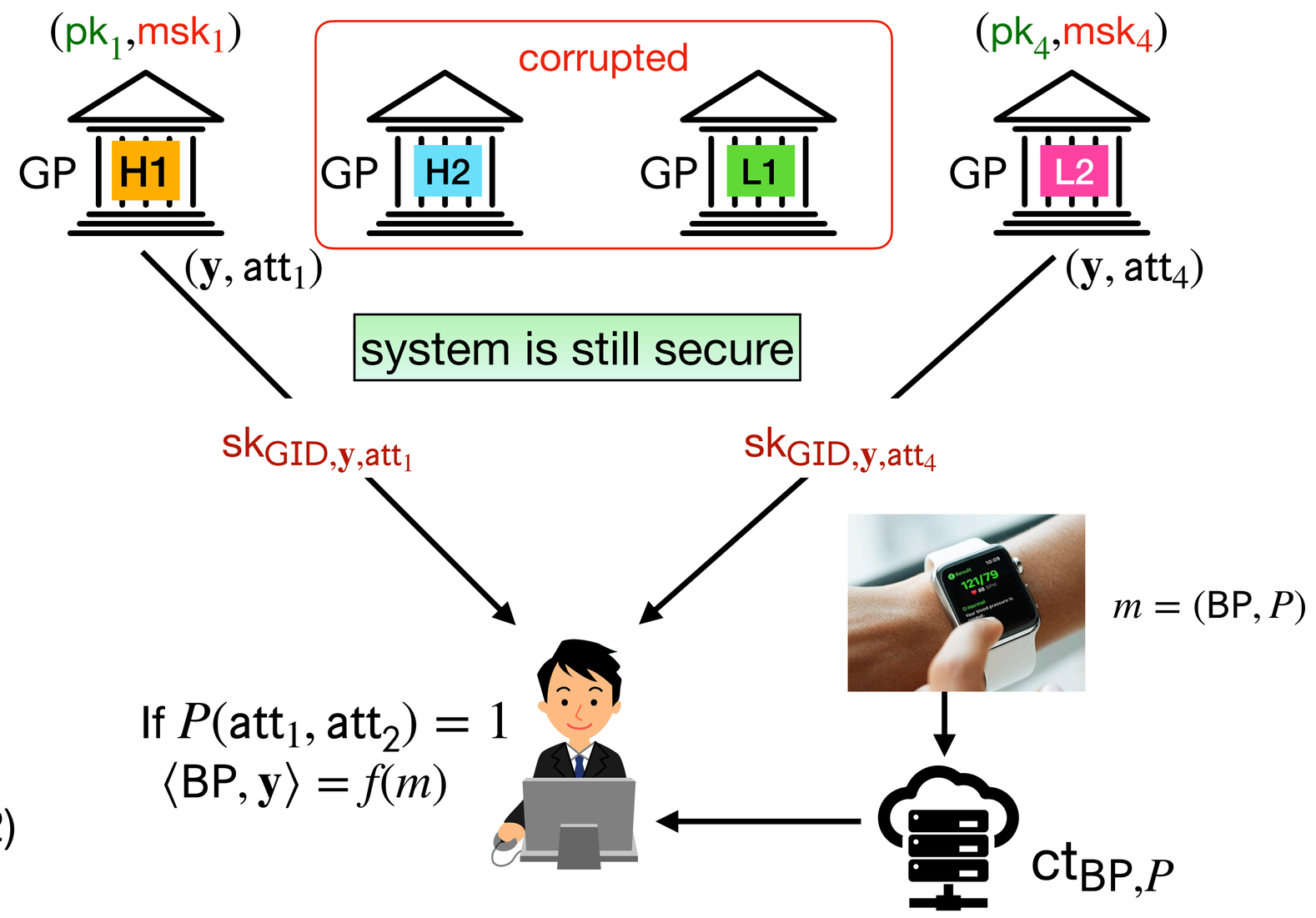

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Limitations of MA-ABIPFE of AGT21: bounded vectors

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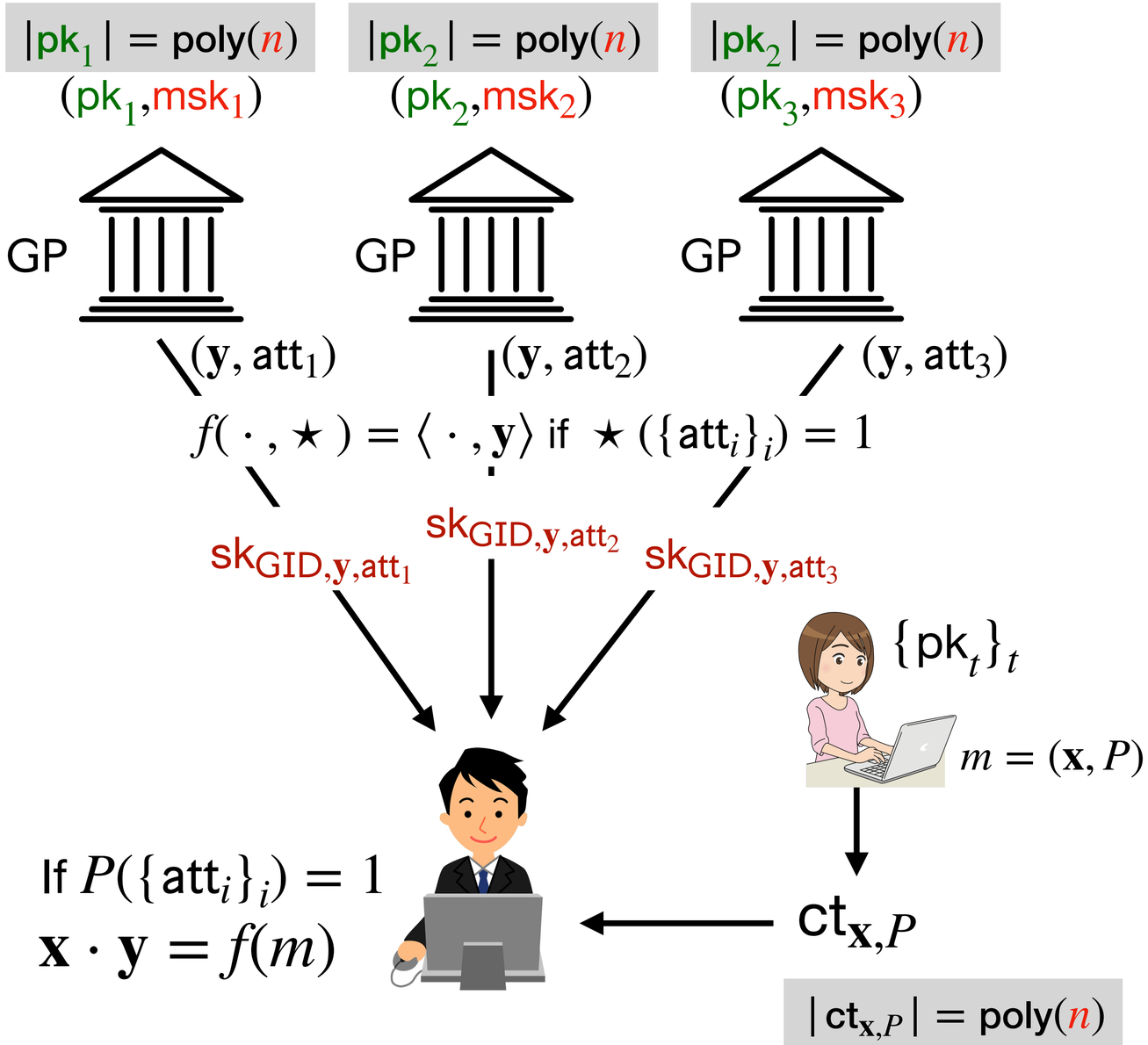
$\mathsf{Enc}(\mathsf{GP}, \{\mathsf{pk}_t\}_t, \mathbf{x}, P) \to \mathsf{ct}_{\mathbf{x},P}$

- n is fixed for all authorities
- $|\mathsf{pk}_t| = \mathsf{poly}(n)$
- $|\mathsf{ct}_{\mathbf{x},P}| = \mathsf{poly}(n)$ even if $|\mathbf{x}| << n$



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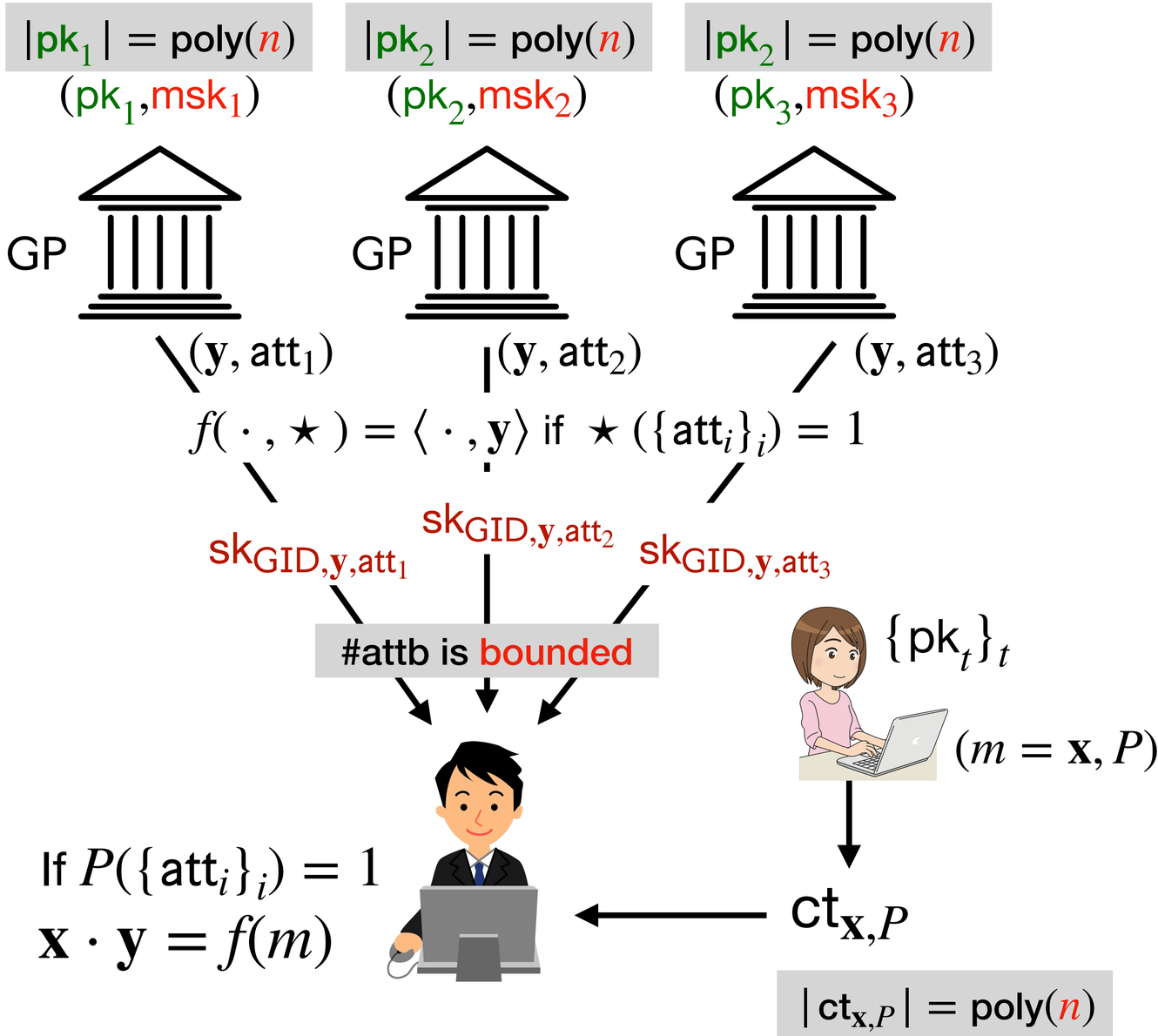
- each auth controls a bounded #attb
- small universe



Limitations of MA-ABIPFE of AGT21: strong assumption/weak performance

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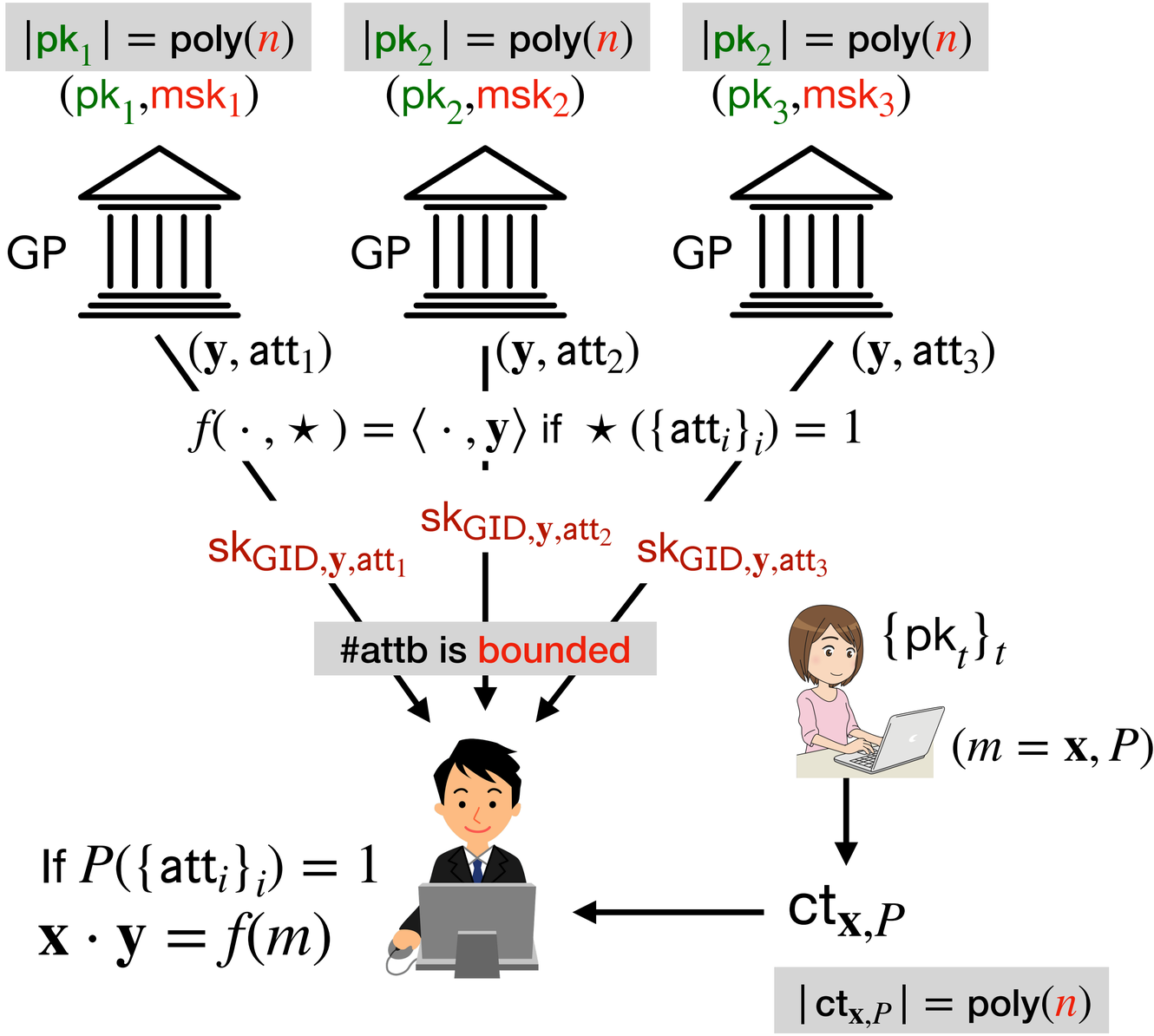
$\mathsf{Enc}(\mathsf{GP}, \{\mathsf{pk}_t\}_t, \mathbf{x}, P) \to \mathsf{ct}_{\mathbf{x},P}$

- Built in composite order groups
- More than 2 hours for encryption
- About 5 days for decryption



Our results

We construct MA-ABIPFE where…

- unbounded length message/key vectors can be processed,

  $\mathsf{LSetup}(\mathsf{GP}, t) \to (\mathsf{pk}_t, \mathsf{msk}_t)$ $|\mathsf{pk}_t| = \mathsf{poly}(\lambda)$ $|\mathsf{ct}_{\mathbf{x},P}| = \mathsf{poly}(|\mathbf{x}|)$

- authorities are allowed to control arbitrary number of attributes
- attributes need not be enumerated at setup, i.e., large universe
- constructed in a prime-order pairing groups
- proven secure under target-group-based assumptions

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- constructed in a prime-order pairing groups
- proven secure under target-group-based assumptions
- static security under DBDH-type assumption

Roadmap

[AGT21]	Our SMA-ABUIPFE	Our LMA-ABUIPFE
small-universe MA-ABIPFE	small-universe MA-ABUIPFE	large-universe MA-ABUIPFE
bounded-length vectors	unbounded-length vectors	unbounded-length vectors
composite-order group-based	prime-order group-based	prime-order group-based
source-group-based assump. e.g., subgroup decision assump.	target-group-based assump. e.g., DBDH	target-group-based assump. e.g., L-DBDH
selective security	static security	static security

Constructing SMA-ABUIPFE

An initial attempt combining techniques of [DKW21] and [DP19]

GP $= (e, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2)$ $s =$ max #AND gates in $P = (\mathsf{M} \in \mathbb{Z}^{\ell \times s}, \rho)$ $m = \mathbf{x} \in \mathbb{Z}^{|I_{\mathbf{x}}|}$ $f = \mathbf{y} \in \mathbb{Z}^{|I_{\mathbf{y}}|}$

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$$\mathsf{sk}_{\mathsf{GID},\mathbf{y},t} = \prod_k \mathsf{H}_1(t \,\|\, k)^{a_t y_k} \cdot \prod_{j=2}^{s} \prod_k \mathsf{H}(\mathsf{GID} \,\|\, \mathbf{y} \,\|\, j \,\|\, k)^{u_{t,j} \cdot y_k} \in \mathbb{G}_2$$

$\{\mathsf{sk}_{\mathsf{GID},\mathbf{y},t}\}_t$

vectorise using hash-and-exponentiation

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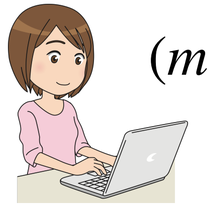

$(m = \mathbf{x}, P = (\mathsf{M}, \rho))$

- $r_1, \ldots, r_\ell \leftarrow \mathbb{Z}_p$
- $\mathbf{V} = (\mathbf{z}, \mathbf{v}_2, \ldots, \mathbf{v}_s) \leftarrow \mathbb{Z}_p^{s \times |I_{\mathbf{x}}|}$
- $\mathbf{X} = (\mathbf{x}_2, \ldots, \mathbf{x}_s) \leftarrow \mathbb{Z}_p^{(s-1) \times |I_{\mathbf{x}}|}$

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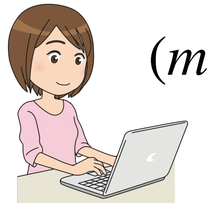

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$\mathsf{ct}_{\mathbf{x},P}$

vectorise using hash-and-pairing

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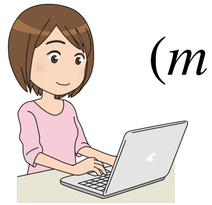

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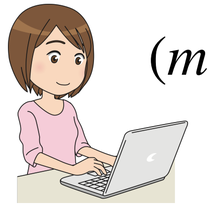

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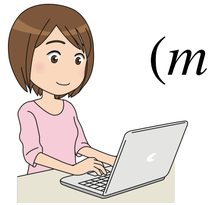

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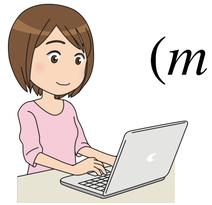

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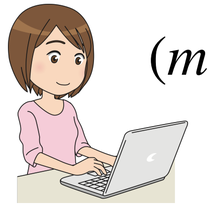

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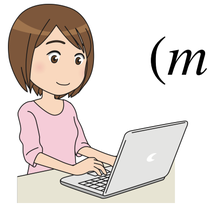

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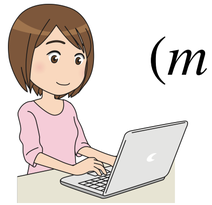

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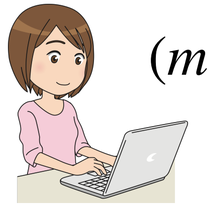

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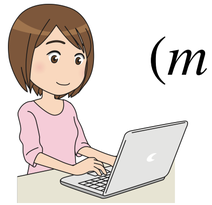

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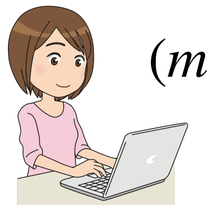

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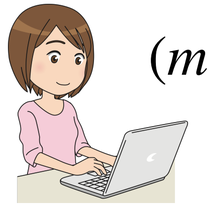

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$$\mathbf{y} \cdot \mathbf{z} = \prod_{i \in I} \left[\frac{C_{1,i} \cdot \mathbf{y} \ \prod_{j=2}^{s} \prod_k \boxed{e(C_{3,i,j,k} \cdot y_k, \mathsf{H}(\mathsf{GID} \parallel \mathbf{y} \parallel j \parallel k))}}{e(C_{2,i}, \mathsf{sk}_{\mathsf{GID},\mathbf{y},t})} \right]^{w_i}$$

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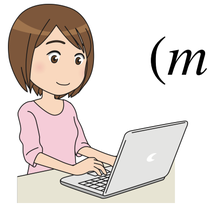

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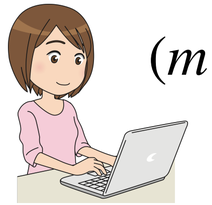

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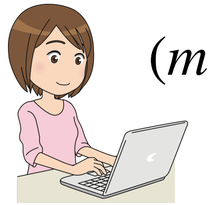

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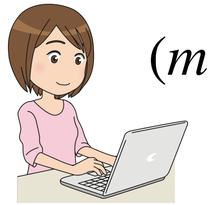

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How to handle $[u^{(3)}_{\rho(i),j,k}]_T$?

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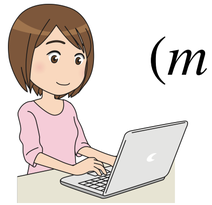

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$\mathsf{ct}_{\mathbf{x},P}$

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$$[u_{t,j}]_1 \longrightarrow [u^{(3)}_{t,j,k}]_T = e([u_{t,j}]_1, \mathsf{H}_3(\boxed{\mathsf{GID} \parallel \mathbf{y}} \parallel j \parallel k))$$

- $\mathbf{f} = (f_2, \ldots, f_s) \leftarrow \mathbb{Z}_p^{s-1}$
- $C_{4,i,j} = [M_{i,j} f_j \; + \; r_i u_{\rho(i),j}]_1$

$[b_{i,k}]_T = e(r_i [a_{\rho(i)}]_1, \mathsf{H}_1(t \parallel k))$

Constructing SMA-ABUIPFE

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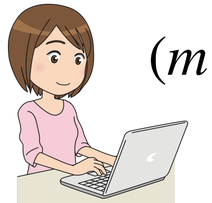

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$\mathsf{ct}_{\mathbf{x},P}$

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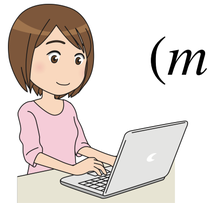

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$$C_{3,i,j,k} \cdot y_k \cdot e(C_{4,i,j} \cdot y_k, \mathsf{H}_3(\mathsf{GID} \,\|\, \mathbf{y} \,\|\, j \,\|\, k))$$

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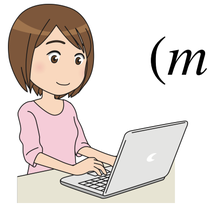

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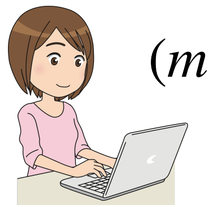

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Conclusion

- Define and Construct **L**arge-universe **M**ulti-**A**uthority **ABUIPFE**
  - ✦ Constant size public-keys for each authority
  - ✦ Succinct secret-keys (only one group element)
  - ✦ Input-specific $|\mathsf{ct}| = \mathsf{poly}(|\mathbf{x}|, s)$
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- Open problems in the area
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  - ✦ Adaptive corruption from standard assumptions
  - ✦ MA-ABIPFE from PQ-assumptions

Thank you!