Multi-Instance Secure Public-Key Encryption

Hans Heum

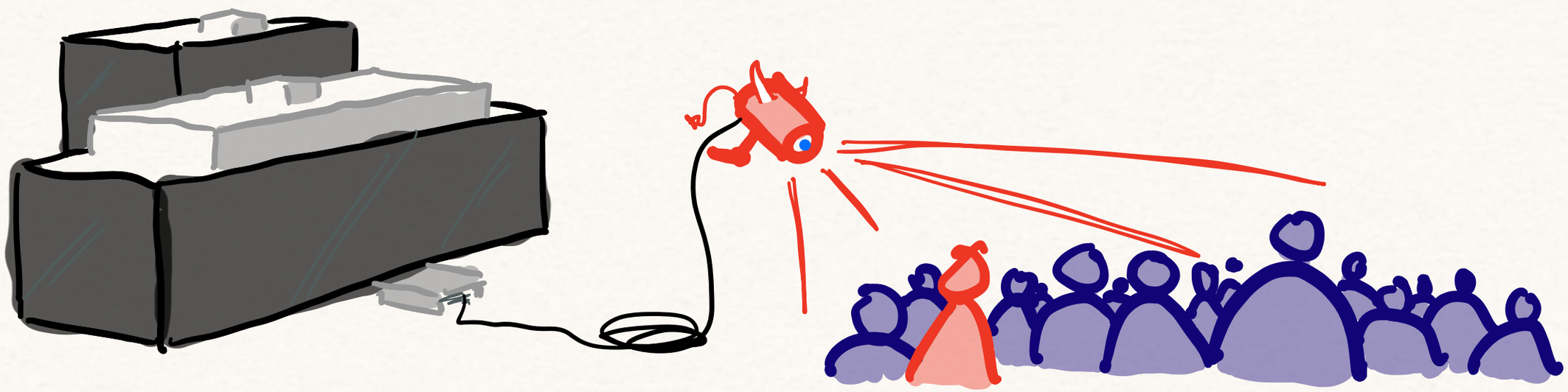
NTNU

with Carlo Brunetta and Martijn Stam

Simula UiB

The Multi-Instance Setting

Bellare, Ristenpart, Tessaro (Crypto'12)



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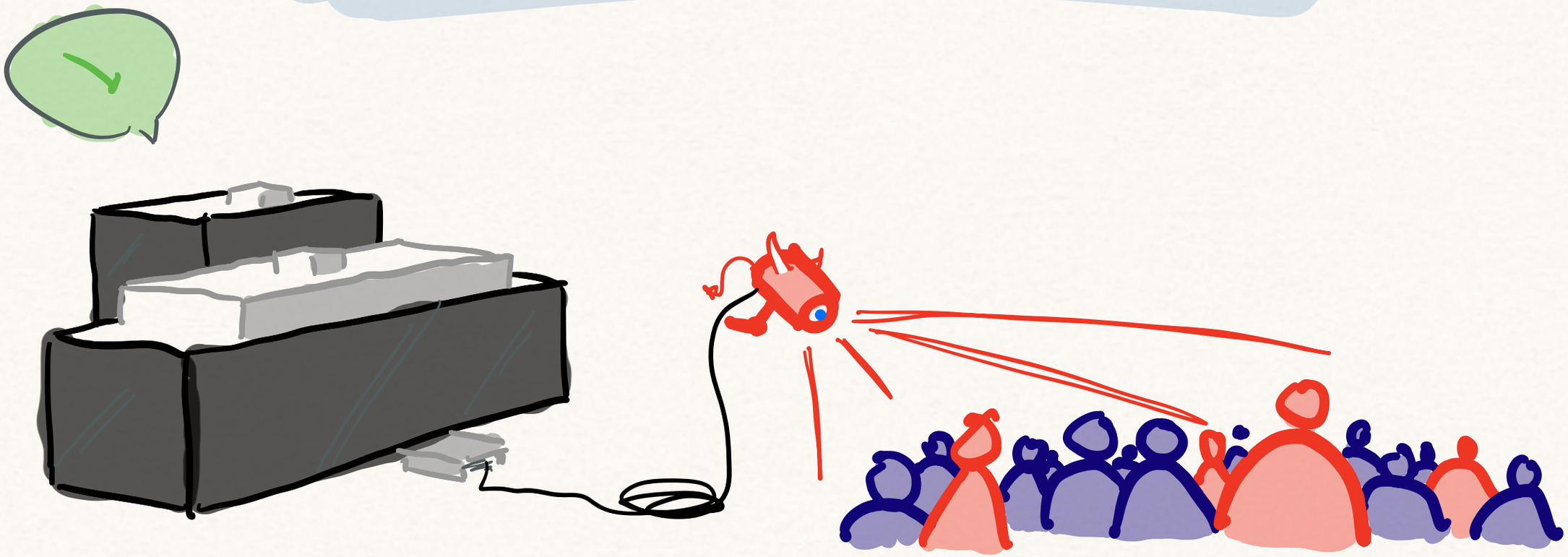

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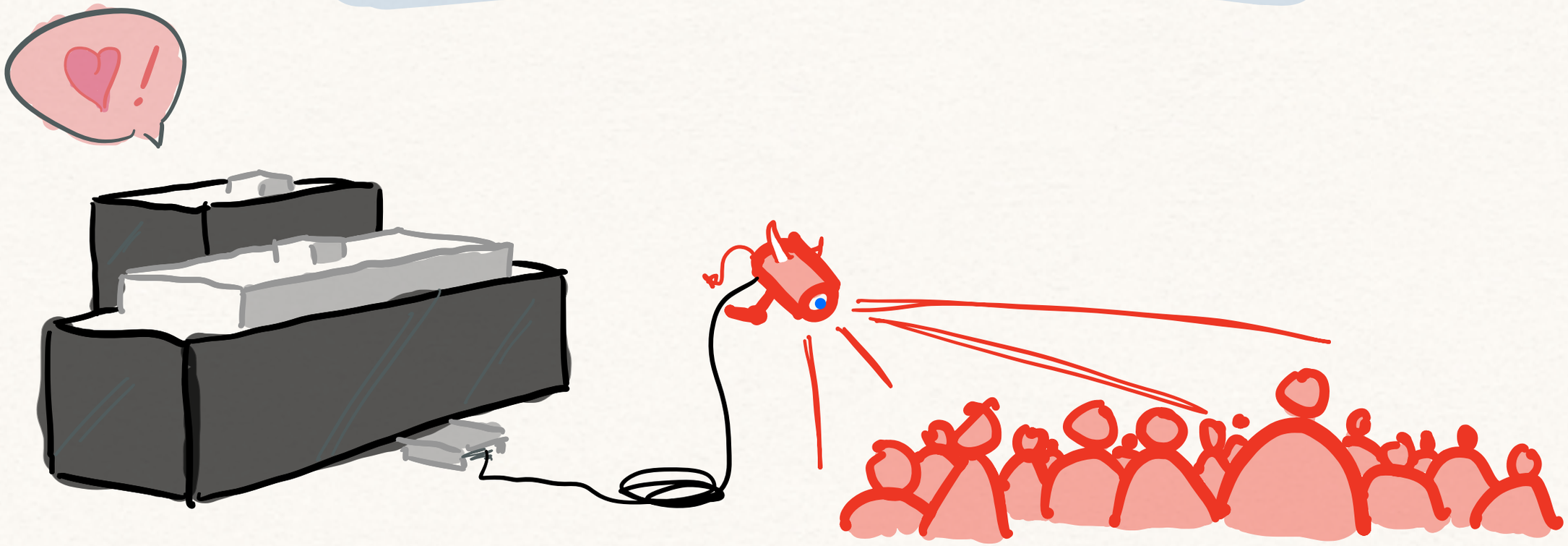
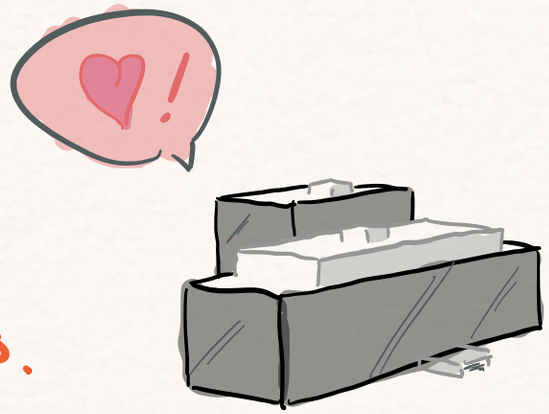


AUERBACH, GIACON, KILZ (EC'20):

EXIST KEM SCHEMES SUCH THAT:



PRE-COMPUTATION EQUIVALENT TO BRUTE-FORCING 1 USER $\Rightarrow$ SUFFICIENT TO BREAK <u>ALL</u> USERS.

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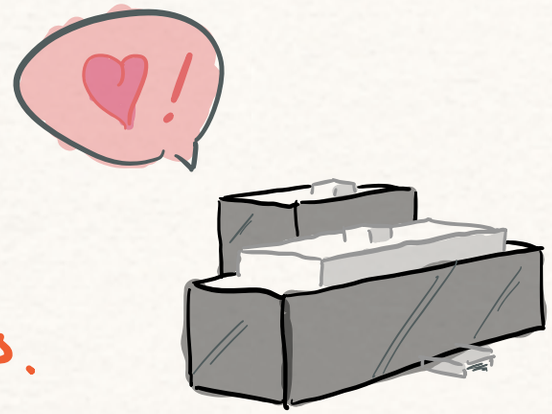

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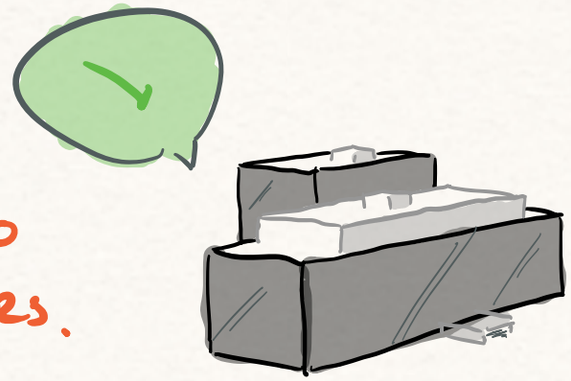

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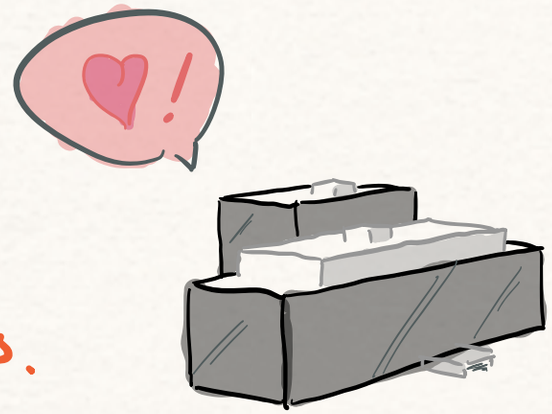

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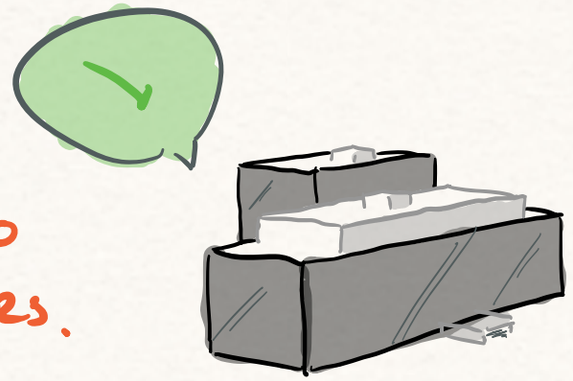

WANT PKE SCHEME S.TH.:

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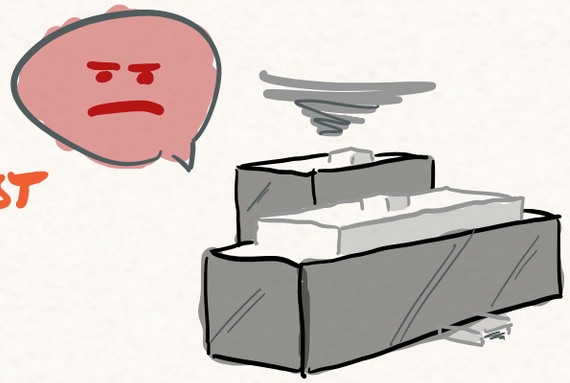

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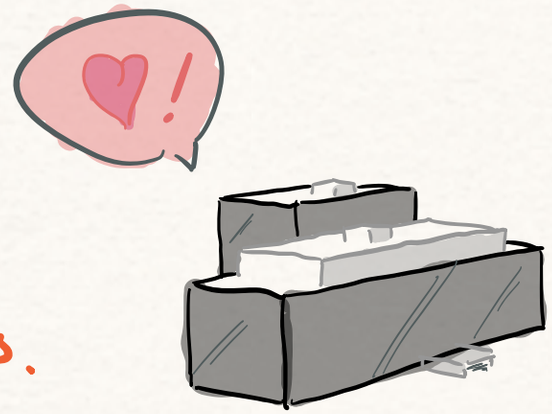

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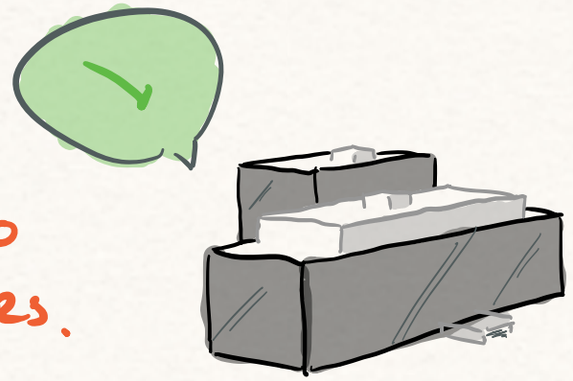

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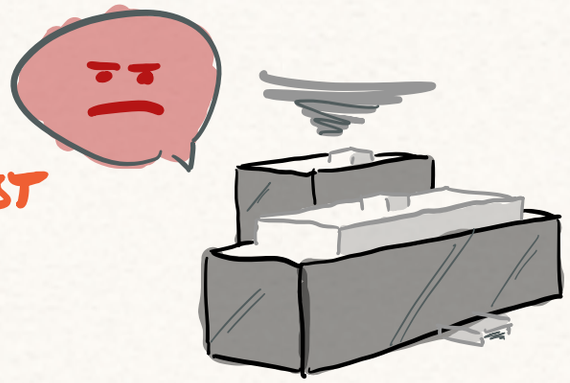
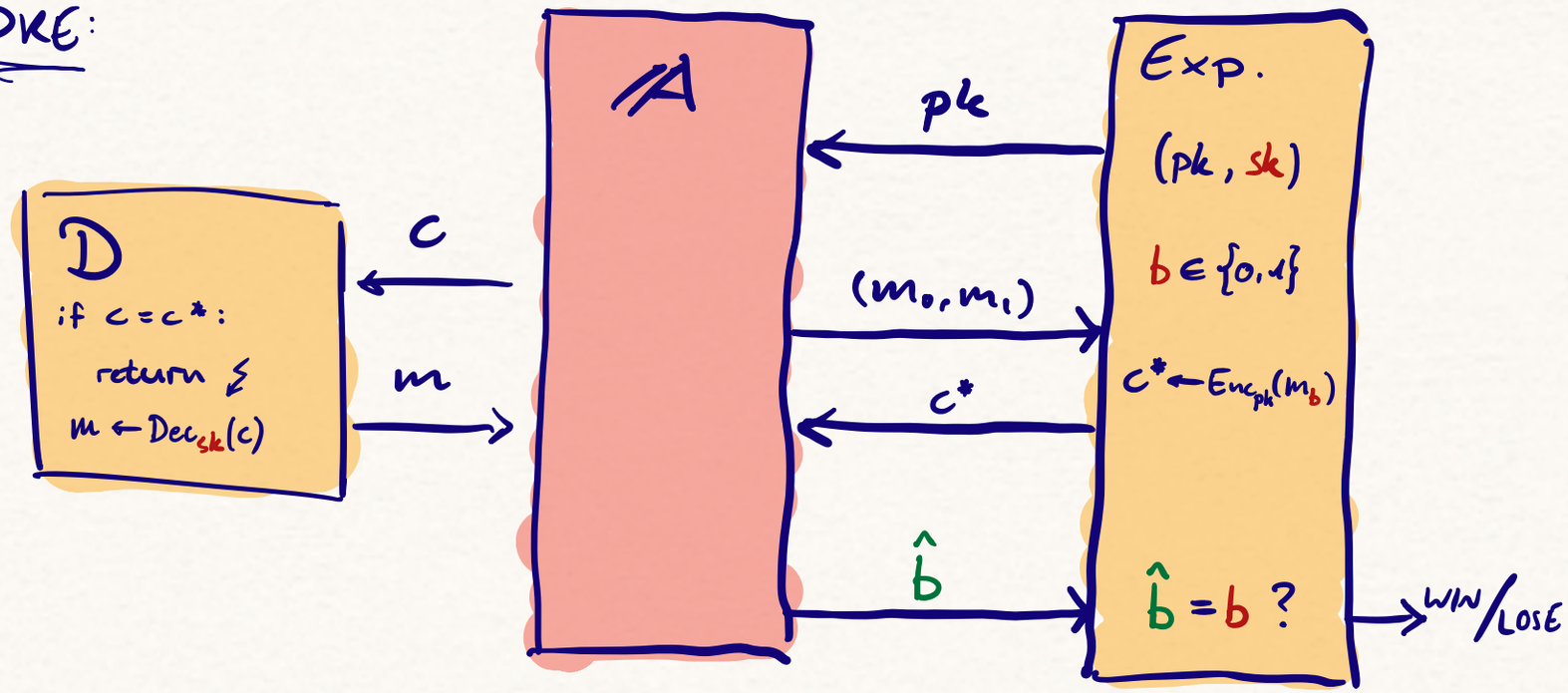


THE SINGLE-INSTANCE IND-CCA EXPERIMENTS:

PKE:



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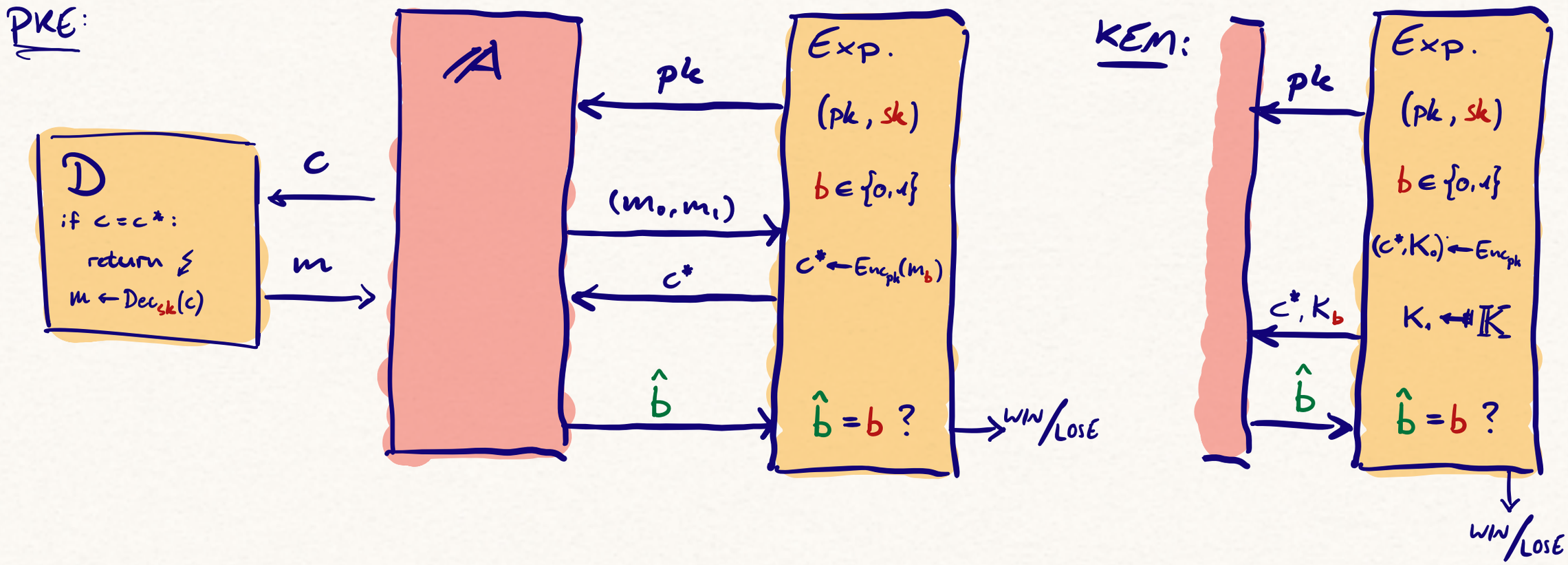

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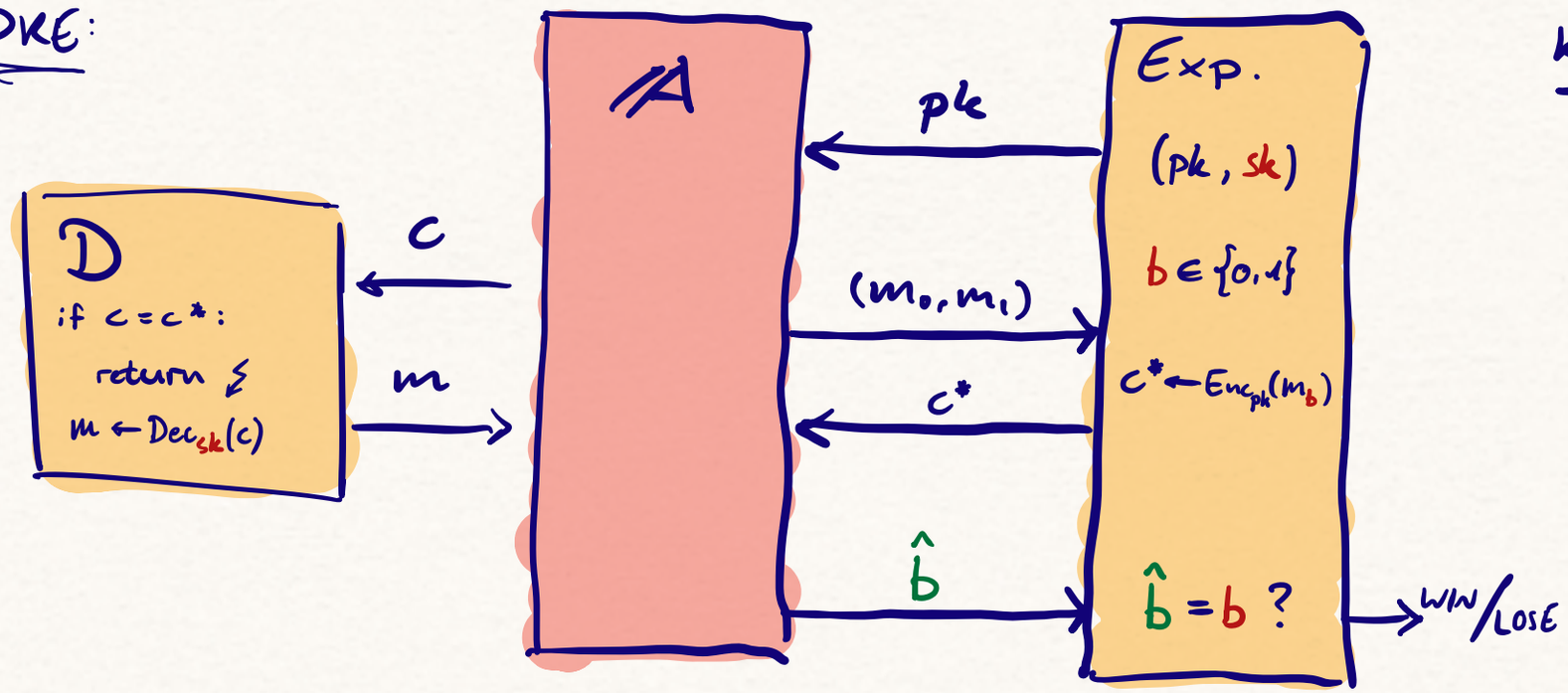
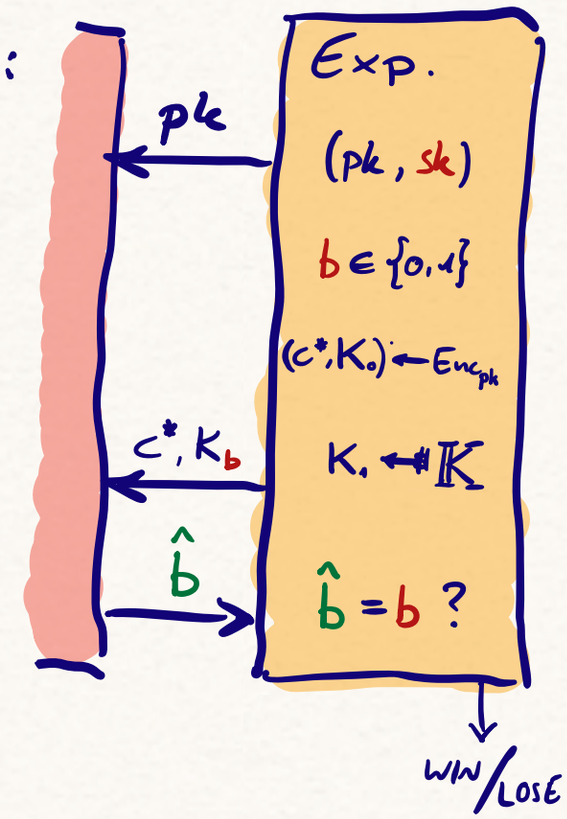
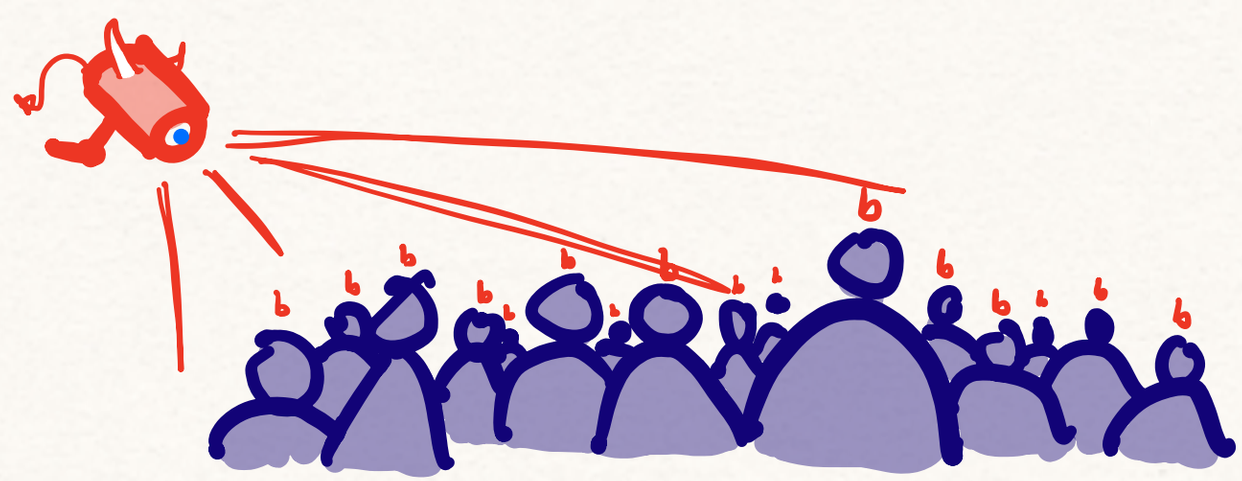


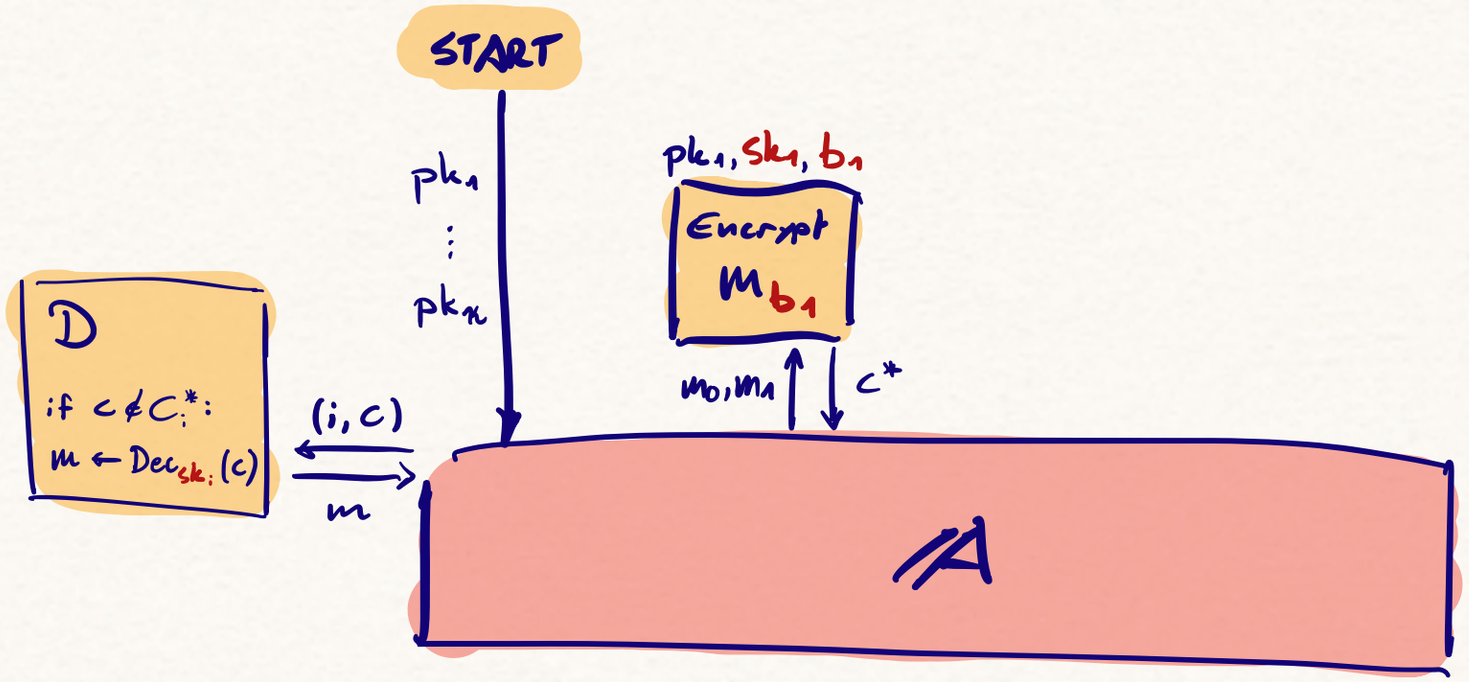
KEM:



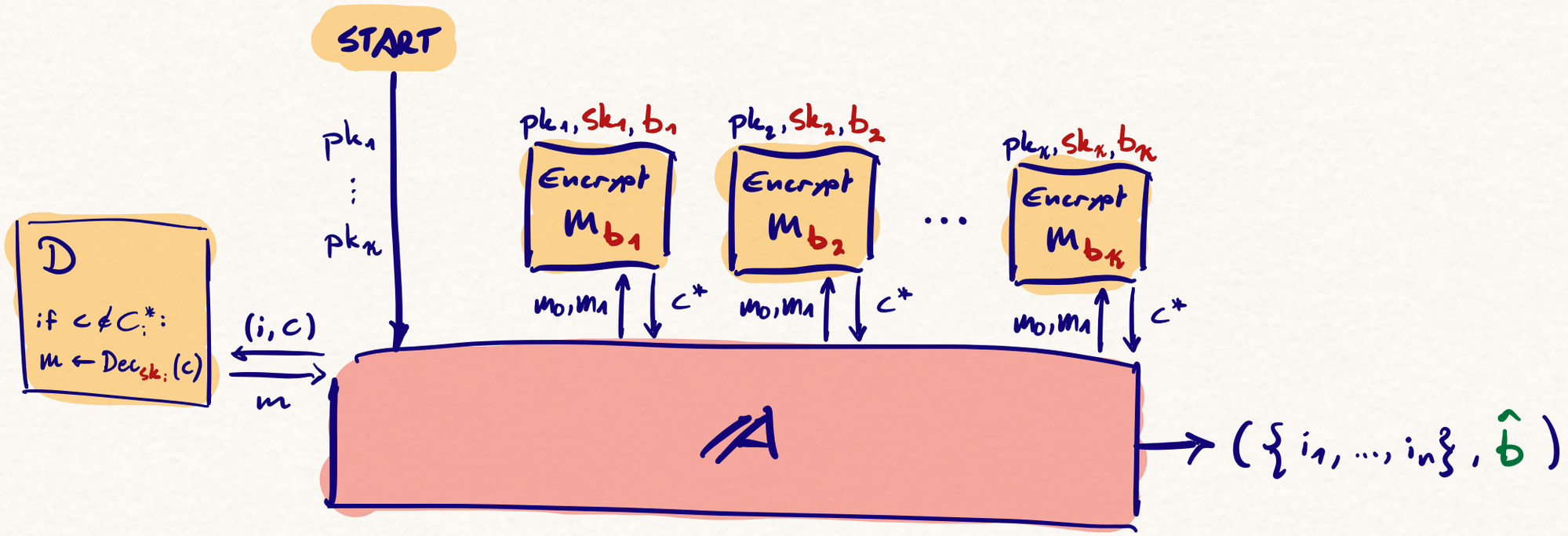
MULTI-USER SECURITY:

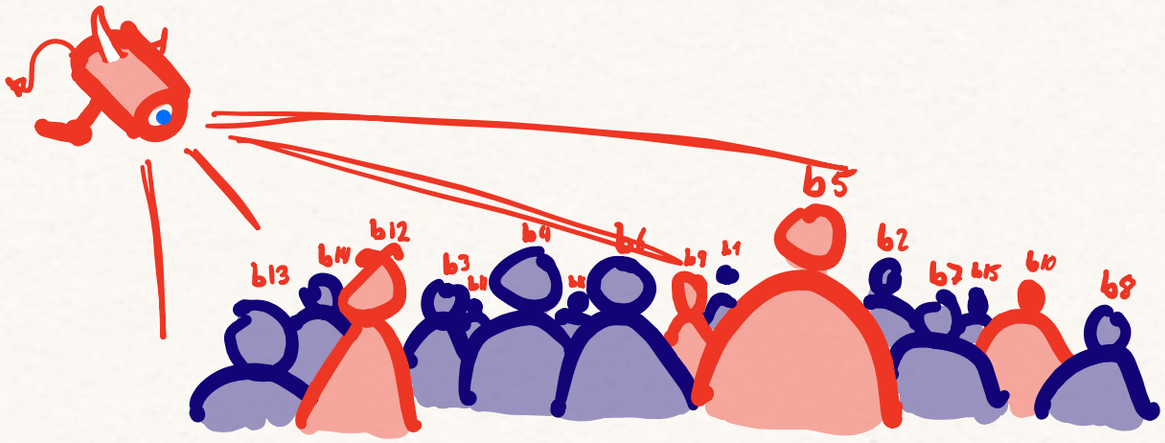


(n-OUT-OF-n) MI-IND-CCA FOR PKE



(n-OUT-OF-κ) MI-IND-CCA FOR PKE





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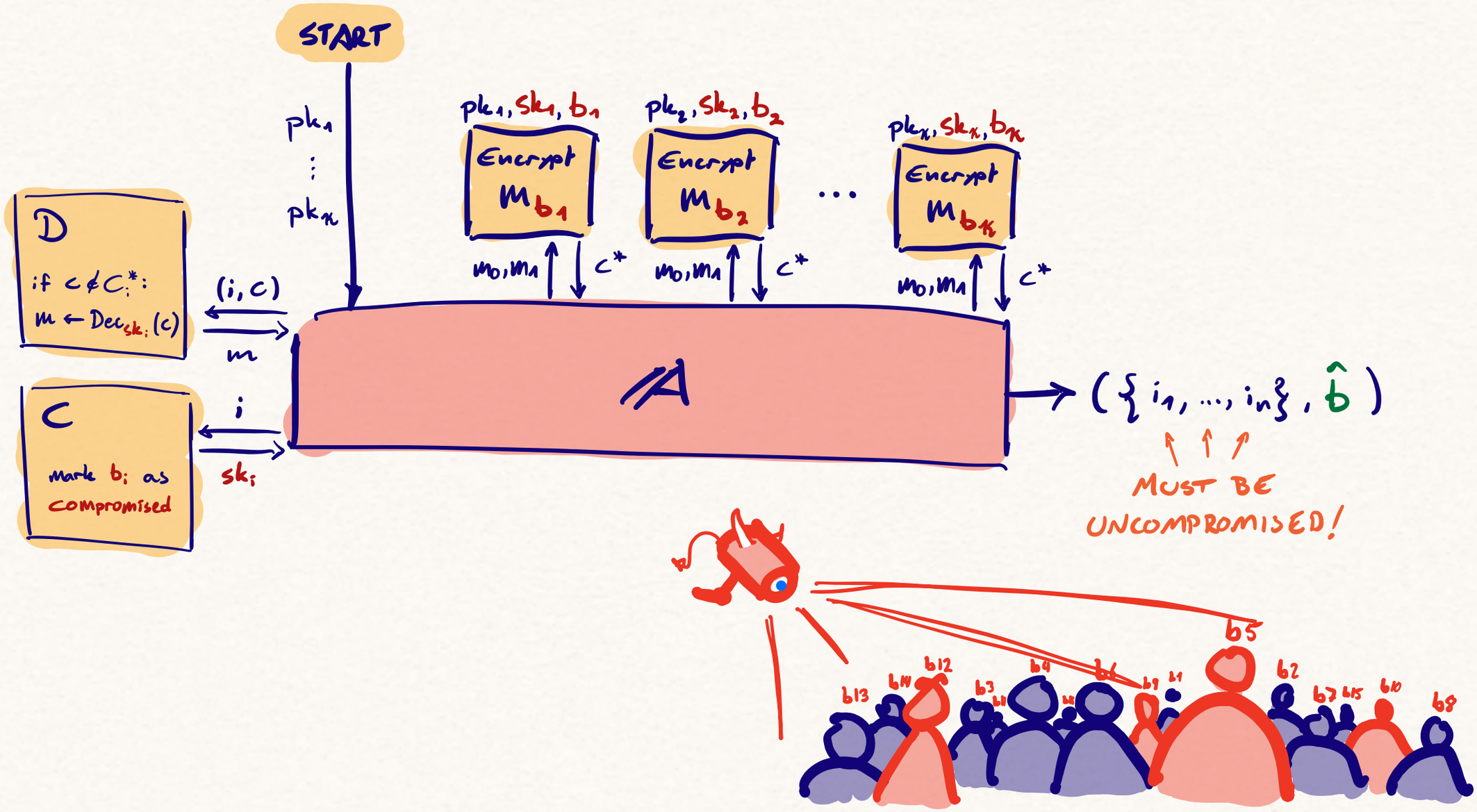

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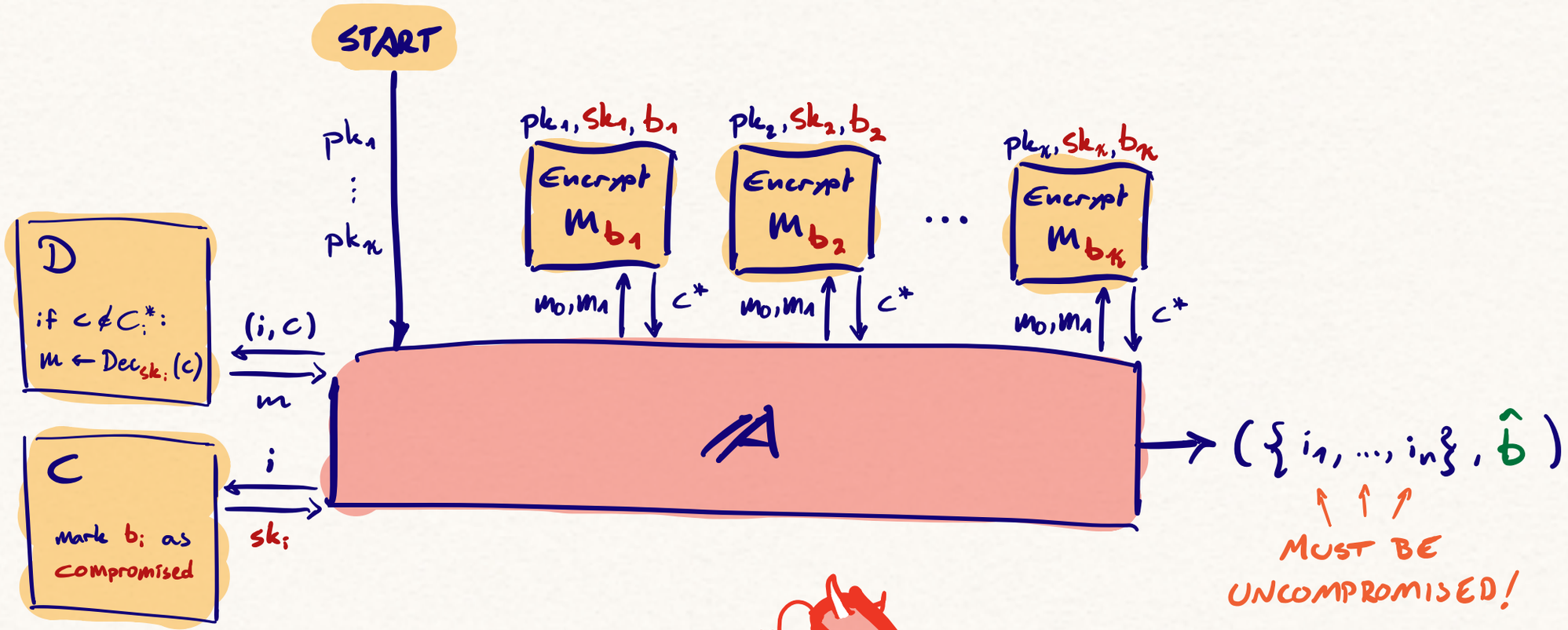
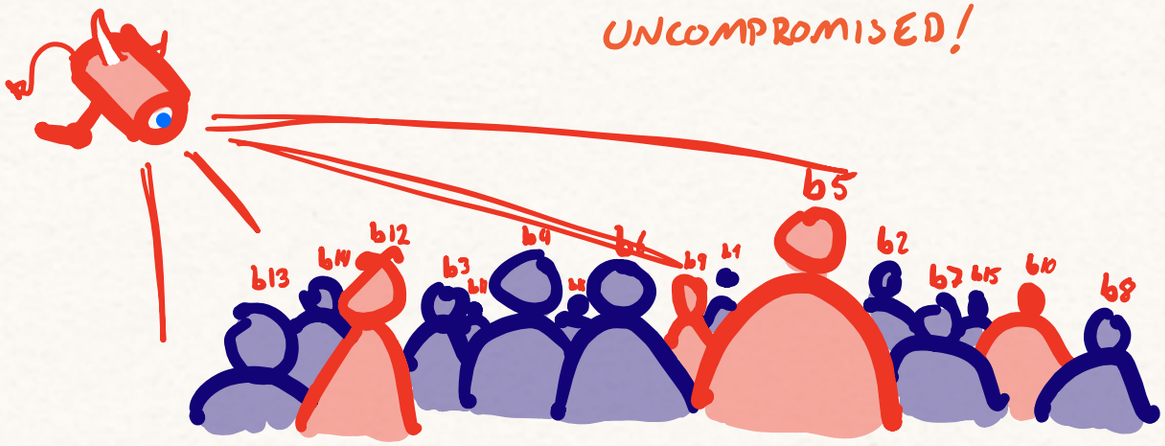


WW IF: $\hat{b} = b_{i_1} \oplus b_{i_2} \oplus \dots \oplus b_{i_n}$



Hybrid Encryption

e.g. AES

Cramer-Shoup (2003): KEM + DEM $\Rightarrow$ PKE

IND-CCA OT-IND-CCA IND-CCA

(almost) tightly!

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AGK (EC'20): KEM with good MI-security

pre-computation equivalent to brute-forcing n users $\Rightarrow$ breaks at most n users.

Hybrid Encryption

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e.g. AES

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PRE-COMPUTATION EQUIVALENT TO BRUTE-FORCING n USERS ⇒ BREAKS AT MOST n USERS.

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(almost) tightly!

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PRE-COMPUTATION EQUIVALENT TO BRUTE-FORCING n USERS ⇒ BREAKS AT MOST n USERS.

PKE with good MI-security?

Nope!

Initial Attempts

1. KEM + DEM $\Rightarrow$ PKE ?

MI-IND-CCA OT-INF.TH.-CCA MI-IND-CCA

Initial Attempts

1. KEM + DEM $\not\Rightarrow$ PKE

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Initial Attempts

1. KEM + DEM $\not\Rightarrow$ PKE

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   MI-IND-CCA OT-INF.TH.-CPA MI-IND-CCA

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↳ MI-IND-CCA ✗

↳ MI-ROR-CCA ✓

Single-instance: $\text{ROR-CCA}_{\text{PKE}} \xrightarrow{\cdot 2} \text{IND-CCA}_{\text{PKE}}$

Multi-instance:

Initial Attempts

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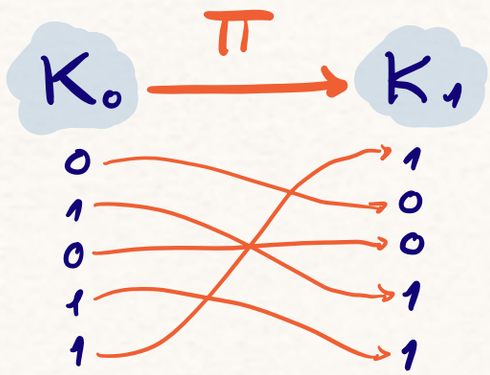
NEED A MORE DIRECT ROUTE ...

NEW KEM NOTION: REAL-OR-PERMUTED



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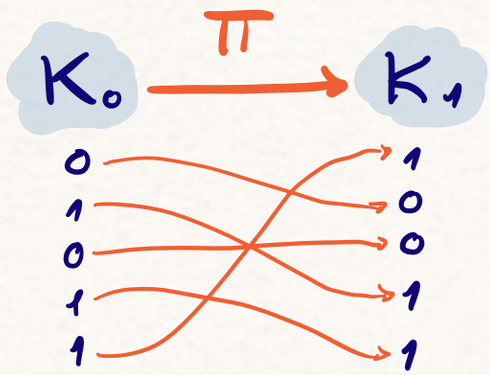
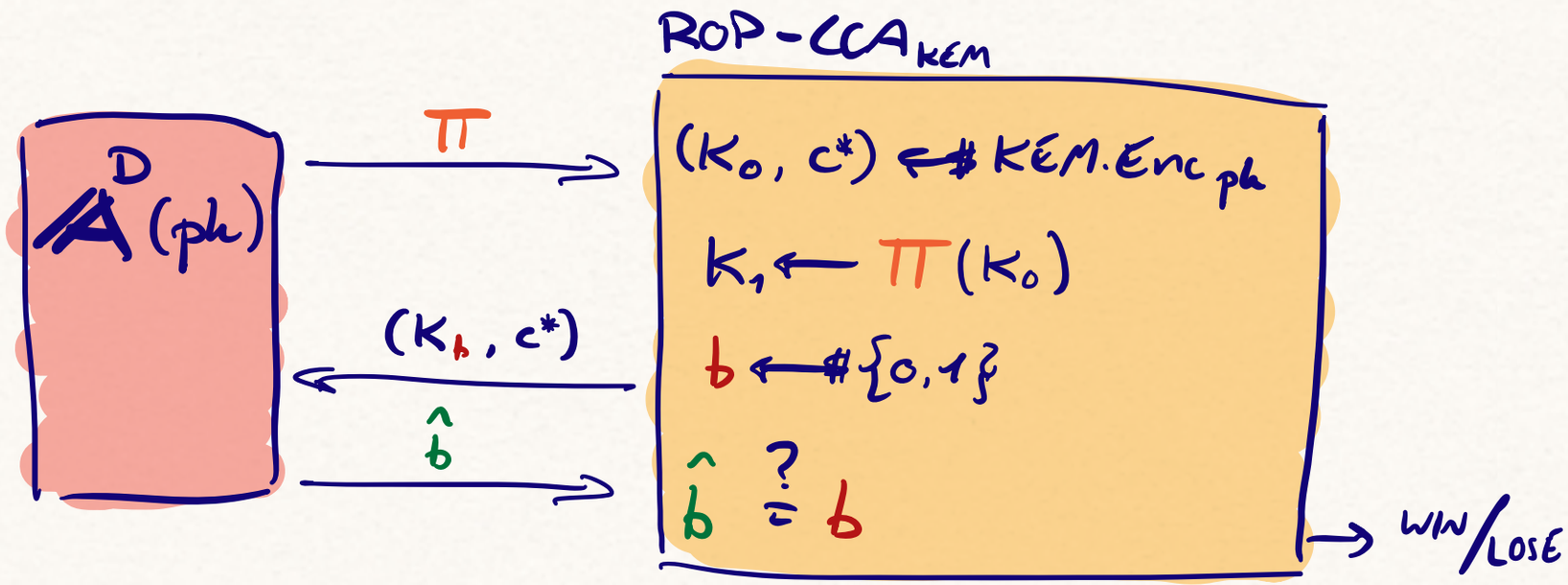
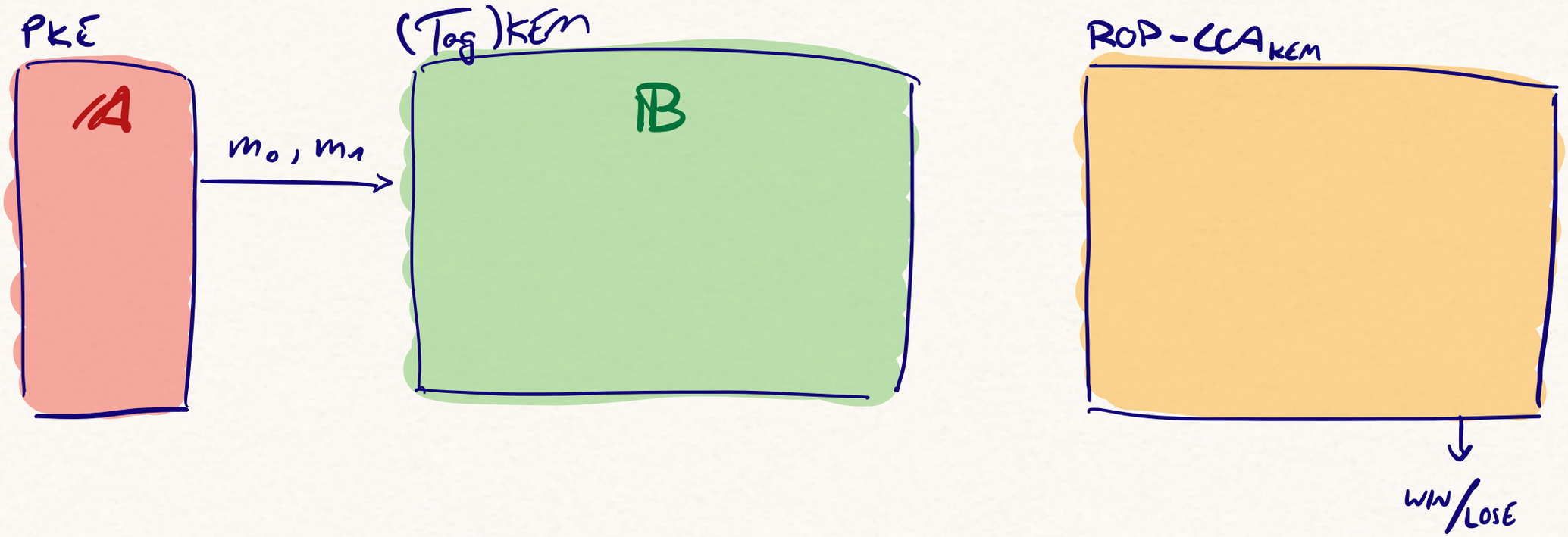




Simulating Left-or-Right IND-CCA



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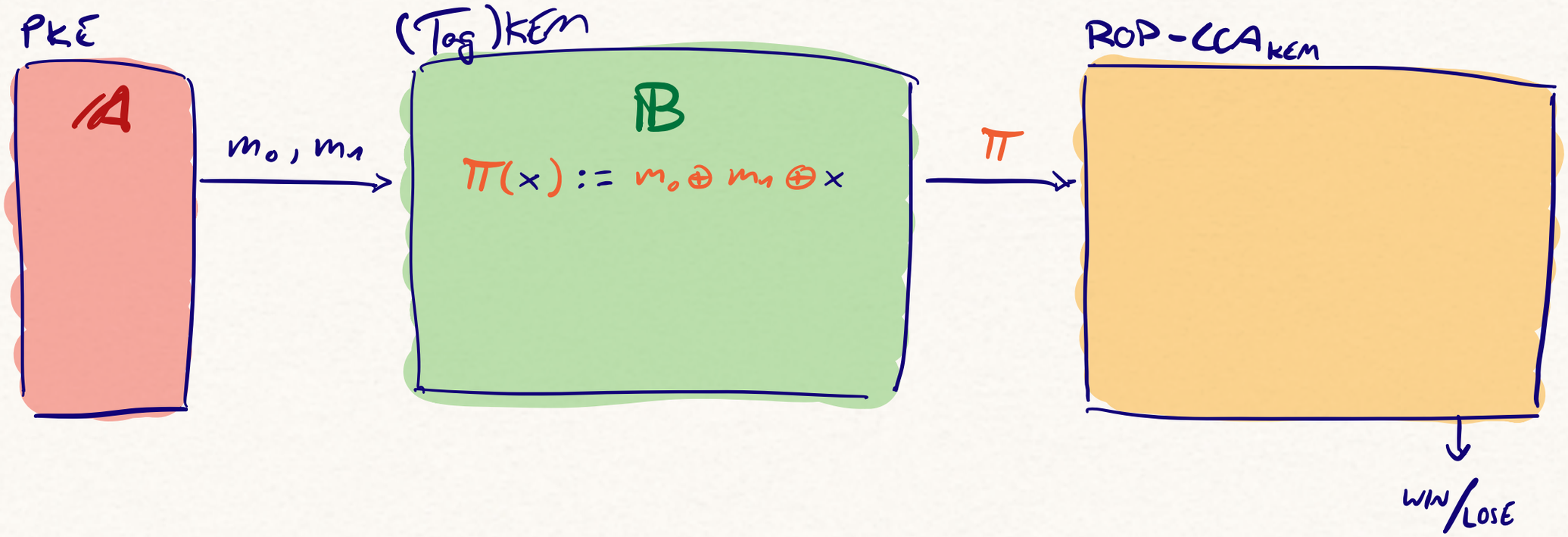

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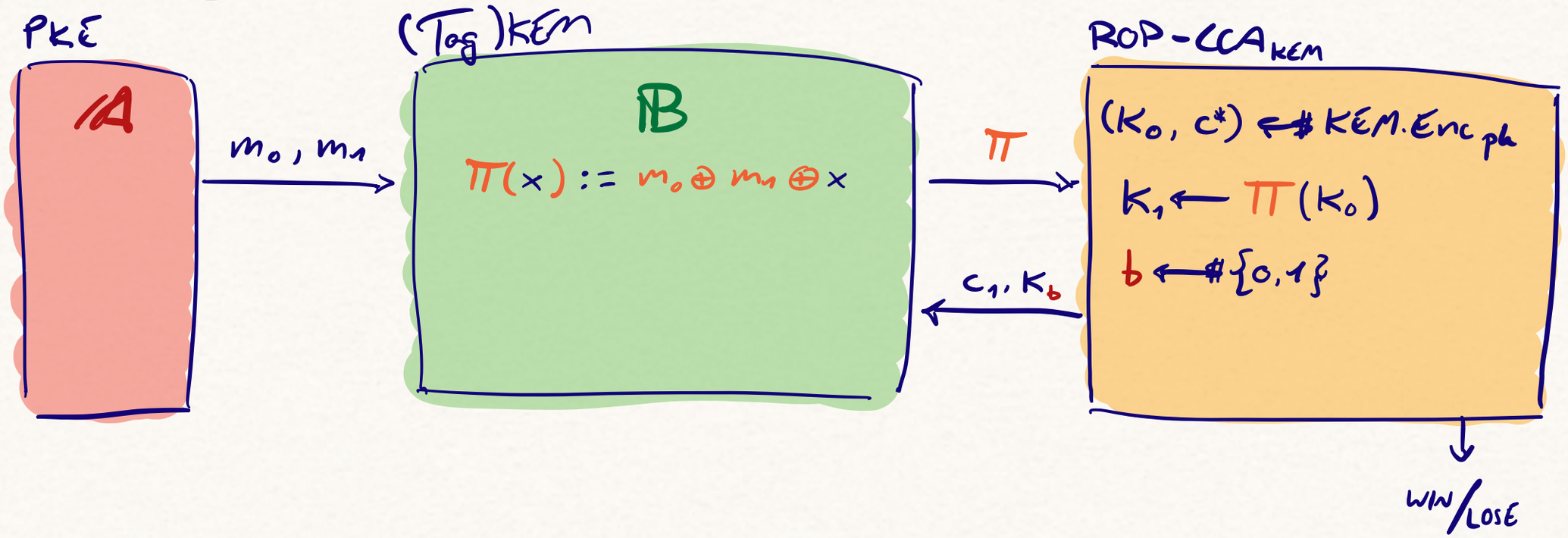

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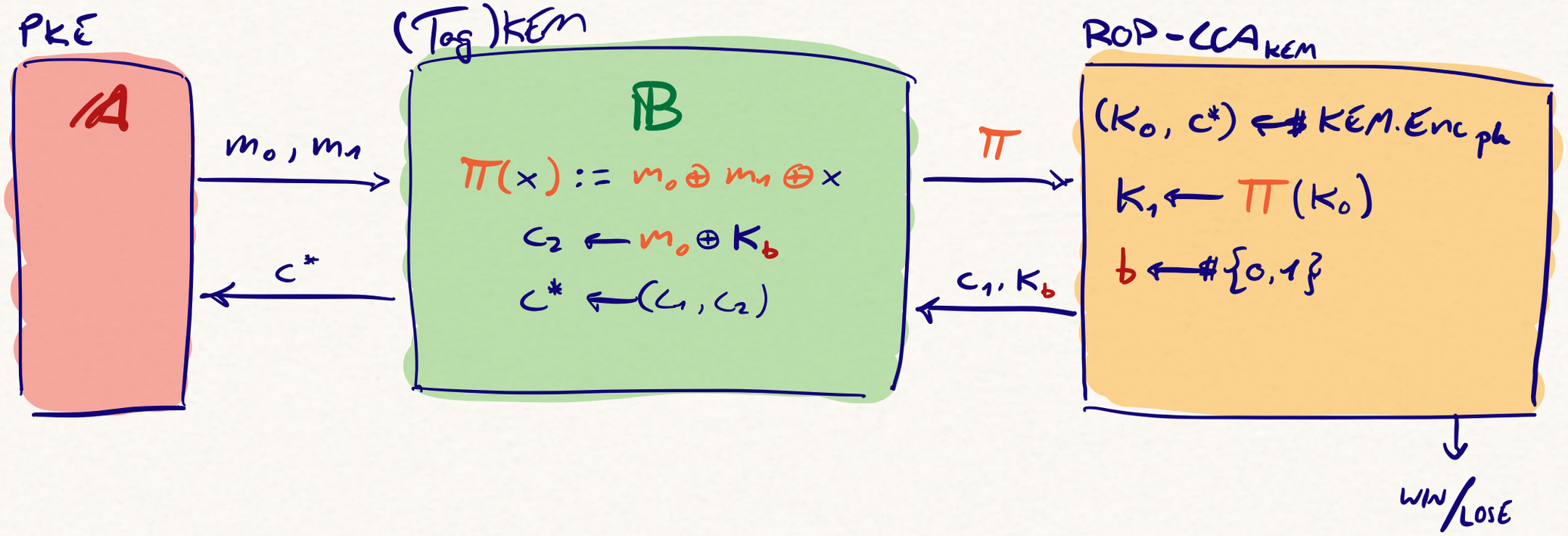

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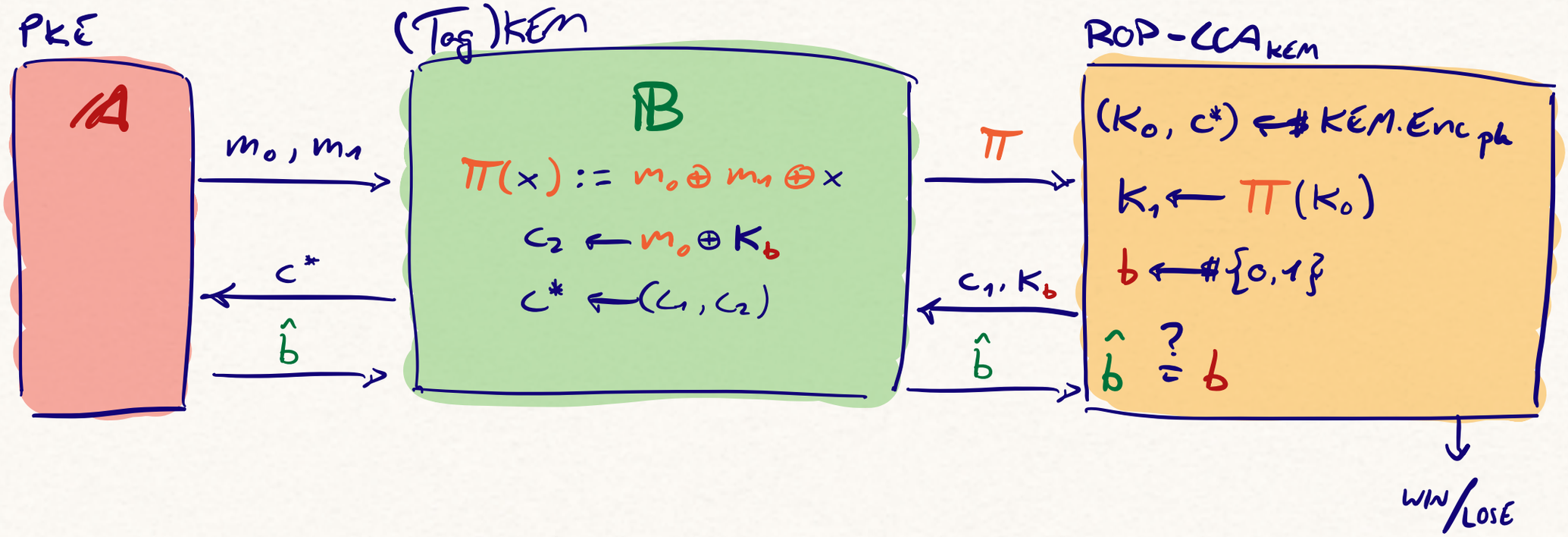

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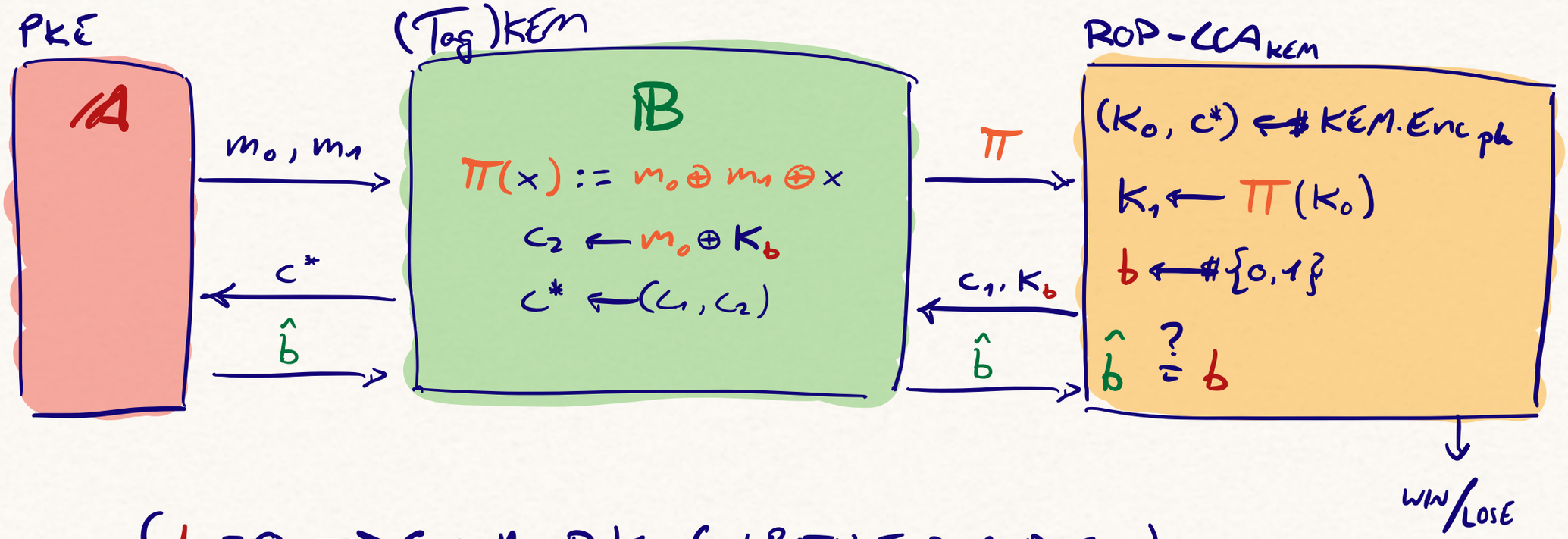
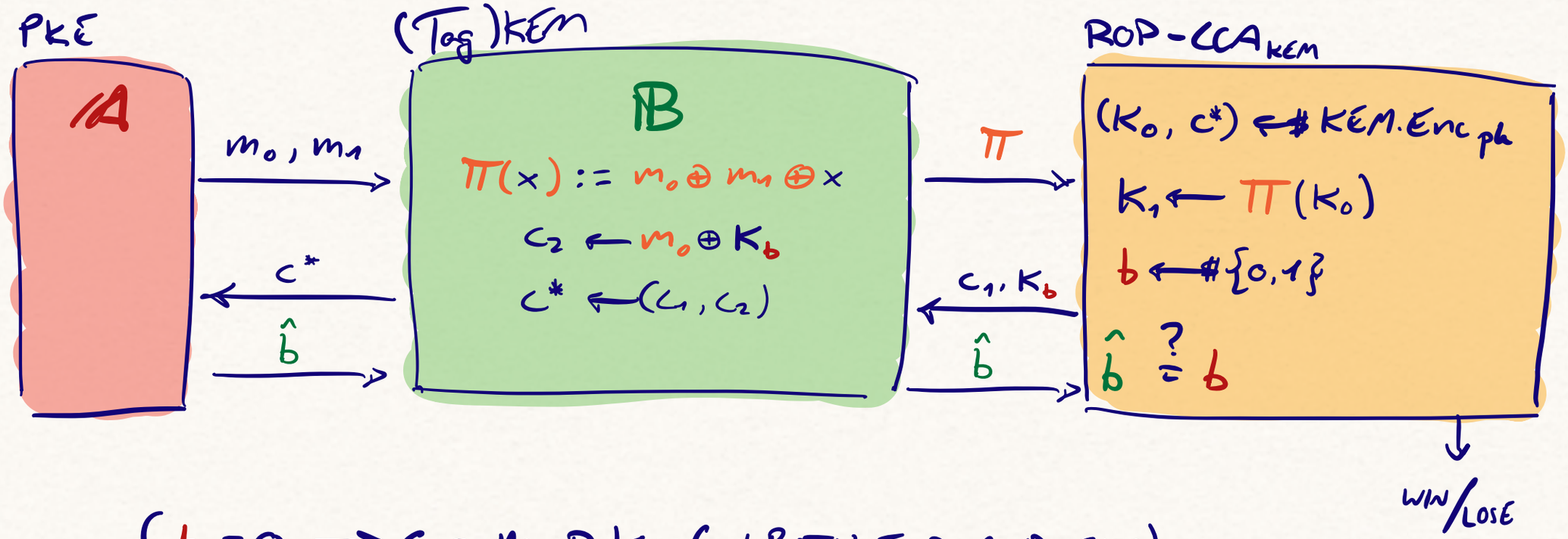


$\Longrightarrow$ $\left\{ b = 0 \longrightarrow c_2 = m_0 \oplus K_0 \text{ (IN BOTH EXP. AND SIM)} \right.$

Simulating Left-or-Right IND-CCA



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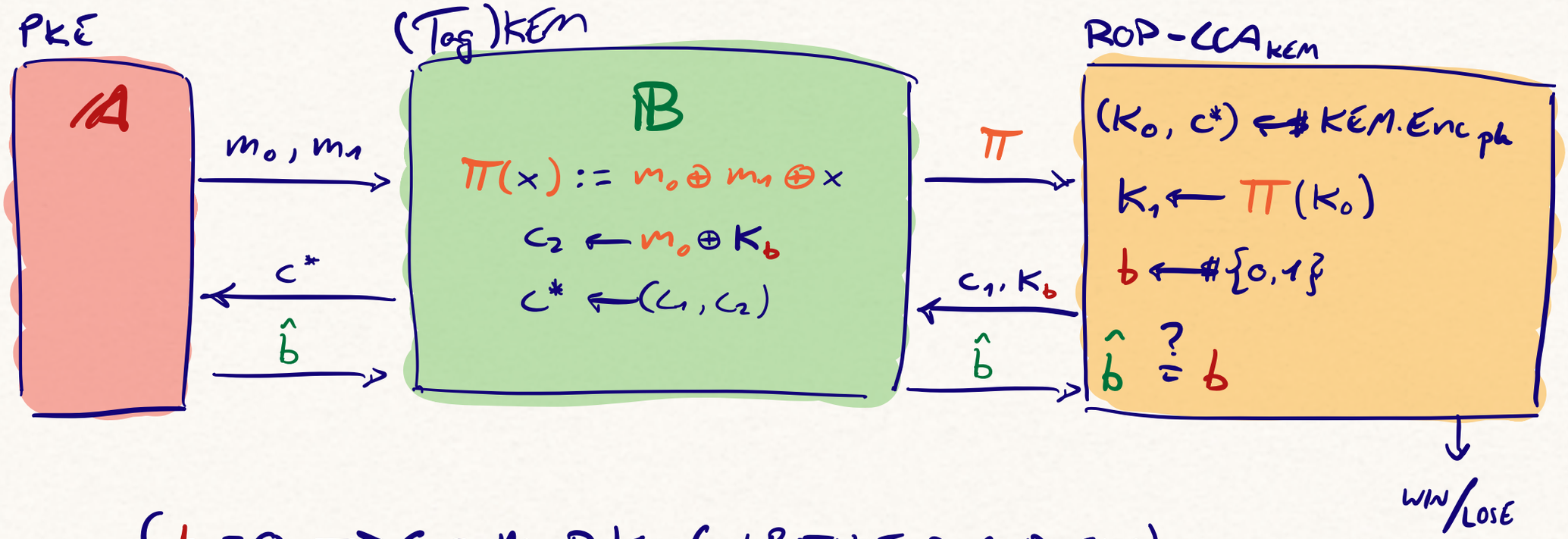

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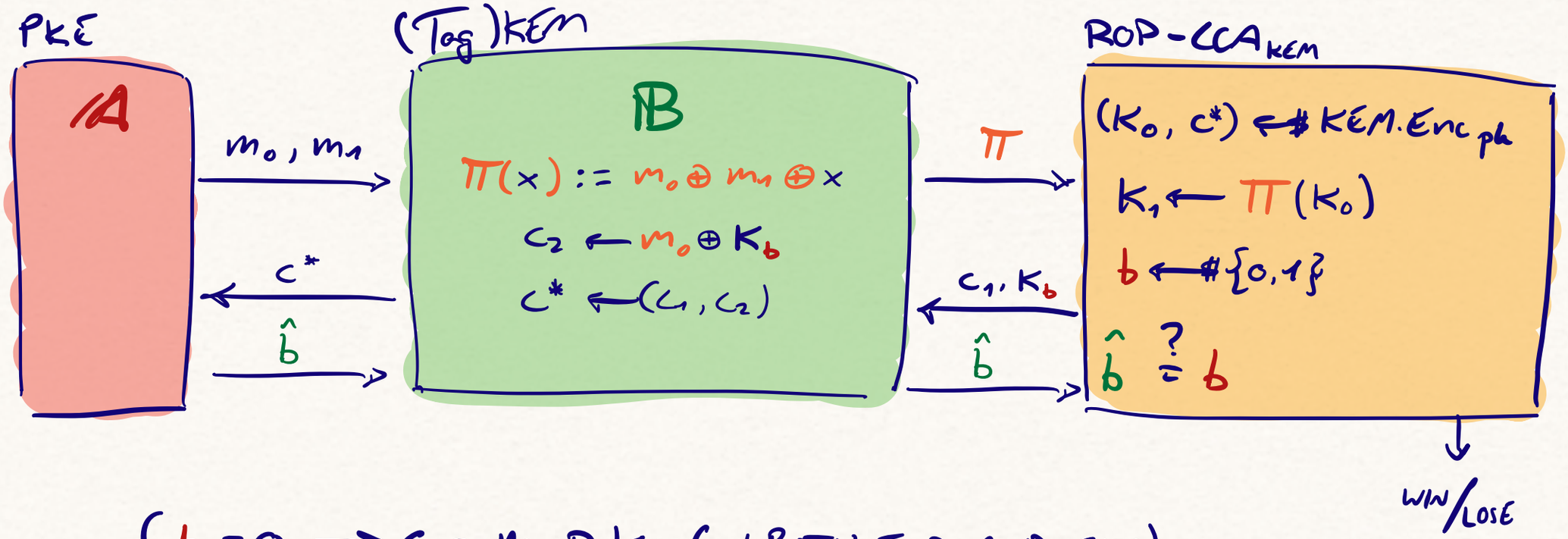

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3rd Attempt

3. TagKEM + OTP ⇒ PKE ✓

MI-ROP-CCA OT-INF.TH.-CPA MI-IND-CCA

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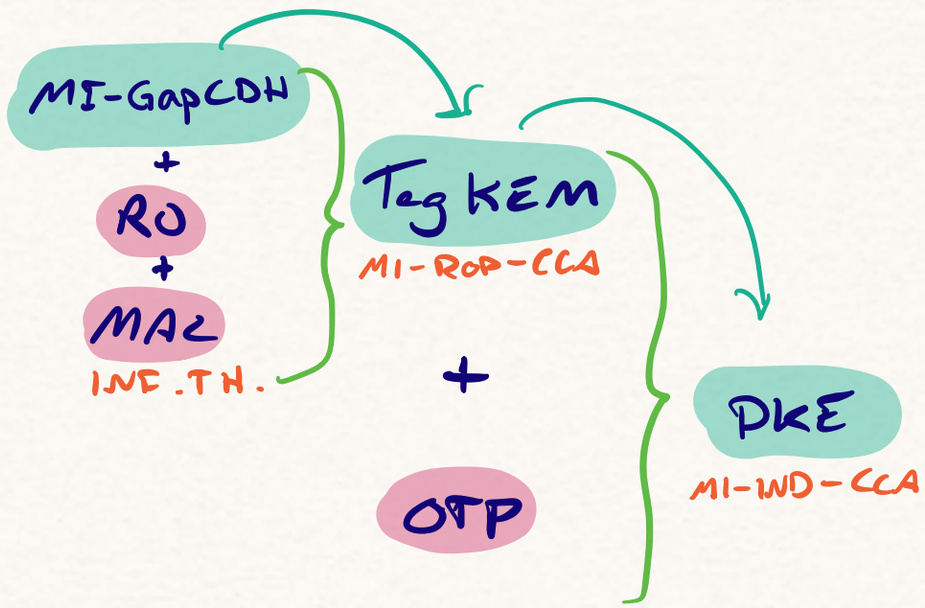

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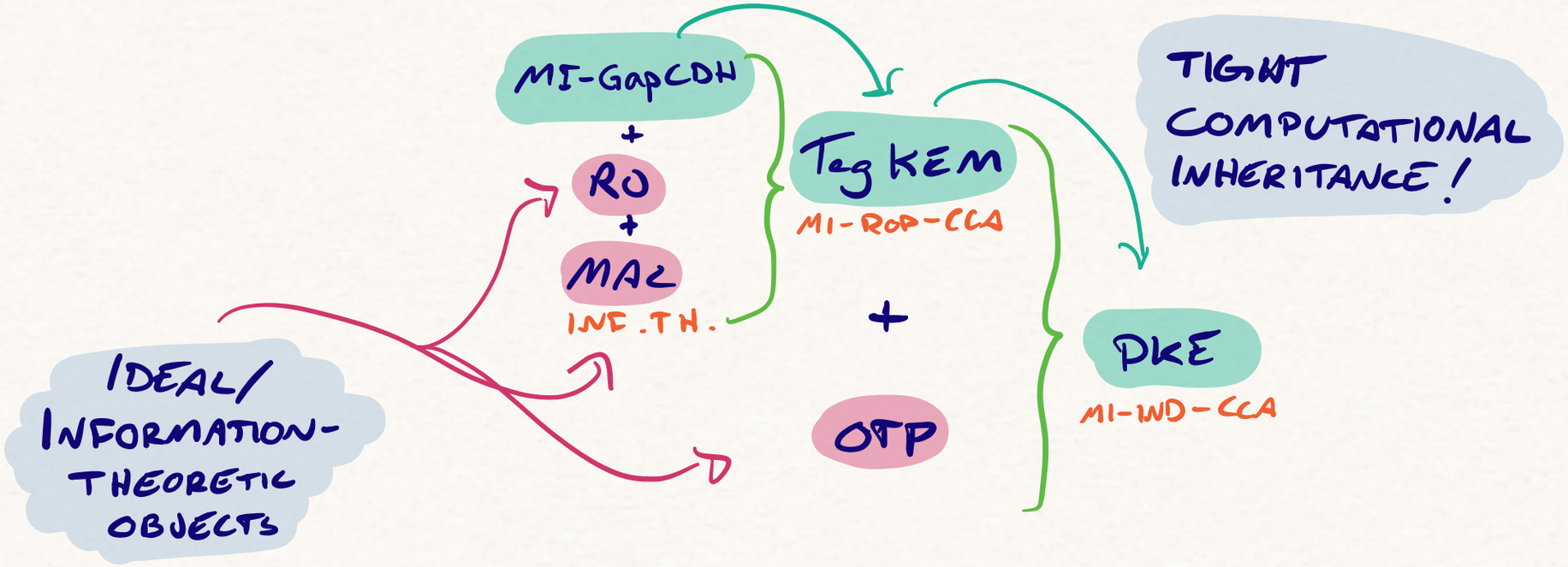

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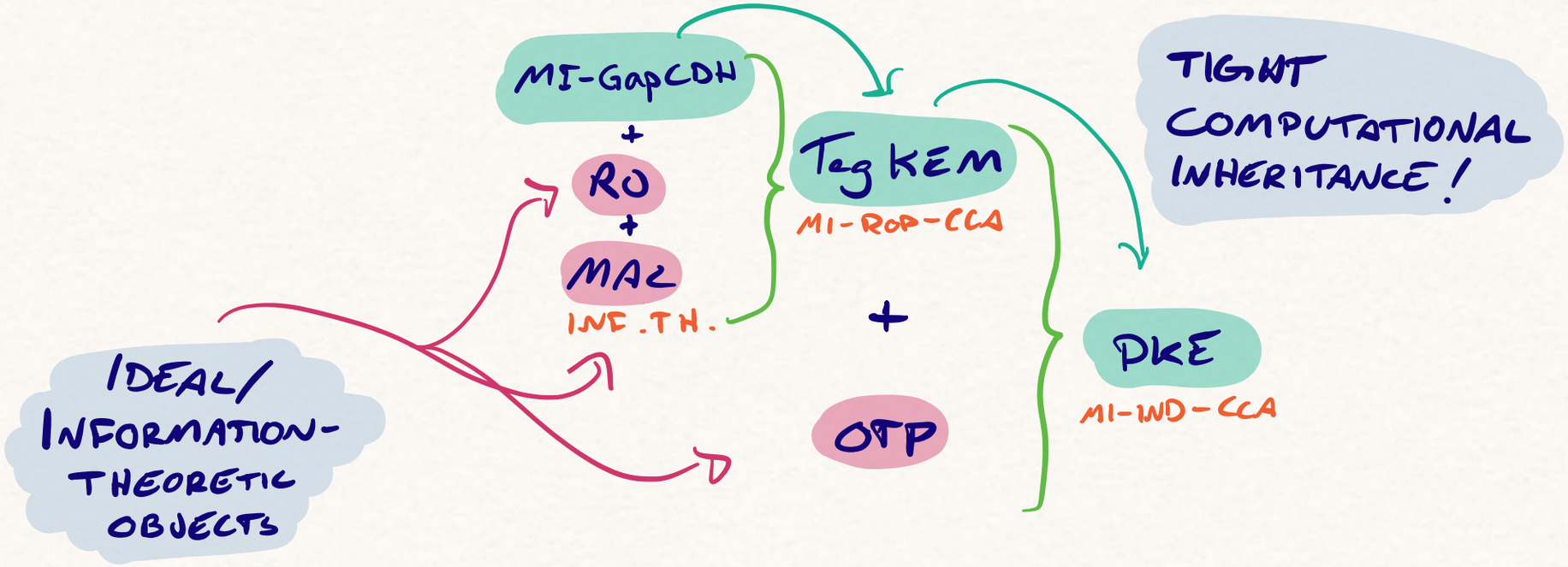

MAIN RESULT:

$$\mathrm{Adv}^{\text{MI-IND-CCA}}_{\mathrm{PKE}}(\mathbb{A}) \leq \mathrm{Adv}^{\text{MI-GapCDH}}(\mathbb{B}) + (\text{COLLISION TERM}) + (\text{MAC FORGERY})$$

CONCLUSION

- TIGHT MI-INHERITANCE FROM KEM TO PKE

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- COROLLARY: PKE WITH OPTIMAL MULTI-INSTANCE RESISTANCE

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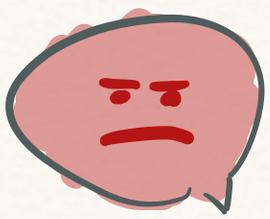
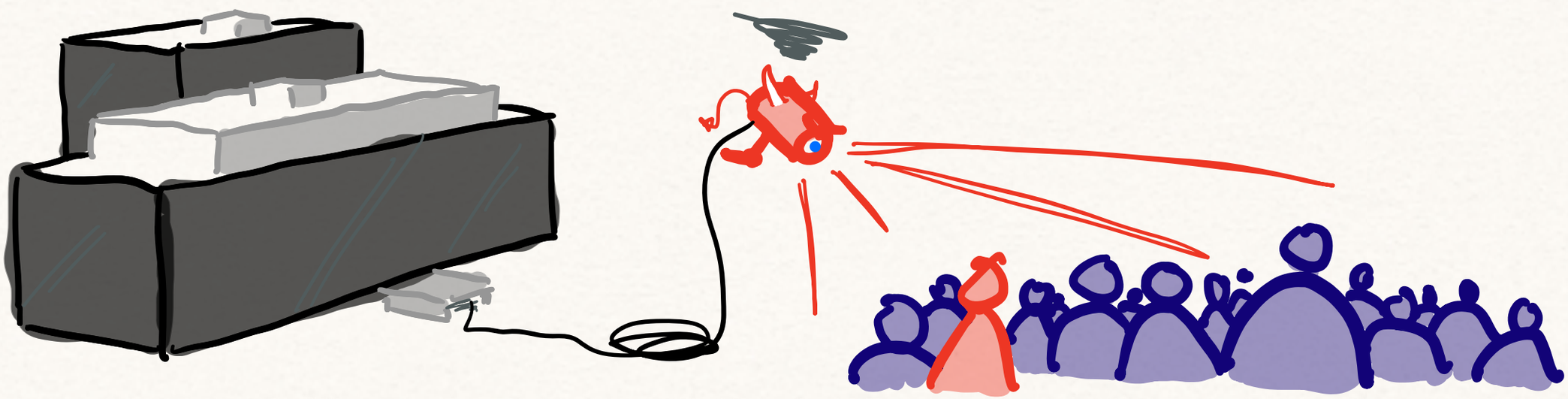
ALSO IN PAPER:

- NOVEL NOTION OF IMPERFECT CORRECTNESS
- NOVEL CCA-NOTION FOR IMPERFECT CORRECTNESS
- NOVEL NOTION OF MI KEY RECOVERY
- RELATIONS BETWEEN ALL RELEVANT NOTIONS
- EXTENDABLE-OUTPUT KEM/TAGKEM: "XEM/TAGXEM"
- (TAG)KEM + XOF = (TAG)XEM
- MORE CONSTRUCTIONS (CPA ONLY AND/OR MI-ROR ONLY)
- NOVEL NOTION OF MI-GAPCDH WITH CORRUPTIONS

Thank You!





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