

Tracing a Linear Subspace

Application to Linearly-Homomorphic Group Signatures

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Outline

- 1 Linearly-Homomorphic Group Signatures
 - Definition
 - Construction
- 2 A Core Technique: Tracing Linear Subspaces
 - Trivial Solution
 - Improved Efficiency via Code-Based Construction

Linearly-Homomorphic Group Signatures

Linearly-Homomorphic

Group Signatures

$\text{KeyGen}(1^\lambda) \rightarrow (\text{sk}, \text{pk})$

$\text{Sign}(\text{sk}, \vec{m}_i) \rightarrow \sigma_i$

$\text{Derive}(\text{pk}, (\omega_i, \vec{m}_i, \sigma_i)_i) \rightarrow \sigma$

$\text{Verify}(\text{pk}, \vec{m}, \sigma) \rightarrow 0 / 1$

Linearly-Homomorphic

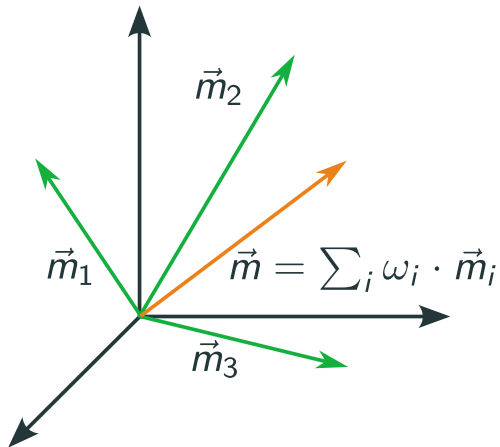
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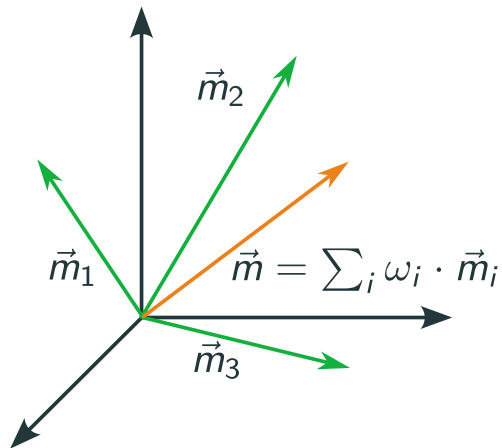
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Security. EUF-CMA without trivial attacks



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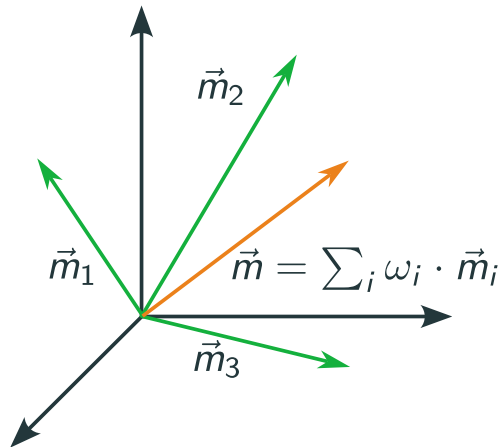
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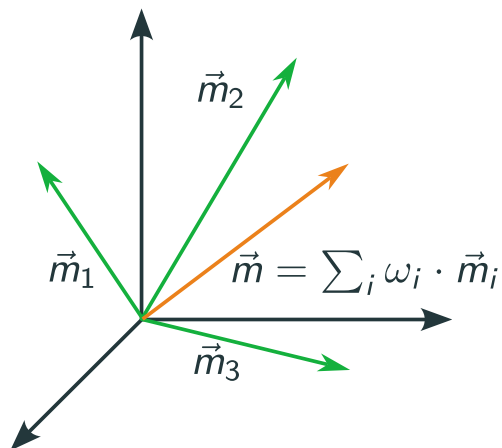
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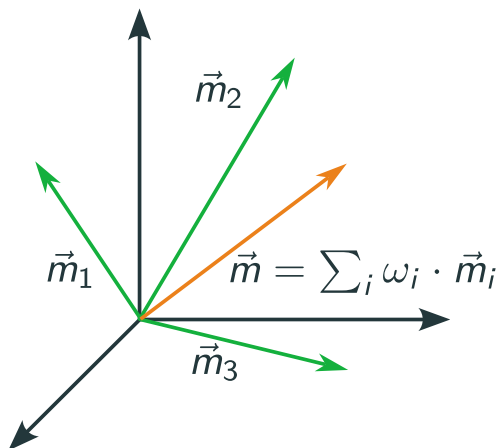
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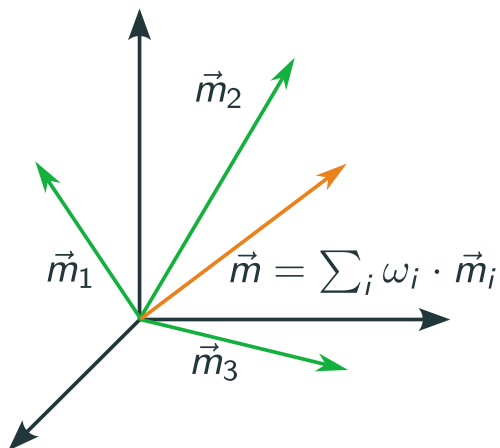
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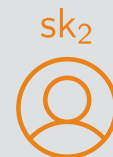
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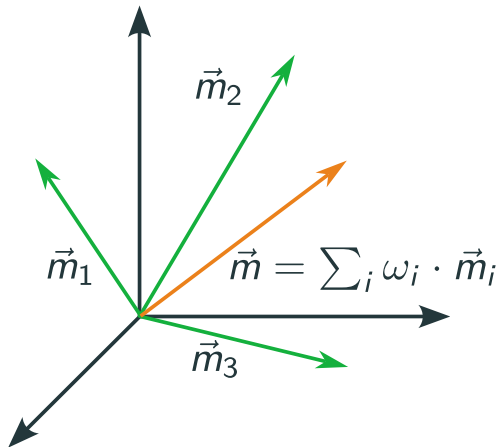
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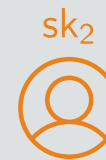
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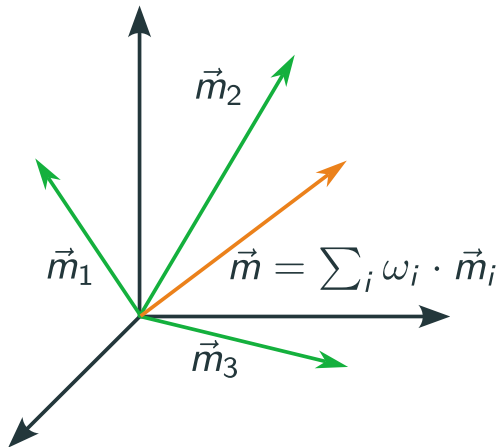
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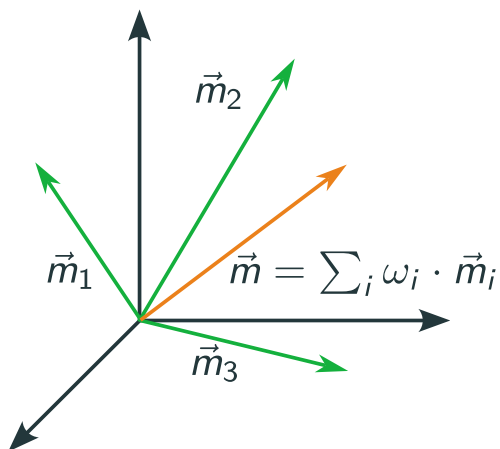
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Anonymity.



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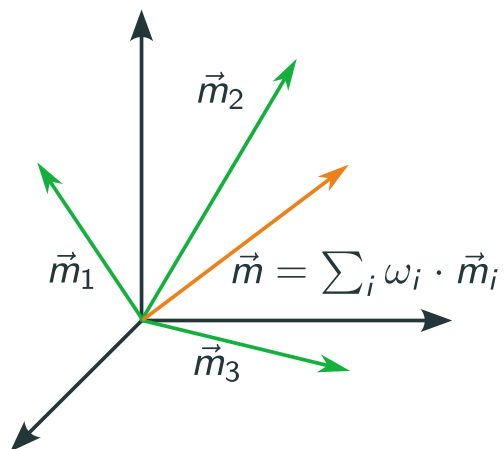
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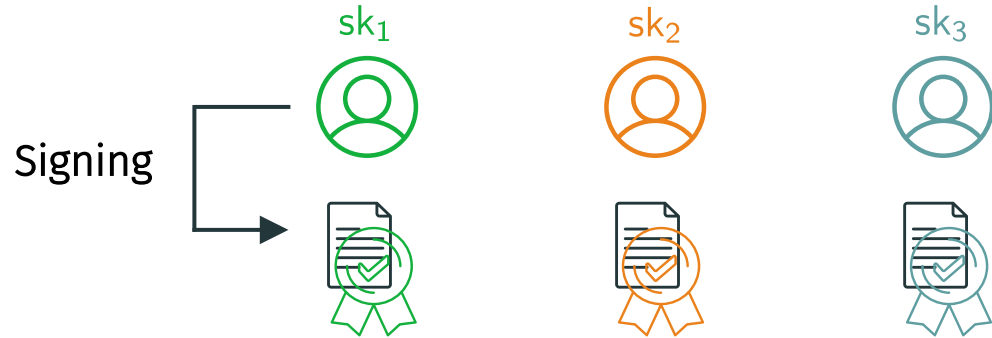
Traceability.



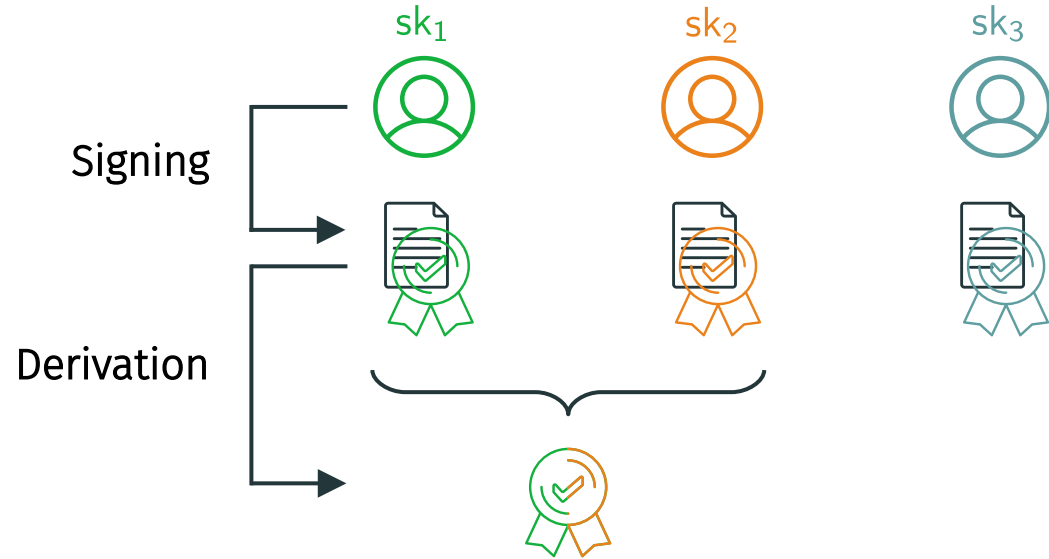
What Makes the Design Non-Trivial?



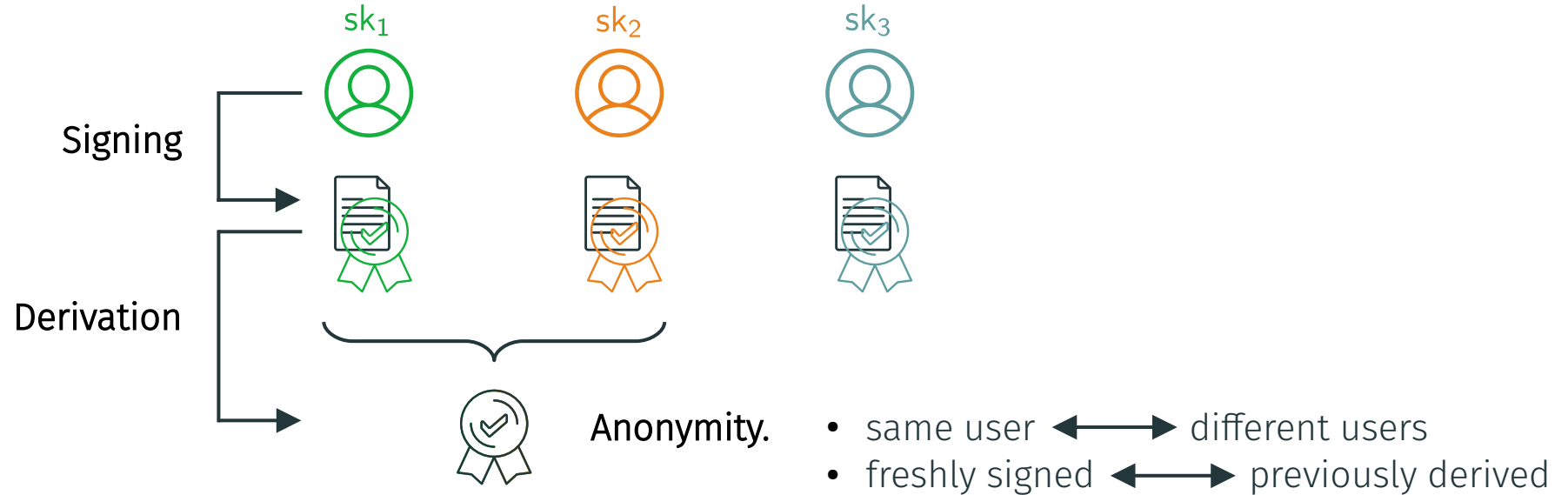
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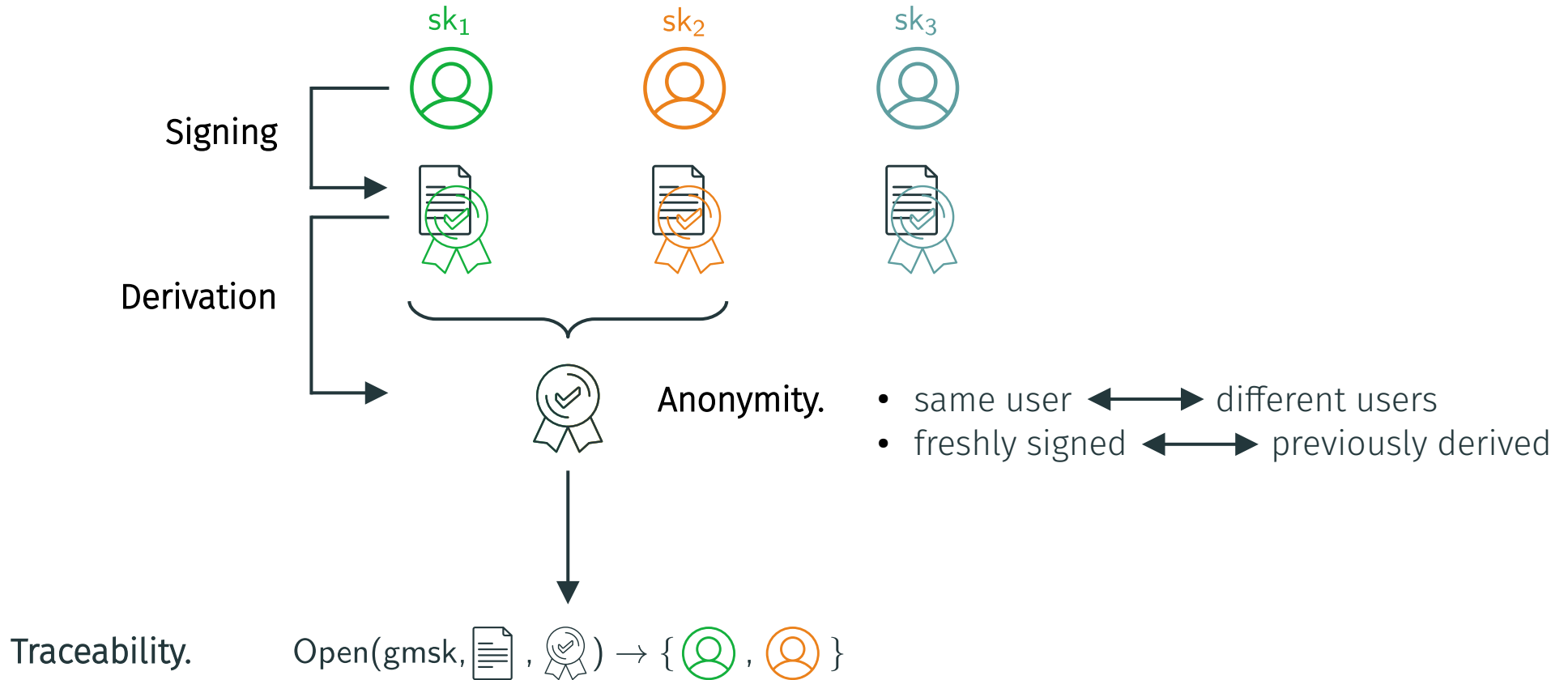
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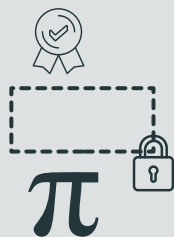


A Framework for Group Signatures [BMW03]

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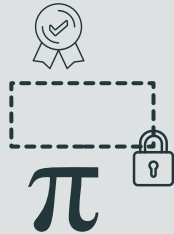
- (traditional) signature
- public-key encryption
- NIZK for NP relations



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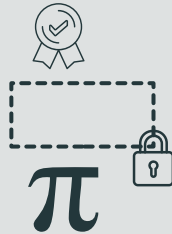
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A Framework for Group Signatures [BMW03]

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$$sk_1 = (\text{signature icon}, \text{person icon with signature})$$



$$sk_2 = (\text{signature icon}, \text{person icon with signature})$$



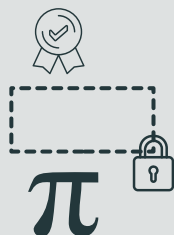
$$sk_3 = (\text{signature icon}, \text{person icon with signature})$$



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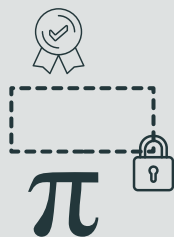


$$gmsk = (\text{lock}, \text{key})$$

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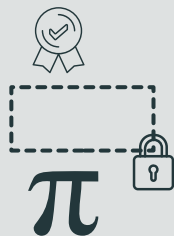
$$gmsk = (\text{lock}, \text{key})$$

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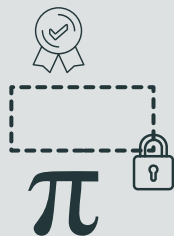
$$G\text{Sign}(sk_1, \text{document icon})$$



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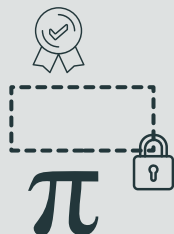
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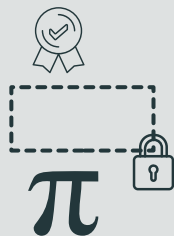
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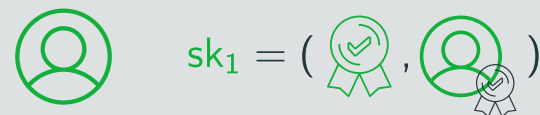
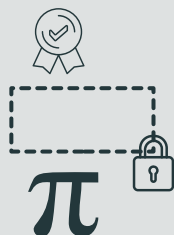
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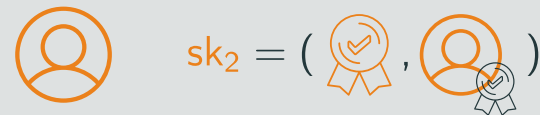
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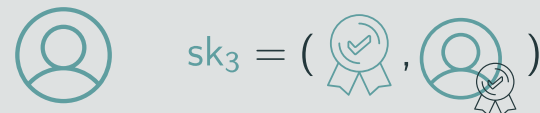
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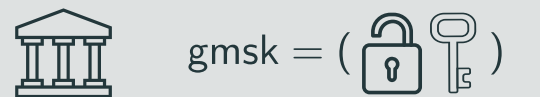
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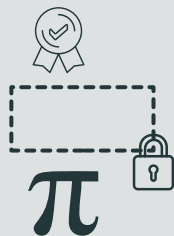


Opening. $\text{Open}(gmsk, \text{document}, (\text{document}, \text{signature}, \pi))$

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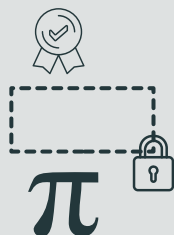
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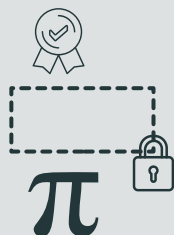
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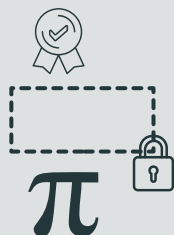
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How to Make This Linearly Homomorphic?



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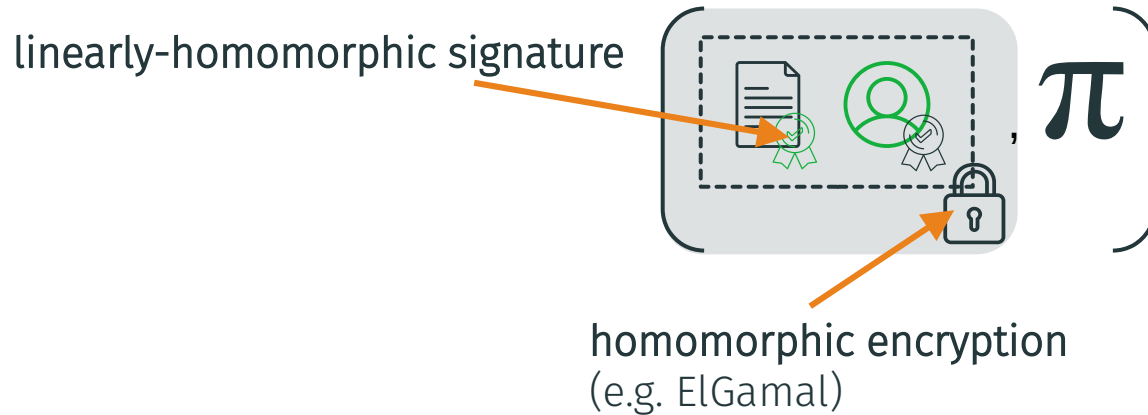


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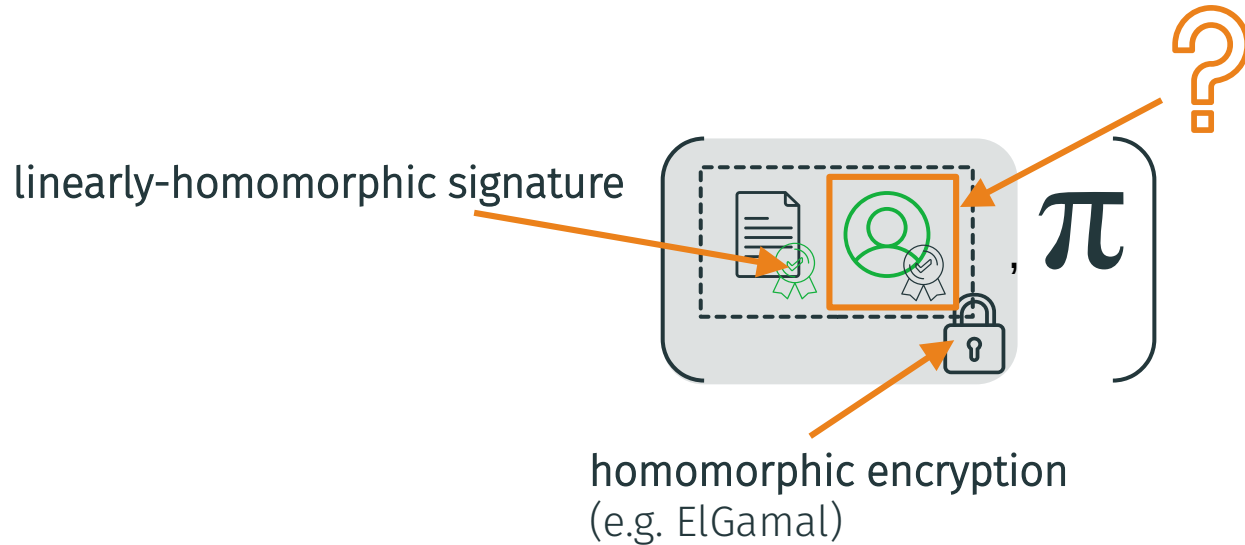


homomorphic encryption
(e.g. ElGamal)

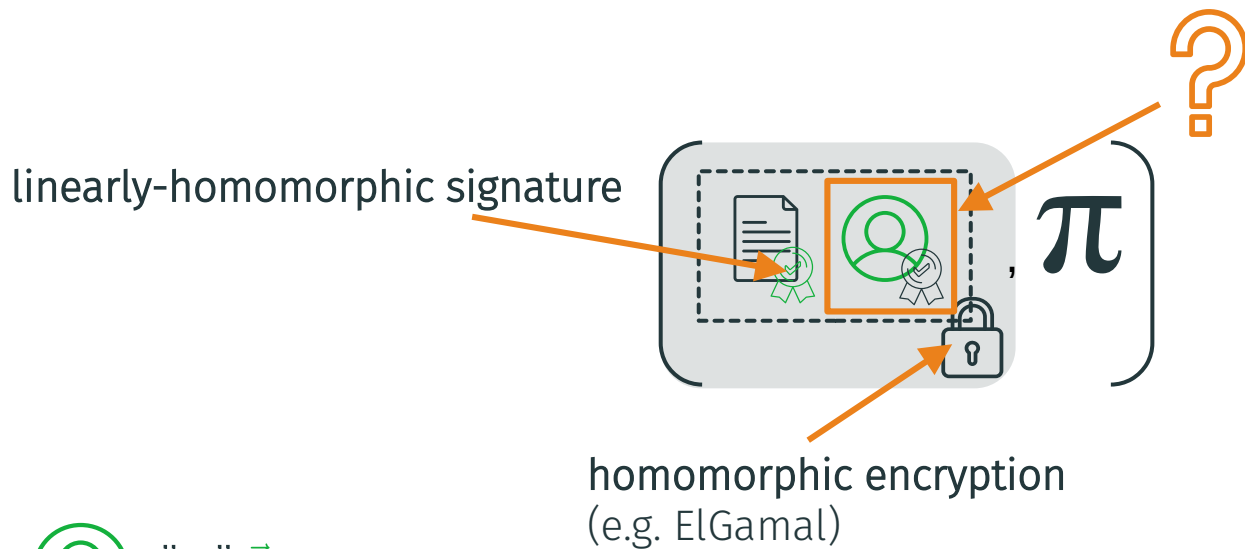
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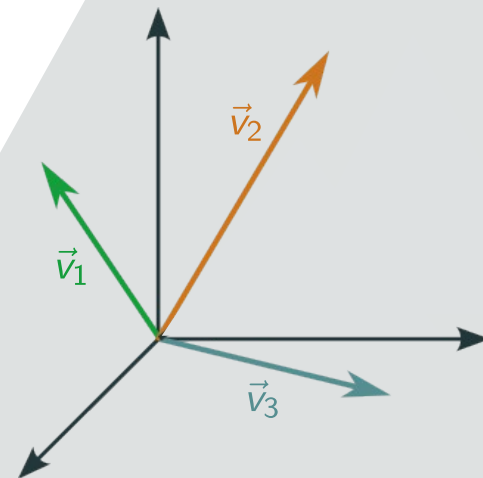
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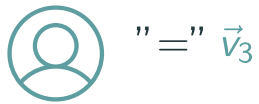
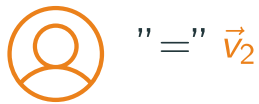
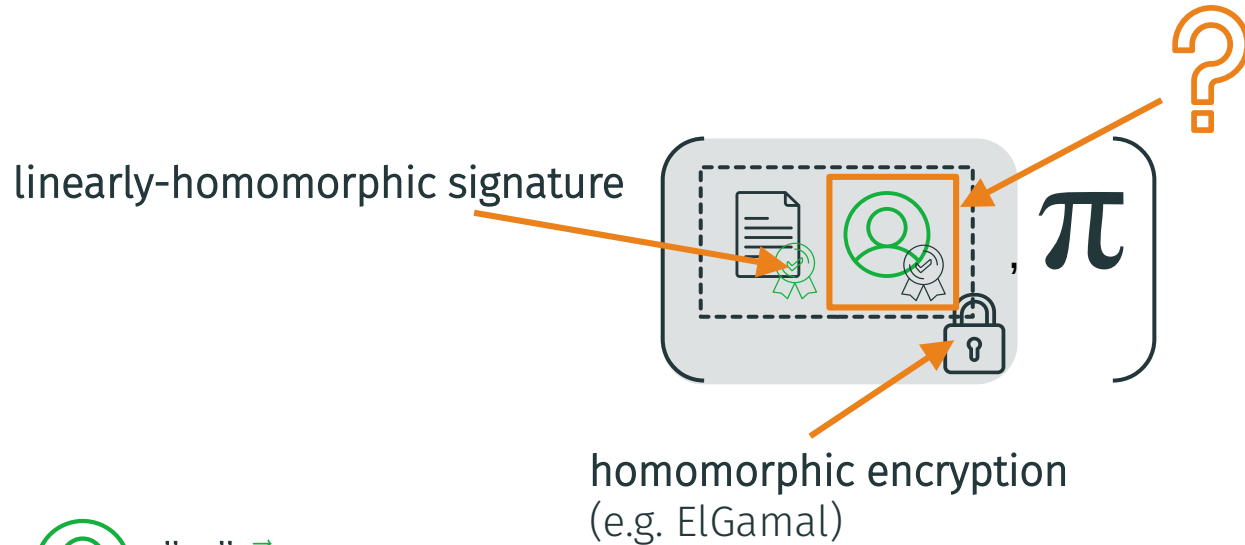
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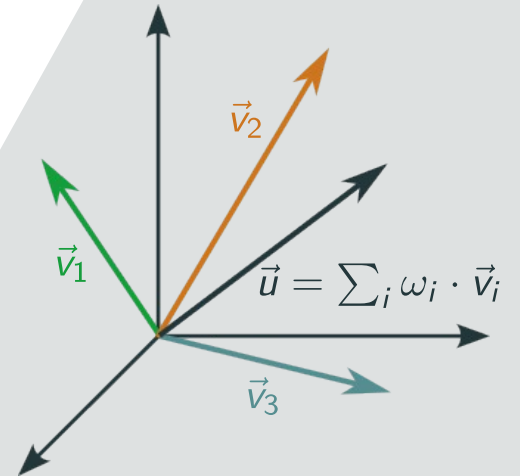


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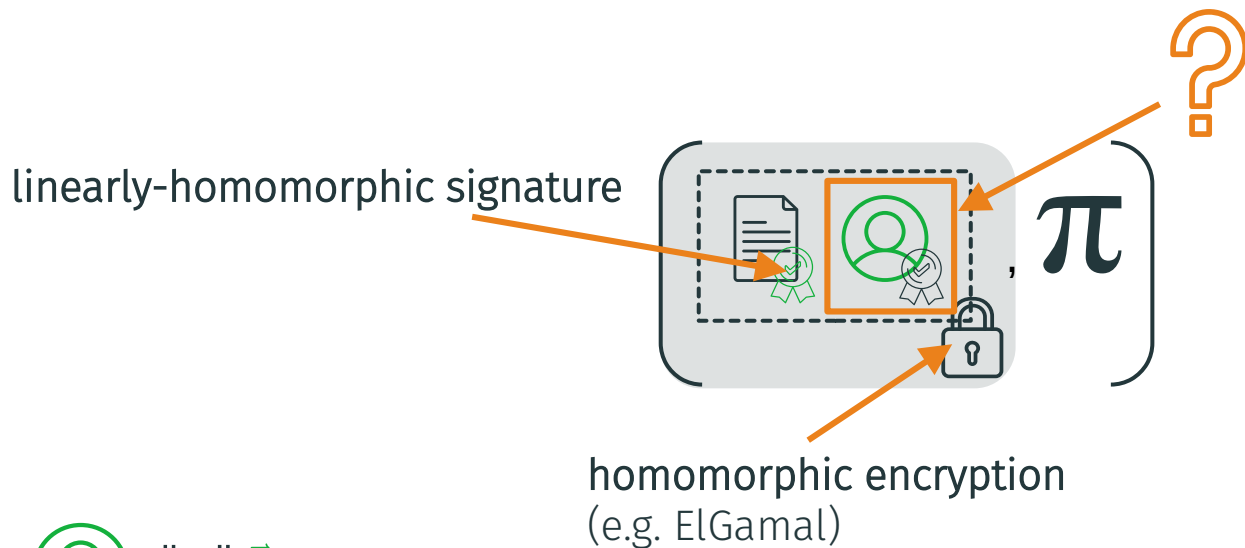


Signature derivation for  and .

$$\vec{u} = \omega_1 \cdot \vec{v}_1 + \omega_2 \cdot \vec{v}_2 + \omega_3 \cdot \vec{v}_3$$




How to Make This Linearly Homomorphic?



 " = " \vec{v}_1

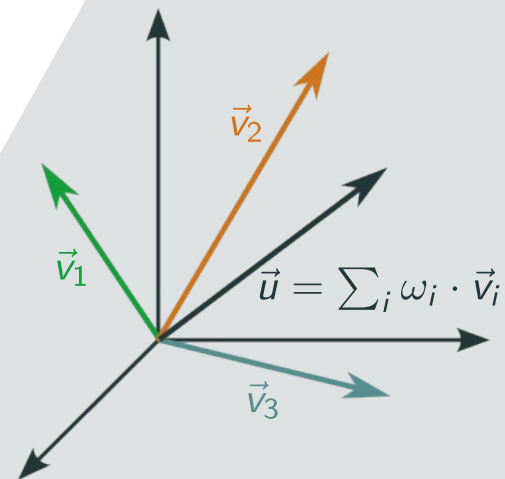
 " = " \vec{v}_2

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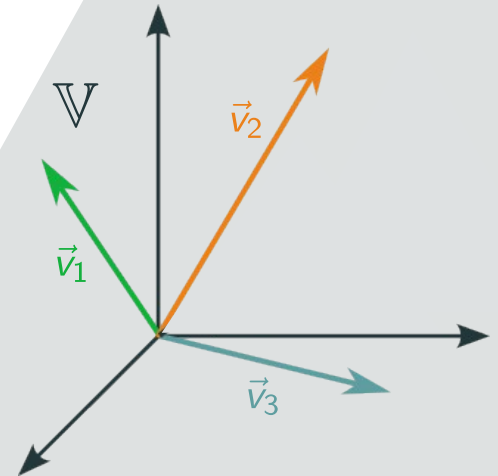
$\omega_1 = 0$



“Tracing” a Linear Subspace

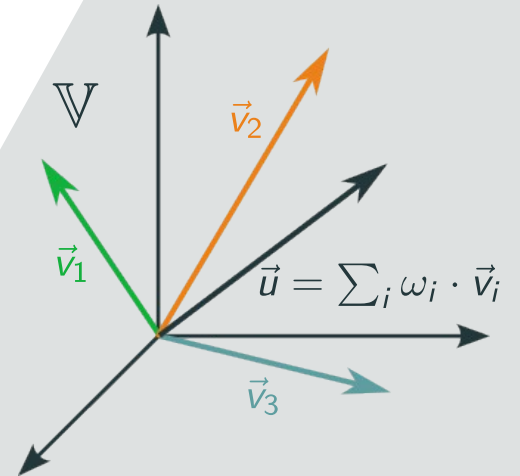
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Given: • basis $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ of vector space $\mathbb{V} = \text{span}(\mathcal{B})$



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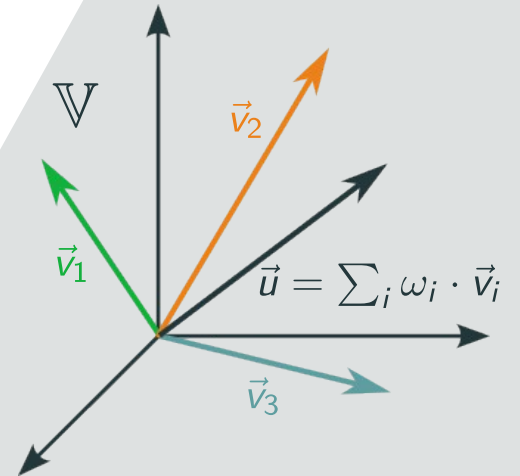
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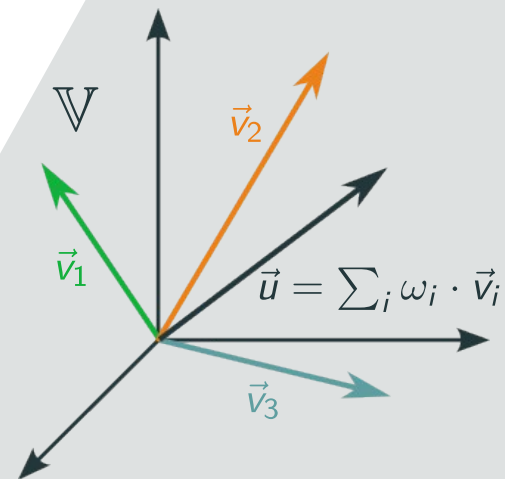
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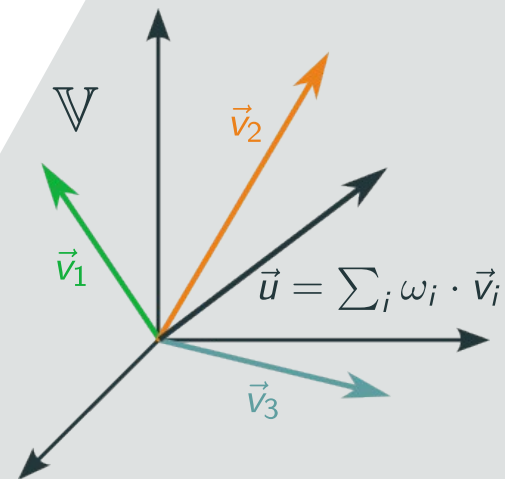
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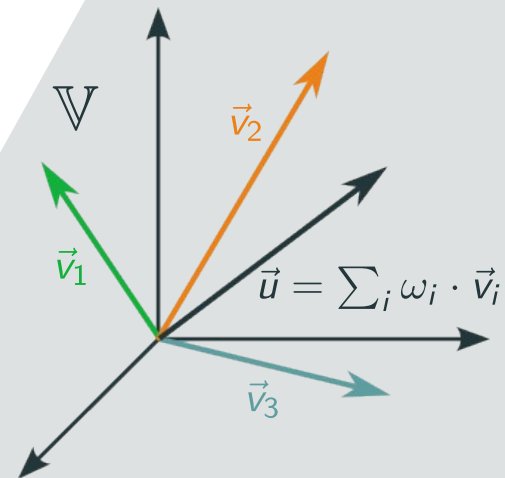
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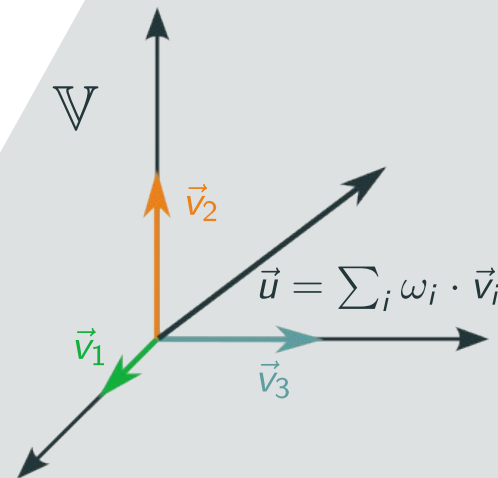
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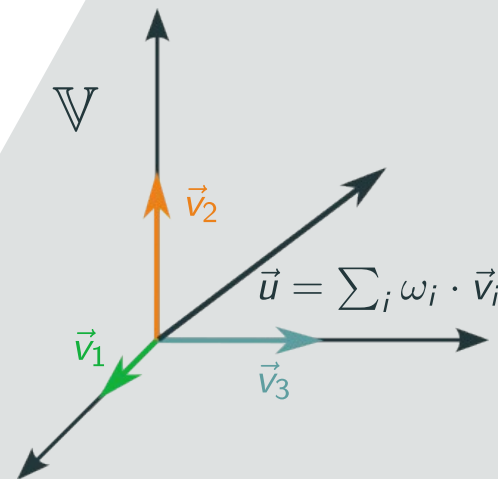
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Note: trivial solution is *optimal* but *inefficient*



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Relaxation of traceability: introduce an upper bound c on the maximum size of collusions

- never accuse honest user
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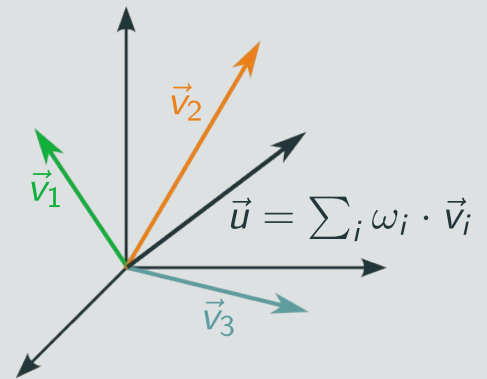
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What does that mean for the subspace-tracing problem?

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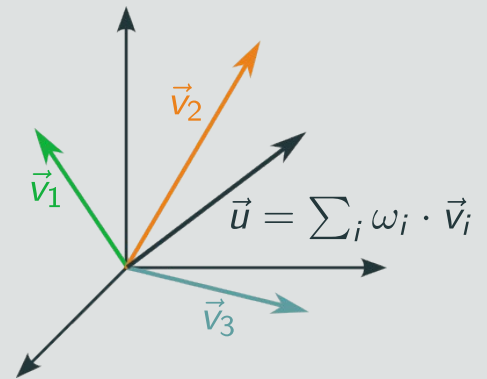
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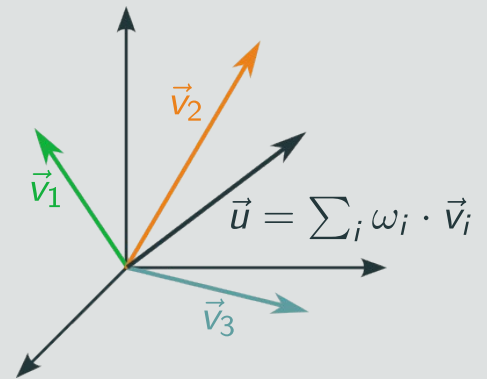
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Efficiency comparison:

- trivial solution $\mathcal{O}(n)$
- our work $\mathcal{O}(c^2 \cdot \log(n/\varepsilon))$ ($\varepsilon =$ maximum acceptable error probability)

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Analogy to IPP (and fingerprinting) codes:

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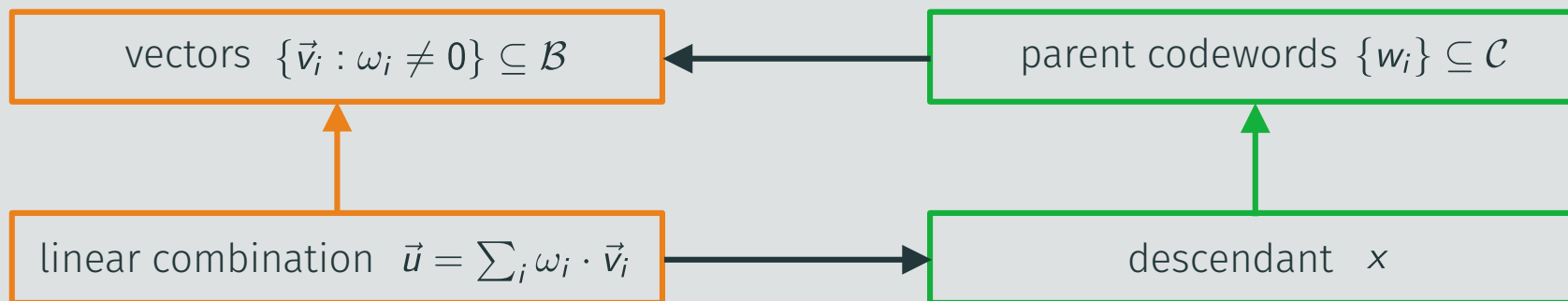
vectors $\{\vec{v}_i : \omega_i \neq 0\} \subseteq \mathcal{B}$

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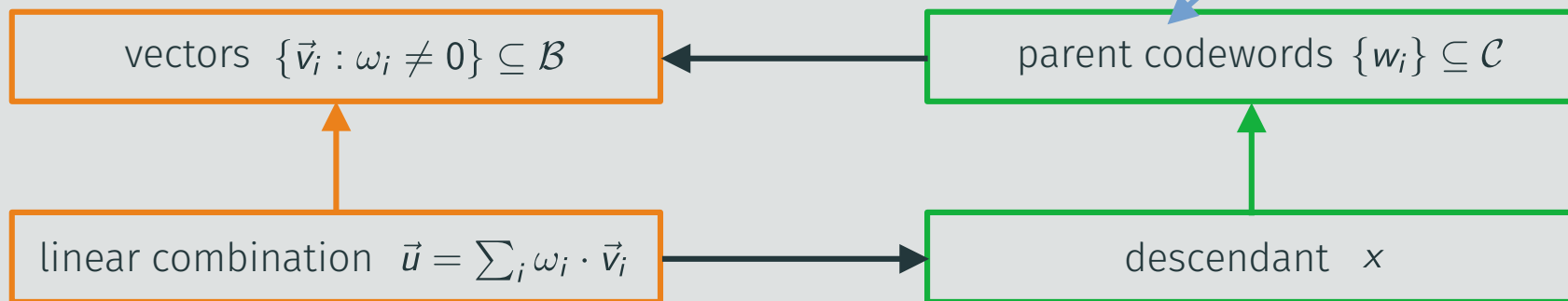


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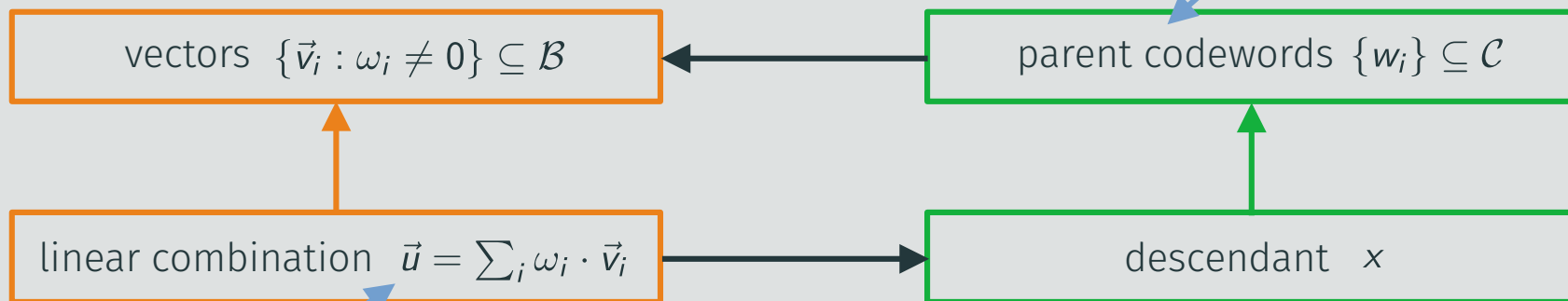


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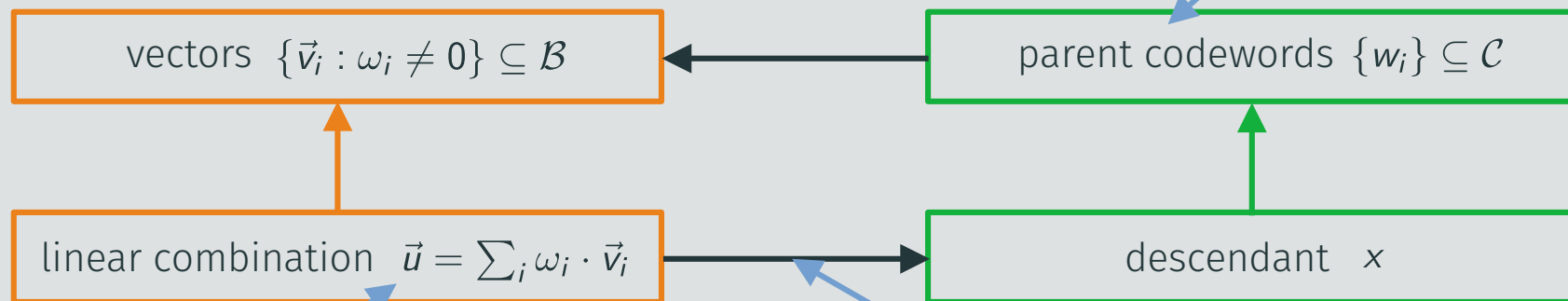
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To hide “bad” linear combinations, we need a hard discrete logarithm ...

... but then extracting the descendant is quite challenging ...

Conclusion



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<https://eprint.iacr.org/2023/138>