Round-Optimal
Oblivious Transfer and MPC from Computational CSIDH

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Joint work with

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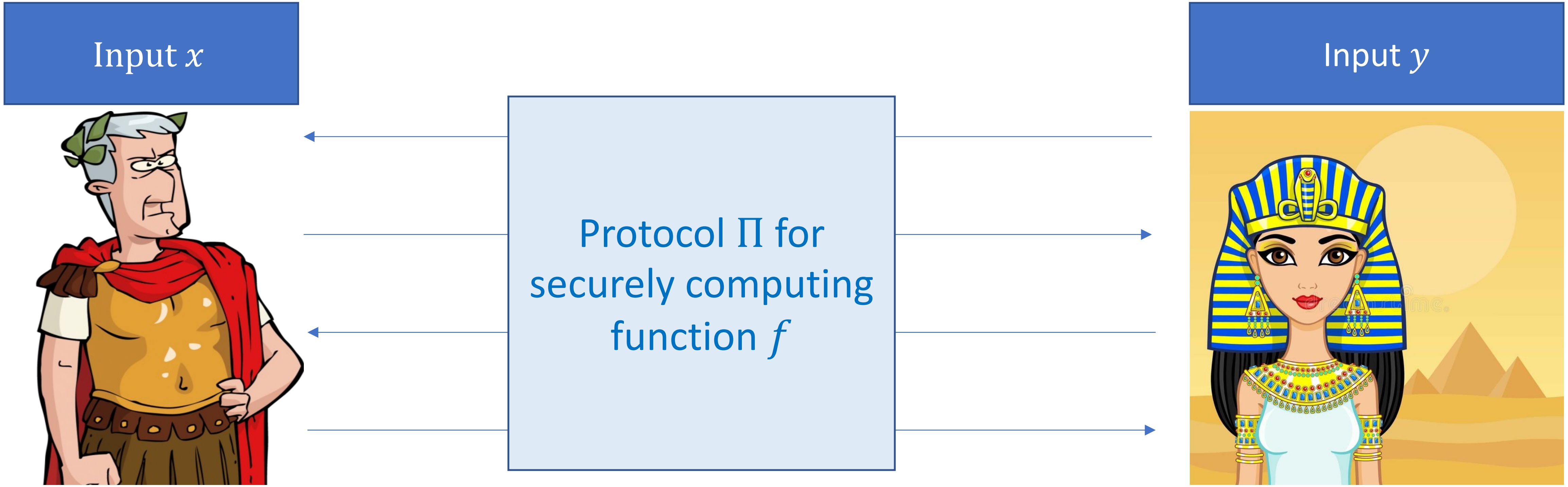
Daniel Masny, Meta
Pratyay Mukherjee, Supra
Sikhar Patranabis, IBM Research India
Srinivasan Raghuraman, Visa Research

Chapter I

Introduction

Secure Two-Party Computation (2-PC)



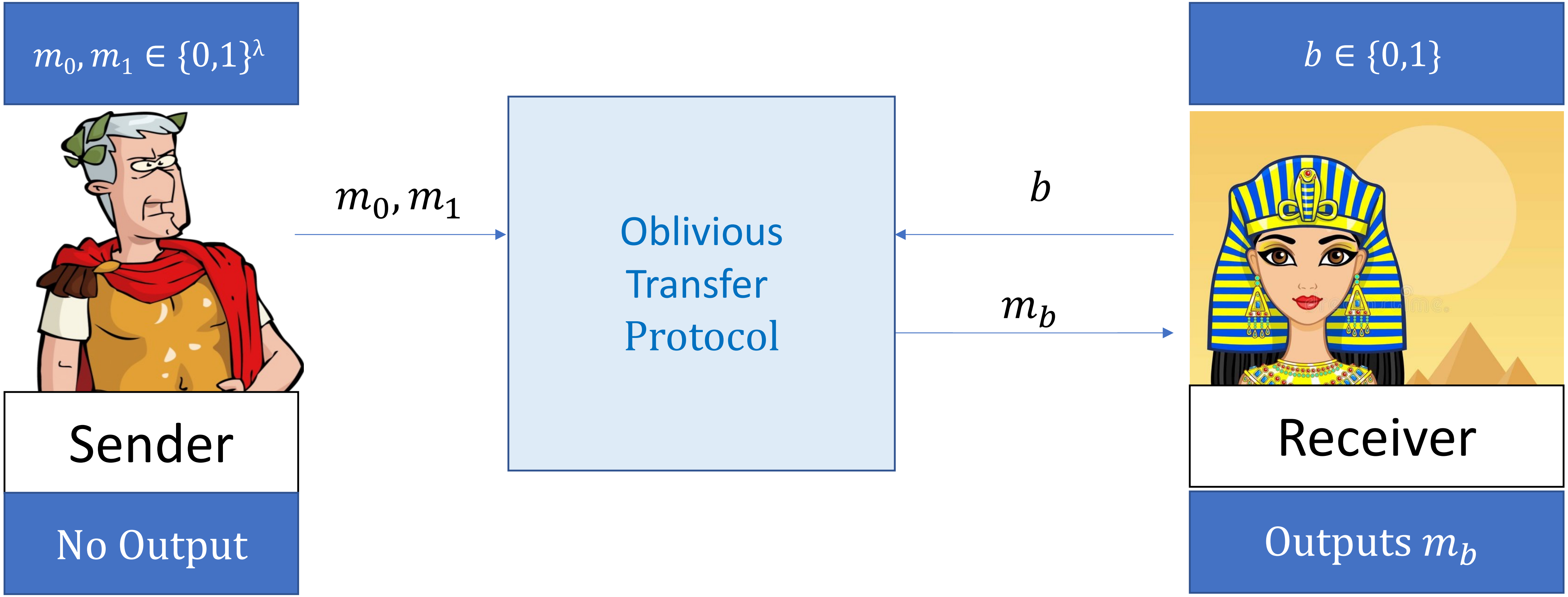
Input x

Protocol Π for securely computing function f

Input y

- Correctness: $\Pi(x, y) = f(x, y)$
- Security: Π leaks no information about x and y beyond $\Pi(x, y)$

Oblivious Transfer (OT)



$m_0, m_1 \in \{0,1\}^\lambda$

Sender

No Output

m_0, m_1

Oblivious Transfer Protocol

b

m_b

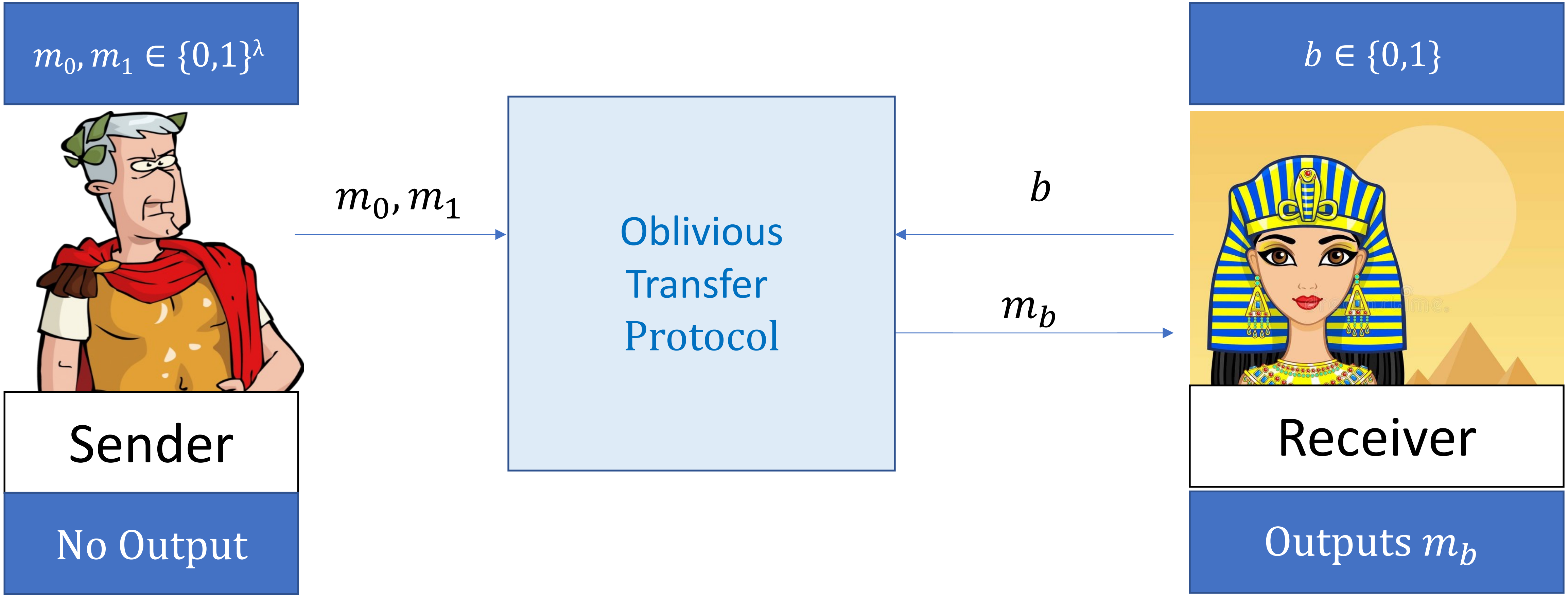
$b \in \{0,1\}$

Receiver

Outputs m_b

Security: Sender does not know b and Receiver does not know m_{1-b}

Oblivious Transfer (OT)



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Security: Sender does not know b and Receiver does not know m_{1-b}

Round-Optimal OT → Round-Optimal MPC [GS18,BL18]

This Talk

Our Focus

OT Protocols in Setup Model and Plain Model:

- Round-Optimal

This Talk

Our Focus

OT Protocols in Setup Model and Plain Model:

- Round-Optimal
- Simulation-Security

This Talk

Our Focus

OT Protocols in Setup Model and Plain Model:

- Round-Optimal
- Simulation-Security
- Weak isogeny-based assumptions

This Talk

Our Focus

OT Protocols in Setup Model and Plain Model:

- Round-Optimal
- Simulation-Security
- Weak isogeny-based assumptions

[BL18,GS18]: MPC in Setup Model and Plain Model:

- Round-Optimal
- Simulation-Security
- Weak isogeny-based assumptions

Chapter II

Contributions and Comparison

Isogeny-based OT Protocols in the Setup Model

Protocol	**Computational Assumptions**	**Rounds**	**Security Model**	**Setup**
[ADMP20]	Decisional CSIDH	2	UC-security	CRS
[BKW20]	Decisional CSIDH	2	UC-security	CRS+Random Oracle
[AMPS21]	Decisional CSIDH	2	UC-security (Adaptive)	CRS
[LGdSG21]	Reciprocal CSIDH	4	UC-security	CRS+Random Oracle
[OZ23]	DLog CSIDH (Knowledge of Exponent)	2	Relaxed UC-security	CRS+Random Oracle

Isogeny-based OT Protocols in the Setup Model

Protocol	Computational Assumptions	Rounds	Security Model	Setup
[ADMP20]	Decisional CSIDH	2	UC-security	CRS
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This Work	**Computational CSIDH**	**2**	**Simulation security**	**CRS+Random Oracle**

Our Contributions

Round Optimal Results in Setup Model:

- 2-round UC-OT in CRS+Random Oracle Model from computational-CSIDH

Our Contributions

Round Optimal Results in Setup Model:

- 2-round UC-OT in CRS+Random Oracle Model from computational-CSIDH
- 2-round MPC in CRS+Random Oracle Model from computational-CSIDH

Isogeny-based OT Protocols in the Plain Model

Protocol	**Computational Assumptions**	**Rounds**	**Security Model**
[ADMP20]	Decisional CSIDH	2	Semantic security
[BPS22]	Reciprocal CSIDH	4	Simulation security
[KM20]	Decisional CSIDH	4	Simulation security

Isogeny-based OT Protocols in the Plain Model

Protocol	**Computational Assumptions**	**Rounds**	**Security Model**
[ADMP20]	Decisional CSIDH	2	Semantic security
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This Work	**Computational CSIDH**	**4**	**Simulation security**

Our Contributions

Round Optimal Results in Setup Model:

- 2-round UC-OT in CRS+Random Oracle Model from computational-CSIDH
- 2-round MPC in CRS+Random Oracle Model from computational-CSIDH

Round Optimal Results in Plain Model:

- 4-round simulation-secure OT without Setup from computational-CSIDH

Our Contributions

Round Optimal Results in Setup Model:

- 2-round UC-OT in CRS+Random Oracle Model from computational-CSIDH
- 2-round MPC in CRS+Random Oracle Model from computational-CSIDH

Round Optimal Results in Plain Model:

- 4-round simulation-secure OT without Setup from computational-CSIDH
- 4-round simulation-secure MPC without Setup from computational-CSIDH

Our Contributions

Round Optimal Results in Setup Model:

- 2-round UC-OT in CRS+Random Oracle Model from computational-CSIDH
- 2-round MPC in CRS+Random Oracle Model from computational-CSIDH

Round Optimal Results in Plain Model:

- 4-round simulation-secure OT without Setup from computational-CSIDH
- 4-round simulation-secure MPC without Setup from computational-CSIDH

Other Results:

- Oblivious Transfer Extension: Each base-OT requires 4 isogeny computations
- Security based on Reciprocal-CSIDH (quantum equivalent to computational-CSIDH)

Chapter III

Isogeny Preliminaries

Group Actions – Basic Definitions

Definition

Group Action of a group $(G,\cdot)$ on a set $\mathcal{X}$ is a function $* : G \times \mathcal{X} \to \mathcal{X}$ such that:

- Letting e be the identity element in G, for every $x \in \mathcal{X}$ we have $e * x = x$
- For every $g, h \in G$ and for every $x \in \mathcal{X}$ we have $(g \cdot h) * x = g * (h * x)$
- G is a commutative/abelian group
- For any $x, x' \in \mathcal{X}$, there exists a **unique** $g \in G$ such that $g * x = x'$

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Effective Group Action (EGA) : Can efficiently compute $g \star x$ for **any** $(g, x) \in G \times \mathcal{X}$

EGA Instantiations: CSIDH [CLMPR18] with known group structure, CSI-Fish [BKV19])

Not broken by the recent attacks on the SIDH family of isogenies!

Group Actions – Computational Assumptions

Definition

ow-EGA (one-way EGA, models DLog-CSIDH):

For $g \leftarrow G$ and $x \leftarrow \mathcal{X}$, given $(x, g * x)$, it is computationally infeasible to compute g

wU-EGA (weak Unpredictable-EGA, models computational-CSIDH):

For $g, h \leftarrow G$ and $x \leftarrow \mathcal{X}$, given $(x, g * x, h * x)$, it is computationally infeasible to compute $(g.h) * x$

Chapter IV

Round Optimal OT in Setup Model

Semi-honest Oblivious Transfer in Setup model

CRS: (x, x_0, x_1)

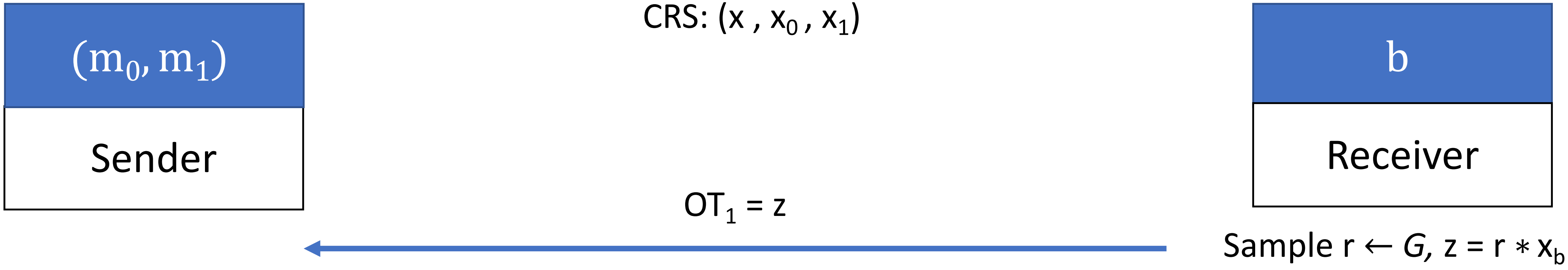
(m_0, m_1)

Sender

b

Receiver

Semi-honest Oblivious Transfer in Setup model



(m_0, m_1)

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CRS: (x, x_0, x_1)

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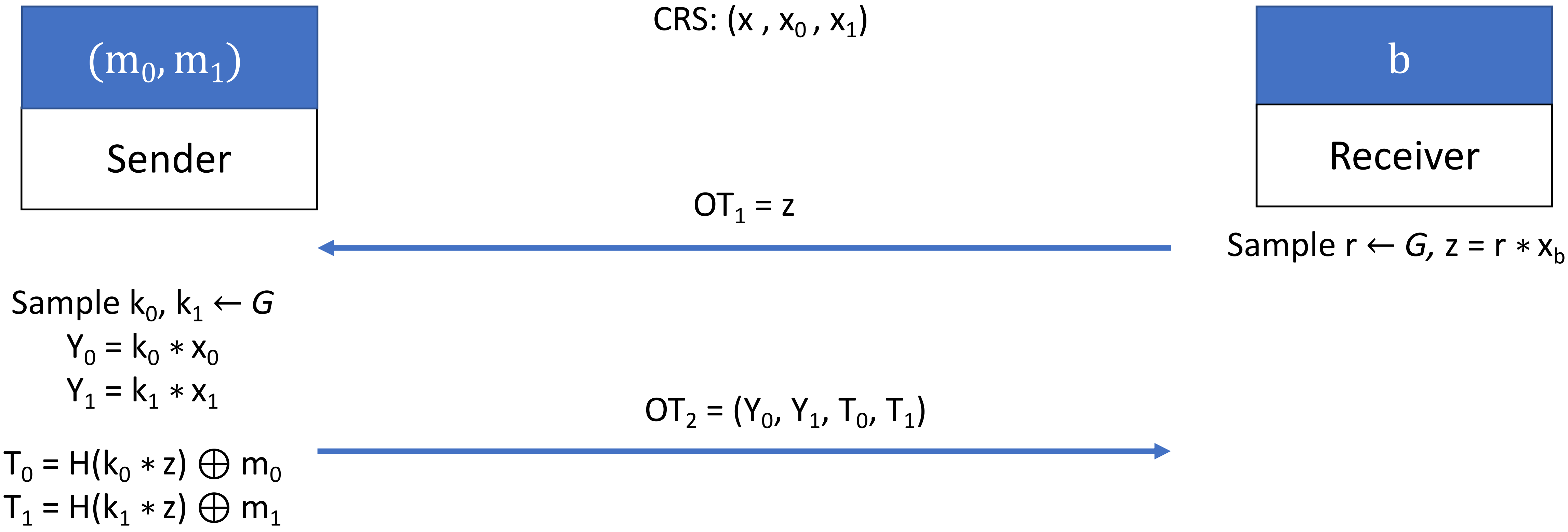
$OT_1 = z$

Sample $r \leftarrow G$, $z = r * x_b$

Semi-honest Oblivious Transfer in Setup model



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Sample $r \leftarrow G$, $z = r * x_b$

Sample $k_0, k_1 \leftarrow G$

$Y_0 = k_0 * x_0$

$Y_1 = k_1 * x_1$

$T_0 = H(k_0 * z) \oplus m_0$

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$OT_2 = (Y_0, Y_1, T_0, T_1)$

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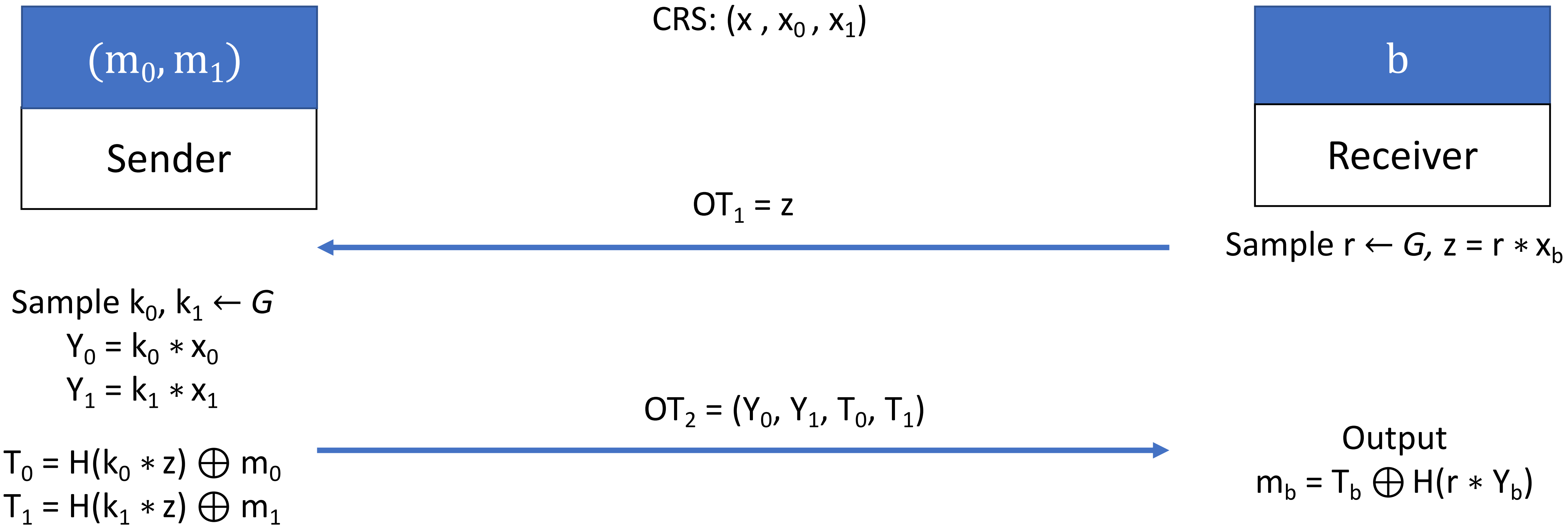

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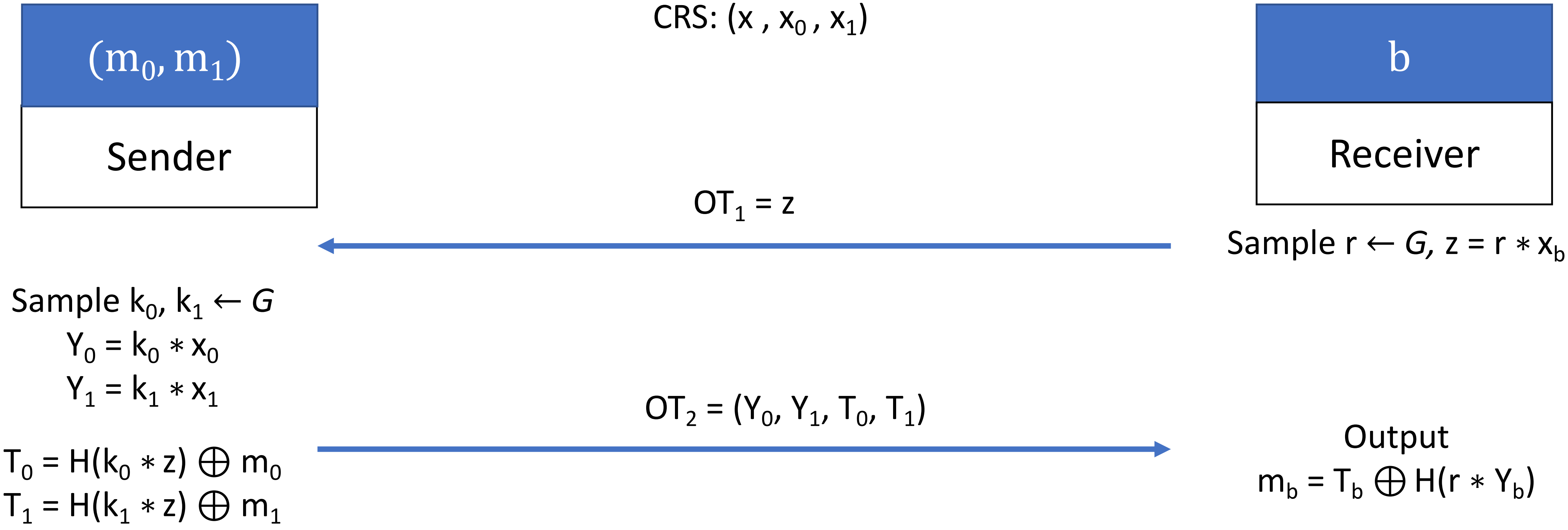

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Output

$m_b = T_b \oplus H(r \ast Y_b)$

Receiver Privacy: Choice bit b is statistically hidden

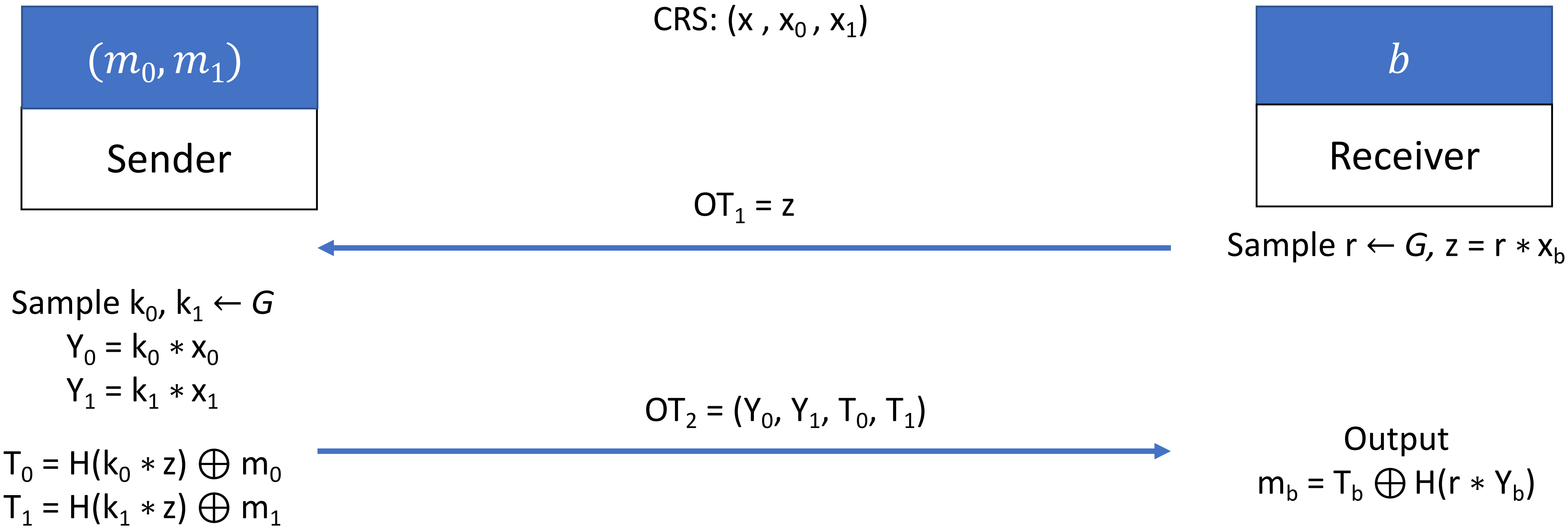
Assuming $b = 1$:

$z = r \ast x_1 = \mathbf{r'} \ast x_0$

where $\mathbf{r' = r\, g_1\, g_0^{-1}}$

for $x_0 = g_0 \ast x$, $x_1 = g_1 \ast x = g_1 \ast g_0^{-1} \ast x_0$

Semi-honest Oblivious Transfer in Setup model



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Sender Privacy: If Receiver computes m_{1-b} then break wu-EGA property
(Need to extract r for the reduction)

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Secure against malicious sender

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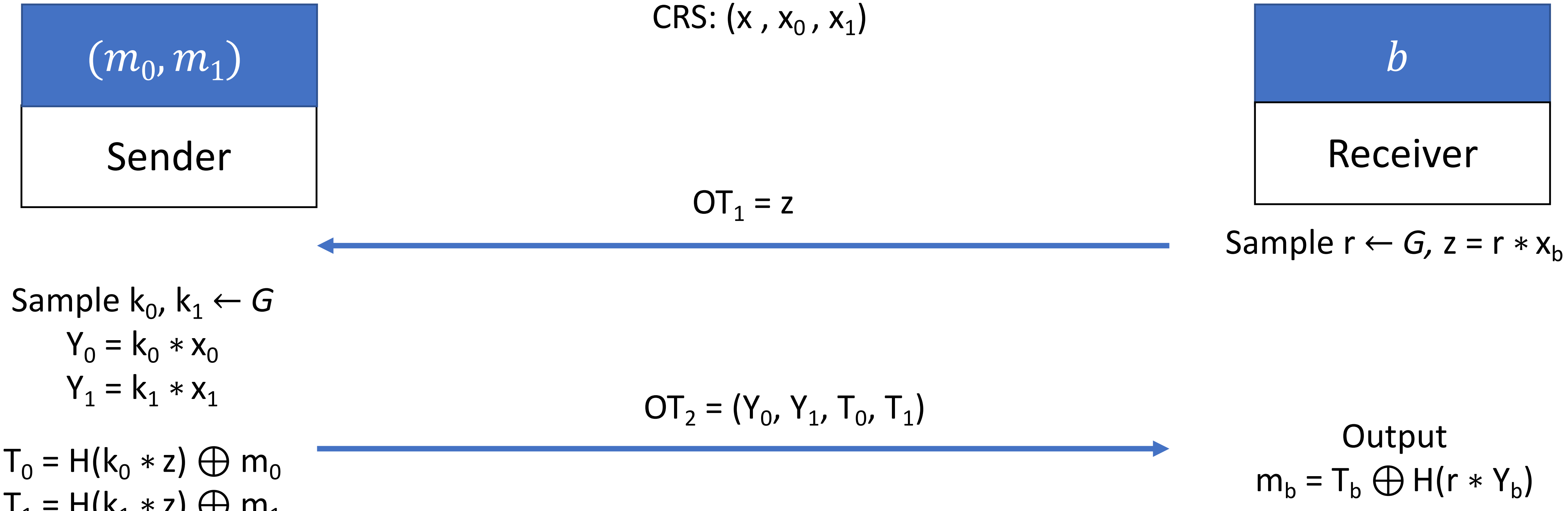

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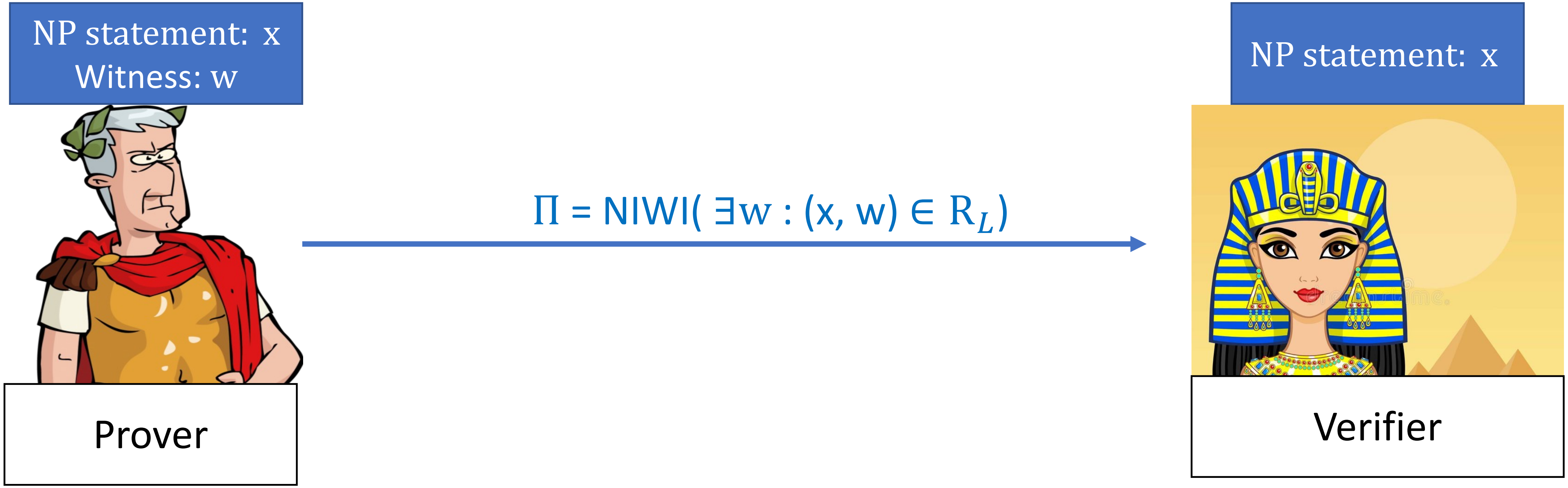
Non-interactive Witness-Indistinguishability Proof-of-Knowledge (NIWI)



NP statement: x
Witness: w

Prover

Π = NIWI($\exists w : (x, w) \in R_L$)

NP statement: x

Verifier

Completeness: Verifier outputs 1 if $(x, w) \in R_L$

Soundness: If $x \notin L$, Verifier outputs 0 with high probability

Witness-Indistinguishability: $\Pi_0 \approx \Pi_1$ where Π_b is generated using witness w_b (where w_0 , w_1 are valid witness)

Proof-of-Knowledge: Witness w can be extracted from an accepting proof

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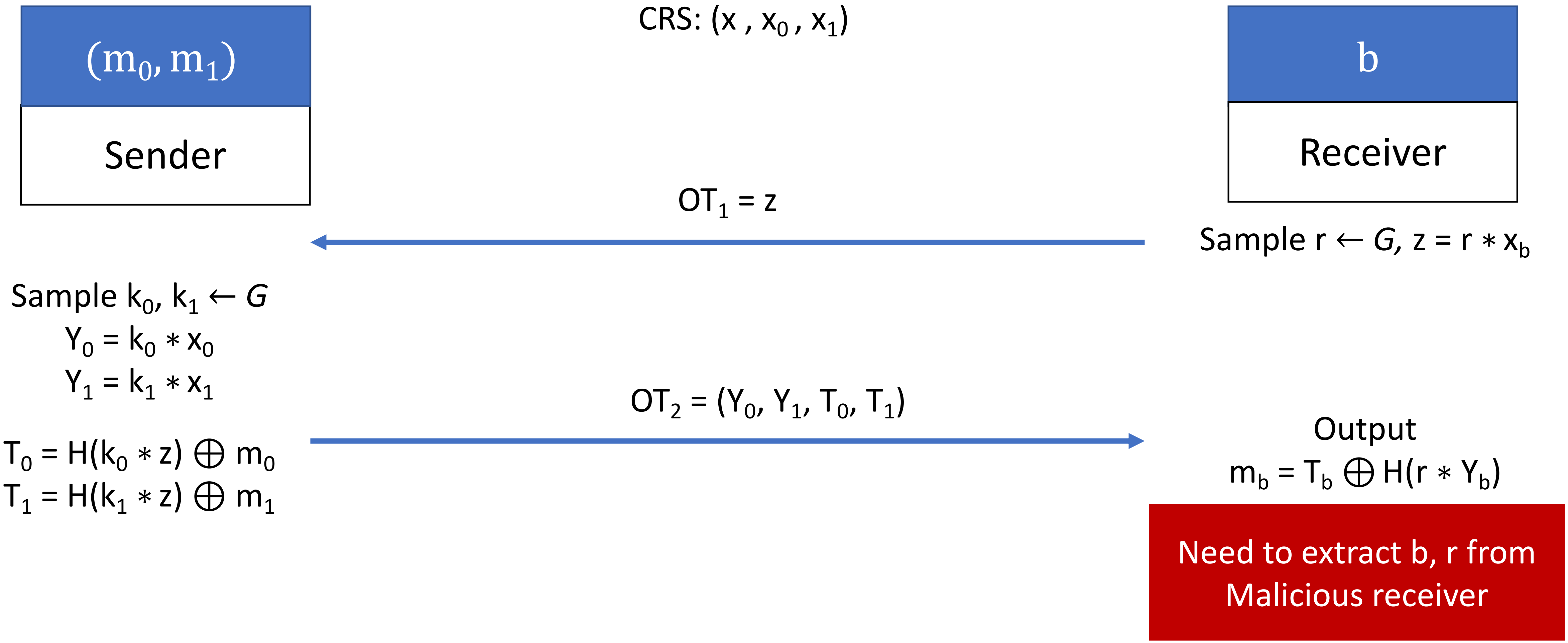
Build from wu-EGA

Maliciously Secure Oblivious Transfer in Setup model



CRS: (x, x_0, x_1)

(m_0, m_1)

Sender

b

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Need to extract b, r from Malicious receiver

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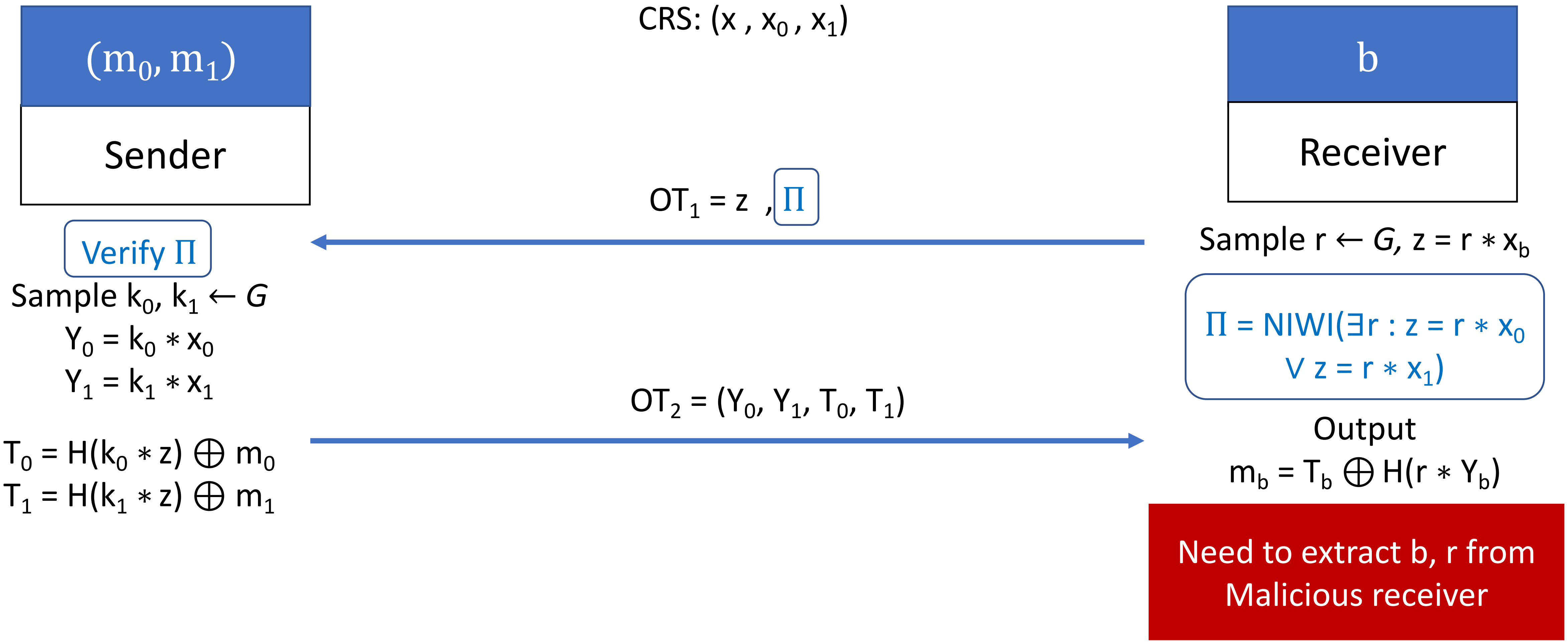

CRS: (x , x_0 , x_1)

(m_0, m_1)

Sender

b

Receiver

Sample $r \leftarrow G$, $z = r * x_b$

$\Pi = \mathrm{NIWI}(\exists r : z = r * x_0 \vee z = r * x_1)$

$OT_1 = z \ , \Pi$

Verify Π

Sample $k_0, k_1 \leftarrow G$

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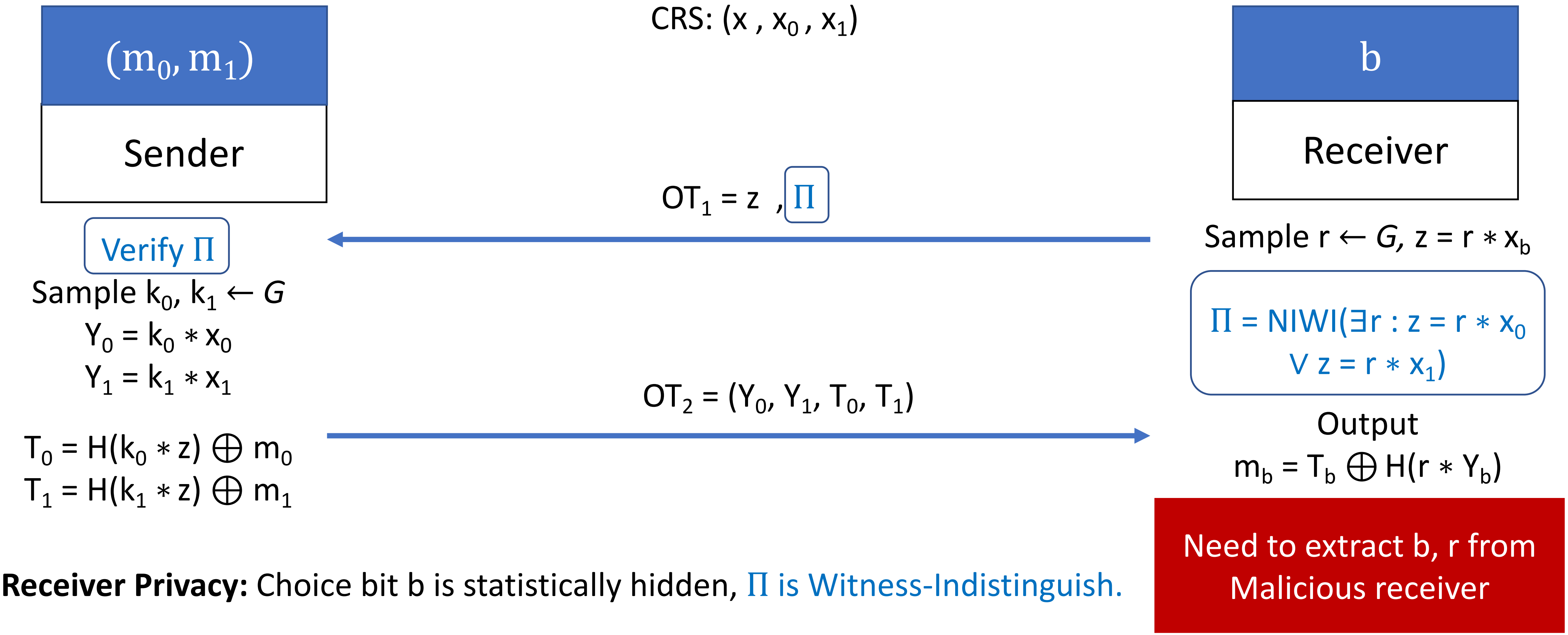
Maliciously Secure Oblivious Transfer in Setup model **(Input Privacy)**



CRS: (x, x_0, x_1)

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Sender

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Receiver

$OT_1 = z$, $\boxed{\Pi}$

Sample $r \leftarrow G$, $z = r * x_b$

Π = NIWI($\exists r : z = r * x_0 \vee z = r * x_1$)

Verify Π

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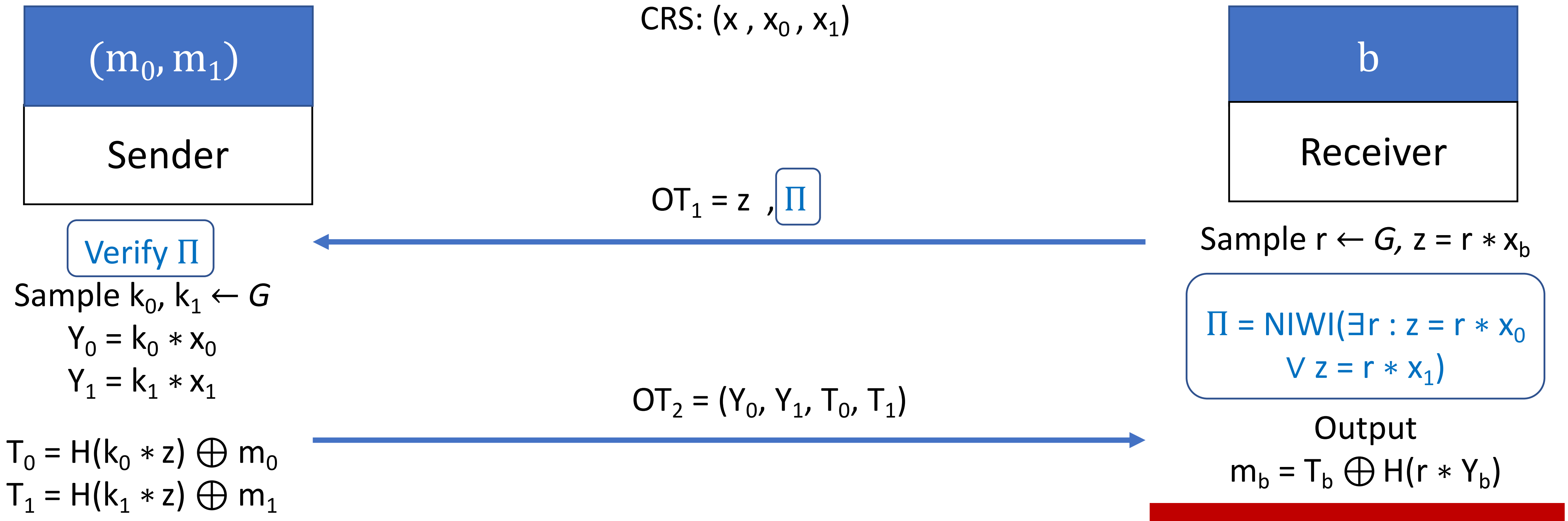
Need to extract b, r from Malicious receiver

Receiver Privacy: Choice bit b is statistically hidden, Π is Witness-Indistinguish.

Sender Privacy: If Receiver computes $m_{1\text{-}b}$ then break wu-EGA property, Π is Sound and extractable
(Need to extract r for the reduction)

Maliciously Secure Oblivious Transfer in Setup model **(Input Extraction)**



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Sender

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Receiver

$OT_1 = z\ ,\boxed{\Pi}$

Sample $r \leftarrow G$, $z = r * x_b$

$\Pi = \text{NIWI}(\exists r : z = r * x_0 \vee z = r * x_1)$

$\boxed{\text{Verify } \Pi}$

Sample $k_0, k_1 \leftarrow G$

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Output

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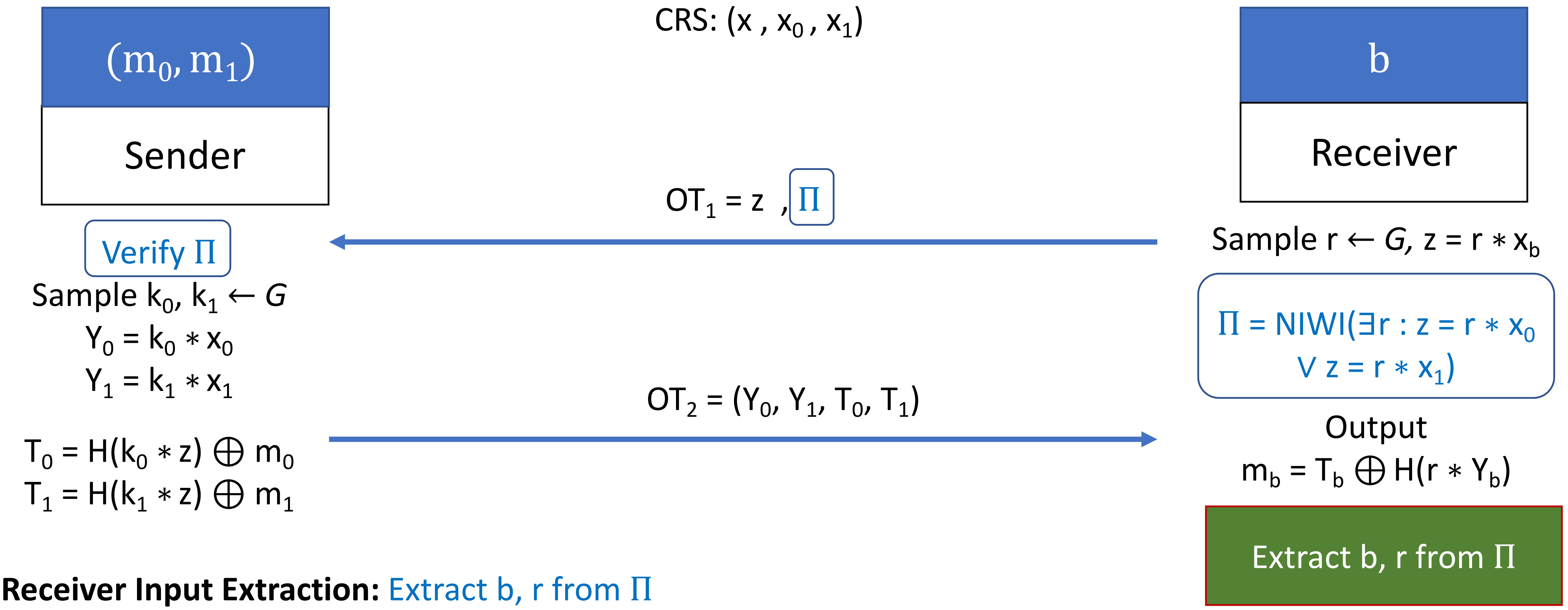
Need to extract b, r from Malicious receiver

Receiver Input Extraction: Extract b, r from Π

Sender Input Extraction: Compute m_1 by setting b = 1, extract m_0 by using the CRS trapdoor ($= g_1 * g_0^{-1}$)

Assuming b = 1: $z = r * x_1 = \mathbf{r'} * x_0$ (where $\mathbf{r' = r\, g_1\, g_0^{-1}}$ for $x_0 = g_0 * x$, $x_1 = g_1 * x = g_1 * g_0^{-1} * x_0$)

Maliciously Secure Oblivious Transfer in Setup model **(Input Extraction)**



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Extract b, r from Π

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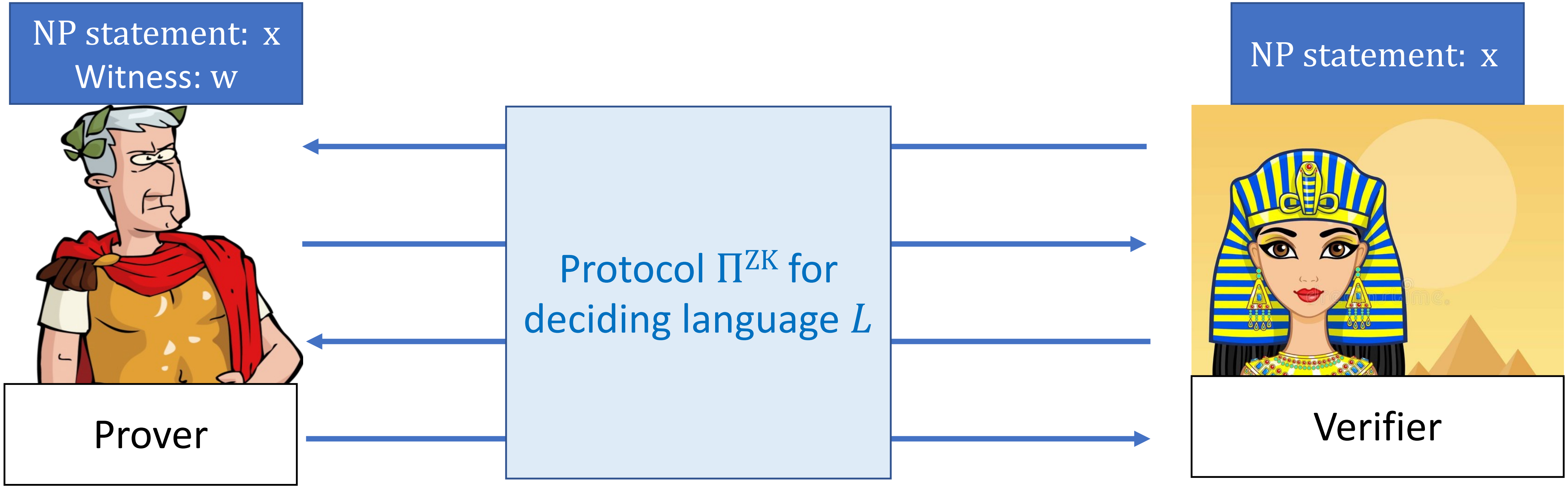
Chapter V

Round Optimal OT in Plain Model

Delayed-Input Zero-Knowledge Proof-of-Knowledge (ZK)



NP statement: x
Witness: w

Prover

Protocol Π^{ZK} for deciding language L

NP statement: x

Verifier

Completeness: Verifier outputs 1 if $(\mathrm{x}, \mathrm{w}) \in \mathrm{R}_L$

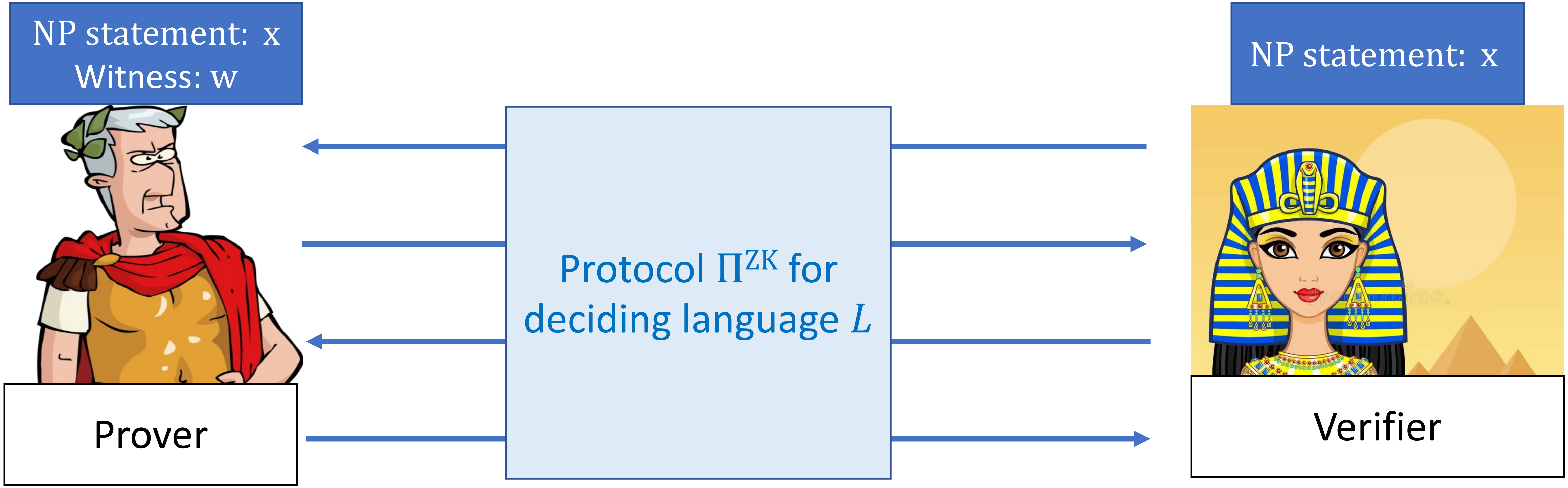
Soundness: If $\mathrm{x} \notin L$, Verifier outputs 0 with high probability

Zero-Knowledge: Π leaks no information about w to the Verifier

Proof-of-Knowledge: Witness w can be extracted from an accepting proof

Delayed-Input: Only the last ZK protocol message depends on statement x

Delayed-Input Zero-Knowledge Proof-of-Knowledge (ZK)



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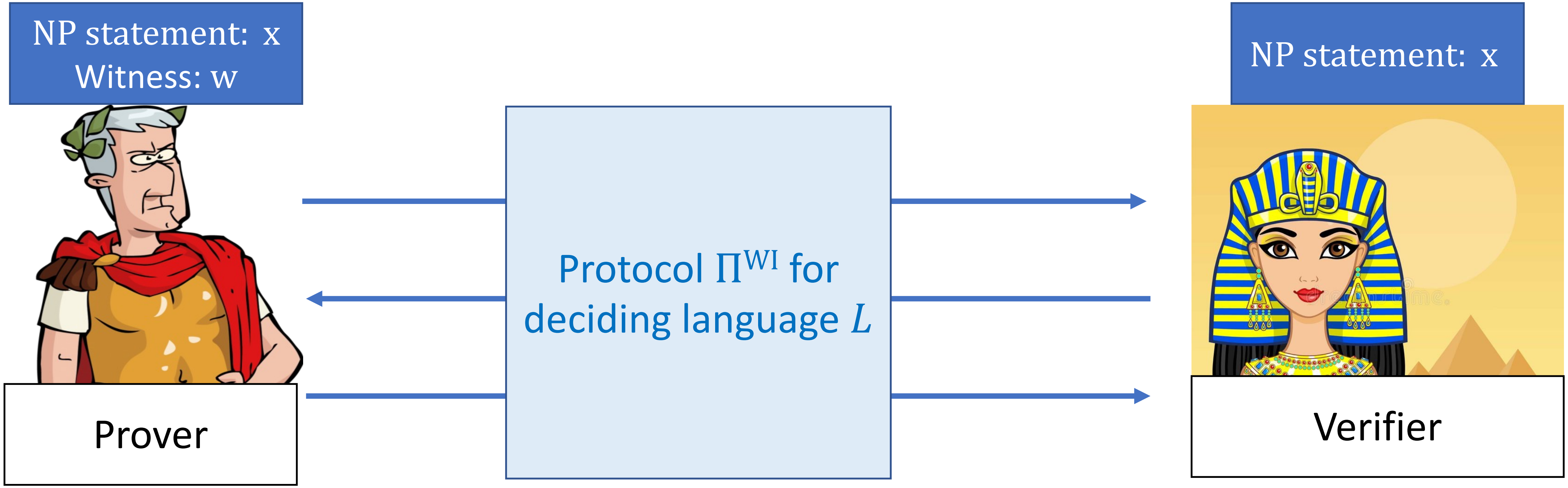
Build from wu-EGA

Delayed-Input Witness-Indistinguishability Proof-of-Knowledge (WI)



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Witness: w

Prover

Protocol Π^{WI} for deciding language L

NP statement: x

Verifier

Completeness: Verifier outputs 1 if $(\mathrm{x}, \mathrm{w}) \in \mathrm{R}_L$

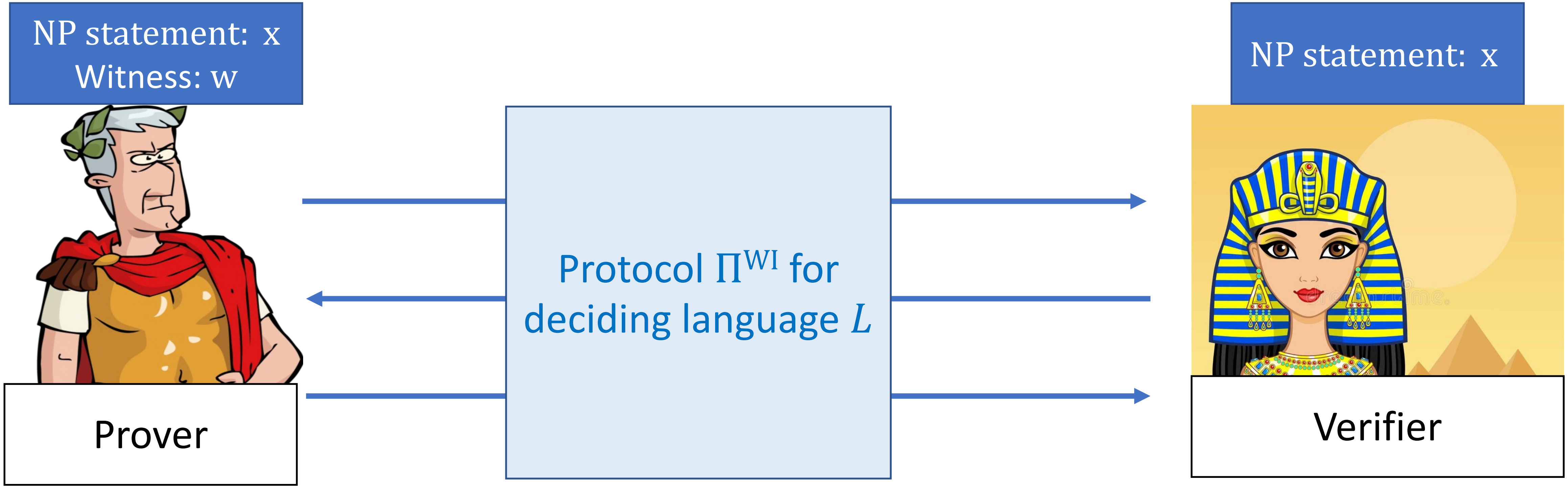
Soundness: If $\mathrm{x} \notin L$, Verifier outputs 0 with high probability

Witness-Indistinguishability: $\Pi_0 \approx \Pi_1$ where Π_b is generated using witness w_b (where w_0 , w_1 are valid witness)

Proof-of-Knowledge: Witness w can be extracted from an accepting proof

Delayed-Input: Only the last WI protocol message depends on statement x

Delayed-Input Witness-Indistinguishability Proof-of-Knowledge (WI)



NP statement: x
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Prover

Protocol Π^{WI} for deciding language L

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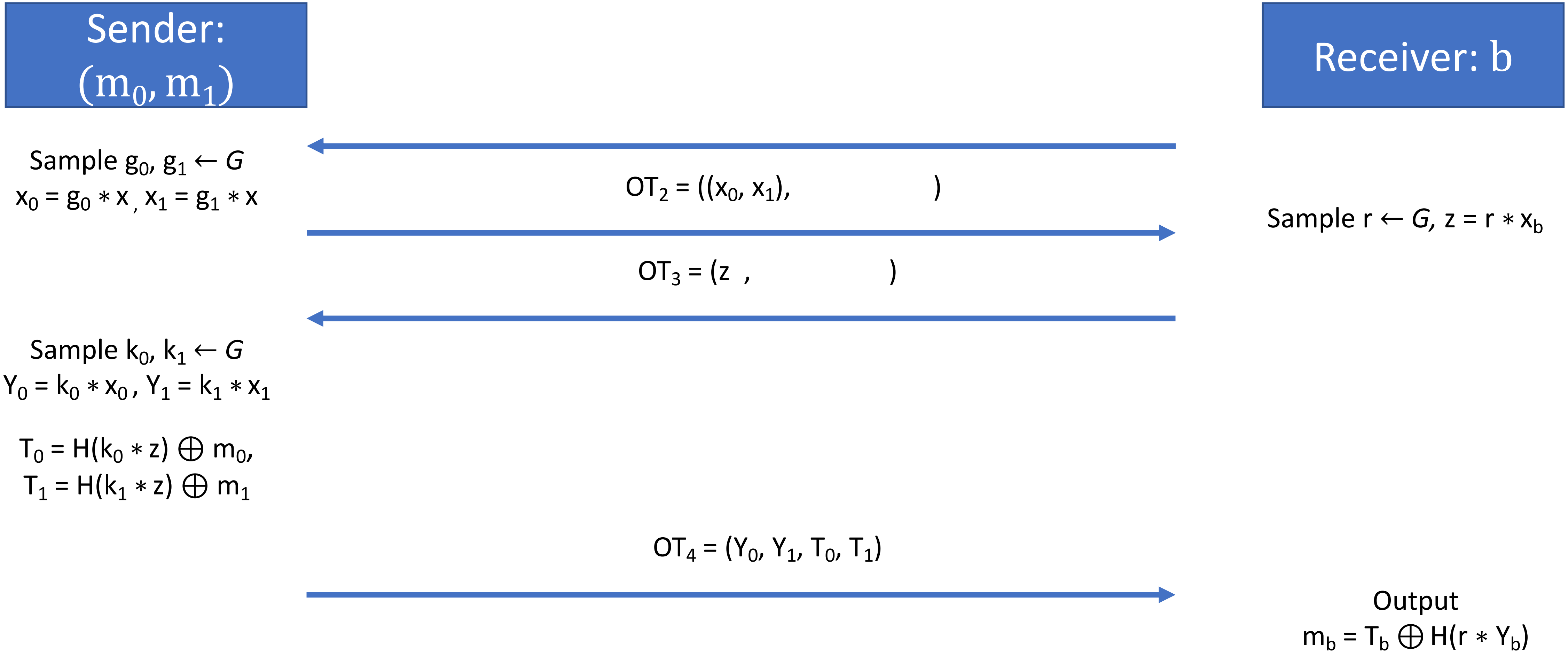
Build from wu-EGA

Maliciously Secure Oblivious Transfer in Plain model



Sender: (m_0, m_1)

Receiver: b

Sample $g_0, g_1 \leftarrow G$
$x_0 = g_0 * x$, $x_1 = g_1 * x$

$OT_2 = ((x_0, x_1),\qquad\qquad)$

Sample $r \leftarrow G$, $z = r * x_b$

$OT_3 = (z\ ,\qquad\qquad)$

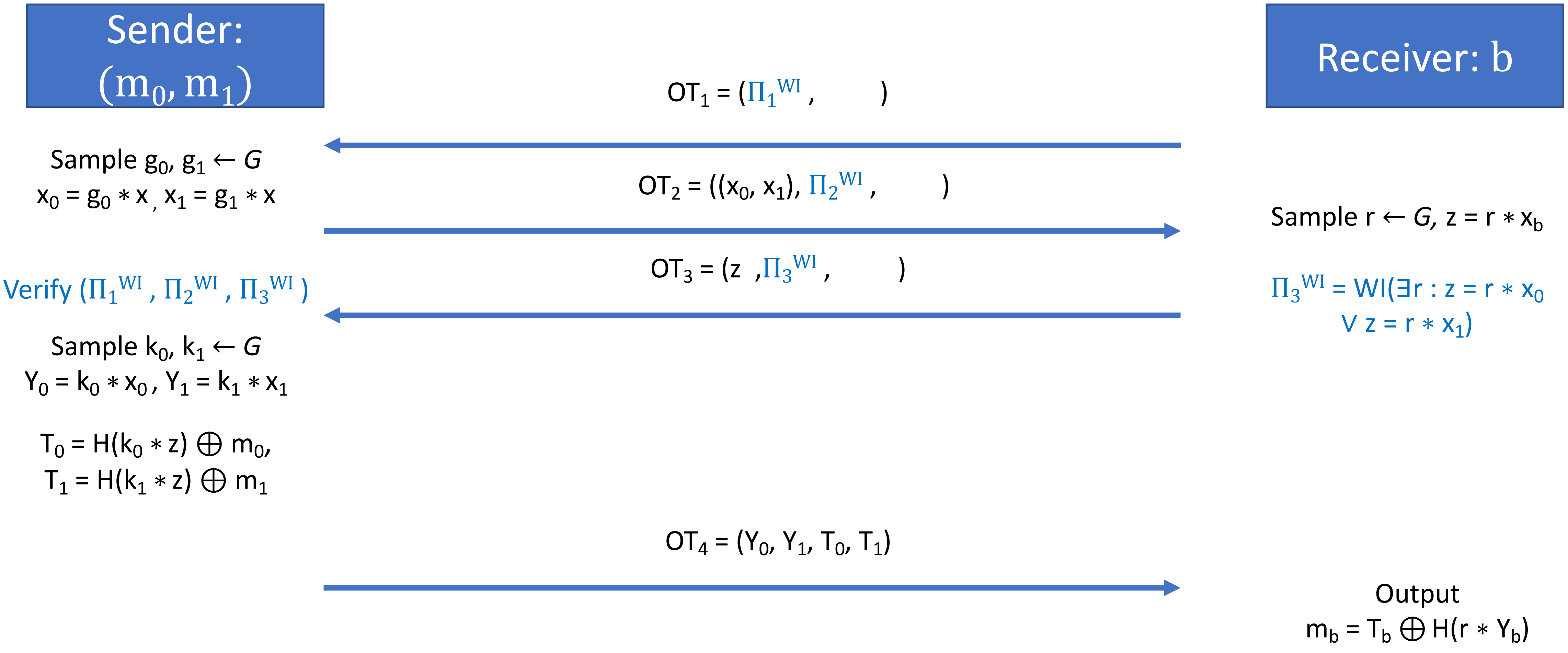
Sample $k_0, k_1 \leftarrow G$
$Y_0 = k_0 * x_0$, $Y_1 = k_1 * x_1$

$T_0 = H(k_0 * z) \oplus m_0$,
$T_1 = H(k_1 * z) \oplus m_1$

$OT_4 = (Y_0, Y_1, T_0, T_1)$

Output
$m_b = T_b \oplus H(r * Y_b)$

Maliciously Secure Oblivious Transfer in Plain model **(Input Privacy)**



Sender: (m_0, m_1)

Receiver: b

$OT_1 = (\Pi_1^{WI} , \quad)$

Sample $g_0, g_1 \leftarrow G$

$x_0 = g_0 * x$, $x_1 = g_1 * x$

$OT_2 = ((x_0, x_1), \Pi_2^{WI} , \quad)$

Sample $r \leftarrow G$, $z = r * x_b$

$OT_3 = (z \ ,\Pi_3^{WI} , \quad)$

$\Pi_3^{WI} = WI(\exists r : z = r * x_0 \vee z = r * x_1)$

Verify $(\Pi_1^{WI} , \Pi_2^{WI} , \Pi_3^{WI})$

Sample $k_0, k_1 \leftarrow G$

$Y_0 = k_0 * x_0 , Y_1 = k_1 * x_1$

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Output

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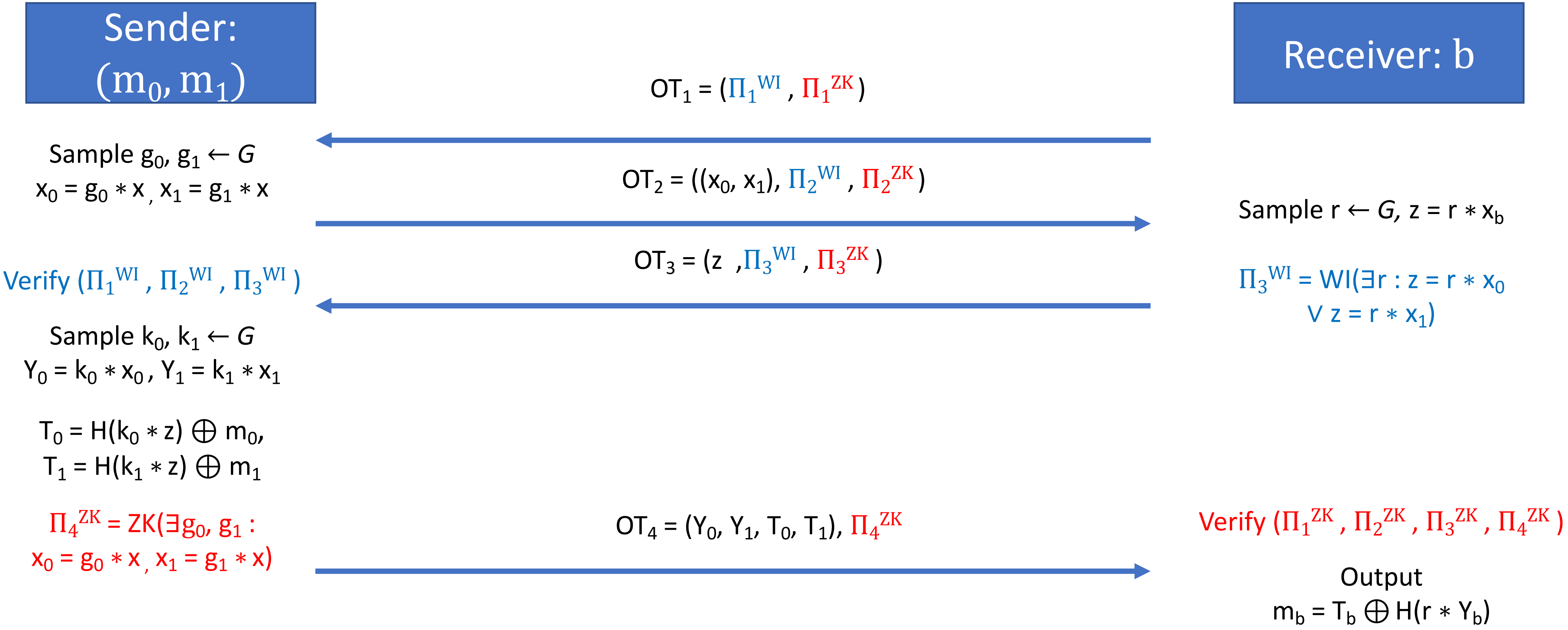
Maliciously Secure Oblivious Transfer in Plain model **(Input Privacy)**



Sender:
(m_0, m_1)

Receiver: b

$OT_1 = (\Pi_1^{WI}, \Pi_1^{ZK})$

Sample $g_0, g_1 \leftarrow G$
$x_0 = g_0 * x , x_1 = g_1 * x$

$OT_2 = ((x_0, x_1), \Pi_2^{WI}, \Pi_2^{ZK})$

Sample $r \leftarrow G, z = r * x_b$

$\Pi_3^{WI} = WI(\exists r : z = r * x_0$
$\vee z = r * x_1)$

$OT_3 = (z ,\Pi_3^{WI}, \Pi_3^{ZK})$

Verify $(\Pi_1^{WI}, \Pi_2^{WI}, \Pi_3^{WI})$

Sample $k_0, k_1 \leftarrow G$
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$T_0 = H(k_0 * z) \oplus m_0,$
$T_1 = H(k_1 * z) \oplus m_1$

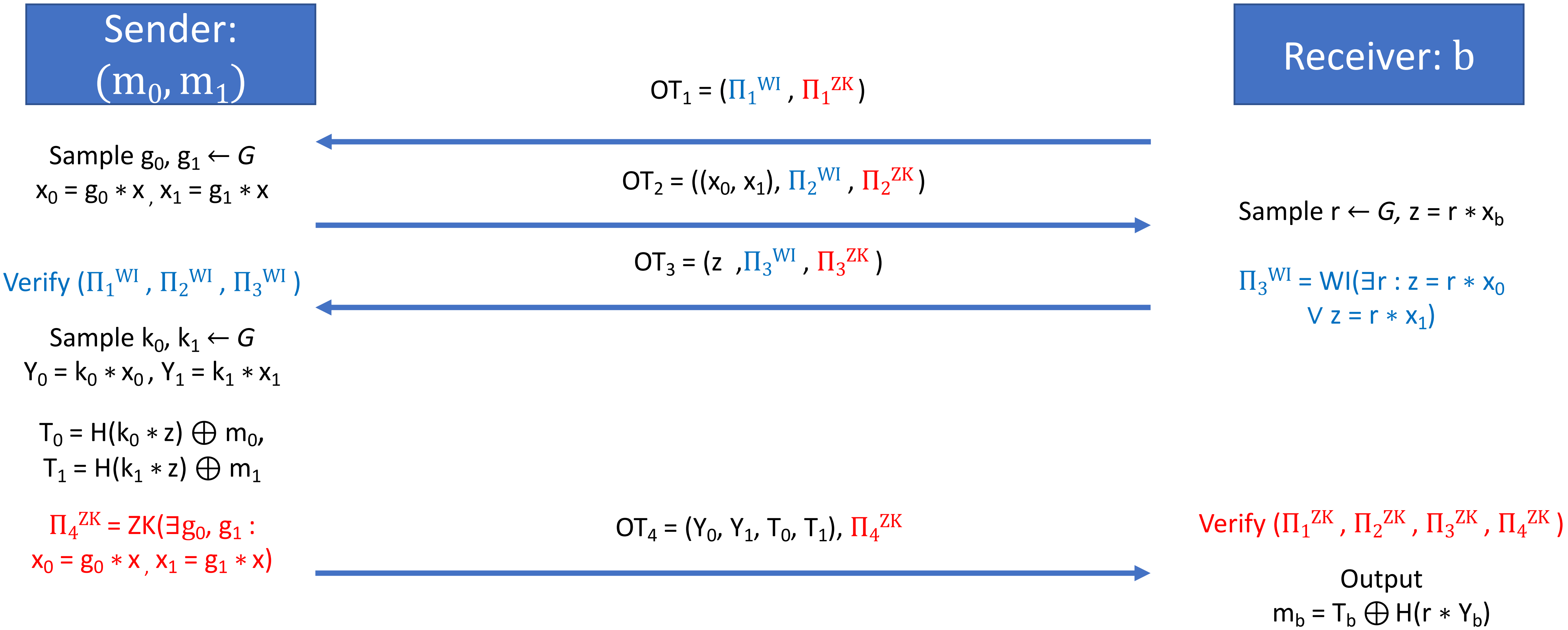
$\Pi_4^{ZK} = ZK(\exists g_0, g_1 :$
$x_0 = g_0 * x , x_1 = g_1 * x)$

$OT_4 = (Y_0, Y_1, T_0, T_1), \Pi_4^{ZK}$

Verify $(\Pi_1^{ZK}, \Pi_2^{ZK}, \Pi_3^{ZK}, \Pi_4^{ZK})$

Output
$m_b = T_b \oplus H(r * Y_b)$

Maliciously Secure Oblivious Transfer in Plain model **(Input Privacy)**



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Verify $(\Pi_1^{WI}, \Pi_2^{WI}, \Pi_3^{WI})$

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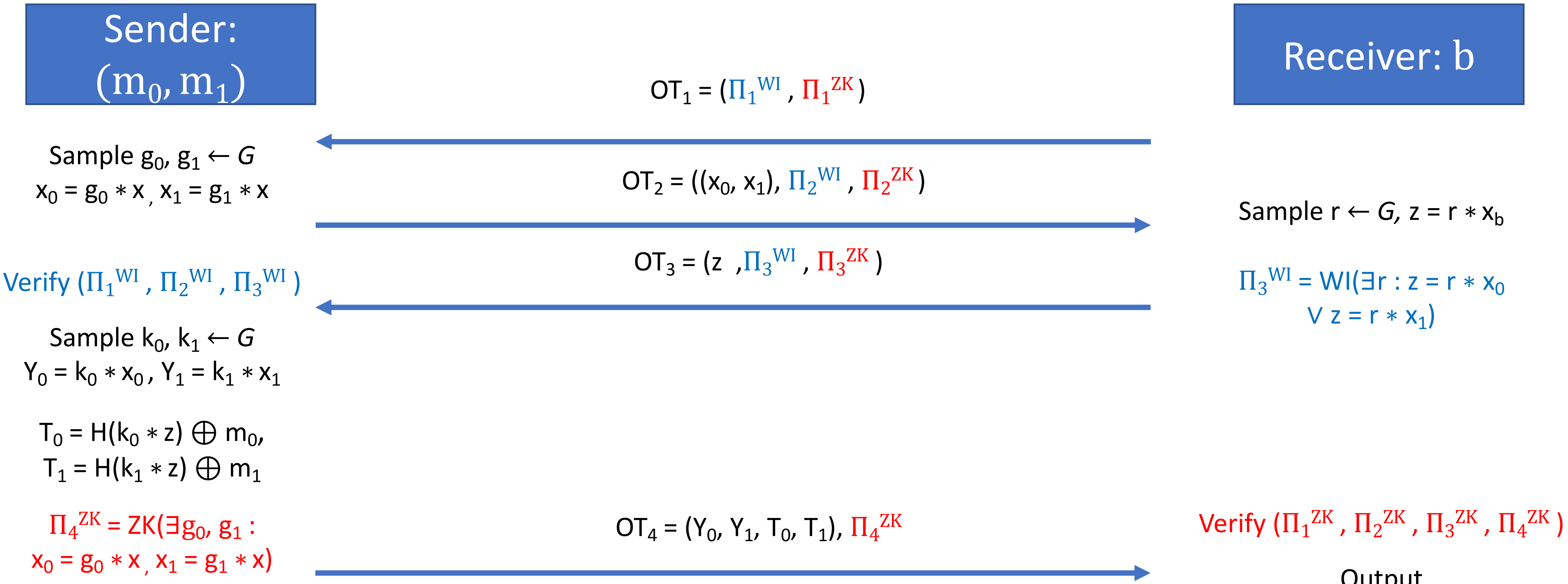
Verify $(\Pi_1^{ZK}, \Pi_2^{ZK}, \Pi_3^{ZK}, \Pi_4^{ZK})$

Output
$m_b = T_b \oplus H(r * Y_b)$

Receiver Privacy: b is statistically hidden, Π^{WI} is Witness-Indistinguishable, Π^{ZK} is Sound

Sender Privacy: If R computes m_{1-b} then break wu-EGA property, Π^{WI} is Sound, Π^{ZK} is Zero Knowledge

Maliciously Secure Oblivious Transfer in Plain model **(Input Extraction)**



Sender: (m_0, m_1)

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$OT_2 = ((x_0, x_1), \Pi_2^{WI}, \Pi_2^{ZK})$

Sample $r \leftarrow G$, $z = r * x_b$

$OT_3 = (z, \Pi_3^{WI}, \Pi_3^{ZK})$

$\Pi_3^{WI} = WI(\exists r : z = r * x_0 \lor z = r * x_1)$

Verify $(\Pi_1^{WI}, \Pi_2^{WI}, \Pi_3^{WI})$

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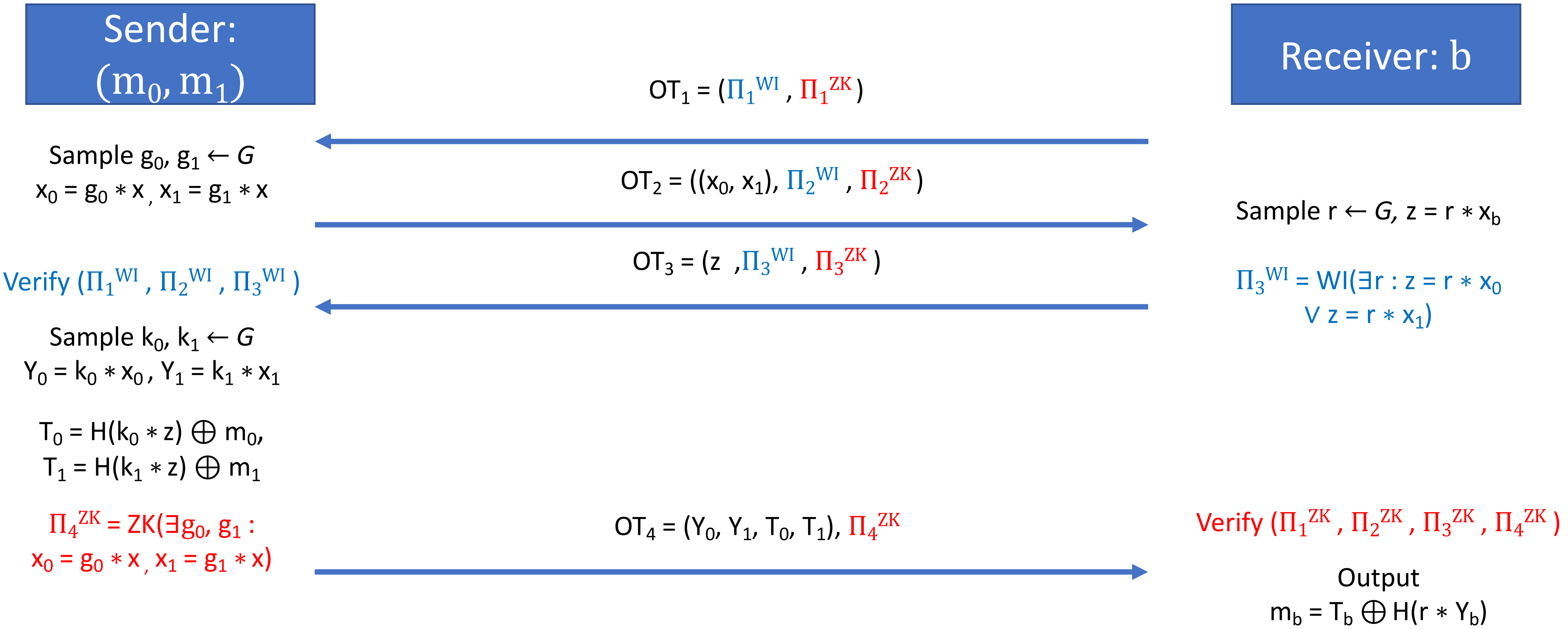
$OT_4 = (Y_0, Y_1, T_0, T_1), \Pi_4^{ZK}$

Verify $(\Pi_1^{ZK}, \Pi_2^{ZK}, \Pi_3^{ZK}, \Pi_4^{ZK})$

Output
$m_b = T_b \oplus H(r * Y_b)$

Receiver Input Extraction: Extract r from Π^{WI}, set b where $z = r * x_b$

Maliciously Secure Oblivious Transfer in Plain model **(Input Extraction)**



Sender: (m_0, m_1)

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Sample $r \leftarrow G$, $z = r * x_b$

$OT_3 = (z, \Pi_3^{WI}, \Pi_3^{ZK})$

Verify $(\Pi_1^{WI}, \Pi_2^{WI}, \Pi_3^{WI})$

$\Pi_3^{WI} = WI(\exists r : z = r * x_0 \lor z = r * x_1)$

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Output

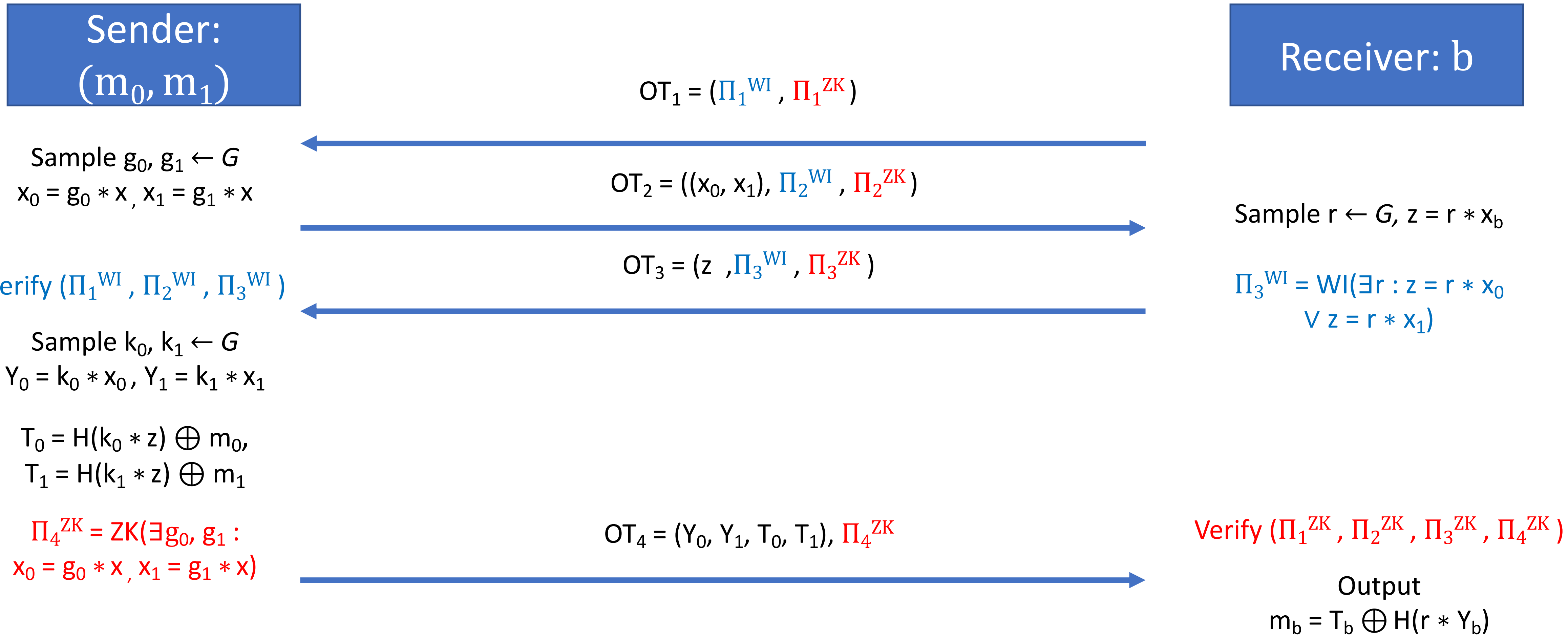
$m_b = T_b \oplus H(r * Y_b)$

Receiver Input Extraction: Extract r from Π^{WI}, set b where $z = r * x_b$

Sender Input Extraction: Extract (g_0, g_1) from Π^{ZK},

Compute m_1 by setting b = 1, Extract m_0 using $g_1 * g_0^{-1}$

Maliciously Secure Oblivious Transfer in Plain model **(Input Extraction)**



Sender: (m_0, m_1)

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Receiver Input Extraction: Extract r from Π^{WI}, set b where $z = r * x_b$

Sender Input Extraction: Extract (g_0, g_1) from Π^{ZK},
Compute m_1 by setting b = 1, Extract m_0 using $g_1 * g_0^{-1}$

Build Π^{ZK}, Π^{WI} from wu-EGA

Chapter VI

Concluding Remarks

Conclusion

- Round Optimal OT/MPC Results in CRS+Random Oracle Model from computational-CSIDH
- Round Optimal OT/MPC Results in Plain Model from computational-CSIDH
- Oblivious Transfer Extension based on Reciprocal-CSIDH

Open Problems:

  - 2-round computational-CSIDH based UC-OT without Random Oracle?
  - Efficient (incurring O(1) isogeny computations) 2-round UC-OT from computational-CSIDH ?

Thank You

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2022/1511