

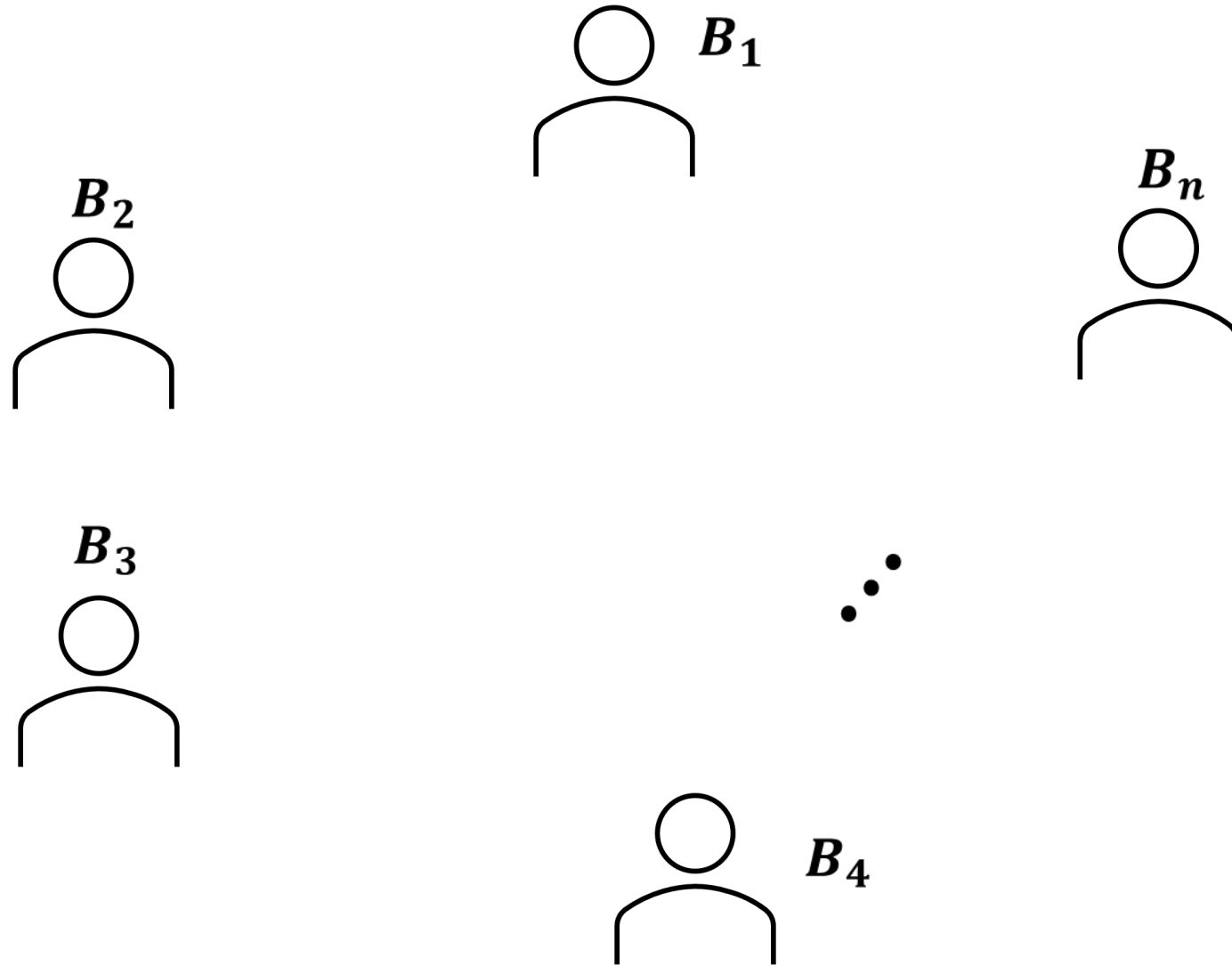
Simple, Fast, Efficient, and Tightly-Secure Non-Malleable Non-Interactive Timed Commitments

Peter Chvojka, Tibor Jager

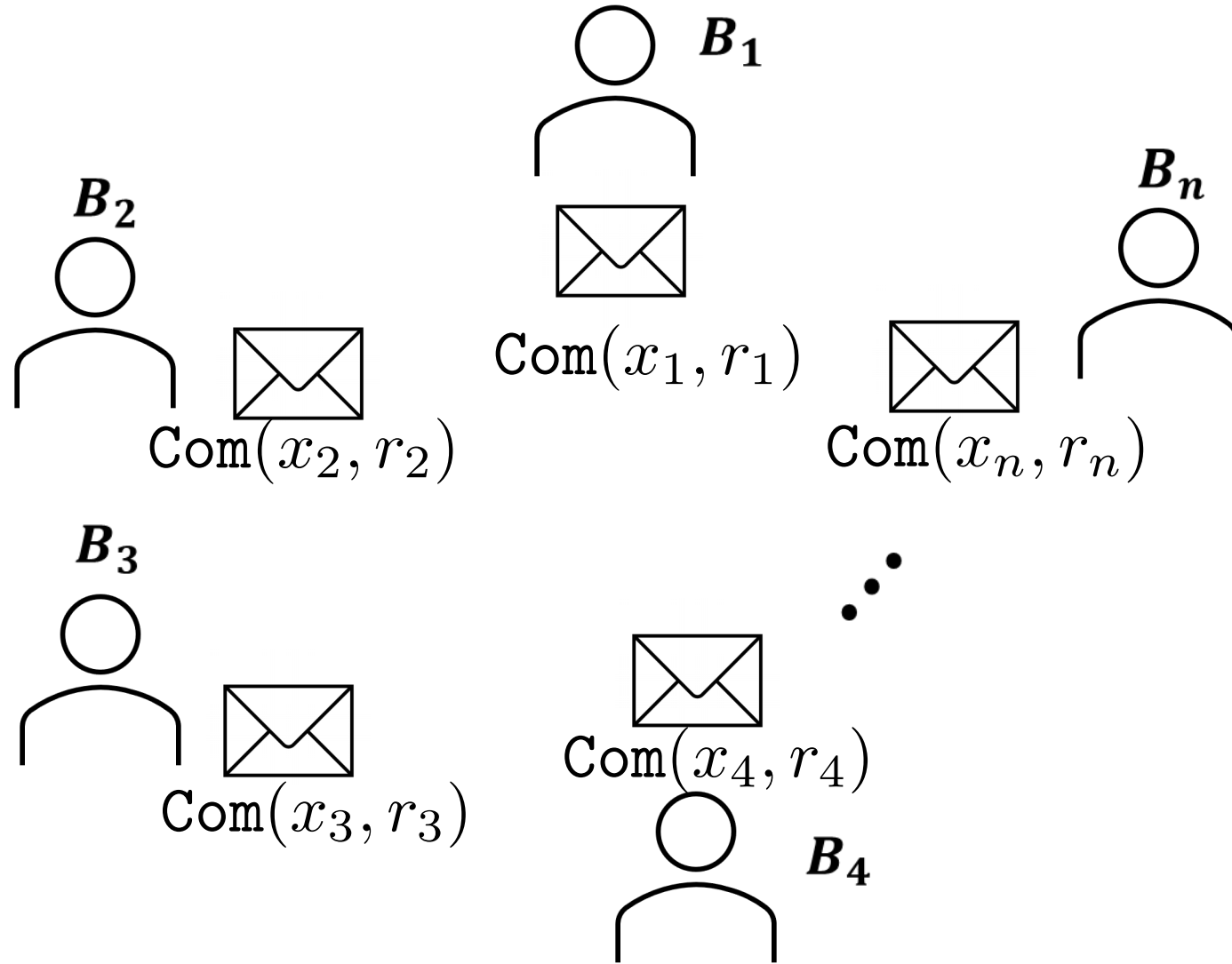


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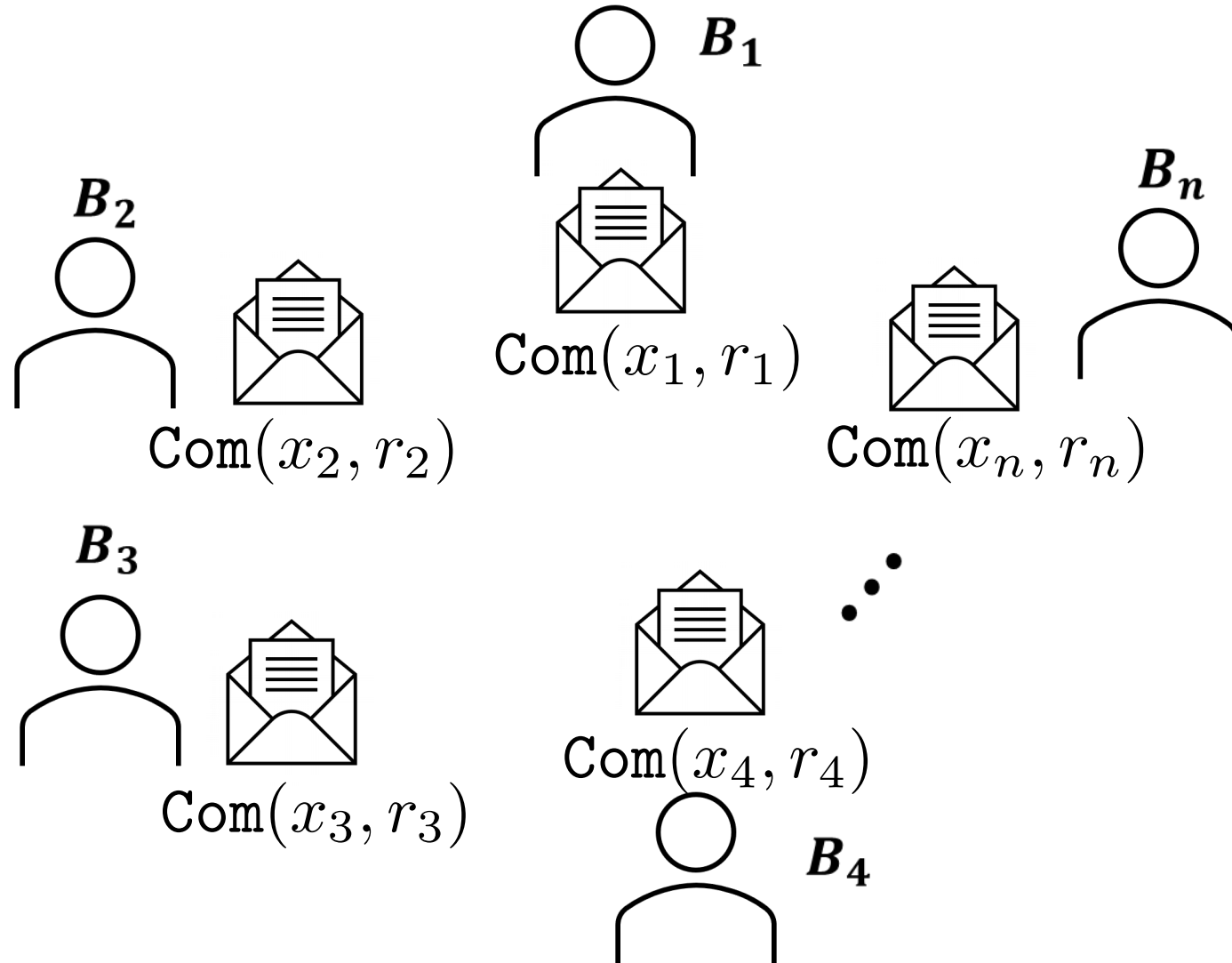
Motivation – Sealed-Bit Auction



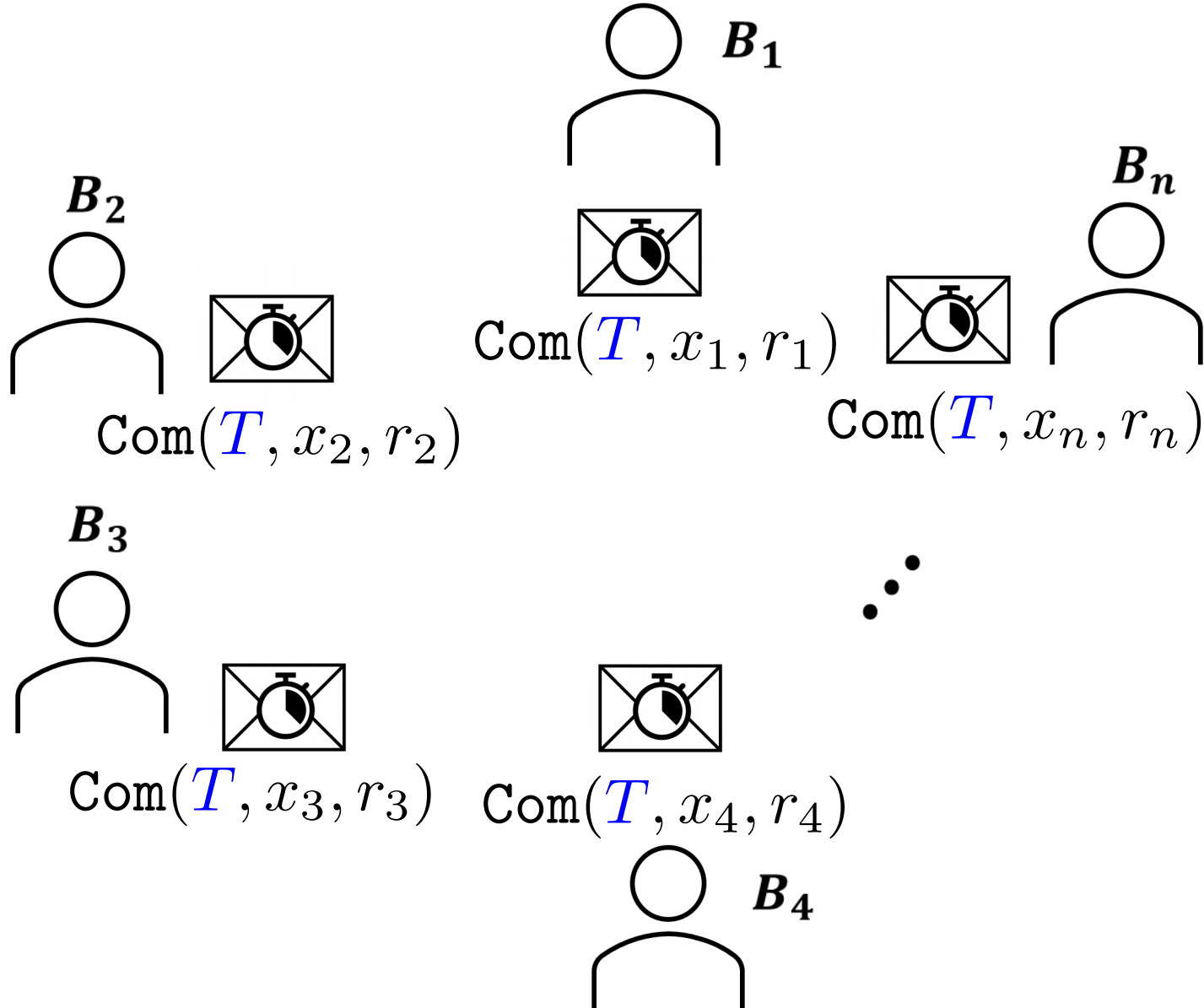
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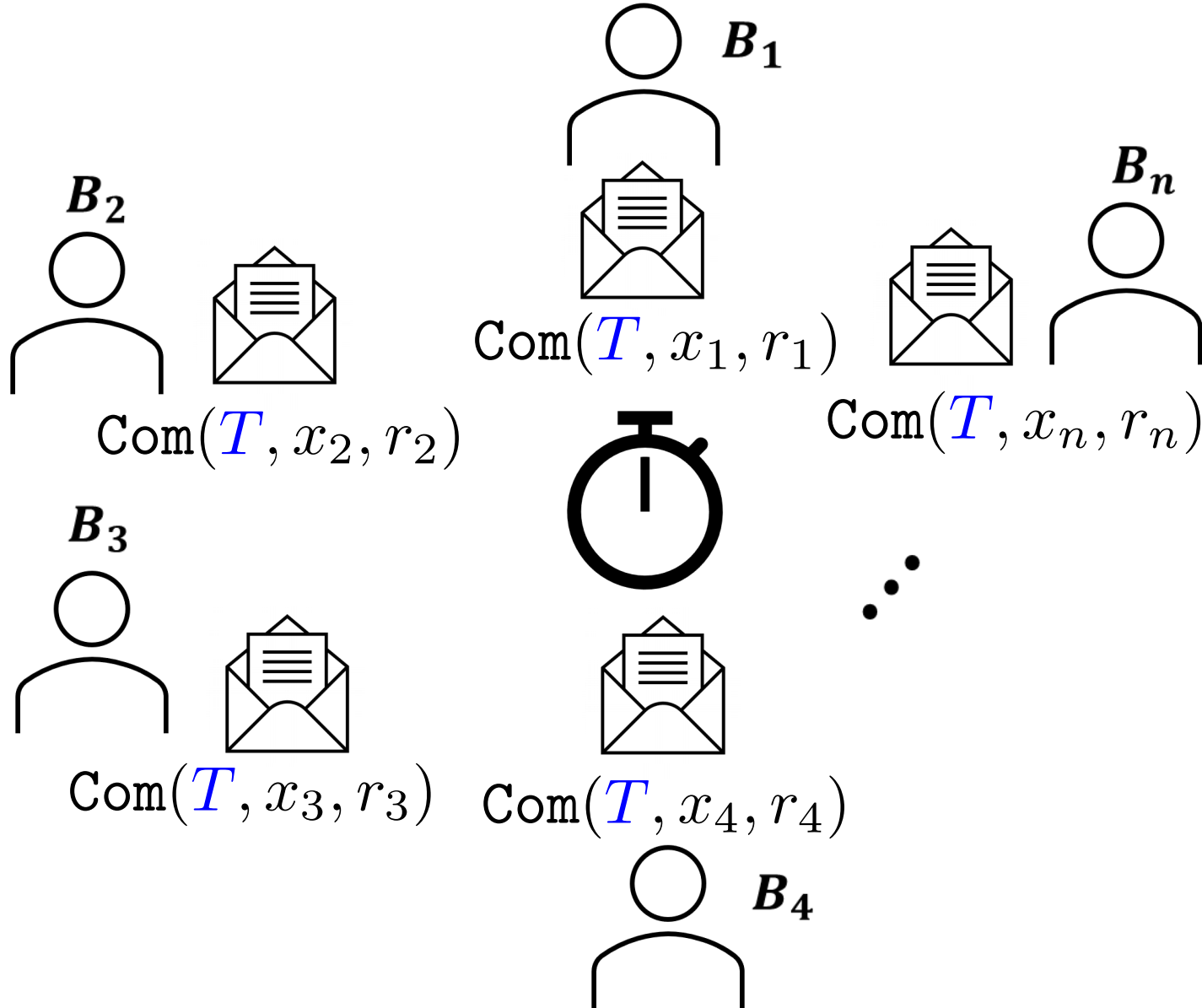
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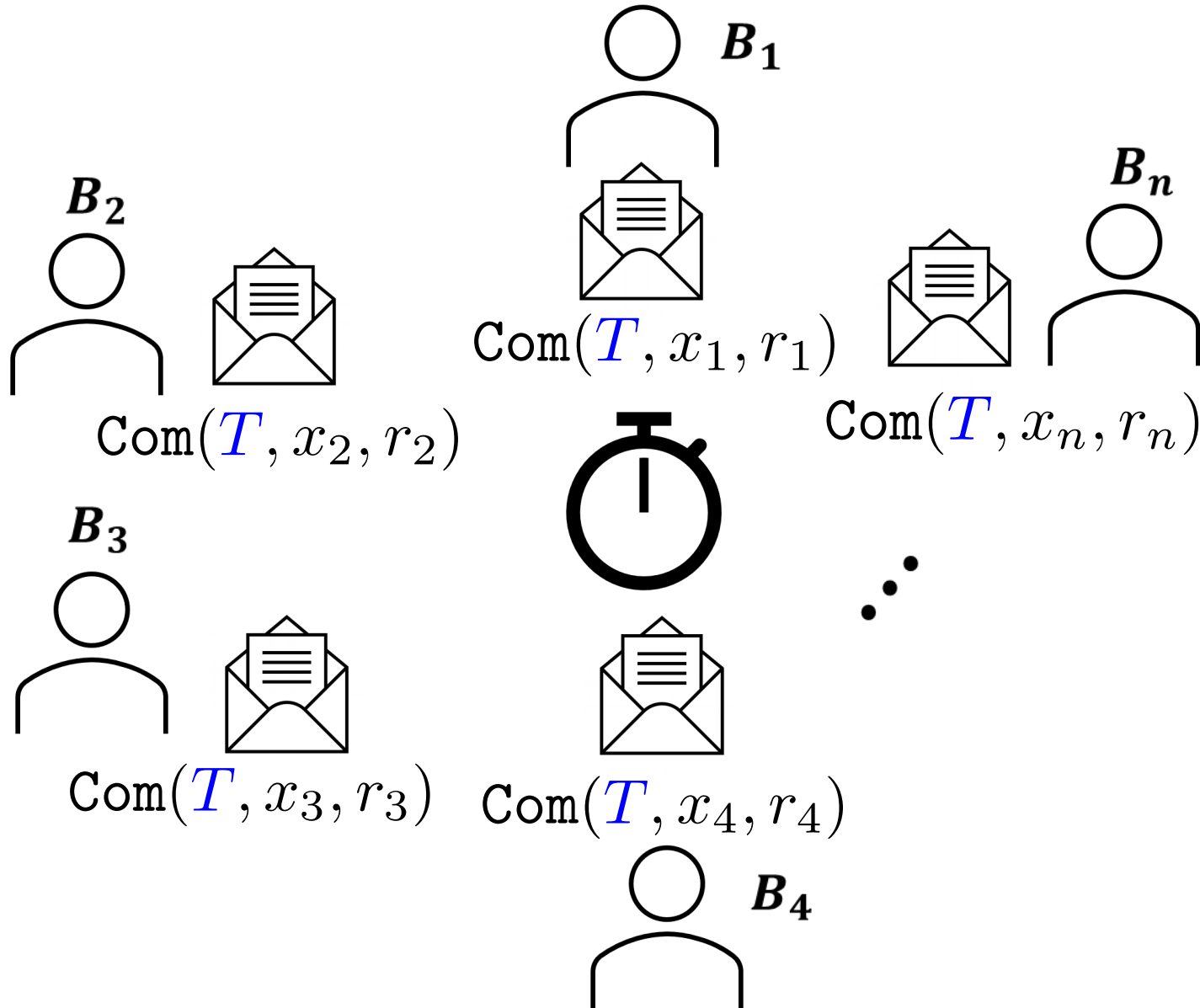
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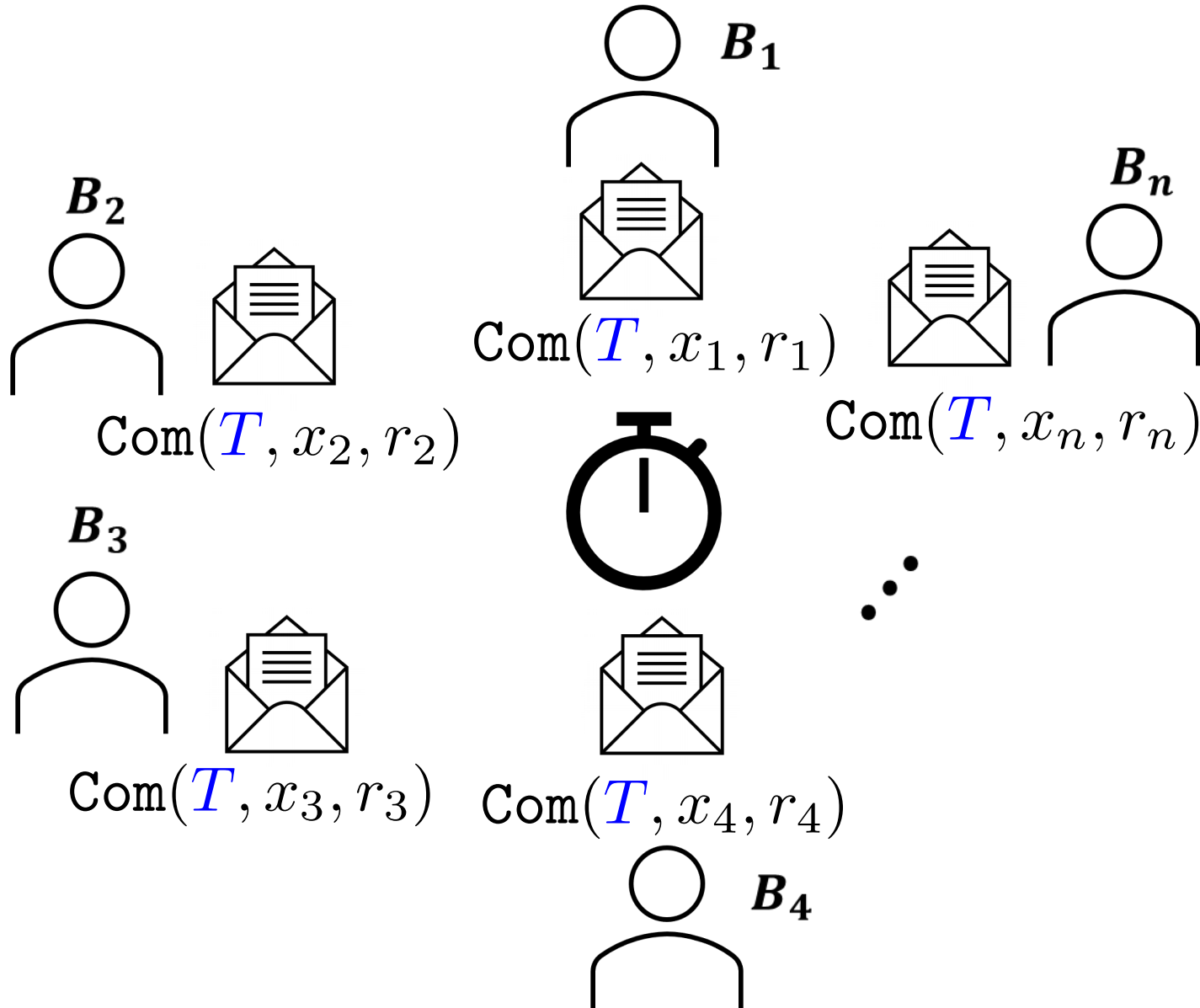


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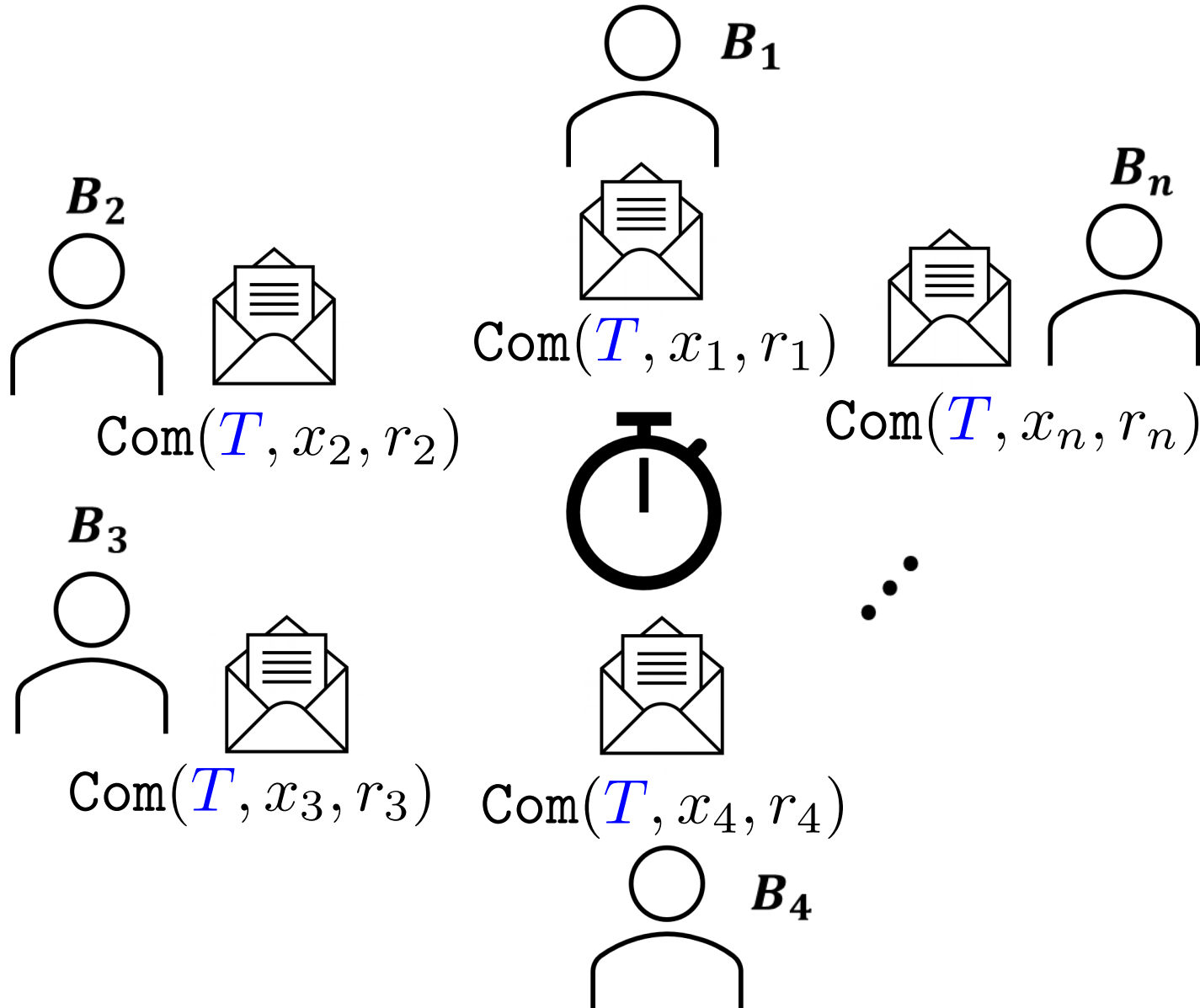
- Public Verifiability of Commitments

Motivation – Sealed-Bit Auction



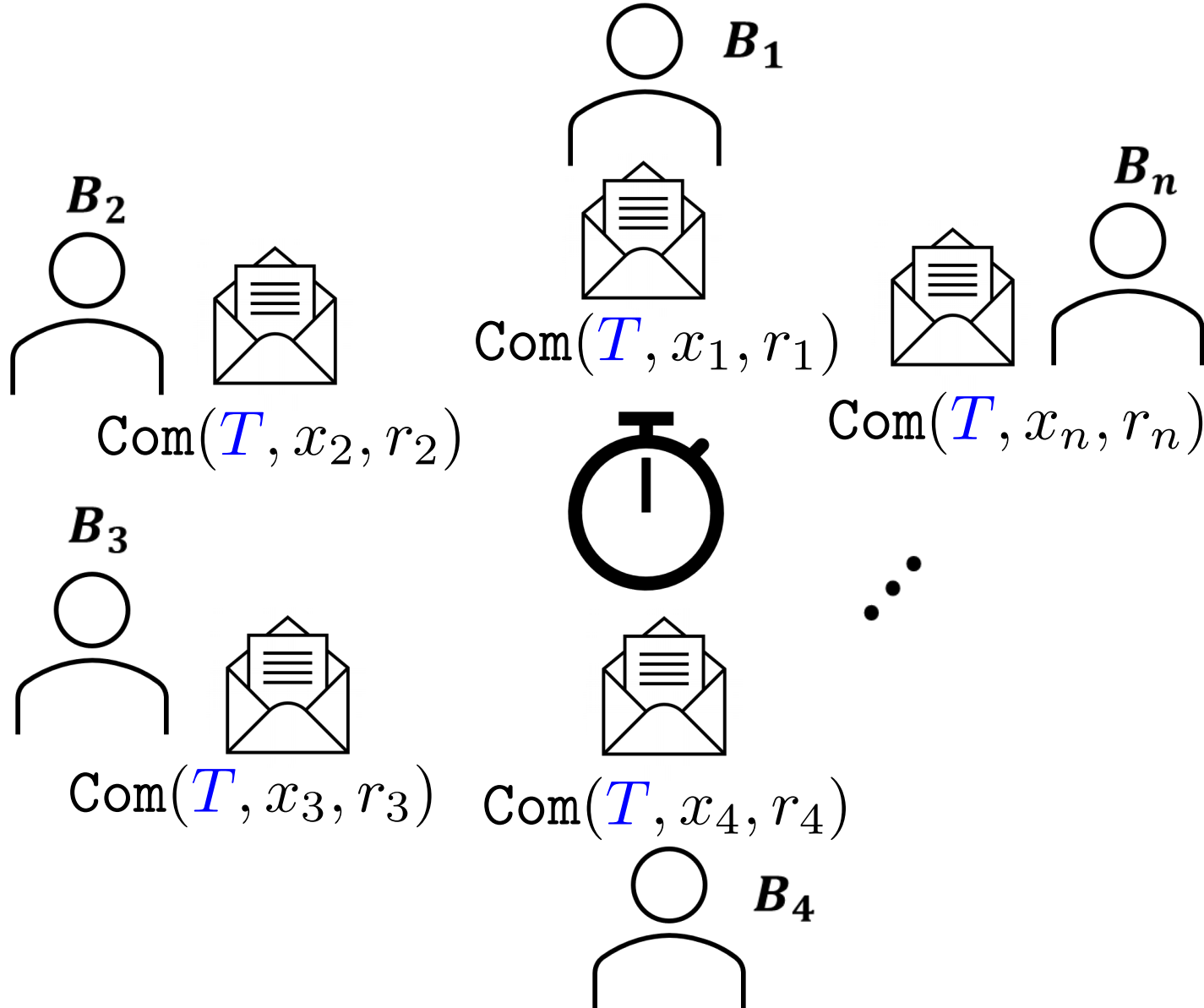
- Public Verifiability of Commitments
- Non-Interactivity

Motivation – Sealed-Bit Auction



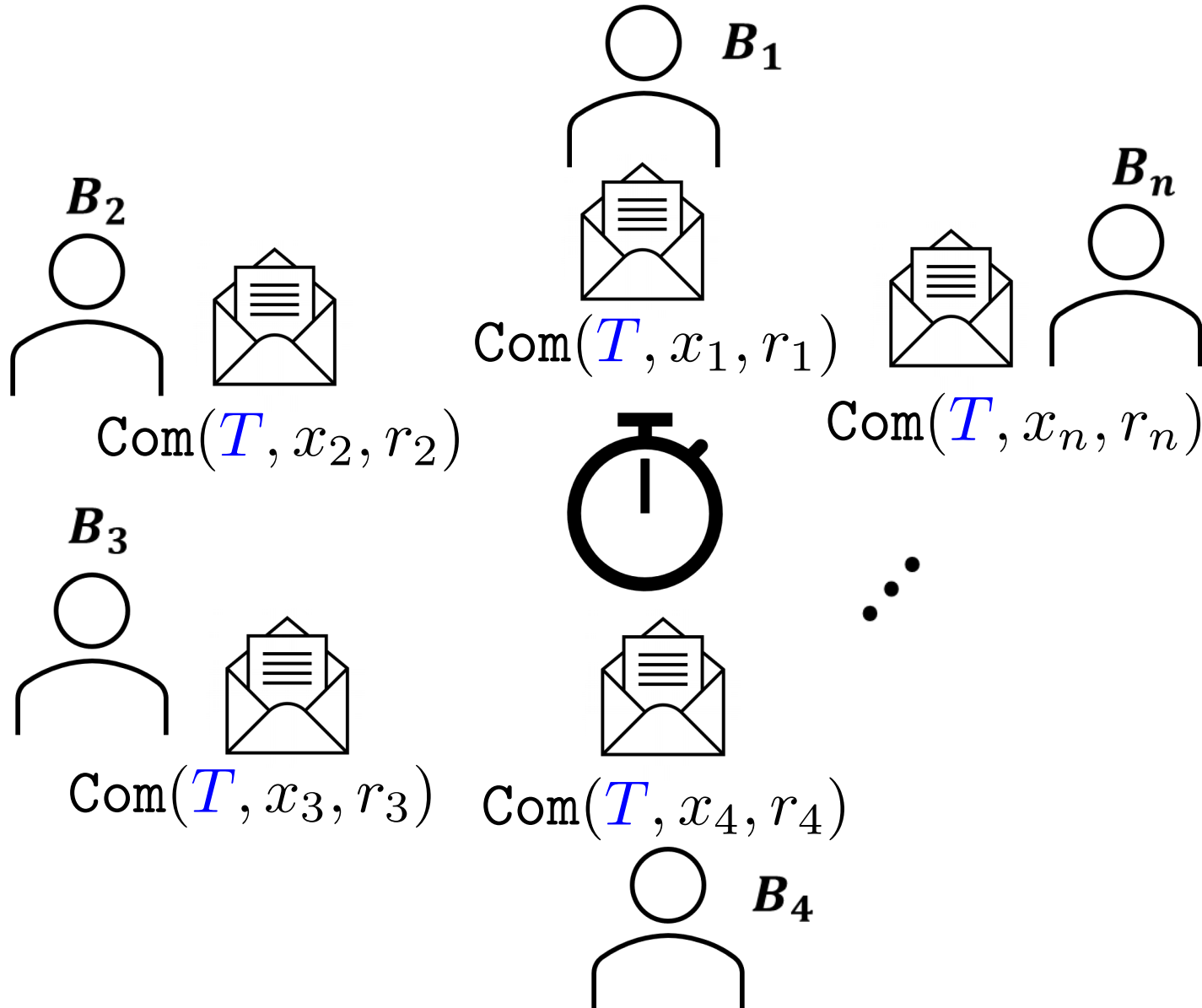
- Public Verifiability of Commitments
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- Non-Malleability/CCA security

Motivation – Sealed-Bit Auction



- Public Verifiability of Commitments
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- Homomorphic Properties

Motivation – Sealed-Bit Auction



- Public Verifiability of Commitments
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- Homomorphic Properties
- **Public Verifiability of Forced Decommitment**

Applications

- Unbiased E-voting (MT19)
- Sealed Bid Auctions (MT19)
- Multi-Party Contract Signing (MT19)
- Fairness in Multi-Party Computation – Fair Coin Flipping (MT19)
- Revealing Census Data
- Responsible Disclosure of Security Flaws

Non-Interactive Timed Commitment (KLX20)

- $\text{crs} \leftarrow \text{PGen}(1^\lambda, T)$
- $(c, \pi_{\text{Com}}, \pi_{\text{Dec}}) \leftarrow \text{Com}(\text{crs}, m)$
- $0/1 \leftarrow \text{ComVrfy}(\text{crs}, c, \pi_{\text{Com}})$
- $0/1 \leftarrow \text{DecVrfy}(\text{crs}, c, m, \pi_{\text{Dec}})$
- $m \leftarrow \text{FDec}(\text{crs}, c)$ - runtime T

Non-Interactive Timed Commitment **Publicly Verifiable**

(KLX20)

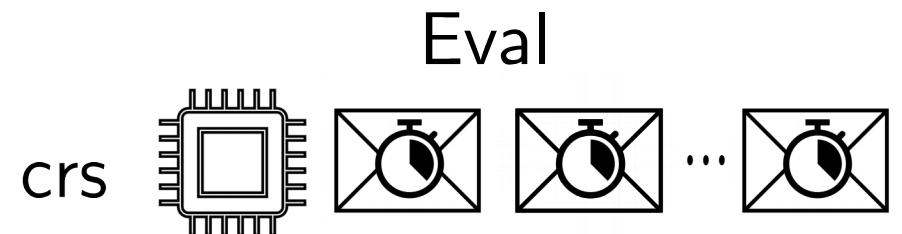
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Non-Interactive Timed Commitment (KLX20) Publicly Verifiable Homomorphic (TCLM21)

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- Eval

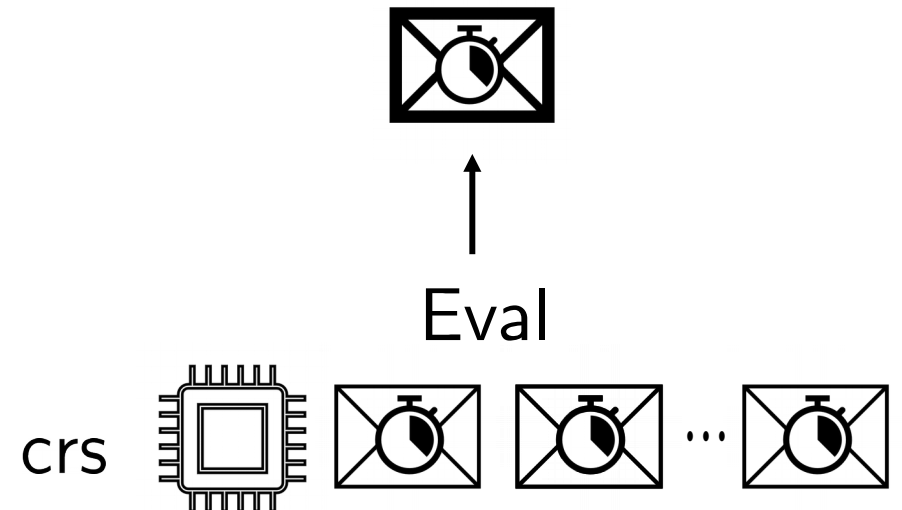
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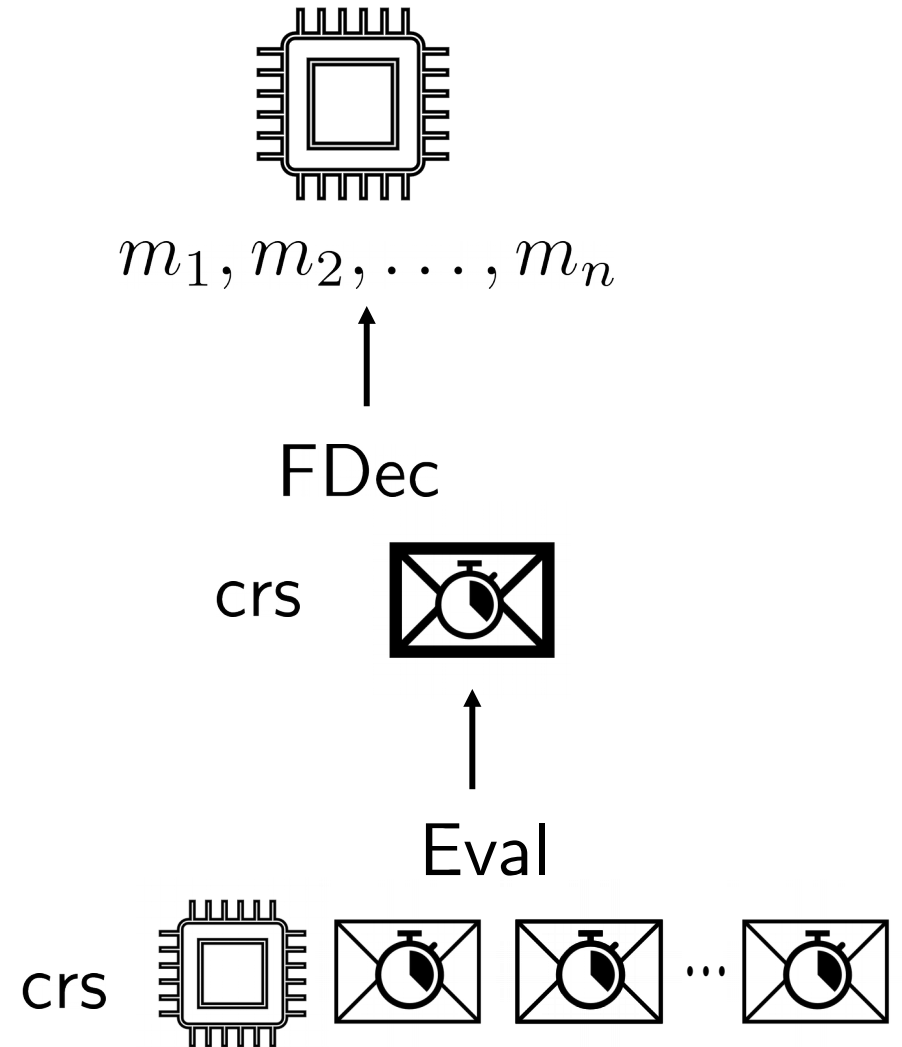
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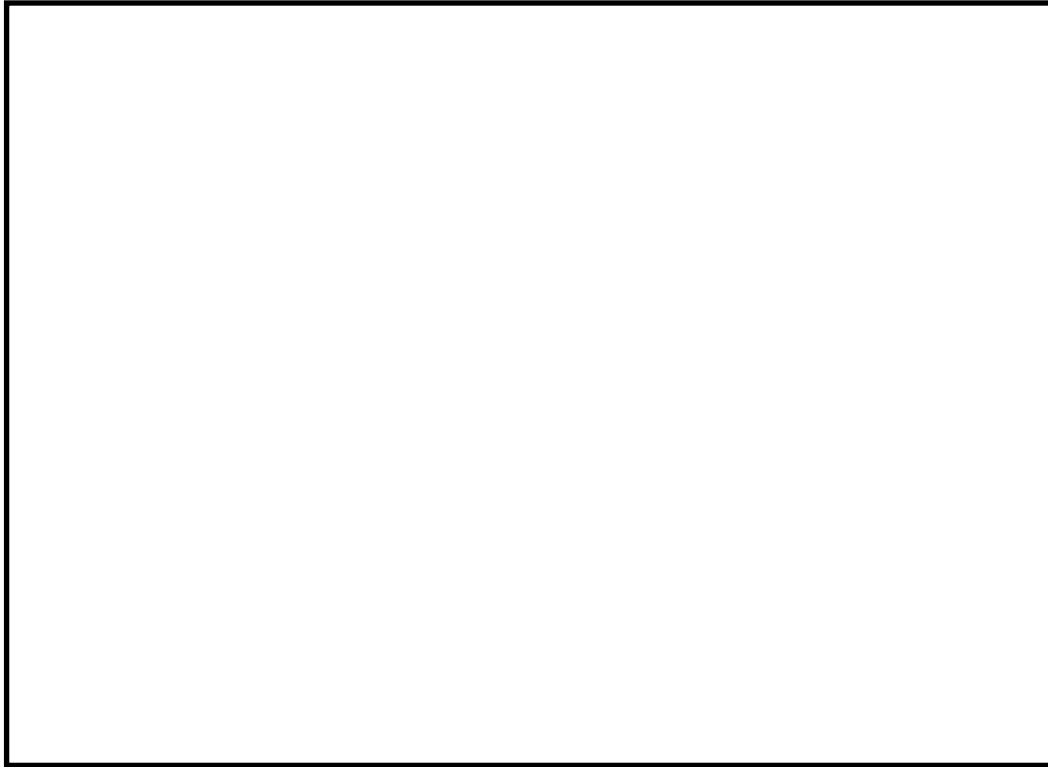
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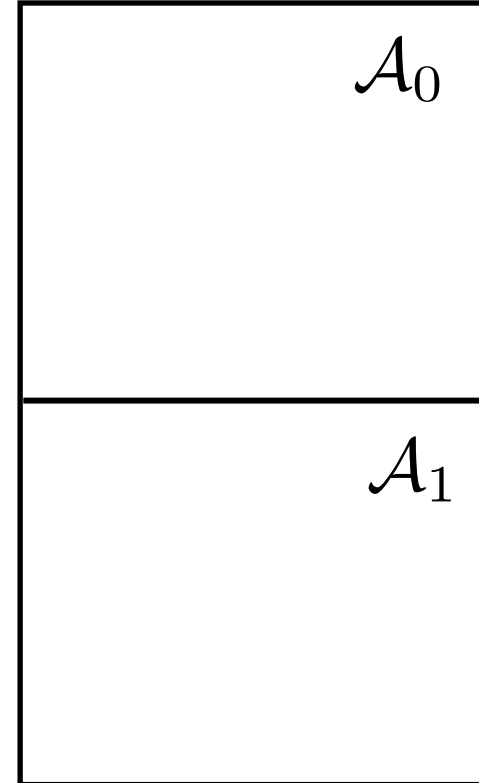
IND-CCA Security (KLX20)

Challenger



$\text{ExpNITC}_{\mathcal{A}}(\lambda)$

\mathcal{A}



IND-CCA Security (KLX20)

Challenger

$\text{crs} \leftarrow \text{PGen}(1^\lambda, T)$

$\text{ExpNITC}_{\mathcal{A}}(\lambda)$

\mathcal{A}

\mathcal{A}_0

\mathcal{A}_1

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\mathcal{A}

$\text{crs} \leftarrow \text{PGen}(1^\lambda, T)$

crs

\mathcal{A}_0

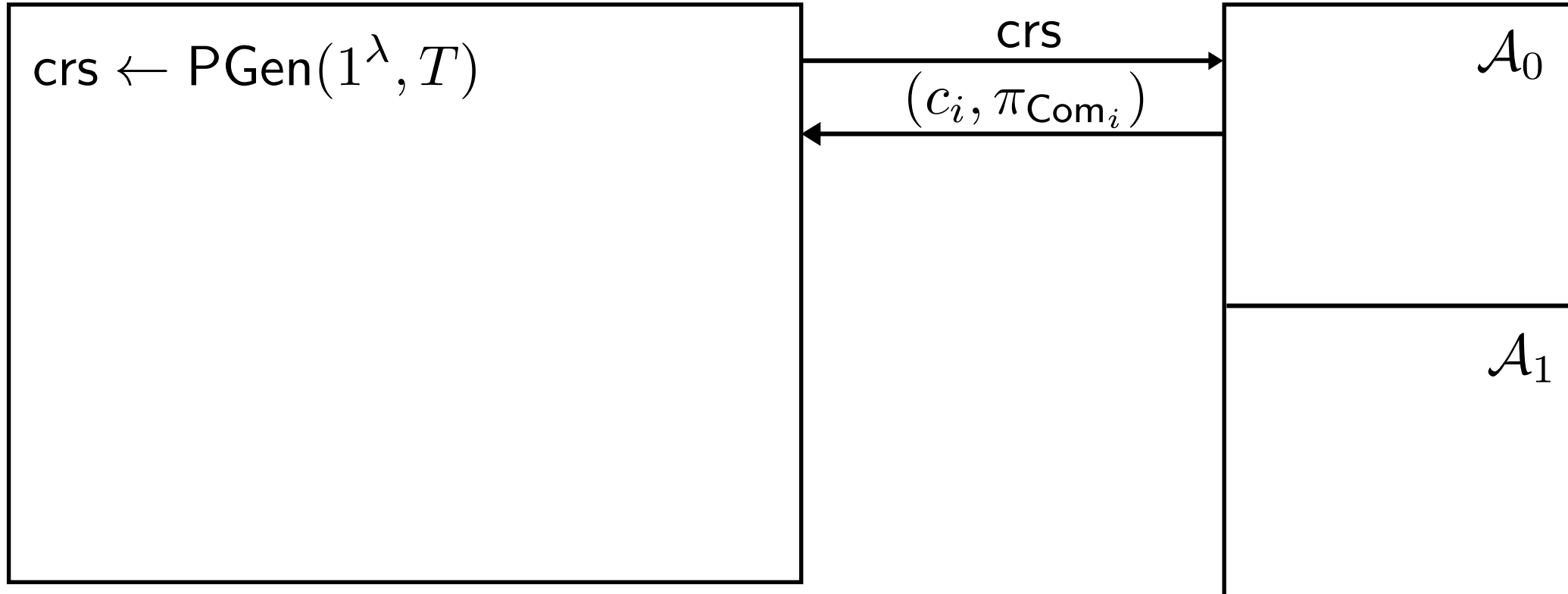
\mathcal{A}_1

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$\text{crs} \leftarrow \text{PGen}(1^\lambda, T)$
if $\text{ComVrfy}(\text{crs}, c_i, \pi_{\text{Com}_i}) = 1$

crs

$(c_i, \pi_{\text{Com}_i})$

\mathcal{A}_0

\mathcal{A}_1

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$\text{FDec}(\text{crs}, c_i)$

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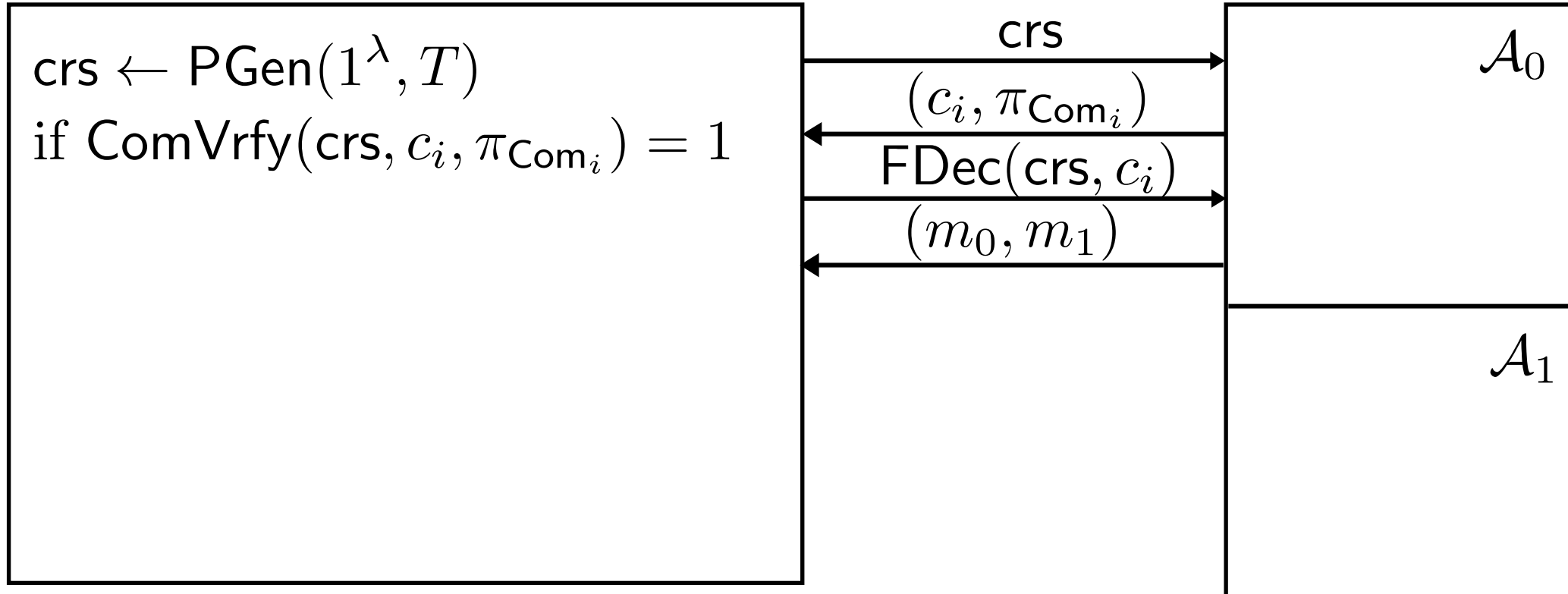
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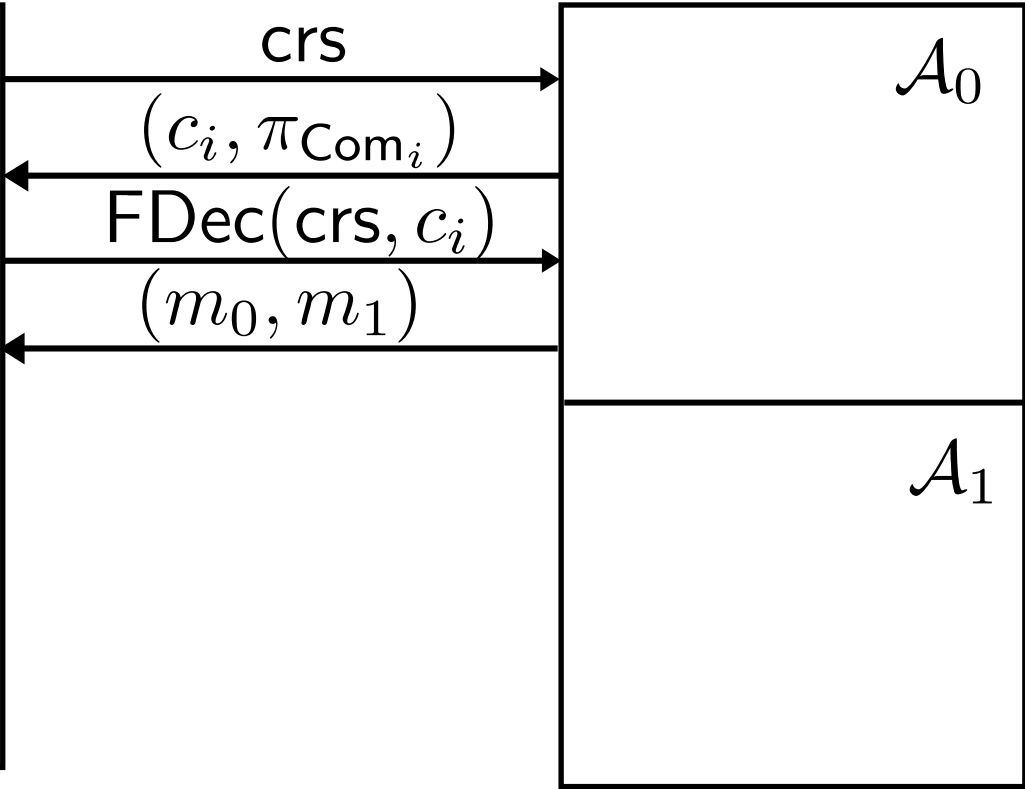
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 $(c, \pi_{\text{Com}}, \pi_{\text{Dec}}) \leftarrow \text{Com}(\text{crs}, m_b)$



IND-CCA Security (KLX20)

Challenger

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crs

$(c_i, \pi_{\text{Com}_i})$

$\text{FDec}(\text{crs}, c_i)$

(m_0, m_1)

(c, π_{Com})

\mathcal{A}_0

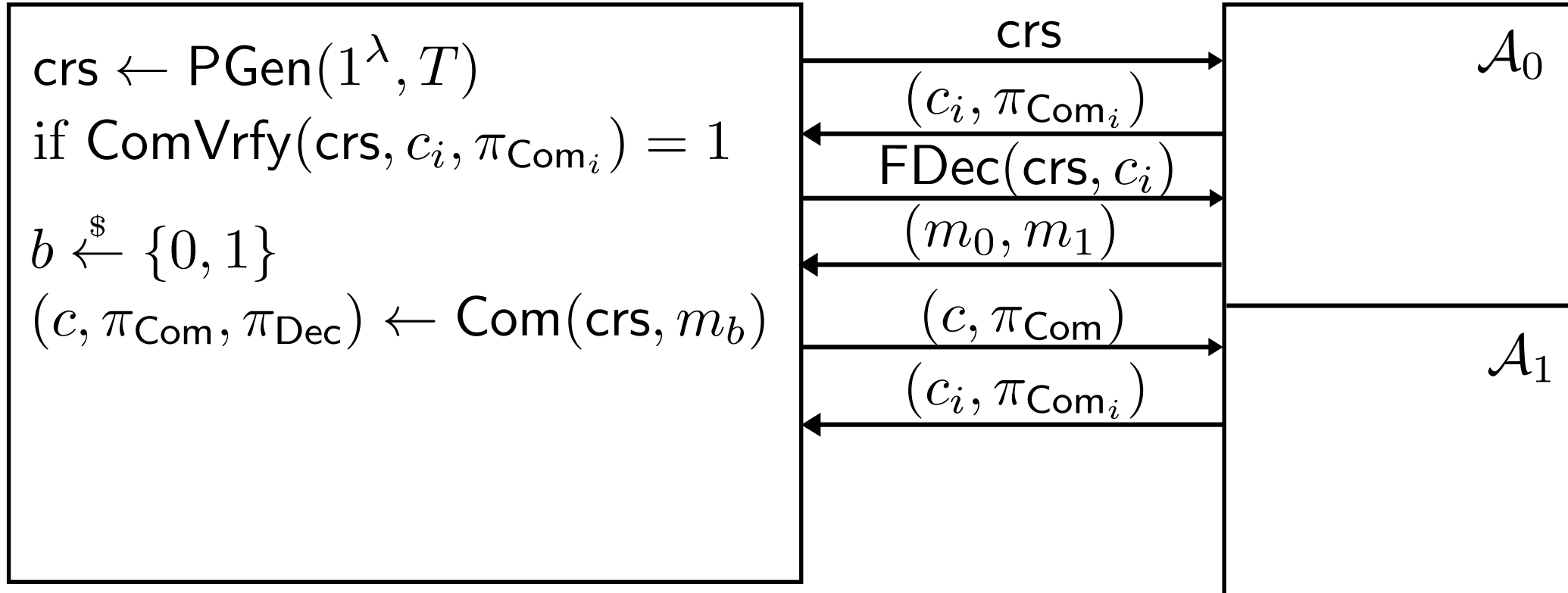
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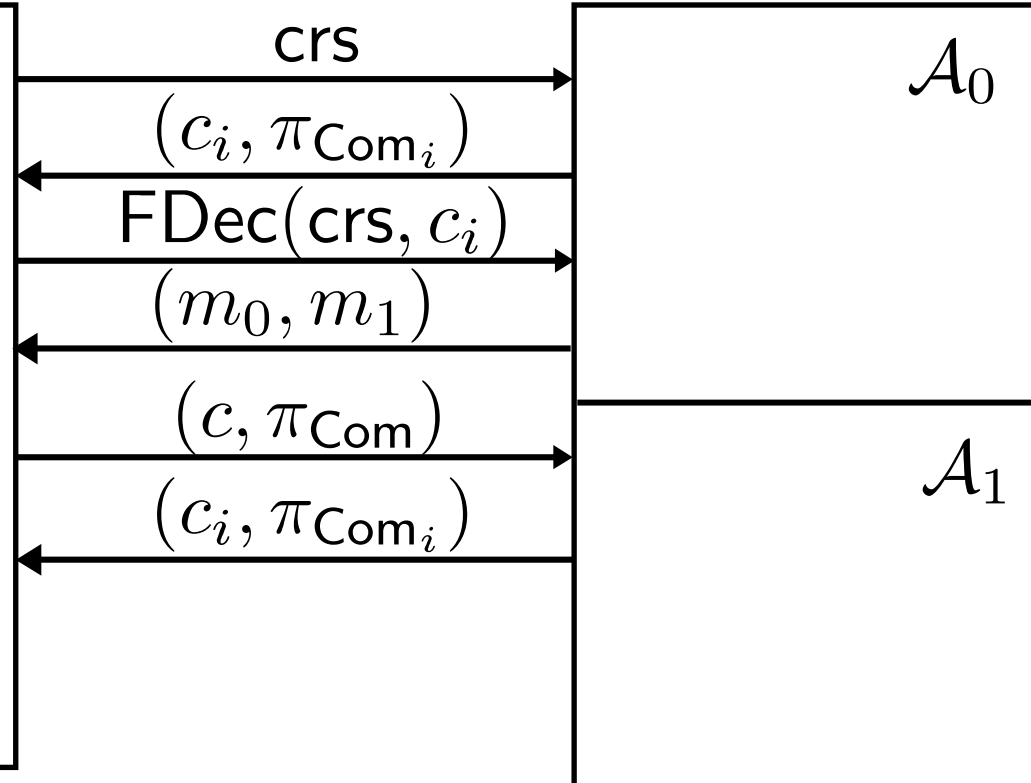
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 $(c, \pi_{\text{Com}}, \pi_{\text{Dec}}) \leftarrow \text{Com}(\text{crs}, m_b)$
if $\text{ComVrfy}(\text{crs}, c_i, \pi_{\text{Com}_i}) = 1 \wedge$
 $(c, \pi_{\text{Com}}) \neq (c_i, \pi_{\text{Com}_i})$

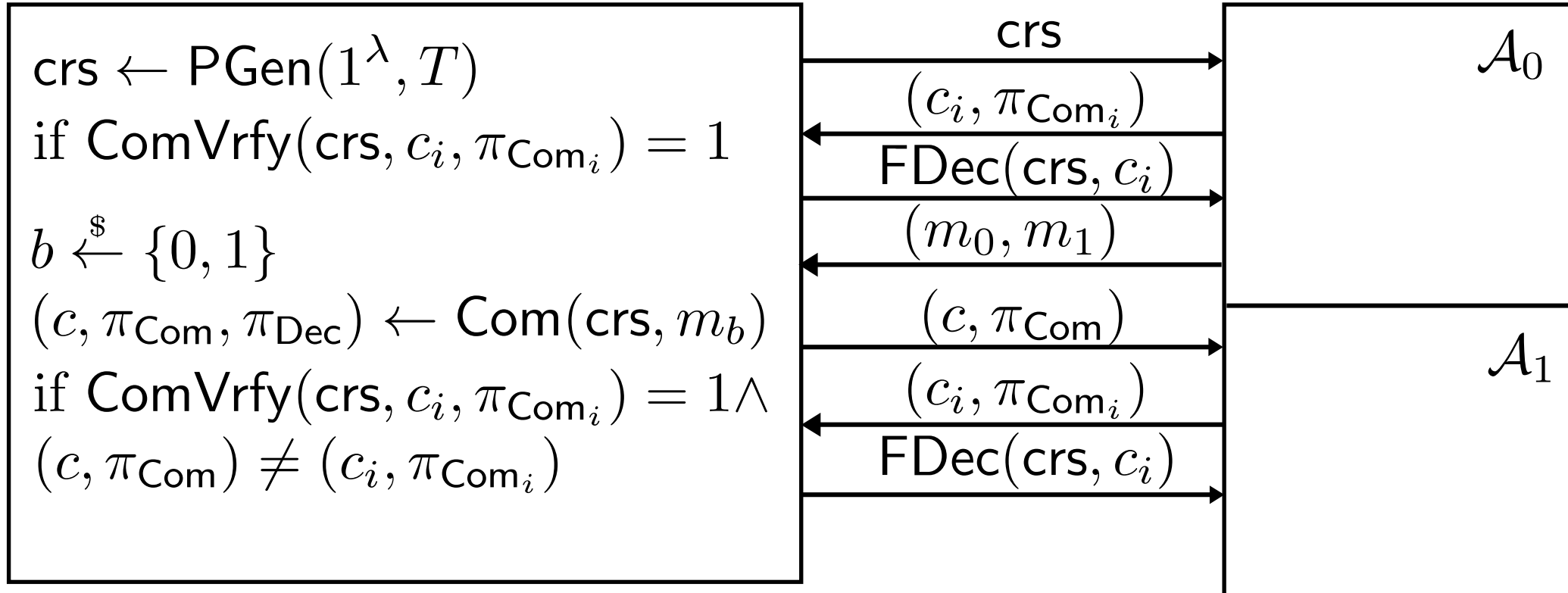


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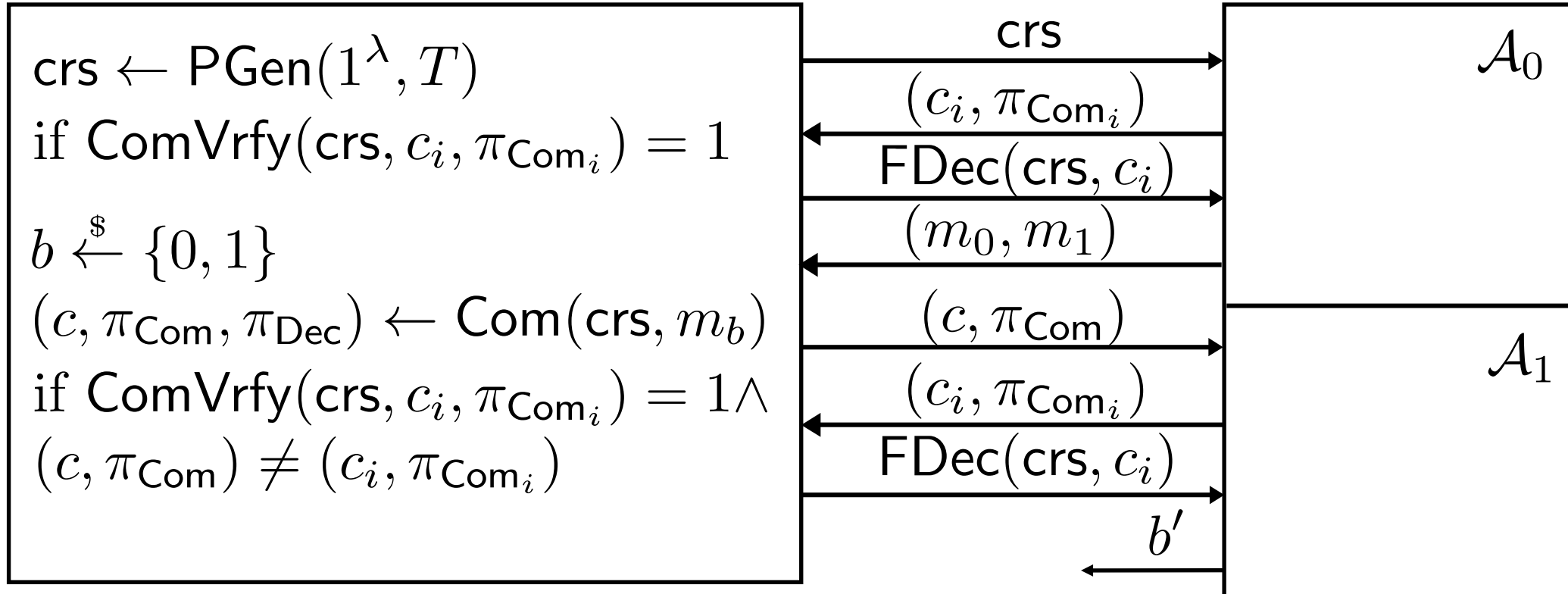


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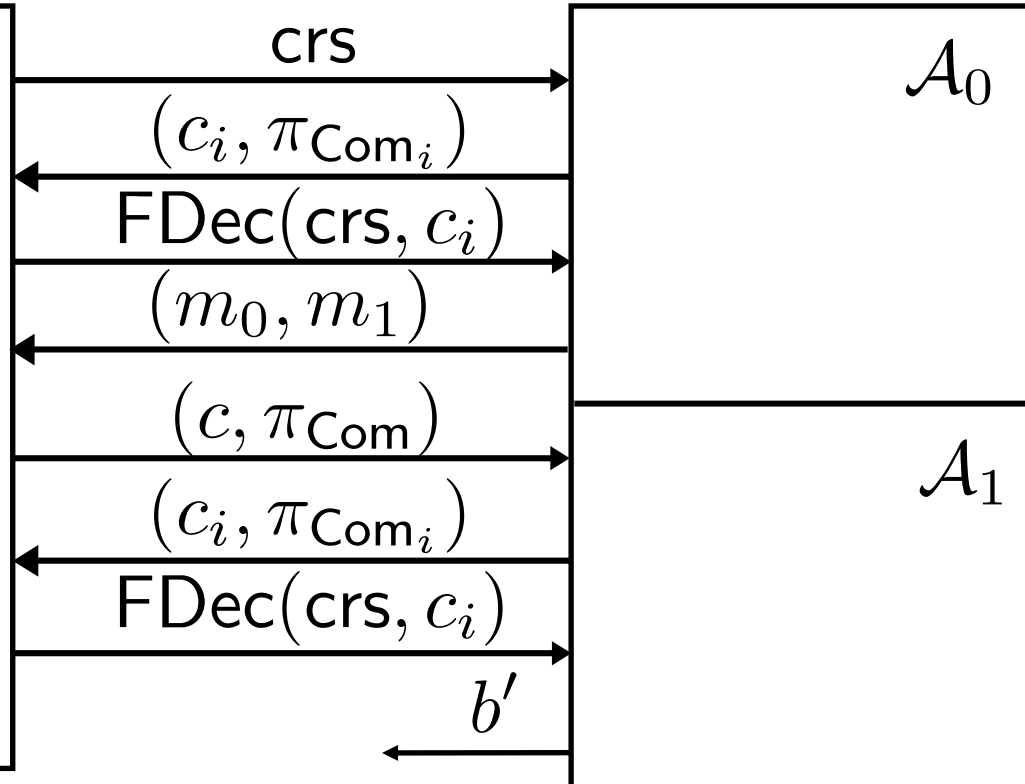
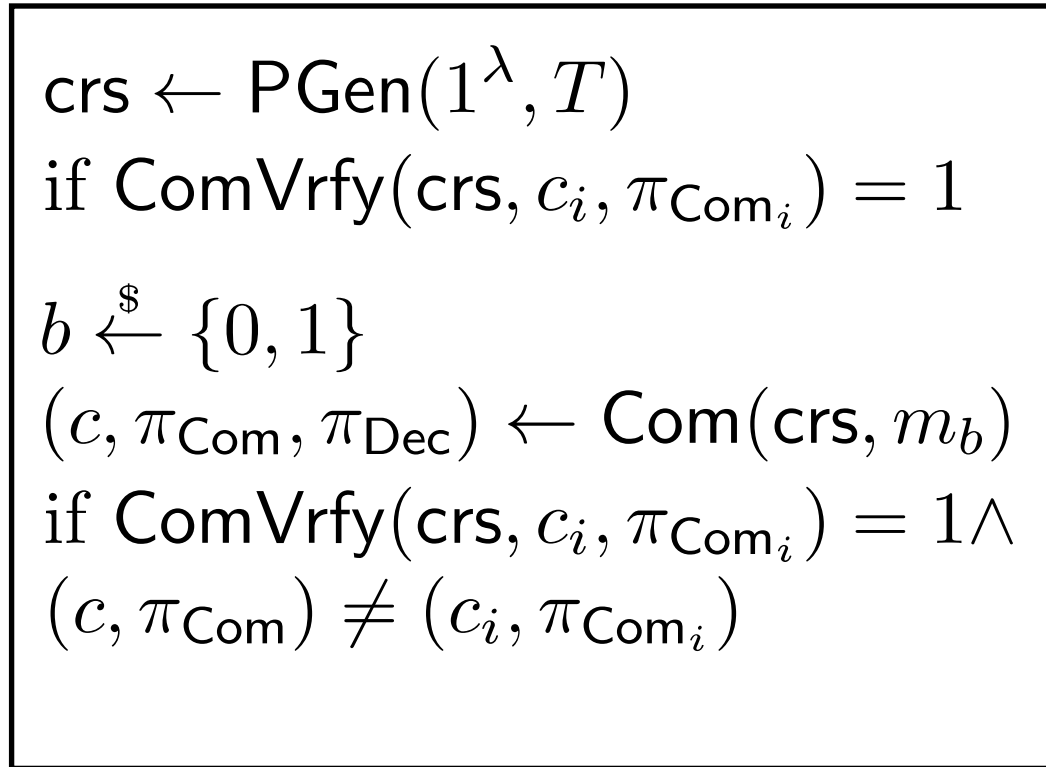


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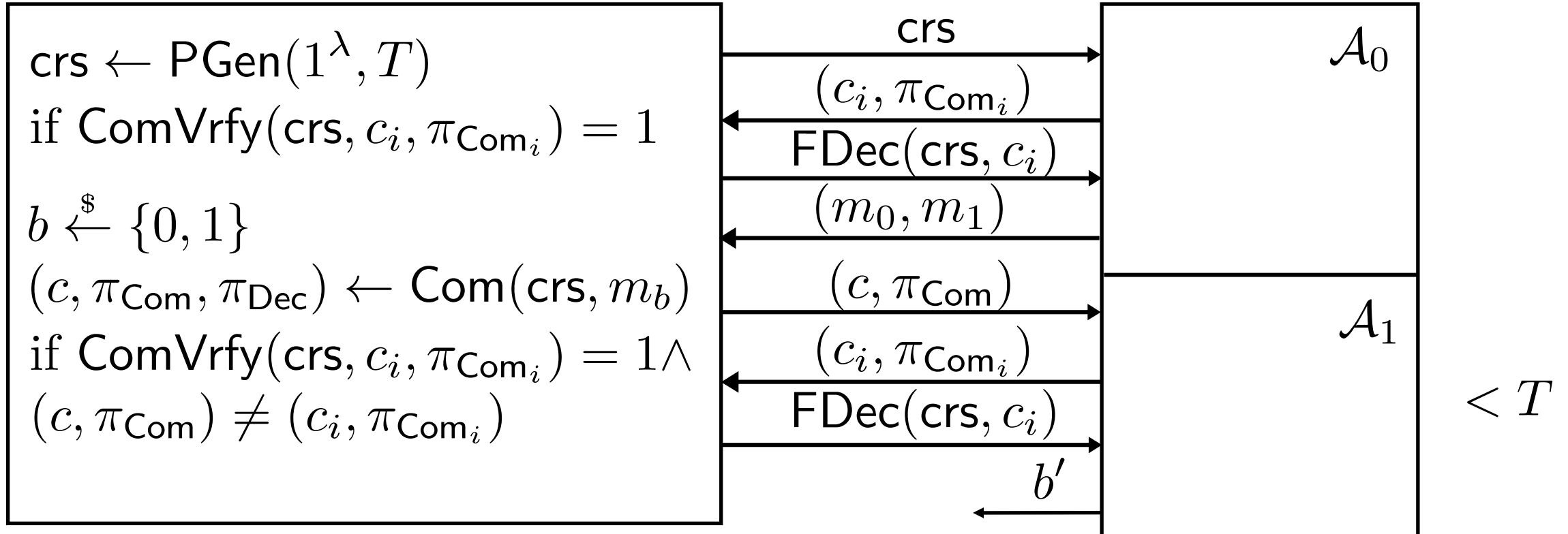
$< T$

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Output of $\text{ExpNITC}_{\mathcal{A}} : b = b'$

$$\text{Adv}_{\mathcal{A}}^{\text{NITC}} = \left| \Pr[\text{ExpNITC}_{\mathcal{A}}(\lambda) = 1] - \frac{1}{2} \right| \leq \text{negl}(\lambda)$$

Publicly Verifiable CCA Secure PKE

Naor-Yung Paradigm (NY90)

- 2x CPA secure PKE and One-Time Simulation Sound NIZK

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Efficient OT-SS NIZK

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- Only for algebraic languages

Publicly Verifiable CCA Secure PKE

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Efficient OT-SS NIZK

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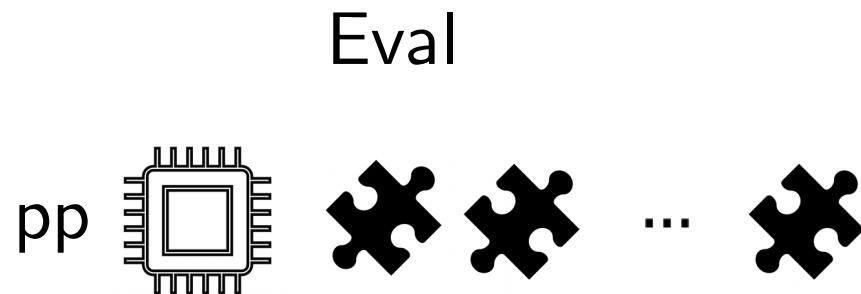
A challenge: Find a replacement to substitute PKE.

Homomorphic TLP (MT19)

- $pp \leftarrow \text{Setup}(1^\lambda, T)$
- $Z \leftarrow \text{Gen}(pp, m)$
- $m \leftarrow \text{Solve}(pp, Z)$
- Eval

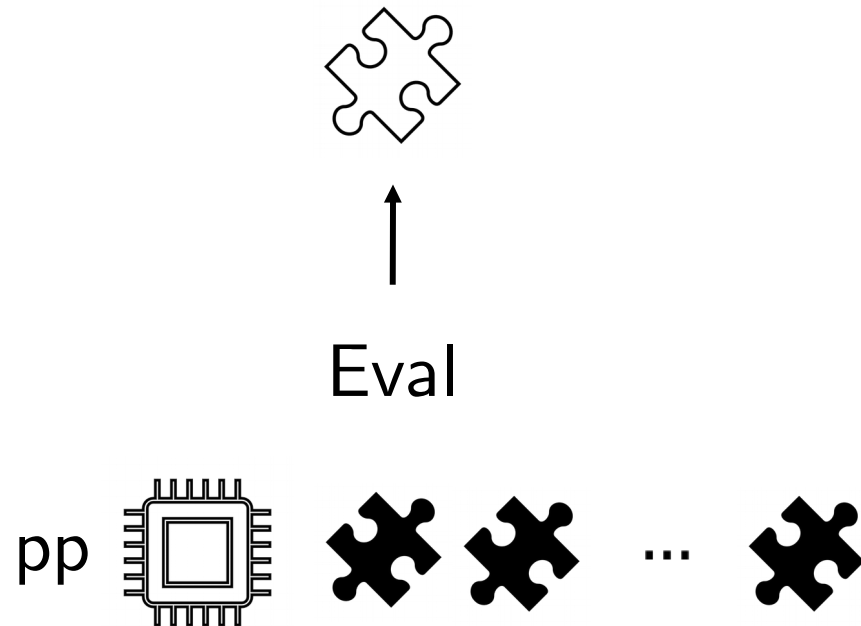
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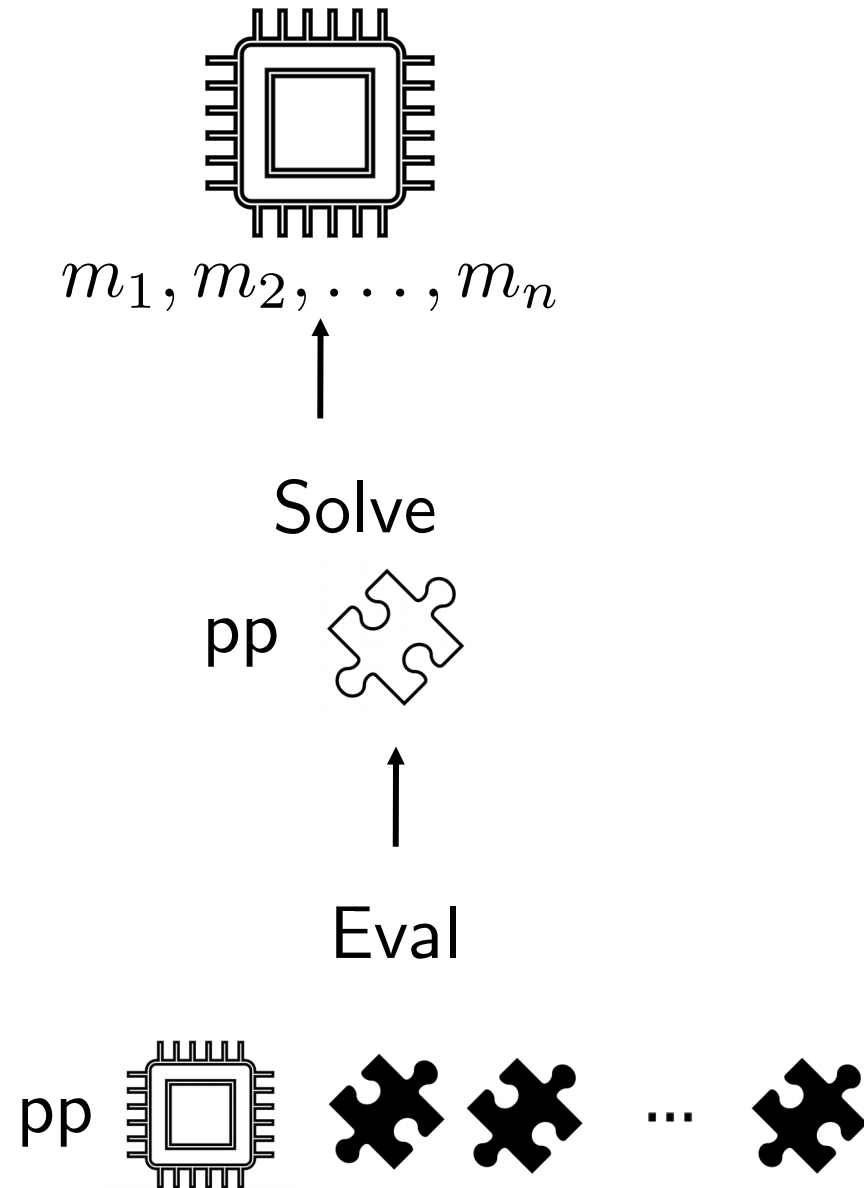
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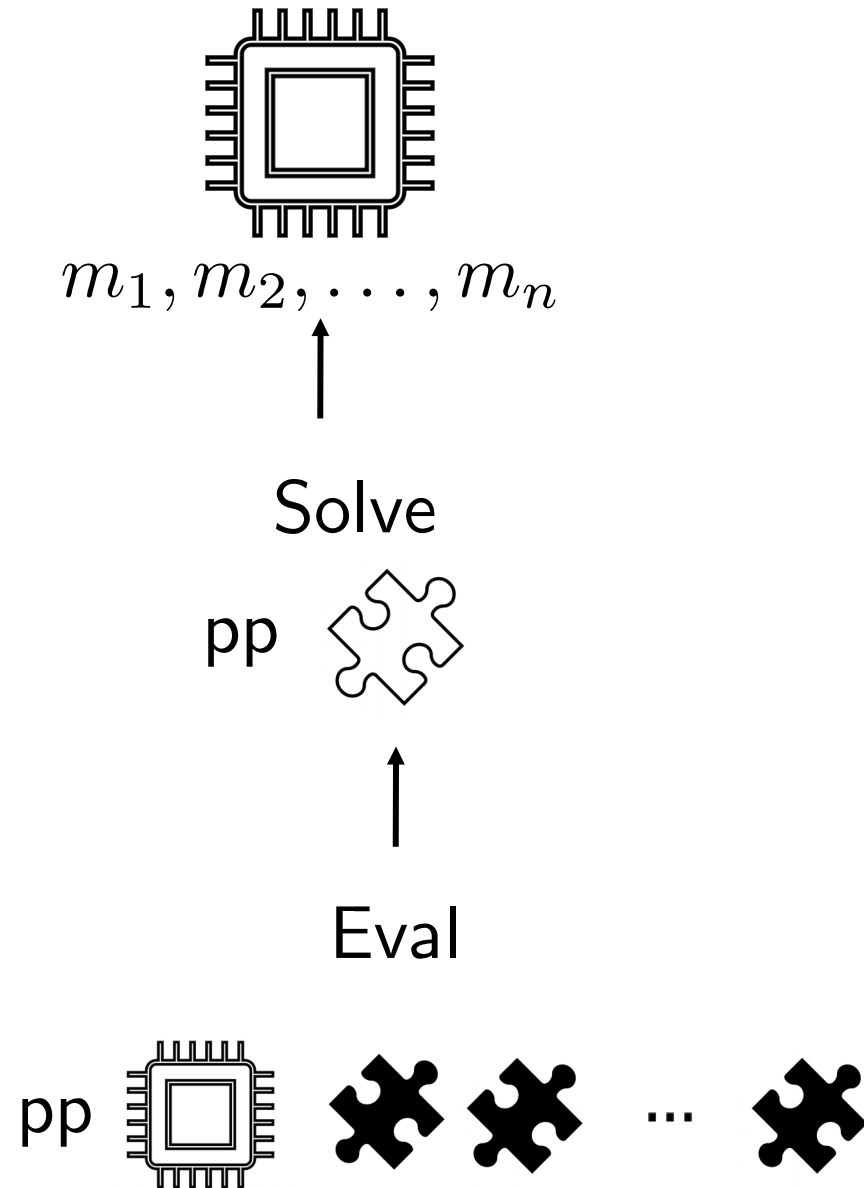
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Homomorphic TLP (MT19)

- $pp \leftarrow \text{Setup}(1^\lambda, T)$ PGen
- $Z \leftarrow \text{Gen}(pp, m)$ Com
- $m \leftarrow \text{Solve}(pp, Z)$ FDec
- Eval



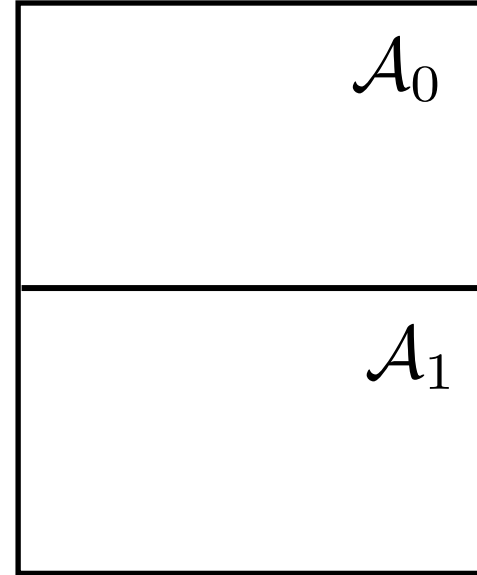
Strong Sequential Squaring Assumption (MT19)

Challenger



$\text{ExpSSS}_{\mathcal{A}}^b(\lambda)$

\mathcal{A}



Strong Sequential Squaring Assumption (MT19)

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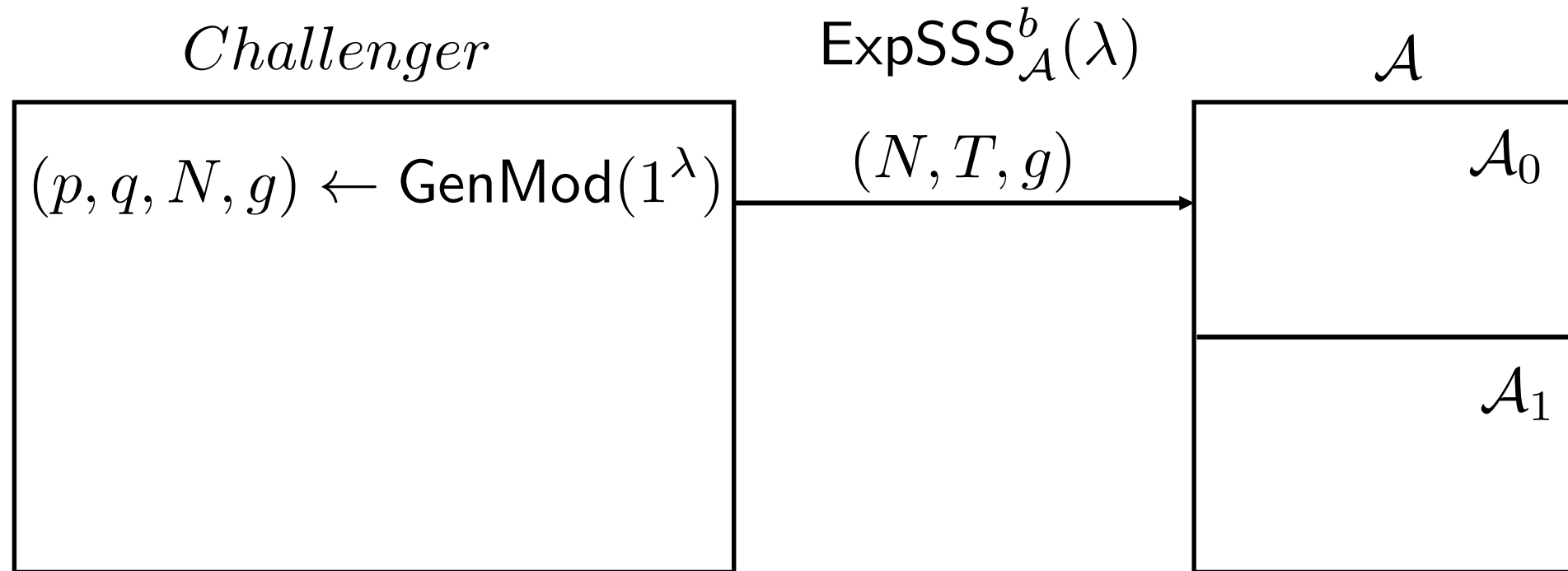
$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

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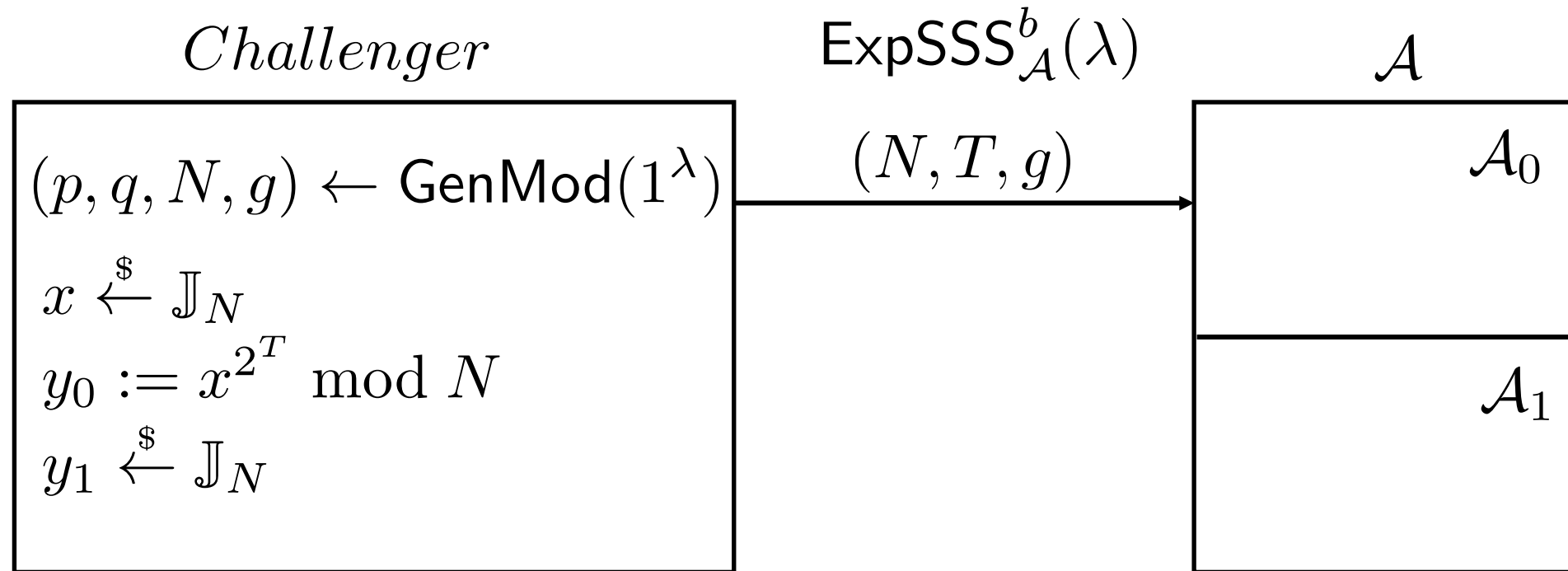
\mathcal{A}

\mathcal{A}_0
\mathcal{A}_1

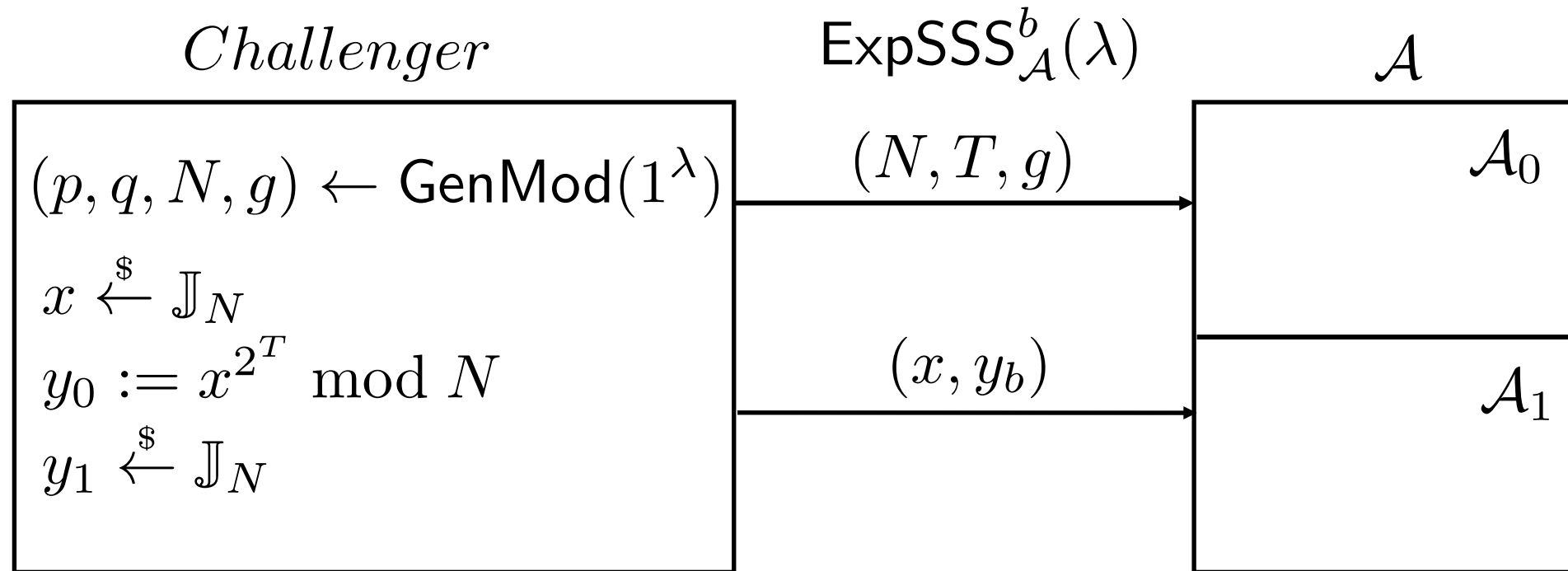
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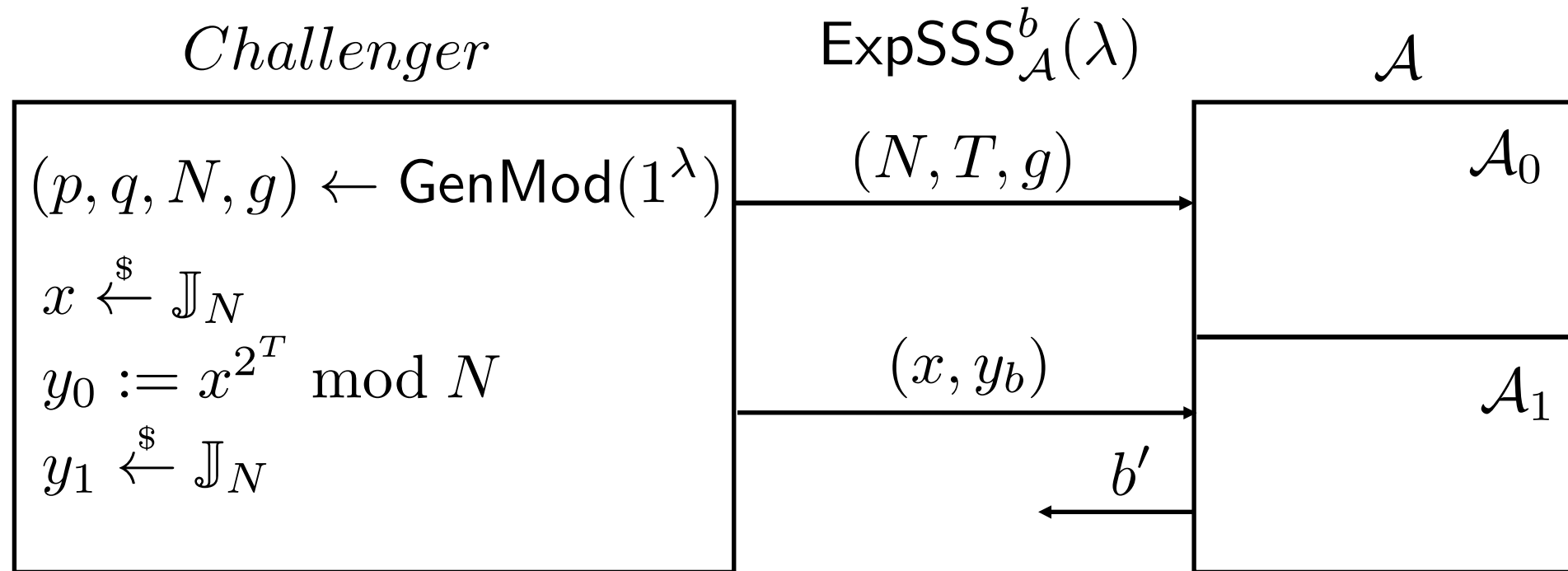
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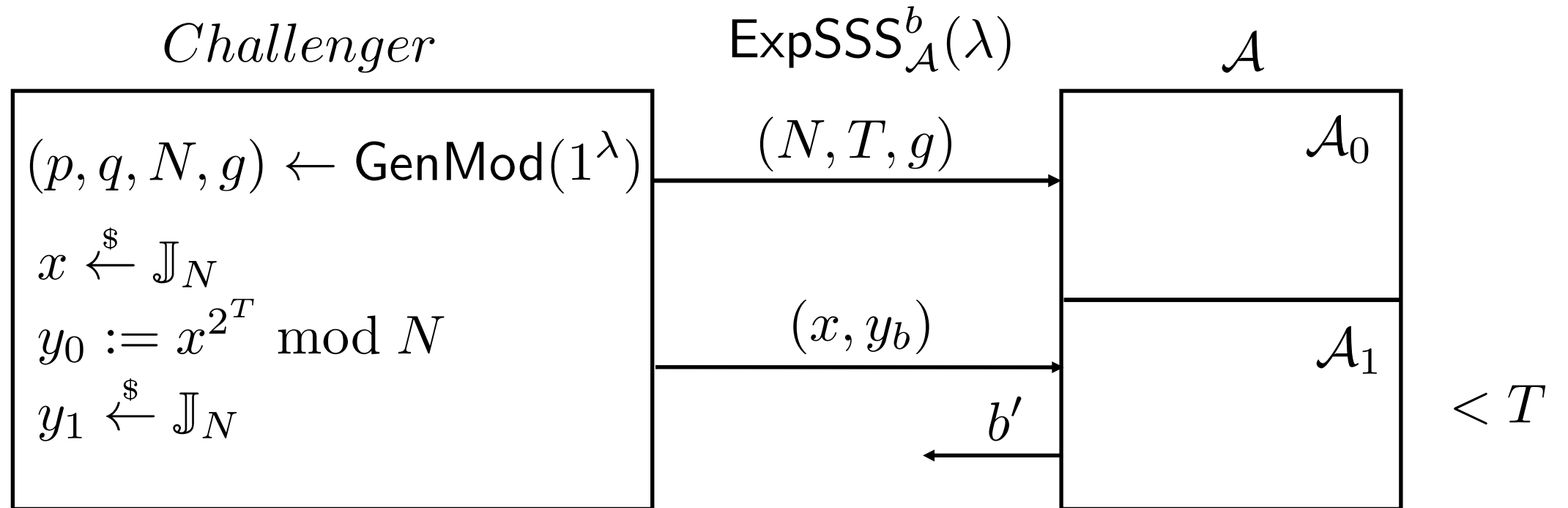
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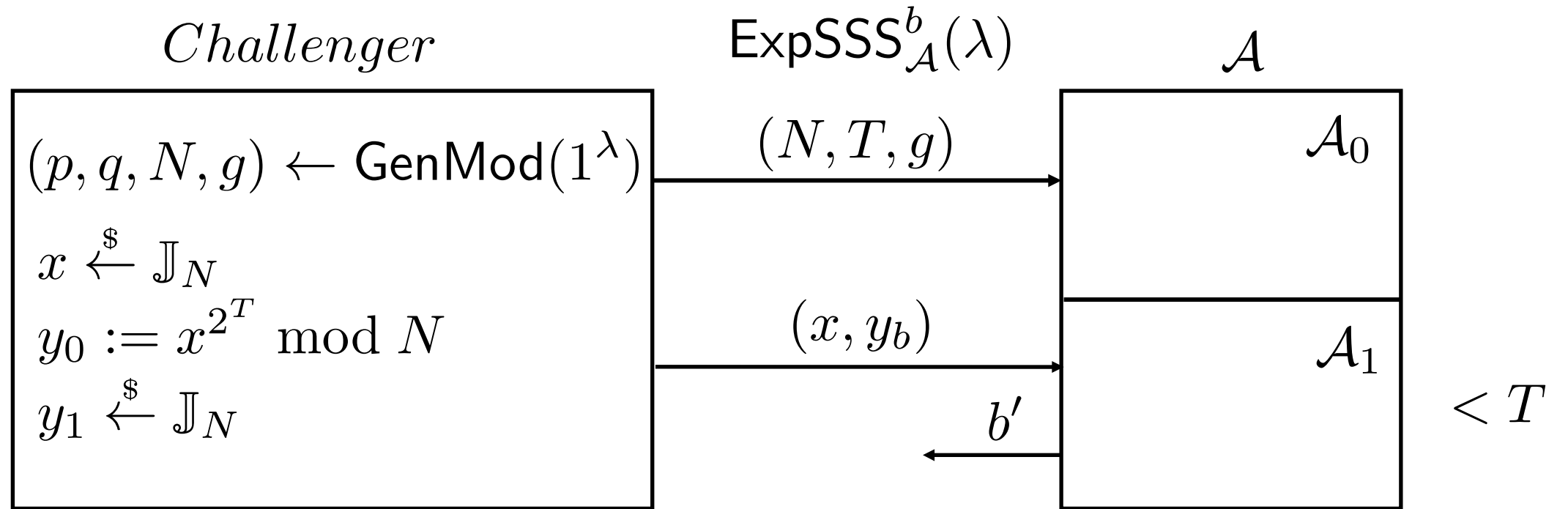
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$$\mathbf{Adv}_{\mathcal{A}}^{\text{SSS}} = |\Pr[\text{ExpSSS}_{\mathcal{A}}^0(\lambda) = 1] - \Pr[\text{ExpSSS}_{\mathcal{A}}^1(\lambda) = 1]| \leq \text{negl}(\lambda)$$

Construction of Linearly Homomorphic TLP (MT19)

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$t := 2^T \bmod \varphi(N)/2$

$h := g^t \bmod N$

return $\text{crs} := (N, T, g, h)$

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Paillier-like encryption

Com(crs, m)

$r \xleftarrow{\$} [N/2]$

$c_0 := g^r \bmod N$

$c_1 := h^{rN} (1 + N)^m \bmod N^2$

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FDec($\text{crs}, (c_0, c_1)$)

$y := c_0^{2^T} \bmod N$

return $\frac{c_1 \cdot y^{-N} \pmod{N^2} - 1}{N}$

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Paillier-like homomorphism

Eval($\text{crs}, \bigoplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$

$c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$

return (c_0, c_1)

Construction of Linearly Homomorphic TLP (MT19)

PGen($1^\lambda, T$)

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Paillier-like homomorphism

Eval($\text{crs}, \bigoplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$
 $c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$
return (c_0, c_1)

DecVrfy($\text{crs}, (c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$
 $c_1 = h_1^{rN} (1 + N)^m \bmod N^2$
 return 1
return 0

Construction of Linearly Homomorphic TLP (MT19)

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$
 $\varphi(N) := (p - 1)(q - 1)$
 $t := 2^T \bmod \varphi(N)/2$
 $h := g^t \bmod N$
return $\text{crs} := (N, T, g, h)$

Paillier-like encryption

Com(crs, m)

$r \xleftarrow{\$} [N/2]$
 $c_0 := g^r \bmod N$
 $c_1 := h^{rN} (1 + N)^m \bmod N^2$
return (c_0, c_1)

FDec($\text{crs}, (c_0, c_1)$)

$y := c_0^{2^T} \bmod N$
return $\frac{c_1 \cdot y^{-N} \pmod{N^2} - 1}{N}$

Paillier-like homomorphism

Eval($\text{crs}, \bigoplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$
 $c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$
return (c_0, c_1)

DecVrfy($\text{crs}, (c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$
 $c_1 = h_1^{rN} (1 + N)^m \bmod N^2$
return 1
return 0

Assumptions:

- Strong Sequential Squaring
- Decisional Composite Residuosity
- Decisional Diffie-Hellman

Naor-Yung (1st Attempt)

$$h_1 := g_1^{t_1} \bmod N_1$$

Com₁(crs₁, m)

$$r_1 \xleftarrow{\$} [N_1/2]$$

$$c_0 := g_1^{r_1} \bmod N_1$$

$$c_1 := h_1^{r_1 N_1} (1 + N_1)^m \bmod N_1^2$$

return (c₀, c₁)

$$h_2 := g_2^{t_2} \bmod N_2$$

Com₂(crs₂, m)

$$r_2 \xleftarrow{\$} [N_2/2]$$

$$c'_0 := g_2^{r_2} \bmod N_2$$

$$c'_1 := h_2^{r_2 N_2} (1 + N_2)^m \bmod N_2^2$$

return (c'₀, c'₁)

Naor-Yung (1st Attempt)

$$h_1 := g_1^{t_1} \bmod N_1$$

$$\underline{\text{Com}_1(\text{crs}_1, m)}$$

$$r_1 \xleftarrow{\$} [N_1/2]$$

$$c_0 := g_1^{r_1} \bmod N_1$$

$$c_1 := h_1^{r_1 N_1} (1 + N_1)^m \bmod N_1^2$$

$$\text{return } (c_0, c_1)$$

$$h_2 := g_2^{t_2} \bmod N_2$$

$$\underline{\text{Com}_2(\text{crs}_2, m)}$$

$$r_2 \xleftarrow{\$} [N_2/2]$$

$$c'_0 := g_2^{r_2} \bmod N_2$$

$$c'_1 := h_2^{r_2 N_2} (1 + N_2)^m \bmod N_2^2$$

$$\text{return } (c'_0, c'_1)$$

Proving that commitments contain the same message using the standard Sigma protocol $\Rightarrow N_1 = N_2$.

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
2	t_1	Simul	m_b	m_b	ZK of NIZK

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
2	t_1	Simul	m_b	m_b	ZK of NIZK
3	t_1	Simul	m_b	m	Sec of HTLP

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
2	t_1	Simul	m_b	m_b	ZK of NIZK
3	t_1	Simul	m_b	m	Sec of HTLP

$$t_1 := 2^T \bmod \varphi(N)/2$$

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
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$$t_1 := 2^T \bmod \varphi(N)/2$$

SSS
DCR
DDH

Naor-Yung (1st Attempt)

Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
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$$t_1 := 2^T \bmod \varphi(N)/2$$

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Game	Dec. Queries	Proof	HTLP ₁	HTLP ₂	Assumption
0	FDec	Real	m_b	m_b	
1	t_1	Real	m_b	m_b	Snd of NIZK
2	t_1	Simul	m_b	m_b	ZK of NIZK
3	t_1	Simul	m_b	m	Sec of HTLP

$$t_1 := 2^T \bmod \varphi(N)/2$$

SSS

DCR

DDH

Knowing the factorization of N does not allow to reduce to SSS!

Construction of Linearly Homomorphic Encryption

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$t := 2^T \bmod \varphi(N)/2$

$h = g^t \bmod N$

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Eval(crs, $\bigoplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$

$c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$

return (c_0, c_1)

Com(crs, m)

$r \xleftarrow{\$} [N/2]$

$c_0 := g^r \bmod N$

$c_1 := h^{rN} (1 + N)^m \bmod N^2$

return (c_0, c_1)

DecVrfy(crs, $(c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$

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return 1

return 0

FDec(crs, (c_0, c_1))

$y := c_0^{2^T} \bmod N$

return $\frac{c_1 \cdot y^{-N} (\bmod N^2) - 1}{N}$

Construction of Linearly Homomorphic Encryption

Setup(1^λ)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$t \xleftarrow{\$} [N/2]$

$h = g^t \bmod N$

return $\text{pk} = (N, g, h), \text{sk} := t$

Eval($\text{crs}, \oplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$

$c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$

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$r \xleftarrow{\$} [N/2]$

$c_0 := g^r \bmod N$

$c_1 := h^{rN} (1 + N)^m \bmod N^2$

return (c_0, c_1)

DecVrfy($\text{crs}, (c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$

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return 0

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Construction of Linearly Homomorphic Encryption

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$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$t \xleftarrow{\$} [N/2]$

$h = g^t \bmod N$

return $\text{pk} = (N, g, h), \text{sk} := t$

Eval($\text{crs}, \oplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$

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$c_0 := g^r \bmod N$

$c_1 := h^{rN} (1 + N)^m \bmod N^2$

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DecVrfy($\text{crs}, (c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$

$c_1 = h_1^{rN} (1 + N)^m \bmod N^2$

return 1

return 0

Dec($\text{sk} = t, (c_0, c_1)$)

$y := c_0^t \bmod N$

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Construction of Linearly Homomorphic Encryption

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$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$
 $\varphi(N) := (p - 1)(q - 1)$
 $t \xleftarrow{\$} [N/2]$
 $h = g^t \bmod N$
return $\text{pk} = (N, g, h), \text{sk} := t$

Eval($\text{crs}, \oplus_N, (c_{i,0}, c_{i,1})_{i \in [n]}$)

$c_0 := \prod_{i=1}^n c_{i,0} \bmod N$
 $c_1 := \prod_{i=1}^n c_{i,1} \bmod N^2$
return (c_0, c_1)

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$r \xleftarrow{\$} [N/2]$
 $c_0 := g^r \bmod N$
 $c_1 := h^{rN} (1 + N)^m \bmod N^2$
return (c_0, c_1)

DecVrfy($\text{crs}, (c_0, c_1), m, r$)

if $c_0 = g^r \bmod N \wedge$
 $c_1 = h_1^{rN} (1 + N)^m \bmod N^2$
return 1
return 0

Dec($\text{sk} = t, (c_0, c_1)$)

$y := c_0^t \bmod N$
return $\frac{c_1 \cdot y^{-N} \pmod{N^2} - 1}{N}$

Assumptions:

- Decisional Composite Residuosity
- Decisional Diffie-Hellman

Naor-Yung (2nd Attempt)

$$h_1 := g_1^k \bmod N$$

Enc(pk, m)

$$r_1 \xleftarrow{\$} [N]$$

$$c_0 := g_1^{r_1} \bmod N$$

$$c_1 := h_1^{r_1 N} (1 + N)^m \bmod N^2$$

return (c_0, c_1)

$$h_2 := g_2^t \bmod N$$

Com(crs₂, m)

$$r_2 \xleftarrow{\$} [N/2]$$

$$c'_0 := g_2^{r_2} \bmod N$$

$$c'_1 := h_2^{r_2 N} (1 + N)^m \bmod N^2$$

return (c'_0, c'_1)

Naor-Yung (2nd Attempt)

$$h_1 := g_1^k \bmod N$$

Enc(pk, m)

$$r_1 \xleftarrow{\$} [N]$$

$$c_0 := g_1^{r_1} \bmod N$$

$$c_1 := h_1^{r_1 N} (1 + N)^m \bmod N^2$$

return (c_0, c_1)

$$h_2 := g_2^t \bmod N$$

Com(crs₂, m)

$$r_2 \xleftarrow{\$} [N/2]$$

$$c'_0 := g_2^{r_2} \bmod N$$

$$c'_1 := h_2^{r_2 N} (1 + N)^m \bmod N^2$$

return (c'_0, c'_1)

$$t := 2^T \bmod \varphi(N)/2$$

Knowing the factorization of N does not allow to reduce to DCR!

Triple Naor-Yung

$$h_1 := g_1^{k_1} \bmod N$$

Enc₁(pk₁, m)

$$r_1 \xleftarrow{\$} [N]$$

$$c_0 := g_1^{r_1} \bmod N$$

$$c_1 := h_1^{r_1 N} (1 + N)^m \bmod N^2$$

return (c₀, c₁)

$$h_2 := g_2^{k_2} \bmod N$$

Enc₂(pk₂, m)

$$r_2 \xleftarrow{\$} [N]$$

$$c'_0 := g_2^{r_2} \bmod N$$

$$c'_1 := h_2^{r_2 N} (1 + N)^m \bmod N^2$$

return (c'₀, c'₁)

$$h_3 := g_3^t \bmod N$$

Com(crs, m)

$$r_3 \xleftarrow{\$} [N/2]$$

$$c''_0 := g_3^{r_3} \bmod N$$

$$c''_1 := h_3^{r_3 N} (1 + N)^m \bmod N^2$$

return (c''₀, c''₁)

Triple Naor-Yung

Game	Dec. Queries	Proof	PKE ₁	PKE ₂	HTLP	Assumption
0	FDec	Real	m_b	m_b	m_b	
1	k_1	Real	m_b	m_b	m_b	Snd of NIZK
2	k_1	Simul	m_b	m_b	m_b	ZK of NIZK
3	k_1	Simul	m_b	m_b	m	Sec of HTLP
4	k_1	Simul	m_b	m	m	Sec of PKE ₂

Triple Naor-Yung

Game	Dec. Queries	Proof	PKE ₁	PKE ₂	HTLP	Assumption
0	FDec	Real	m_b	m_b	m_b	
1	k_1	Real	m_b	m_b	m_b	Snd of NIZK
2	k_1	Simul	m_b	m_b	m_b	ZK of NIZK
3	k_1	Simul	m_b	m_b	m	Sec of HTLP
4	k_1	Simul	m_b	m	m	Sec of PKE ₂
5	k_2	Simul	m_b	m	m	OT-SimSnd
6	k_2	Simul	m	m	m	Sec of PKE ₁

Shrinking CRS and Commitment Size - BMV16

crs	Commitment
g_1 $h_1 := g_1^{k_1}$	$g_1^{r_1}$ $h_1^{r_1 N} (1 + N)^m$
g_2 $h_2 := g_2^{k_2}$	$g_2^{r_2}$ $h_2^{r_2 N} (1 + N)^m$
g_3 $h_3 := g_3^t$	$g_3^{r_3}$ $h_3^{r_3 N} (1 + N)^m$

Shrinking CRS and Commitment Size - BMV16

crs	Commitment
g_1 $h_1 := g_1^{k_1}$	$g_1^{r_1}$ $h_1^{r_1 N} (1 + N)^m$
g_2 $h_2 := g_2^{k_2}$	$g_2^{r_2}$ $h_2^{r_2 N} (1 + N)^m$
g_3 $h_3 := g_3^t$	$g_3^{r_3}$ $h_3^{r_3 N} (1 + N)^m$



crs	Commitment
g $h_1 := g^{k_1}$ $h_2 := g^{k_2}$ $h_3 := g^t$	g^r $h_1^{r N} (1 + N)^m$ $h_2^{r N} (1 + N)^m$ $h_3^{r N} (1 + N)^m$

Shrinking CRS and Commitment Size - BMV16

crs	Commitment
g_1 $h_1 := g_1^{k_1}$	$g_1^{r_1}$ $h_1^{r_1 N} (1 + N)^m$
g_2 $h_2 := g_2^{k_2}$	$g_2^{r_2}$ $h_2^{r_2 N} (1 + N)^m$
g_3 $h_3 := g_3^t$	$g_3^{r_3}$ $h_3^{r_3 N} (1 + N)^m$



crs	Commitment
g $h_1 := g^{k_1}$ $h_2 := g^{k_2}$ $h_3 := g^t$	g^r $h_1^{r N} (1 + N)^m$ $h_2^{r N} (1 + N)^m$ $h_3^{r N} (1 + N)^m$

One-Time Simulation Sound NIZK

$$L = \left\{ (c_0, c_1, c_2, c_3) \mid \exists (m, r) : \left(\bigwedge_{i=1}^3 c_i = h_i^{r N} (1 + N)^m \pmod{N^2} \right) \wedge c_0 = g^r \pmod{N} \right\}$$

Construction of Linearly Homomorphic PVNITC

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$k_1, k_2 \xleftarrow{\$} [N/2]$

$t := 2^T \bmod \varphi(N)/2$

For $i \in [2] : h_i := g^{k_i} \bmod N$

$h_3 := g^t \bmod N$

$\text{crs}_{\text{NIZK}} \leftarrow \text{NIZK.Setup}(1^\lambda, L)$

return $\text{crs} := (N, T, g, h_1, h_2, h_3, \text{crs}_{\text{NIZK}})$

Construction of Linearly Homomorphic PVNITC

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$k_1, k_2 \xleftarrow{\$} [N/2]$

$t := 2^T \bmod \varphi(N)/2$

For $i \in [2] : h_i := g^{k_i} \bmod N$

$h_3 := g^t \bmod N$

$\text{crs}_{\text{NIZK}} \leftarrow \text{NIZK.Setup}(1^\lambda, L)$

return $\text{crs} := (N, T, g, h_1, h_2, h_3, \text{crs}_{\text{NIZK}})$

Com(crs, m)

$r \xleftarrow{\$} [N/2]$

$c_0 := g^r \bmod N$

For $i \in [3] : c_i := h_i^{r_i} (1 + N)^m \bmod N^2$

$c := (c_0, c_1, c_2, c_3), w := (m, r)$

$\pi_{\text{Com}} \leftarrow \text{NIZK.Prove}(\text{crs}_{\text{NIZK}}, c, w)$

$\pi_{\text{Dec}} := r$

return $(c, \pi_{\text{Com}}, \pi_{\text{Dec}})$

Construction of Linearly Homomorphic PVNITC

PGen($1^\lambda, T$)

$(p, q, N, g) \leftarrow \text{GenMod}(1^\lambda)$

$\varphi(N) := (p - 1)(q - 1)$

$k_1, k_2 \xleftarrow{\$} [N/2]$

$t := 2^T \bmod \varphi(N)/2$

For $i \in [2] : h_i := g^{k_i} \bmod N$

$h_3 := g^t \bmod N$

$\text{crs}_{\text{NIZK}} \leftarrow \text{NIZK.Setup}(1^\lambda, L)$

return $\text{crs} := (N, T, g, h_1, h_2, h_3, \text{crs}_{\text{NIZK}})$

Com(crs, m)

$r \xleftarrow{\$} [N/2]$

$c_0 := g^r \bmod N$

For $i \in [3] : c_i := h_i^{r_i} (1 + N)^m \bmod N^2$

$c := (c_0, c_1, c_2, c_3), w := (m, r)$

$\pi_{\text{Com}} \leftarrow \text{NIZK.Prove}(\text{crs}_{\text{NIZK}}, c, w)$

$\pi_{\text{Dec}} := r$

return $(c, \pi_{\text{Com}}, \pi_{\text{Dec}})$

ComVrfy, DecVrfy, FDec, FDecVrfy, Eval

Conclusion/Comparison to Prior Work

Construction	Hom.	Std.	Setup	Com?	FDec?	Com	$ \pi_{\text{Com}} $	t_{com}	Tight
EFKP20	-	✗	-	✗	✓	$O(1)$	-	$O(\log T)$	✓
KLX20	-	✓	priv.	✓	✗	$O(1)$	$O(1)$	$O(T)$	✓
TCLM21	lin.	✗	pub.	✓	✗	$O(\lambda)$	$O(\lambda)$	$O(1)$	✗
Our work	lin.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	mult.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	lin.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓
Our work	mult.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓

Conclusion/Comparison to Prior Work

Construction	Hom.	Std.	Setup	Com?	FDec?	Com	$ \pi_{\text{Com}} $	t_{com}	Tight
EFKP20	-	✗	-	✗	✓	$O(1)$	-	$O(\log T)$	✓
KLX20	-	✓	priv.	✓	✗	$O(1)$	$O(1)$	$O(T)$	✓
TCLM21	lin.	✗	pub.	✓	✗	$O(\lambda)$	$O(\lambda)$	$O(1)$	✗
Our work	lin.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	mult.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	lin.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓
Our work	mult.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓

- Full paper <https://eprint.iacr.org/2022/1498>
- Contact: chvojka.p@gmail.com

Conclusion/Comparison to Prior Work

Construction	Hom.	Std.	Setup	Com?	FDec?	Com	$ \pi_{\text{Com}} $	t_{com}	Tight
EFKP20	-	✗	-	✗	✓	$O(1)$	-	$O(\log T)$	✓
KLX20	-	✓	priv.	✓	✗	$O(1)$	$O(1)$	$O(T)$	✓
TCLM21	lin.	✗	pub.	✓	✗	$O(\lambda)$	$O(\lambda)$	$O(1)$	✗
Our work	lin.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	mult.	✓	priv.	✓	✓	$O(1)$	$O(\log \lambda)$	$O(1)$	✓
Our work	lin.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓
Our work	mult.	✗	priv.	✓	✓	$O(1)$	$O(1)$	$O(1)$	✓

- Full paper <https://eprint.iacr.org/2022/1498>
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Thank you for your attention.