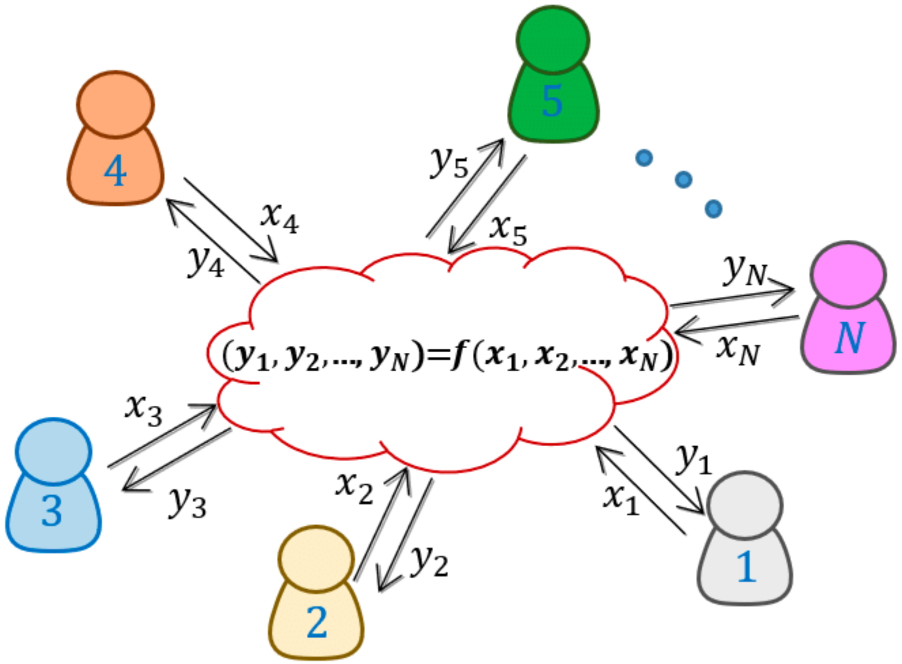
Private set intersection (PSI)
from pseudorandom correlation generator (PCG)

Geoffroy Couteau, **Dung Bui**

Secure computation (MPC)



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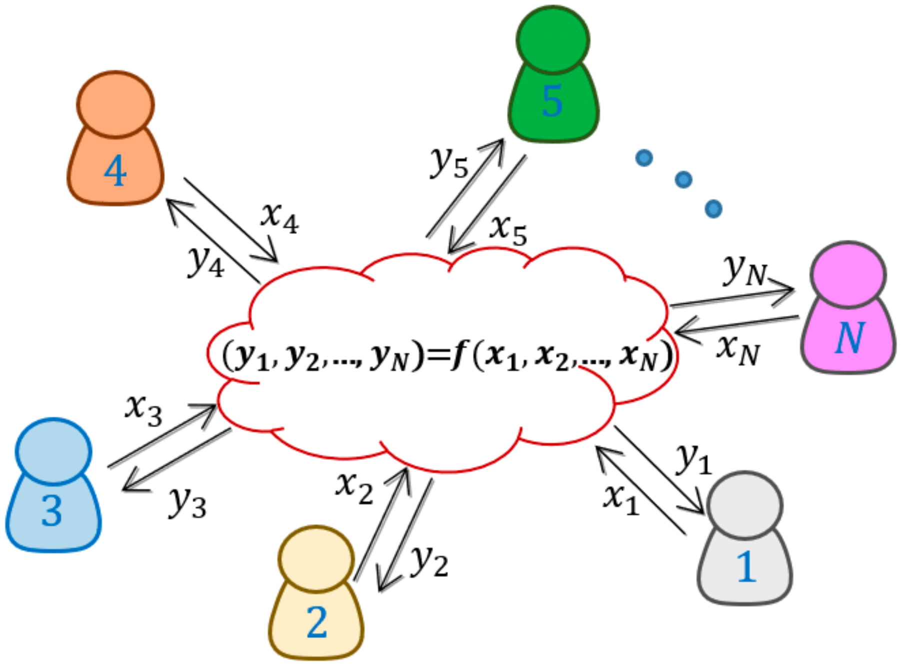
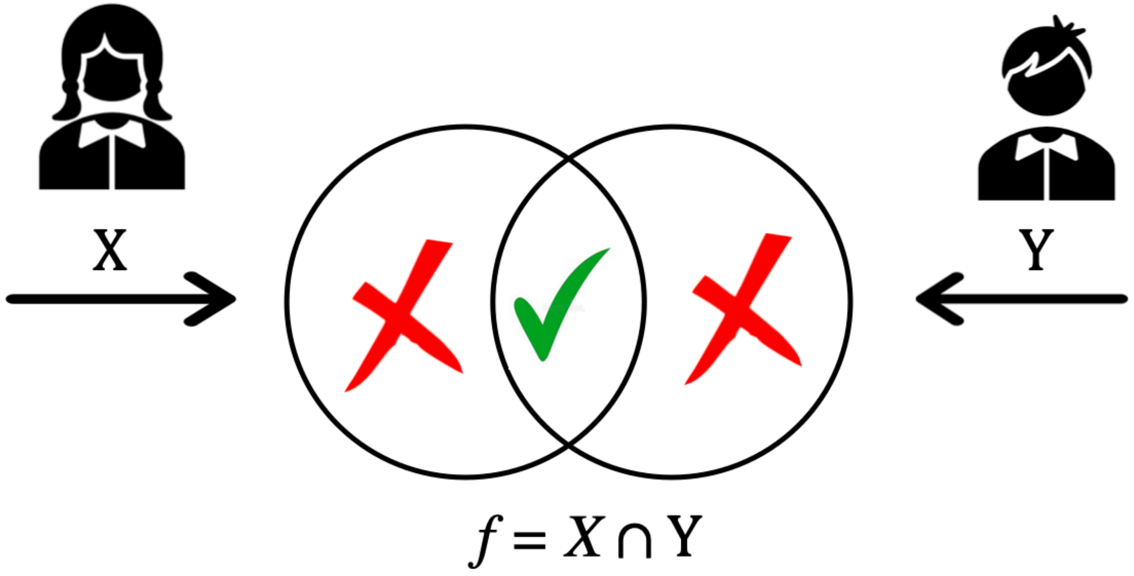


Private Set Intersection (PSI)



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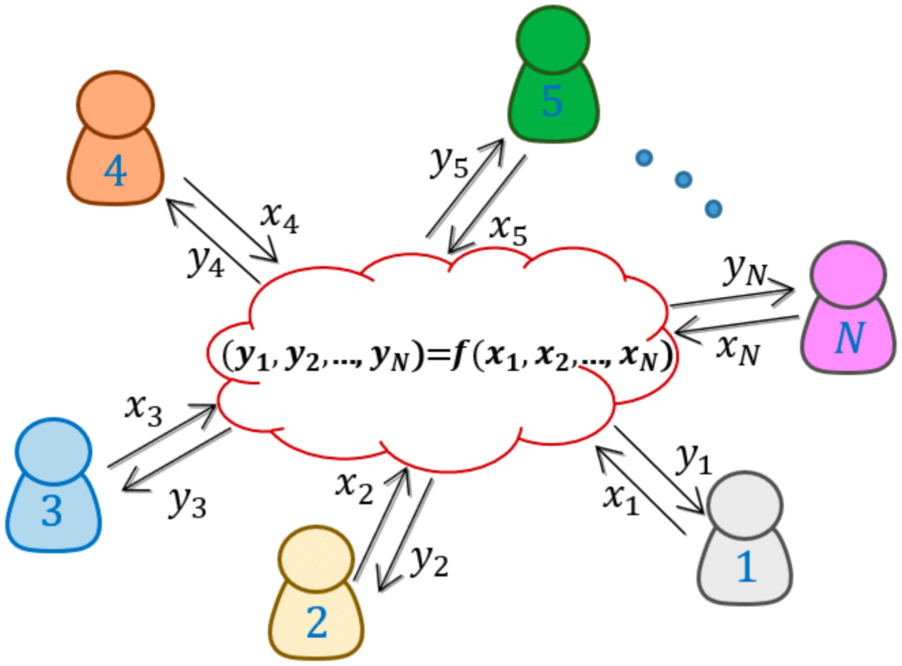

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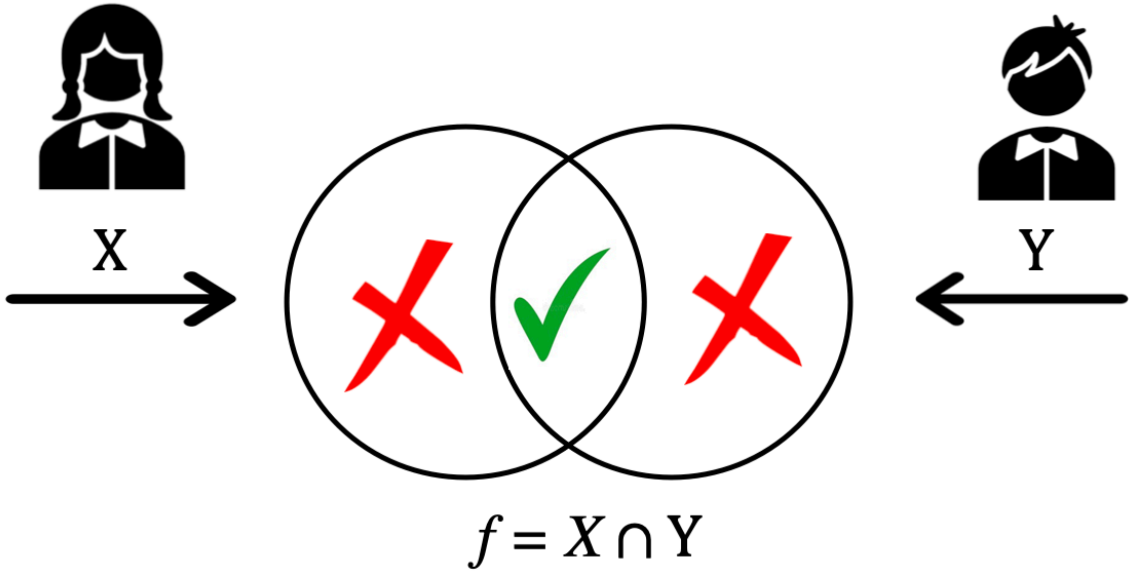
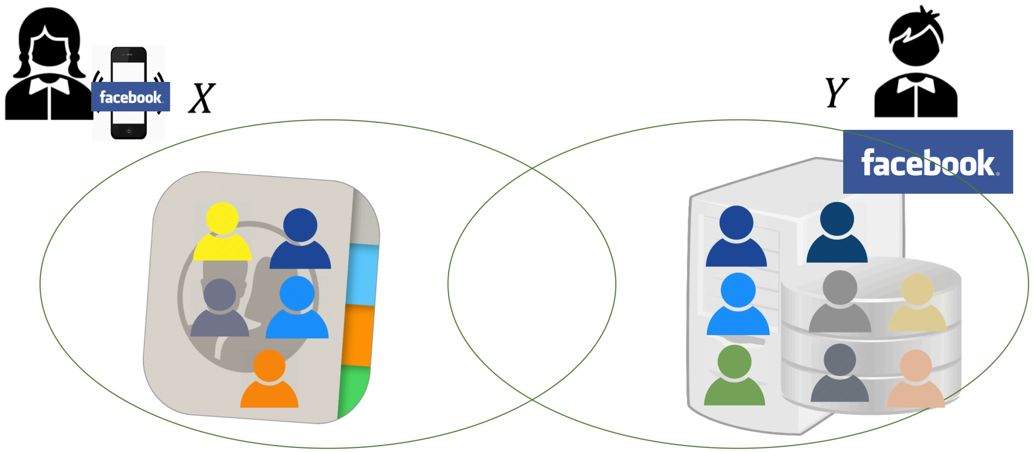
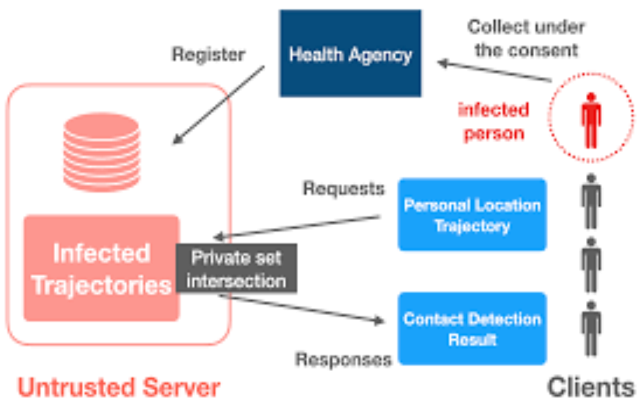


Adversary types:

- Semi-honest : following the protocol
- Malicious: may deviate from the protocol



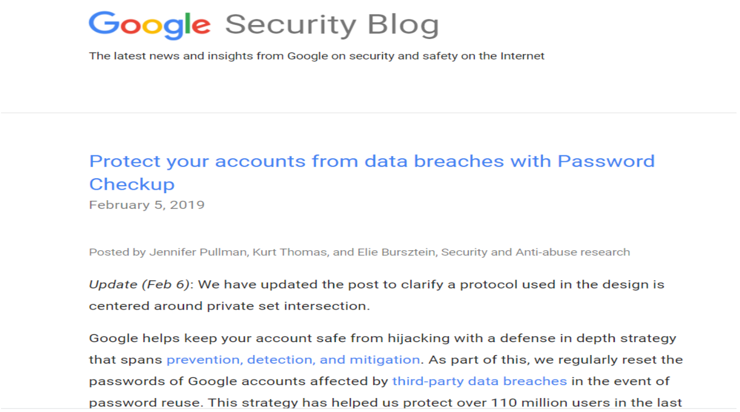
Contact Discovery



Contact Tracing

PSI

Ad Efficiency



Google Security Blog

The latest news and insights from Google on security and safety on the Internet

Protect your accounts from data breaches with Password Checkup

February 5, 2019

Posted by Jennifer Pullman, Kurt Thomas, and Elie Bursztein, Security and Anti-abuse research

Update (Feb 6): We have updated the post to clarify a protocol used in the design is centered around private set intersection.

Google helps keep your account safe from hijacking with a defense in depth strategy that spans prevention, detection, and mitigation. As part of this, we regularly reset the passwords of Google accounts affected by third-party data breaches in the event of password reuse. This strategy has helped us protect over 110 million users in the last

Google Password Checkup

Paradigms for PSI

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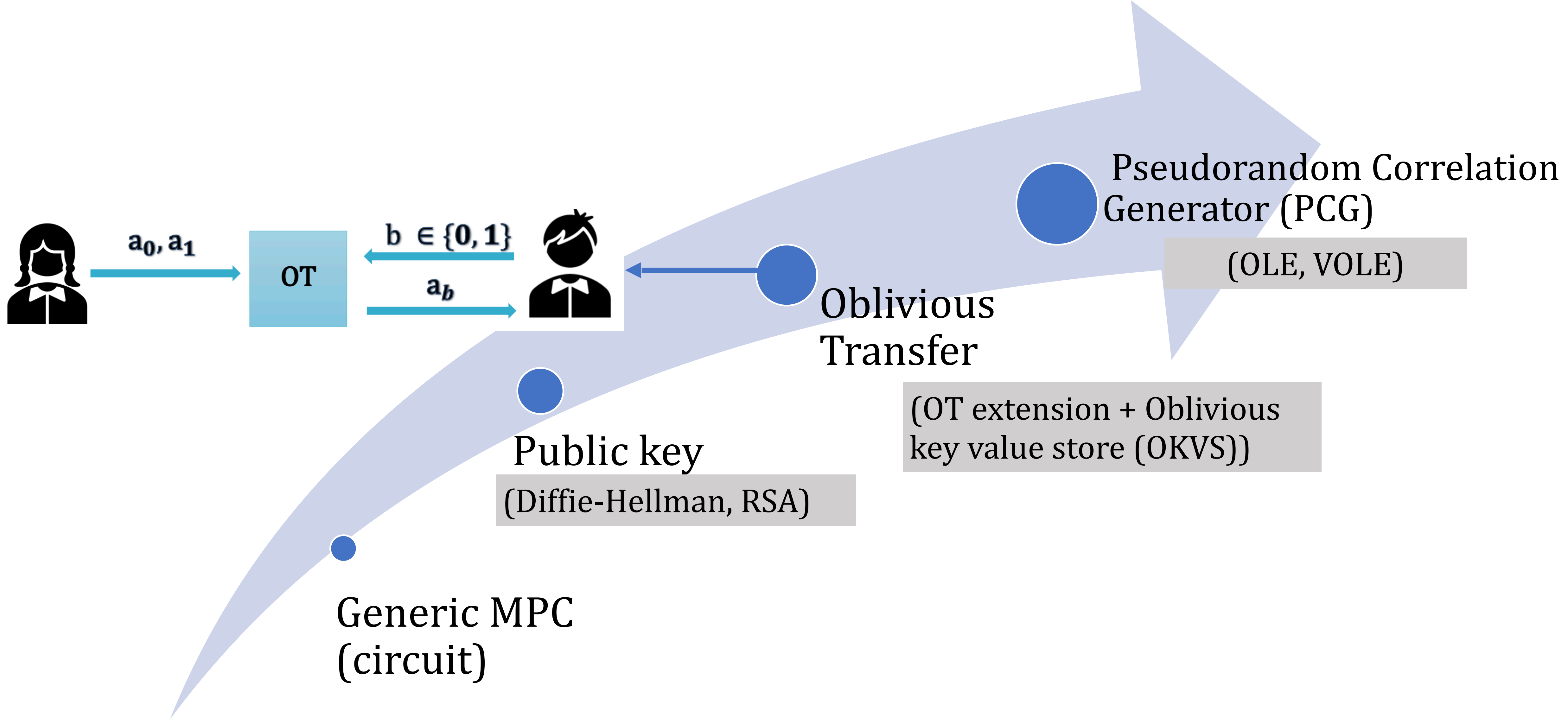

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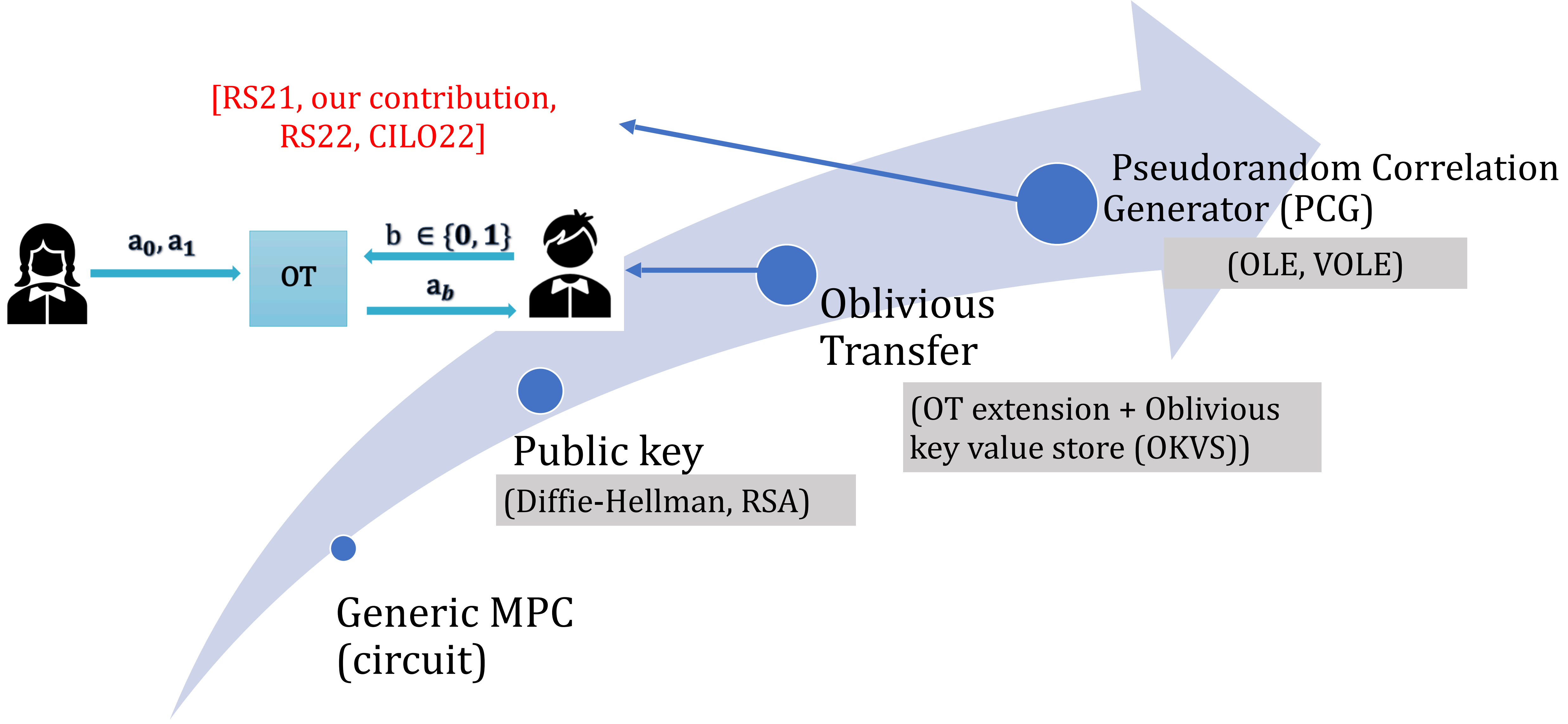

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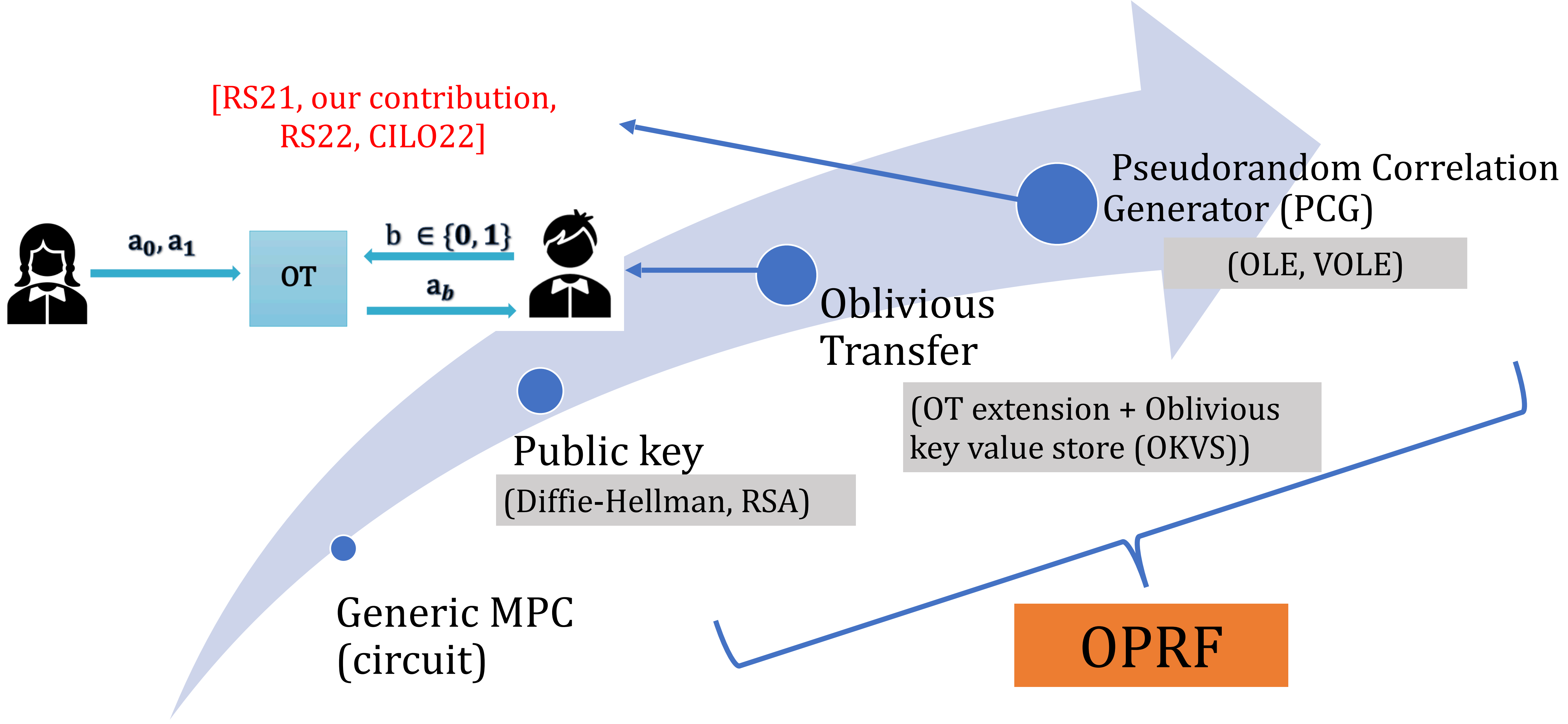

Oblivious PRF (OPRF)

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- A PRF is a keyed function $F_k()$

If k is random and kept secret then a polynomial-time adversary can not distinguish between the output of $F_k()$ and a truly random function

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$X = \{x_1, x_2, \ldots, x_n\}$

$Y = \{y_1, y_2, \ldots, y_n\}$

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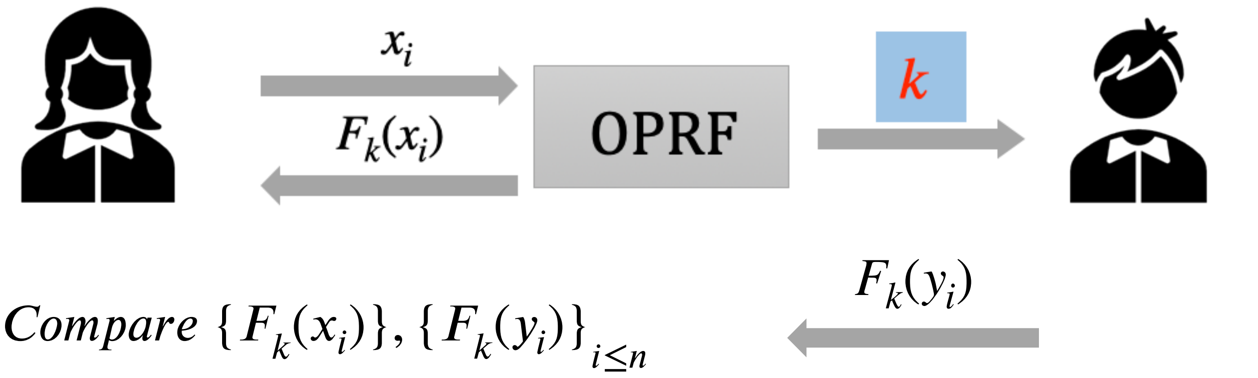
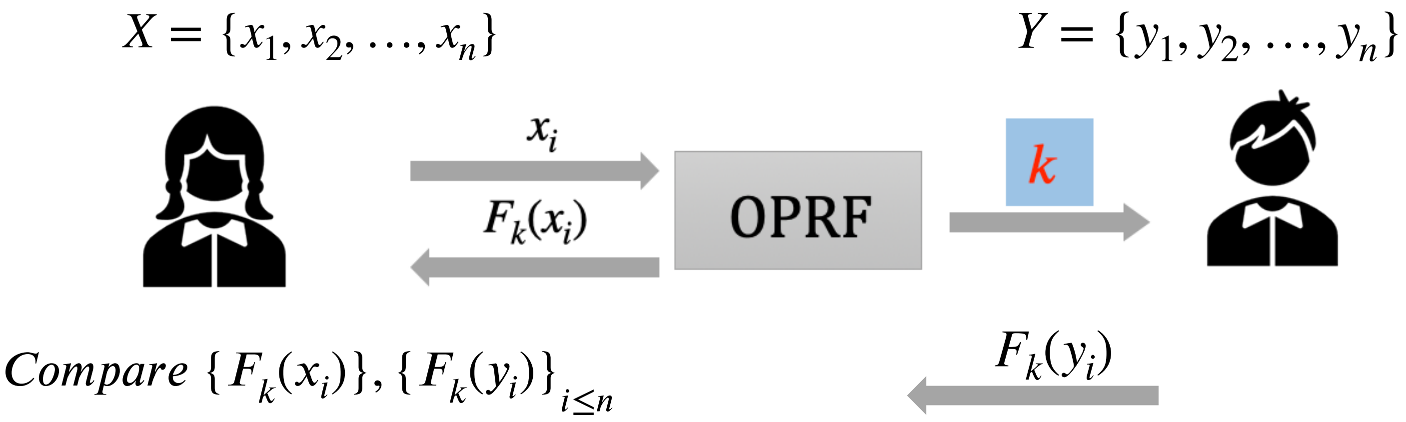


PSI from OPRF



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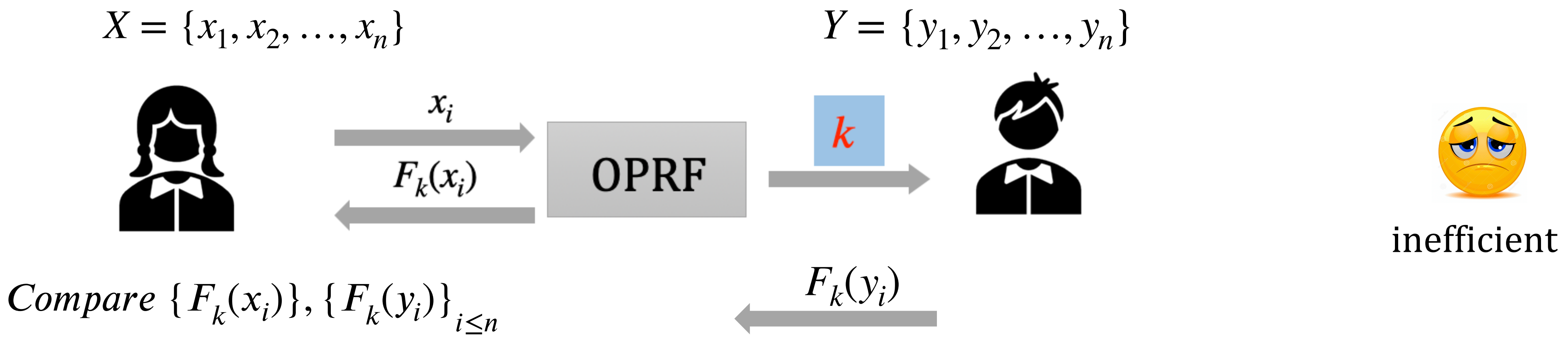

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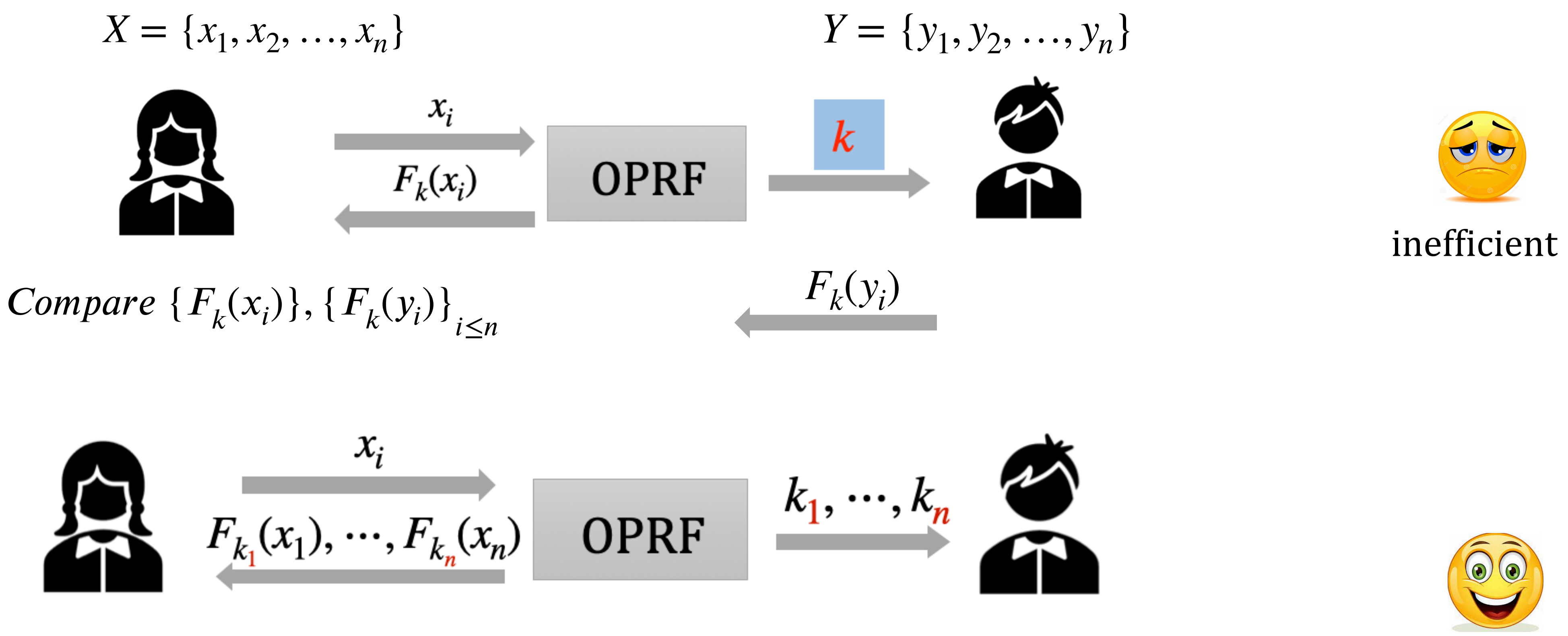

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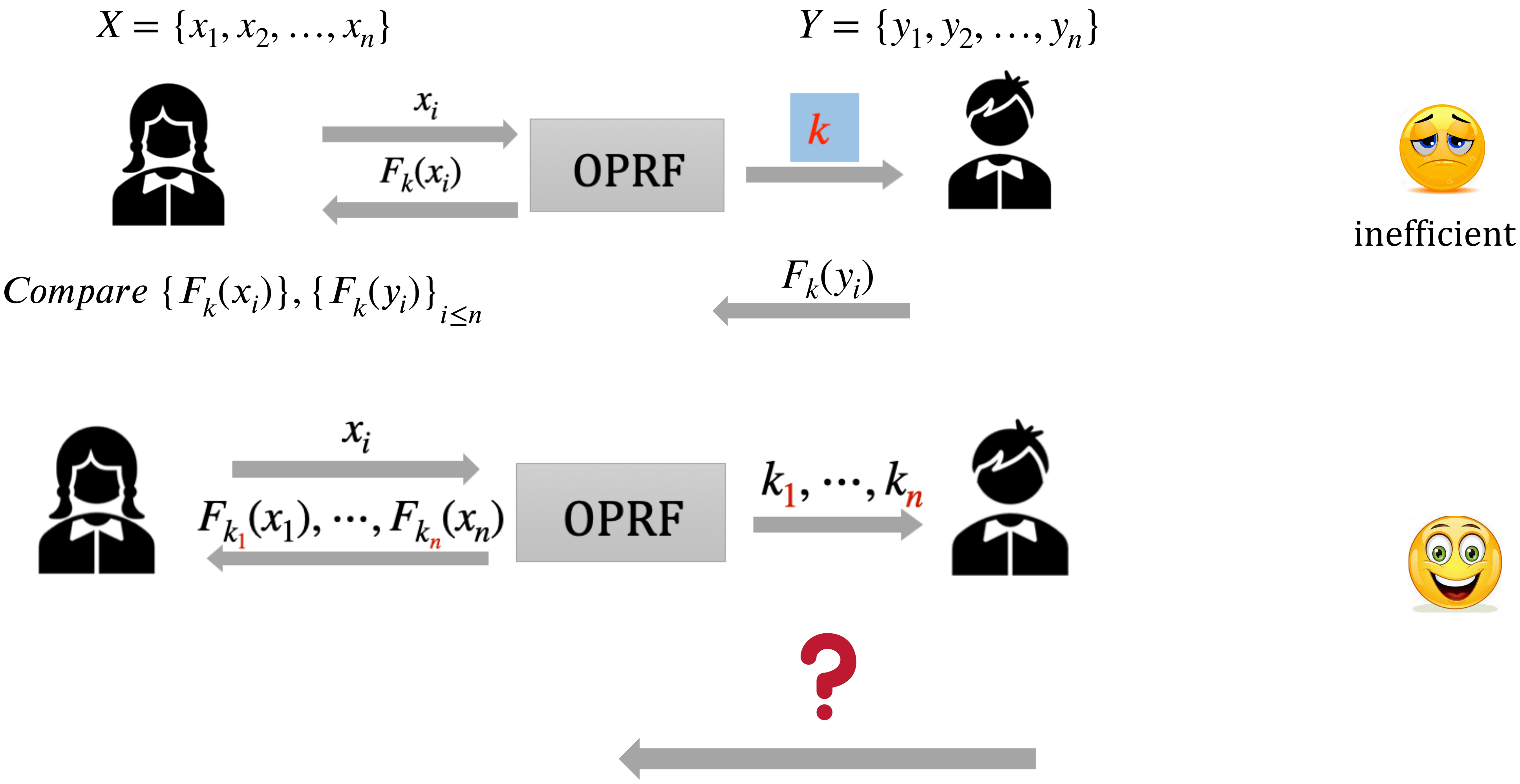

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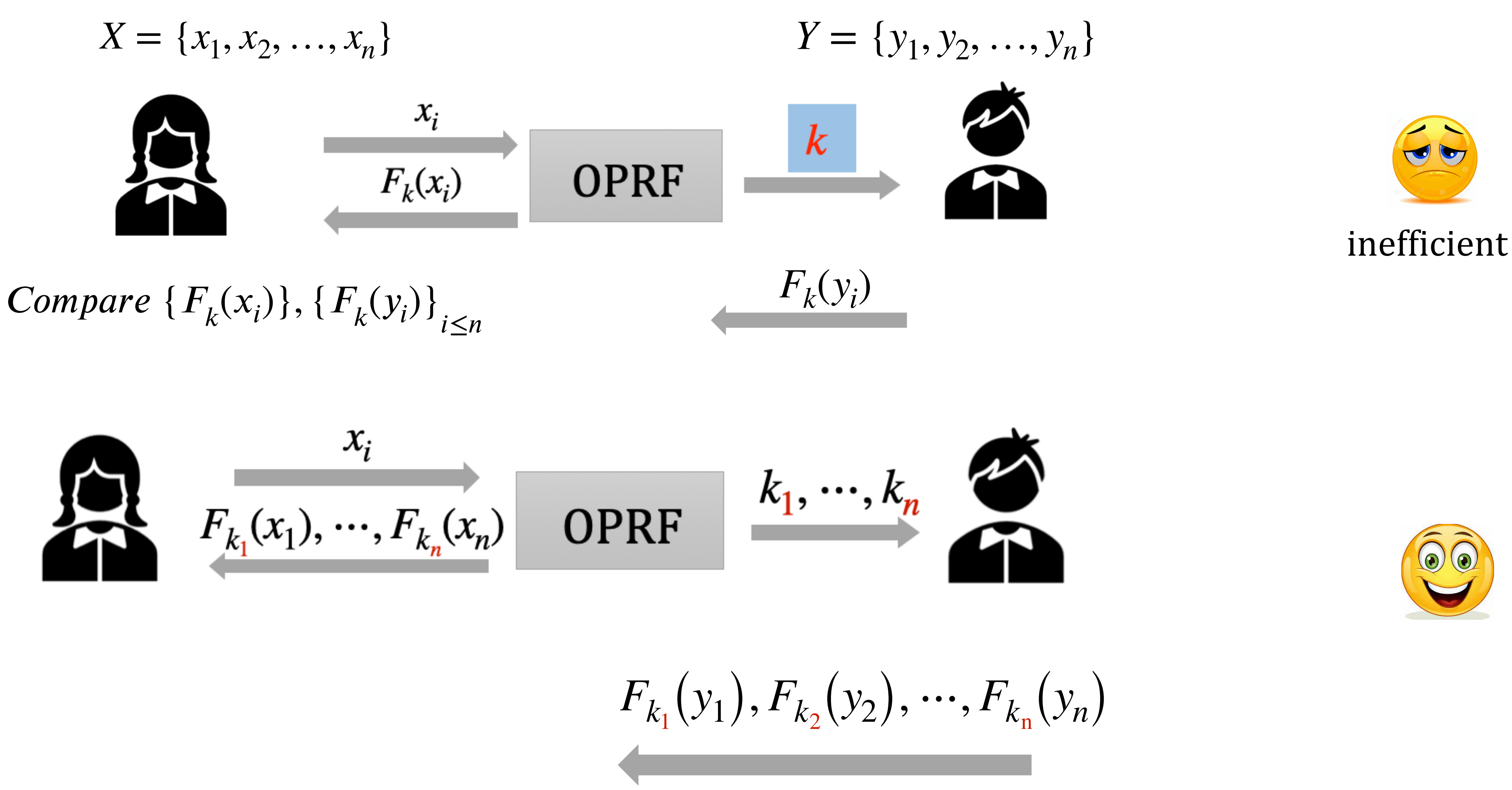

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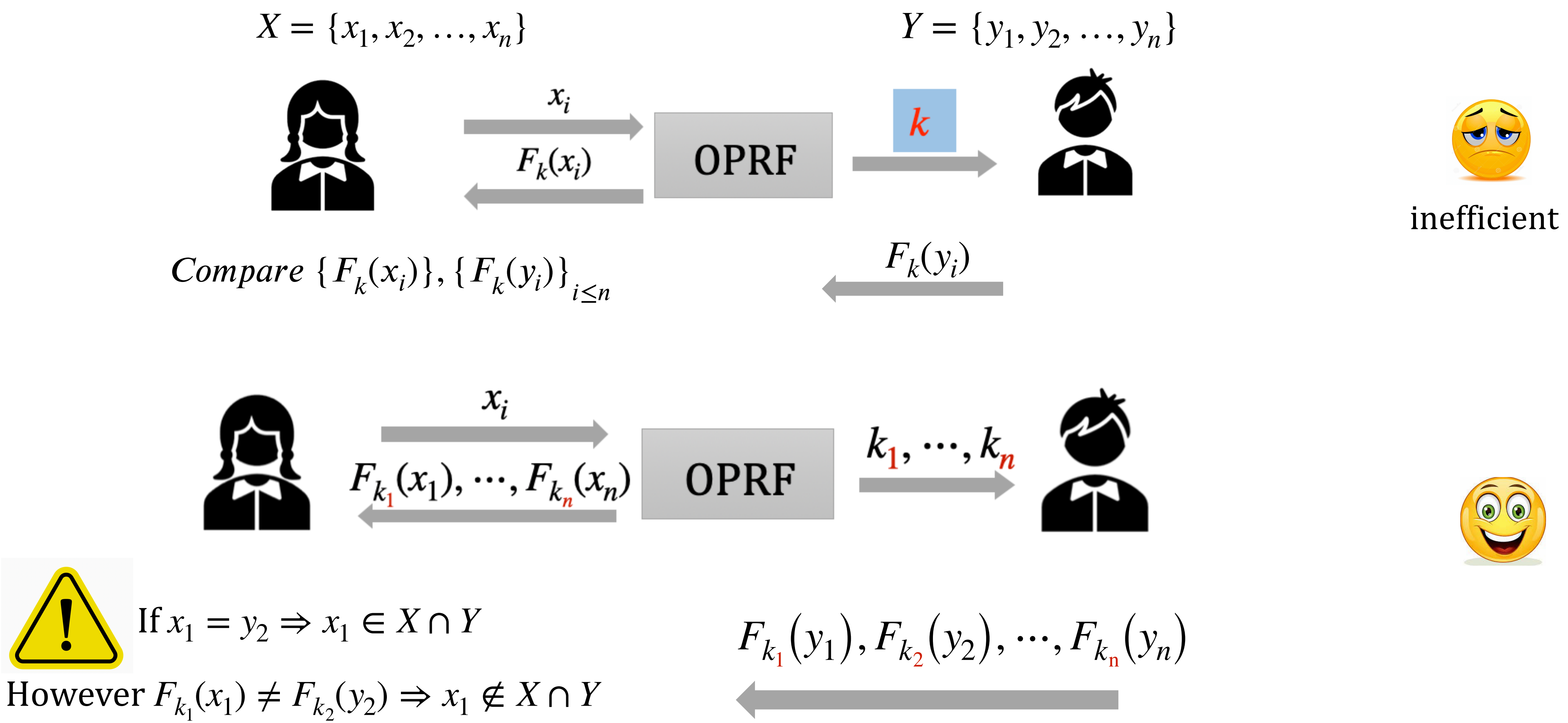

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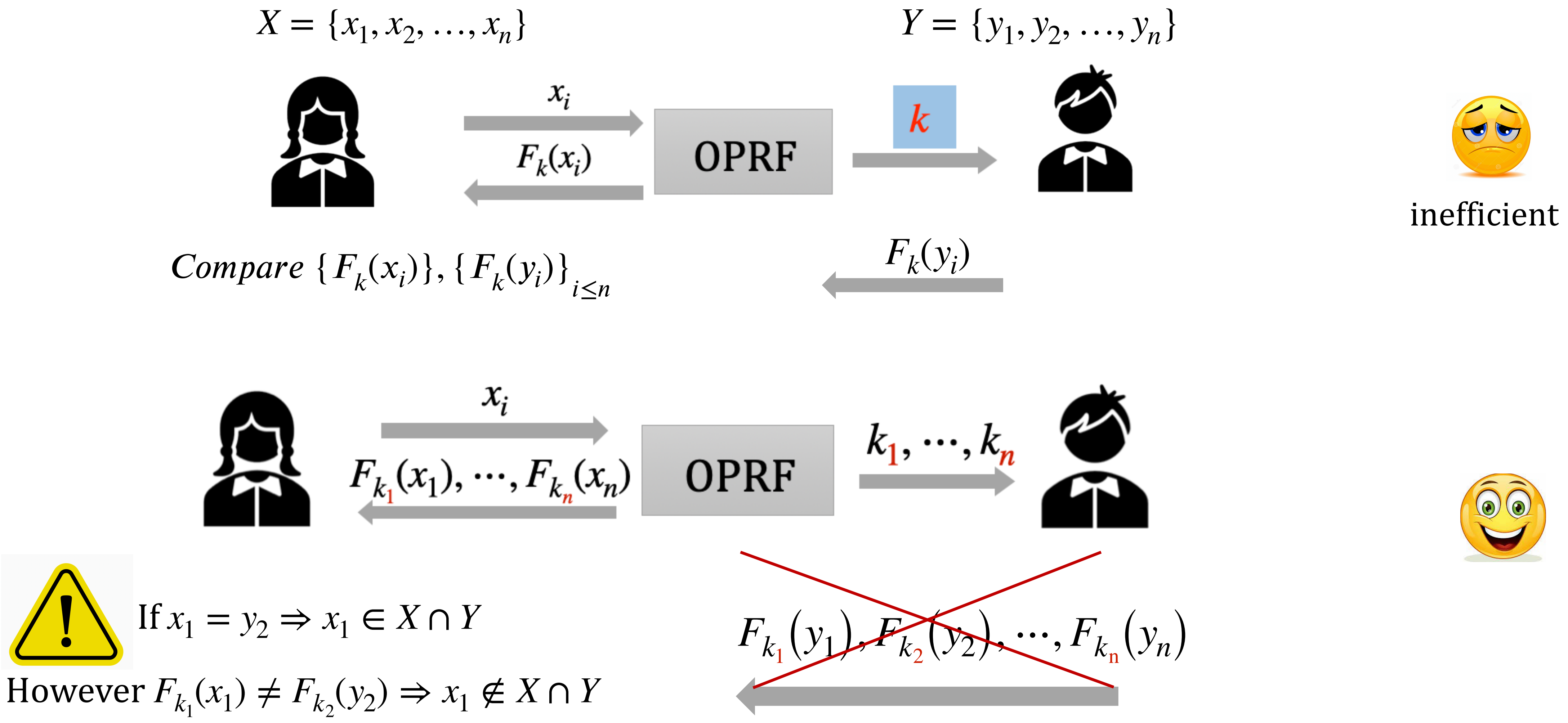

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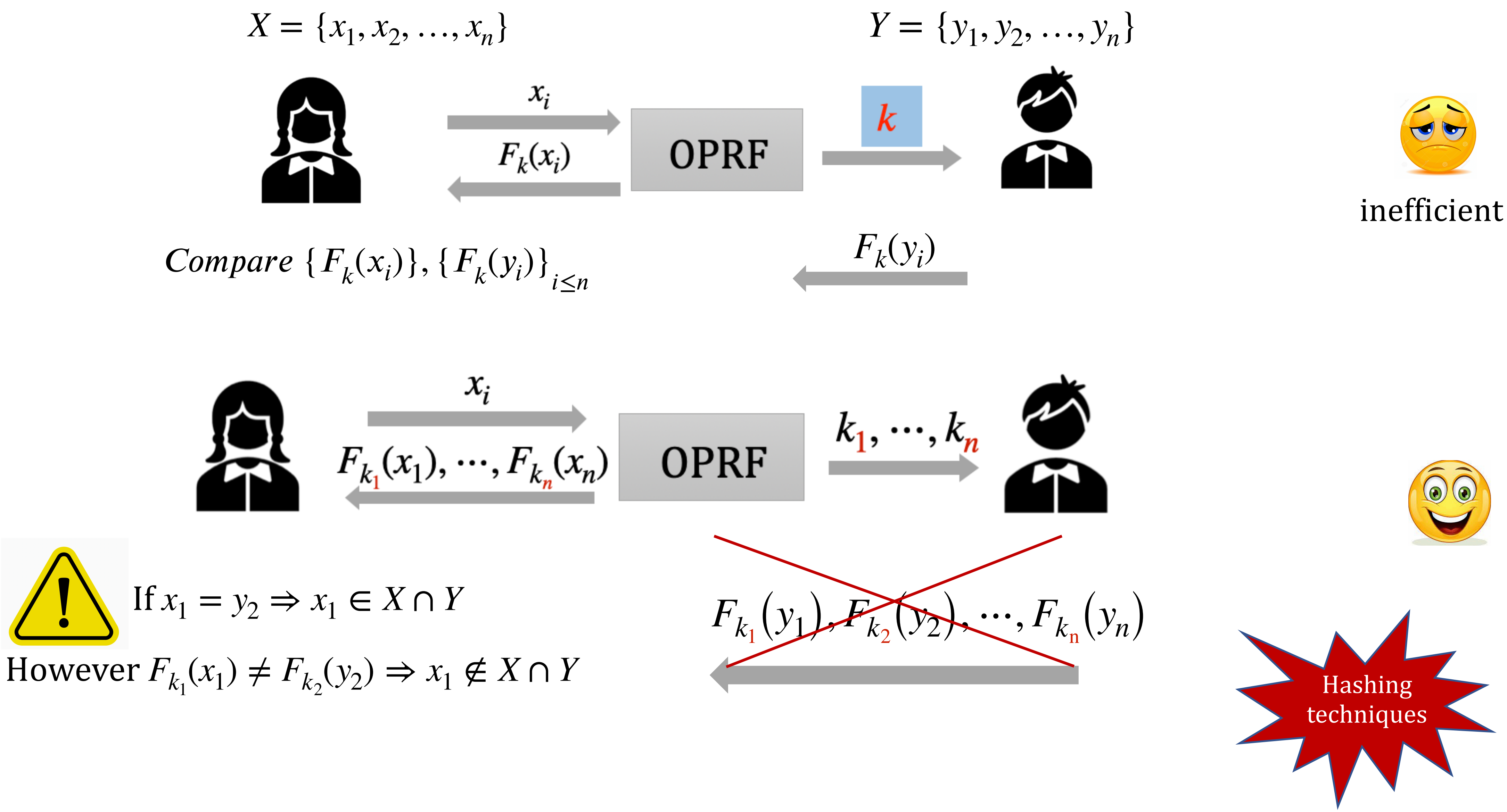

Pseudorandom correlation generator (PCG)

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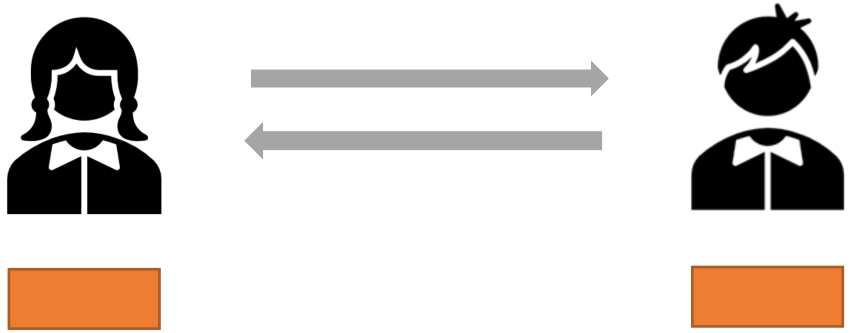
Secure computation ➡ Correlated random strings

Pseudorandom correlation generator (PCG)

Secure computation → Correlated random strings

Preprocessing phase:
Interactive protocol with short communication and computation. Alice and Bob store a small seed afterwards



Short correlated seeds

Pseudorandom correlation generator (PCG)

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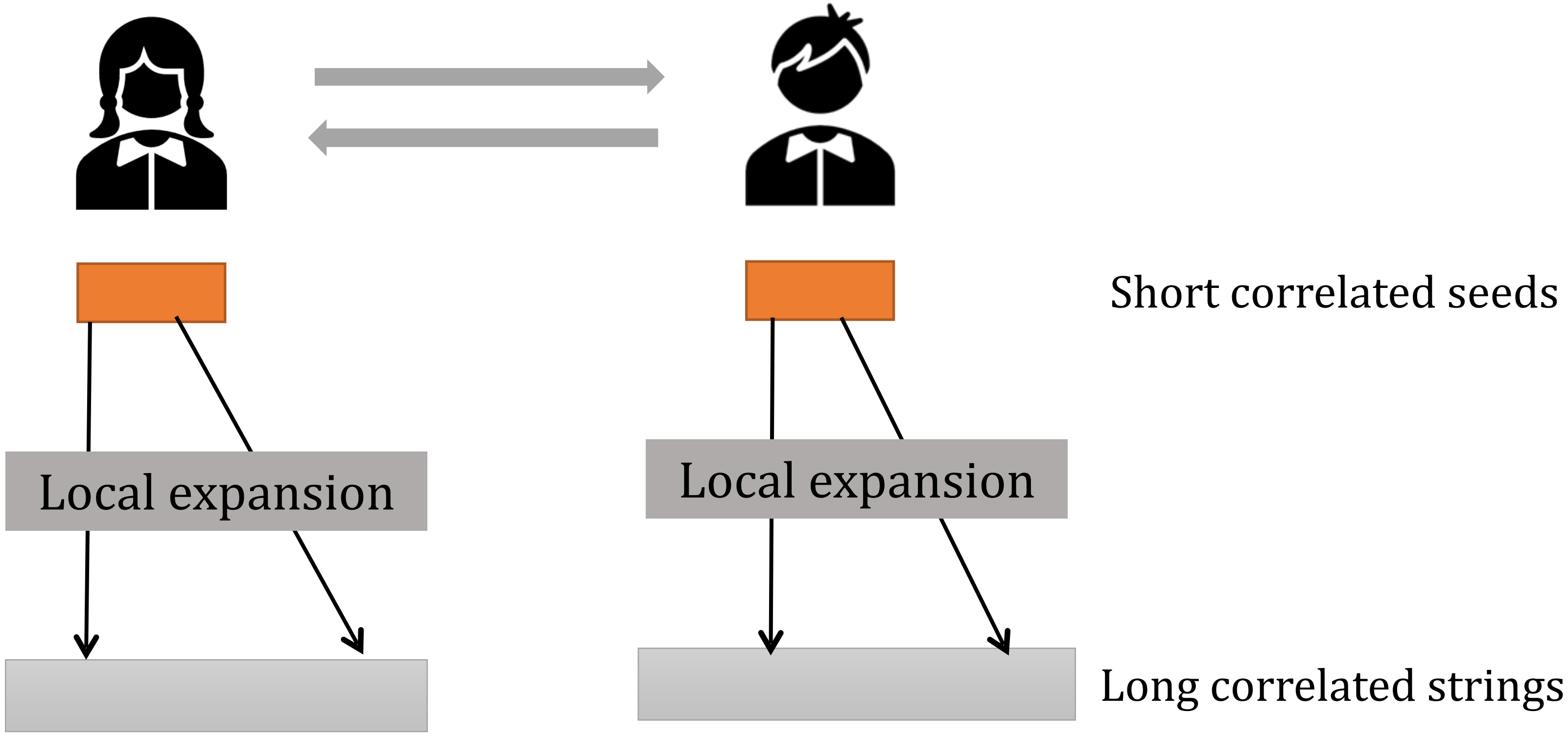
'Silent' computation:
Alice and Bob
stretch their seeds into large pseudorandom correlated strings



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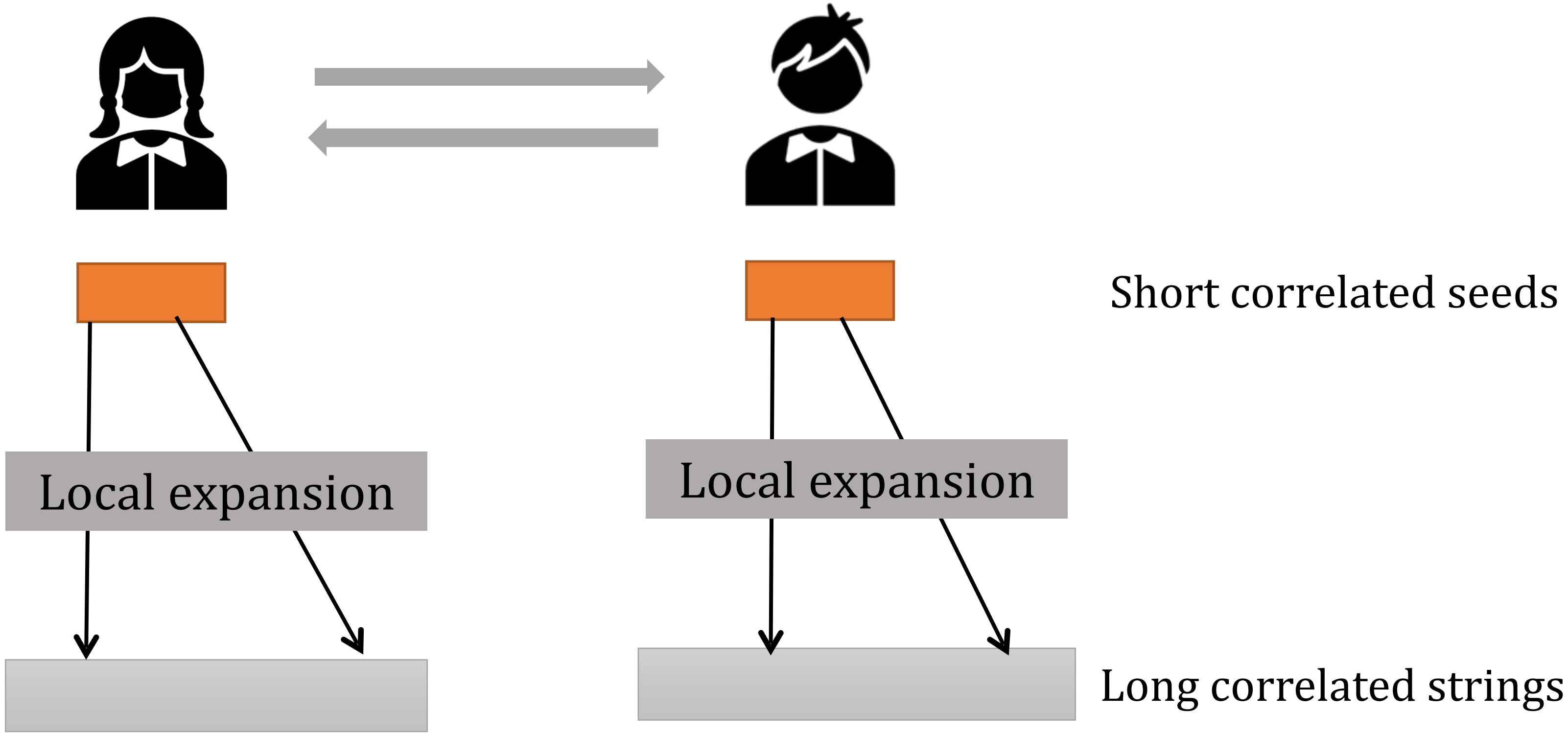

Seeds can be silently expanded to long correlated strings

→ Parties just need to communicate and store a short correlated seeds

Pseudorandom correlation generator (PCG)

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Batch-OLE

A finite field $E \subset F$

Vector OLE

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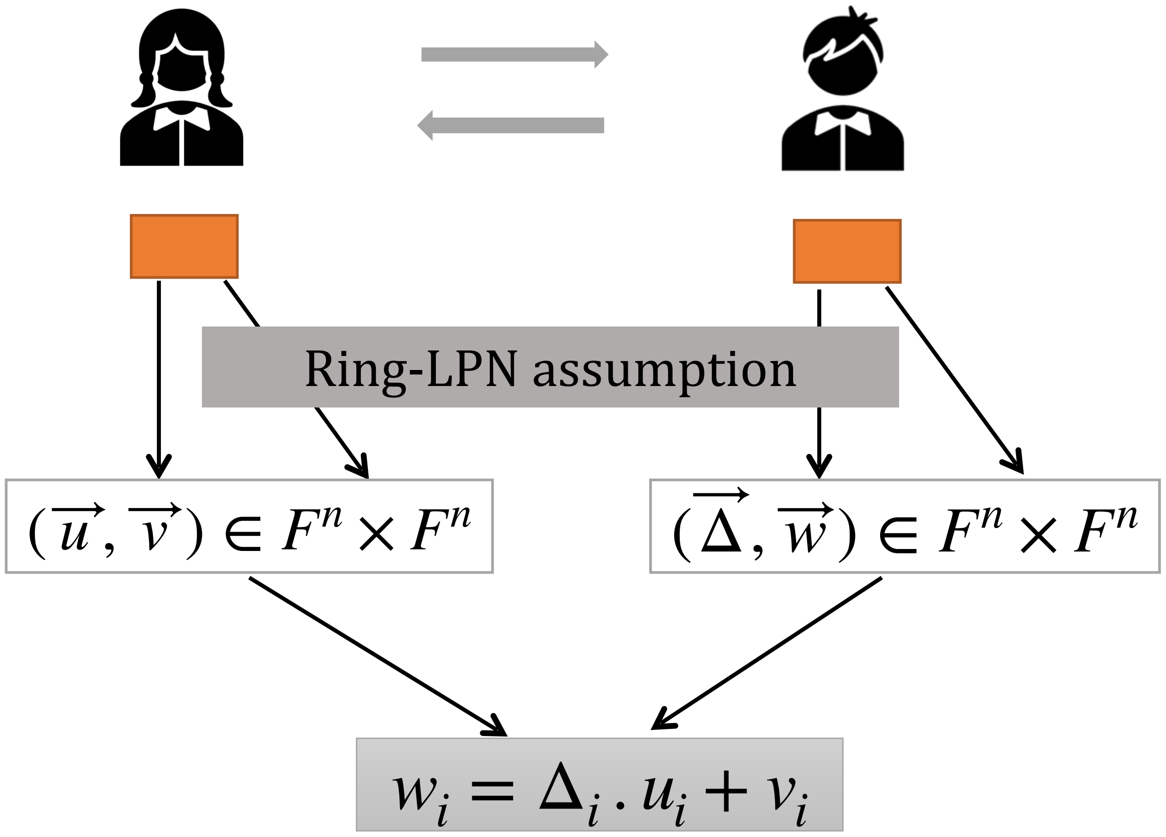

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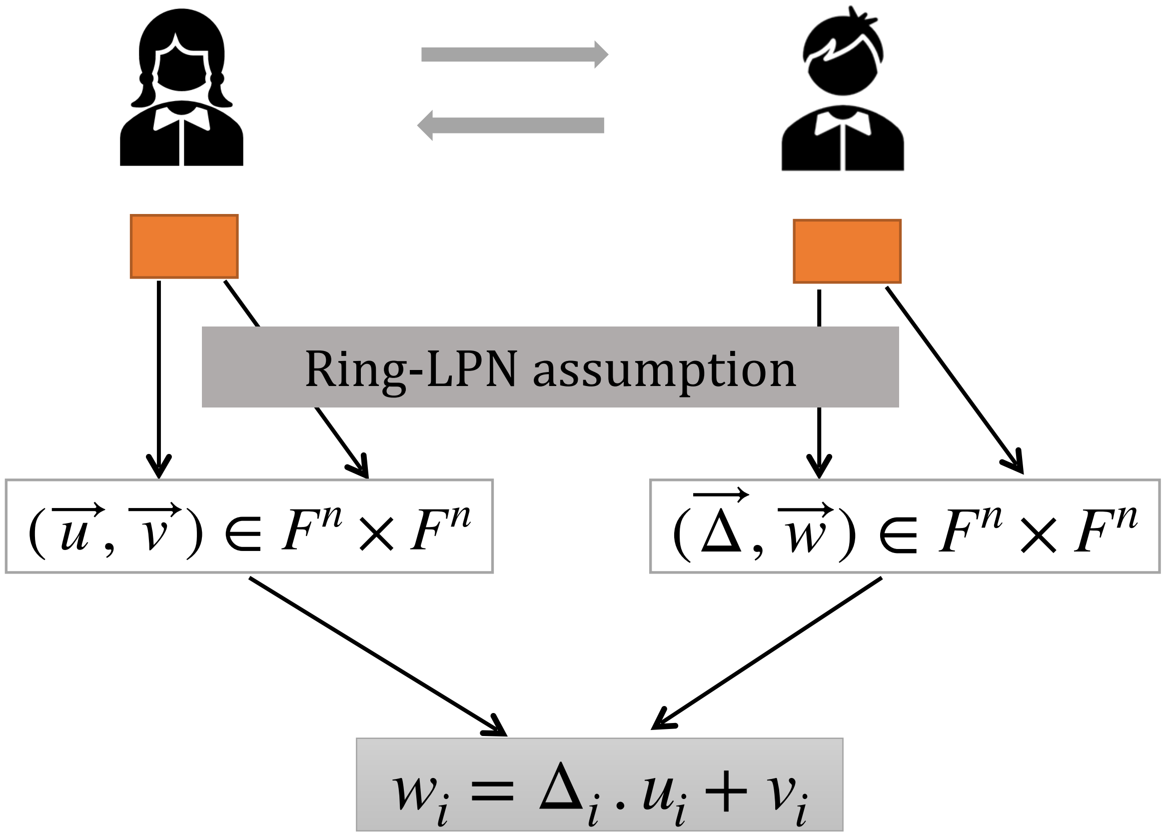

$\overrightarrow{\Delta} \in E^n$ - subfield-batch OLE

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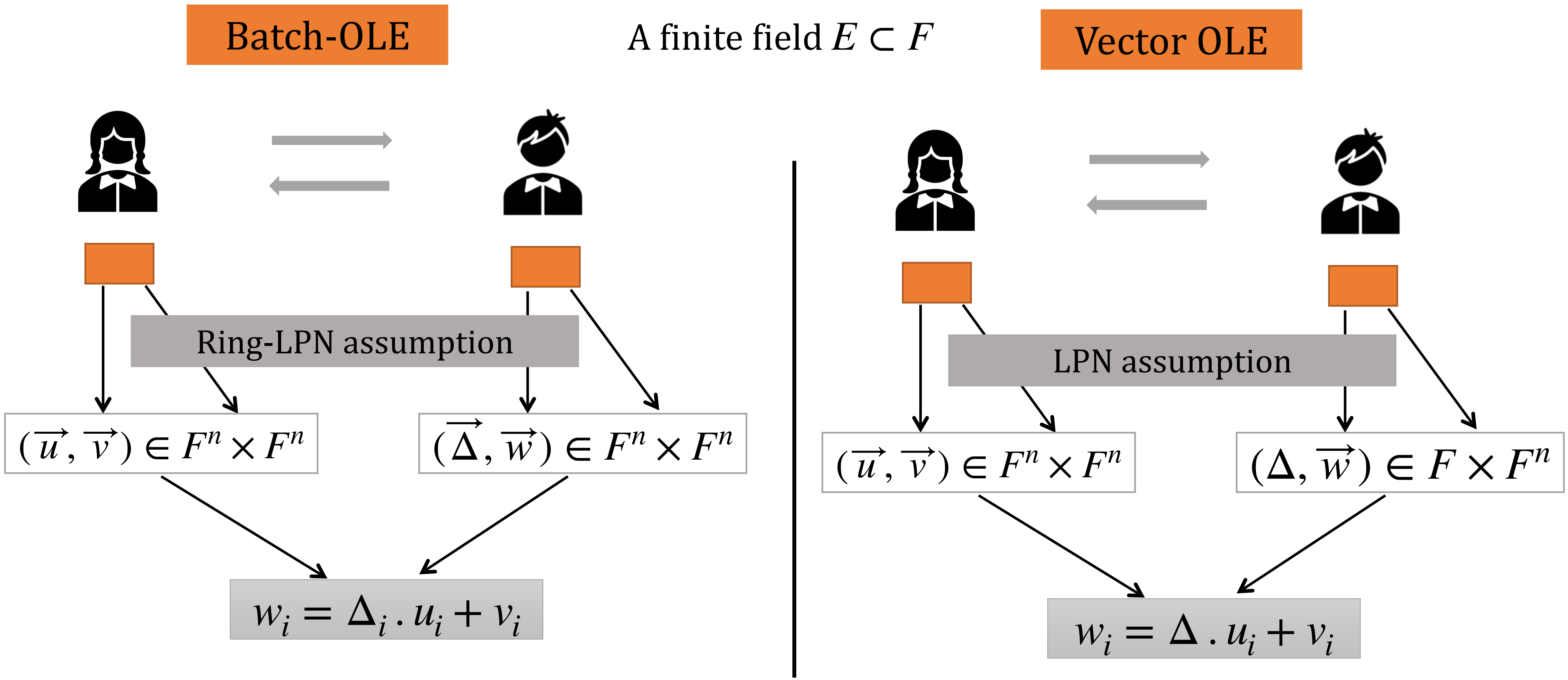

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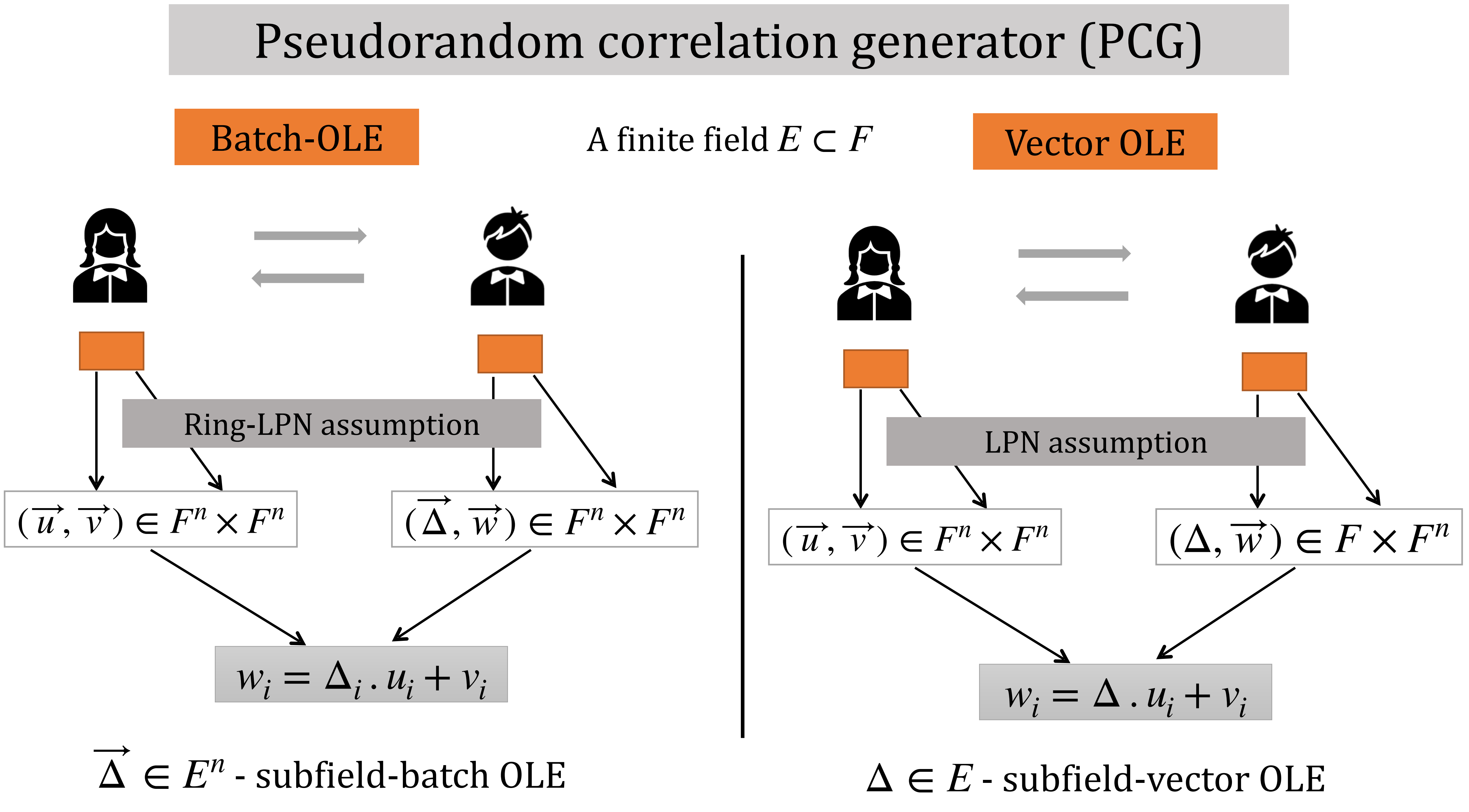
Pseudorandom correlation generator (PCG)
Batch-OLE
A finite field $E \subset F$
Vector OLE
Ring-LPN assumption
$(\overrightarrow{u}, \overrightarrow{v}) \in F^n \times F^n$
$(\overrightarrow{\Delta}, \overrightarrow{w}) \in F^n \times F^n$
$w_i = \Delta_i . u_i + v_i$
$\overrightarrow{\Delta} \in E^n$ - subfield-batch OLE
LPN assumption
$(\overrightarrow{u}, \overrightarrow{v}) \in F^n \times F^n$
$(\Delta, \overrightarrow{w}) \in F \times F^n$
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$\Delta \in E$ - subfield-vector OLE

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- **A semi-honest PSI protocol** from subfield-vector OLE and generalized Cuckoo hashing:
  - The communication depends only on the input set and statistic security parameter
  - Handle Cuckoo hashing having multiple items per bin by a new OPRF
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- **A malicious PSI protocol** from subfield-batch OLE and polynomial structure:
  - Secure without random oracle model or any tailor-made correlation robustness assumptions
  - Based on subfield-batch OLE on polynomial
  - Competitive communication even with the best ROM-based PSI

OPRF Construction

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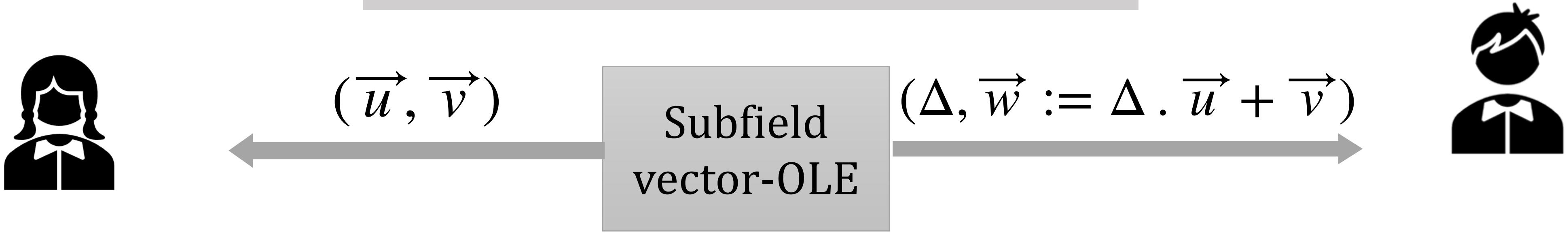

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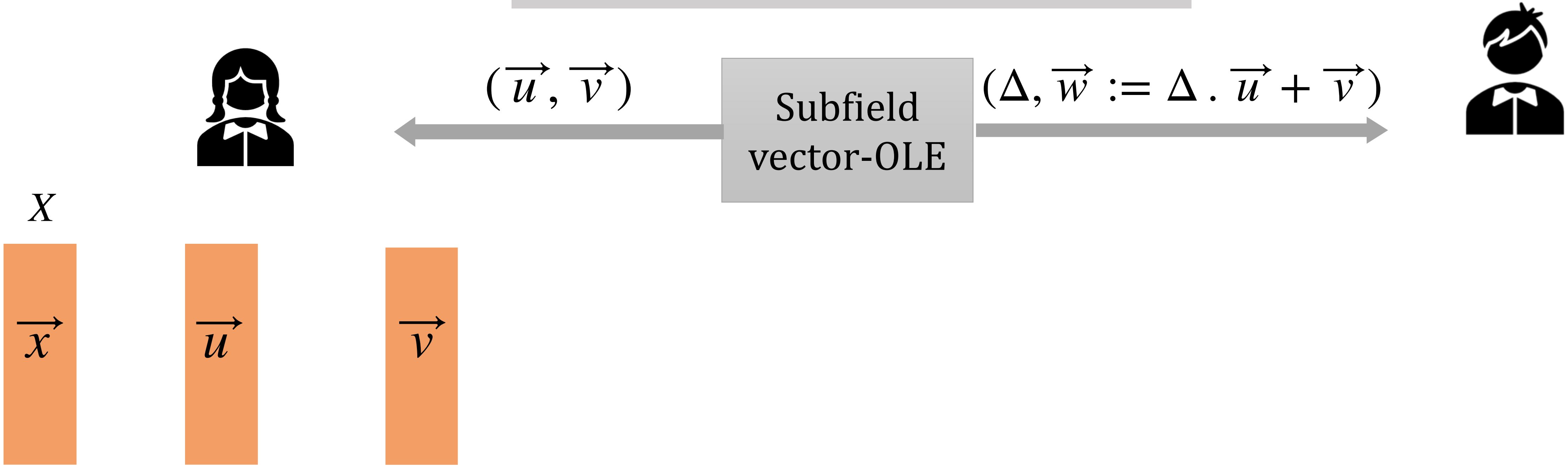

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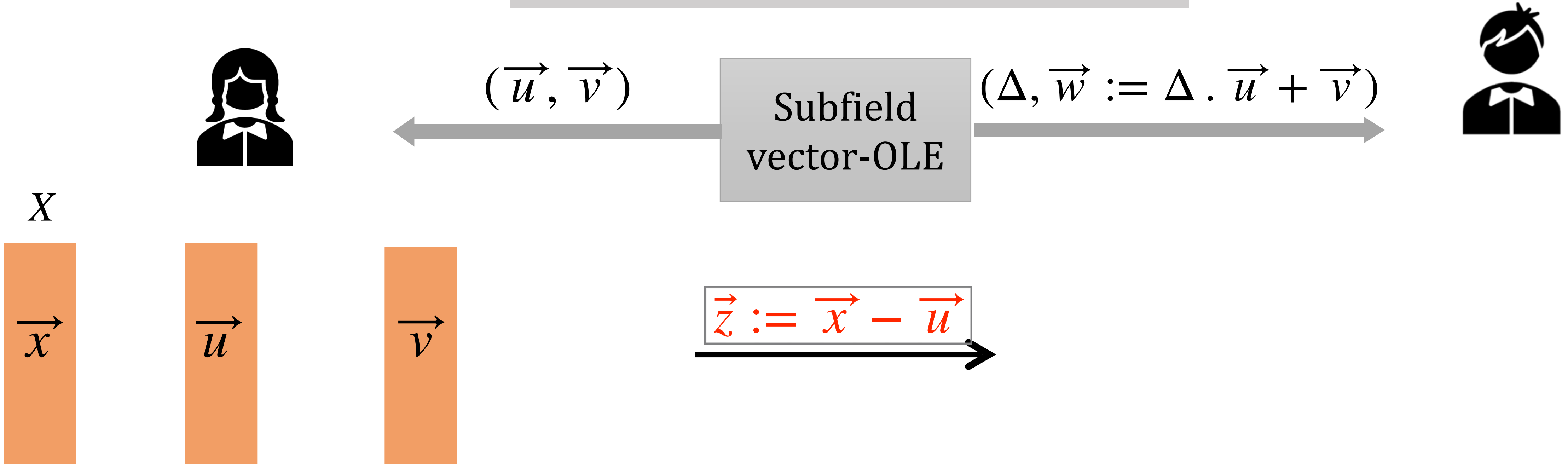

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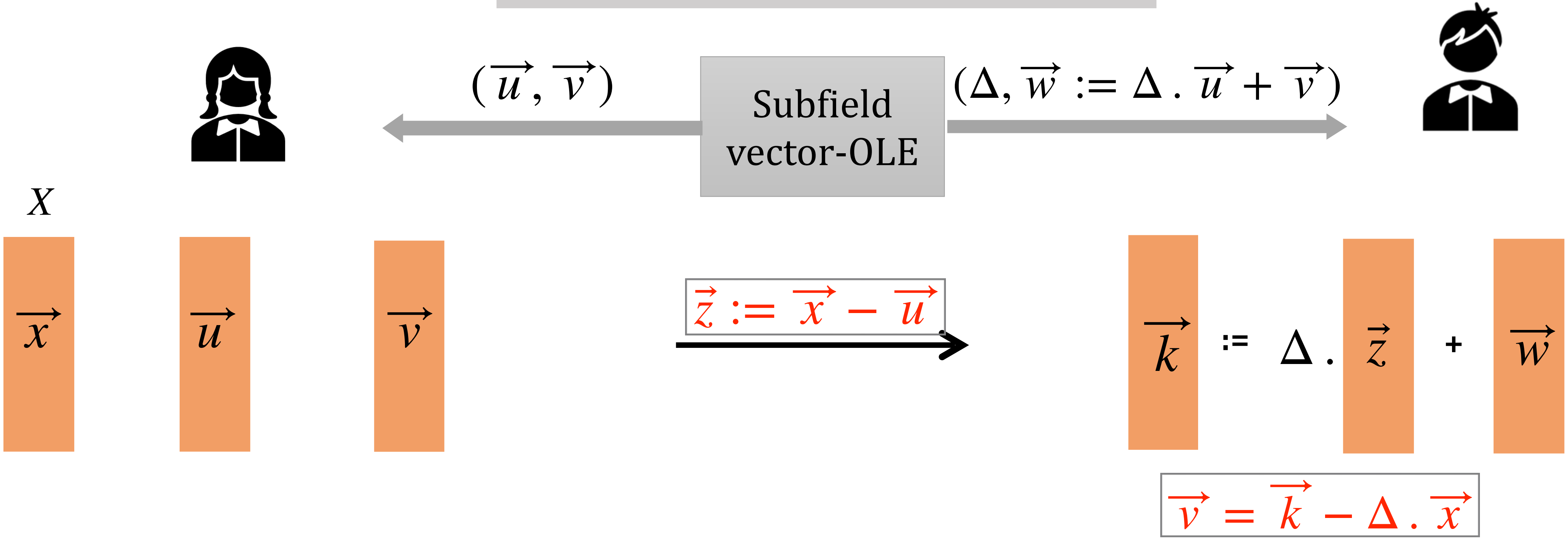

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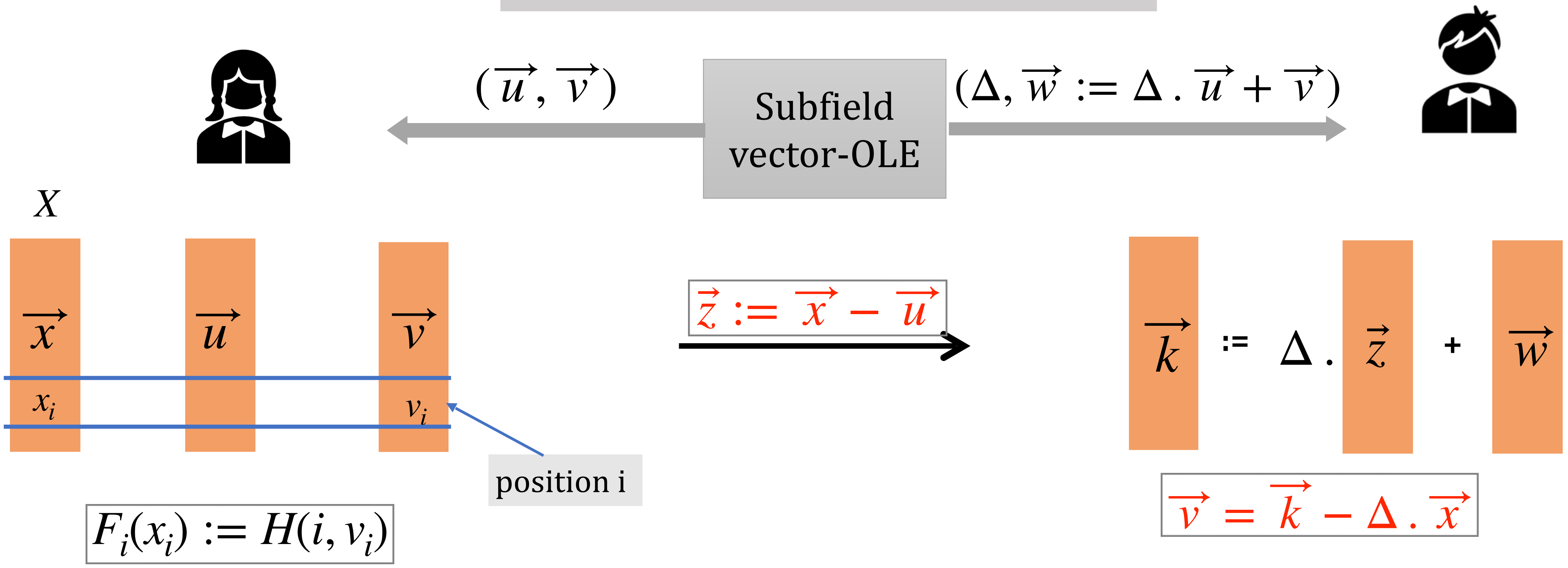

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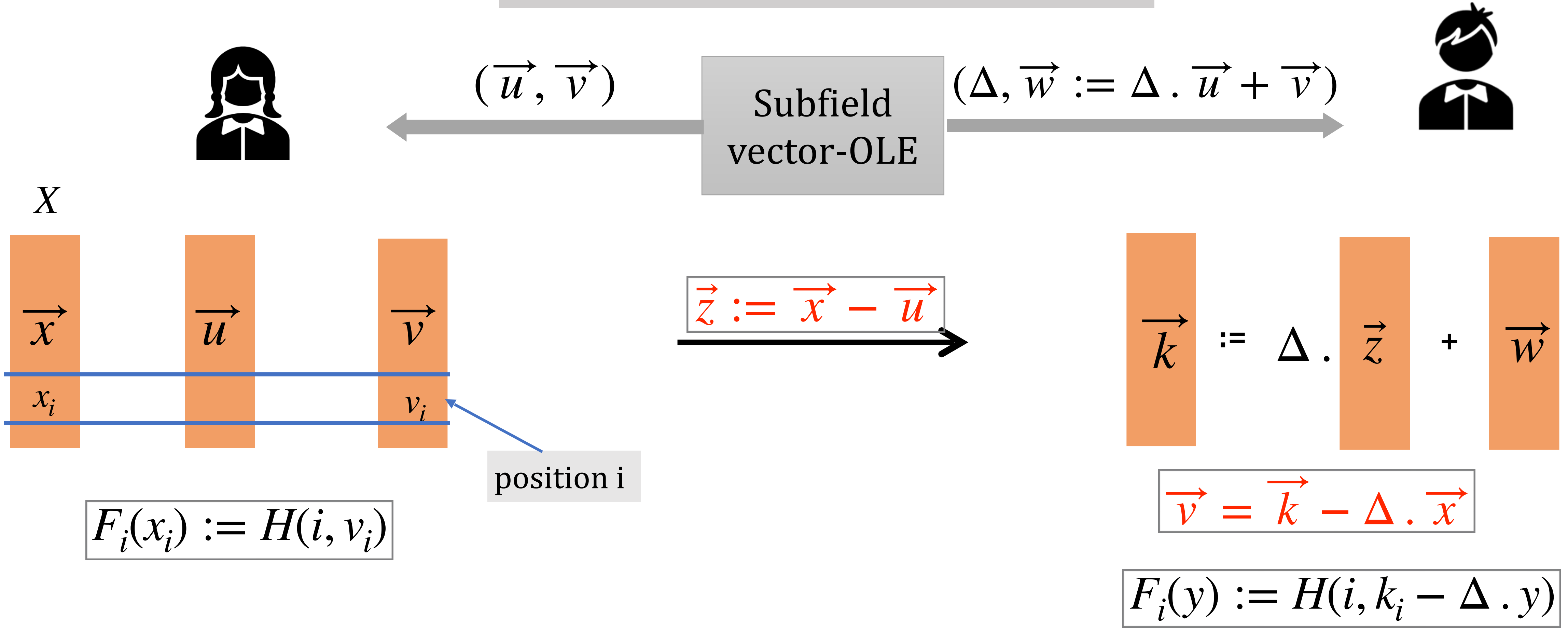

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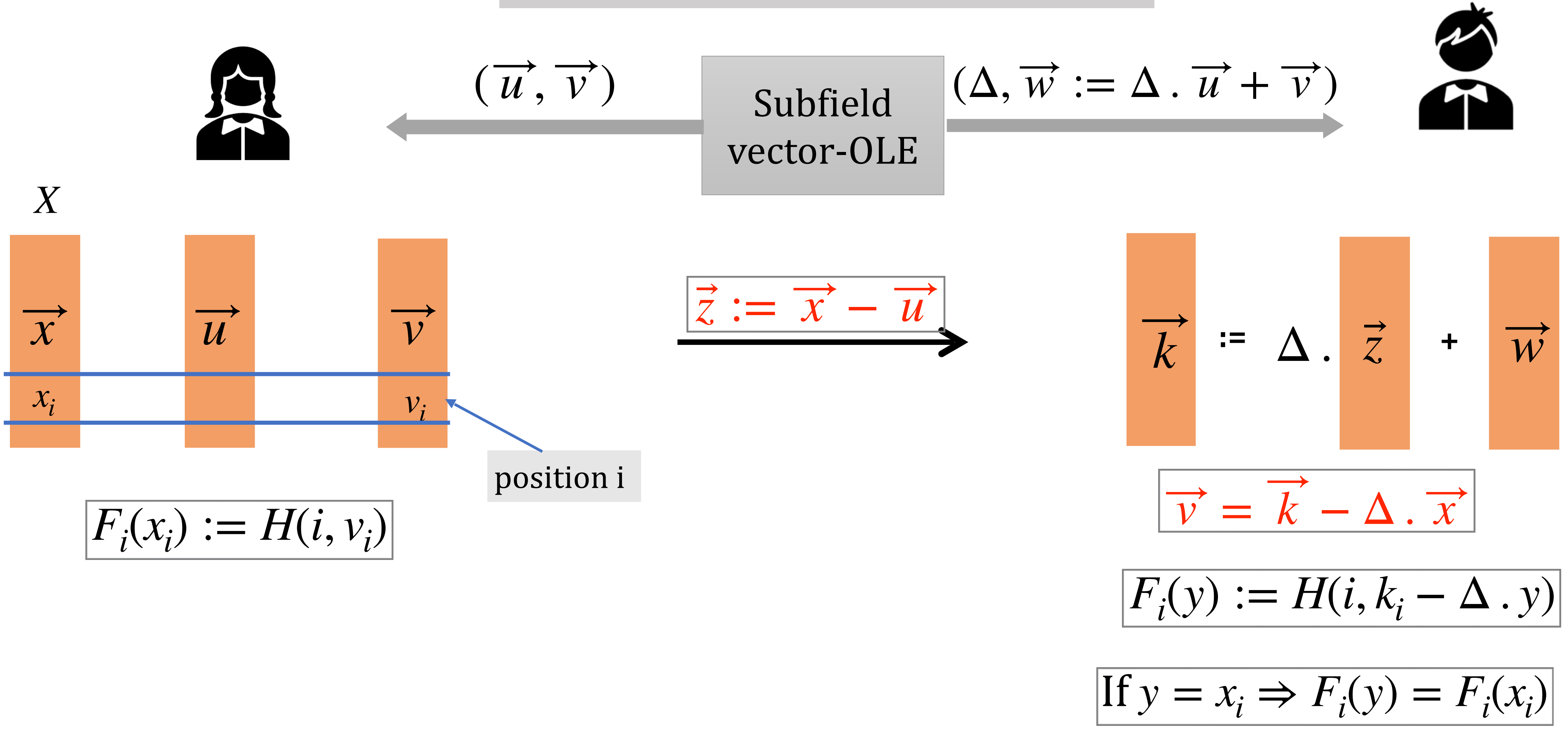

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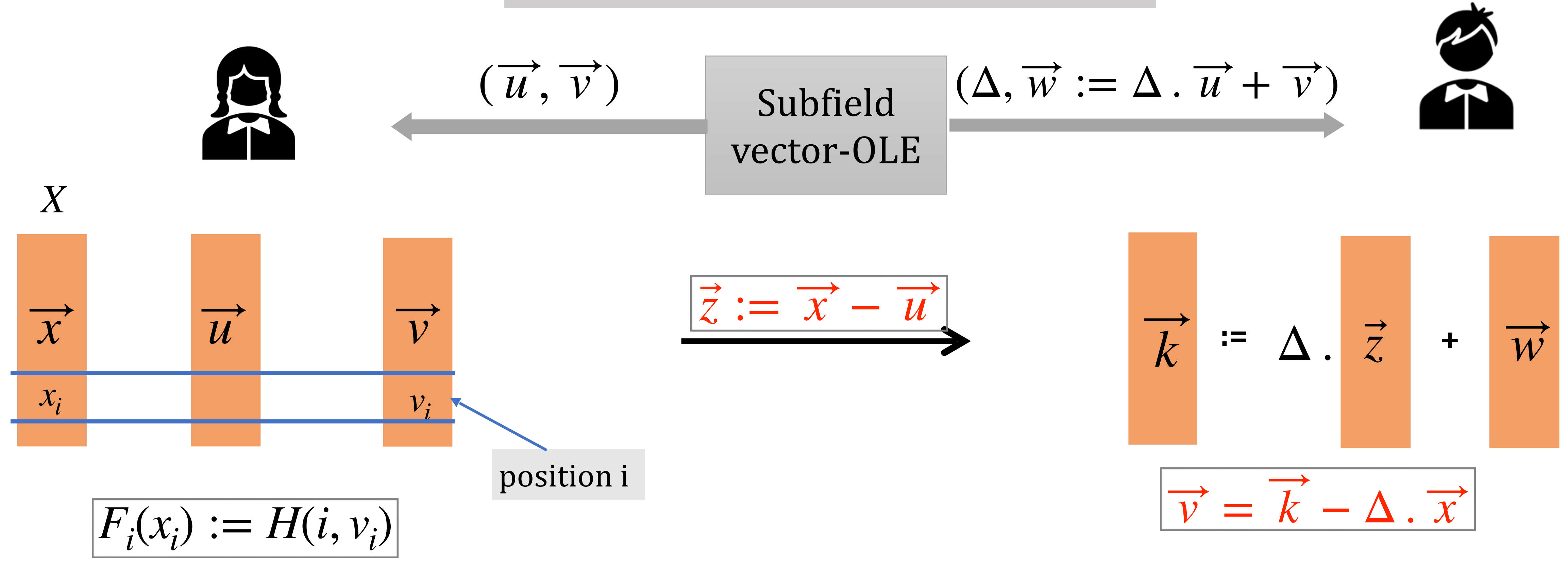

$$F_i(y) := H(i, k_i - \Delta . y)$$

If $y = x_i \Rightarrow F_i(y) = F_i(x_i)$

- Secure against semi-honest setting.
- Communication cost depends only on the bit-length of the input set.

Hashing scheme

Map n elements to m bins ($m > n$) by k hash functions $h_1, h_2, \ldots, h_k : \{0,1\}^* \rightarrow [m]$

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Each bin contains at most one element

One element x is put into exactly one bin $h_i(x)$ for $i \in [k]$

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In our work, each bin contains at most d elements

➡ (d,k)- general Cuckoo hashing

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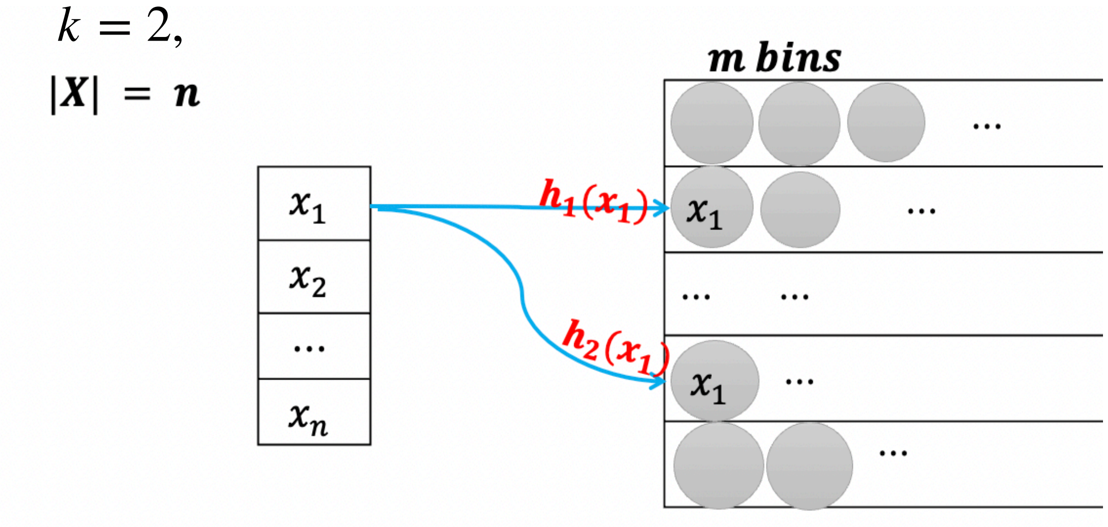
(d,k)- general Cuckoo hashing

Simple hashing for Bob

Any element x is put into all k bins $h_i(x)$



Semi-honest PSI

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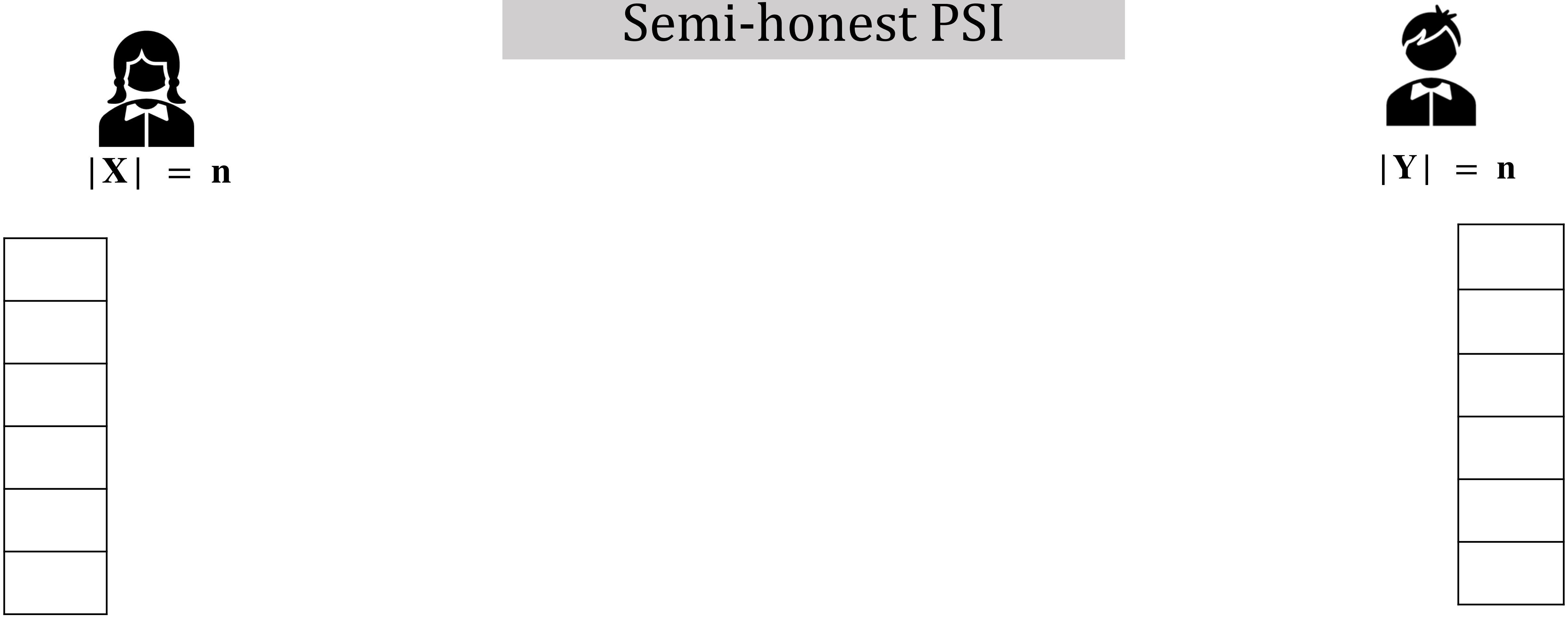

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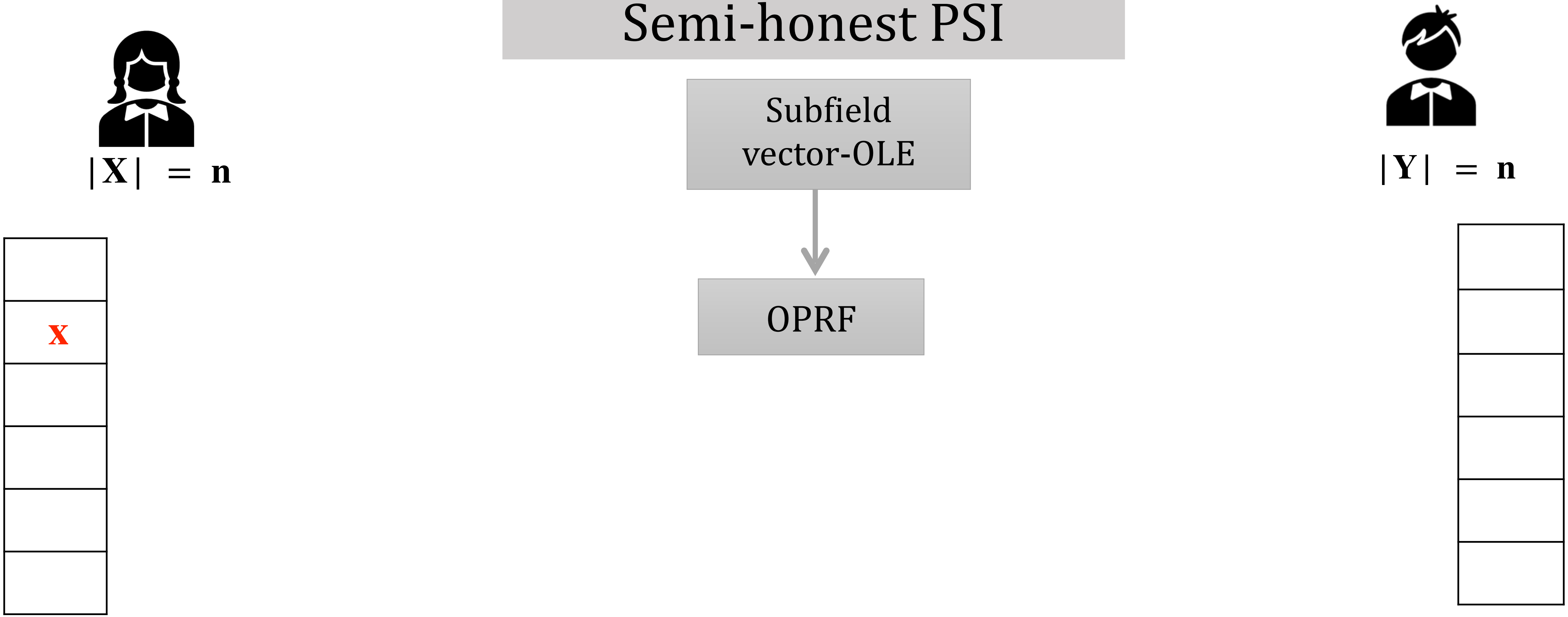

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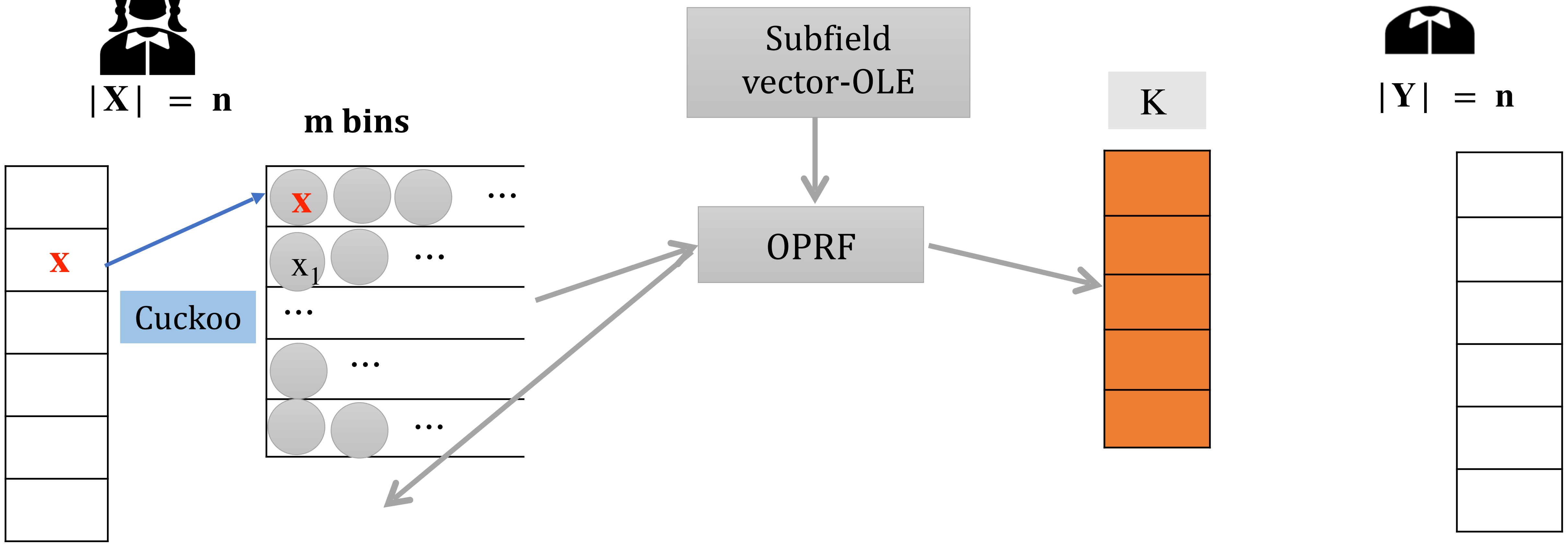
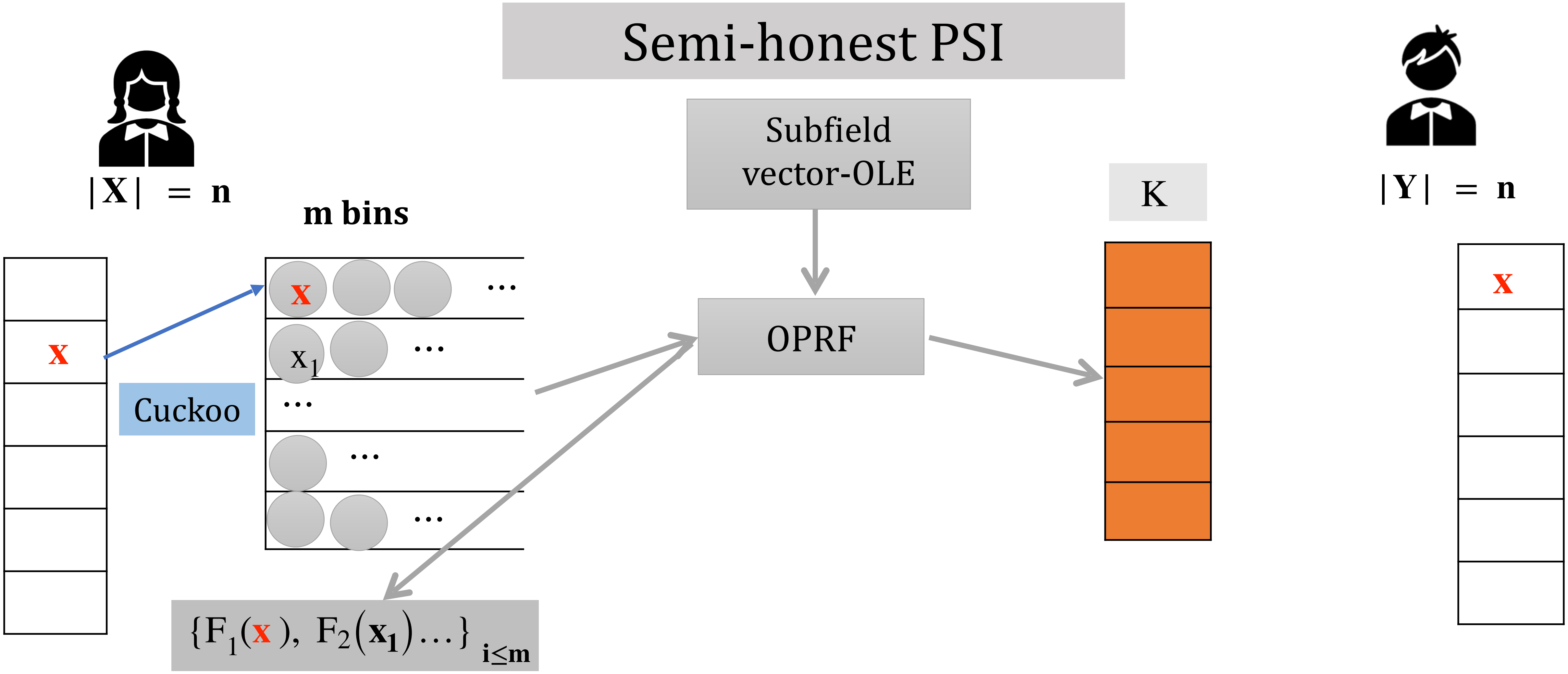
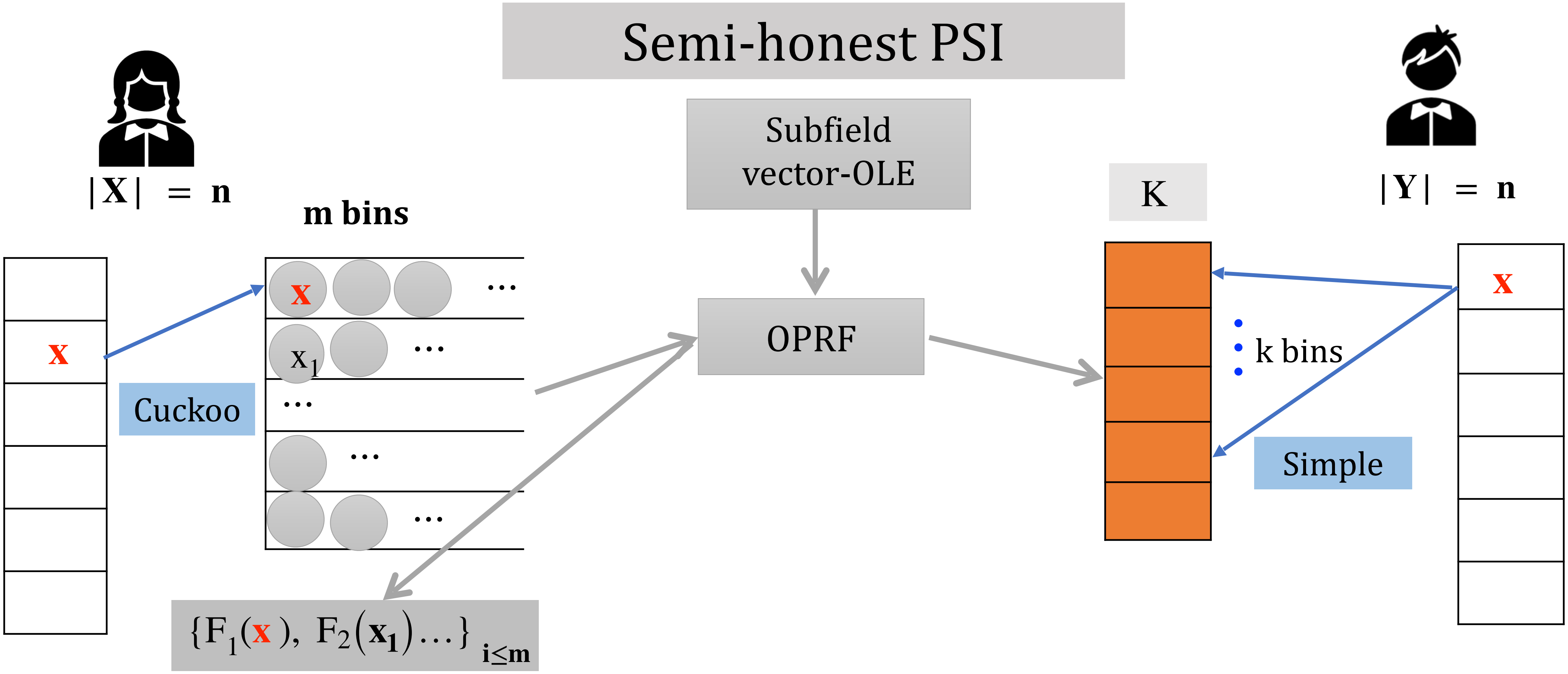




Semi-honest PSI
|X| = n
m bins
x
x
x1
…
Cuckoo
Subfield vector-OLE
OPRF
K
|Y| = n
x
{F1(x), F2(x1)…} i≤m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|X| = n
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m bins
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x
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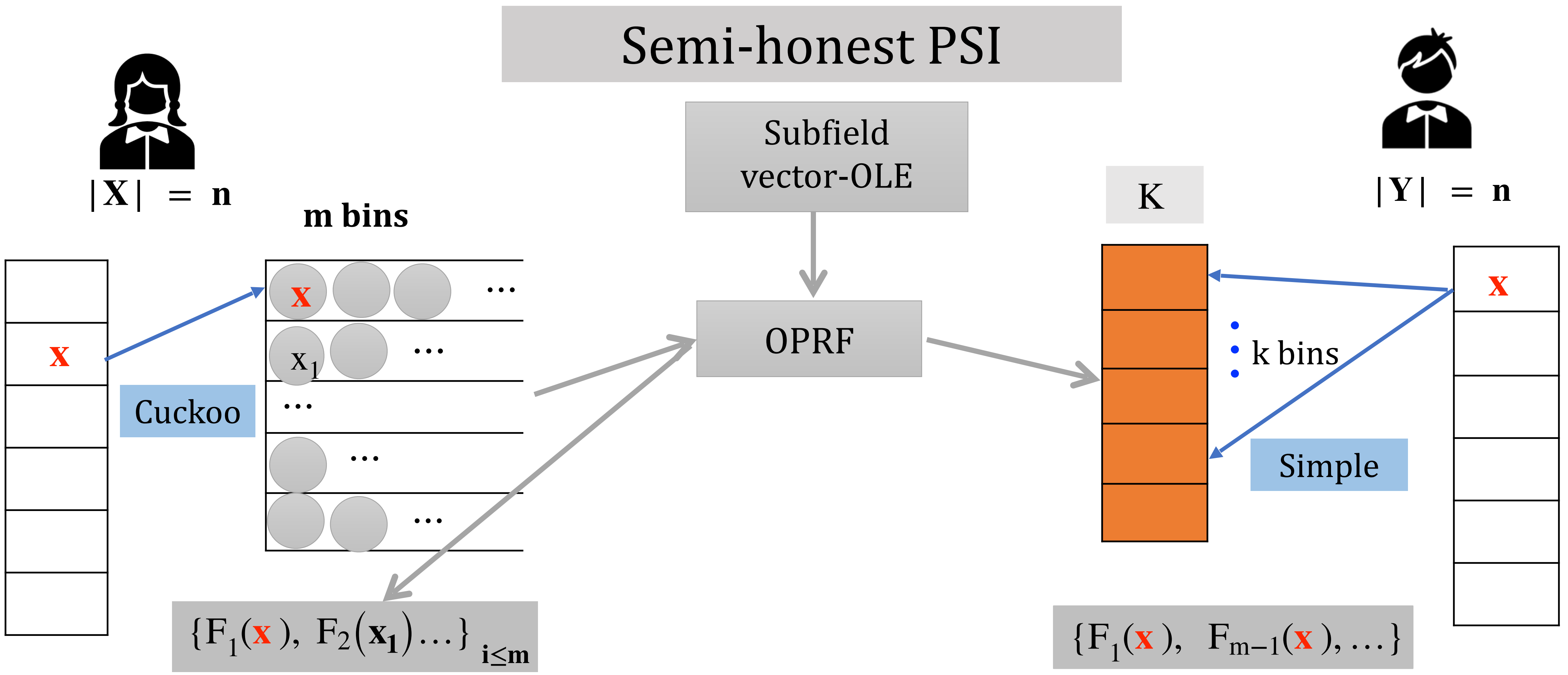
k bins
Simple
$\{F_1(x), F_2(x_1) \ldots\}_{i \le m}$


Semi-honest PSI
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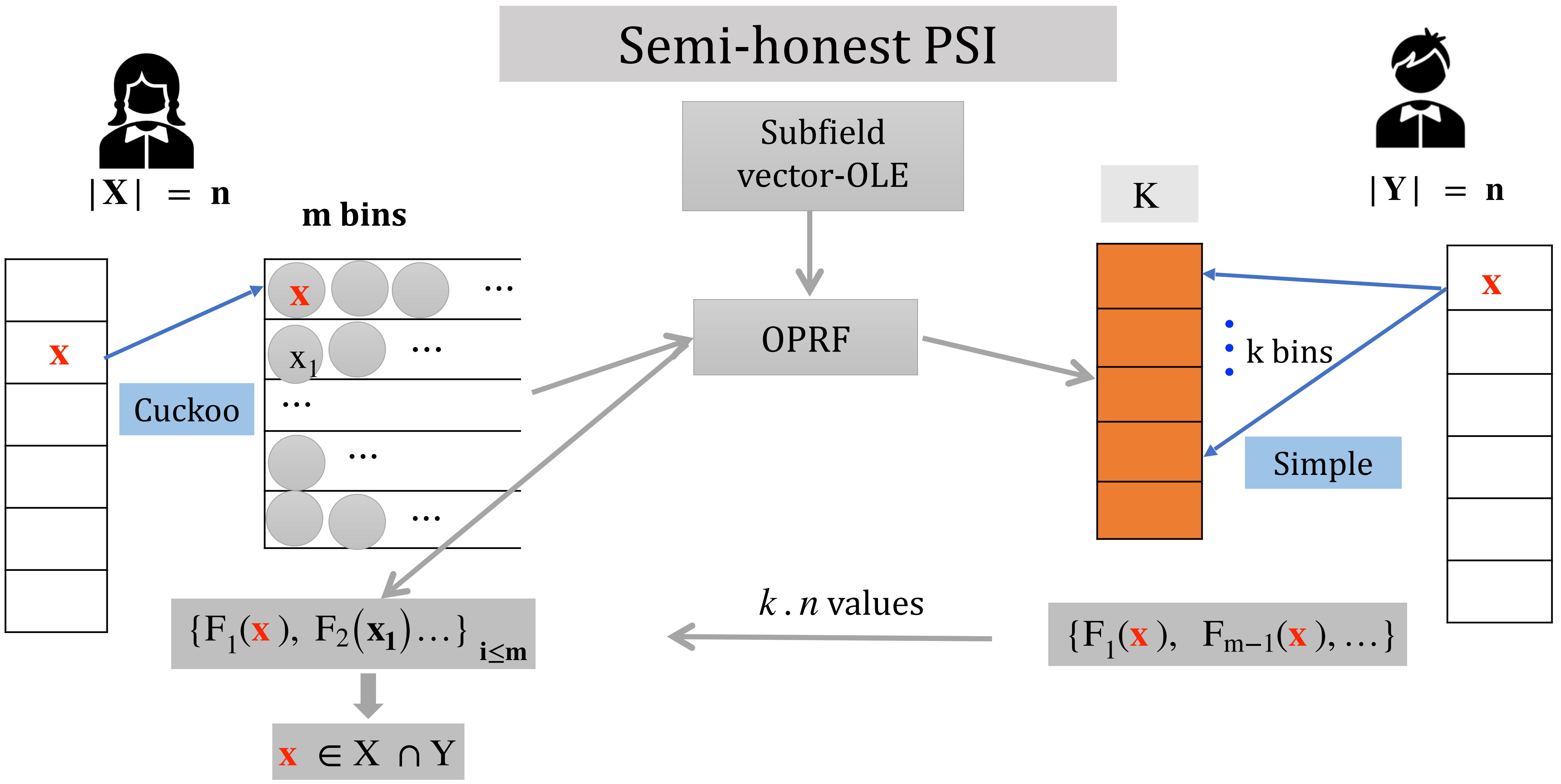
$\{F_1(x), F_2(x_1) \dots\}_{i \leq m}$
$\{F_1(x), F_{m-1}(x), \dots\}$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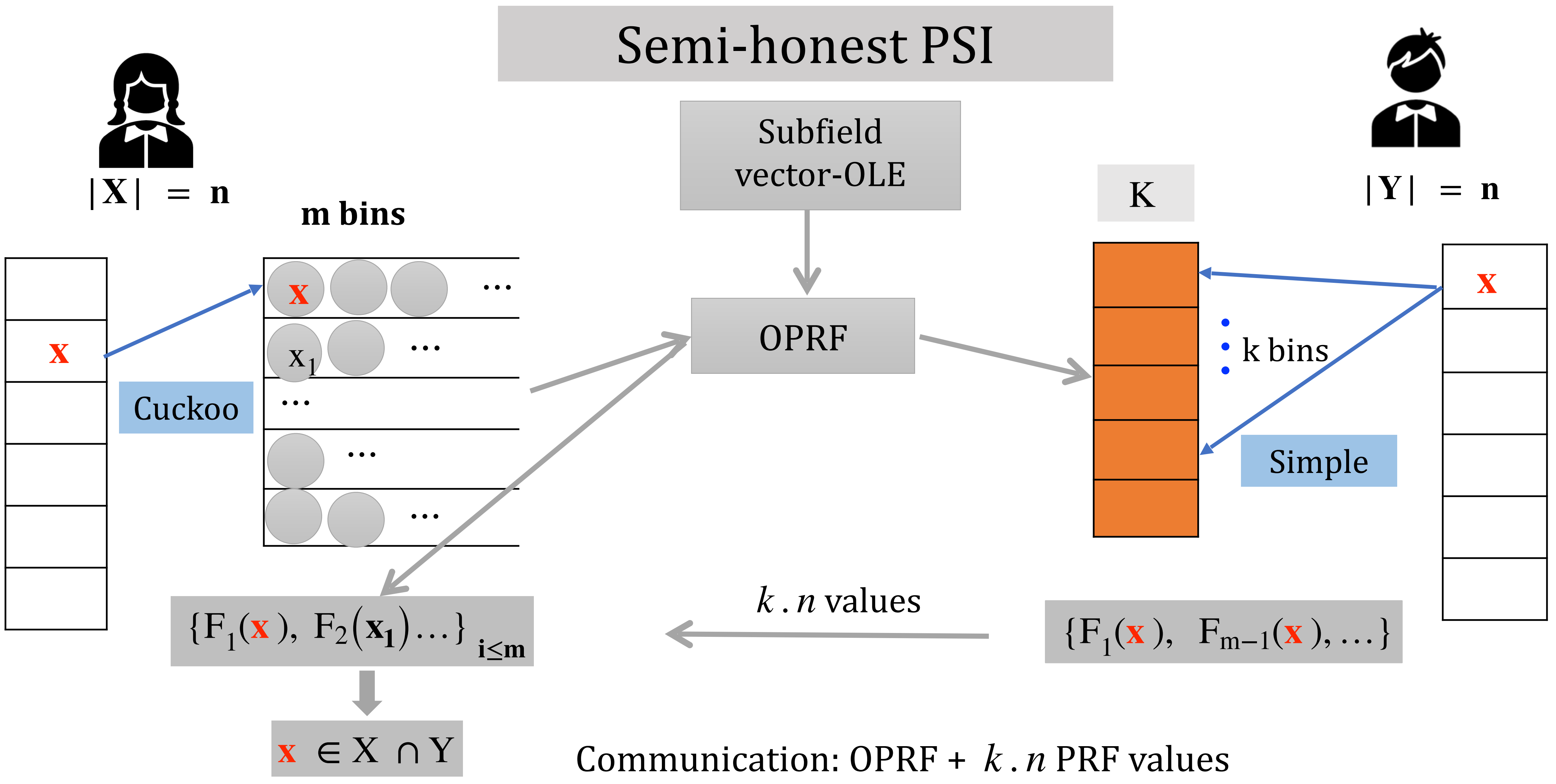
k . n values
{F1(x), Fm−1(x), ...}
x ∈ X ∩ Y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Subfield vector-OLE
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K
|Y| = n
x
x
x₁
Cuckoo
k bins
Simple
x
{F₁(x), F₂(x₁)…} i≤m
k . n values
{F₁(x), Fm−1(x), …}
x ∈ X ∩ Y
Communication: OPRF + k . n PRF values

Theoretical comparison

	$n = 2^{14}$	$n = 2^{16}$	$n = 2^{20}$	$n = 2^{24}$
Semi-honest setting				
KKRT16 [KKRT16]	$930n$	$936n$	$948n$	$960n$
PRTY19 [PRTY19] low*	$491n$	$493n$	$493n$	$494n$
PRTY19 [PRTY19] fast*	$560n$	$571n$	$579n$	$587n$
CM20 [CM20]	$668n$	$662n$	$674n$	$676n$
PRTY20 [PRTY20]	$1244n$	$1192n$	$1248n$	$1278n$
RS21 [RS21]	$2024n$	$898n$	$406n$	$374n$
RS21 [RS21] enhanced**	$280n$	$260n$	$263n$	$275n$
Ours ($\ell = 64$, GCH)	$246n$	$220n$	$210n$	$209n$
Ours ($\ell = 48$, GCH)	$215n$	$189n$	$179n$	$178n$
Ours ($\ell = 32$, GCH)	$184n$	$158n$	$148n$	$147n$
Ours ($\ell = 64$, 2CH)	$214n$	$190n$	$183n$	$185n$
Ours ($\ell = 48$, 2CH)	$193n$	$169n$	$162n$	$164n$
Ours ($\ell = 32$, 2CH)	$171n$	$148n$	$141n$	$142n$
Ours ($\ell = 64$, SH, $N = n/10$)	$332n$	$302n$	$284n$	$276n$
Ours ($\ell = 48$, SH, $N = n/10$)	$261n$	$230n$	$209n$	$198n$
Ours ($\ell = 32$, SH, $N = n/10$)	$191n$	$158n$	$133n$	$120n$
Ours ($\ell = 64$, SH, $N = 1$) ***	$154n$	$131n$	$125n$	$128n$
Ours ($\ell = 48$, SH, $N = 1$) ***	$138n$	$115n$	$109n$	$112n$
Ours ($\ell = 32$, SH, $N = 1$) ***	$122n$	$99n$	$93n$	$96n$

n: the size of the database.

l: bit-length of the inputs in the database.

[*] PRTY19 has two variants:
- SpOT-low (lowest communication, higher computation).
- SpOT-fast (higher communication, better computation).

[**] Using the 3H-GCT OKVS of GPRTY21 instead of PaXoS, and the VOLE of CRR21 instead of the one from WYKW21.

[***] Using $n = 1$ requires an expensive degree-n polynomial interpolation.

Thank you, Questions? ☺

For more details: ia.cr/2022/334