Time-Space Tradeoff for Collision Finding in Sponge Functions

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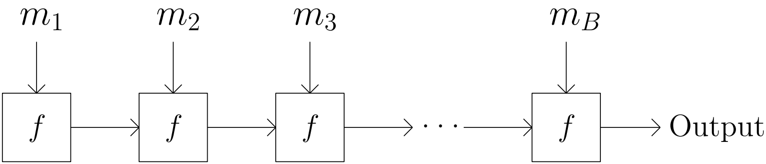
Domain Extension Hash Functions



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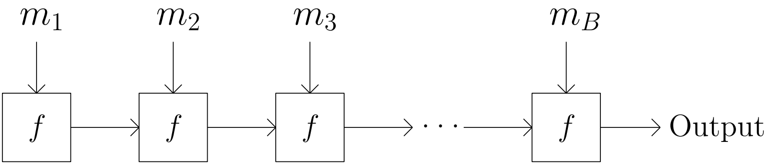

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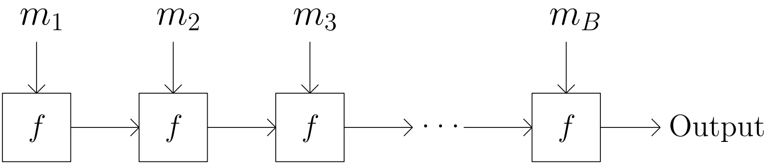

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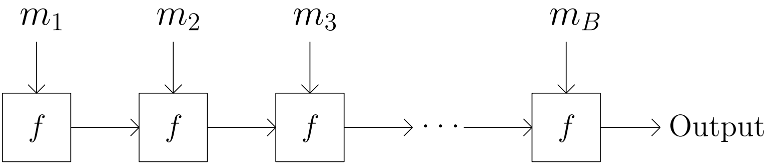

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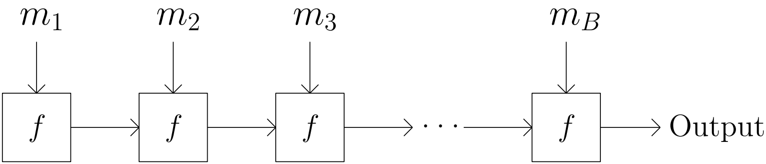

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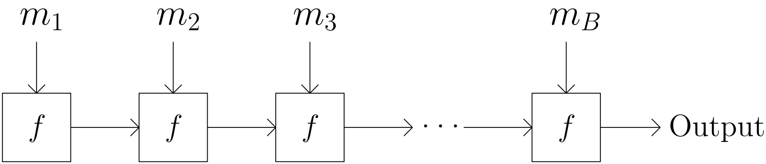

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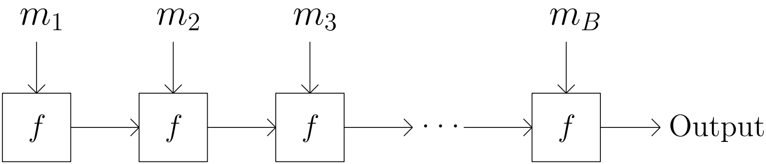

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Trivial birthday attack advantage: T^2/N (MD) or T^2/R (Sponge)
Exist non-trivial attacks!

Short Collision Finding in Merkel Damgård [CDGS18,ACDW20,GK22,AGL22]

Message Length	Best Known Attack
$B = 1$	$S/N + T^2/N$
$B = 2$	$ST/N + T^2/N$
$3 \leq B \leq T$	$STB/N + T^2/N$
$B > T$	ST^2/N

Short Collision Finding in Sponge [FGK22]

Message Length	Known Attack (MD)	Known Attack (Sponge)
$B = 1$	$S/N + T^2/N$	$\min((ST/C)^2, (S^2T/C^2)^{\frac{2}{3}})$ $+S/C + T^2/R$
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Utilizes the inverse oracle

Multi-Instance Games

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Old Techniques [DGK17,CDGS18]: presampling, compression

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Multi-Instance Games (MI): A recent technique for proving security bounds for preprocessing attacks [IK10,CGLQ20,ACDW20,AGL22,FGK22]

Upper Bounds in Merkel Damgård [ACDW20,GK22,AGL22]

Message Length	Best Known Attack	Upper Bound Tight?
$B = 1$	$S/N + T^2/N$	✓
$B = 2$	$ST/N + T^2/N$	✓
$3 \leq B \leq T$	$STB/N + T^2/N$	$\times$
$B > T$	ST^2/N	✓

MI works pretty well here

Upper Bounds in Sponge [FGK22]

Message Length	Best Known Attack	Upper Bound Tight?
$B = 1$	$\min((ST/C)^2, (S^2T/C^2)^{\frac{2}{3}})$ $+S/C + T^2/R$	$\times$
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$B > T$	$ST^2/C + T^2/R$	$\checkmark$

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What happens at sponge?

Can we prove better bounds (via MI)?

Our Results

Message Length	Best Known Attack	Upper Bound Tight?
$B = 1$	$\min((ST/C)^2, (S^2T/C^2)^{\frac{2}{3}})$ $+S/C + T^2/R$	Almost
$B = 2$	$ST/C + T^2/\min(C, R)$	$\times$
$3 \leq B \leq T$	$STB/C + T^2/\min(C, R)$	$\times$
$B > T$	$ST^2/C + T^2/R$	$\checkmark$

Better bounds for $B = 1$, Simpler proofs for $B = 2$

Our Results

Message Length	Upper Bound Tight?	Better bounds for MI?
$B = 1$	Almost	$\times$
$B = 2$	$\times$	$\times$
$3 \leq B \leq T$	$\times$	$\times$
$B > T$	$\checkmark$	-

Limit of MI games

MI and AI

Advantages between MI and AI adversaries [AGL22]

We can reduce an AI adversary with success probability 2ϵ to an MI adversary with probability $\tilde{O}(\epsilon^S)$.

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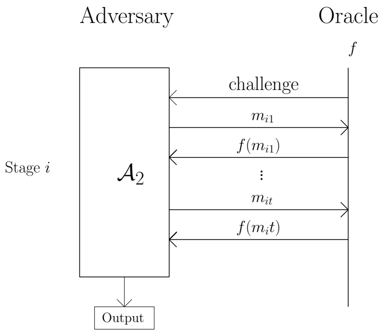
Proof Idea. Guess the S-bit advice and run AI with that advice each round. The 2^{-S} guessing probability will be amortized into ϵ^S.

Multi-Instance Games

Repeat S times:



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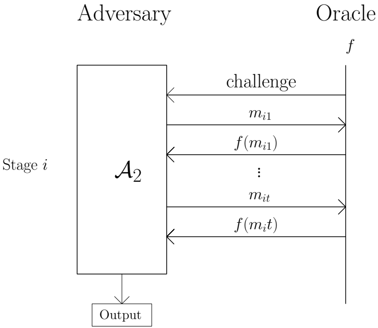

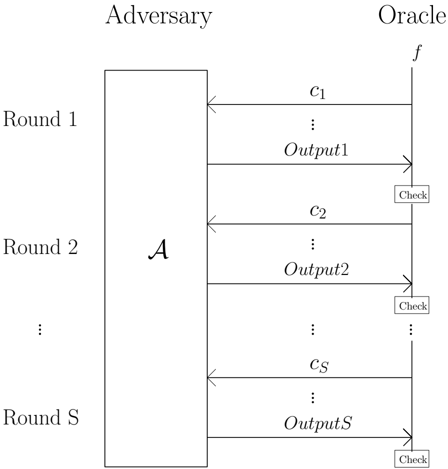

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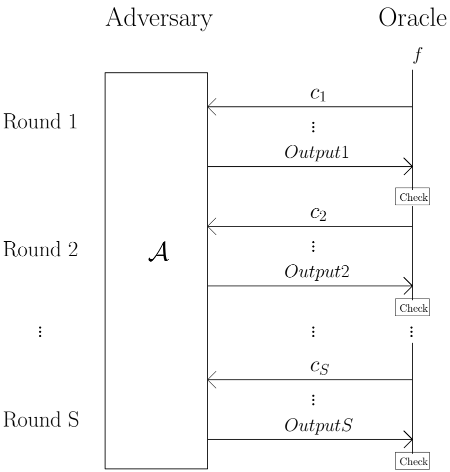

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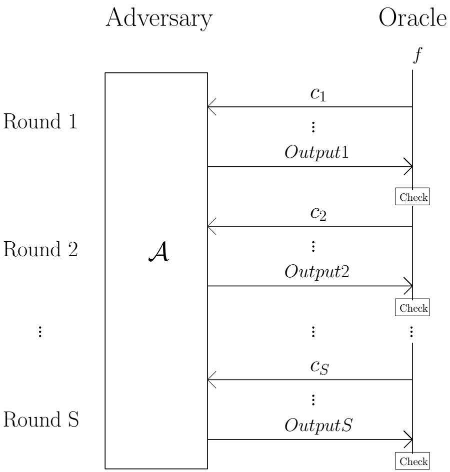

Something to mention:

- No advice string
- f doesn't change within rounds
- Has "memory" of previous rounds
- Need to win all S rounds

Multi-Instance Techniques

Main idea: By bounding the success probability of the MI game, we directly have upper bound for the original AI adversary.

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- No advice bits
- Ability to use lazy sampling and other techniques

Often easier to find upper bounds

Our Results

	Upper Bound	Known Attack
$B = 1$	$S^2T^2/C^2 + T^2/R$ $+S/C + T/C$	$\min((ST/C)^2, (S^2T/C^2)^{\frac{2}{3}})$ $+S/C + T^2/R$
$B = 2$	$ST/C + S^2T^4/C^2$ $+T^2/\min(C, R)$	$ST/C + T^2/\min(C, R)$
$B \geq 3$	$ST^2/C + T^2/R$	$STB/C + T^2/\min(C, R)$

Our proof uses Multi-Instance Games technique
and highly non-trivial compression argument (please refer to original paper)

Our Results

Showed limitations of MI Techniques:

	Upper Bound Given by MI	Best Attack in MI
$B = 1$	$(\tilde{O}(S^2T^2/C^2 + T^2/R + S/C + T/C))^S$	$(\tilde{\Omega}(S^2T^2/C^2))^S$
$B = 2$	$(\tilde{O}(ST/C + S^2T^4/C^2 + T^2/\min(C,R)))^S$	$(\tilde{\Omega}(S^2T^4/C^2))^S$
$B \geq 3$	$(\tilde{O}(ST^2/C + T^2/R))^S$	$(\tilde{\Omega}(ST^2/C))^S$

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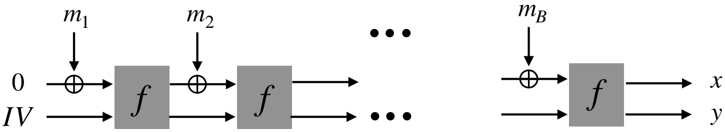
$B \geq 3$	$(\tilde{O}(ST^2/C + T^2/R))^S$	$(\tilde{\Omega}(ST^2/C))^S$

It means we can't use MI to further bridge the gaps.

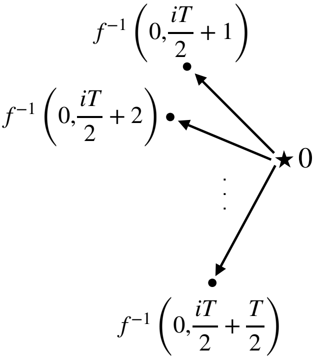
Sponge Hash Functions

Input $m = m_1 || \ldots || m_B$, $m_i \in [R]$



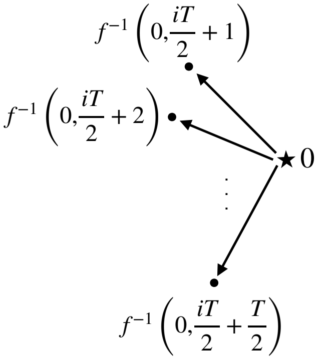
$$\textbf{Sponge}^f(IV, m) := x$$

where $f : [R] \times [C] \to [R] \times [C]$ is a permutation



$f^{-1}\left(0, \frac{iT}{2} + 1\right)$
$f^{-1}\left(0, \frac{iT}{2} + 2\right)$
$\star\, 0$
$f^{-1}\left(0, \frac{iT}{2} + \frac{T}{2}\right)$

MI Attack, B=2



(1) Query $f^{-1}(0, i)$ for different i

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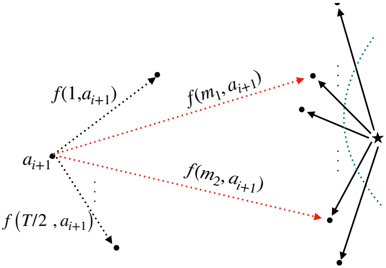

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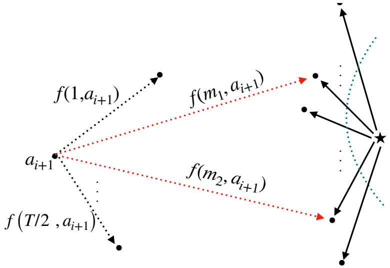

(1) Query $f^{-1}(0,i)$ for different i
(2) For challenge salt a, query $f(j,a)$ for different j
If two queries in (2) hits two salts visited in (1),

$$f^{-1}(0,i_1) = (m_1,a_1)$$
$$f(m_3,a) = (m_5,a_1)$$
$$f^{-1}(0,i_2) = (m_2,a_2)$$
$$f(m_4,a) = (m_6,a_2)$$

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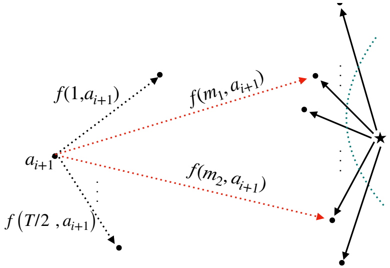

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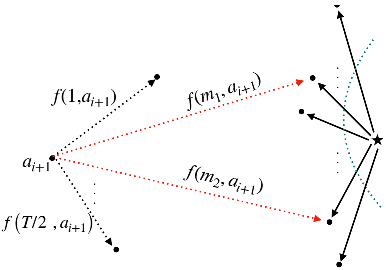
then we have found valid collisions on challenge salt a
$(m_3 | m_5 \oplus m_1), (m_4 | m_6 \oplus m_2)$

MI Attack, B=2



Each round: wins if we hit two old salts within $T/2$ queries

# of different salts in (1):

$$\tilde{\Omega}(iT)$$

Winning Probability this round:

$$\tilde{O}((iT^2/C)^2)$$

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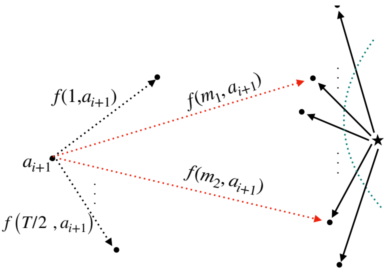

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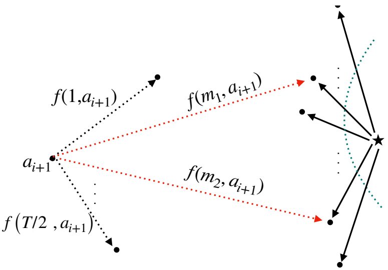

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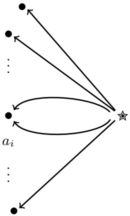
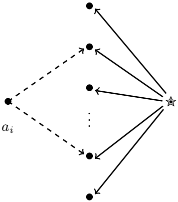
Matches current upper bound (proved by MI)

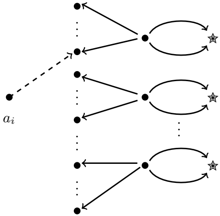
MI Attacks



(a) 1-Block collision attack

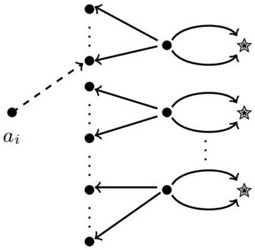


(b) 2-Block collision attack



(c) 3-Block collision attack

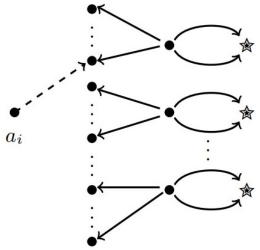
MI Attack, B=3



(c) 3-Block collision attack

(1) Query out collisions at some salts a (via birthday attack)
(2) Query $f^{-1}(*, a)$ on these salts
(3) Query $f(*, a_i)$ for challenge salt a_i

MI Attack, B=3



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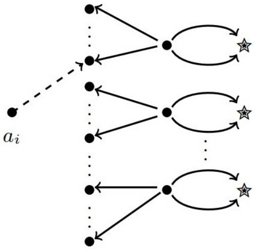
Wins if one query in (3) hits one salt in step (2)

Winning Probability for MI-game:

$$(\tilde{O}(ST^2/C))^S$$

MI Attack, B=3



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Matches current upper bound (proved by MI)

Recap

Better bounds for $B = 1$, Showed limitation of MI

Message Length	Upper Bound Tight?	Better bounds for MI?
$B = 1$	Almost	$\times$
$B = 2$	$\times$	$\times$
$3 \leq B \leq T$	$\times$	$\times$
$B > T$	$\checkmark$	-

Open problems:

- Tight bounds (even for $B = 2$)?
- Better methods than MI?
- Better attacks?