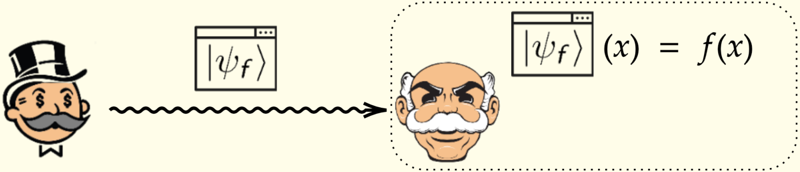
Semi-Quantum Copy-Protection and More

Céline Chevalier, Paul Hermouet and Quoc-Huy Vu

Copy-Protection

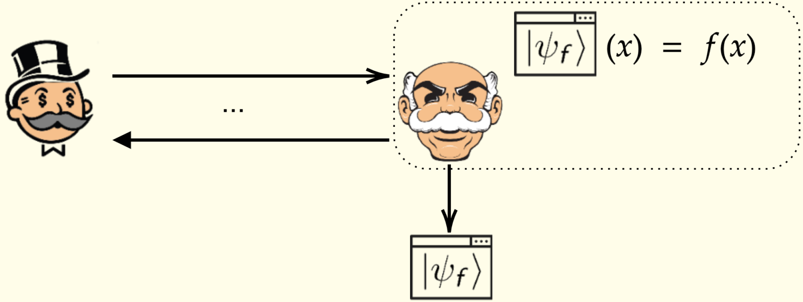
Copy-protection: Vendor and Client are quantum; quantum communications

Copy-Protection



Copy-protection: Vendor and Client are quantum; quantum communications

Semi-Quantum Copy-Protection



Copy-protection: Vendor and Client are quantum; quantum communications

Semi-quantum Copy-protection: Vendor is classical; classical communications

Coset States

For $A \subset \mathbb{F}_2^n$, $s, s' \in \mathbb{F}_2^n$

$$|A_{ss'}\rangle = \frac{1}{\sqrt{|A|}} \sum_{a \in A} (-1)^{a \cdot s'} |a + s\rangle$$

Holds information on both $A + s$ and $A^{\perp} + s'$

[1] [CLLZ21]

Coset States

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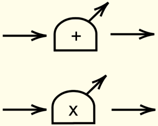

$u \in A + s$

$v \in A^{\perp} + s'$

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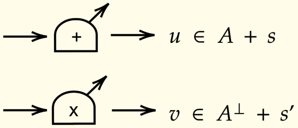

Holds information on both $A + s$ and $A^\perp + s'$

Direct product hardness:
No adversary can, given $|A_{s,s'}\rangle$ return $u \in A + s$ and $v \in A^\perp + s'$.

[1][CLLZ21]

Monogamy-of-Entanglement

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$$p_{win} = \mathsf{negl}(\lambda)$$

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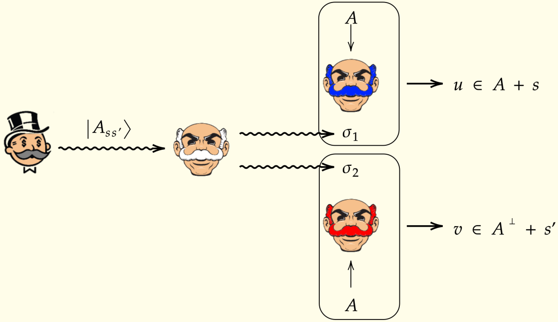

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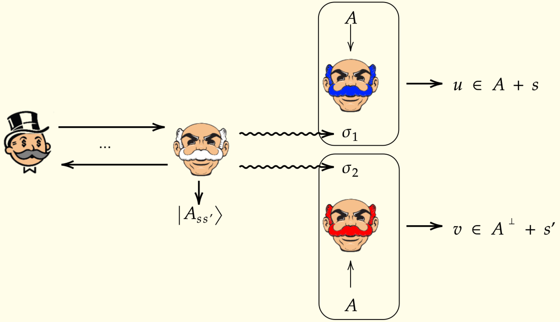


Semi-Quantum Monogamy-of-Entanglement

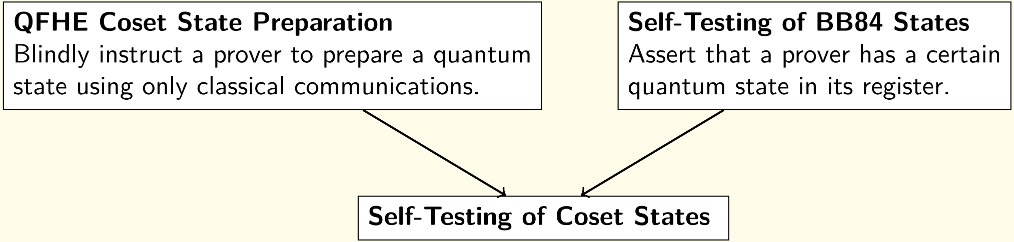
$$p_{win} = \mathsf{negl}(\lambda) \text{ ?}$$

Construction Overview

QFHE Coset State Preparation
Blindly instruct a prover to prepare a quantum state using only classical communications.

Self-Testing of BB84 States
Assert that a prover has a certain quantum state in its register.

Construction Overview



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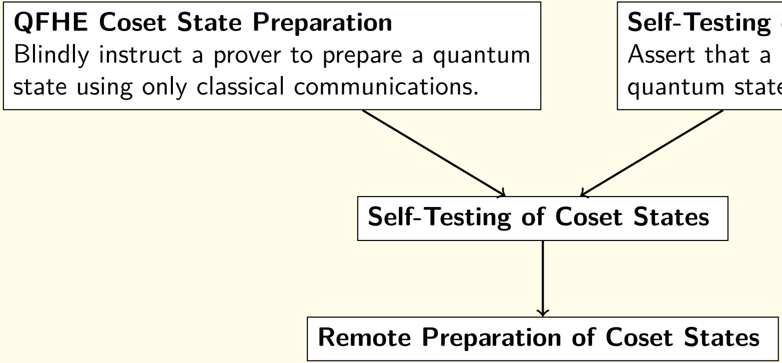

QFHE Coset State Preparation

Q(uantum)FHE: $\mathsf{Enc}(x) \rightarrow$ x $\mathsf{Eval}(C,$ x $) \rightarrow \mathsf{QOTP}_{s,s'} C(x),$ s, s'

[1] [Shm22]

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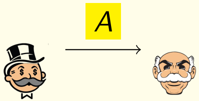

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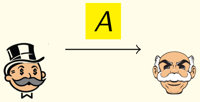

$$\text{Eval}(C, A)$$
$$\downarrow$$
$$|A_{s,s'}\rangle, s, s'$$

$C(A) \rightarrow |A\rangle = \sum_{a \in A} |a\rangle$

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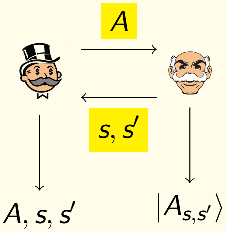

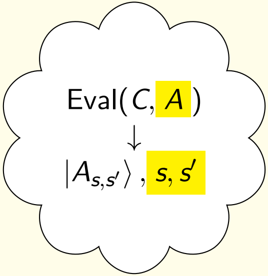

$C(A) \to |A\rangle = \sum_{a \in A} |a\rangle$

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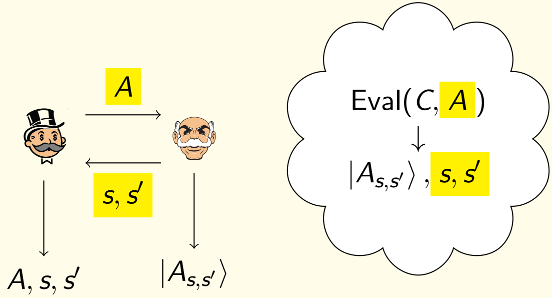

- **Problem:** there is a simple "cloning" attack in our case...

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QFHE Coset State Preparation

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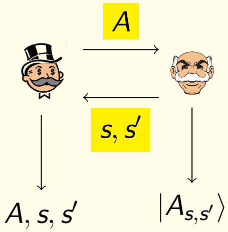


$\text{Eval}(C, A)$

$\downarrow$

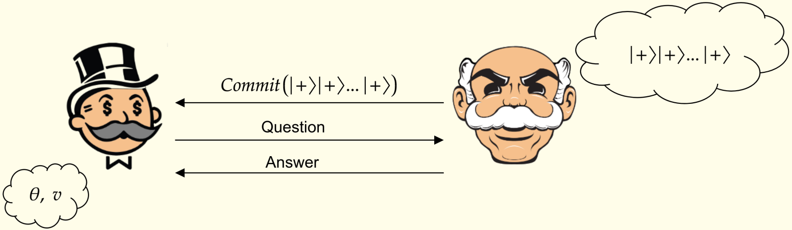
$|A_{s,s'}\rangle\,, s, s'$

$C(A) \rightarrow |A\rangle = \sum_{a \in A} |a\rangle$

- **Problem:** there is a simple "cloning" attack in our case...
- **Solution:** use self-testing !

[1] [Shm22]

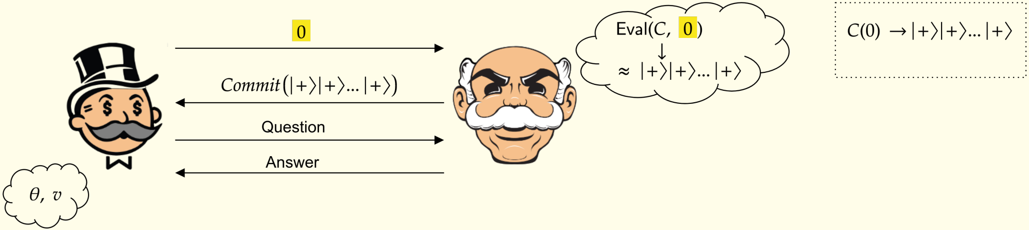
Self-Testing of BB84 States



Soundness: If the Verifier accepts, then the state in the Prover's register before the last message is $H^{\theta}|v\rangle$.

[0] [GMP22, GV19, Mah18]

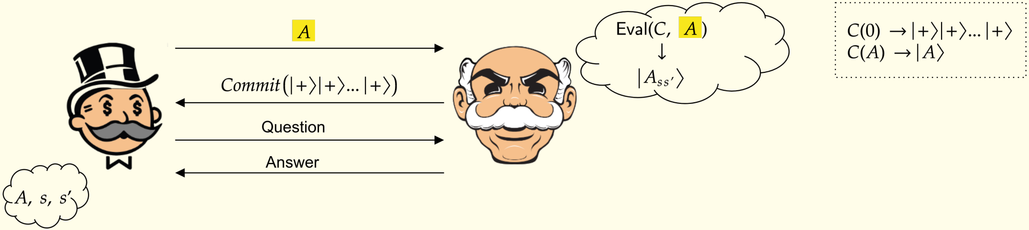
Self-Testing of BB84 States



QFHE preparation: Using QFHE for $|+\rangle$ preparation does not change the correctness and soundness.

[0] [GMP22, GV19, Mah18]

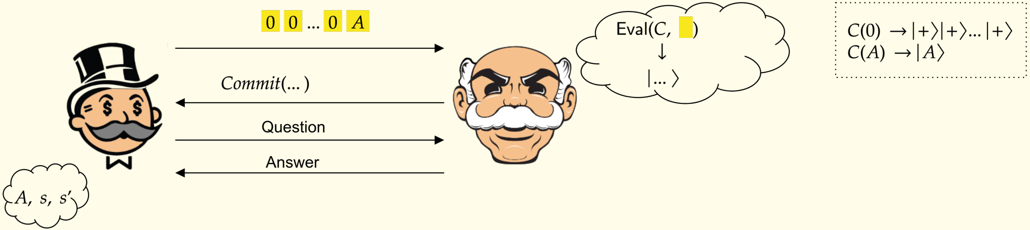
Self-Testing of Coset States



Using A: Replacing 0 by A is indistinguishable from the Prover's point of view.

[0] [GMP22, GV19, Mah18]

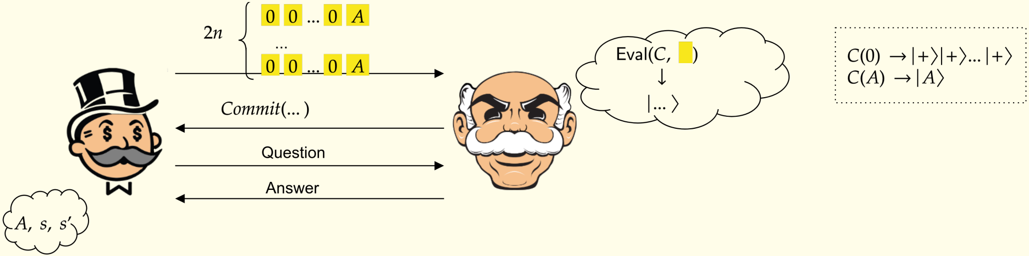
Self-Testing of Coset States



Self-testing: Run BB84 instances until we are sure the Prover is honest, then run a coset instance.

[0] [GMP22, GV19, Mah18]

Remote Preparation of Coset States



From self-testing to remote preparation: Self-testing destroys the state. Solution: run the protocol in a n-among-$2n$ cut-and-choose way.

[0] [GMP22, GV19, Mah18]

Semi-Quantum Monogamy-of-Entanglement

Soundness is not perfect: If the Verifier accepts, then the state in the Prover's register before the last message is $|A_{s,s'}\rangle$ (with probability $1 - 1/\text{poly}(\lambda)$).

Solution: We actually do not need negligible error: only that the prover cannot win the semi-quantum monogamy-of-entanglement $\rightarrow$ we reduce this semi-quantum monogamy-of-entanglement to the original monogamy-of-entanglement.

Conclusion

Contributions:

- Remote coset state preparation $\rightarrow$ semi-quantum copy-protection.
- Copy-protection for point functions in the plain model (for a specific distribution).
- Tokenized signature scheme with strong unforgeability property.

Thank You !

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