Beyond "MPC in the Head": Black-Box Constructions of Short Zero-Knowledge Proofs

Carmit Hazay — Bar-Ilan University

Muthuramakrishnan Venkitasubramaniam — Georgetown University

Mor Weiss — Bar-Ilan University

What This Talk Is About

- **Part (1): Beyond MPC in the Head**
  - New paradigm for ZKP design, extends [IKOS07]
  - Based on computations “in the head” from weak primitives
  - **Versatile:** applicable to many primitives (FHE, FE, FSS, HSS, RE, LFE) and protocols (IP, IOP), extends to commit-and-prove functionalities

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- **Part (2): Constructions of short (almost witness length) ZKPs**
  - New constructions for NC^1
  - Black-box alternatives to existing (non-BB) ZKPs for NC^1, NP and more
  - Casting some existing BB ZKPs as special cases of the paradigm

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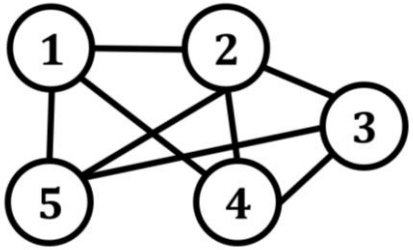
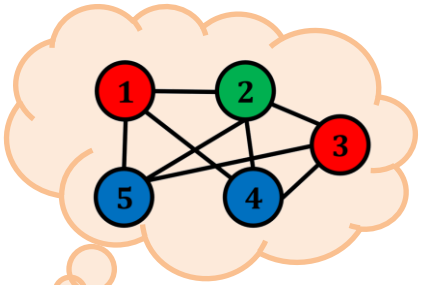
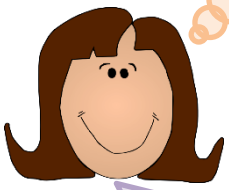
Scary table coming up!

Zero-Knowledge Proofs







Graph is 3-colorable!

I’m convinced!

Beyond MPC in the Head

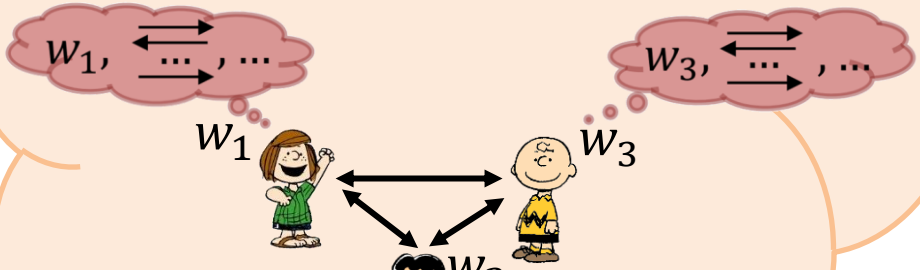
MPC in the Head 101 [IKOS07]

Claim: $C(w) = 1$ for some w

Witness Secret Share

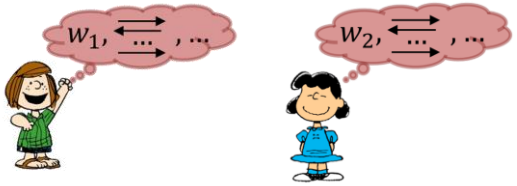
$$w = w_1 \oplus w_2 \oplus w_3$$

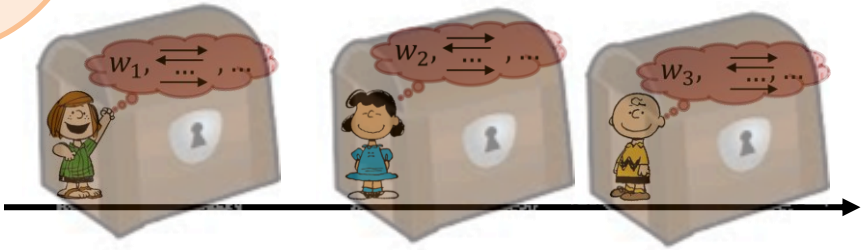
$$f(w_1, w_2, w_3) = C(w_1 \oplus w_2 \oplus w_3)$$

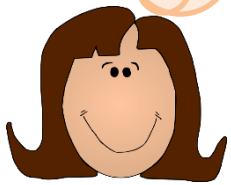


Check correctness of execution

$$i \neq j \leftarrow \{1,2,3\}$$







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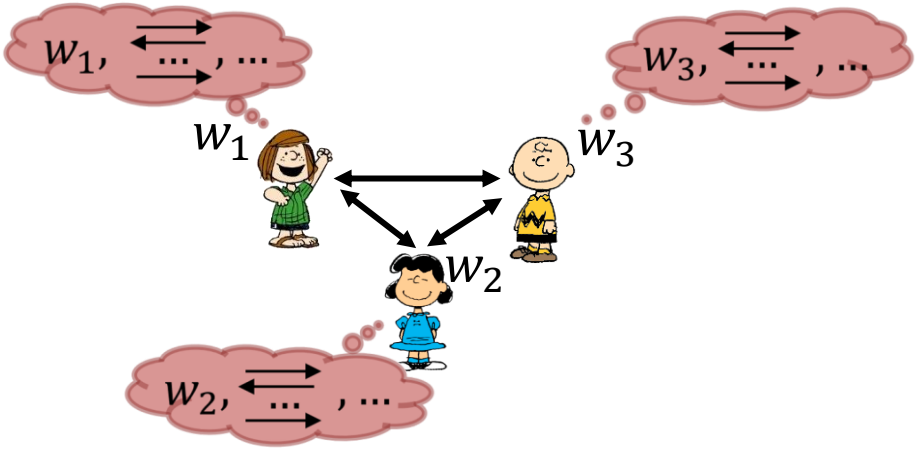

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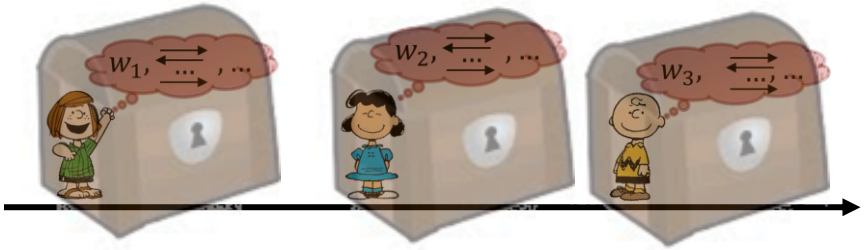

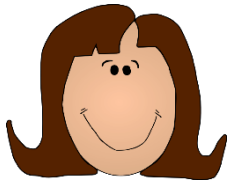

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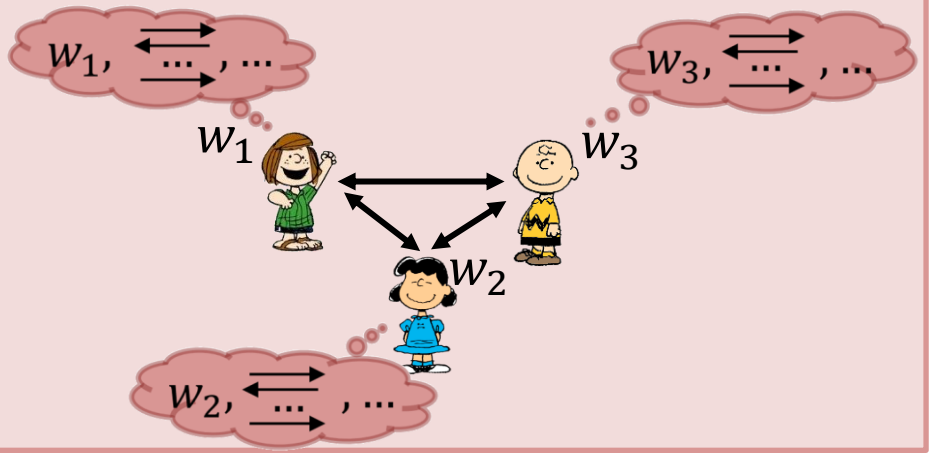

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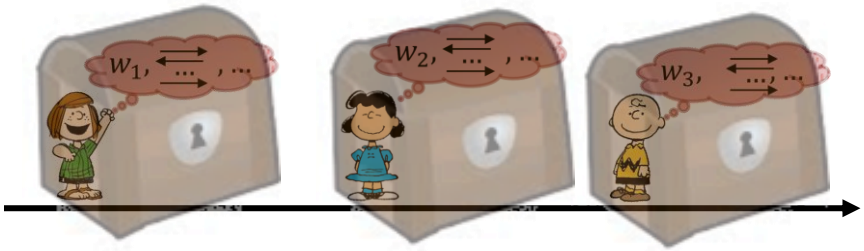

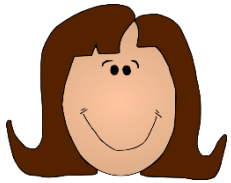

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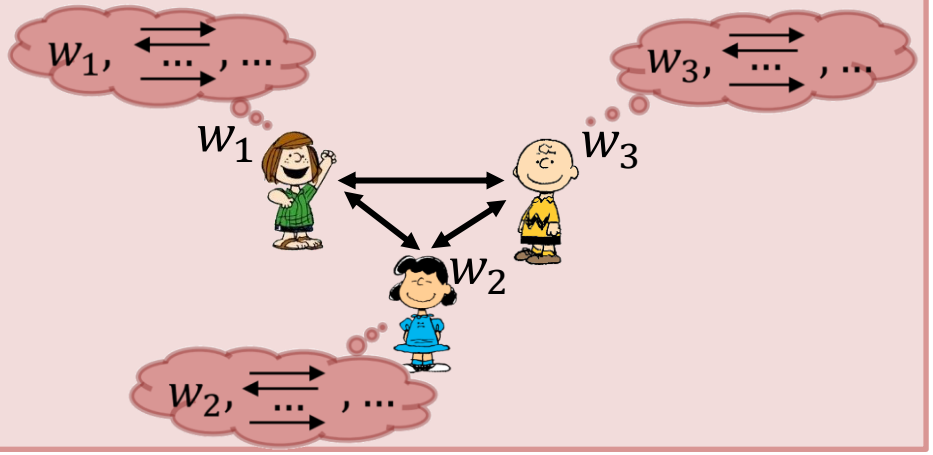

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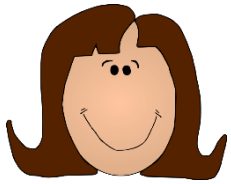
MPC correctness ⇒ soundness

MPC privacy ⇒ only 2 witness shares revealed ⇒ ZK



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- Main ingredients:
  - Witness secret sharing
  - Evaluation of relation on secret shares
  - Correctness and privacy
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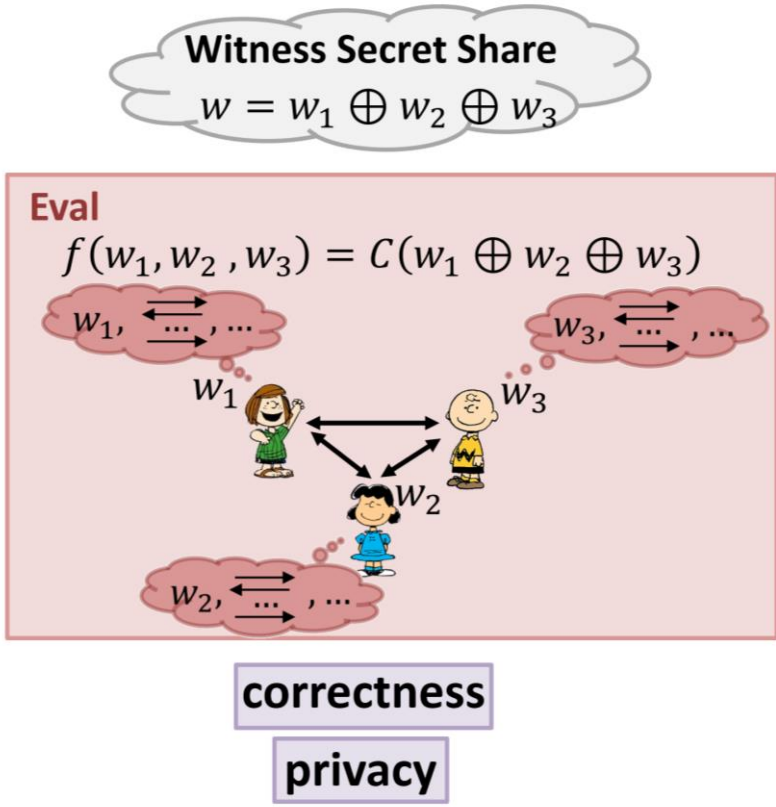

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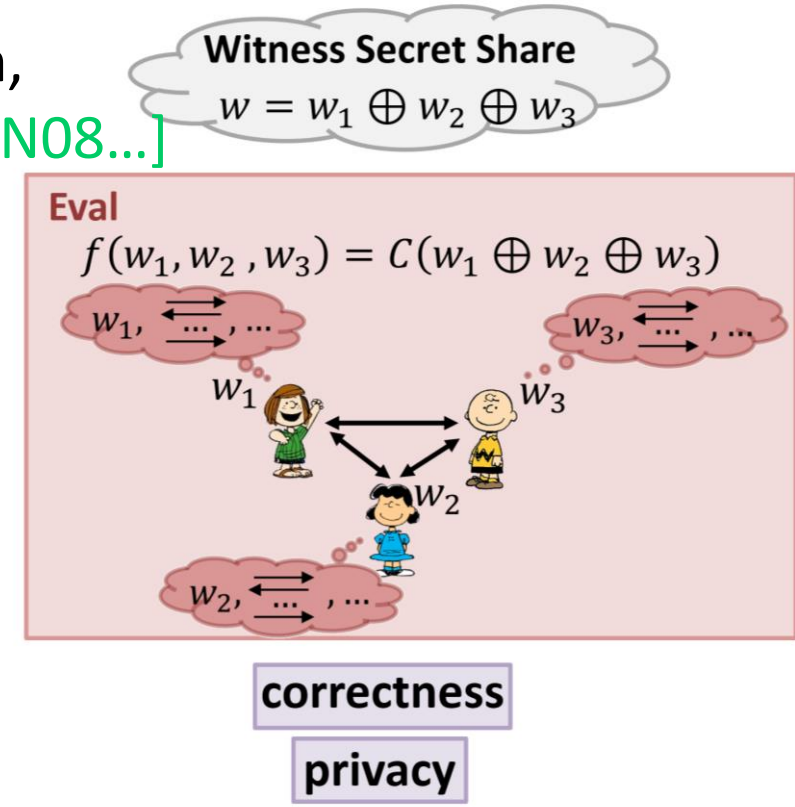

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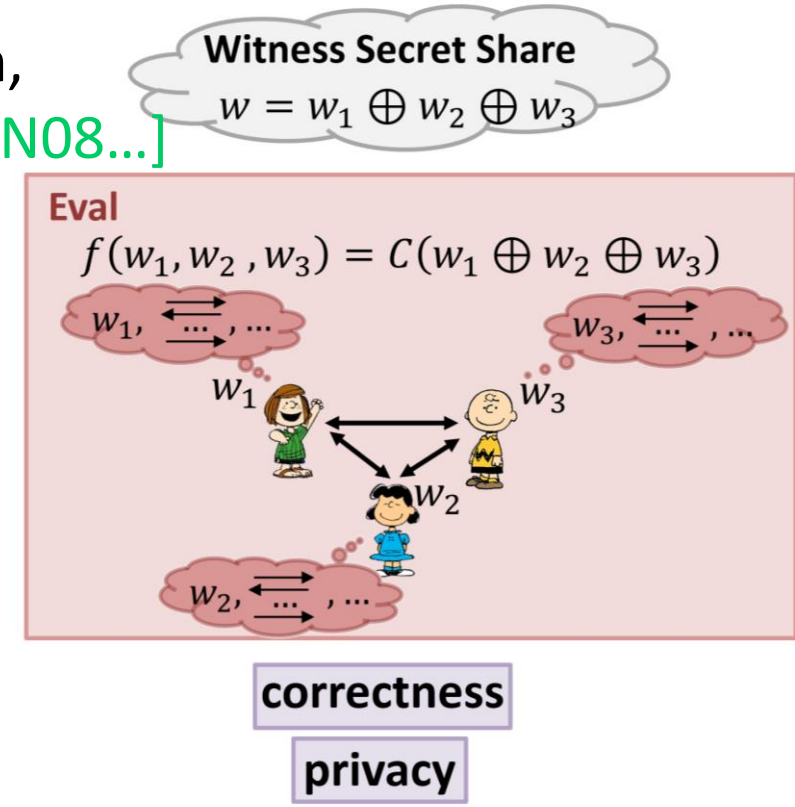

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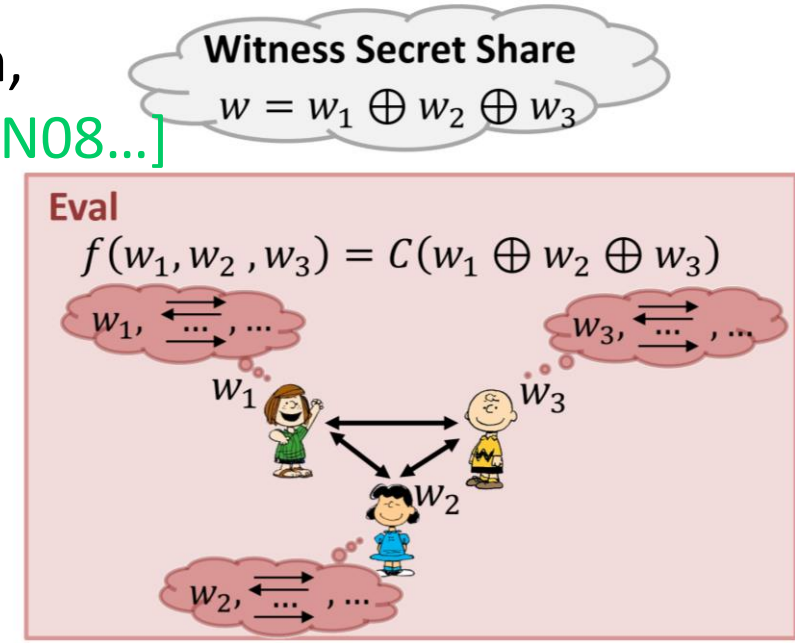

- **Today:** Beyond MPC in the Head
  - Generalized paradigm
  - From **game-based** primitives, enabling encrypting secrets and homomorphic computations

correctness

privacy

Example: ZKPs from FHE

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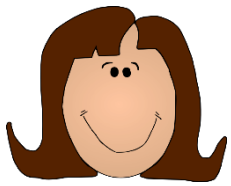

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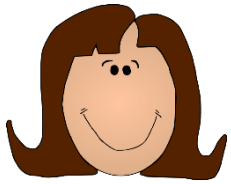

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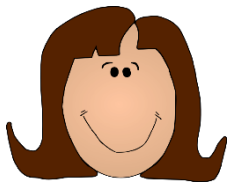

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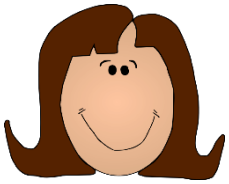

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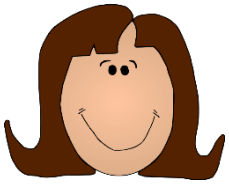

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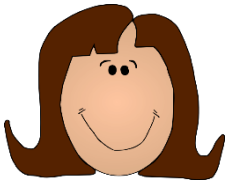

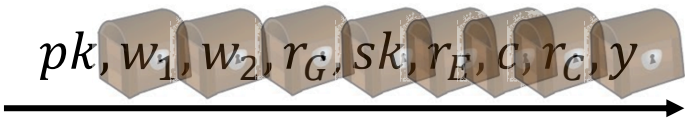

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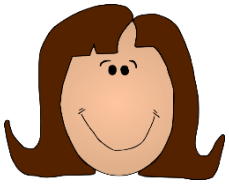

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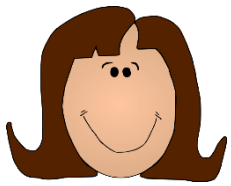

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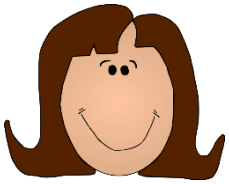

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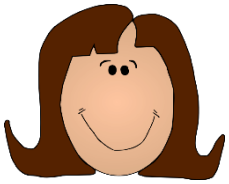
	Output read sk, y



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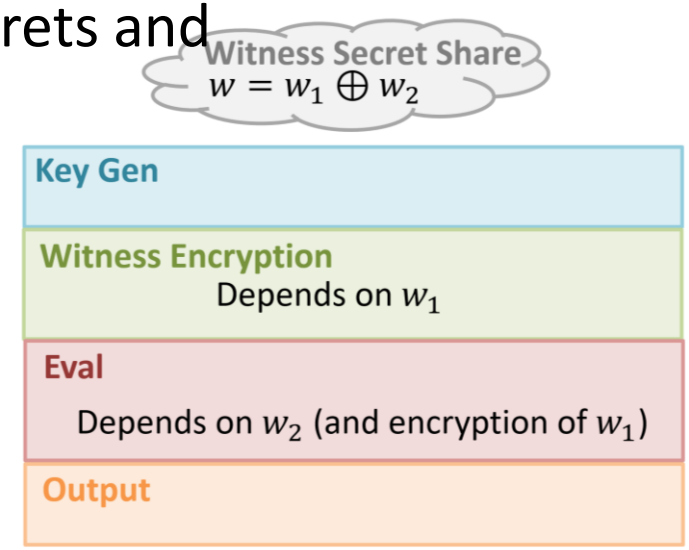
- Instead of MPC, use weaker primitive
  - MPC execution replaced with executing primitive algorithms
- **Primitive syntax:** enables encrypting secrets and homomorphic computations



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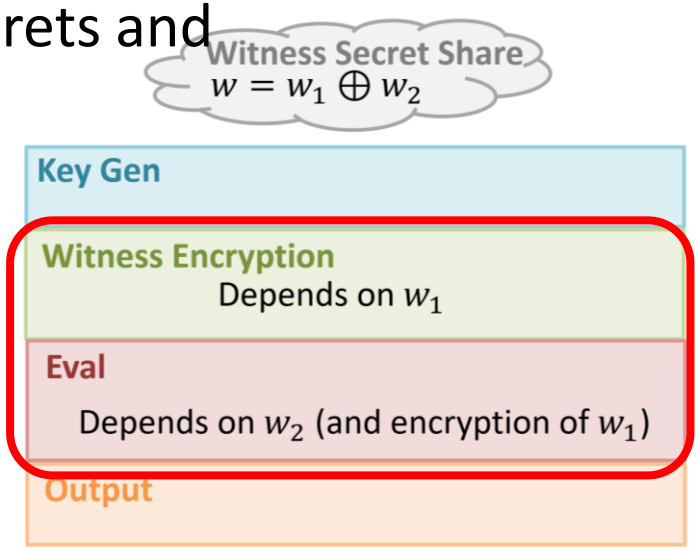

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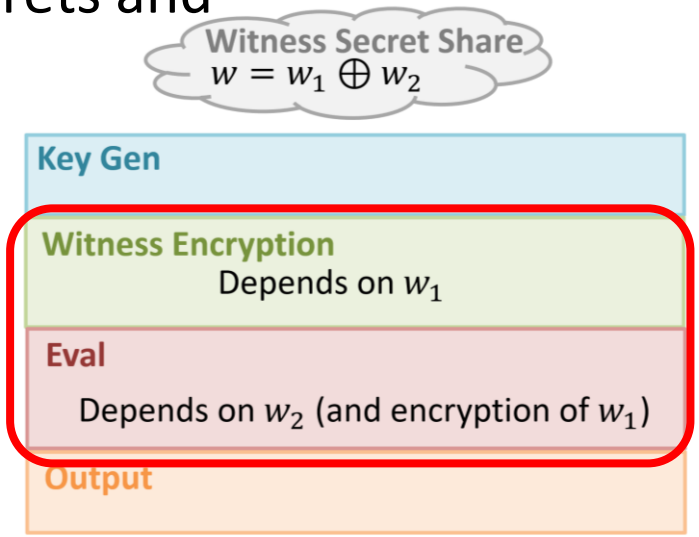
- **Versatile:** primitive can be
  - 1-party (FHE, FE, LFE, RE) or multi-party (HSS, FSS)
  - Interactive (IP, IOP)
  - Secret- or public-key
  - With imperfect correctness
- Extends also to commit-and-prove functionalities



New ZKP Constructions

- **ZKPs with $O(1)$ rounds and soundness error, BB in underlying primitive**

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Circuit Class	Communication	Assumption	SotA (Same Params, NBB)
NC^1	$n \cdot poly(\kappa)$	DCR (BB in HSS)	AC^0, BB from OWFs [IKOS07]
NC^1	$n \cdot poly(\kappa)$	OWF	[GR20] ($O(\log n)$-round, NC [GKR08])
NP	$O(n + poly(\kappa))$ $O(\kappa \cdot S)$	FHE OWF	[GGIPSS15] [HV16] (BB in OWF)
$poly(m)$-size, $d(m)$-depth NP	$n \cdot poly(\kappa, d(m))$ $\boldsymbol{O(d_m)}$ **rounds**	OWF	[GKR08]
$poly(m)$-time, m^δ-space NP	$O(n) + m^\beta \cdot poly(\kappa)$	OWF	[NR22] (with communication $(1+\gamma)n + m^\beta \cdot poly(\kappa)$)

- **ZKPs with $\boldsymbol{O(1)}$ rounds and soundness error, BB in underlying primitive**
- n = witness length, κ = sec param, S = verification circuit size, m = instance length
- NC^1 results for poly-time uniform relations
- Depth-$d(m)$ result for logspace uniform relations

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Summary

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  - With homomorphic computation properties
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Thank you!