On the Multi-User Security of LWE-based NIKE

Roman Langrehr, ETH Zurich

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Non-Interactive Key Exchange (NIKE) [CKS08, FHKP13]

Alice

Bob

$$\mathsf{pp} \leftarrow \mathsf{Setup}(1^\lambda)$$

$$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(\mathsf{pp}) \qquad\qquad (\mathsf{pk}_B, \mathsf{sk}_B) \leftarrow \mathsf{KeyGen}(\mathsf{pp})$$

$$\xrightarrow{\mathsf{pk}_A} \quad \text{PKI} \quad \xleftarrow{\mathsf{pk}_B}$$

$$\xleftarrow[\mathsf{pk}_B]{} \quad \text{PKI} \quad \xrightarrow[\mathsf{pk}_A]{}$$

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$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(\mathsf{pp})$

$(\mathsf{pk}_B, \mathsf{sk}_B) \leftarrow \mathsf{KeyGen}(\mathsf{pp})$

$$\text{Alice} \overset{\mathsf{pk}_A}{\longrightarrow} \text{PKI} \overset{\mathsf{pk}_B}{\longleftarrow} \text{Bob}$$

$$\text{Alice} \underset{\mathsf{pk}_B}{\longleftarrow} \text{PKI} \underset{\mathsf{pk}_A}{\longrightarrow} \text{Bob}$$

$K_{AB} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_A, \mathsf{pk}_B)$

$K_{BA} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_B, \mathsf{pk}_A)$

Non-Interactive Key Exchange (NIKE) [CKS08, FHKP13]

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$$\underset{\xleftarrow[\mathsf{pk}_B]{}}{\xrightarrow{\mathsf{pk}_A}} \quad \text{PKI} \quad \underset{\xrightarrow[\mathsf{pk}_A]{}}{\xleftarrow{\mathsf{pk}_B}}$$

$K_{AB} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_A, \mathsf{pk}_B)$ $\quad = \quad$ $K_{BA} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_B, \mathsf{pk}_A)$

Non-Interactive Key Exchange (NIKE) [CKS08, FHKP13]

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$$\xrightarrow{\mathsf{pk}_A} \text{ PKI } \xleftarrow{\mathsf{pk}_B}$$

$$\xleftarrow{\mathsf{pk}_B} \text{ PKI } \xrightarrow{\mathsf{pk}_A}$$

$$K_{AB} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_A, \mathsf{pk}_B) \overset{\text{w.h.p.}}{=} K_{BA} \leftarrow \mathsf{SharedKey}(\mathsf{sk}_B, \mathsf{pk}_A)$$

Light security

Adversary

- gets public keys of 2 users and

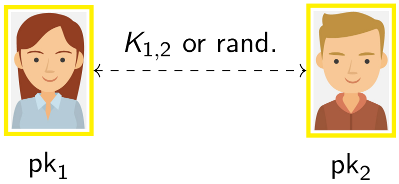
pk_1

pk_2

Light security

Adversary

- gets public keys of 2 users and
- real or random shared key



Adaptive Honest Key Registration (HKR) Security

Adversary can adaptively

- spawn new users

pk_1

pk_2

pk_3

pk_4

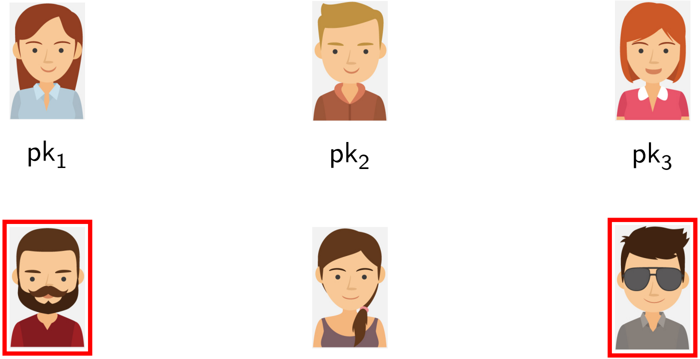
pk_5

pk_6

Adaptive Honest Key Registration (HKR) Security

Adversary can adaptively

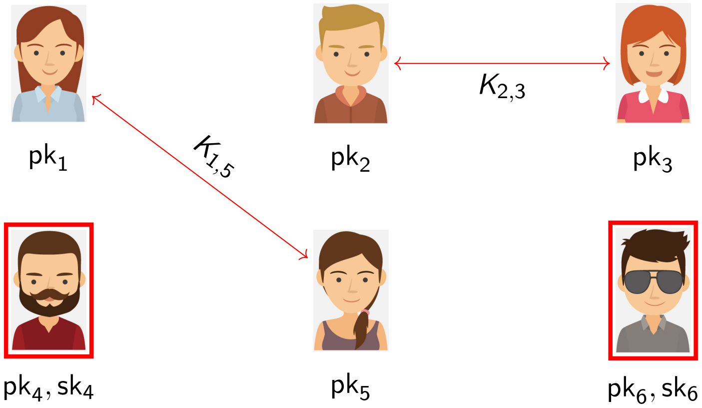
- spawn new users
- corrupt users



Adaptive Honest Key Registration (HKR) Security

Adversary can adaptively

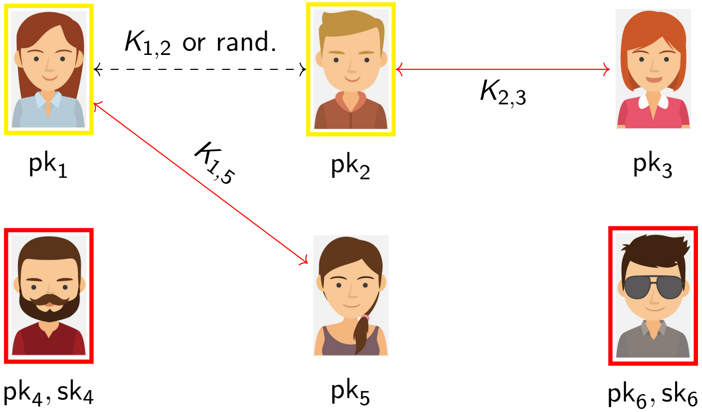
- spawn new users
- corrupt users
- reveal shared keys, even those computed with a challenge users secret key



Adaptive Honest Key Registration (HKR) Security

Adversary can adaptively

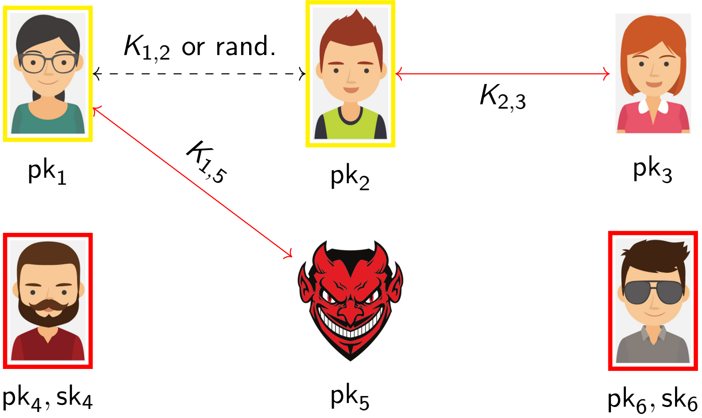
- spawn new users
- corrupt users
- reveal shared keys, even those computed with a challenge users secret key
- get challenged on one uncorrupted shared key



Adaptive Dishonest Key Registration (DKR) Security

Adversary can adaptively

- spawn new users
- corrupt users
- reveal shared keys, even those computed with a challenge users secret key
- get challenged on (one) uncorrupted shared key
- introduce maliciously generated public keys



Relations between these security notions

Light $\leftarrow$ - - - - - - - - - - - Adaptive HKR $\leftarrow$ - - - - - - - Adaptive DKR

Relations between these security notions

For negligible correctness error

$$\begin{tikzcd} \text{Light} \arrow[r, bend left, "\text{[FHKP13]}"] & \text{Adaptive HKR} \arrow[l, dashed] & \text{Adaptive DKR} \arrow[l, dashed] \end{tikzcd}$$

Relations between these security notions

This work: This is necessary for a generic argument

For negligible correctness error

$$\begin{tikzcd} \text{Light} \arrow[r, bend left, "\text{[FHKP13]}"] & \text{Adaptive HKR} \arrow[l, dashed] & \text{Adaptive DKR} \arrow[l, dashed] \end{tikzcd}$$

Relations between these security notions

This work: This is necessary for a generic argument

For negligible correctness error

For perfect correctness

$$\begin{tikzcd}[column sep=huge]
\text{Light} \arrow[r, bend left, "\text{[FHKP13]}"] & \text{Adaptive HKR} \arrow[l, dashed] \arrow[r, bend left, "\substack{\text{[HHK18]} \\ +\ \text{NIZKPoK}}"] & \text{Adaptive DKR} \arrow[l, dashed]
\end{tikzcd}$$

LWE-based NIKE

Alice

Bob

$$\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times n}$$

$\mathbf{s}_A \xleftarrow{\$} \{0,1\}^n$; $\mathbf{e}_A \xleftarrow{\$} \mathcal{E}^n$

$\mathrm{pk}_A := \mathbf{s}_A^\top \mathbf{A} + \mathbf{e}_A^\top, \mathrm{sk}_A := \mathbf{s}_A$

$\mathbf{s}_B \xleftarrow{\$} \{0,1\}^n$; $\mathbf{e}_B \xleftarrow{\$} \mathcal{E}^n$

$\mathrm{pk}_B := \mathbf{A}\mathbf{s}_B + \mathbf{e}_B, \mathrm{sk}_A := \mathbf{s}_B$

$$\xrightarrow[\mathrm{pk}_B]{\mathrm{pk}_A} \text{PKI} \xrightarrow[\mathrm{pk}_A]{\mathrm{pk}_B}$$

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$$\text{Alice} \underset{\mathsf{pk}_B}{\overset{\mathsf{pk}_A}{\rightleftarrows}} \text{PKI} \underset{\mathsf{pk}_A}{\overset{\mathsf{pk}_B}{\rightleftarrows}} \text{Bob}$$

$e'_A \xleftarrow{\$} \mathcal{E}$

$e'_B \xleftarrow{\$} \mathcal{E}$

$$K_{AB} := \mathbf{s}_A^\top \mathsf{pk}_B + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_B + \mathbf{s}_A^\top \mathbf{e}_B + e'_A$$

$$K_{BA} := \mathsf{pk}_A \mathbf{s}_B + e'_B = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_B + \mathbf{e}_A^\top \mathbf{s}_B + e'_B$$

LWE-based NIKE

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$\xrightarrow{\mathsf{pk}_A}$ PKI $\xrightarrow{\mathsf{pk}_B}$

$\xleftarrow[\mathsf{pk}_B]{}$ PKI $\xleftarrow[\mathsf{pk}_A]{}$

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$\mathsf{Round}(K_{AB})$ $\qquad$ $\mathsf{Round}(K_{BA})$

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$$\text{Alice} \xrightarrow{\mathsf{pk}_A} \text{PKI} \xrightarrow{\mathsf{pk}_B} \text{Bob}$$

$$\text{Alice} \xleftarrow{\mathsf{pk}_B} \text{PKI} \xleftarrow{\mathsf{pk}_A} \text{Bob}$$

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$$\mathrm{Round}(K_{AB}) \overset{\text{w.h.p.}}{=} \mathrm{Round}(K_{BA})$$

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio $\implies$ non-neglible correctness error

LWE-based NIKE & Our results

$$\begin{aligned} \text{polynomial modulus-to-noise ratio} \quad &\implies \quad \text{non-neglible correctness error} \\ \text{super-polynomial modulus-to-noise ratio} \quad &\implies \quad \text{neglible correctness error} \end{aligned}$$

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This correctness error is inherent [GKRS20]

LWE-based NIKE & Our results

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Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

LWE-based NIKE & Our results

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super-polynomial modulus-to-noise ratio $\Longrightarrow$ neglible correctness error

This correctness error is inherent [GKRS20]
Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.			
super-poly.			

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio $\Longrightarrow$ non-neglible correctness error
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Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓		
super-poly.	✓		

[DXL12, Pei14]

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio $\implies$ non-neglible correctness error
super-polynomial modulus-to-noise ratio $\implies$ neglible correctness error

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Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓		
super-poly.	✓	Generic ✓	

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio	$\Longrightarrow$	non-neglible correctness error
super-polynomial modulus-to-noise ratio	$\Longrightarrow$	neglible correctness error

This correctness error is inherent [GKRS20]
Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓		
super-poly.	✓	✓	✓ with (Q)ROM

Concurrent work [GdKQ$^+$23]

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio $\implies$ non-neglible correctness error
super-polynomial modulus-to-noise ratio $\implies$ neglible correctness error

This correctness error is inherent [GKRS20]
Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓	✓ (bounded) ? (unbounded)	
super-poly.	✓	✓	✓ with (Q)ROM

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio $\implies$ non-neglible correctness error
super-polynomial modulus-to-noise ratio $\implies$ neglible correctness error

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Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓	✓ (bounded) ? (unbounded)	✗
super-poly.	✓	✓	✓ with (Q)ROM

LWE-based NIKE & Our results

polynomial modulus-to-noise ratio	$\Longrightarrow$	non-neglible correctness error
super-polynomial modulus-to-noise ratio	$\Longrightarrow$	neglible correctness error

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Potential errors can be corrected with one bit of interaction [DXL12, Pei14]

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓	✓ (bounded) ? (unbounded)	✗
super-poly.	✓	✓	✗ poly. noise ✓ super-poly. noise ✓ with (Q)ROM

Alice

$\boxed{\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n\times n}}$

Bob

$\mathbf{s}_A \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_A \xleftarrow{\$} \mathcal{E}^n$

$\boxed{\mathsf{pk}_A := \mathbf{s}_A^\top \mathbf{A} + \mathbf{e}_A^\top}, \mathsf{sk}_A := \mathbf{s}_A$

$\mathbf{s}_B \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_B \xleftarrow{\$} \mathcal{E}^n$

$\boxed{\mathsf{pk}_B := \mathbf{A}\mathbf{s}_B + \mathbf{e}_B}, \mathsf{sk}_A := \mathbf{s}_B$

$\boxed{\mathsf{Round}(K_{AB})}$ where $K_{AB} := \mathbf{s}_A^\top \mathsf{pk}_B + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_B + \mathbf{s}_A^\top \mathbf{e}_B + e'_A,\ \ e'_A \xleftarrow{\$} \mathcal{E}$

G_0 Real light security game

Light security [DXL12, Pei14]

Alice

$\boxed{\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n\times n}}$

Bob

$\mathbf{s}_A \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_A \xleftarrow{\$} \mathcal{E}^n$

$\boxed{\mathsf{pk}_A := \mathbf{s}_A^\top \mathbf{A} + \mathbf{e}_A^\top}, \mathsf{sk}_A := \mathbf{s}_A$

$\cancel{\mathbf{s}_B \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_B \xleftarrow{\$} \mathcal{E}^n}$

$\cancel{\boxed{\mathsf{pk}_B := \mathbf{A}\mathbf{s}_B + \mathbf{e}_B}, \mathsf{sk}_A := \mathbf{s}_B}$

$\boxed{\mathsf{pk}_B \xleftarrow{\$} \mathbb{Z}_q^n}$

$\boxed{\mathsf{Round}(K_{AB})}$ where $K_{AB} := \mathbf{s}_A^\top \mathsf{pk}_B + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_B + \mathbf{s}_A^\top \mathbf{e}_B + e'_A,\ \ e'_A \xleftarrow{\$} \mathcal{E}$

G_0 Real light security game

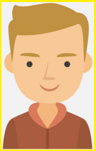
G_1 pk_B is uniformly random

LWE: $\mathbf{A}\mathbf{s}_B + \mathbf{e}_B \approx_c$ uniform

Light security [DXL12, Pei14]

Alice

$\boxed{\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n\times n}}$



Bob

$\cancel{\mathbf{s}_A \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_A \xleftarrow{\$} \mathcal{E}^n \quad \boxed{\mathrm{pk}_A := \mathbf{s}_A^\top \mathbf{A} + \mathbf{e}_A^\top}, \mathrm{sk}_A := \mathbf{s}_A}$ $\boxed{\mathrm{pk}_A \xleftarrow{\$} \mathbb{Z}_q^n}$

$\cancel{\mathbf{s}_B \xleftarrow{\$} \{0,1\}^n;\ \mathbf{e}_B \xleftarrow{\$} \mathcal{E}^n \quad \boxed{\mathrm{pk}_B := \mathbf{A}\mathbf{s}_B + \mathbf{e}_B}, \mathrm{sk}_A := \mathbf{s}_B}$ $\boxed{\mathrm{pk}_B \xleftarrow{\$} \mathbb{Z}_q^n}$

$\boxed{\mathrm{Round}(K_{AB})}$ where $\cancel{K_{AB} := \mathbf{s}_A^\top \mathrm{pk}_B + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_B + \mathbf{s}_A^\top \mathbf{e}_B + e'_A,\ e'_A \xleftarrow{\$} \mathcal{E}}$ $K_{AB} \xleftarrow{\$} \mathbb{Z}_q$

- G_0 Real light security game
- G_1 pk_B is uniformly random
- G_2 pk_A and K_{AB} are uniformly random

LWE: $\mathbf{A}\mathbf{s}_B + \mathbf{e}_B \approx_c$ uniform

LWE: $\begin{pmatrix} \mathbf{A}^\top \\ \mathrm{pk}_B^\top \end{pmatrix} \mathbf{s}_A + \begin{pmatrix} \mathbf{e}_A \\ e'_A \end{pmatrix} \approx_c$ uniform

Adaptive HKR security

- Reduction guesses the two challenge users

Adaptive HKR security

- Reduction guesses the two challenge users
- Problem: Shared key queries with a challenge users secret key

Alice

$\boxed{\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times n}}$

Charlie

$\mathbf{s}_A \xleftarrow{\$} \{0,1\}^n$; $\mathbf{e}_A \xleftarrow{\$} \mathcal{E}^n$

$\boxed{\mathsf{pk}_A := \mathbf{s}_A^\top \mathbf{A} + \mathbf{e}_A^\top}$, $\mathsf{sk}_A := \mathbf{s}_A$

$\mathbf{s}_C \xleftarrow{\$} \{0,1\}^n$; $\mathbf{e}_C \xleftarrow{\$} \mathcal{E}^n$

$\boxed{\mathsf{pk}_C := \mathbf{A}\mathbf{s}_C + \mathbf{e}_C}$, $\boxed{\mathsf{sk}_C := \mathbf{s}_C}$

$\boxed{\mathsf{Round}(K_{AC})}$ where $K_{AC} := \mathbf{s}_A^\top \mathsf{pk}_C + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_C + \mathbf{s}_A^\top \mathbf{e}_C + e'_A$, $e'_A \xleftarrow{\$} \mathcal{E}$

Trick 1: Using leakage

Needed: $K_{AC} := \mathbf{s}_A^\top \mathsf{pk}_C + e'_A = \mathbf{s}_A^\top \mathbf{A}\mathbf{s}_C + \ \mathbf{s}_A^\top \mathbf{e}_C \ + e'_A, \ e'_A \xleftarrow{\$} \mathcal{E}$

Trick 1: Using leakage

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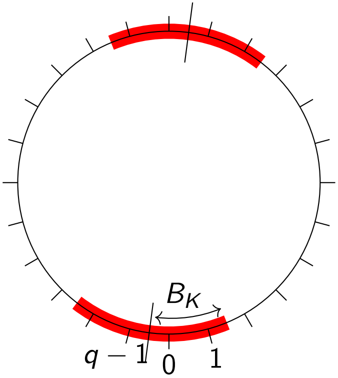

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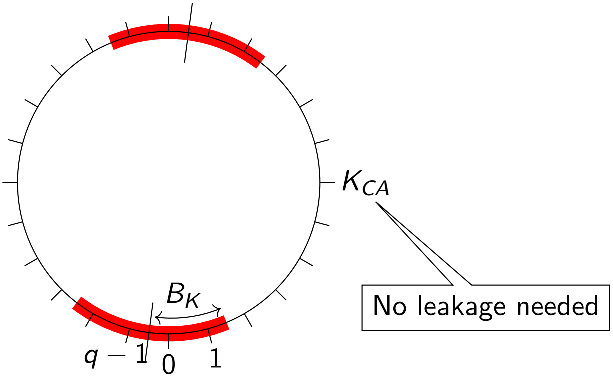

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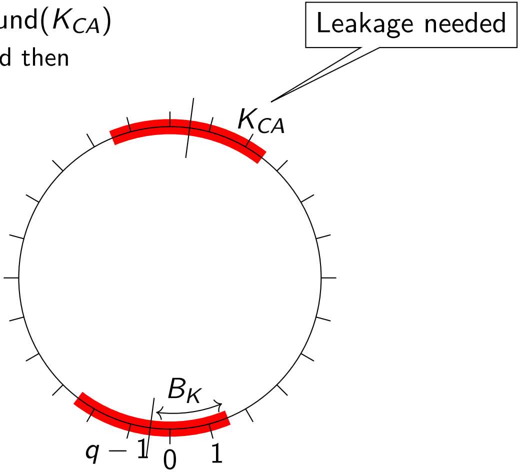

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$$\Longrightarrow \text{number of users can grow polynomially in } n$$

Summary

m-t-n ratio	Light security	Adaptive HKR security	Adaptive DKR security with NIZKPoK
poly.	✓	✓ (bounded) ? (unbounded)	✗
super-poly.	✓	✓	✗ poly. noise ✓ super-poly. noise ✓ with (Q)ROM

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Pictures

Alice, Bob, and other faces: freepik.com
Devil face: vecteezy.com

Trick 2: Using leakage more economically (Full detail)

The difficulty:

- Leakage on $\mathbf{s}_A$ must not depend on $\mathbf{A}$
- Whether K_{CA} is in the red zone does depend on $\mathbf{A}$

The solution:

- We need leakage $\mathbf{s}_A^\top \mathbf{e}_C$
- $\mathbf{e}_C$ has only small influence on K_{CA}.
- Use leakage $\mathbf{s}_A^\top \mathbf{e}_i$ for several $\mathbf{e}_i \xleftarrow{\$} \mathcal{E}^n$
- If $\mathbf{s}_A^\top \mathbf{A} \mathbf{s}_C$ is in the red zone $\Rightarrow$ use one of the $\mathbf{e}_i$ as Charlie's error vector.
- Otherwise sample a fresh error vector for Charlie

$$\implies \text{number of users can grow polynomially in } n$$

DKR insecurity

Use of NIZKPoKs $\Rightarrow \mathcal{A}$ can register malicious public key only with a valid secret key.

- Adversary can create $(\mathsf{pk}, \mathsf{sk})$ s.t. $\mathbf{s}_A^\top \mathsf{pk} \approx 0$

$\Rightarrow$ High likelihood of correctness error with Alice

Example:

- Assume for simplicity Alice does not add noise to the shared key before rounding
- Register for $i \in [n]$ user with $\mathsf{pk}_i = -e_i$, $\mathsf{sk}_i = \mathbf{0}$
- If $(\mathbf{s}_A)_i = 0$, $\mathsf{Round}(K_{A,i}) = \mathsf{Round}(0) = 0$
- If $(\mathbf{s}_A)_i = 1$, $\mathsf{Round}(K_{A,i}) = \mathsf{Round}(q-1) = 1$

$\Rightarrow \mathcal{A}$ can extract $\mathbf{s}_A$ with n malicious users

- Attack can be extended to
  - shared keys with noise
  - different distributions of LWE secrets
  - different rounding functions (with polynomial modulus-to-noise ratio)

DKR security

For malicious user Charlie:

- Extract sk_C from NIZKPoK
- Compute K_{CA} with sk_C
- B: maximum difference between K_{AC} and K_{CA}
- Use noise super-polynomial in B for the shared keys

$\Rightarrow$ $K_{AC} \approx_s K_{CA}$