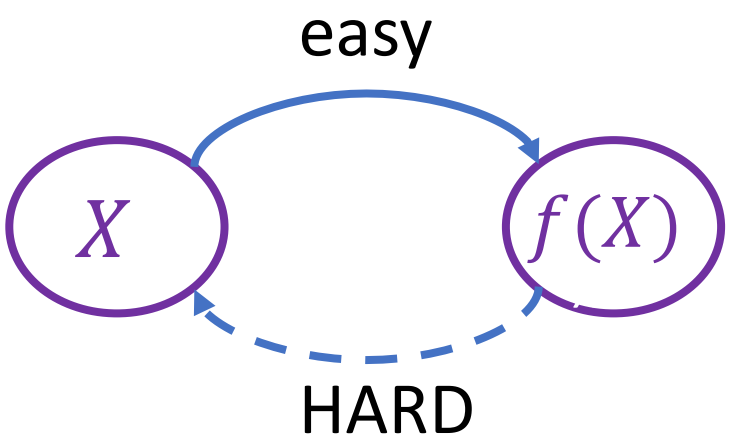
Counting Unpredictable Bits: Simple PRG from OWFs

Noam Mazor (Cornell Tech)
Rafael Pass (Tel Aviv University & Cornell Tech)

One-Way Functions



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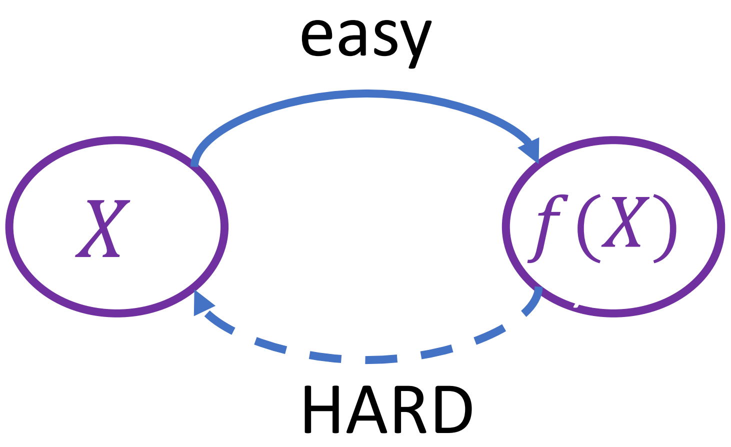

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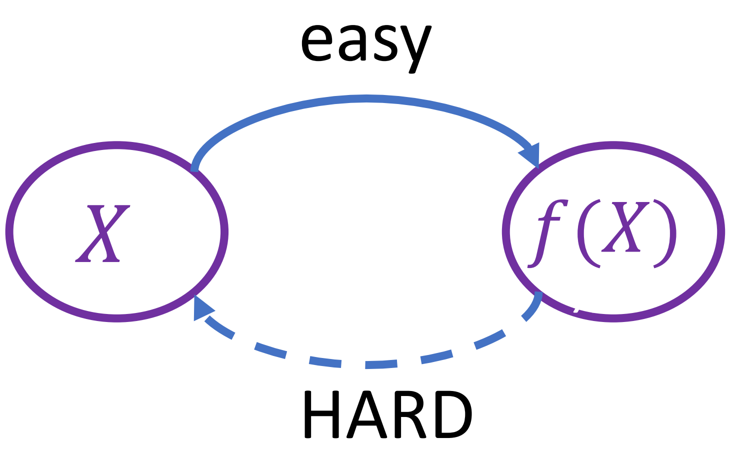

- The minimal assumption for cryptography
- Can be used to construct many useful primitives

OWF $\Rightarrow$ PRG

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This work: Simple proof for the non-uniform setting
with better parameters

Proving OWF $\Rightarrow$ PRG

Proving OWF ⇒ PRG

General approach:

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1. Start with a weaker notion of pseudorandomness

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<u>Key insight</u>: A simple weak notion of pseudorandomness

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Bits Unpredictability - Counting unpredictable bits

Next-Bit Unpredictability

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- Want $|S|$ to be large

Proof Overview

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Relying on simple tools (Goldriech-Levin, Chernoff, ...)

Part I:
OWF $\Rightarrow$ Bits Unpredictability

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$$\boxed{\begin{array}{c} M(x) = M \cdot x = \langle M_1, x\rangle, \dots, \langle M_n, x\rangle \\ \text{For } M = (M_1, \dots, M_n) \end{array}}$$

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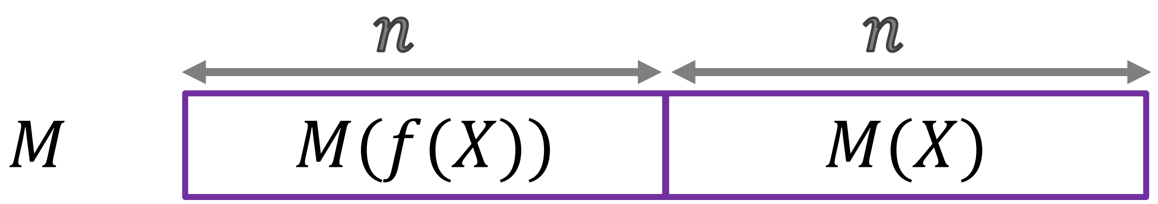
$$g_M(X) = M\big(f(X)\big), M(X)$$



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- Can find X from $M, M(X)$

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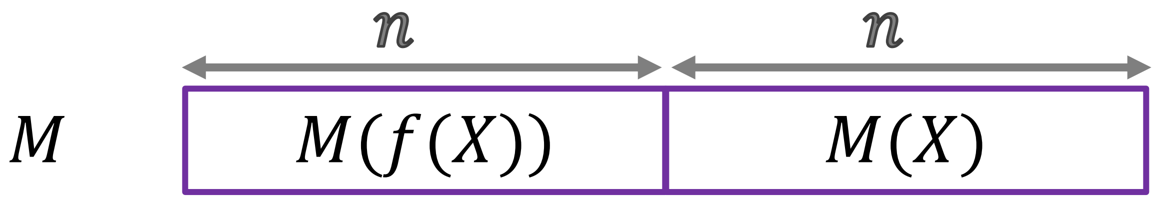

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<u>Thm</u>:

$g_M(X)$ has $(n + \log n)$ Bits Unpredictability (given M).

Proof: Bits Unpredictability from Regular OWF

	n	n
M	$M(f(X))$	$M(X)$

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Assume $f:\{0,1\}^n \to \{0,1\}^n$ is r-regular ($\left|f^{-1}\left(f(x)\right)\right| = 2^r$) [Goldreich-Krawczyk-Luby]

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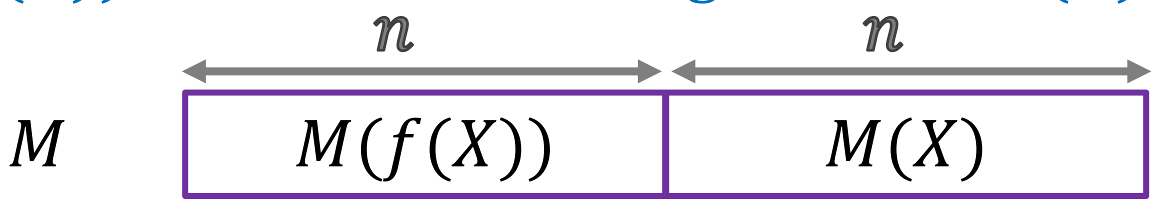
<u>Claim</u>: The first $\approx n - r$ bits of $M(f(X))$, and the first $r + \log n$ bits of $M(X)$ are unpredictable.



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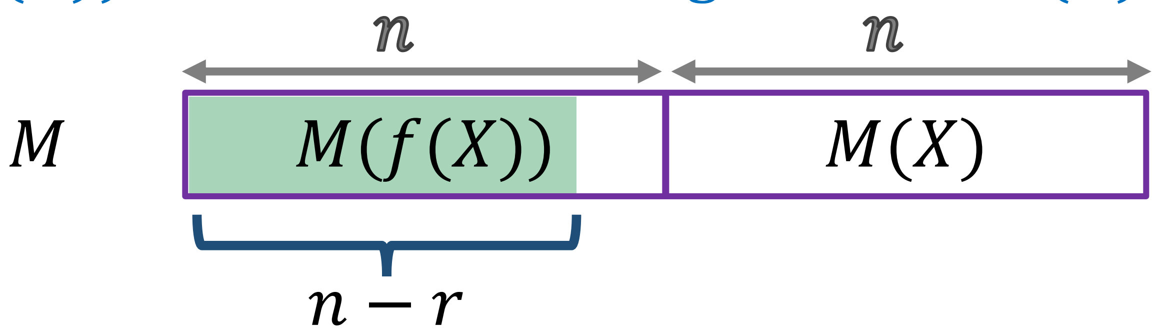

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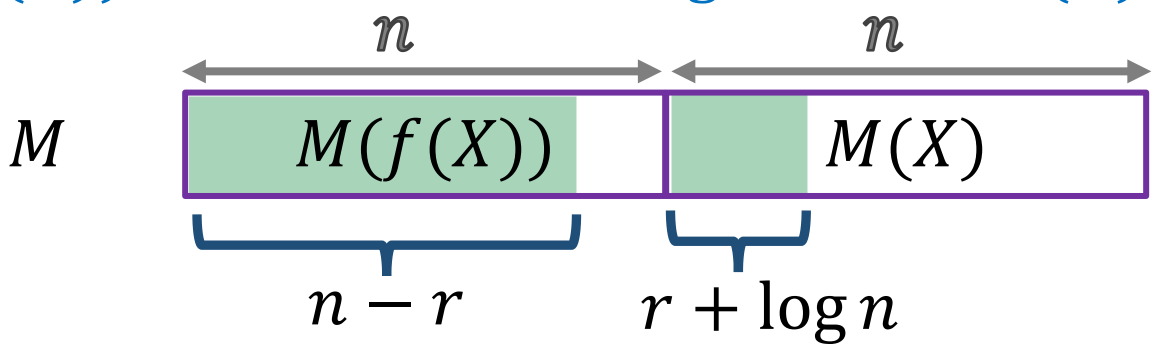

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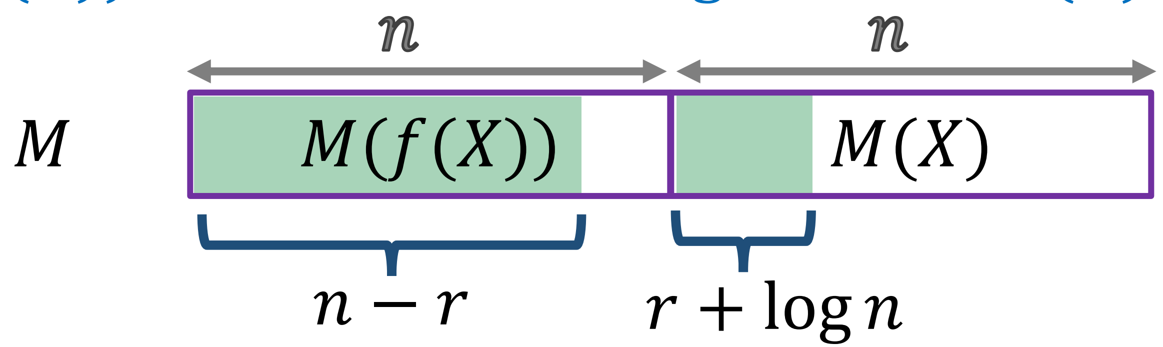

Pf: Similar to construction of PRG from regular one-way functions.

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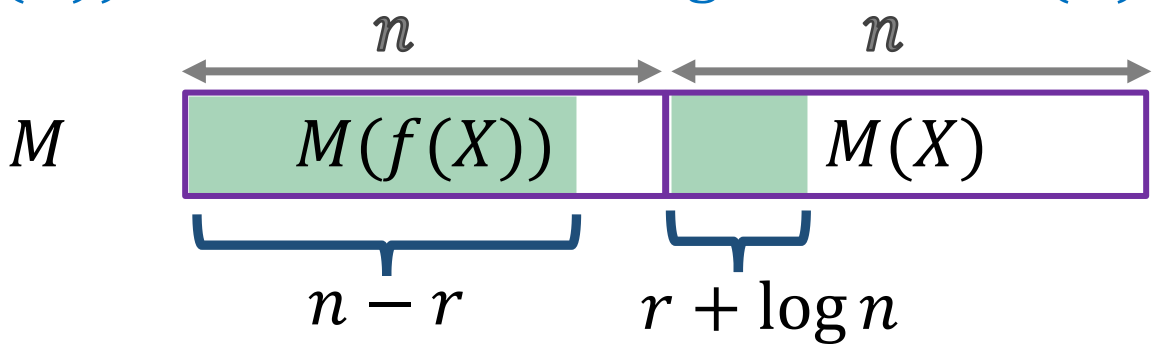

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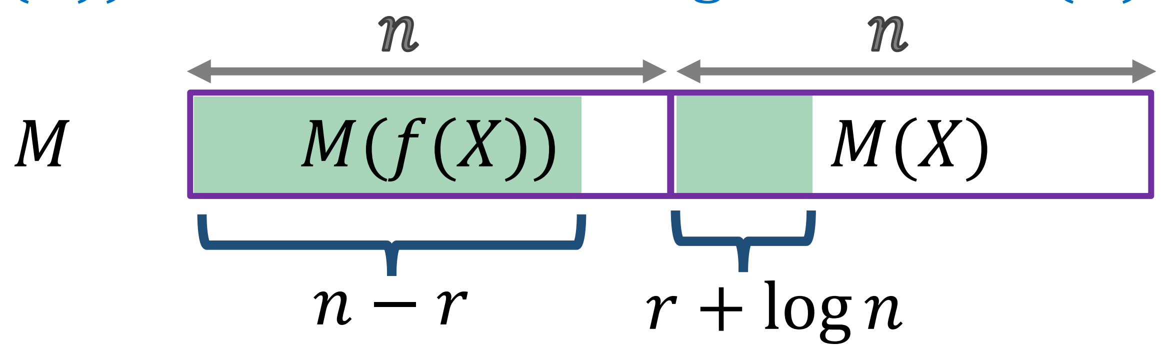

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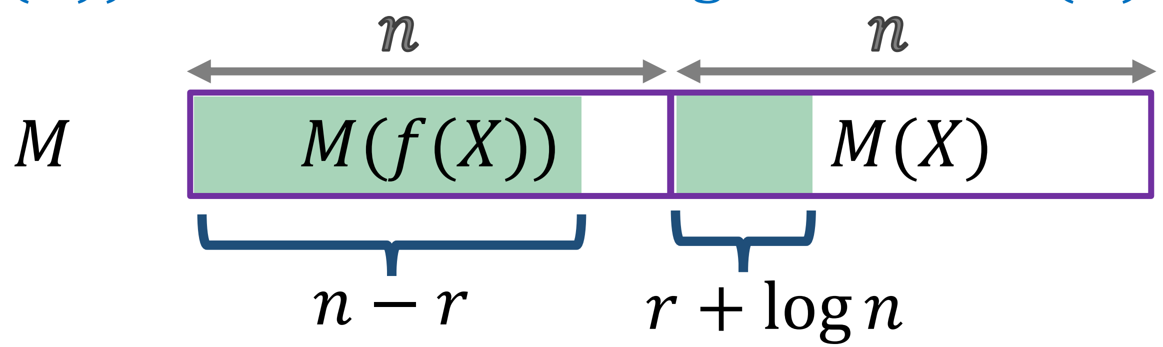

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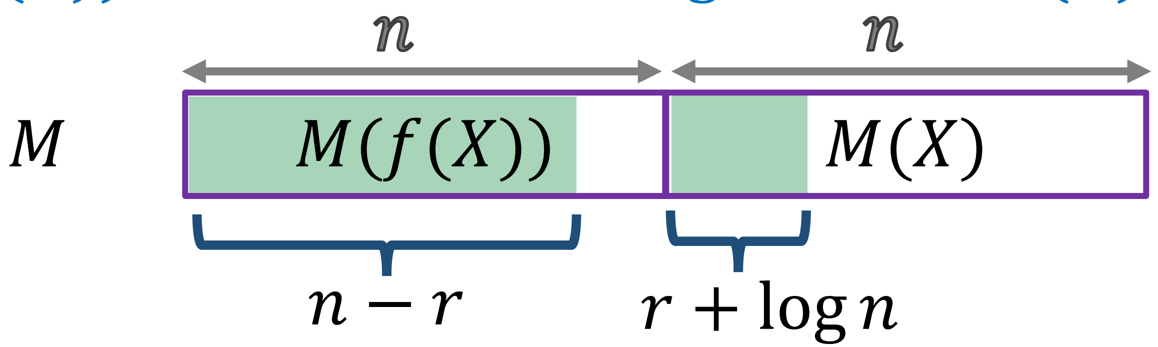

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  - Can extract r random bits $+ \log n$ pseudorandom bits (GL)

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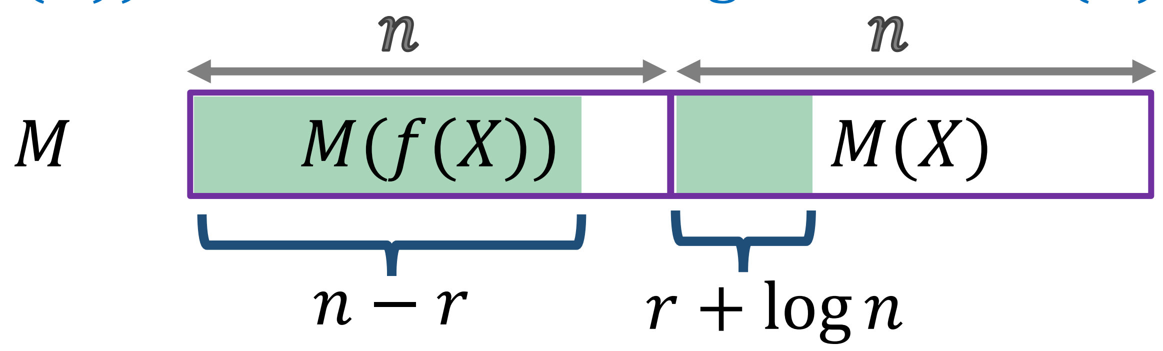

When reading $g_M(X)$ bit-by-bit, there are $n + \log n$ unpredictable bits!

Proof: Bits Unpredictability from Any OWF

What about arbitrary OWFs?

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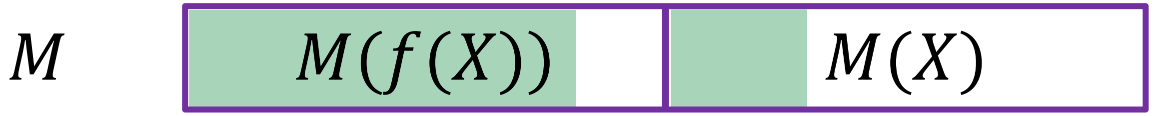

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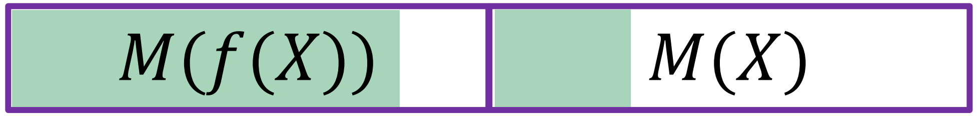
M 

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$g_M(X)$ is a convex combination of distributions with $(n + \log n)$ Bits Unpredictability.

Part II:
Bits Unpredictability $\Rightarrow$ PRG

Bits Unpredictability ⇒ Pseudorandomness

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For any OWF f, $g_M(X)$ has $(n + \log n)$-bits unpredictability

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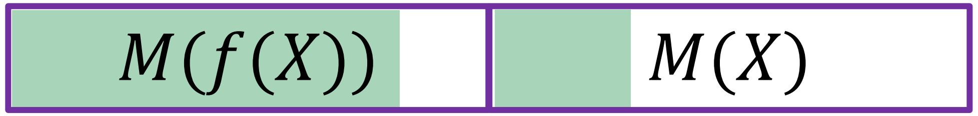
$g_M(X)$ is not pseudorandom

M



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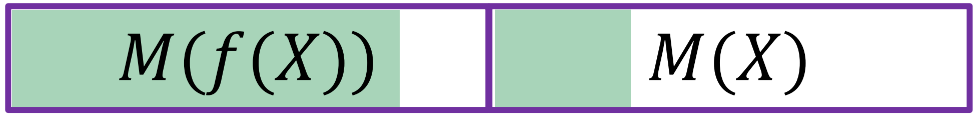

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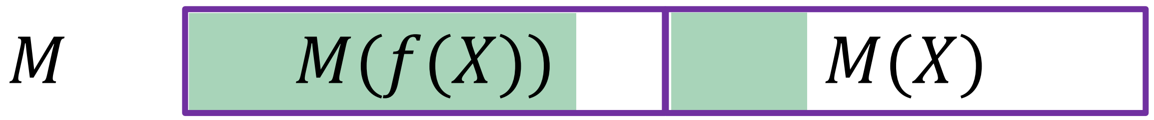

Simple construction ([HRV])

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Problem: We don’t know which bits are unpredictable

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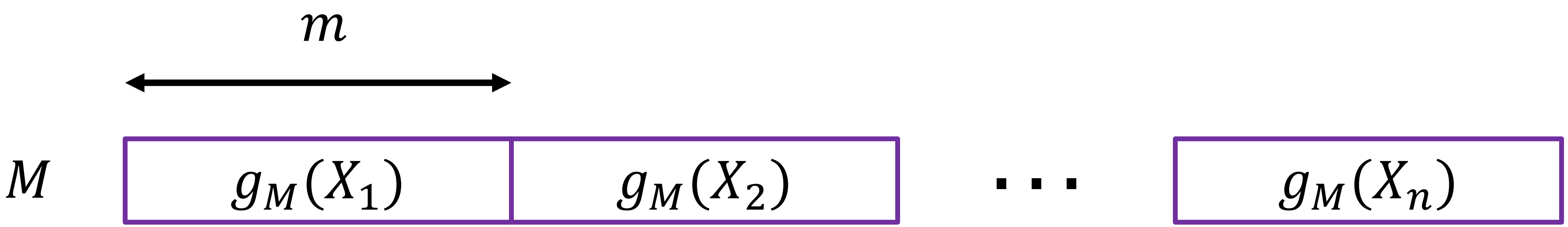

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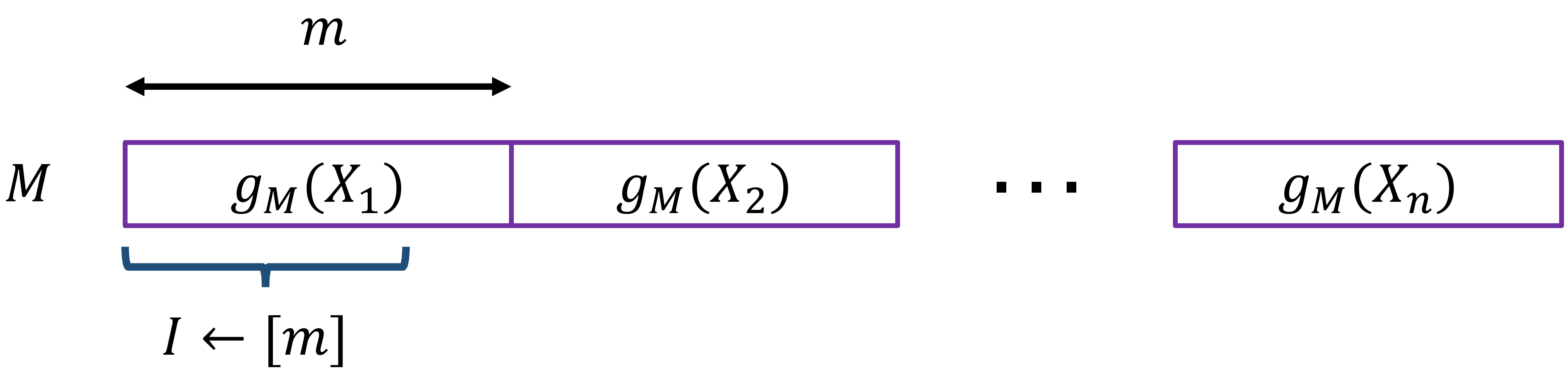

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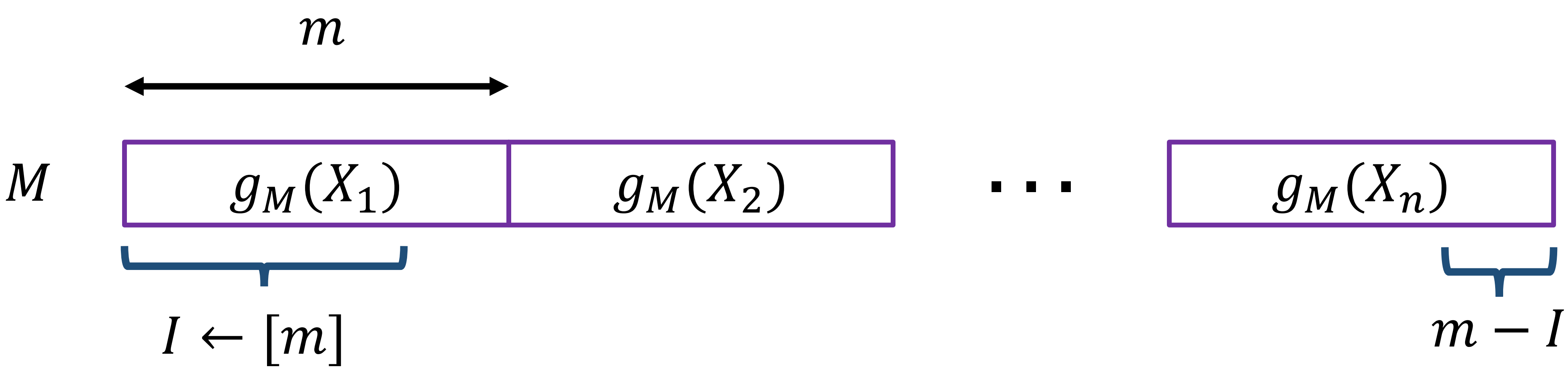

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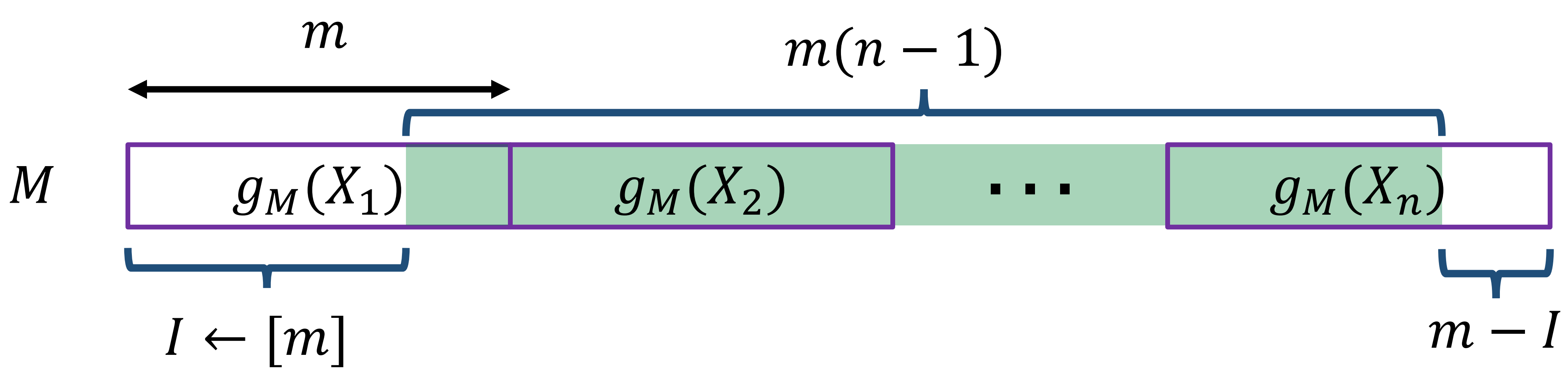

$$g_M^n(X_1, \ldots, X_n, I) = g_M(X_1)_{\geq I}, g_M(X_2), \ldots, g_M(X_n)_{<I}$$

Step 2: Extraction

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$$
t\left\{
\begin{array}{c}
\boxed{g_M^n(X_1^1, \dots, X_n^1, I_1)} \\
\boxed{g_M^n(X_1^2, \dots, X_n^2, I_2)} \\
\vdots \\
\boxed{g_M^n(X_1^t, \dots, X_n^t, I_t)}
\end{array}
\right.
$$

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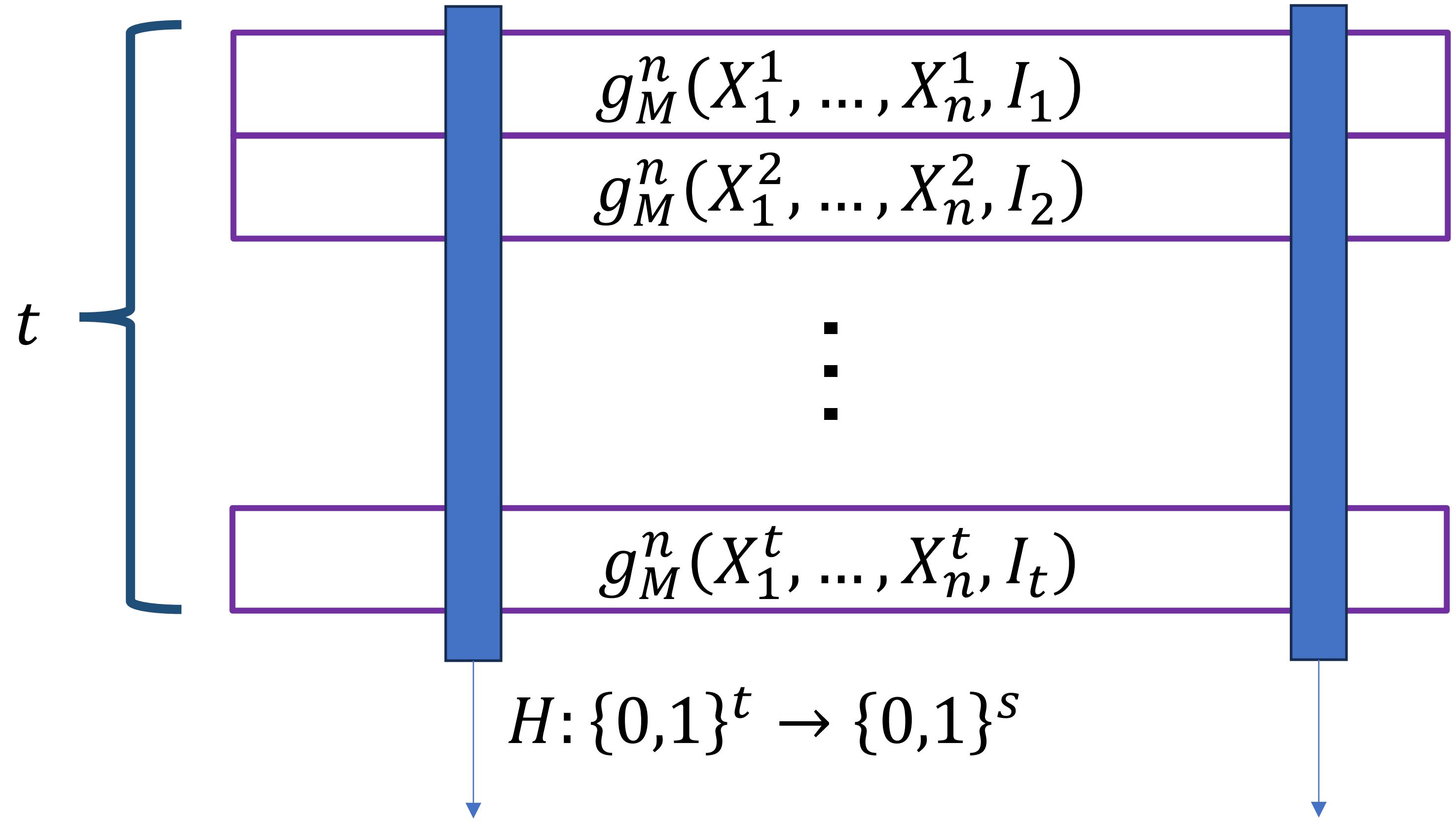

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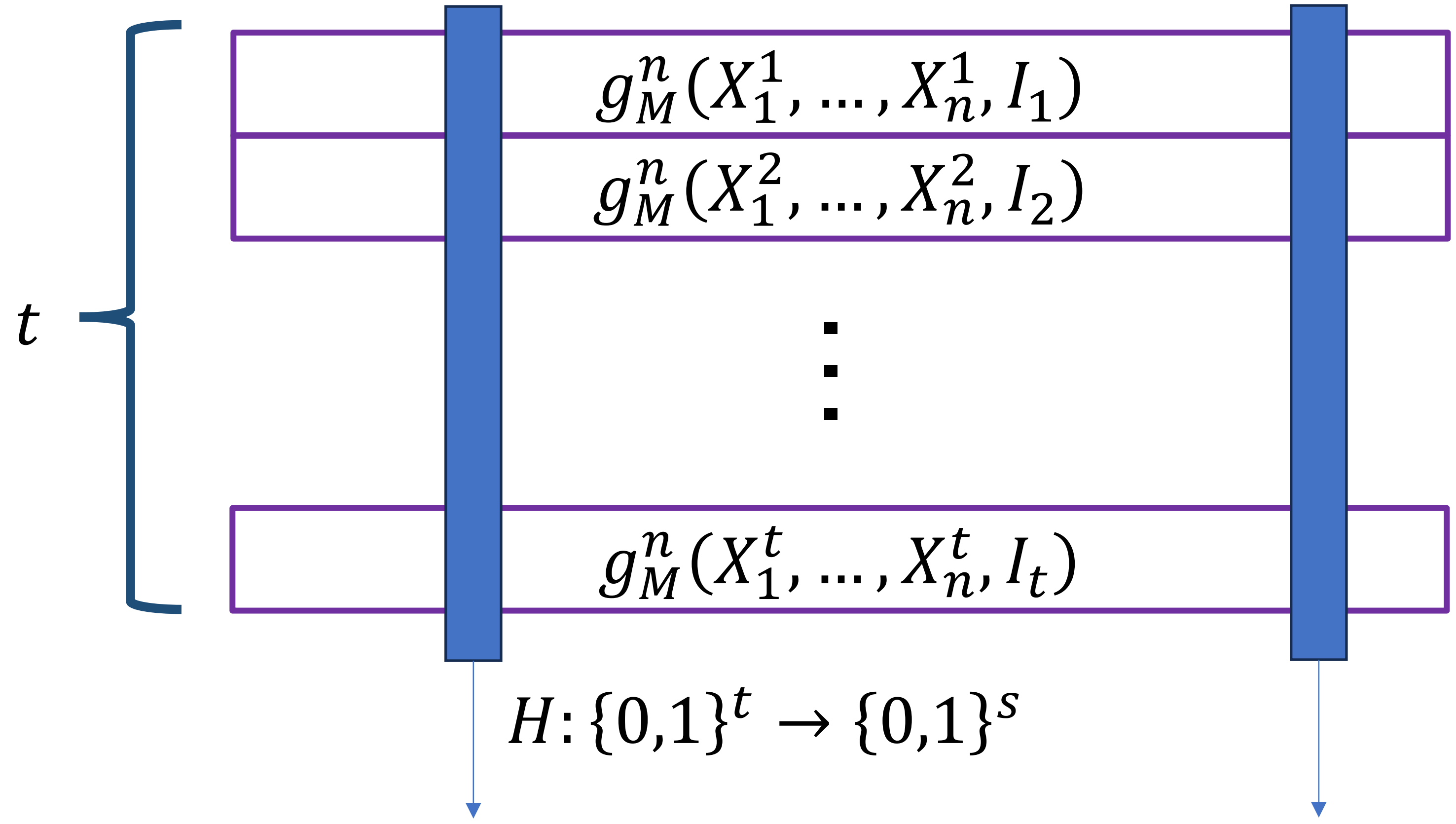

Here we improve parameters over [HRV]

Summary

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Bits Unpredictabilty

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- Simpler Proof

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- $g_M(\mathrm{X}) = \mathrm{M}\big(\mathrm{f}(\mathrm{X})\big), \mathrm{M}(\mathrm{X})$

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OWF $\rightarrow$ Bits Unpredictability

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<u>Question</u>: Simplifying other proofs/constructions

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- Simpler Proof
- Better Parameters

OWF → Bits Unpredictability

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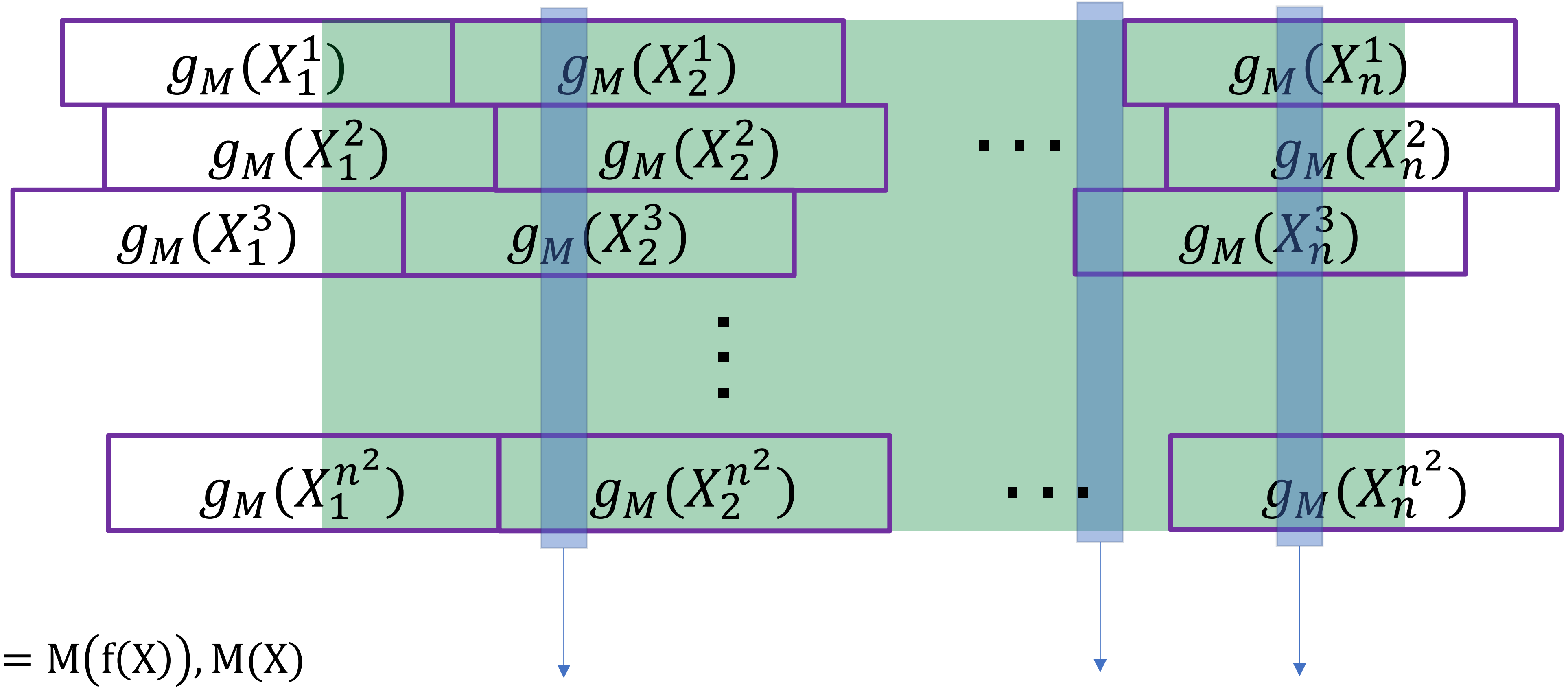
Bits Unpredictabity → PRG

<u>Question</u>: Simplifying other proofs/constructions

Thanks!

The Final Construction



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Bits Unpredictability – Formal Definition

Def:

$g_M: \{0,1\}^n \to \{0,1\}^m$ has k bit unpredictability if the following holds for every $\epsilon \in 1/poly$.

For every $x \in \{0,1\}^n$ there exists a set $S_x \subseteq [m]$, such that $|S_x| \geq k$, and,

$$\Pr_{\mathrm{M,X}}[P(M, g_M(X)_{<i}) = g_M(X)_i \mid i \in S_X] \leq \frac{1}{2} + \epsilon$$

For *any* PPT P.