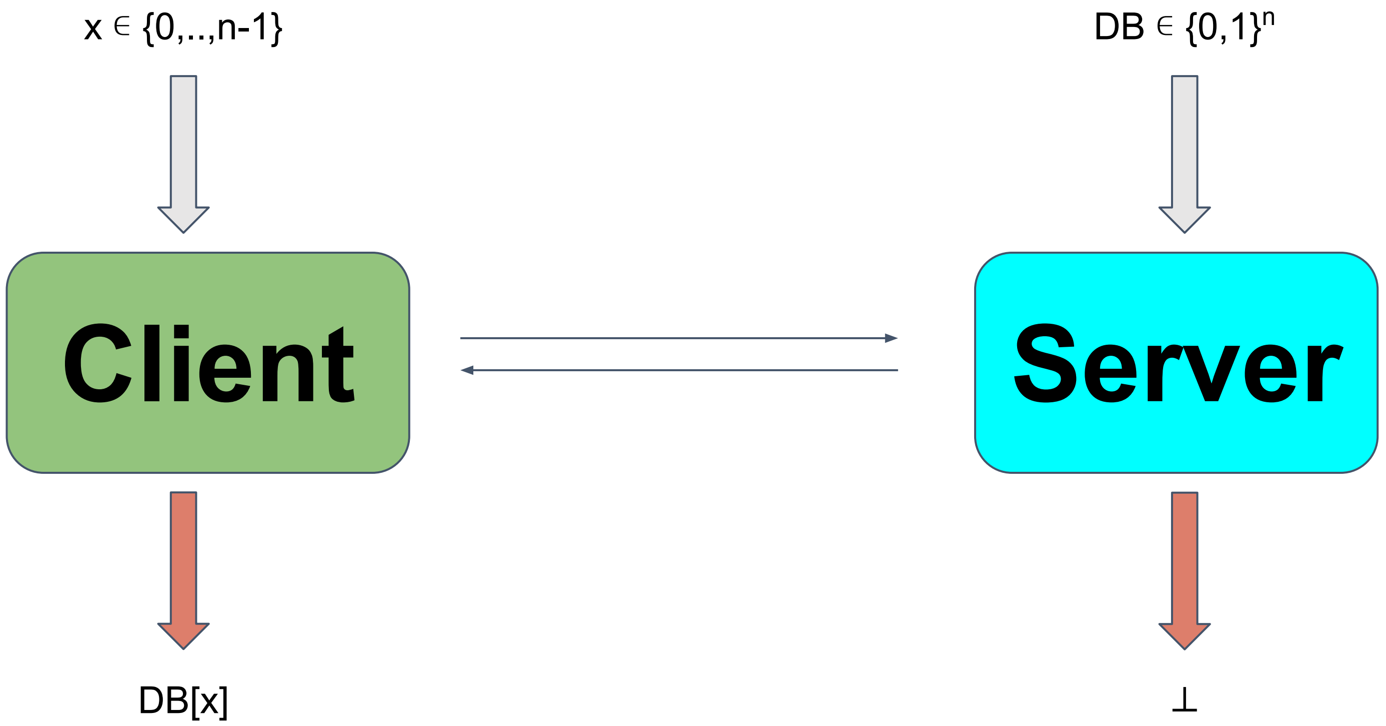
Near-Optimal Private Information Retrieval with Preprocessing

Arthur Lazzaretti and Charalampos Papamanthou

Private Information Retrieval [CGKM ‘95, KO ‘97,....]



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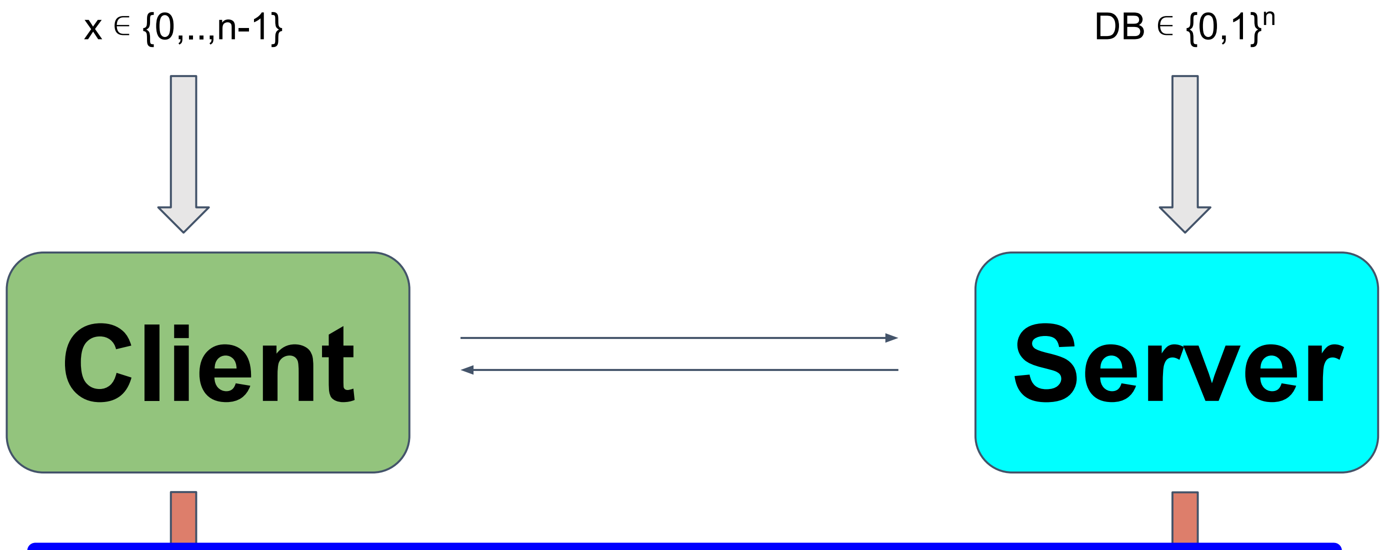

Privacy:
There exists a PPT simulator A* with no knowledge of x that can act as Client and Server cannot tell the difference

Applications

- Building block for different applications:

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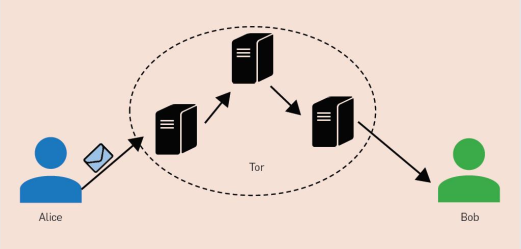
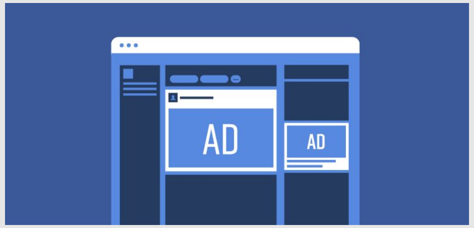


Meta-data hiding messaging
[Angel et al., OSDI ‘16]

Private movie streaming
[Gupta et al., USENIX ‘16]



Private ad serving
[Zhong et al., USENIX ‘21]

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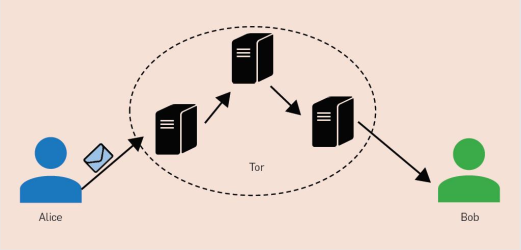

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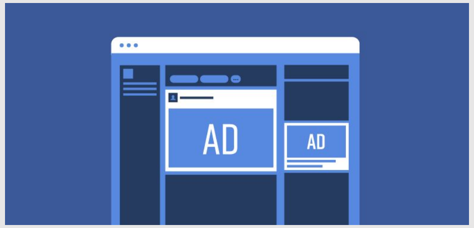

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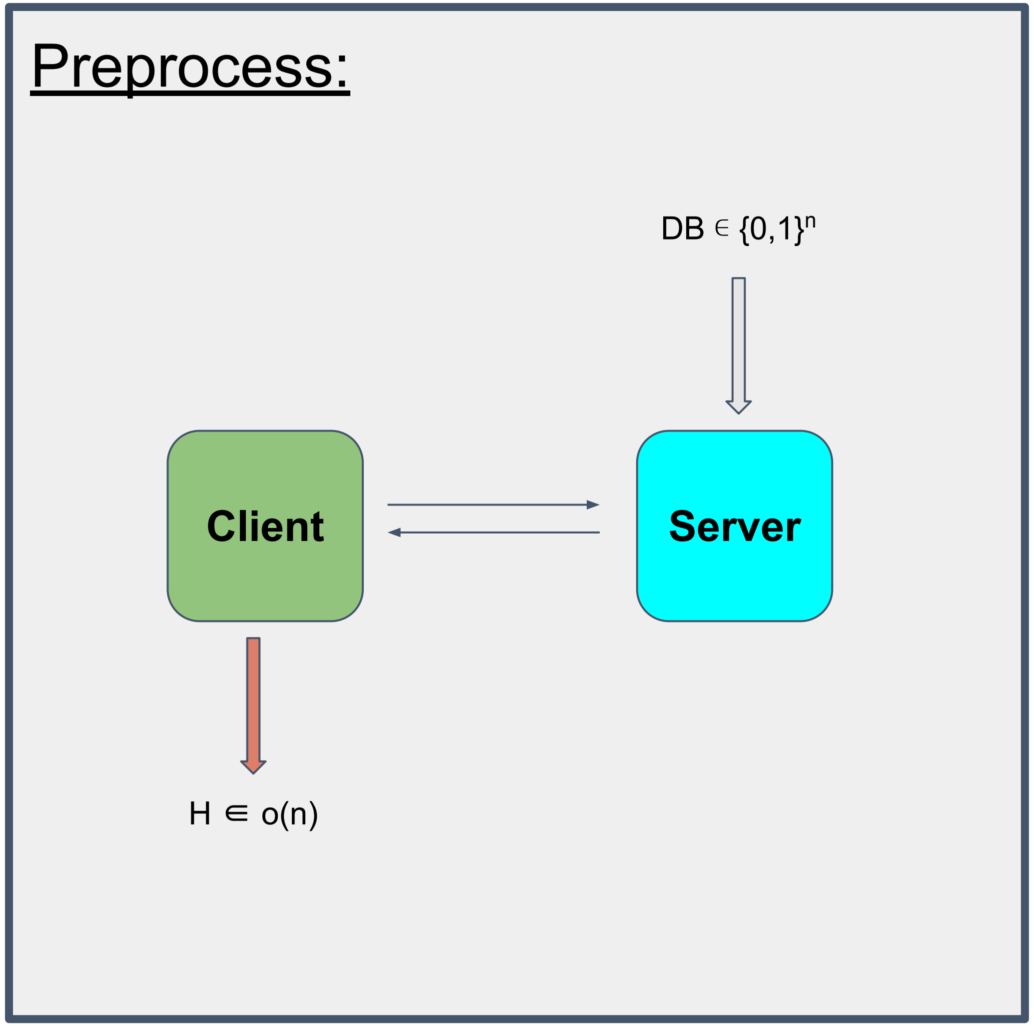
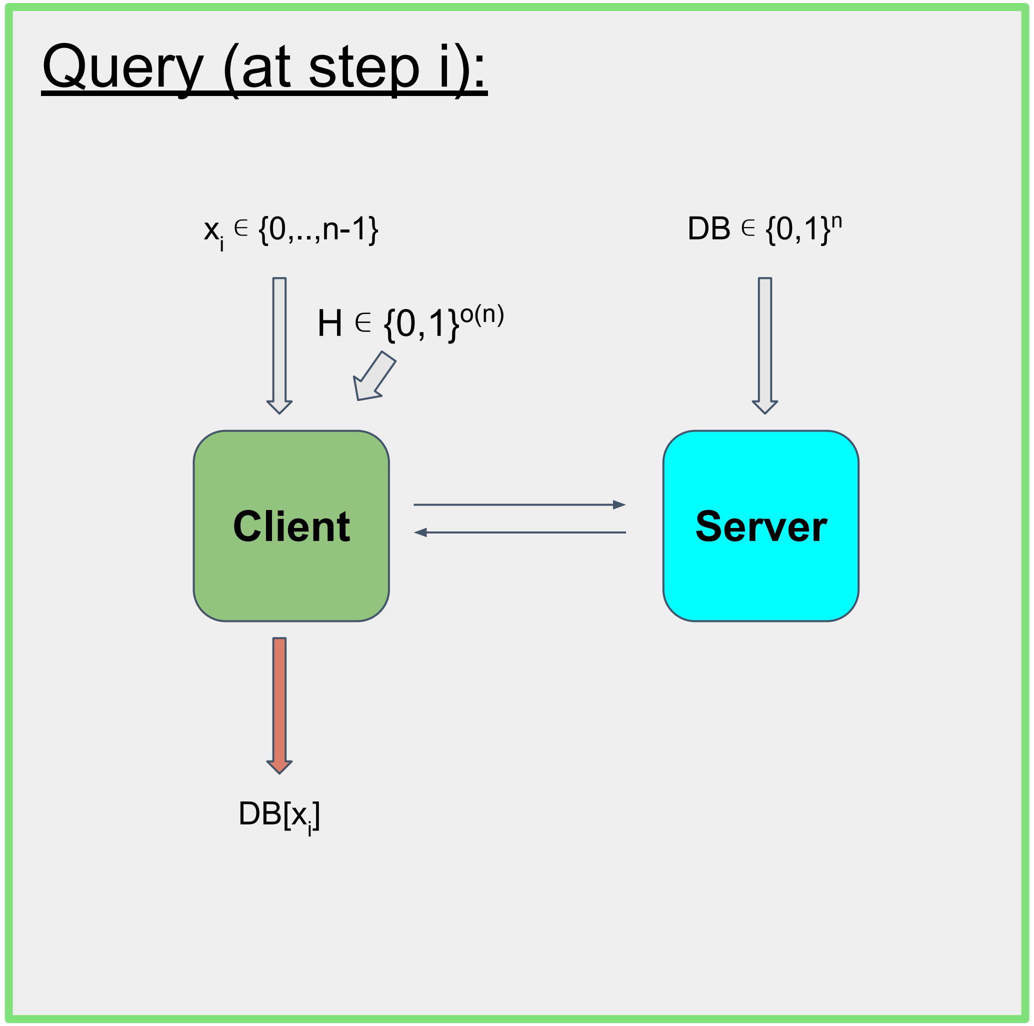
Bottleneck: Server computation

PIR with Client Preprocessing [BIM ‘04, …, CK ‘20, ...]



PIR with Client Preprocessing [BIM '04, …, CK '20, ...]



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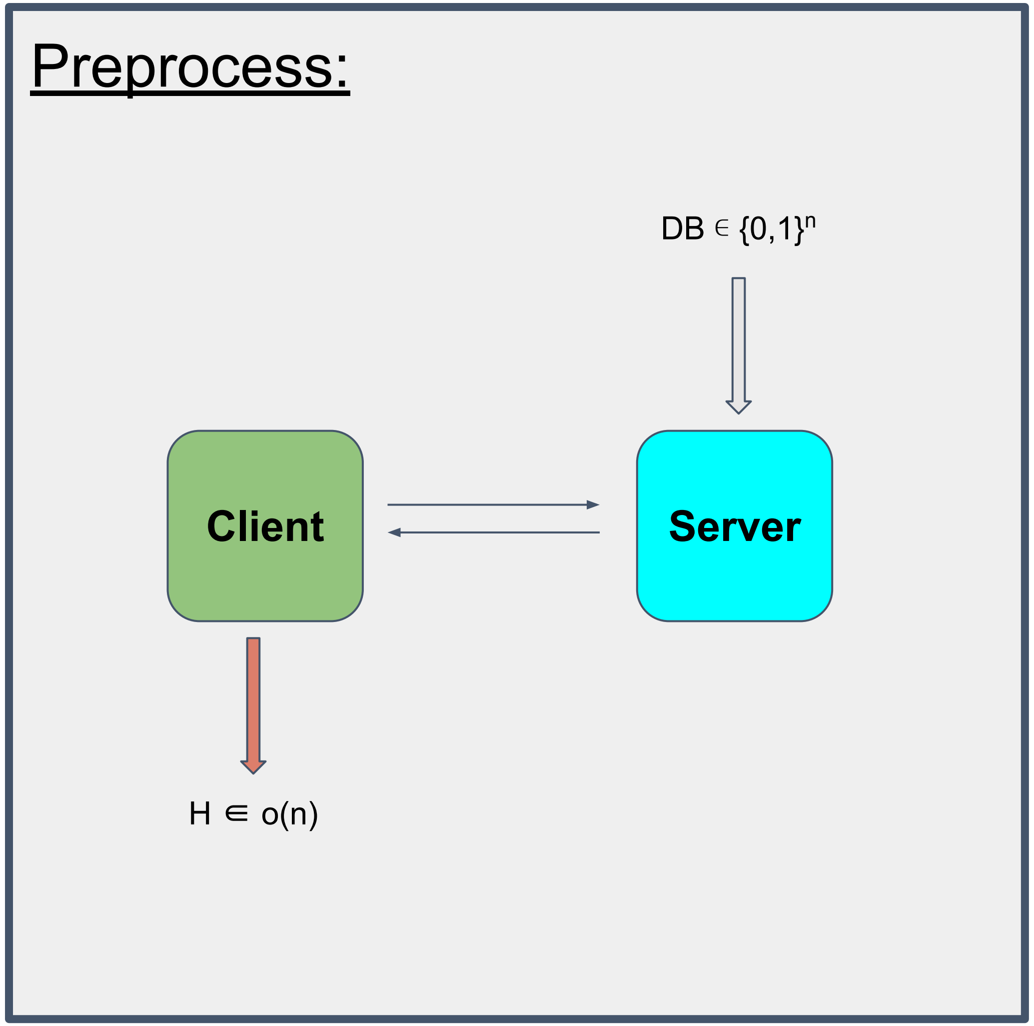

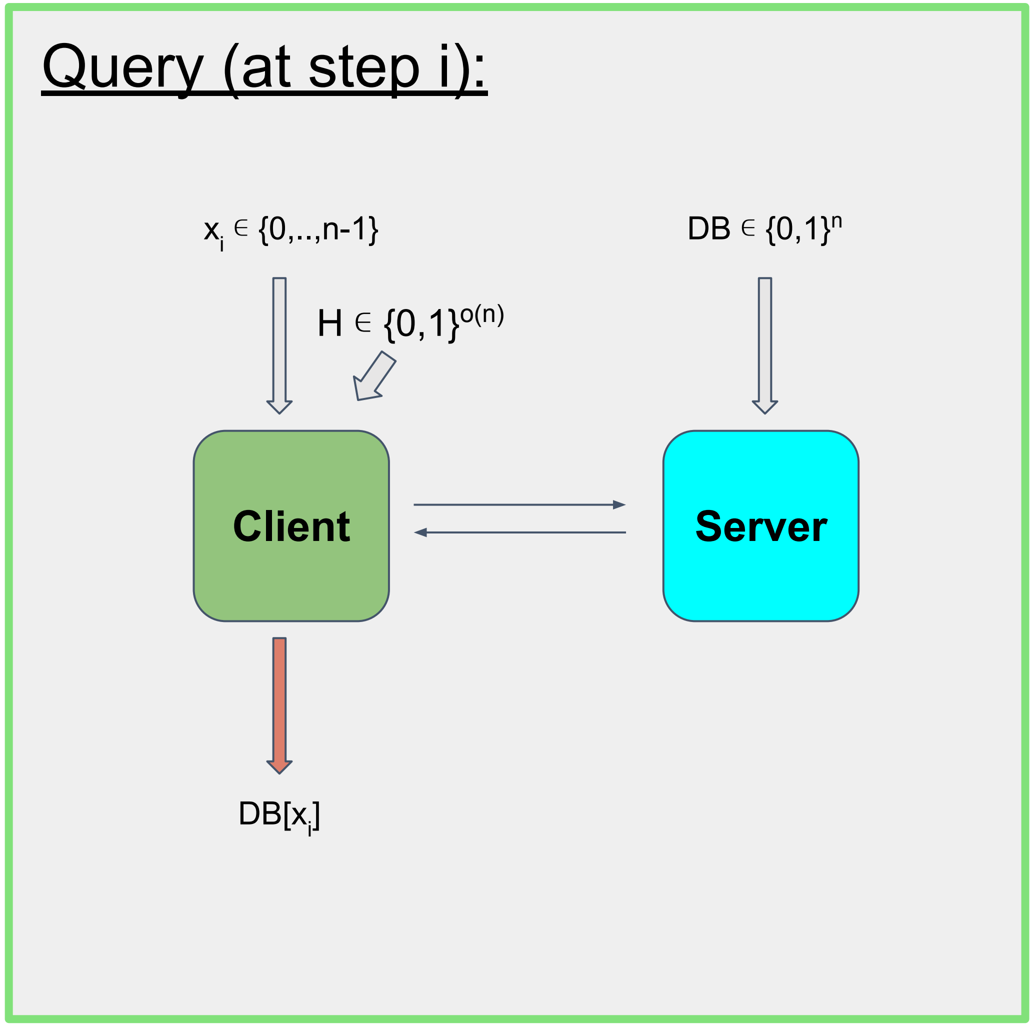

Our results

Scheme	Amortized Server Computation	Amortized Bandwidth	Client space	Number of servers
Ours	$\tilde{O}(\sqrt{n})$	$\tilde{O}(1)$	$\tilde{O}(\sqrt{n})$	1
[Shi et al., CRYPTO ‘21]	$\tilde{O}(\sqrt{n})$	$\tilde{O}(1)$	$\tilde{O}(\sqrt{n})$	2
[Corrigan Gibbs et al., EUROCRYPT ‘22]	$\tilde{O}(\sqrt{n})$	$\tilde{O}(\sqrt{n})$	$\tilde{O}(\sqrt{n})$	1

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Concurrent work [Zhou et al., EUROCRYPT ‘23] achieves same asymptotics from different techniques

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1. Construct pseudorandom sets with the following properties:
   a. Concise representation
   b. Fast membership testing
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Resulting key must hide which elements were operated on.

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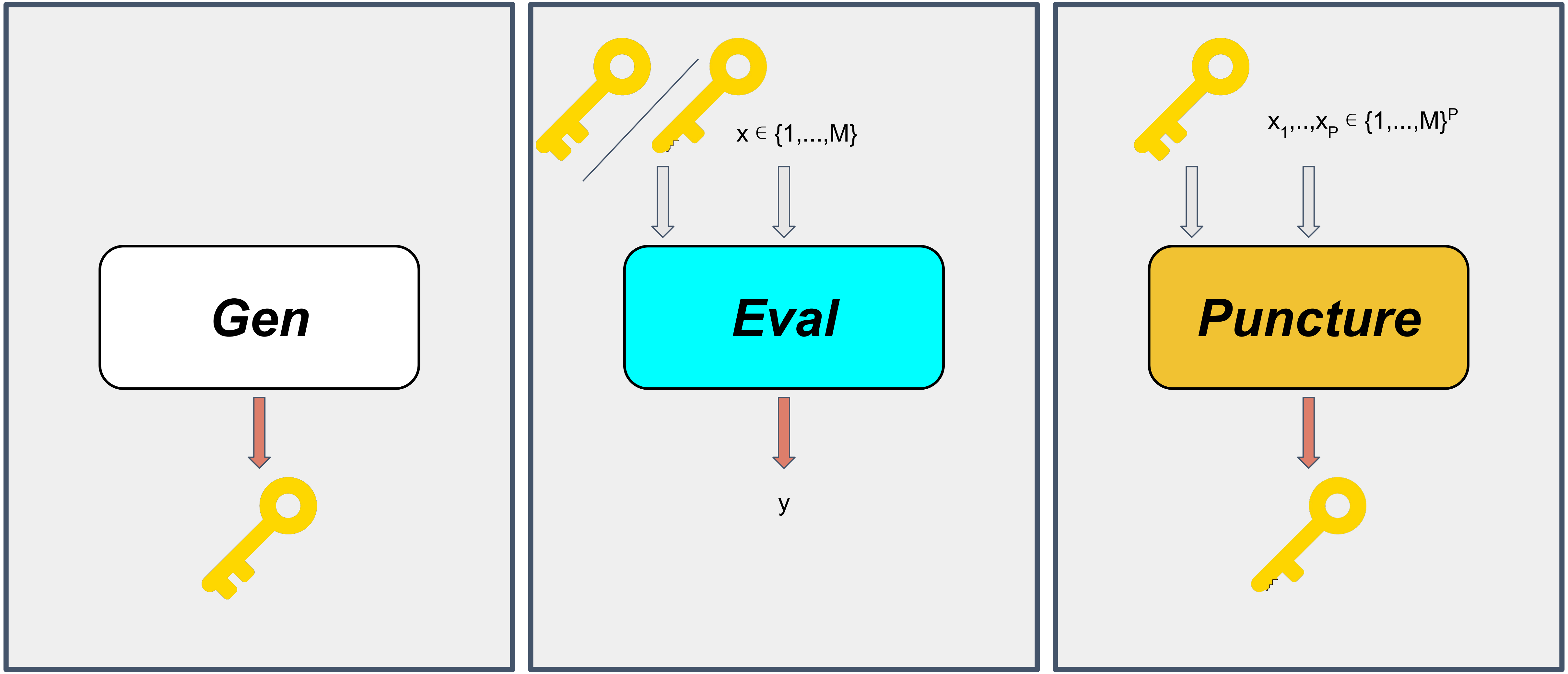
[Shi et al., CRYPTO ‘21] previously achieved sets supporting (a)(b)(c) while allowing for a single removal, our construction allows for any constant number of additions and removals.

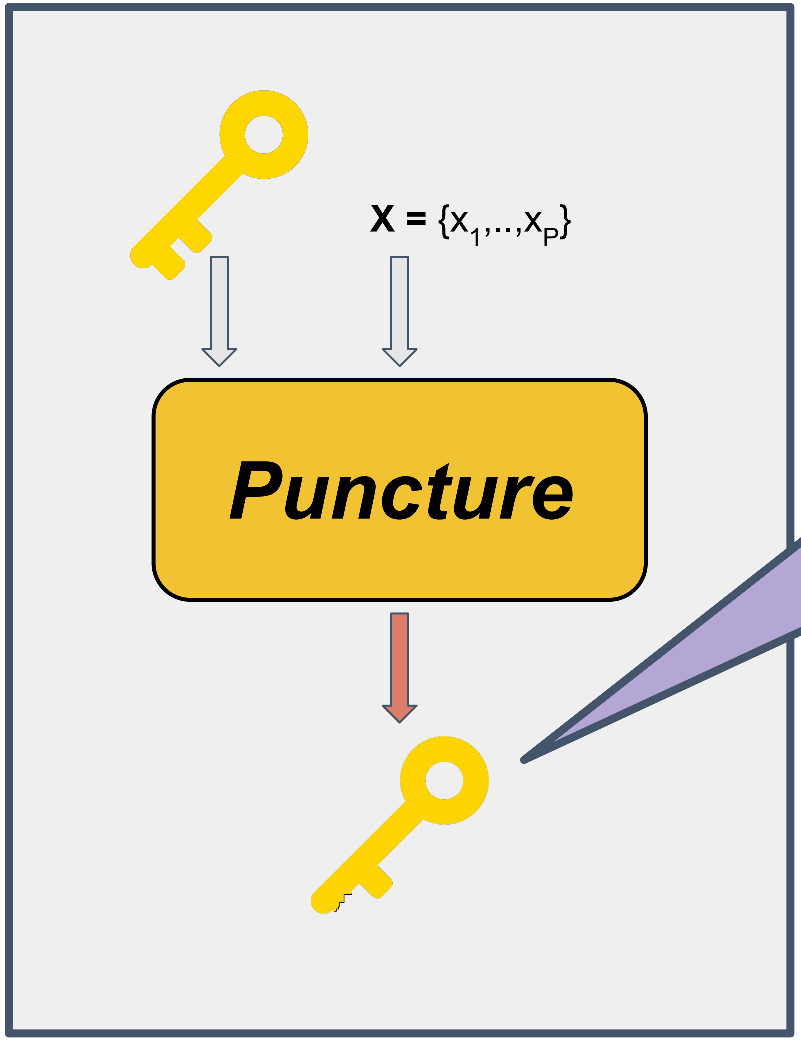
Starting point

- Our starting point is the privately puncturable PRF primitive.

Tool: Privately Puncturable PRF [BLW ‘15, BKM ‘17, CC ‘17, ...]



Puncture on Privately Puncturable PRF [BLW ‘15, BKM ‘17, CC ‘17, ...]



For this presentation: Puncture returns a key k’ where:

1. Outputs for points $x_1,\ldots,x_P$ are ‘re-randomized’.
2. Given k’ adversary cannot figure out which points were punctured.

Set definition [SACM '21]

- Given: Privately puncturable PRF F: $\{0,1\}^\lambda \times \{0,1\}^* \rightarrow \{0,1\}$.
- Want: set containing approximately $\sqrt{n}$ elements in $\{0,...,n-1\}$ picked from some sampling distribution over $\{0,...,n-1\}$.

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- For each x in $\{0,...,n-1\}$:
  - $x \in S_k$ **iff** $F_k(x[i:]) = 1 \ \forall i \in \{0,...,\log(n)/2\}$.

Set definition (by example)

- Let n = 16, $k \leftarrow$ppPRF.Gen() represent our set, and let x = 7 = 0111.

x in base 2

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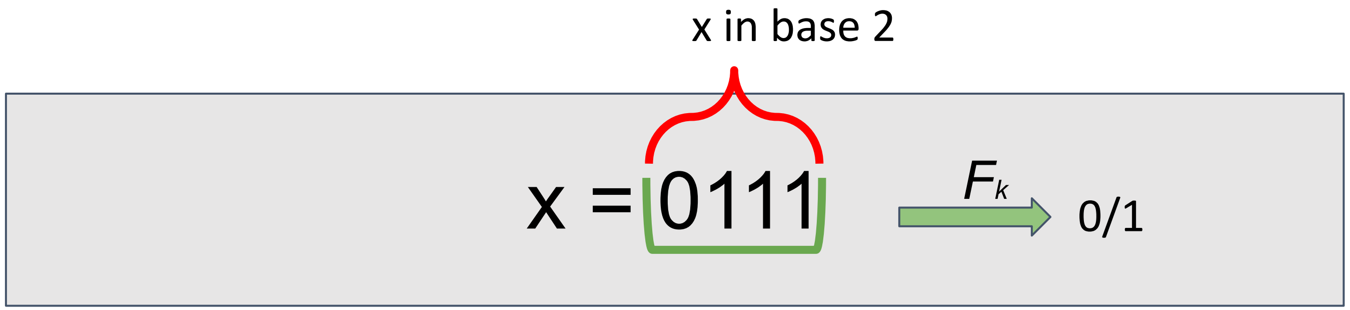

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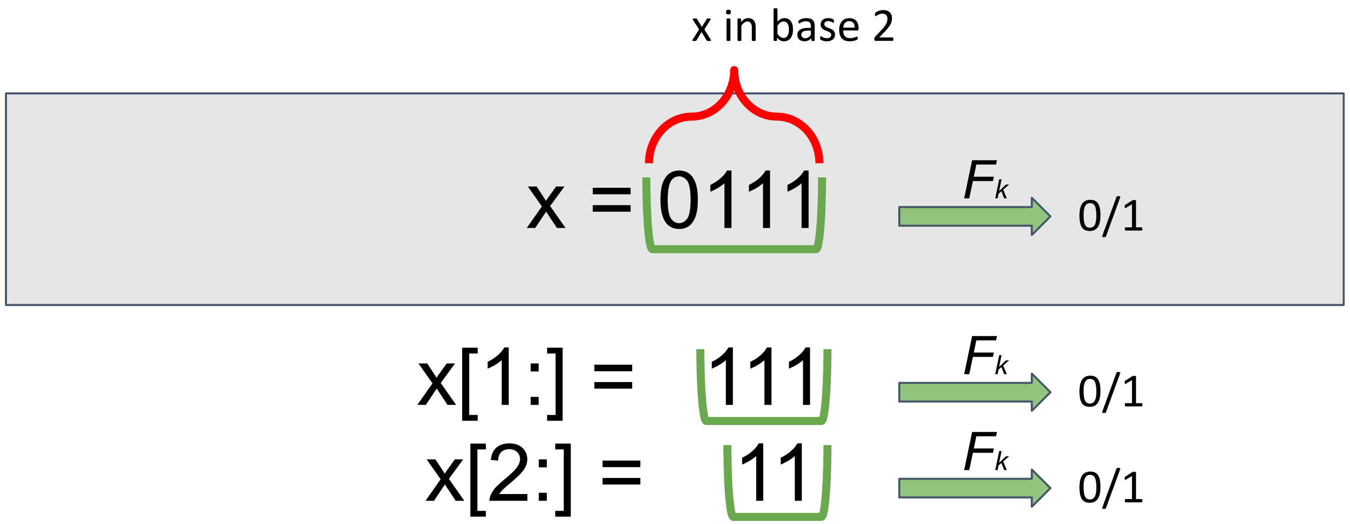

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$S_k = \{x : \forall i \in \{0,..,\log(n)/2\}, F_k(x[i:]) = 1 \wedge x \in \{0,...,n-1\}\}$

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Enumerate in Õ(√n) time by starting from all strings of size log(n)/2 and appending 0/1 only to those that evaluate to 1.

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Adaptability ?

Adding and removing elements

Remove (k, x):
- Let **Z** be set of points that define x’s membership.
- Output new set key k’ = ppPRF.Puncture(k, **Z**).

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With some probability, does not remove or removes other elements.

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<u>Add (k, x):</u>

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- Run k' = ppPRF.Puncture(k, **Z**) *until* $F(k', z) = 1\ \forall z \in \mathbf{Z}$. Output k'.

Adding and removing elements

Uses privately puncturable PRF where puncture operation is randomized [Canetti and Chen '17].

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How to make both Add and Remove work together?
Both access the same PRF operation.

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    - $x \in S_{\mathbf{k}}$ **iff** $\forall i \in \{0,..,\log(n)/2\}$, $F_{k_1}(x[i:]) \oplus \ldots \oplus F_{k_C}(x[i:]) = 1$.
  - We show additions and removals can be done sequentially on each key $k_1,\ldots,k_C$ and satisfy appropriate notions of privacy.

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Thank you!

Image sources:

https://sproutsocial.com/insights/facebook-ad-examples/

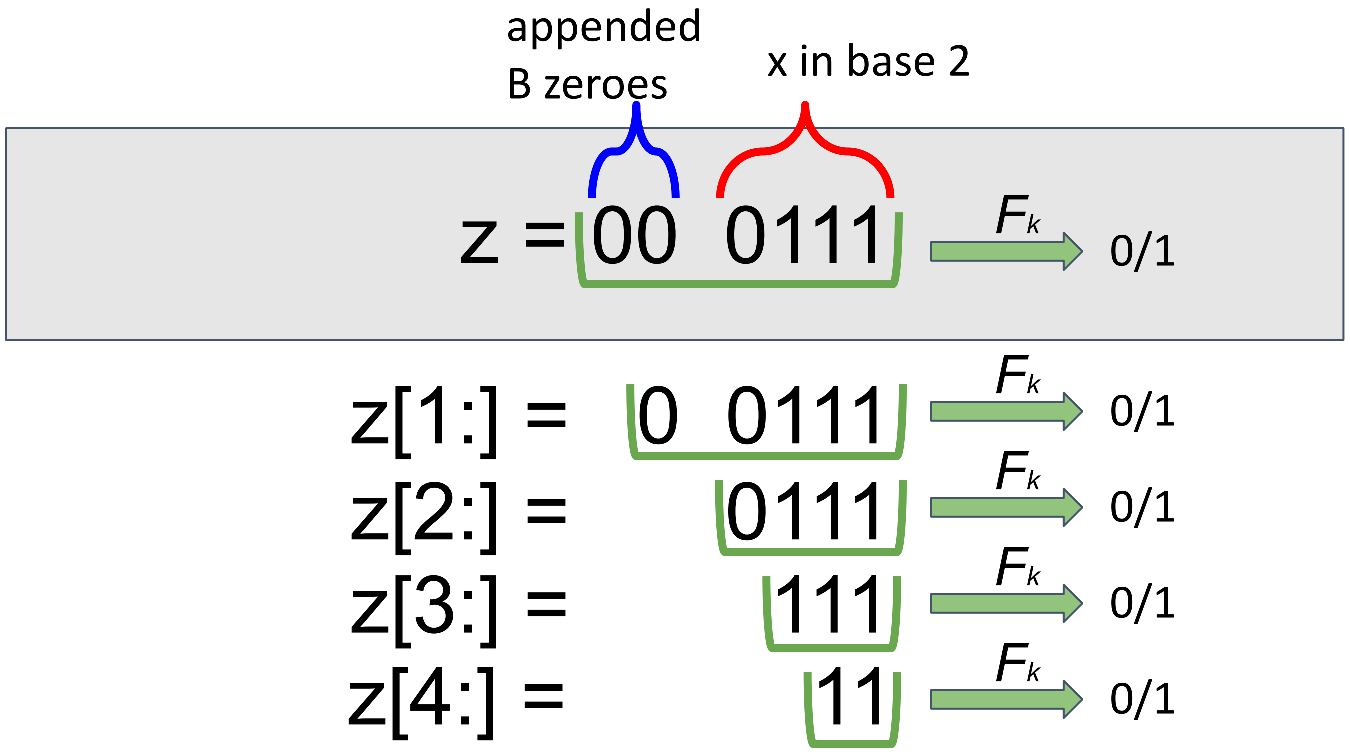
https://icon-library.com/icon/key-icon-png-7.html.html

https://www.freepnglogos.com/images/tick-33835.html

https://cacm.acm.org/magazines/2019/9/238971-metadata-private-communication-for-the-99/abstract

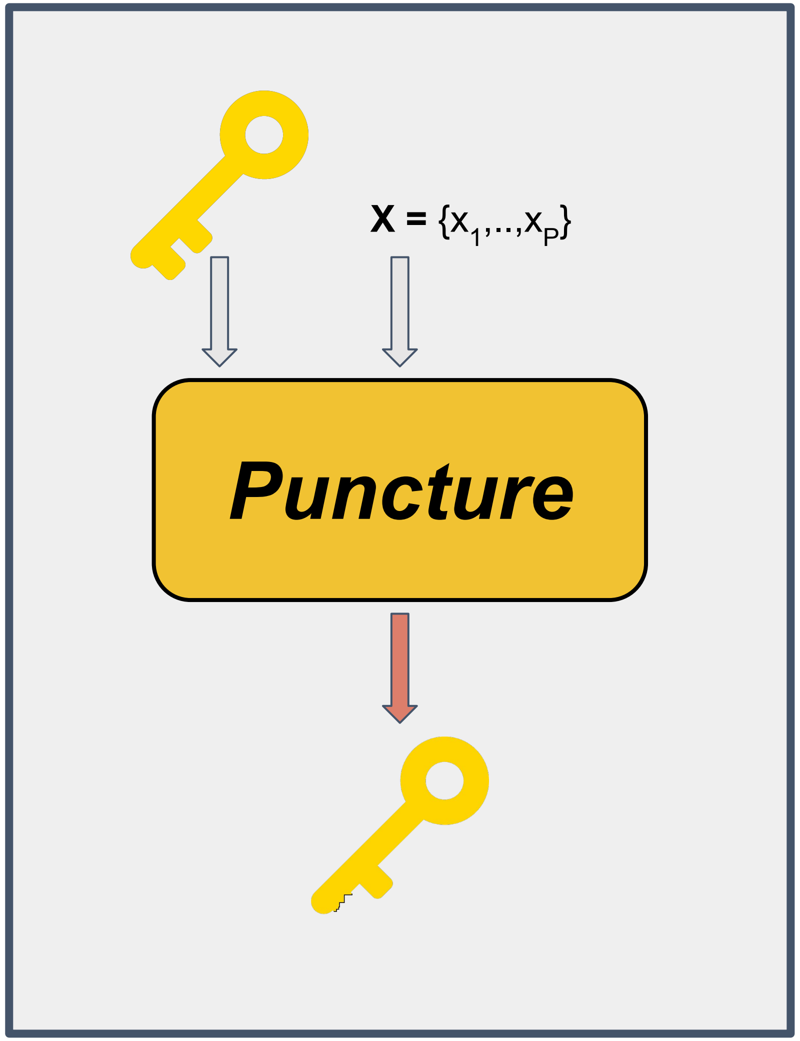
Set definition (by example)

- Let $n = 16$, $k \leftarrow$ppPRF.Gen() represent our set, and let $x = 7 = 0111$.
- Let $z = 0^B || x$, $B = \log(\log(n))$.



$S_k = \{x : \forall i \in \{0,..,\log(n)/2 + B\}, F_k(z[i:]) = 1 \text{ for } z = 0^B \,||\, x \wedge x \in \{0,...,n-1\}\}$

Puncture on Privately Puncturable PRF [BLW ‘15, BKM ‘17, CC ‘17, ...]



Correctness:

For any input $x' \notin \mathbf{X}$, punctured key evaluates to same output as original key.

Security:

New key contains no information about original evaluation at punctured points.

Privacy:

New key contains no information about what points were punctured.