Searching for ELFs in the Cryptographic Forest

TCC 2023

Marc Fischlin **Felix Rohrbach**

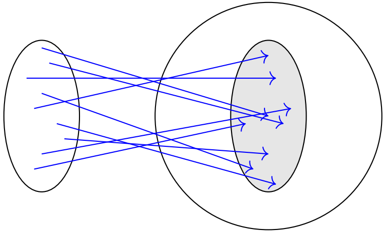
Technische Universität Darmstadt, Germany

Extremely Lossy Functions (ELFs)

Mark Zhandry: The Magic of ELFs, Crypto 2016

Extremely Lossy Functions (ELFs)

Injective Mode



pk_{inj}

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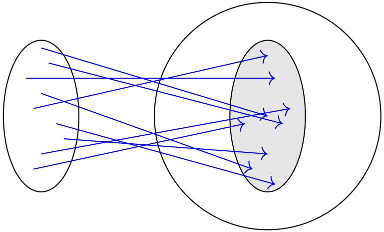
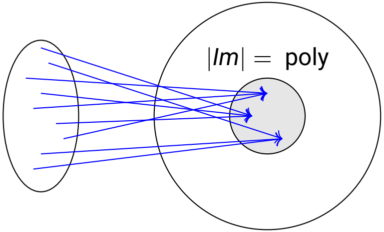


pk_{inj} pk_{loss}



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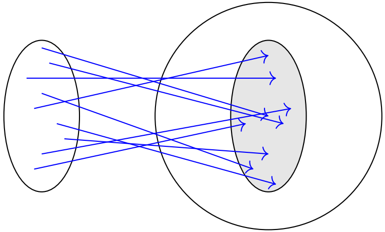

$$\mathrm{pk}_{\mathrm{inj}} \overset{c}{\approx} \mathrm{pk}_{\mathrm{loss}}$$

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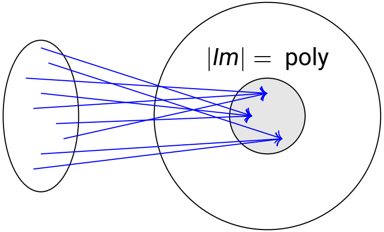

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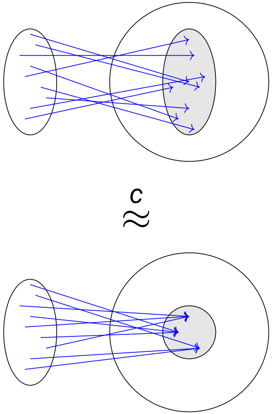
ELFs and the Random-Oracle Model

- ELFs can be used to replace ROM



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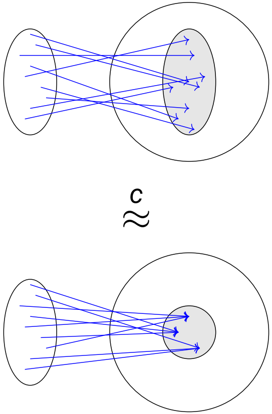

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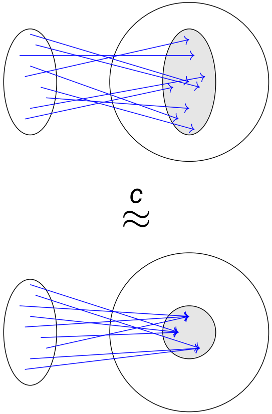

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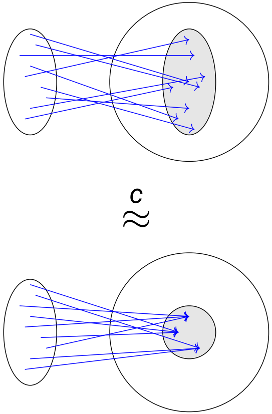
- ELFs can be used to replace ROM
- Many attempts to replace ROM
  - Correlation Intractability, Universal Computational Extractors
- **Extremely Lossy Functions:**
  - Standard-ish assumptions
  - Useful for many applications



Constructing ELFs

- Exponential decisional *k*-linear assumption:

$$\left(g, g^{a_1}, \ldots, g^{a_k}, g^{\sum_i b_i}, g^{a_1 b_1}, \ldots, g^{a_k b_k}\right) \overset{c_e}{\approx} \left(g, g^{a_1}, \ldots, g^{a_k}, g^{c}, g^{a_1 b_1}, \ldots, g^{a_k b_k}\right)$$

Generalized version of exponential DDH

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- Claim: True for e.g. elliptic curves

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  - Zhandry’16: eOWFs, eCRH might be enough
  - Holmgren, Lombardi’18: ELFs from One-Way Product Functions?

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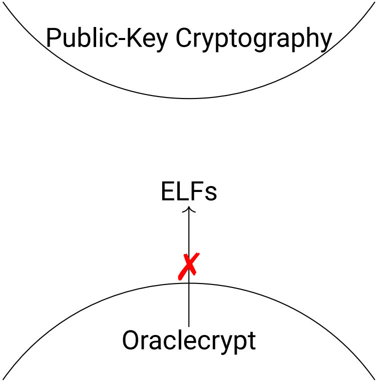
What are the minimal assumptions for building ELFs?

Our Results

- No fully black-box construction of ELFs from eOWFs, eCRHFs, OWPFs, . . .



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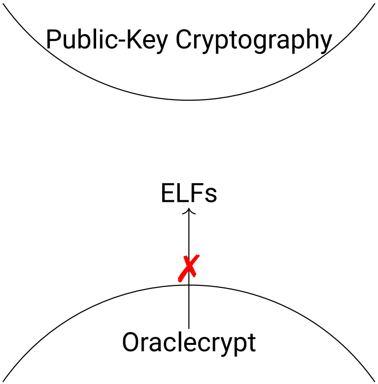
  - Even holds for (moderately) lossy functions!



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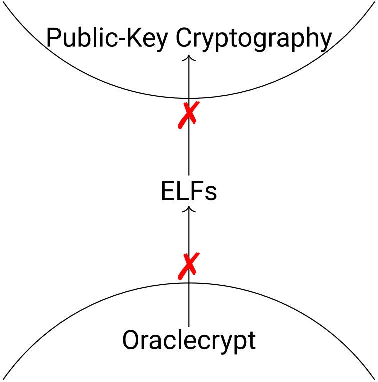

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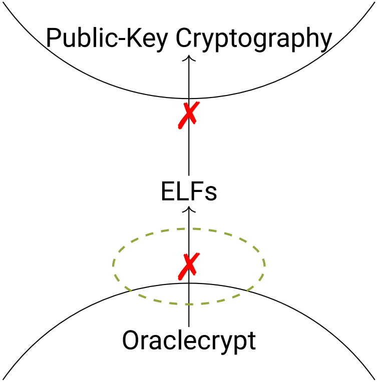

Oracle Separation

There exist oracles $\mathcal{O}$, PSPACE^+, such that relative to them:

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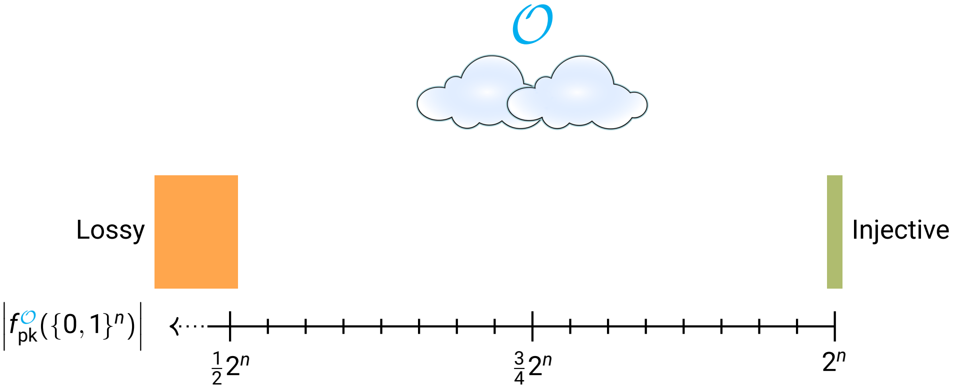
- but lossy functions and ELFs do not
- Idea similar to *Pietrzak, Rosen, Segev, TCC'12*

Inefficient Distinguisher



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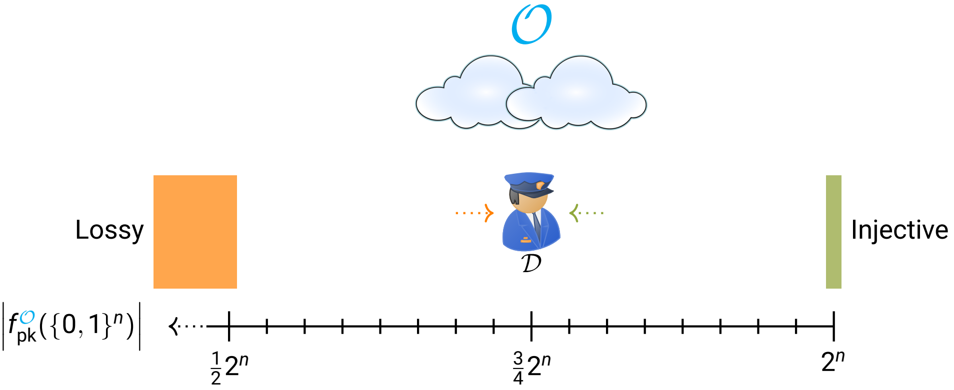

Heavy Queries are Important

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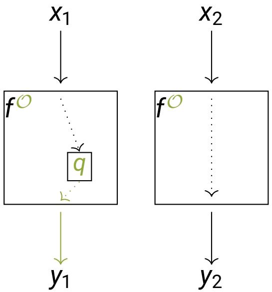

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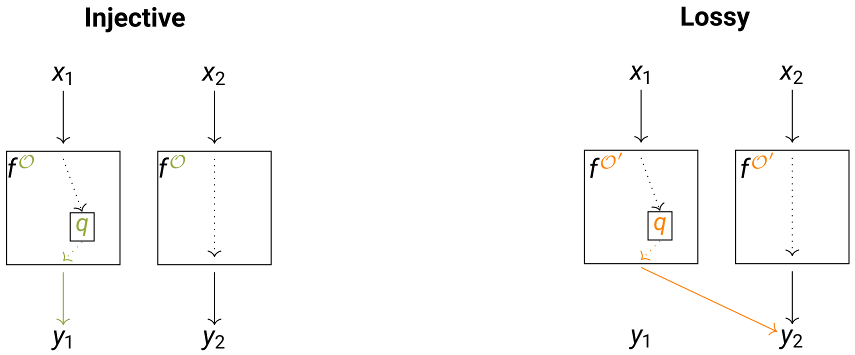
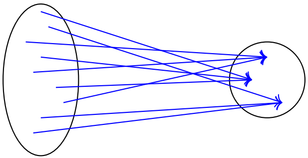
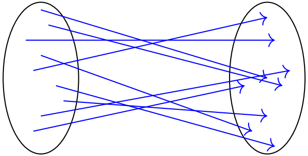


Observations

Observation 1: Lossiness is a global property.





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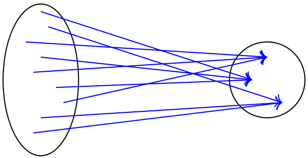

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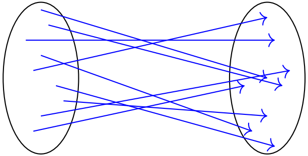

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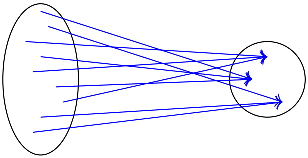

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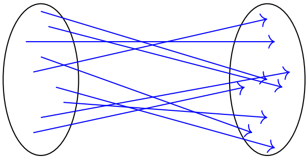
- Other positions cannot influence mode (w.h.p.)



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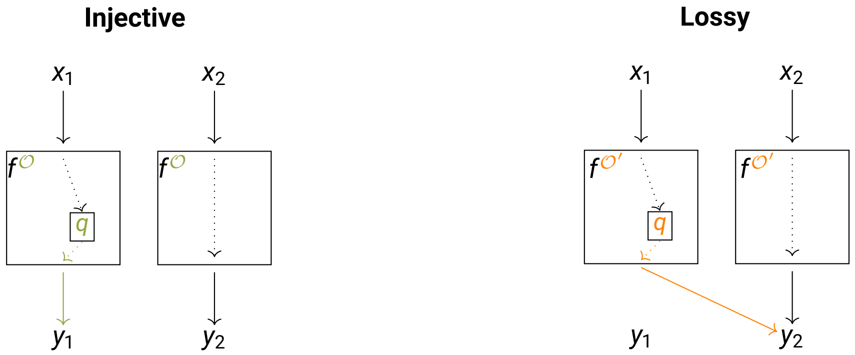

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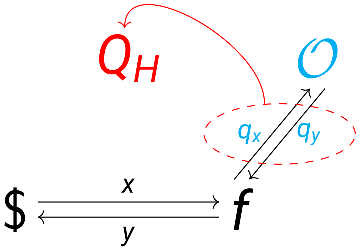

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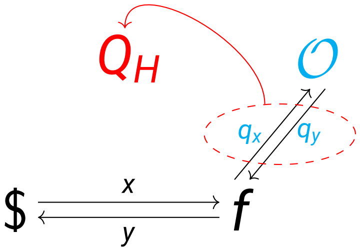

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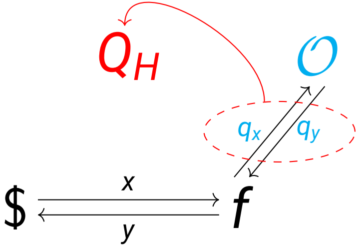
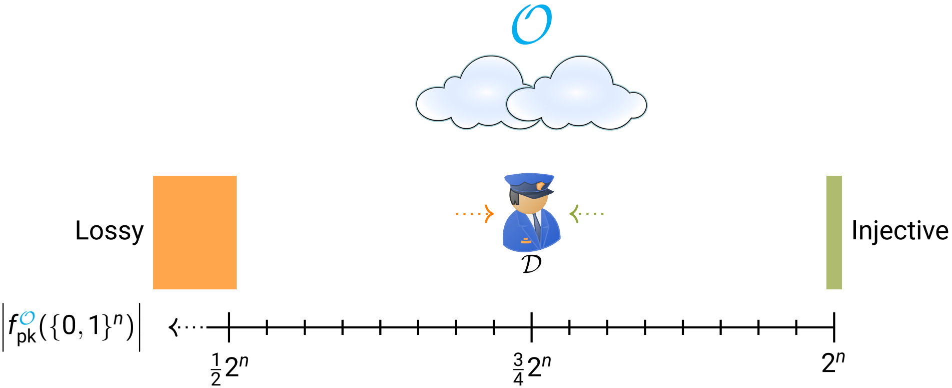


- $|Q_H|$ polynomial
- With overwhelming probability: All heavy queries are in Q_H

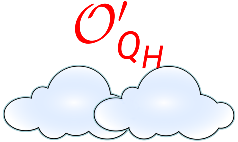
Cryptoplexity
Cryptography & Complexity Theory
Technische Universität Darmstadt
www.cryptoplexity.de

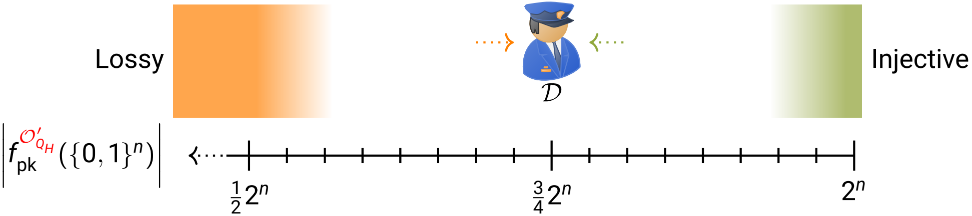
Efficient Distinguisher



Efficient Distinguisher

$$\mathcal{O}'_{Q_H}(x) = \begin{cases} \mathcal{O}(x) & x \in Q_H \\ \mathcal{O}'(x) & x \notin Q_H \end{cases}$$





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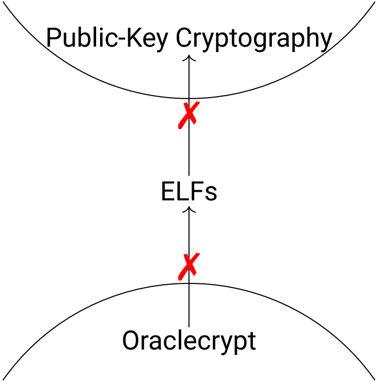
$\Rightarrow$ No fully BB construction of ELFs from anything in Oraclecrypt

Overview



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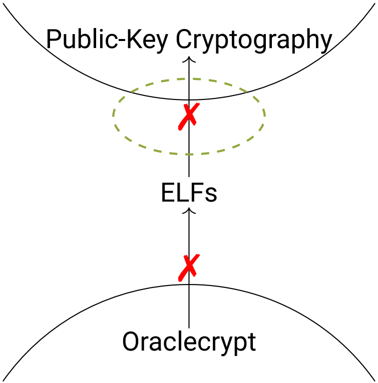

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- Reuse Impagliazzo–Rudich result (No KA relative to a random permutation)

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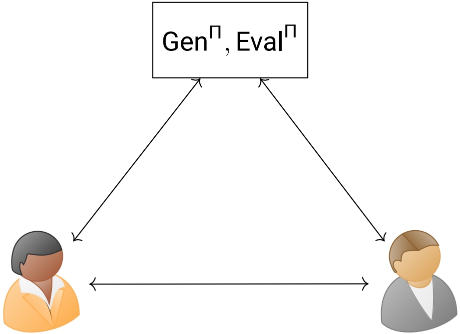
Lemma (Simulation Lemma, informal)

There exists an efficient algorithm Wrap^{Π} *such that access to* Wrap^{Π} *or the oracles* $\mathsf{Gen}^{\Pi}, \mathsf{Eval}^{\Pi}$ *is indistinguishable. Further,* Wrap *has no (global) state.*

No Key Agreement from ELFs

- Assume KA exists



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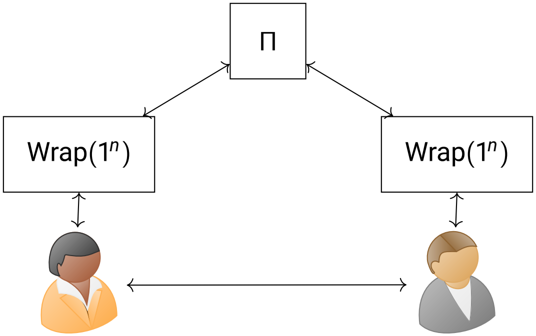

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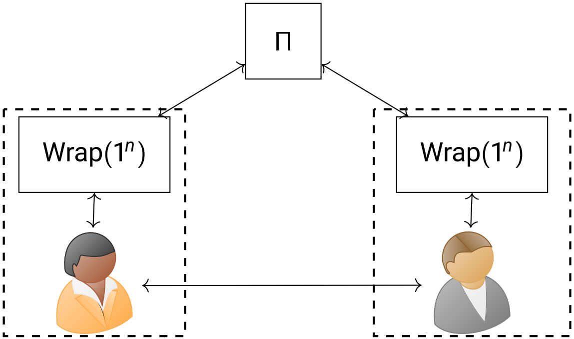

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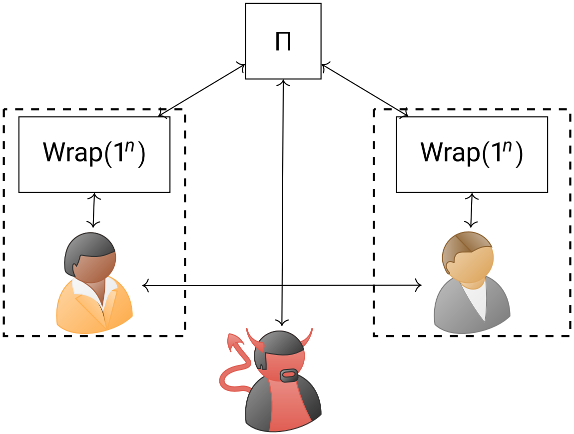

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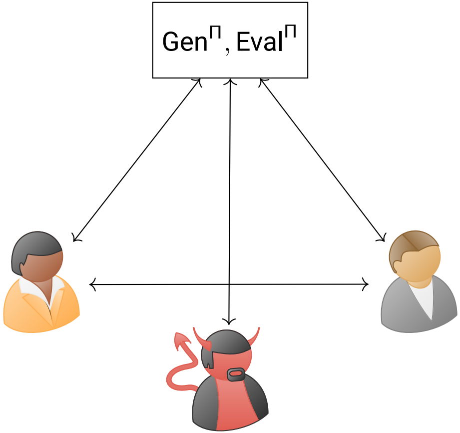
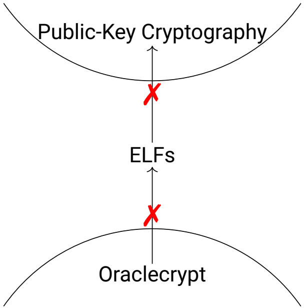
- Assume KA exists
- Introducing Wrap does not break completeness
- Successful adversary exists (*Impagliazzo, Rudich, STOC'89*)
- Removing Wrap does not break attack ↯



Conclusion

- No fully black-box construction of ELFs from Oraclecrypt primitives
- No fully black-box construction of KA from ELFs



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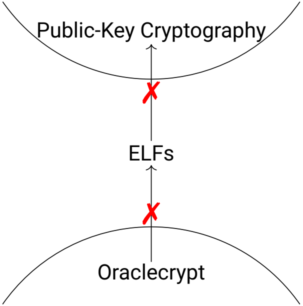

Thank you!

https://ia.cr/2023/1403