CRYPTOGRAPHY FROM PLANTED GRAPHS:

SECURITY WITH LOG-SIZED MESSAGES

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YUVAL ISHAI

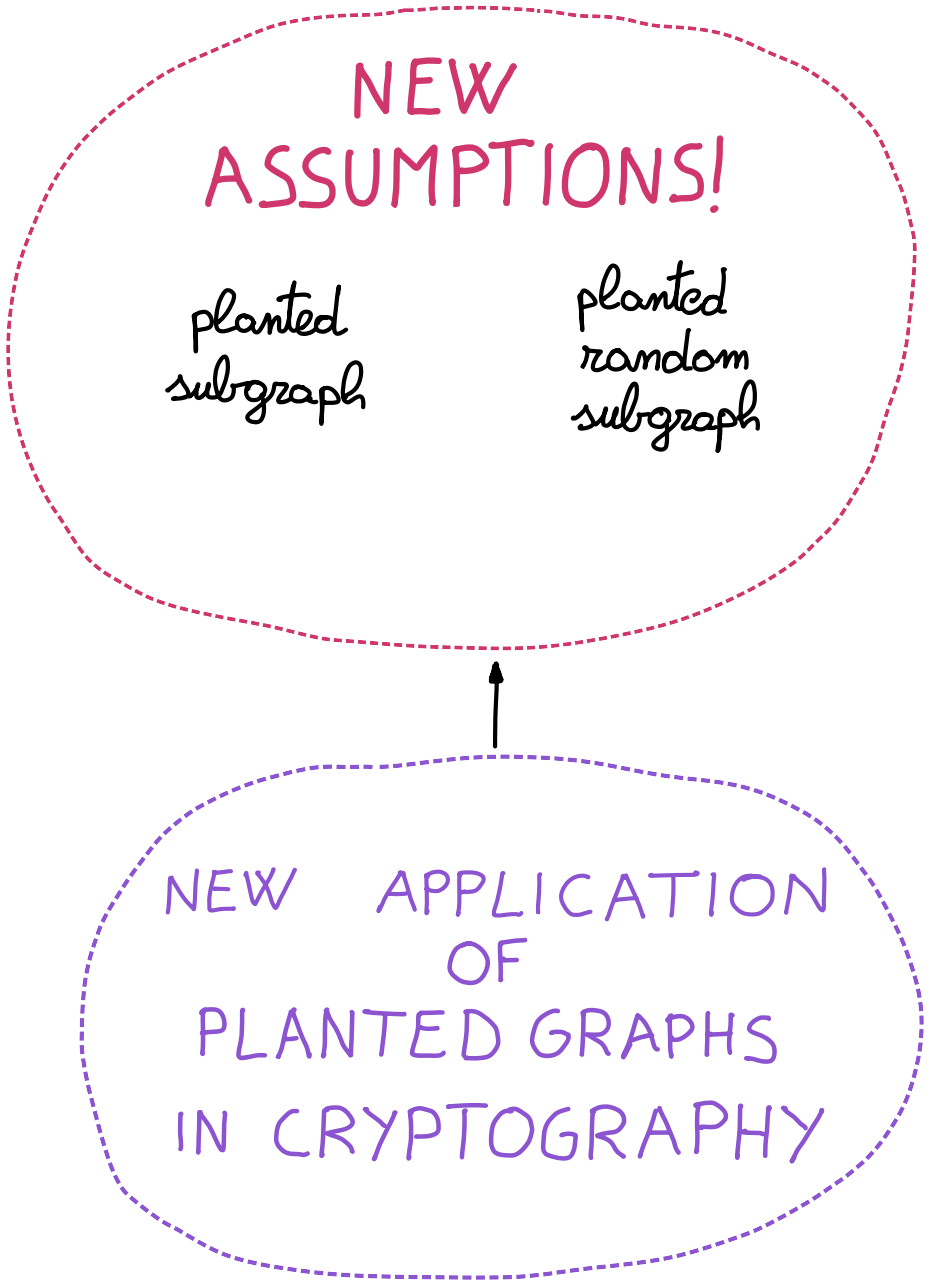
EYAL KUSHILEVITZ

VARUN NARAYANAN

CONTRIBUTIONS

NEW APPLICATION
OF
PLANTED GRAPHS
IN CRYPTOGRAPHY

CONTRIBUTIONS



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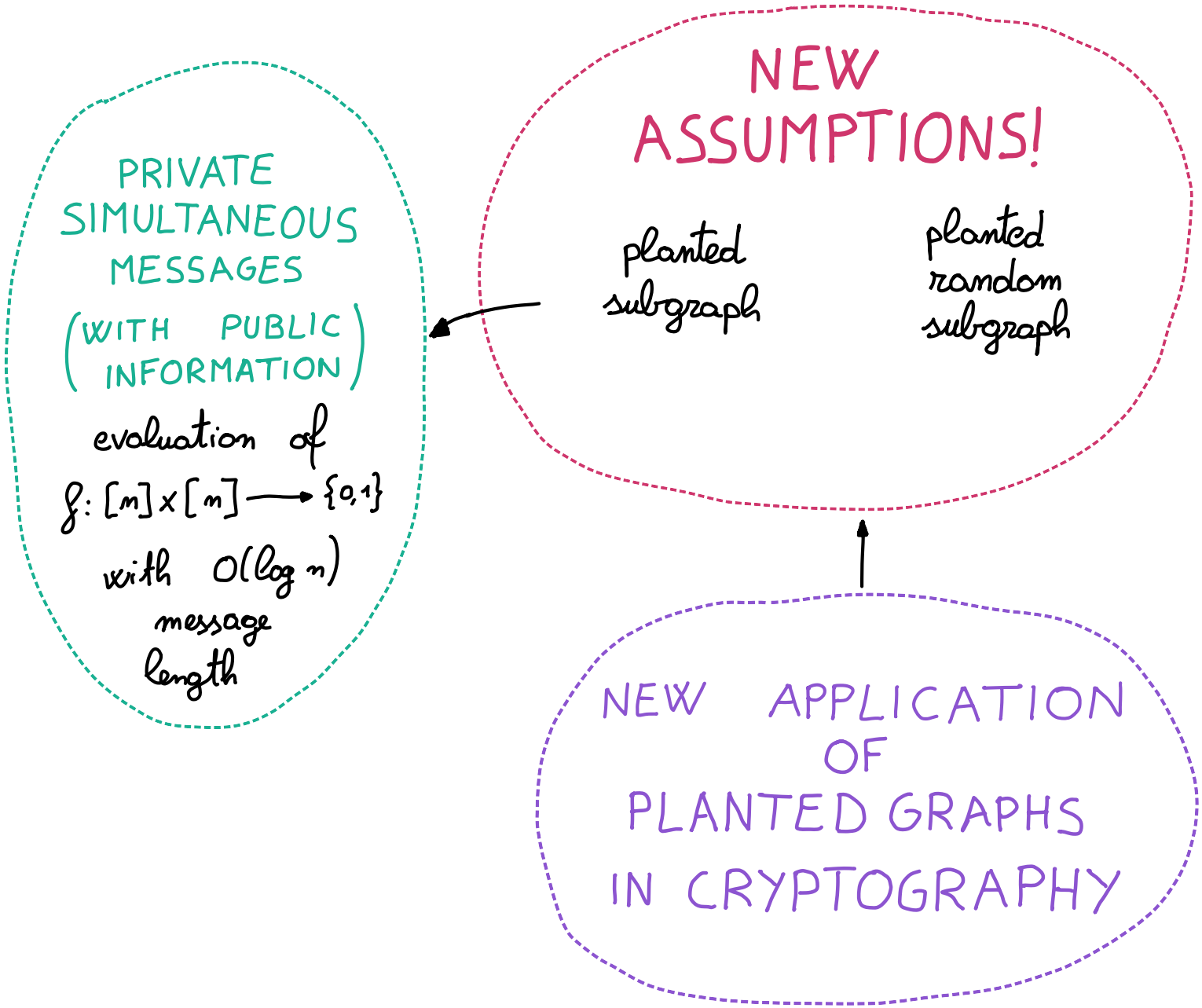

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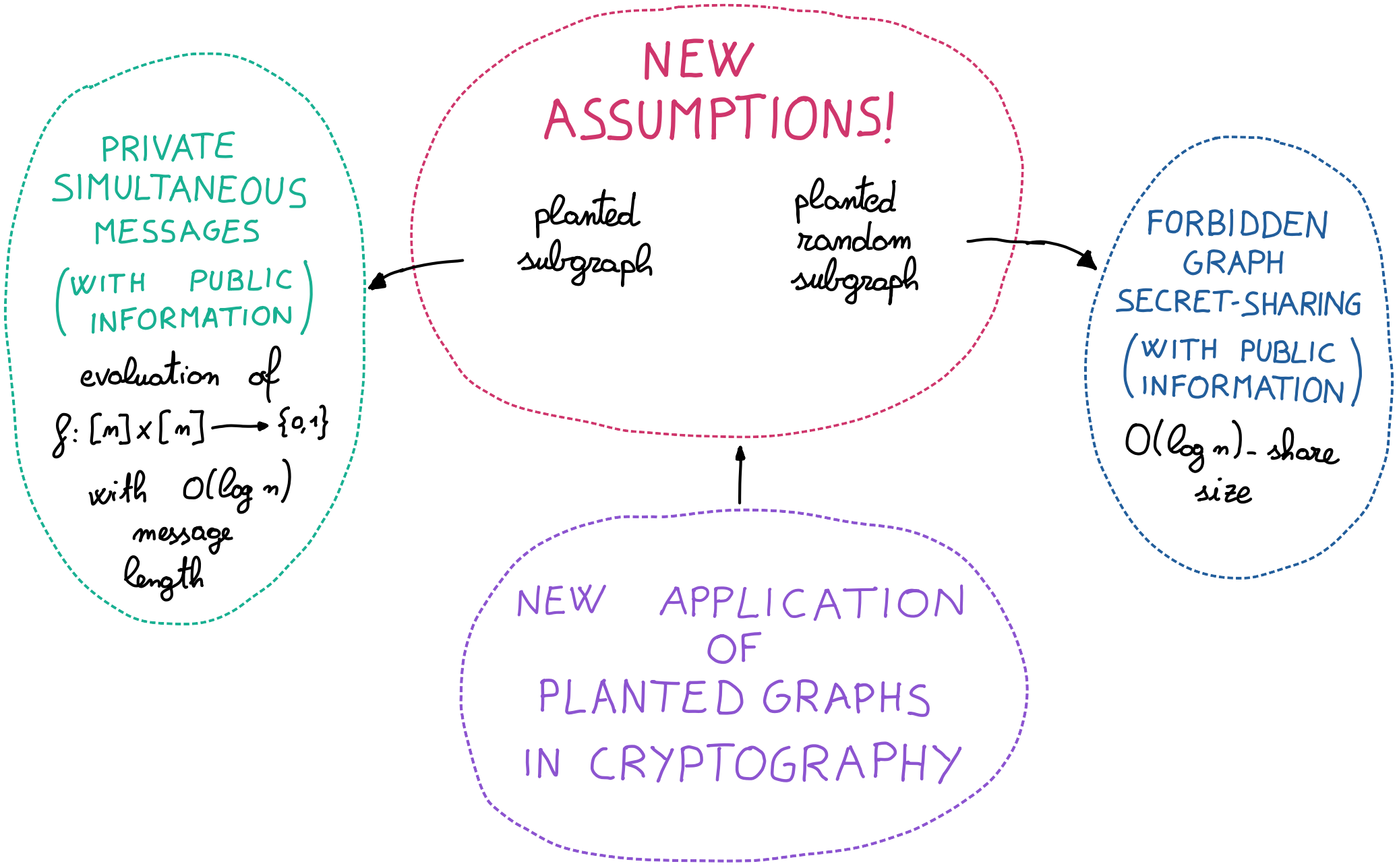


CONTRIBUTIONS

NEW ASSUMPTIONS!

planted subgraph

planted random subgraph

PRIVATE SIMULTANEOUS MESSAGES (WITH PUBLIC INFORMATION)

evaluation of $f: [n] \times [n] \longrightarrow \{0,1\}$ with $O(\log n)$ message length

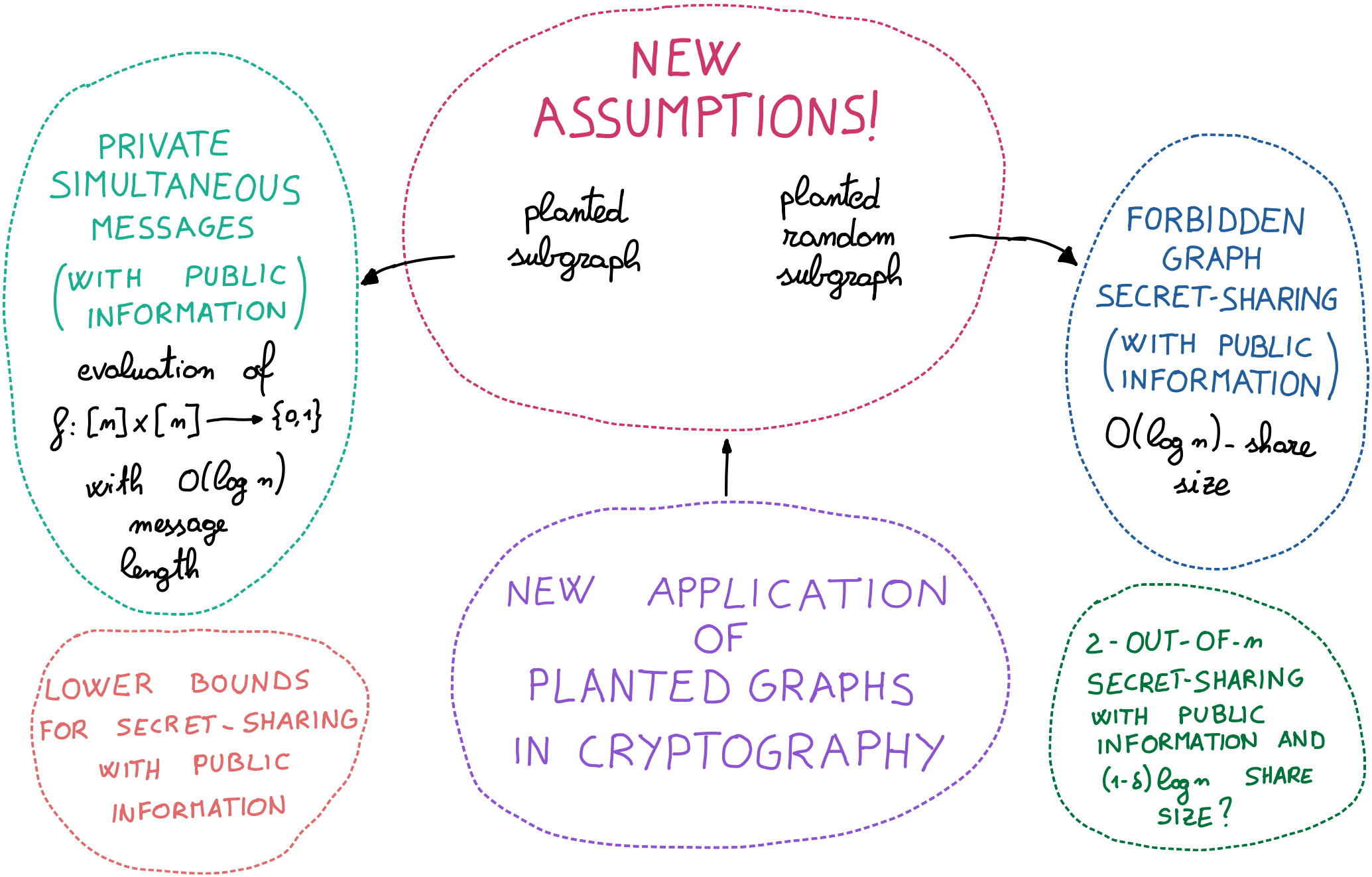
FORBIDDEN GRAPH SECRET-SHARING (WITH PUBLIC INFORMATION)

$O(\log n)$ - share size

NEW APPLICATION OF PLANTED GRAPHS IN CRYPTOGRAPHY

LOWER BOUNDS FOR SECRET-SHARING WITH PUBLIC INFORMATION

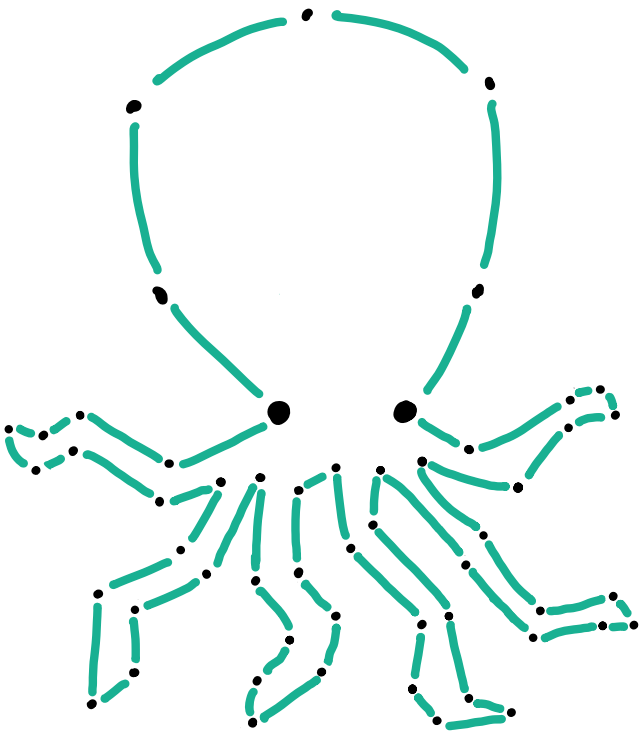
CONTRIBUTIONS



PLANTED GRAPH PROBLEMS

D_H

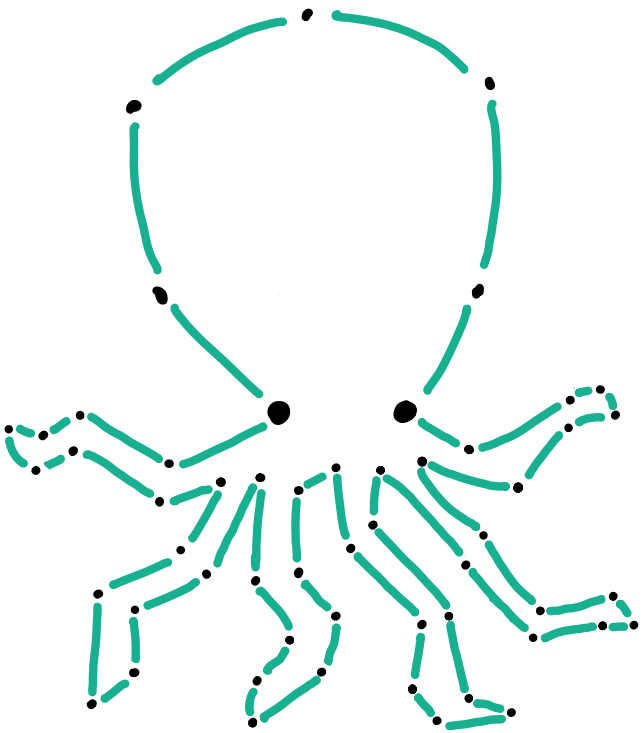
n nodes

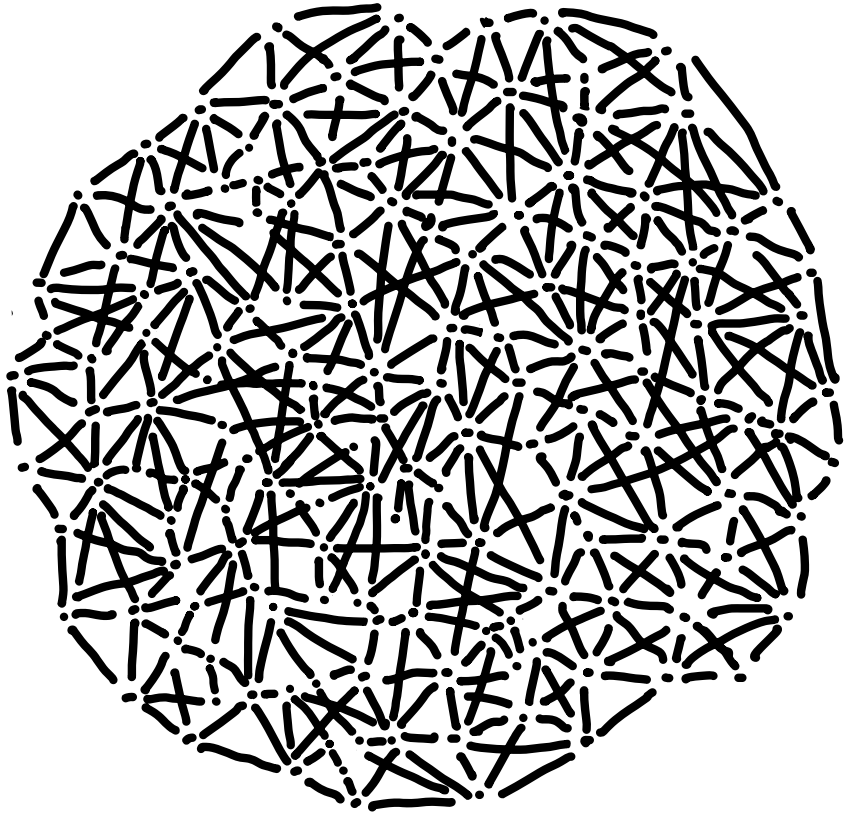


PLANTED GRAPH PROBLEMS

D_H D_A

n nodes $<$ N nodes

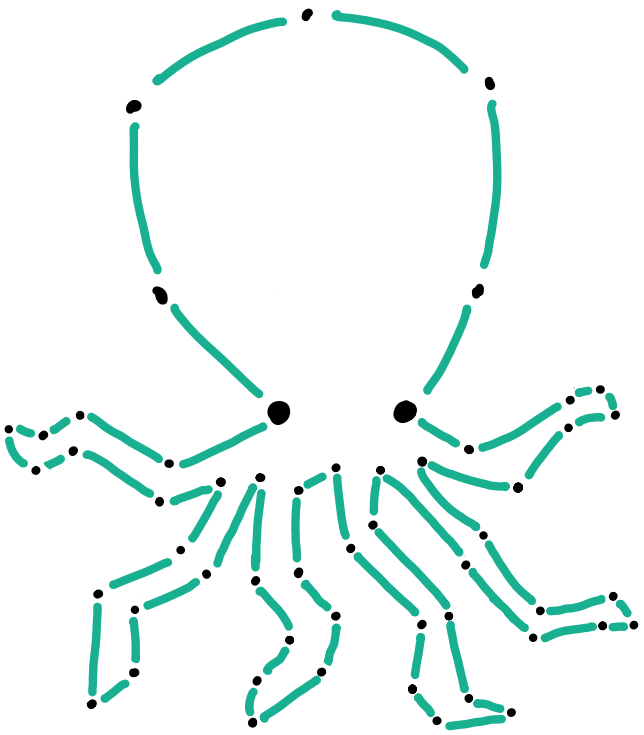


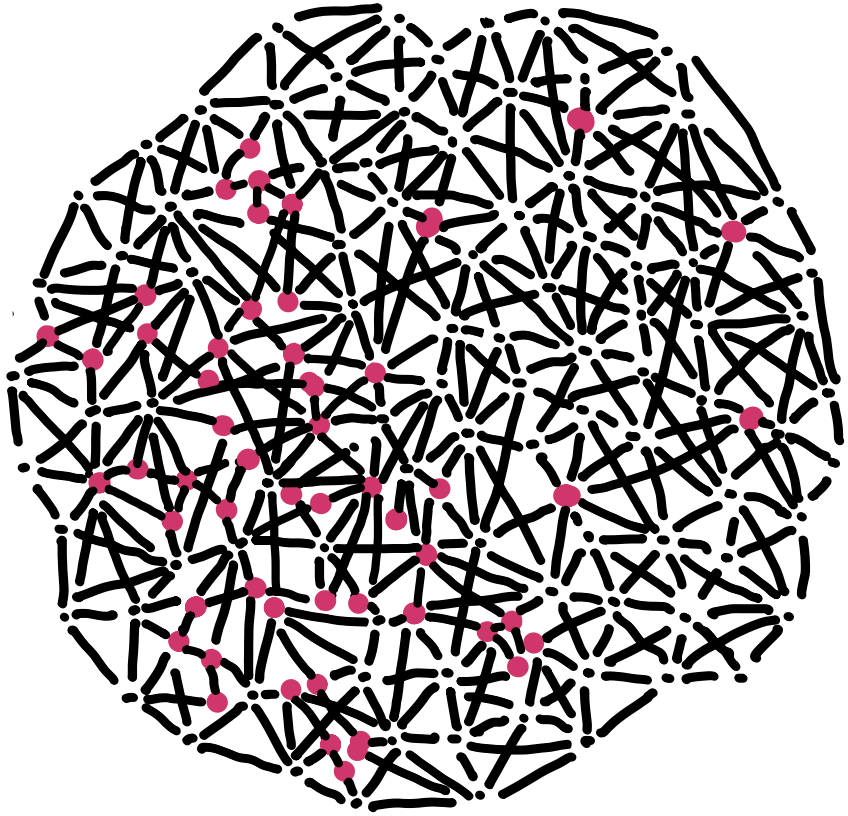


PLANTING OPERATION

D_H $<$ D_A

n nodes $<$ N nodes

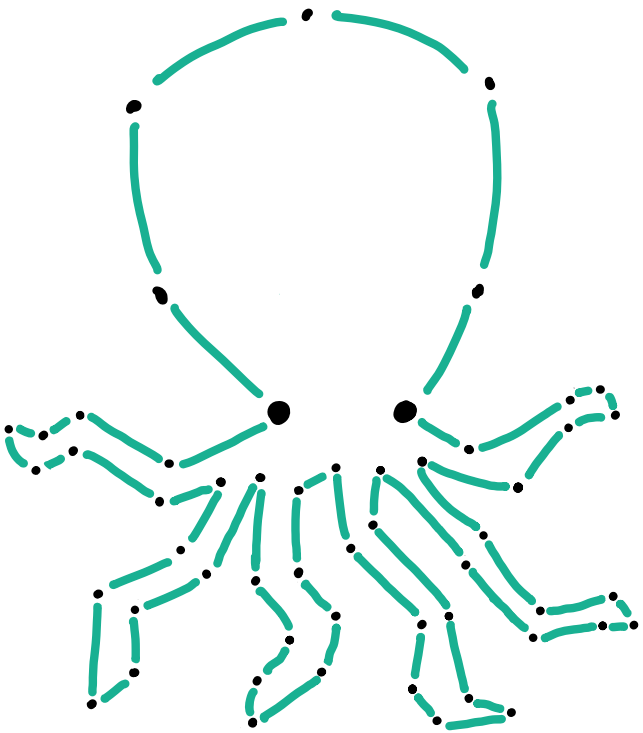


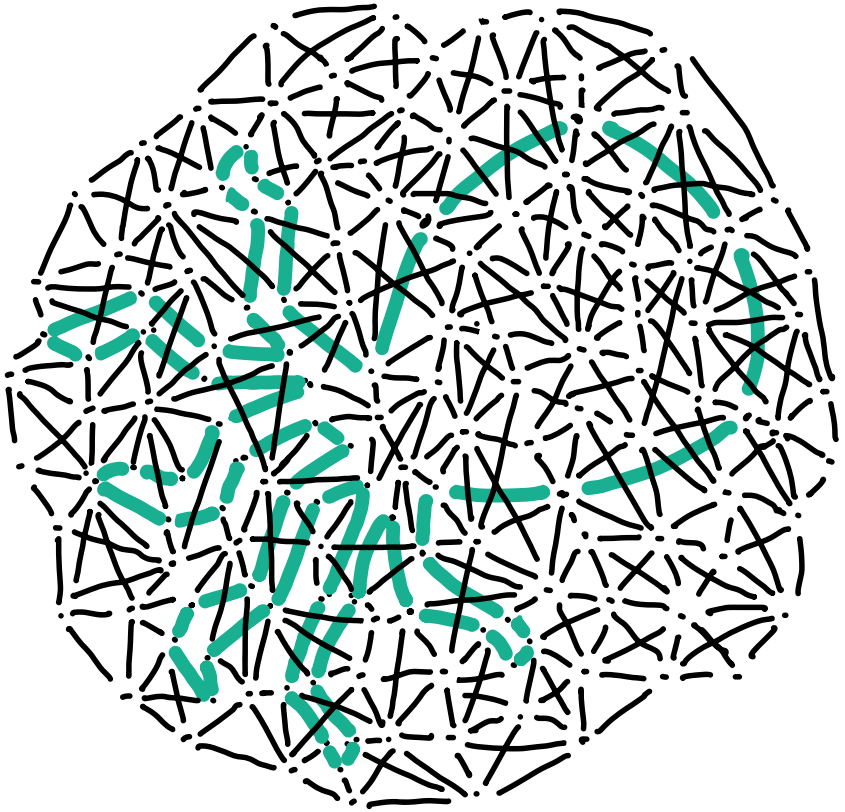


PLANTING OPERATION

D_H $<$ D_A

n nodes $<$ N nodes





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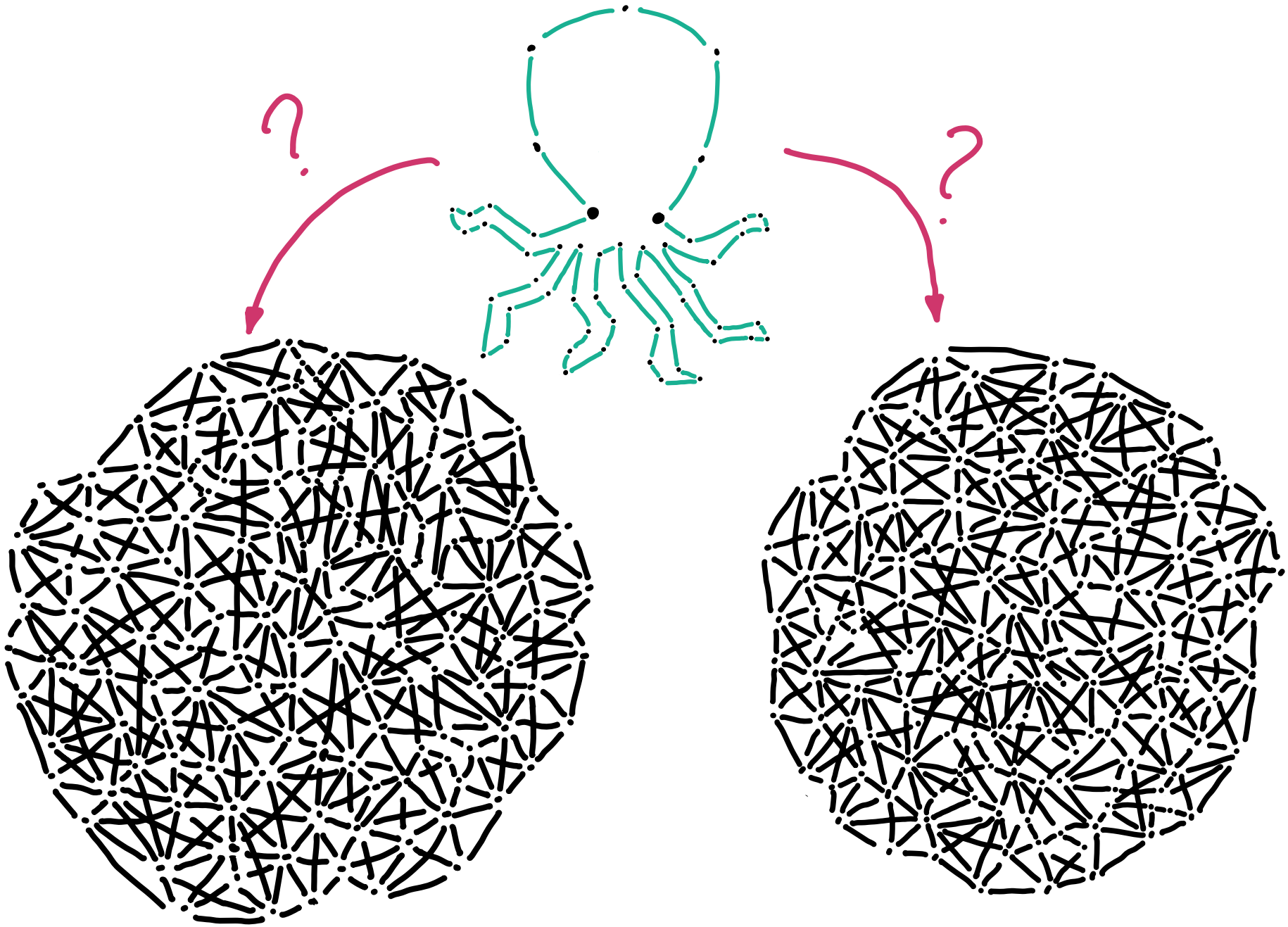

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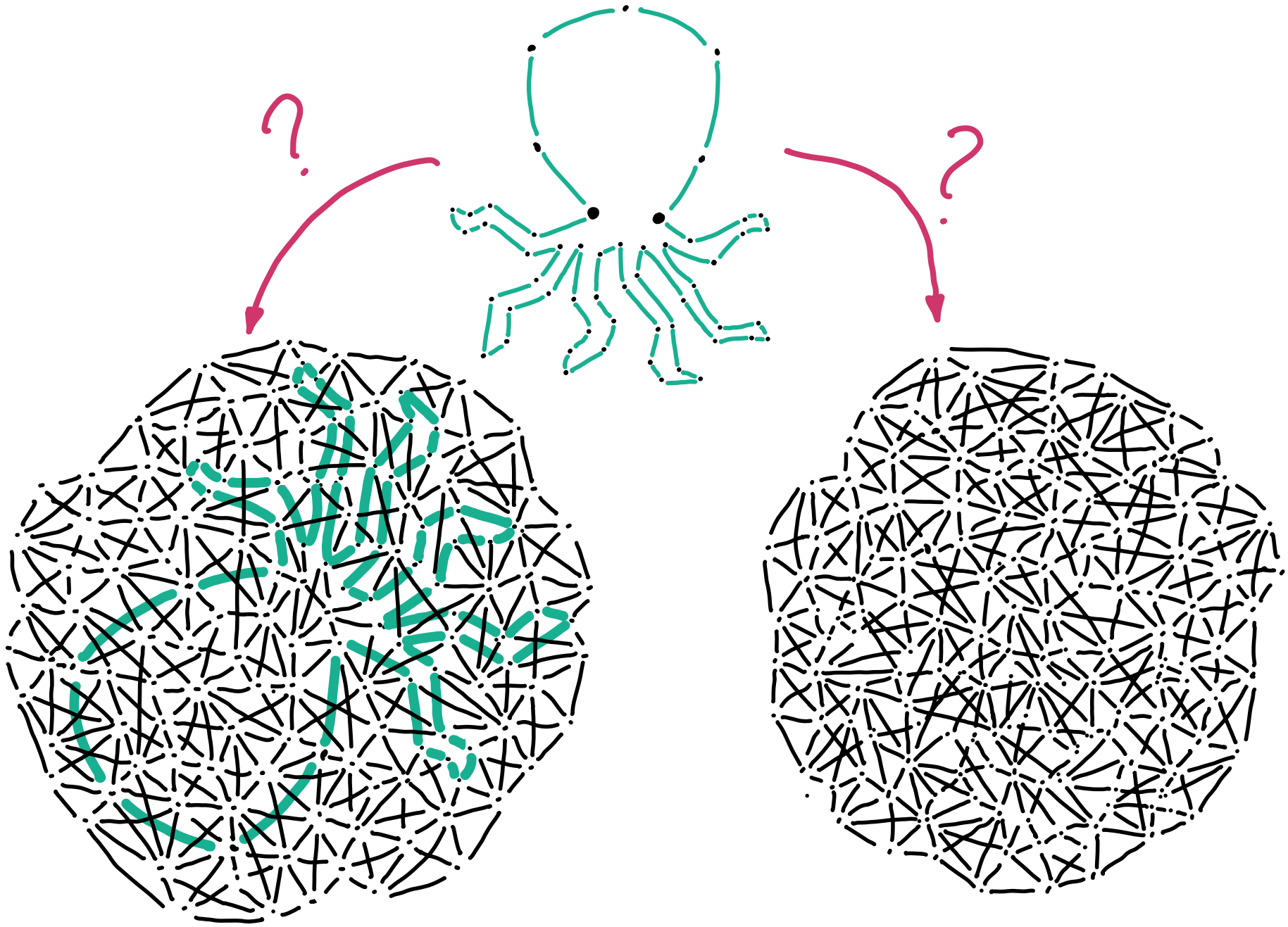
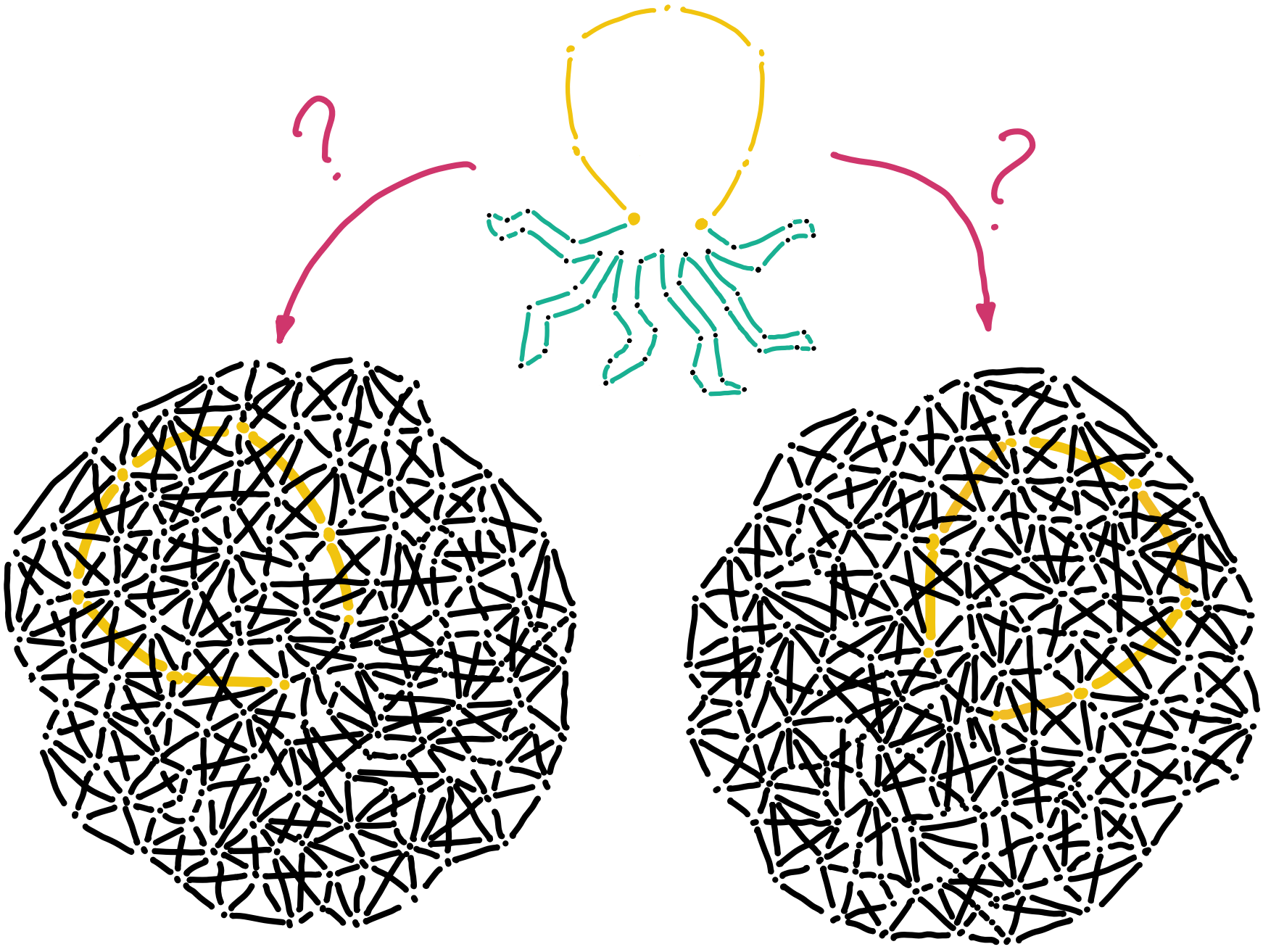
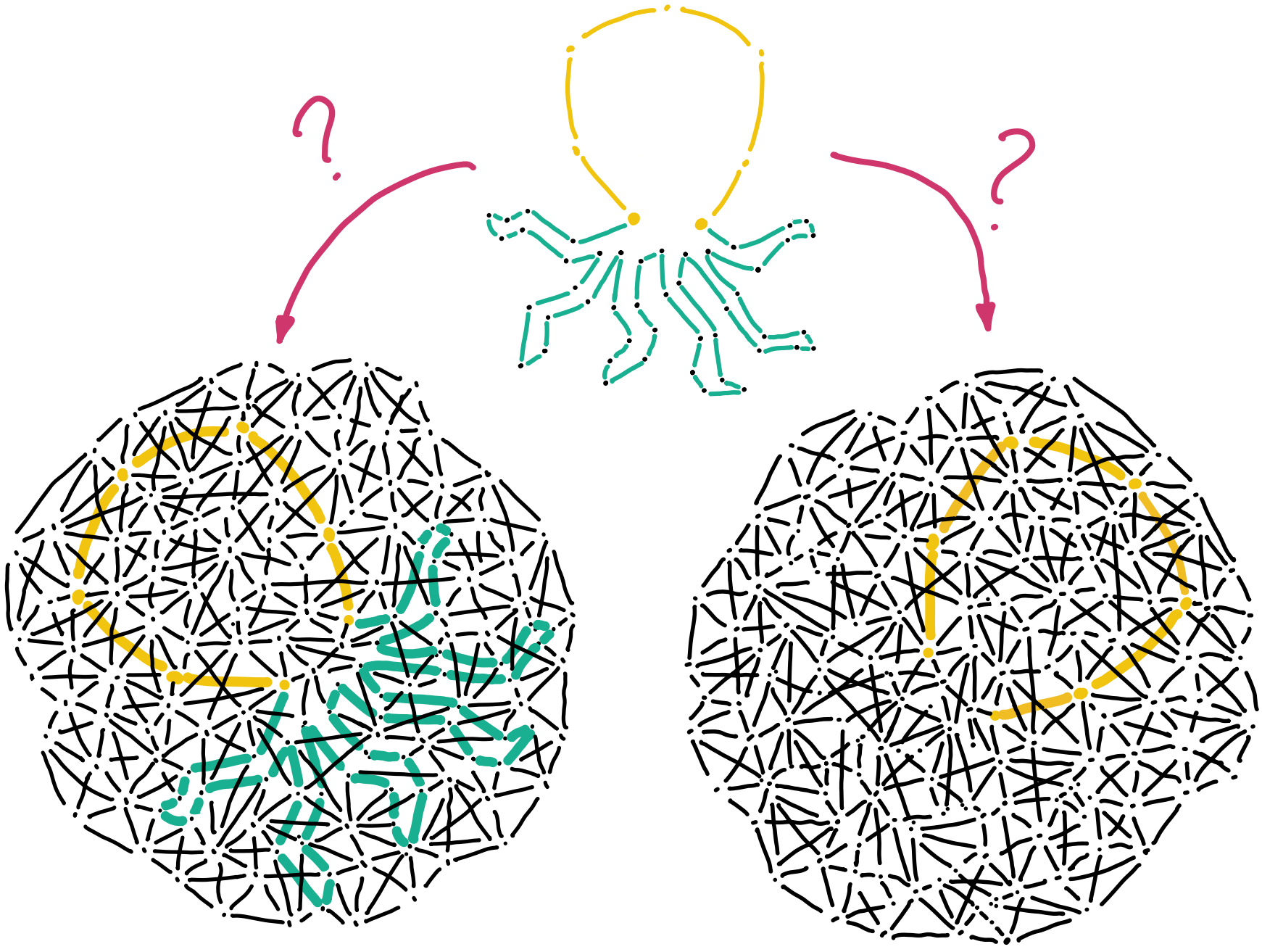


PLANTED GRAPHS WITH HINTS



PLANTED GRAPHS WITH HINTS



PLANTED GRAPH PROBLEMS

PLANTED
CLIQUE

PLANTED GRAPH PROBLEMS

PLANTED CLIQUE

← WELL STUDIED ASSUMPTION

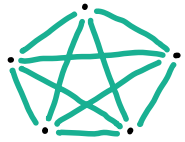
[JERRUM 92
KUČERA 95
BHK+16]

PLANTED GRAPH PROBLEMS

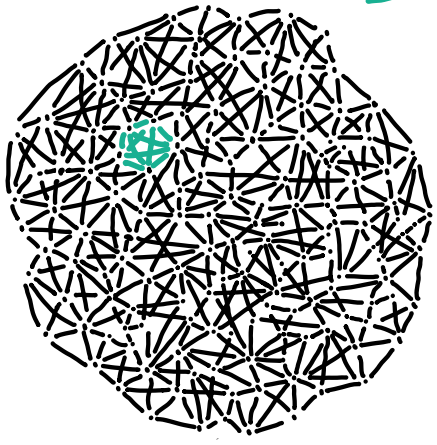
PLANTED CLIQUE

← WELL STUDIED ASSUMPTION

[JERRUM 92
KUČERA 95
BHK+16]

D_H 

RANDOM AMBIENT GRAPHS



PLANTED GRAPH PROBLEMS

PLANTED CLIQUE

← WELL STUDIED ASSUMPTION [JERRUM 92, KUČERA 95, BHK+16]

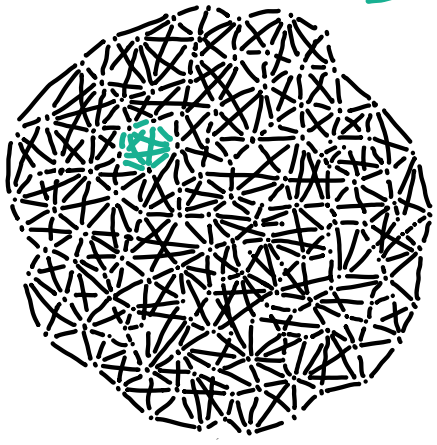
PLANTED SUBGRAPH

THIS WORK

PLANTED RANDOM SUBGRAPH

D_H

RANDOM AMBIENT GRAPHS



PLANTED GRAPH PROBLEMS

PLANTED CLIQUE

← WELL STUDIED ASSUMPTION

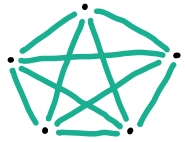
[JERRUM 92
KUČERA 95
BHK+16]

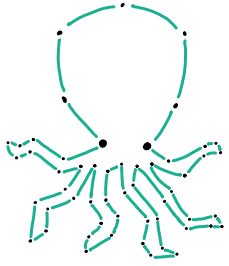
PLANTED SUBGRAPH

THIS WORK

PLANTED RANDOM SUBGRAPH

DETERMINISTIC

D_H 

D_H 

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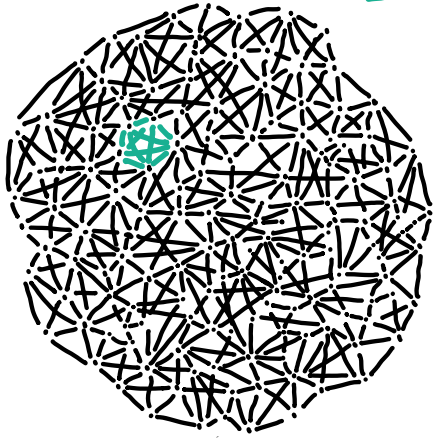
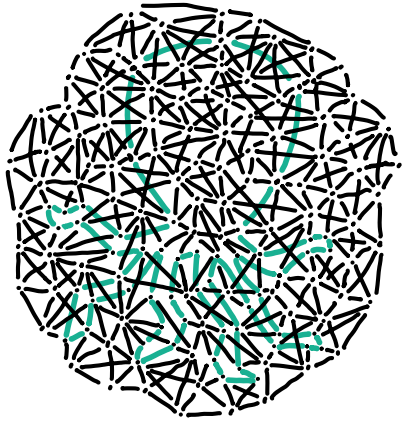




PLANTED GRAPH PROBLEMS

PLANTED CLIQUE

← WELL STUDIED ASSUMPTION

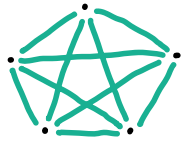
[JERRUM 92
KUČERA 95
BHK+16]

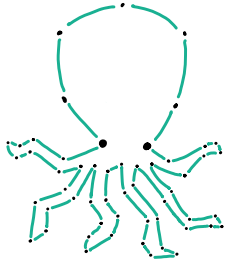
THIS WORK

PLANTED SUBGRAPH

PLANTED RANDOM SUBGRAPH

DETERMINISTIC

D_H 

D_H 

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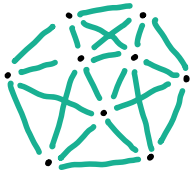
D_H 

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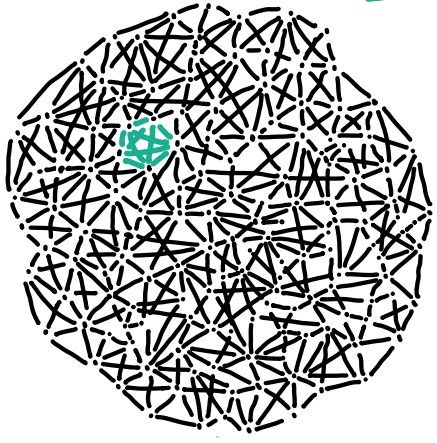
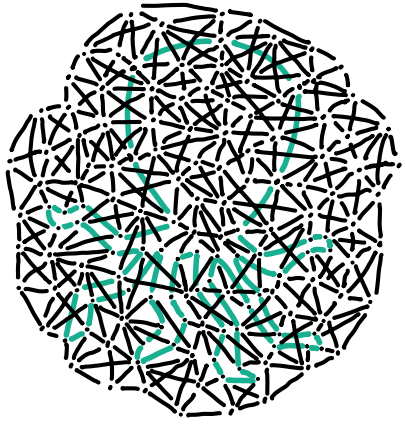
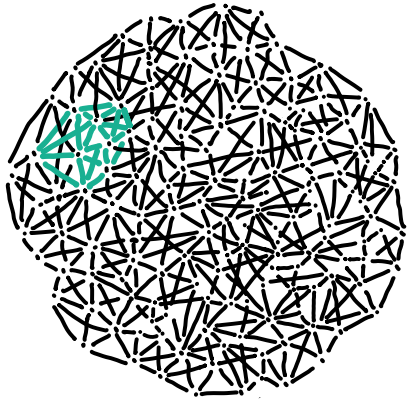






PLANTED GRAPH PROBLEMS

PLANTED CLIQUE

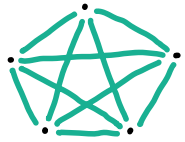
← WELL STUDIED ASSUMPTION [JERRUM 92, KUČERA 95, BHK+16]

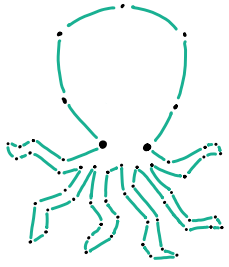
PLANTED SUBGRAPH

THIS WORK

PLANTED RANDOM SUBGRAPH

DETERMINISTIC

D_H 

D_H 

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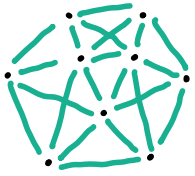
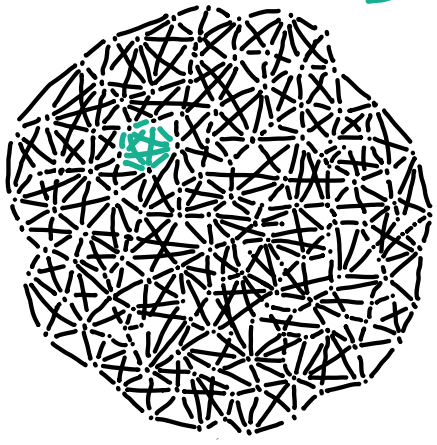
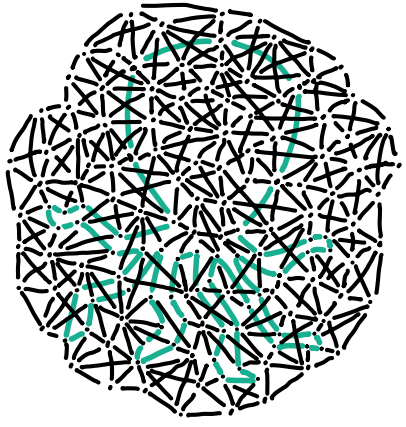
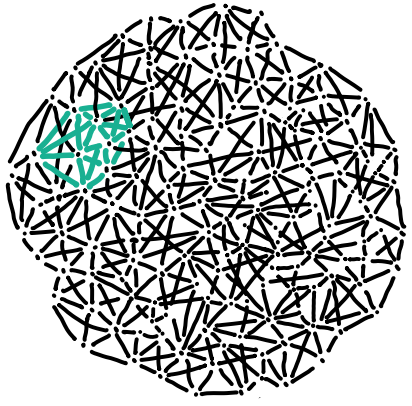
D_H 

RANDOM AMBIENT GRAPHS







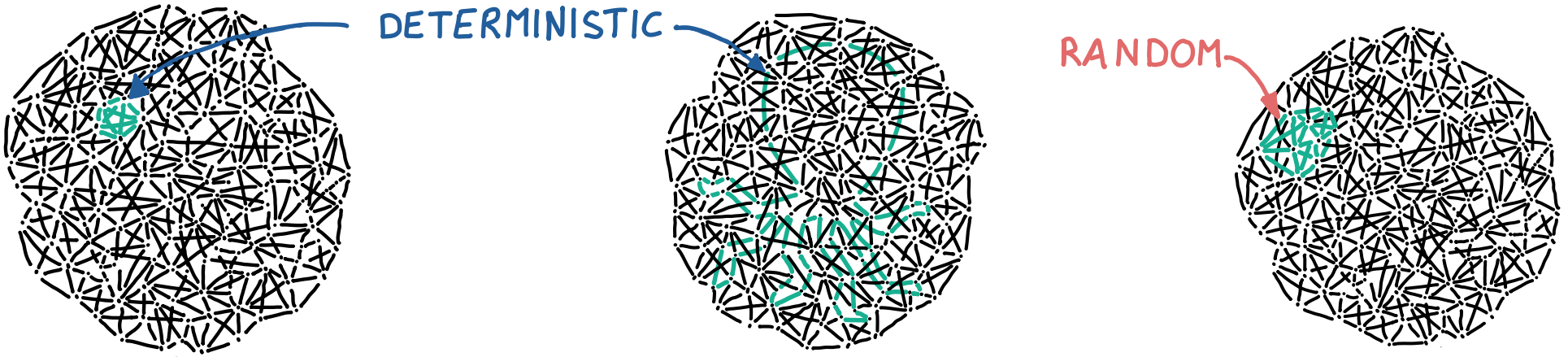
CONJECTURED HARDNESS (LOW-DEGREE ANALYSIS)

PLANTED GRAPH PROBLEMS PARAMETERS

PLANTED CLIQUE

PLANTED SUBGRAPH

PLANTED RANDOM SUBGRAPH

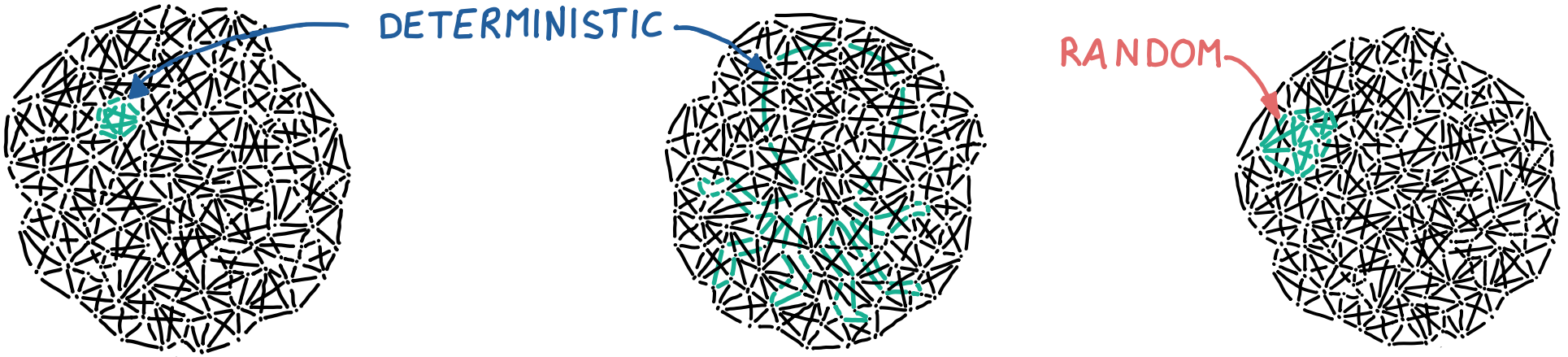


PLANTED GRAPH PROBLEMS PARAMETERS

PLANTED CLIQUE

PLANTED SUBGRAPH

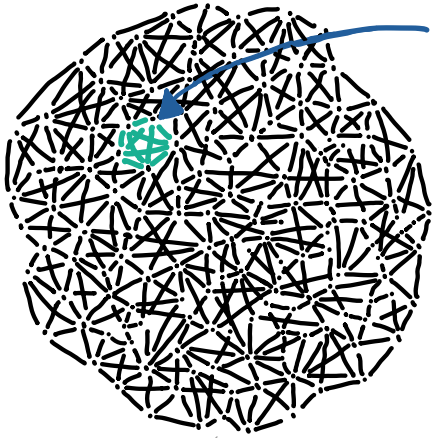
PLANTED RANDOM SUBGRAPH



$N = n^{2+\delta}$

PLANTED GRAPH PROBLEMS PARAMETERS

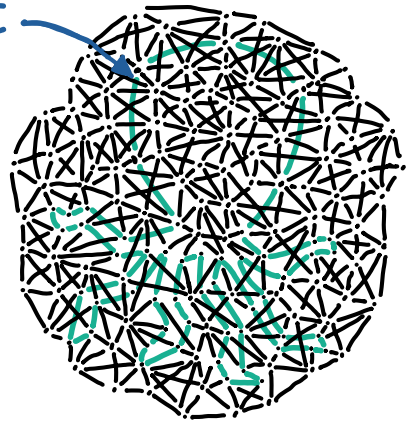
PLANTED CLIQUE



$N = n^{2+\delta}$

PLANTED SUBGRAPH

DETERMINISTIC

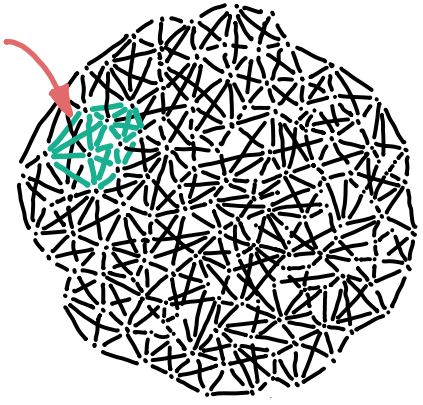


$N = n^{1+\delta}$

(for 1-negl(n) fraction of hidden graphs!)

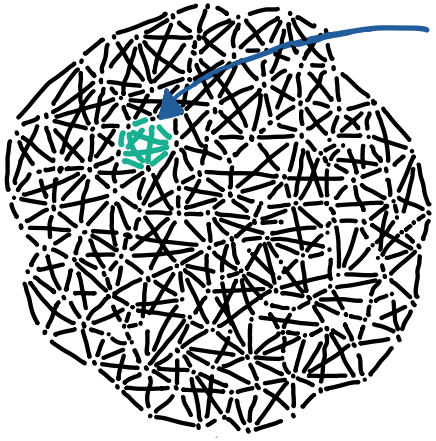
PLANTED RANDOM SUBGRAPH

RANDOM



PLANTED GRAPH PROBLEMS PARAMETERS

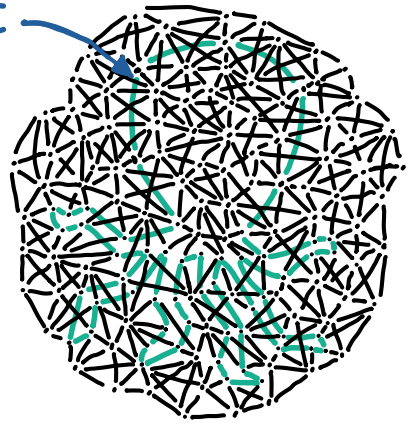
PLANTED CLIQUE



$N = n^{2+\delta}$

PLANTED SUBGRAPH

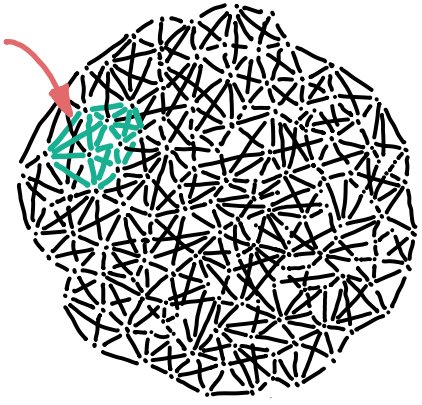
DETERMINISTIC



$N = n^{1+\delta}$

PLANTED RANDOM SUBGRAPH

RANDOM

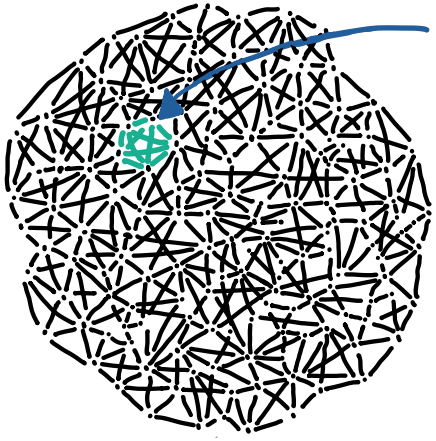


$N = n^{1+\delta}$

(for 1-negl(n) fraction of hidden graphs!)

PLANTED GRAPH PROBLEMS PARAMETERS

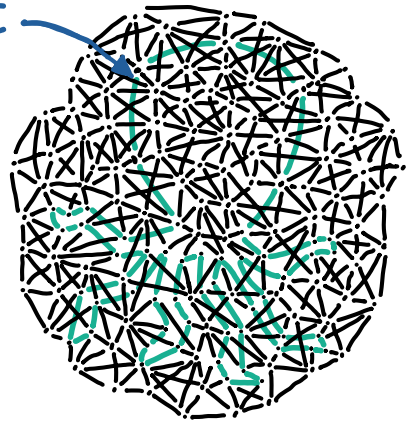
PLANTED CLIQUE



$N = n^{2+\delta}$

PLANTED SUBGRAPH

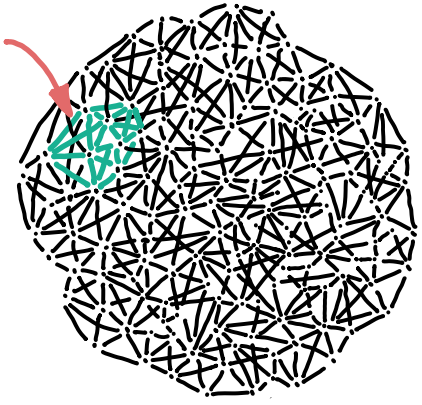
DETERMINISTIC



$N = n^{1+\delta}$

PLANTED RANDOM SUBGRAPH

RANDOM



$N = n^{1+\delta}$

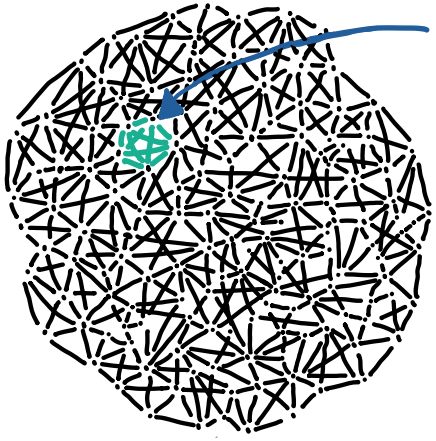
(for 1-negl(n) fraction of hidden graphs!)

CAVEAT

SECURITY AGAINST $n^{o(\log n)}$-TIME ADVERSARIES!

PLANTED GRAPH PROBLEMS PARAMETERS

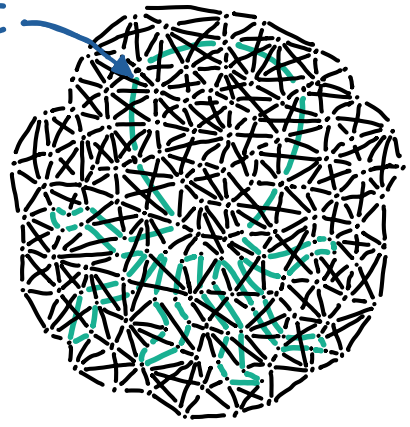
PLANTED CLIQUE



$N = n^{2+\delta}$

PLANTED SUBGRAPH

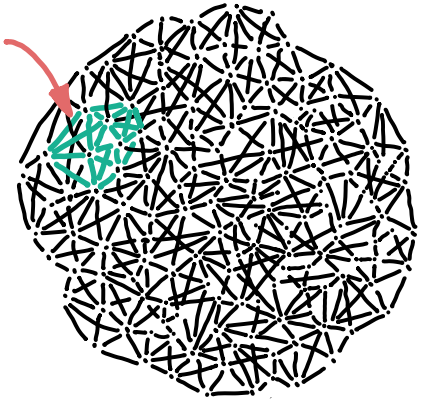
DETERMINISTIC



$N = n^{1+\delta}$

PLANTED RANDOM SUBGRAPH

RANDOM



$N = n^{1+\delta}$

(for 1-negl(n) fraction of hidden graphs!)

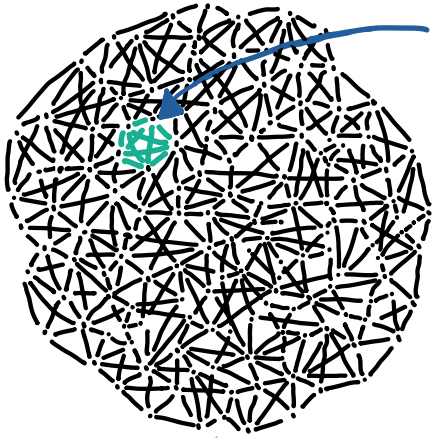
CAVEAT

SECURITY AGAINST $n^{o(\log n)}$-TIME ADVERSARIES!

INVERSE-POLYNOMIAL ADVANTAGE!

PLANTED GRAPH PROBLEMS PARAMETERS

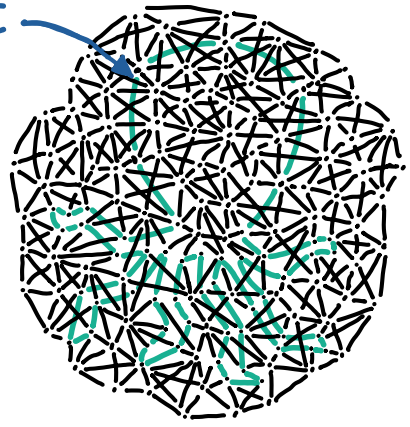
PLANTED CLIQUE



$N = n^{2+\delta}$

PLANTED SUBGRAPH

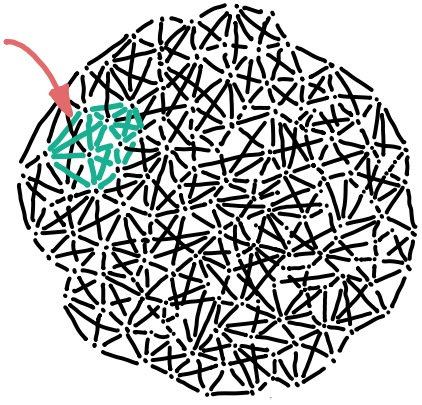
DETERMINISTIC



$N = n^{1+\delta}$

PLANTED RANDOM SUBGRAPH

RANDOM



$N = n^{1+\delta}$

(for $1 - \text{negl}(n)$ fraction of hidden graphs!)

CAVEAT

SECURITY AGAINST $n^{o(\log n)}$-TIME ADVERSARIES!

INVERSE-POLYNOMIAL ADVANTAGE!

EVEN WITH $t = O(1)$ HINTS!

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$$f : [n] \times [n] \to \{0,1\}$$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$$f : [n] \times [n] \to \{0,1\}$$

ALICE

BOB

CAROL

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$$f : [n] \times [n] \to \{0,1\}$$

ALICE

$x \in [n]$

BOB

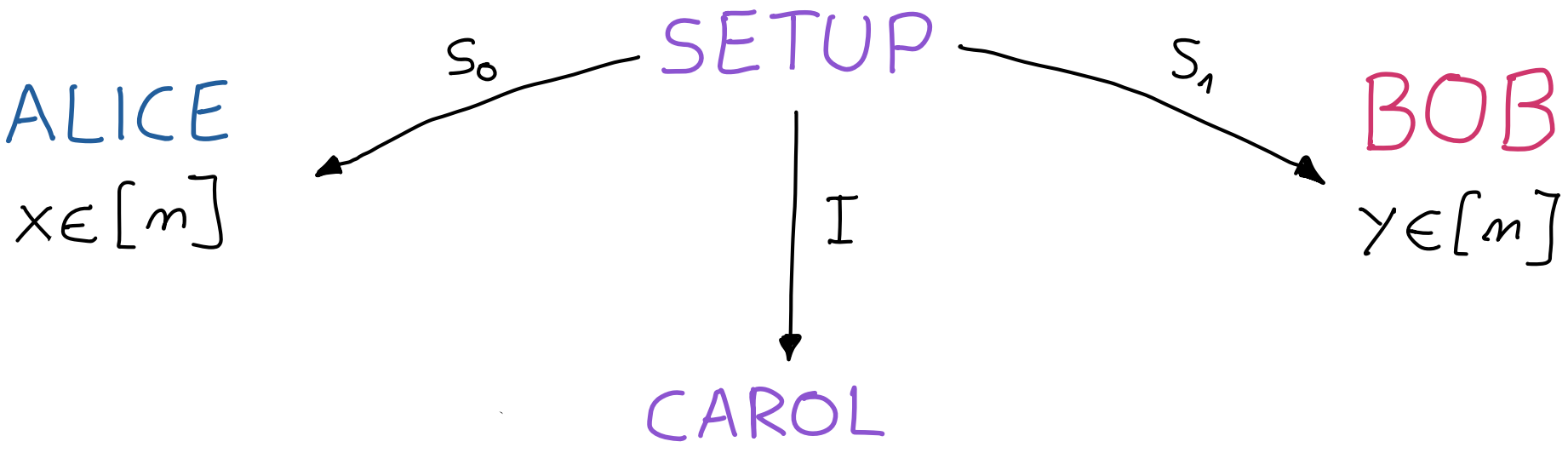
$y \in [n]$

CAROL

$f(x,y)$ ← NO ADDITIONAL LEAKAGE!

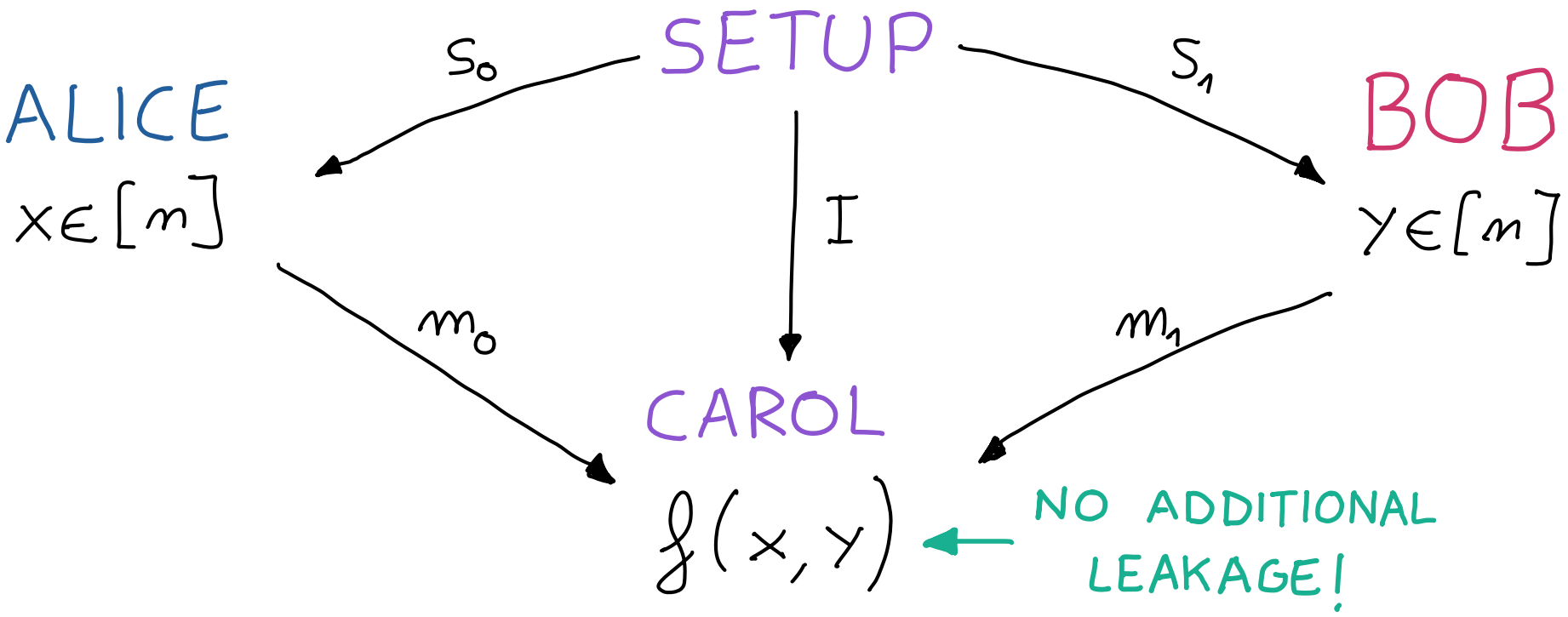
PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$$f : [n] \times [n] \to \{0,1\}$$



PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$$f : [n] \times [n] \to \{0,1\}$$



PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE

$x \in [n]$

BOB

$y \in [n]$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE

$x \in [n]$

0
1
2
⋮
$n-1$

BOB

$y \in [n]$

0
1
2
⋮
$n-1$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

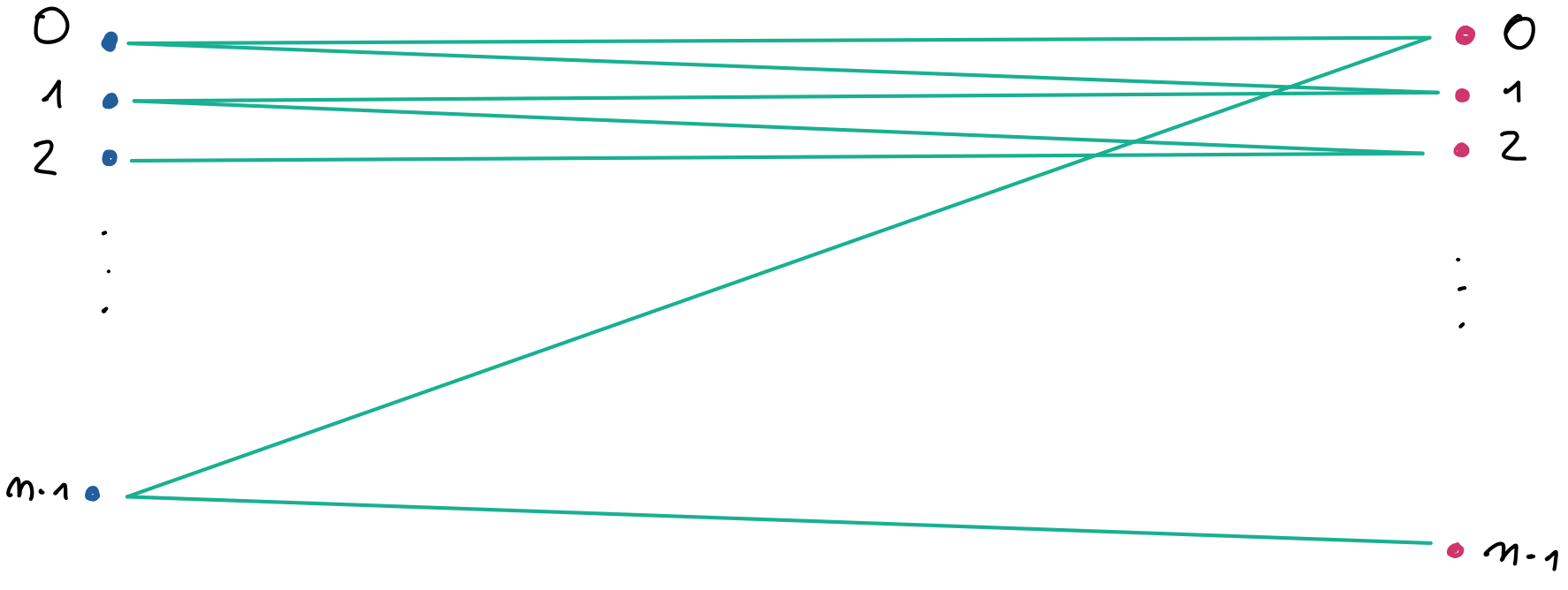
ALICE

$x \in [n]$

OCTOPUS GRAPH!

BOB

$y \in [n]$



PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

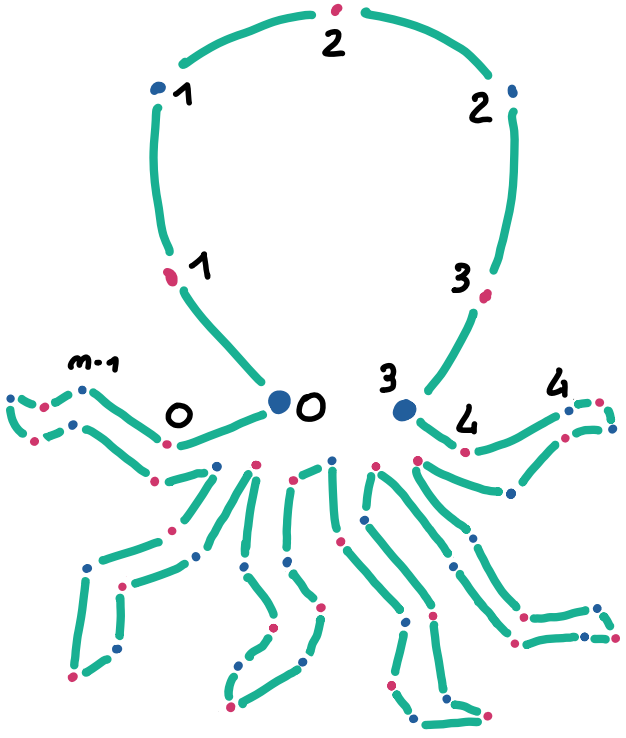
$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE

$x \in [n]$

BOB

$y \in [n]$



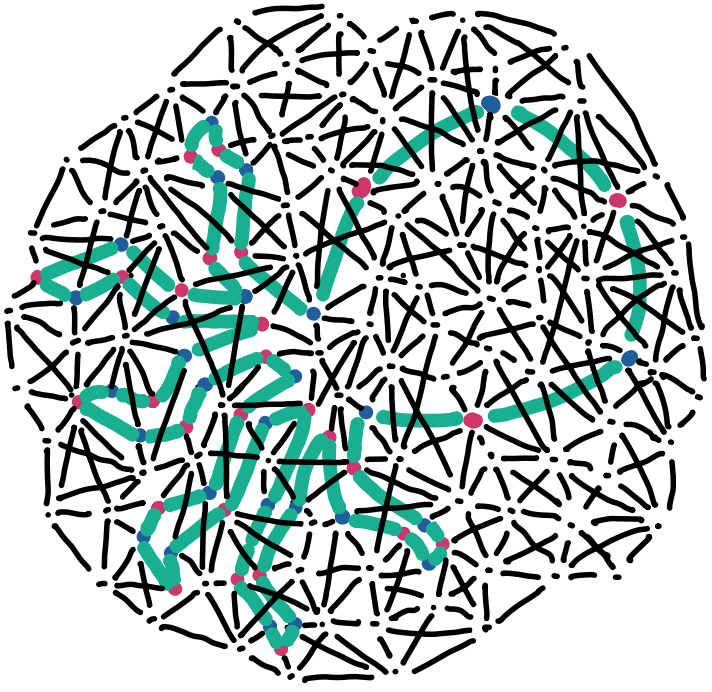
PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

SETUP

ALICE

$x \in [n]$



BOB

$y \in [n]$

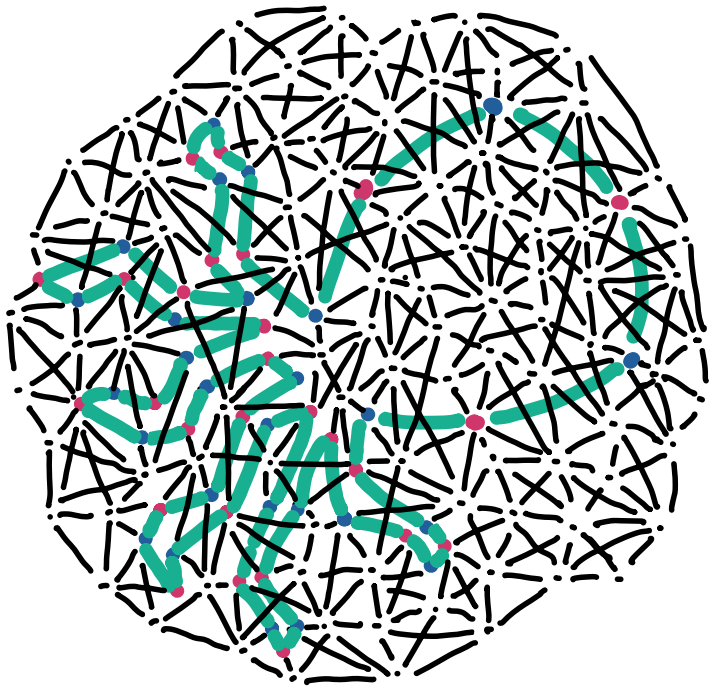
PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

SETUP

ALICE

$x \in [n]$



public information I

BOB

$y \in [n]$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

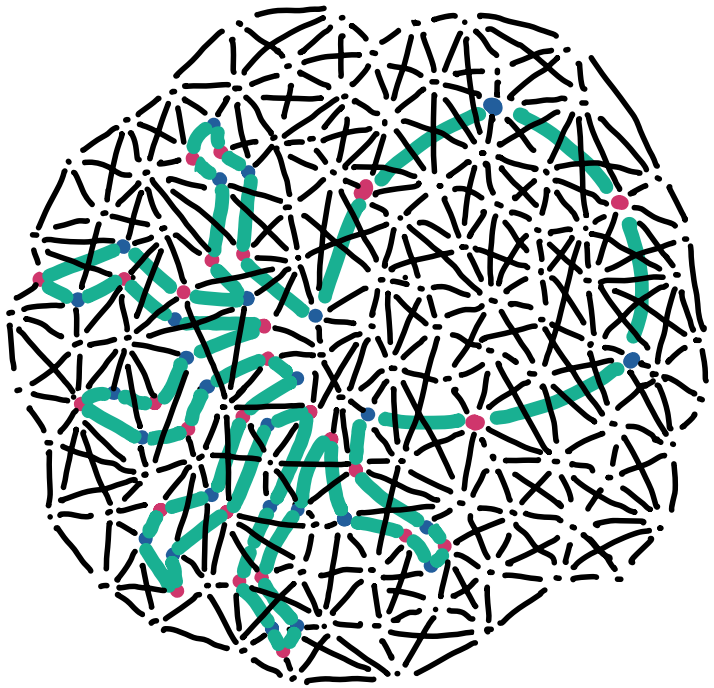
SETUP

ALICE

$x \in [n]$

s_0

position of blue nodes in I



public information I

s_1

position of red nodes in I

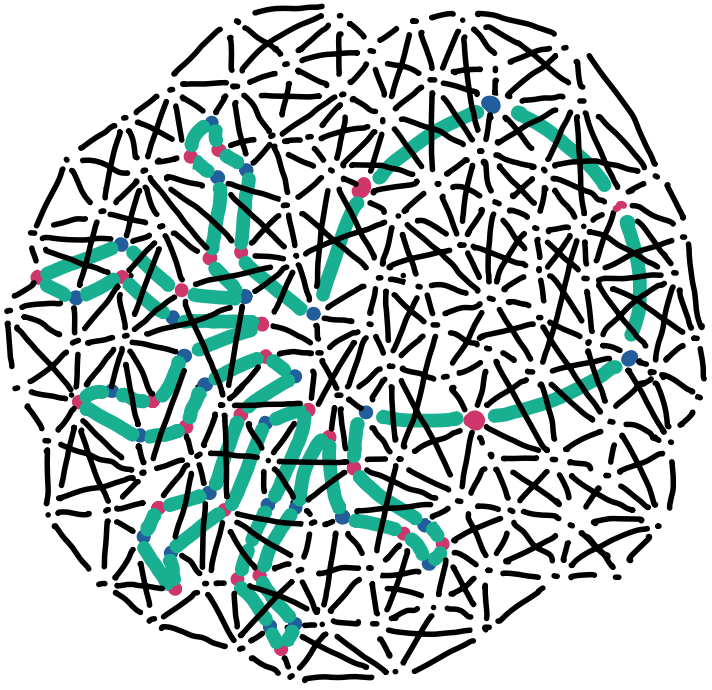
BOB

$y \in [n]$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE
$x = 3$



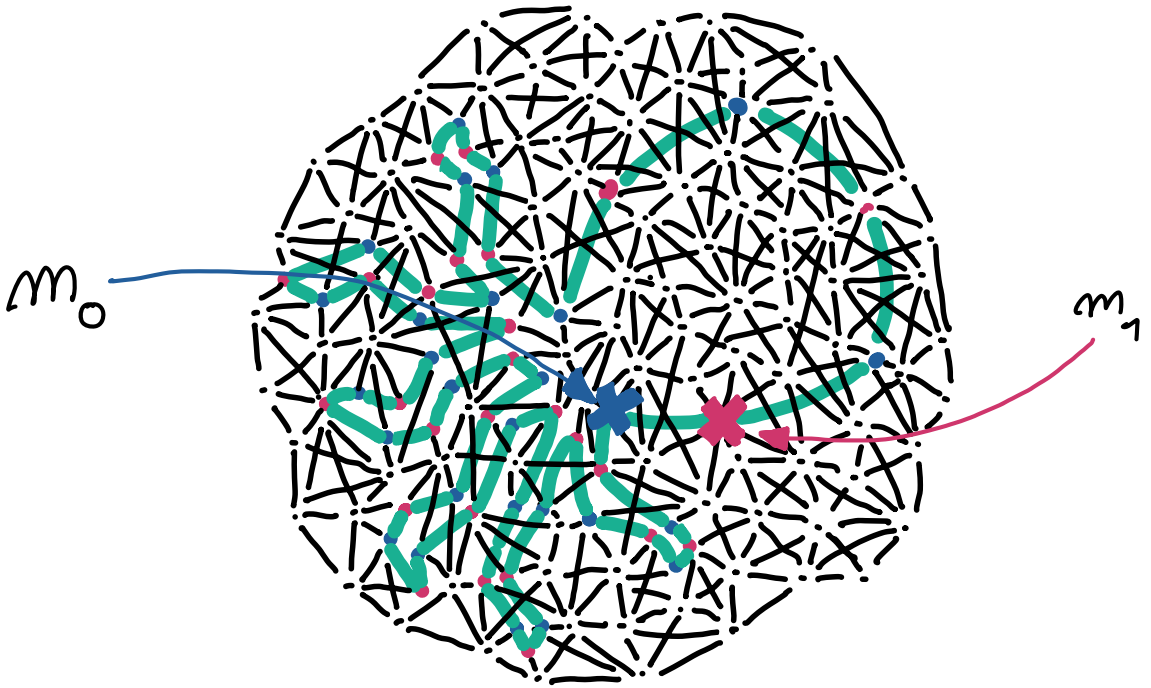
public information I

BOB
$y = 3$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x, y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE
$x = 3$



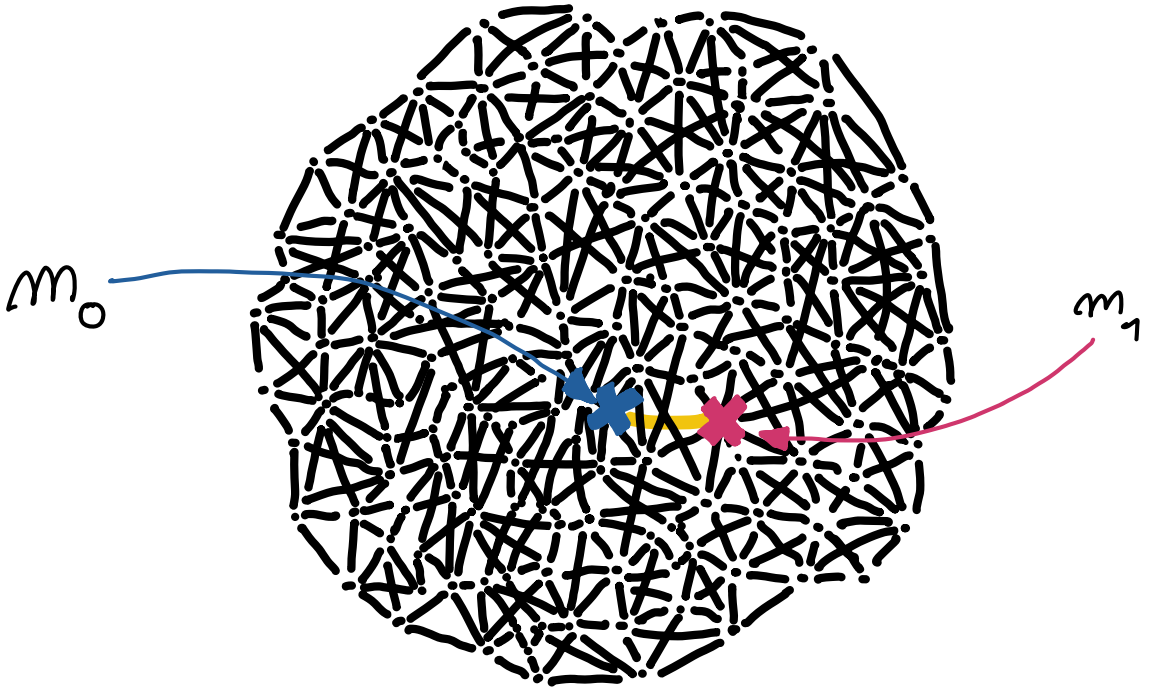
BOB
$y = 3$

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE

$x = 3$



BOB

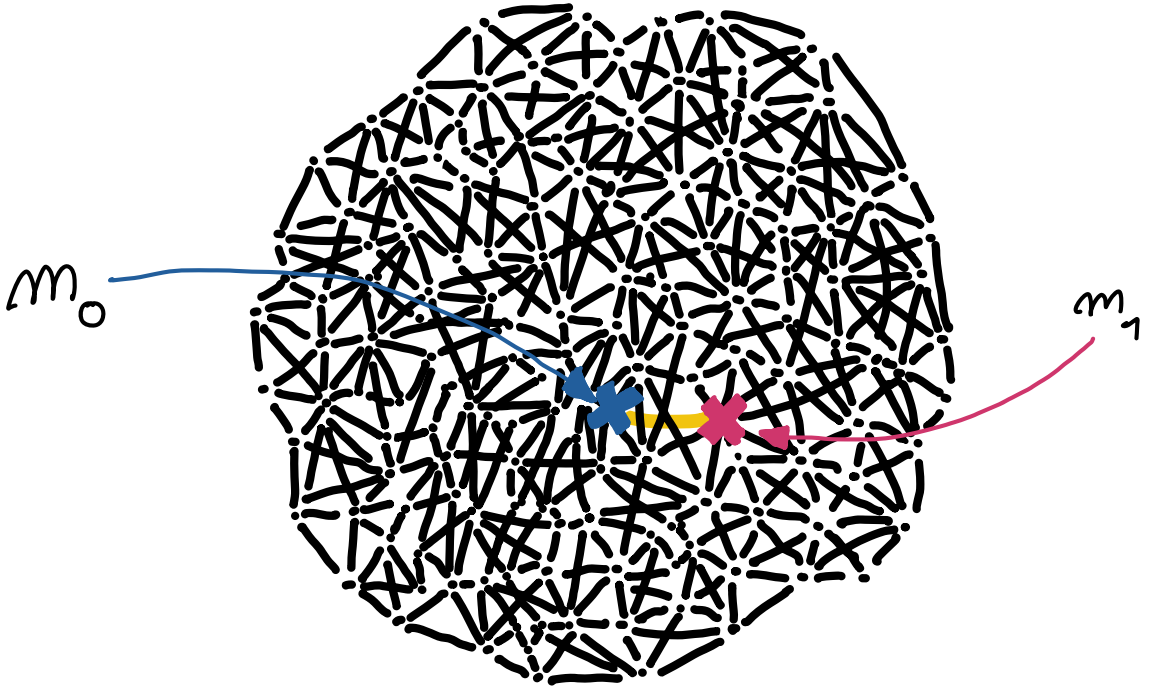
$y = 3$

CAROL

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x, y)$: is $y \in \{x, (x+1) \bmod n\}$?

ALICE
$x = 3$



BOB
$y = 3$

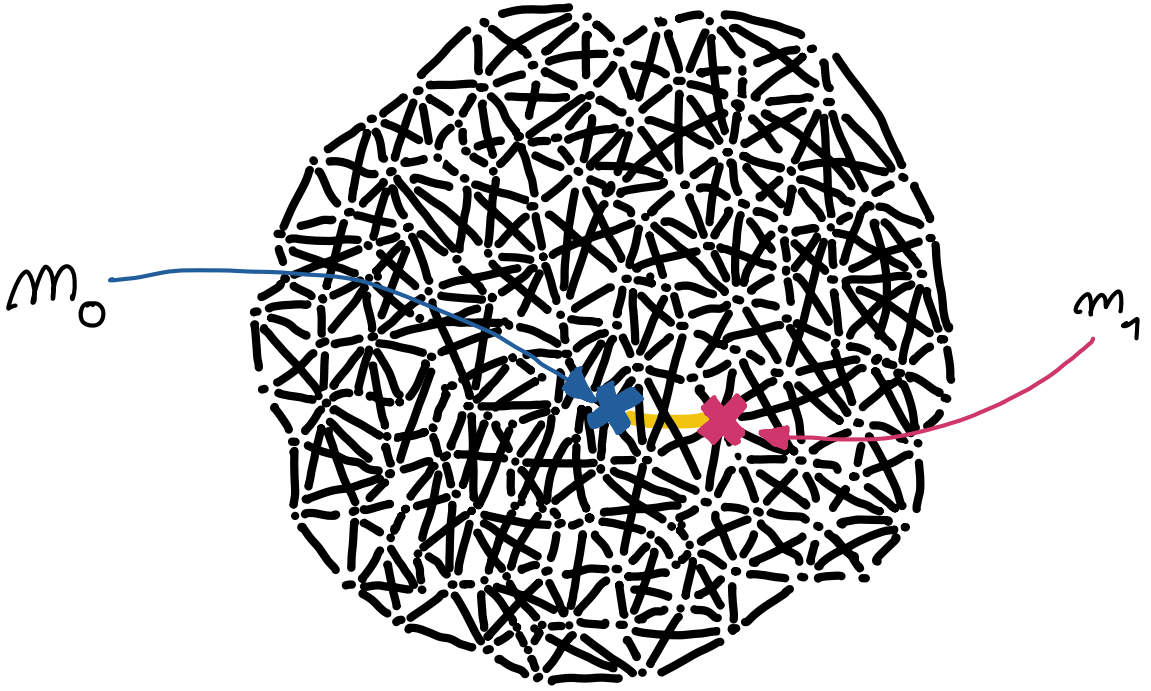
CAROL: what part of the octopus have I received?

PRIVATE SIMULTANEOUS MESSAGES [WITH PUBLIC INFORMATION]

$f(x,y)$: is $y \in \{x, (x+1) \bmod n\}$?

$$|m_0| = |m_1| = \log\binom{N}{2} = O(\log n)!$$

ALICE
$x = 3$



BOB
$y = 3$

CAROL: what part of the octopus have I received?

SUMMARY

NEW CRYPTOGRAPHIC ASSUMPTIONS BASED ON PLANTED GRAPHS:

planted subgraph (with hints) – PS(H)

planted random subgraph (with hints) – PRS(H)

OPEN QUESTION: are there better planting procedures? →

CAVEAT: SECURITY AGAINST $n^{o(\log n)}$-TIME ADVERSARIES! INVERSE-POLYNOMIAL ADVANTAGE!

POSSIBLE GAP! EQUIVALENT PLANTED GRAPH PROBLEM

APPLICATIONS

	INFORMATION THEORETIC		COMPUTATIONAL WITH PUBLIC INFORMATION	
PSM $f: [n]\times[n] \to \{0,1\}$	$\leq \sqrt{n}$	[BIKK14]	$\leq 1.01 \log n$	(under PSH)
	$\geq (1.5 - o(1)) \cdot \log n$	[AHMS18]	$\geq \log n$	
FORBIDDEN GRAPH SECRET-SHARING	$\leq 2^{\tilde{O}(\sqrt{\log n})}$	[LVW17]	$\leq 1.01 \log n$	(under PRSH)
	$\geq \log n$	[KN90]	$\Omega(\log\log n)$	
2-OUT-OF-n SECRET-SHARING	$\leq \log n$	[SHAMIR]	$\log n$	[SHAMIR]
	$\geq \log n$	[KN90]	$\Omega(\log\log n)$	
NON-IDEAL BINARY SECRET-SHARING	≥ 1		≥ 1	