Towards Topology-Hiding Computation from Oblivious Transfer

Marshall Ball, Alexander Bienstock, Lisa Kohl, Pierre Meyer

TCC 2023

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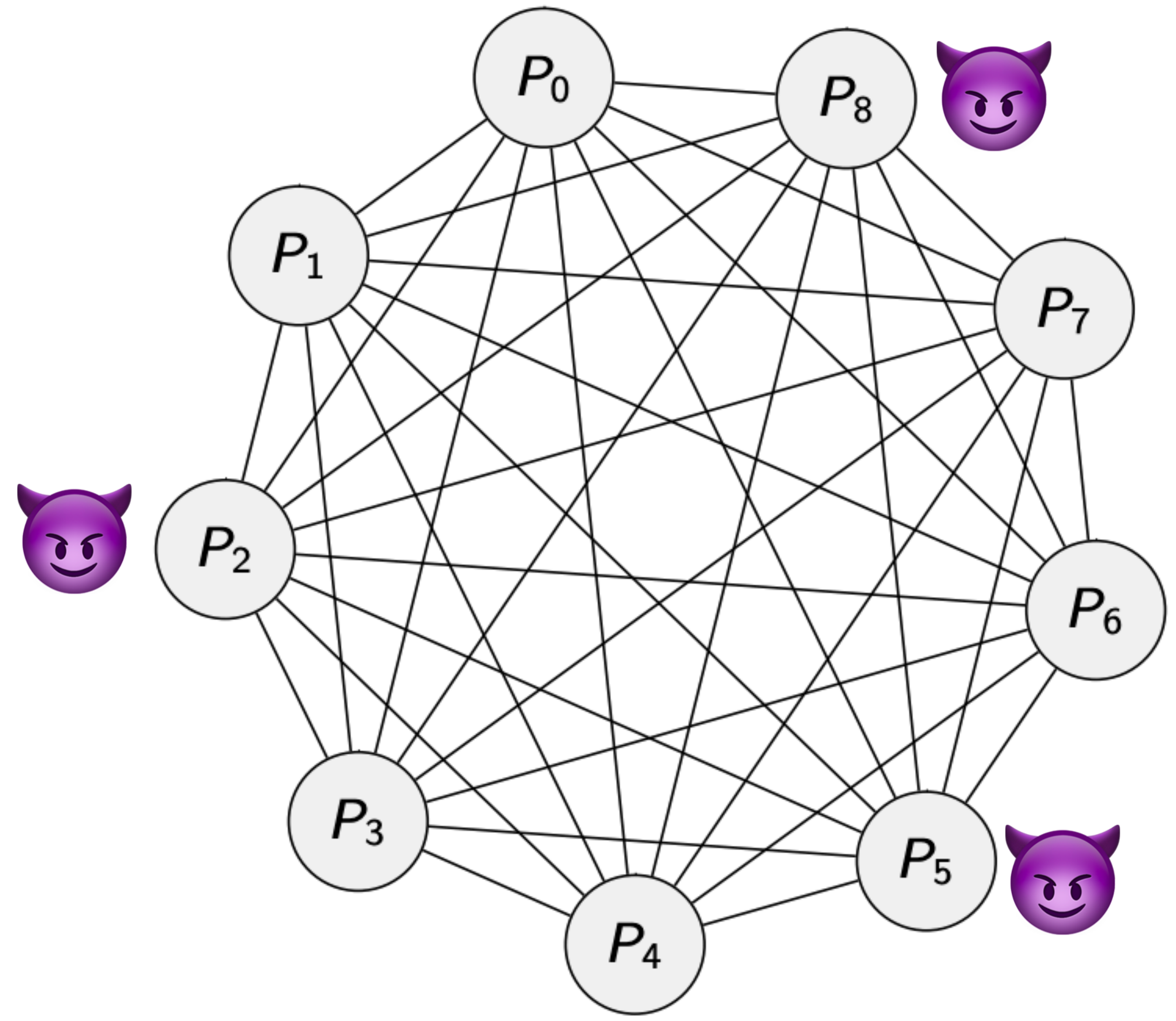
    - Adversary (corrupting possibly many parties) learns nothing beyond inputs of corrupted parties and output



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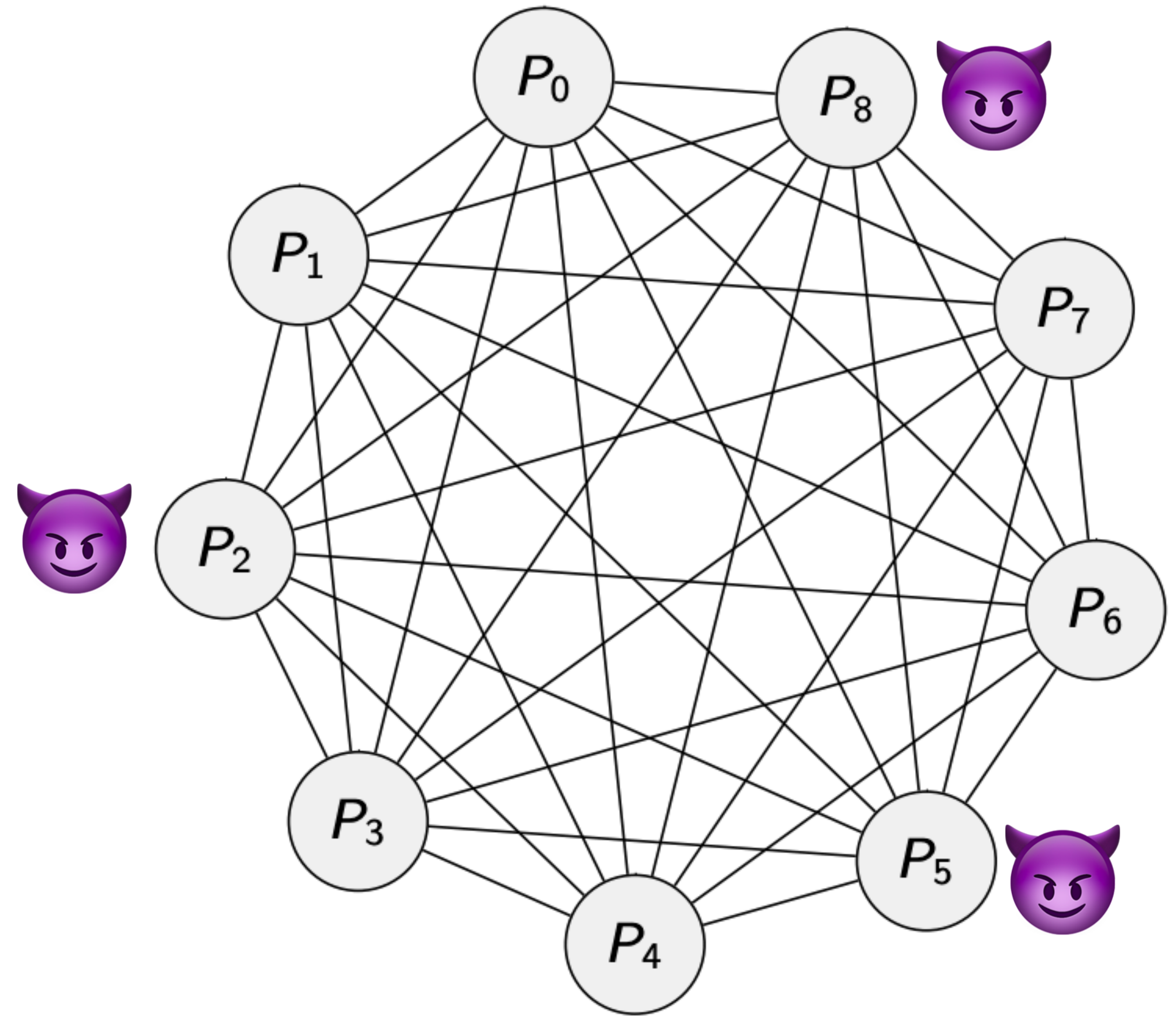
- Typically -- assume secure point-to-point communication channels between all parties



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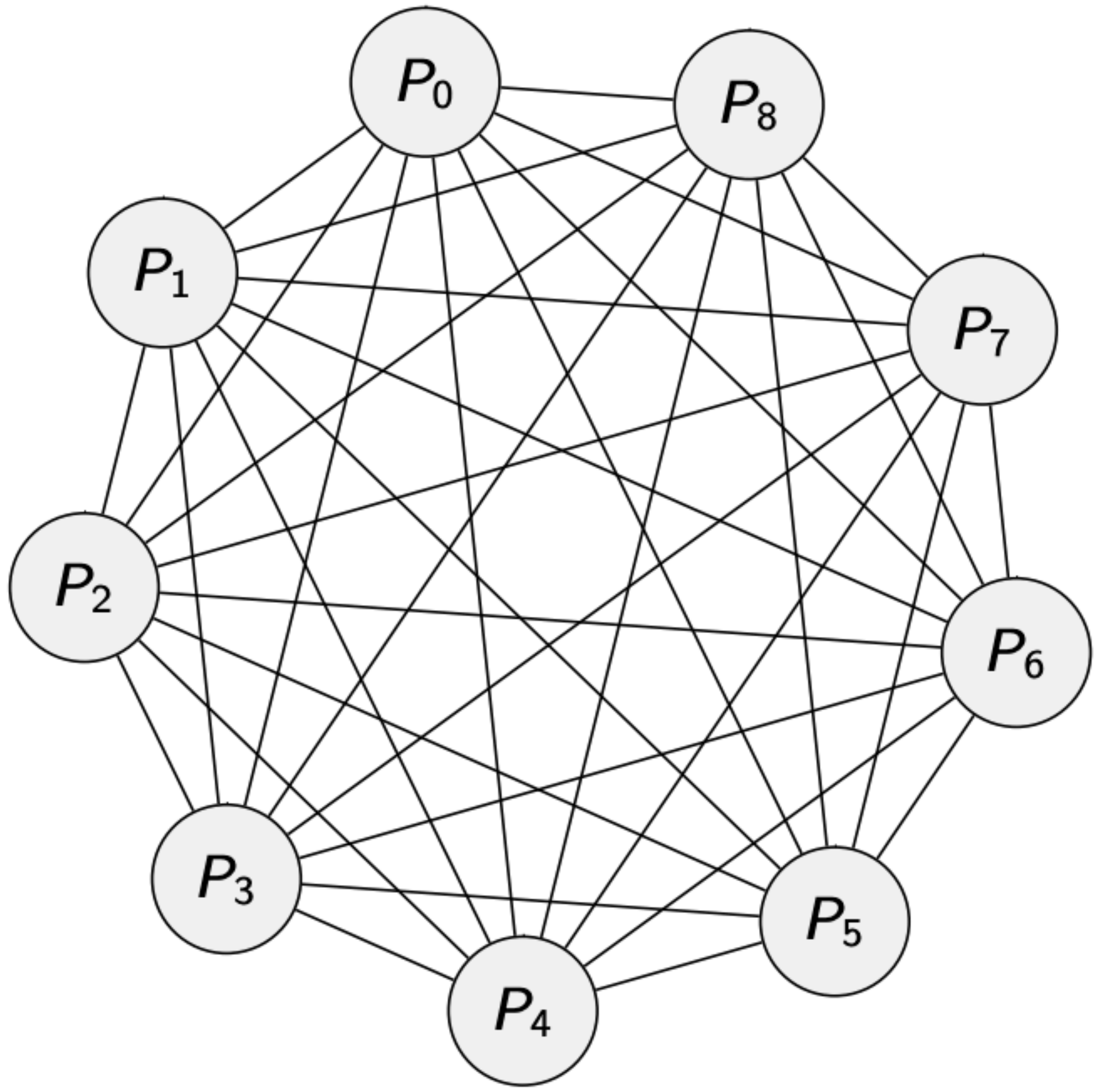
Connectivity in MPC



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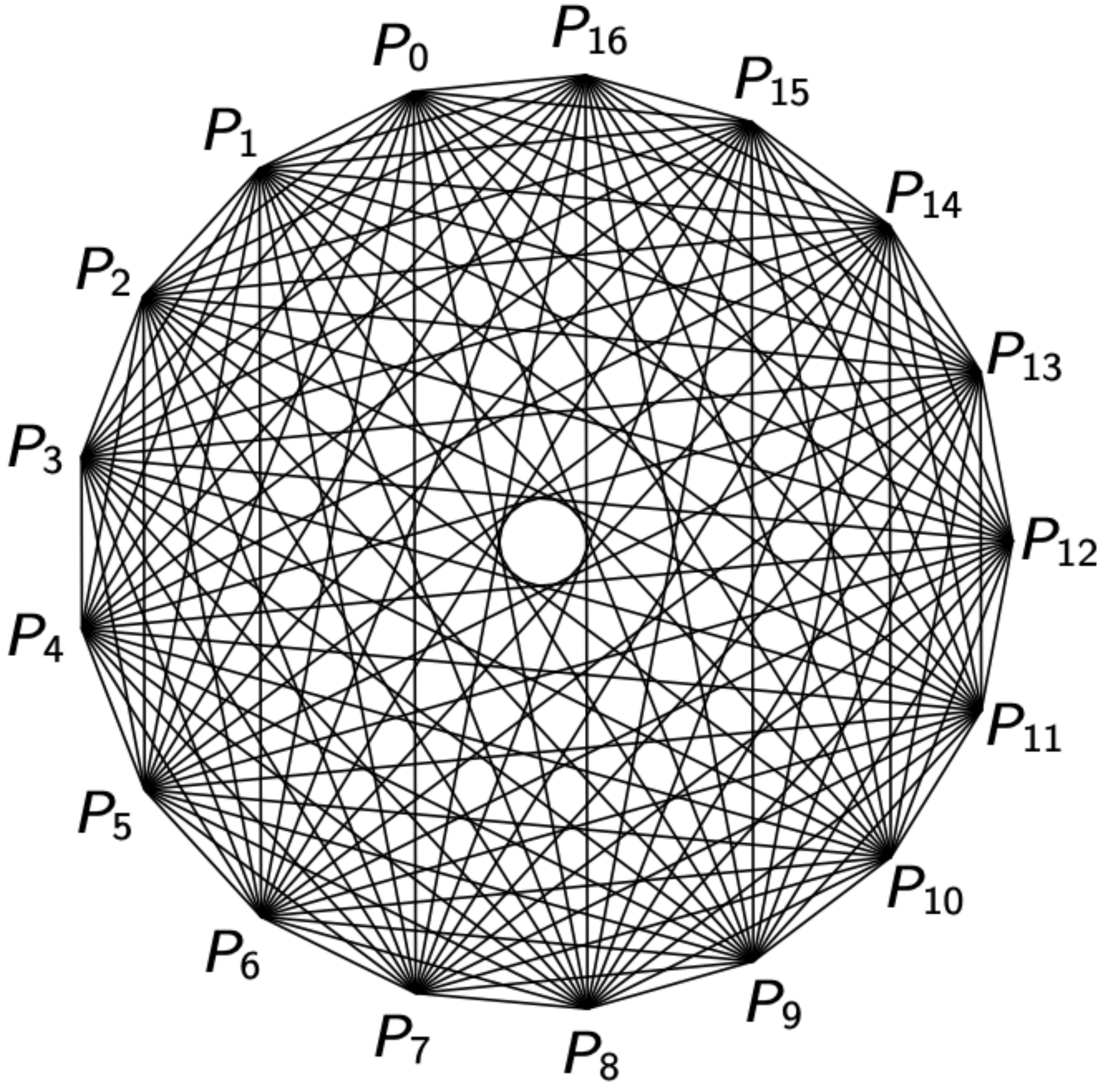

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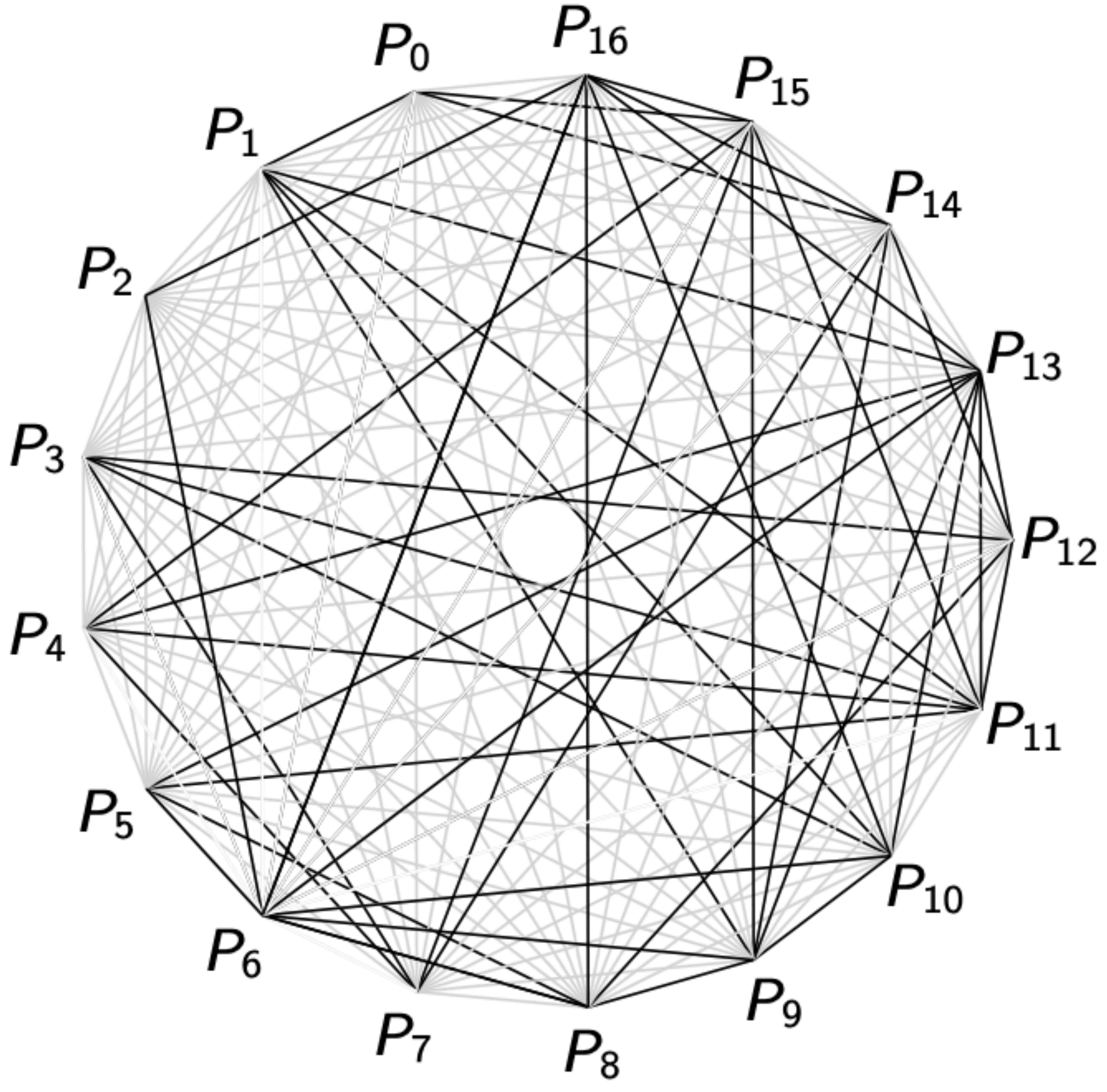

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- What does this mean?

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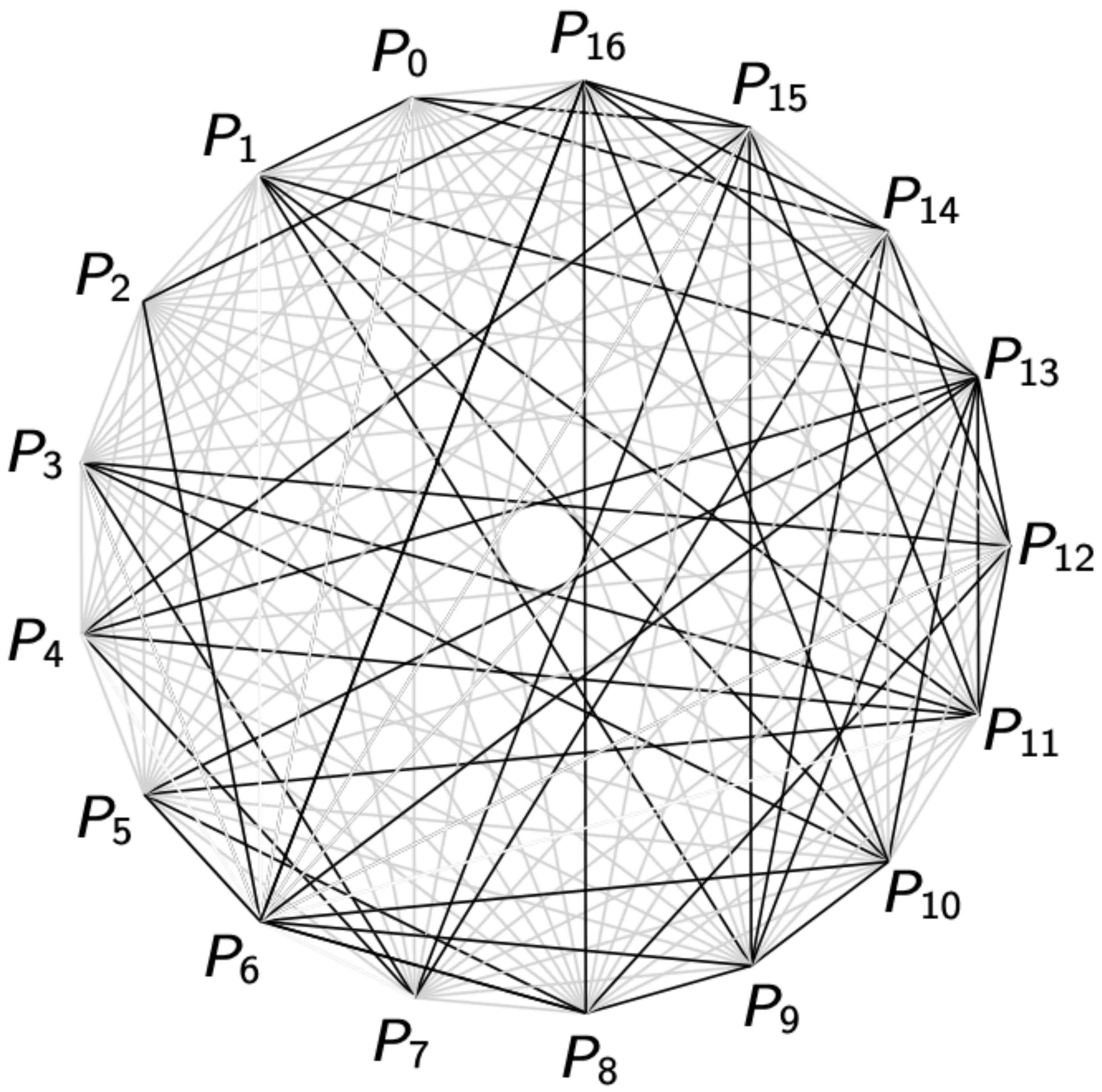

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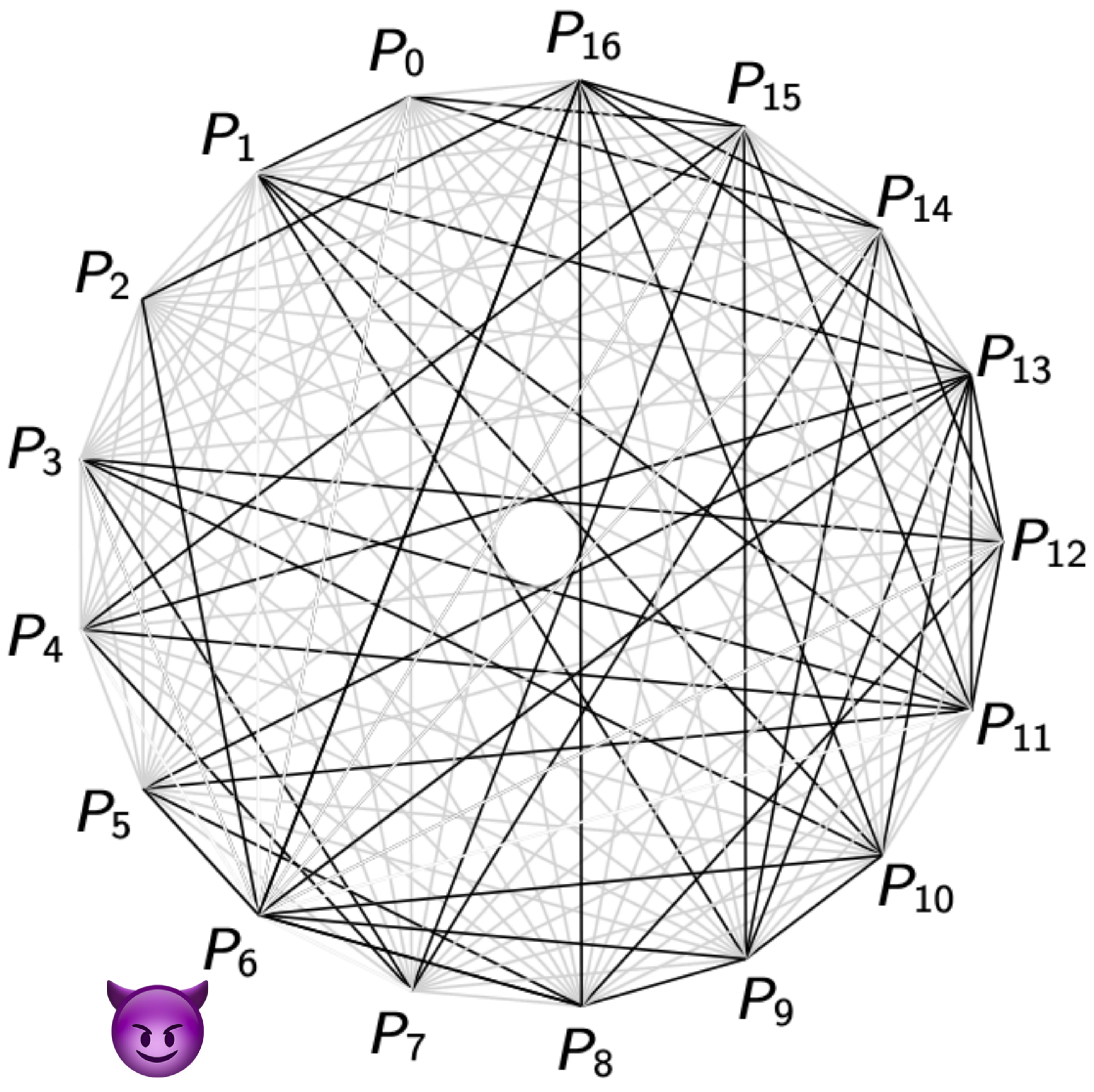

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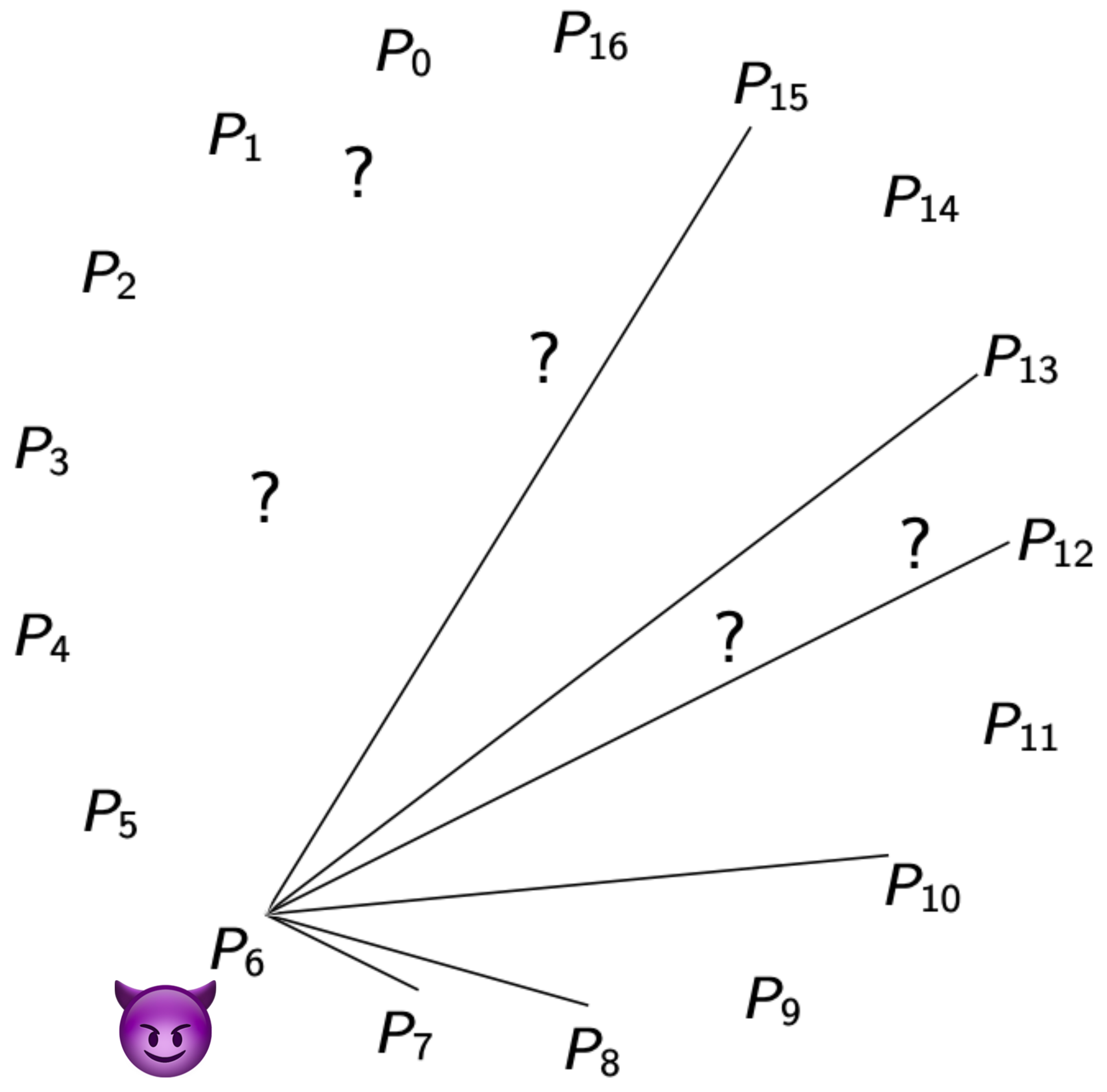

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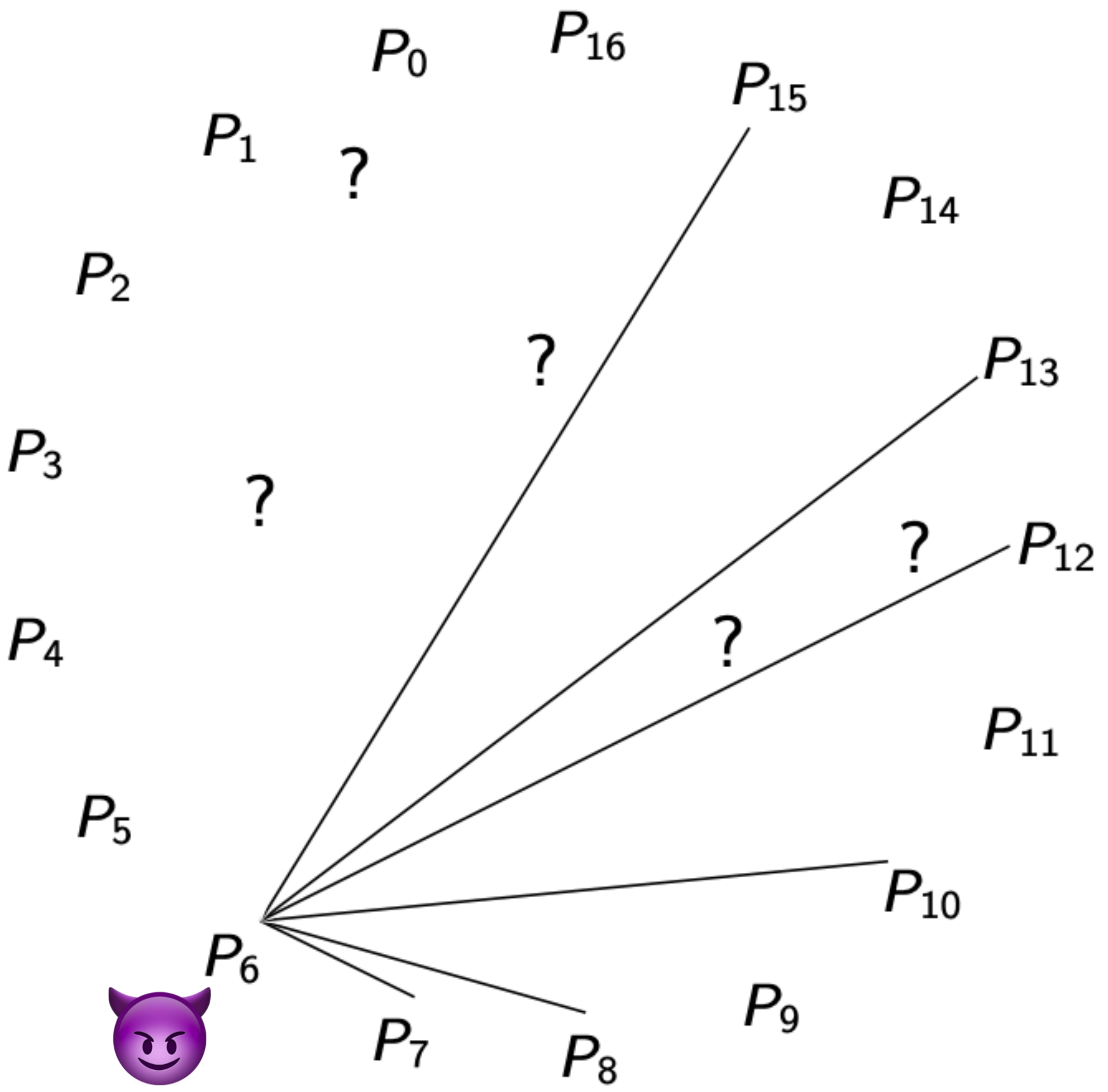

Goal: P_6 should learn nothing else beyond its local view (its neighbors)

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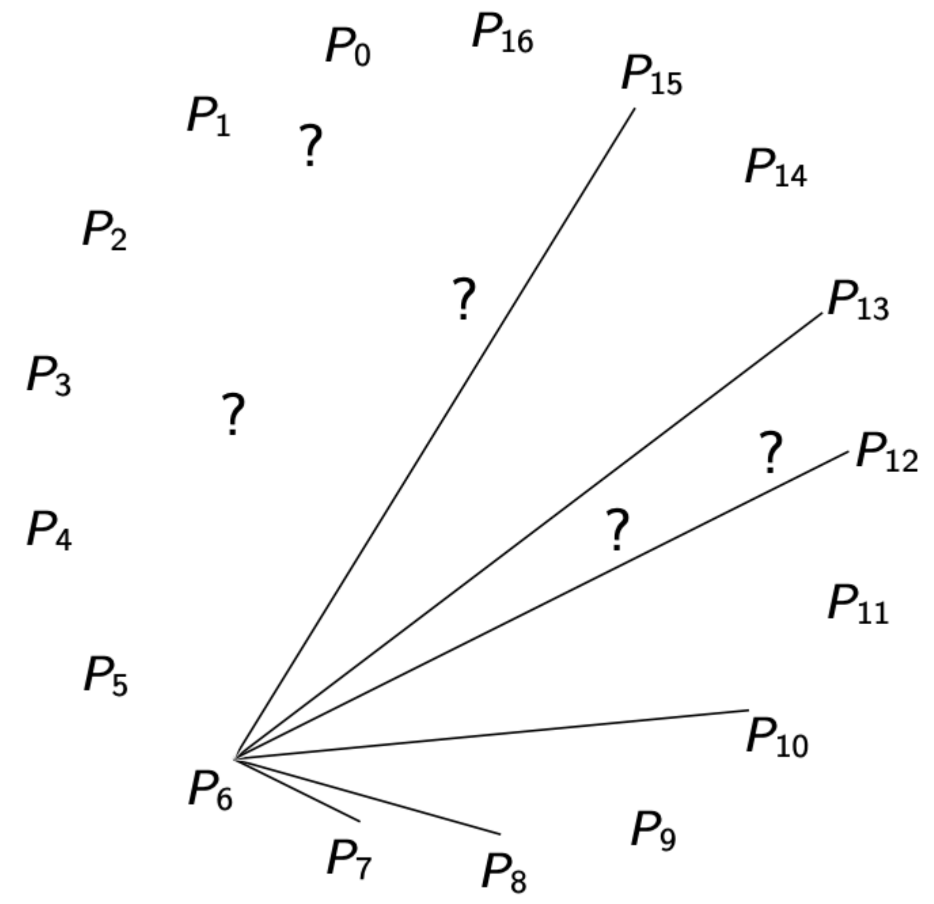

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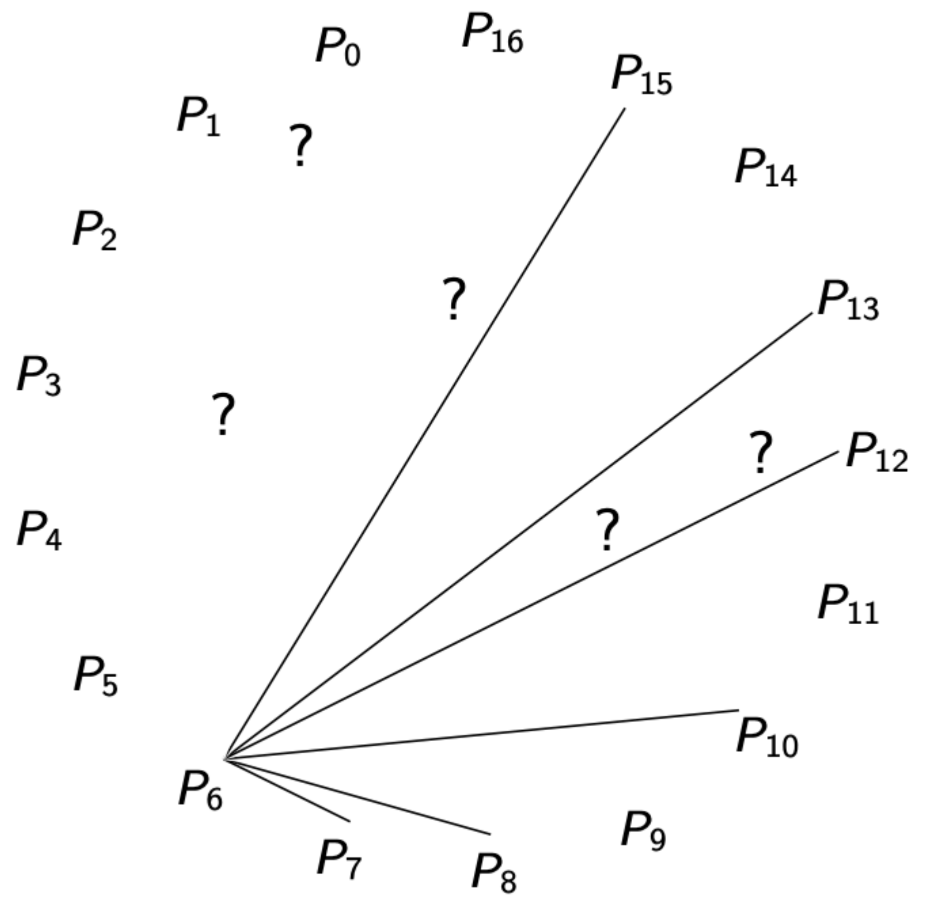

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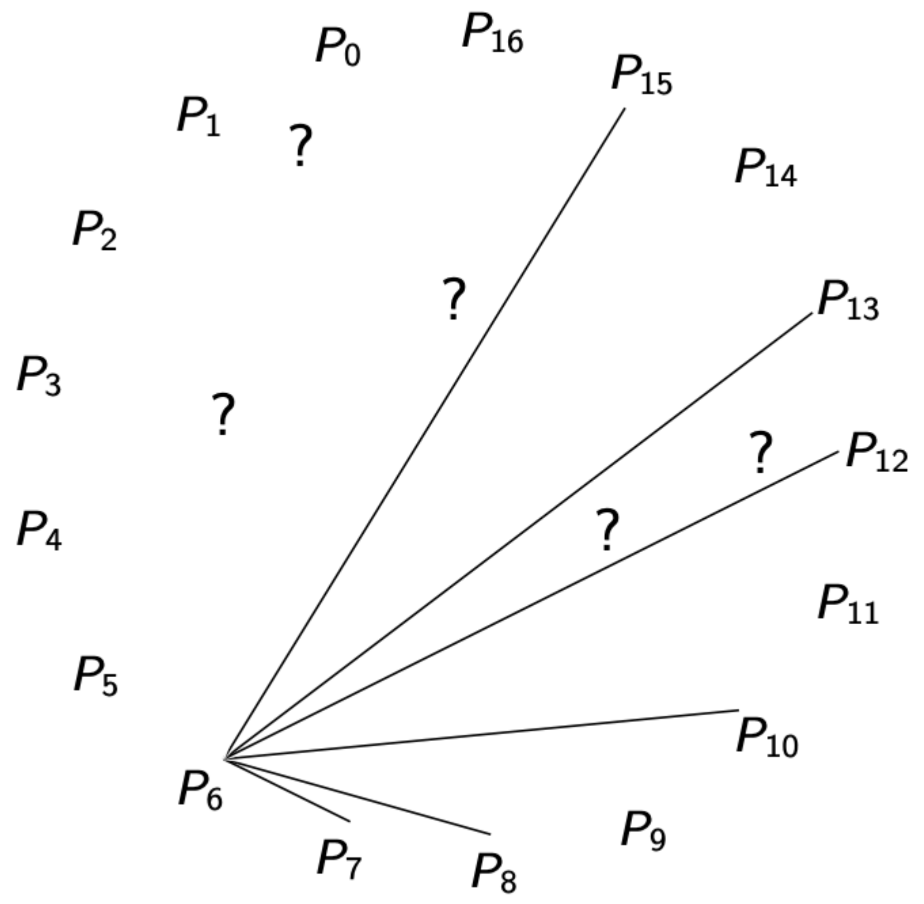

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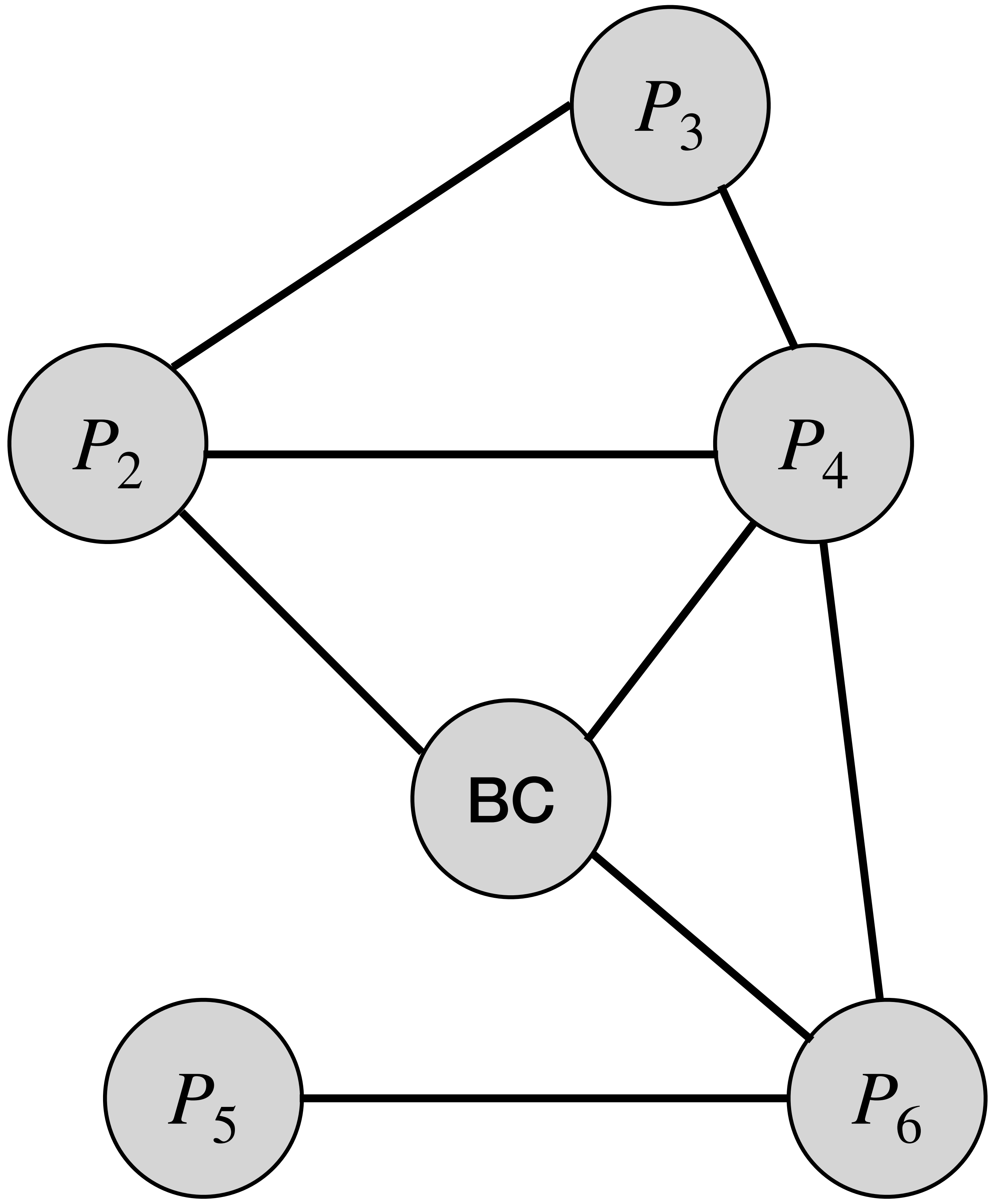
- (Protocol reveals nothing beyond input/output, and neighbors in graph)
- Can consider different number of corruptions, passive/active, static/adaptive

Why is THC hard (even with passive adversaries)?

Flooding Broadcast



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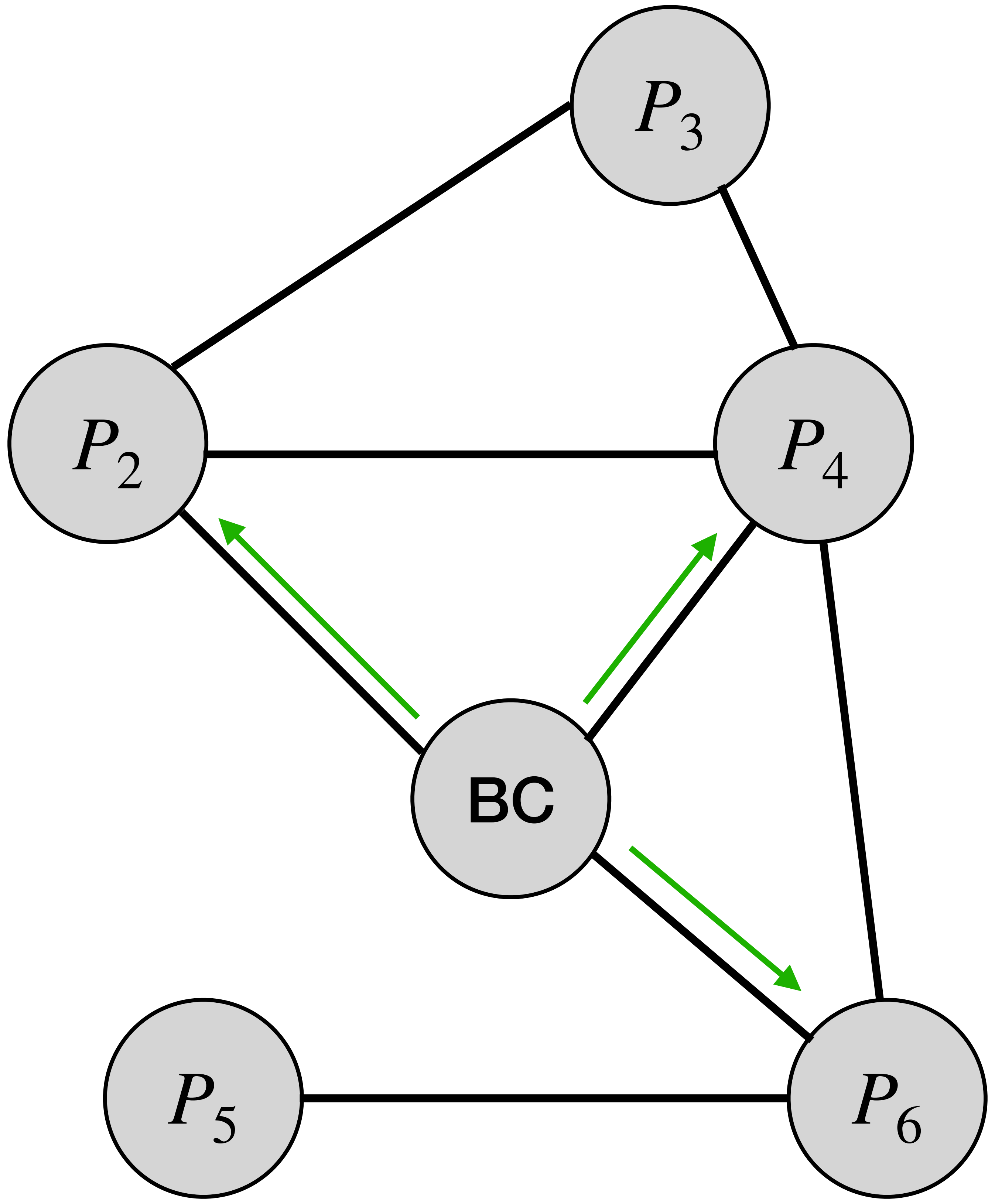

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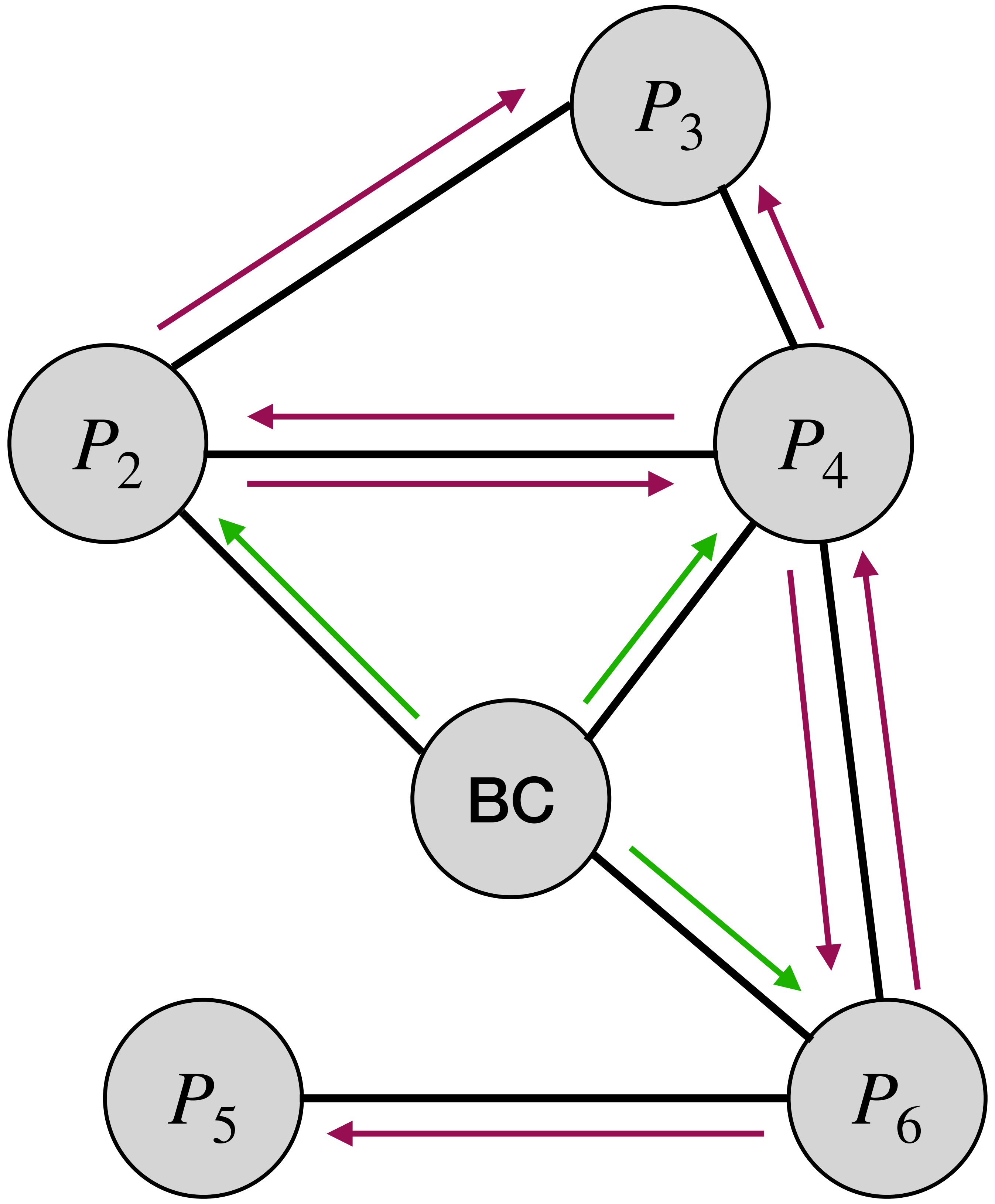

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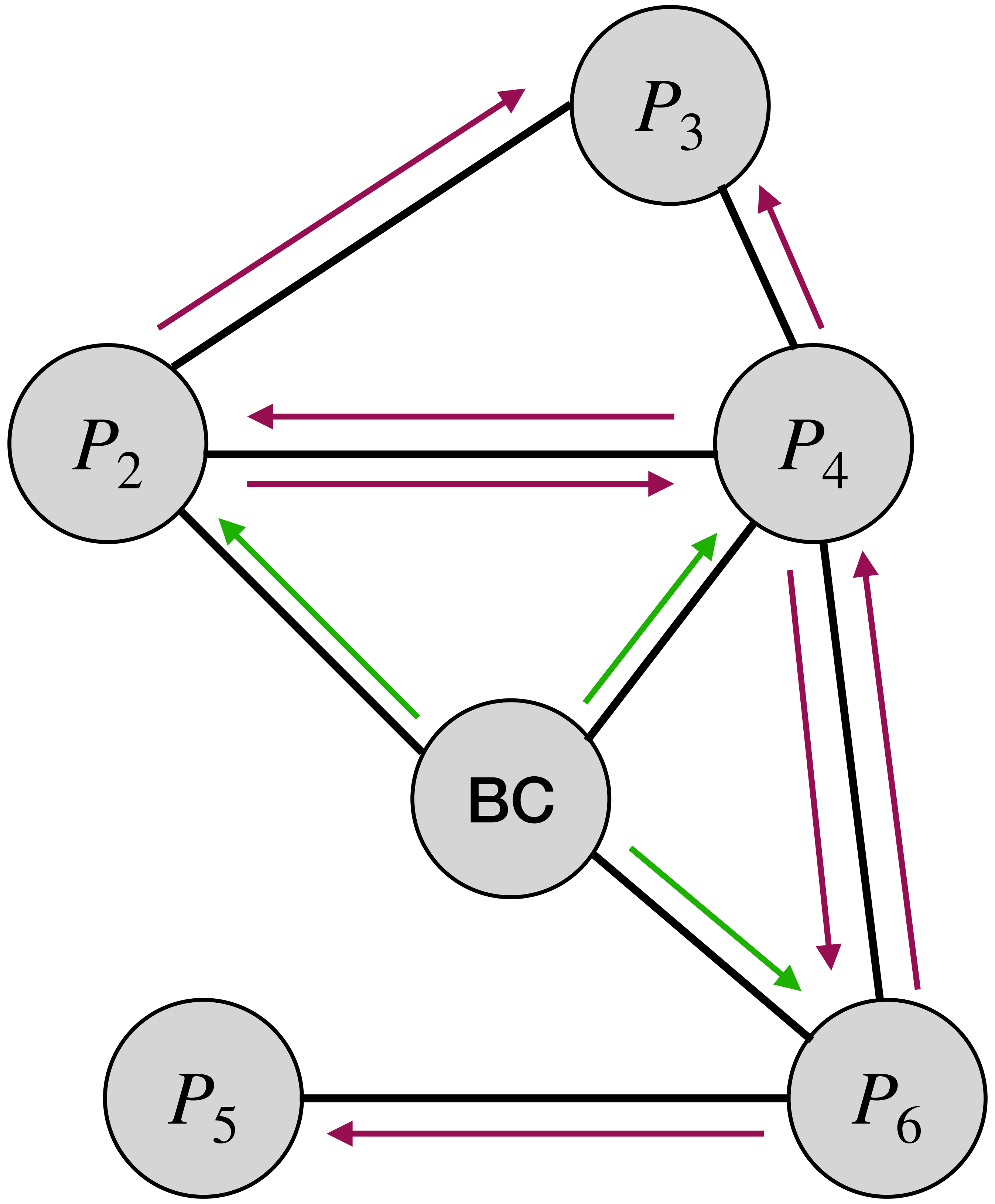

Each party learns:

- Distance to BC
- Nbrs' distance to BC

Not Topology-Hiding!

Our Question

THC Oblivious Transfer MPC

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THC

Oblivious Transfer ⇄ MPC

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THC → Oblivious Transfer ⇄ MPC

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Known Results

Semi-Honest, Static, Arbitrary Number of Corruptions

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- [MOR15]: Constant Round MPC with Constant Overhead => THC for graphs with constant degree and logarithmic diameter
- [AM17,ALM17,LZM+18]: Key-homomorphic, re-randomizable encryption => THC for all graphs
  - Only known from structured algebraic assumptions; e.g., QR, DDH, LWE

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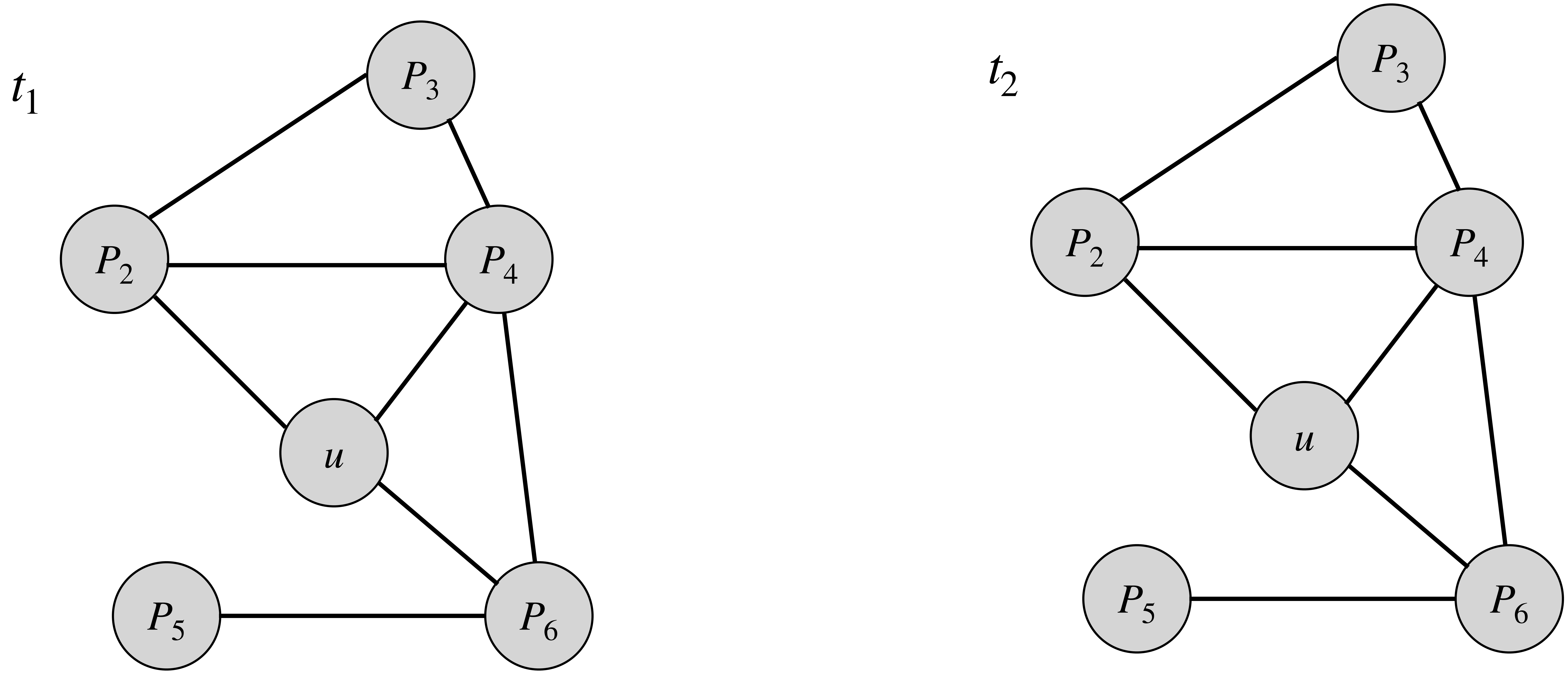
  1. Define Locally-Simulatable MPC (for any fixed Graph)

  2. Use Locally-Simulatability + Correlated Random Walks [ALM17] to reduce THC to Locally-Simulatable OR on a Path

  3. Construct Locally-Simulatable OR on a Path from constant round 2PC with constant overhead

Correlated Random Walks [ALM17]

For each time step, each node u defines a permutation on its neighbors



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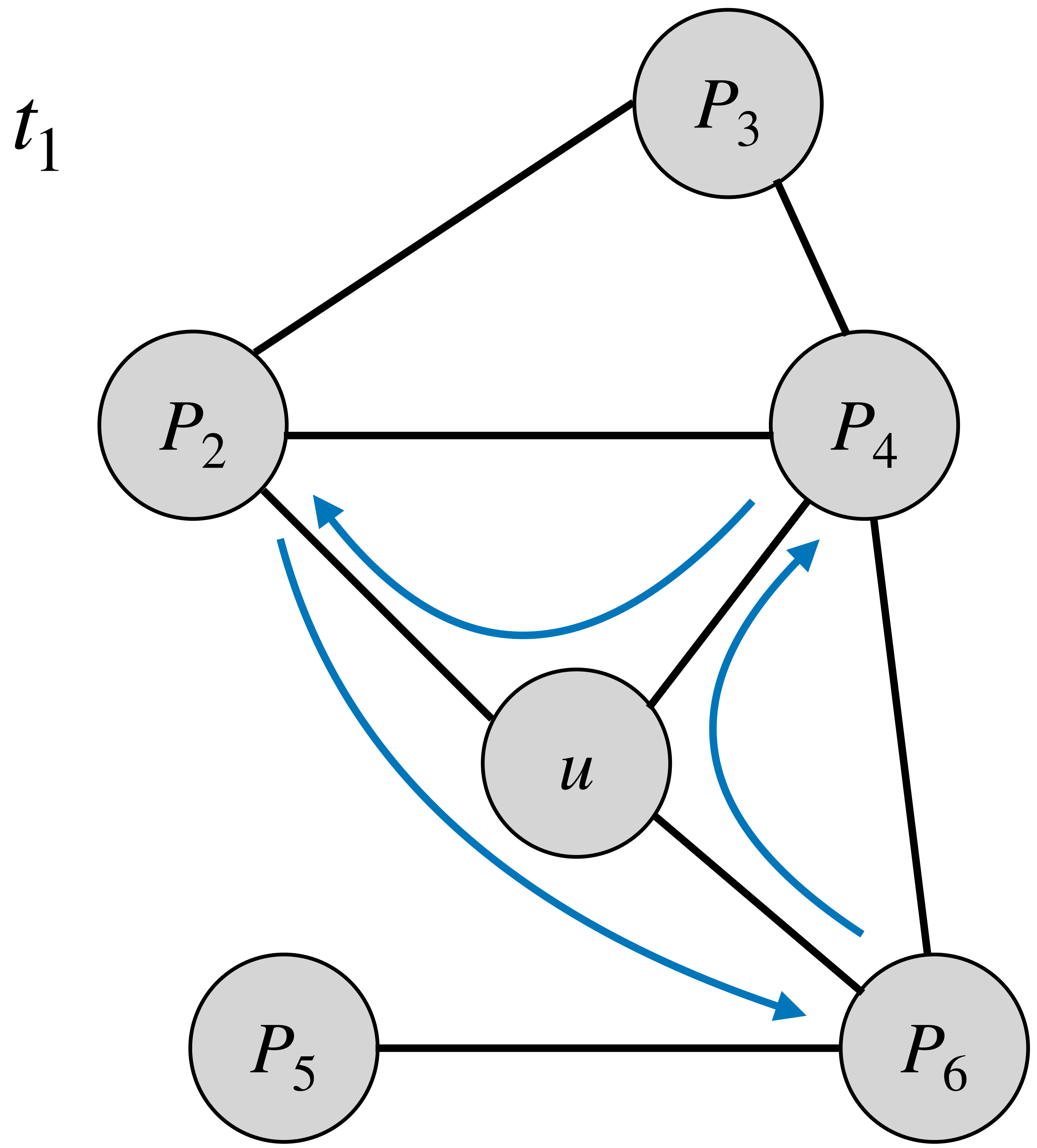

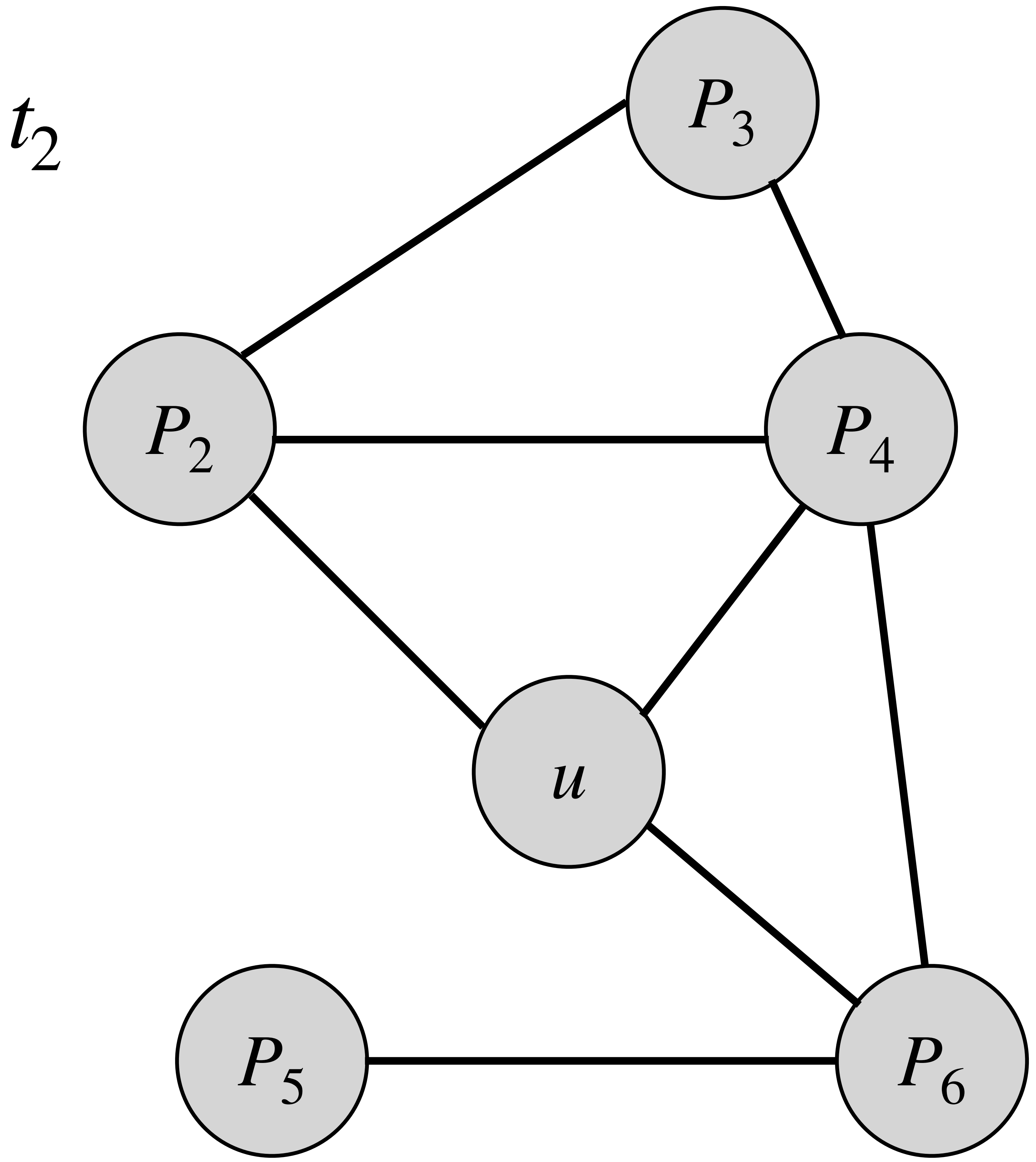

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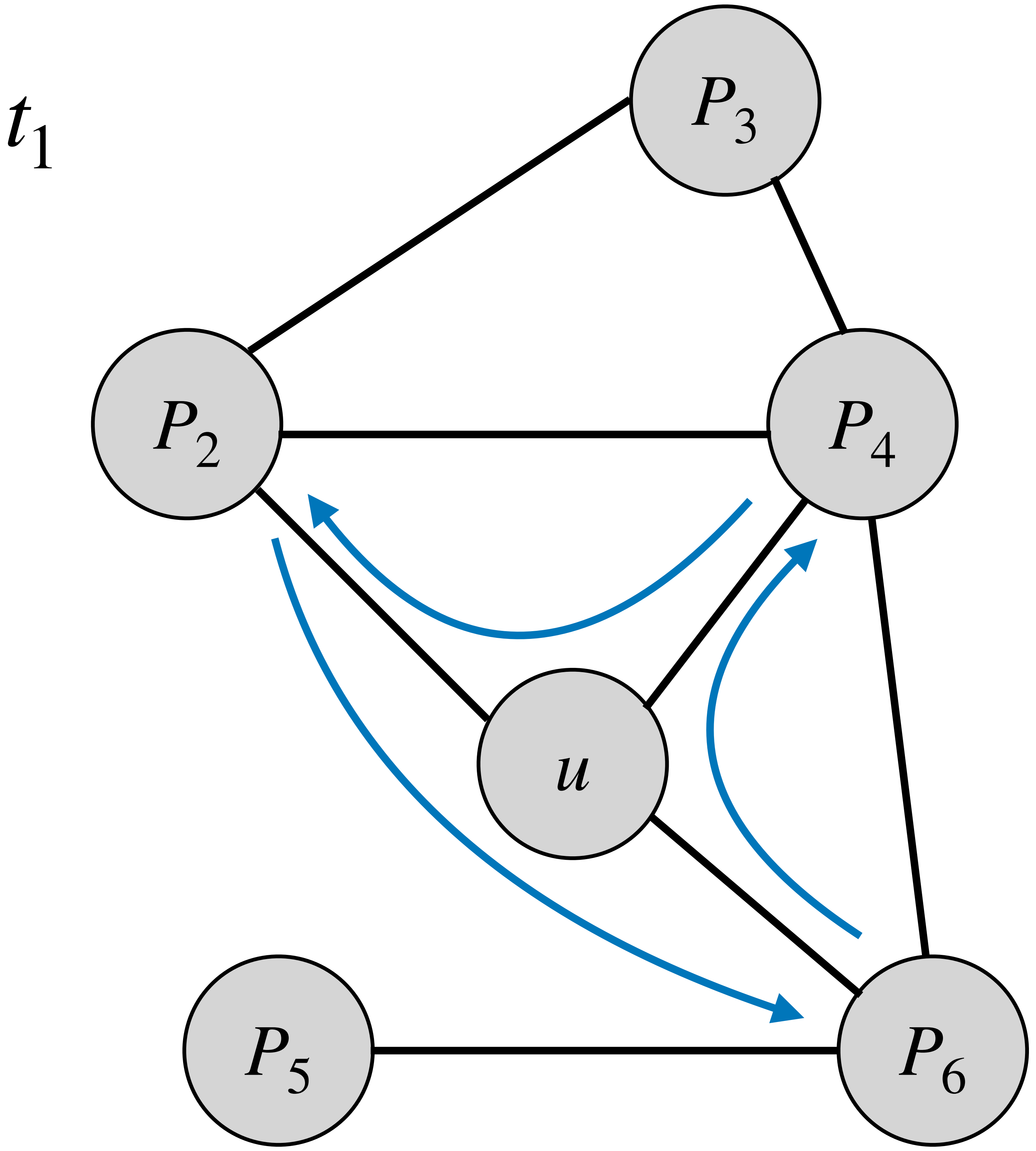

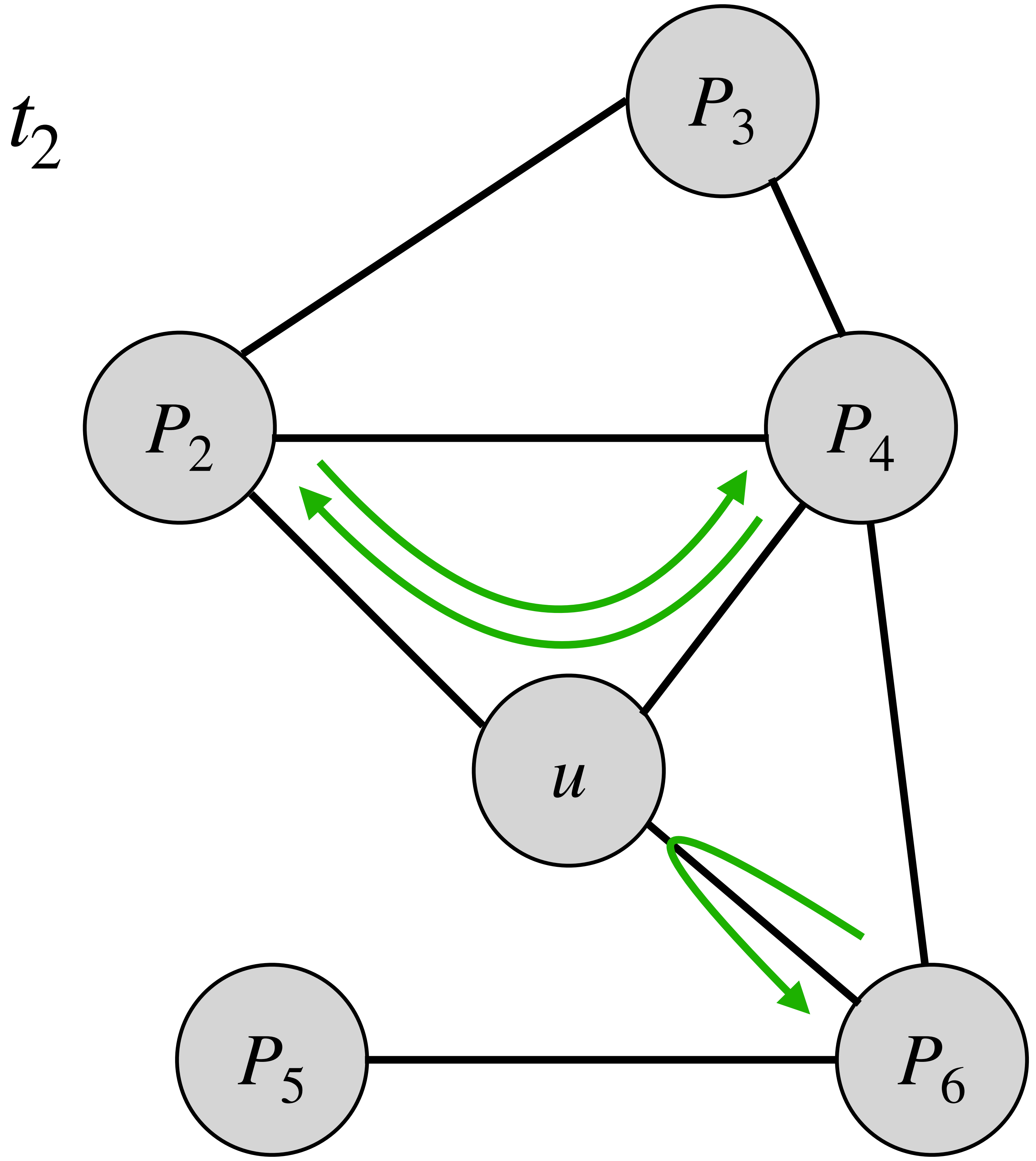

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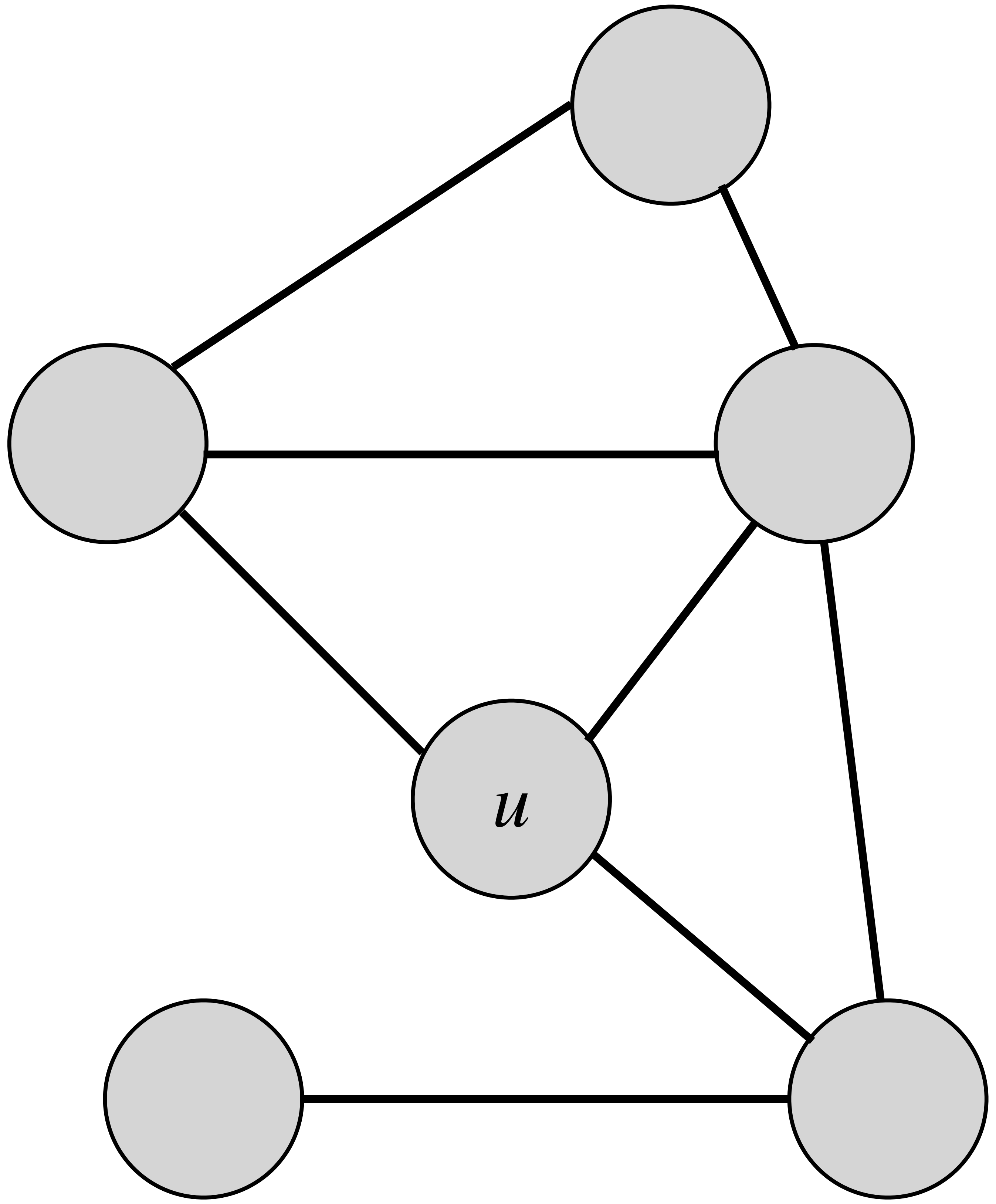

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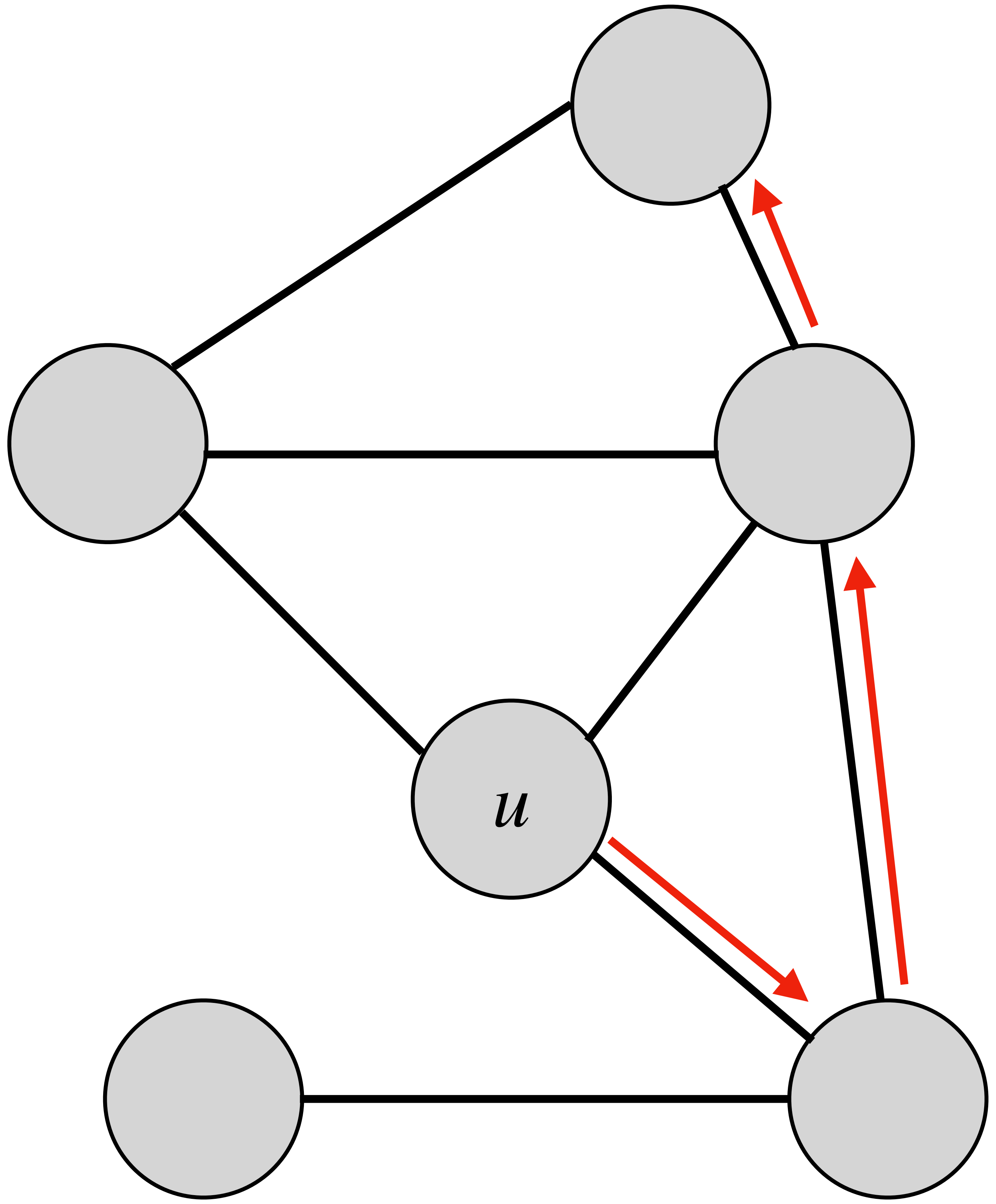

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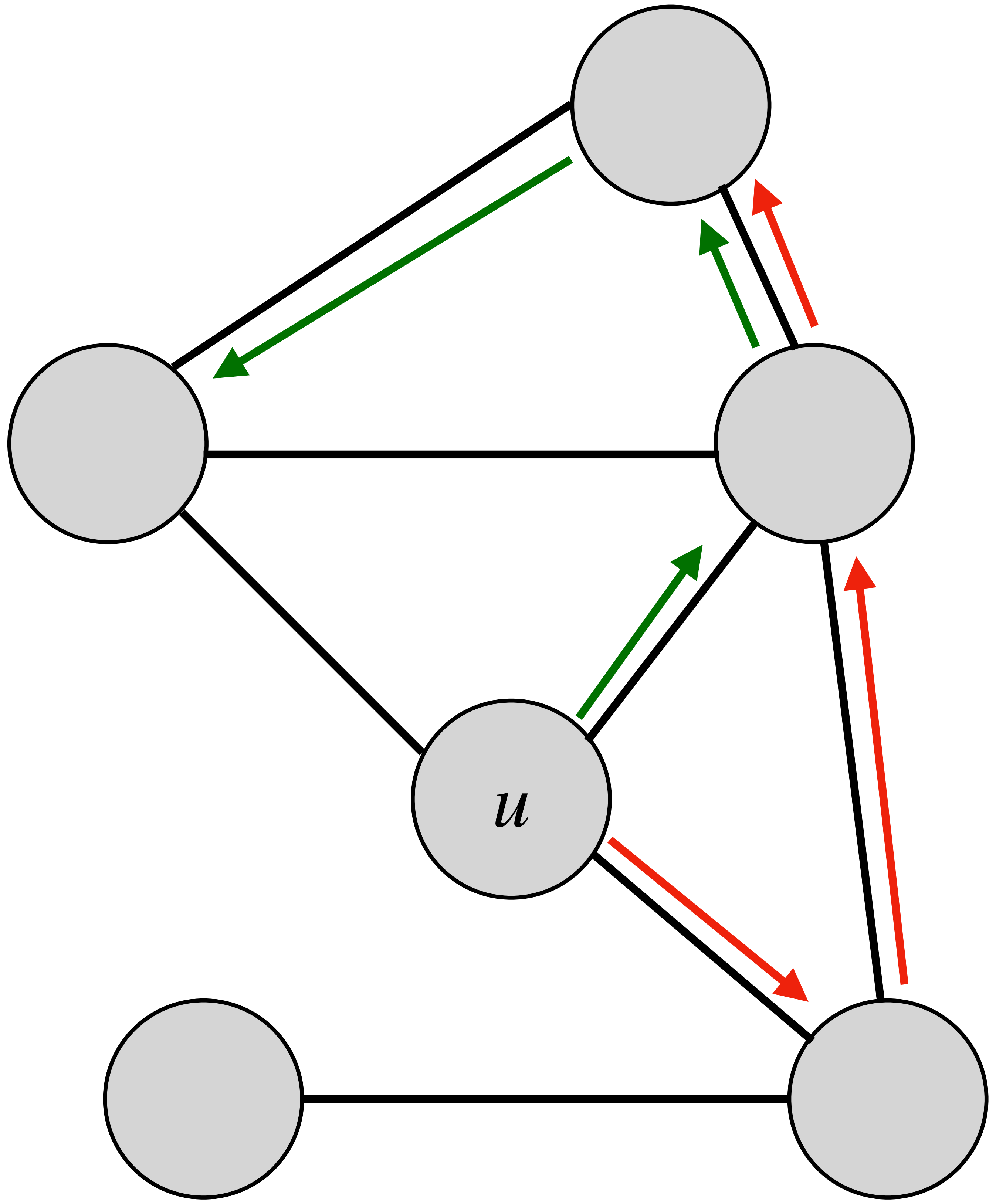

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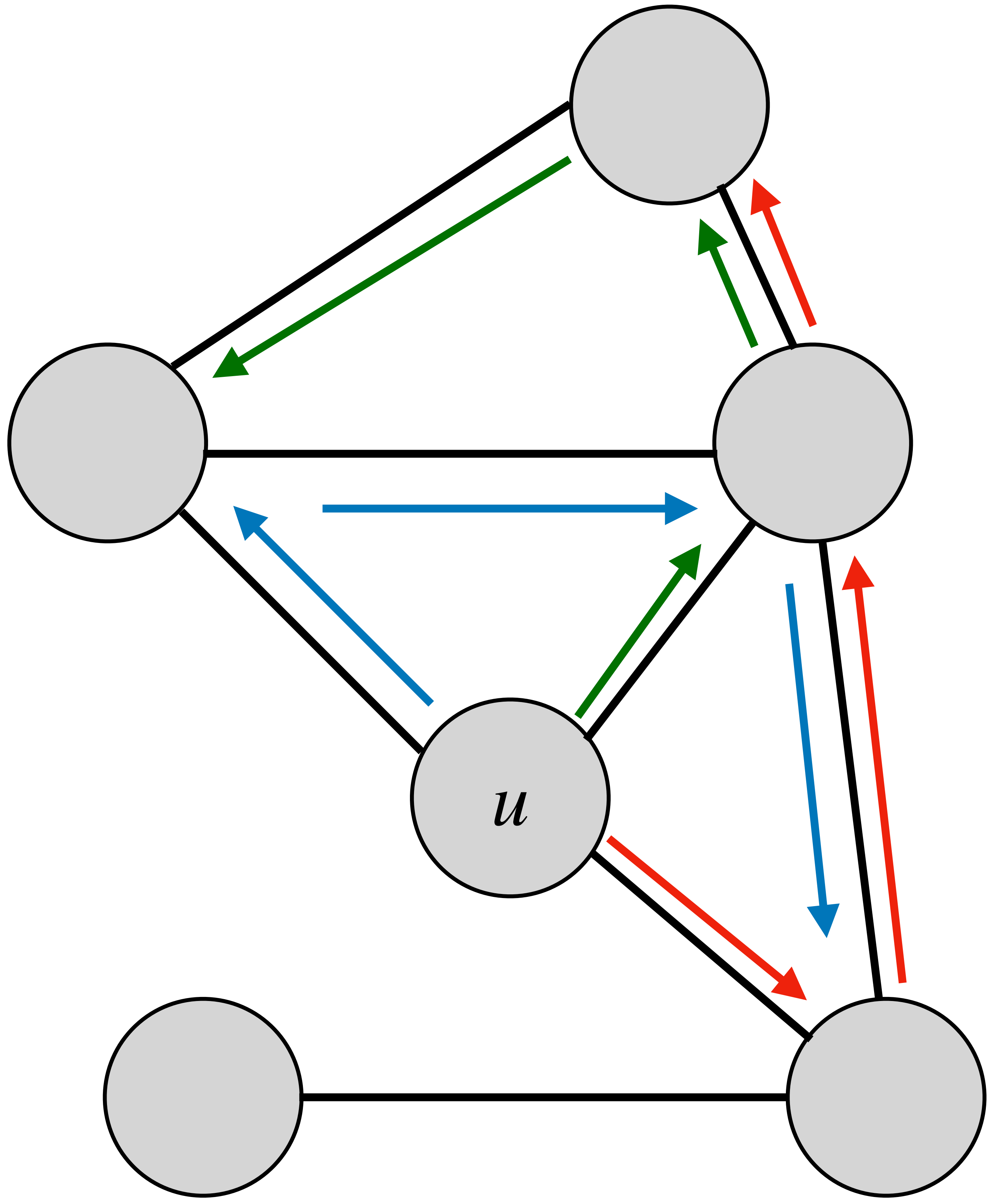

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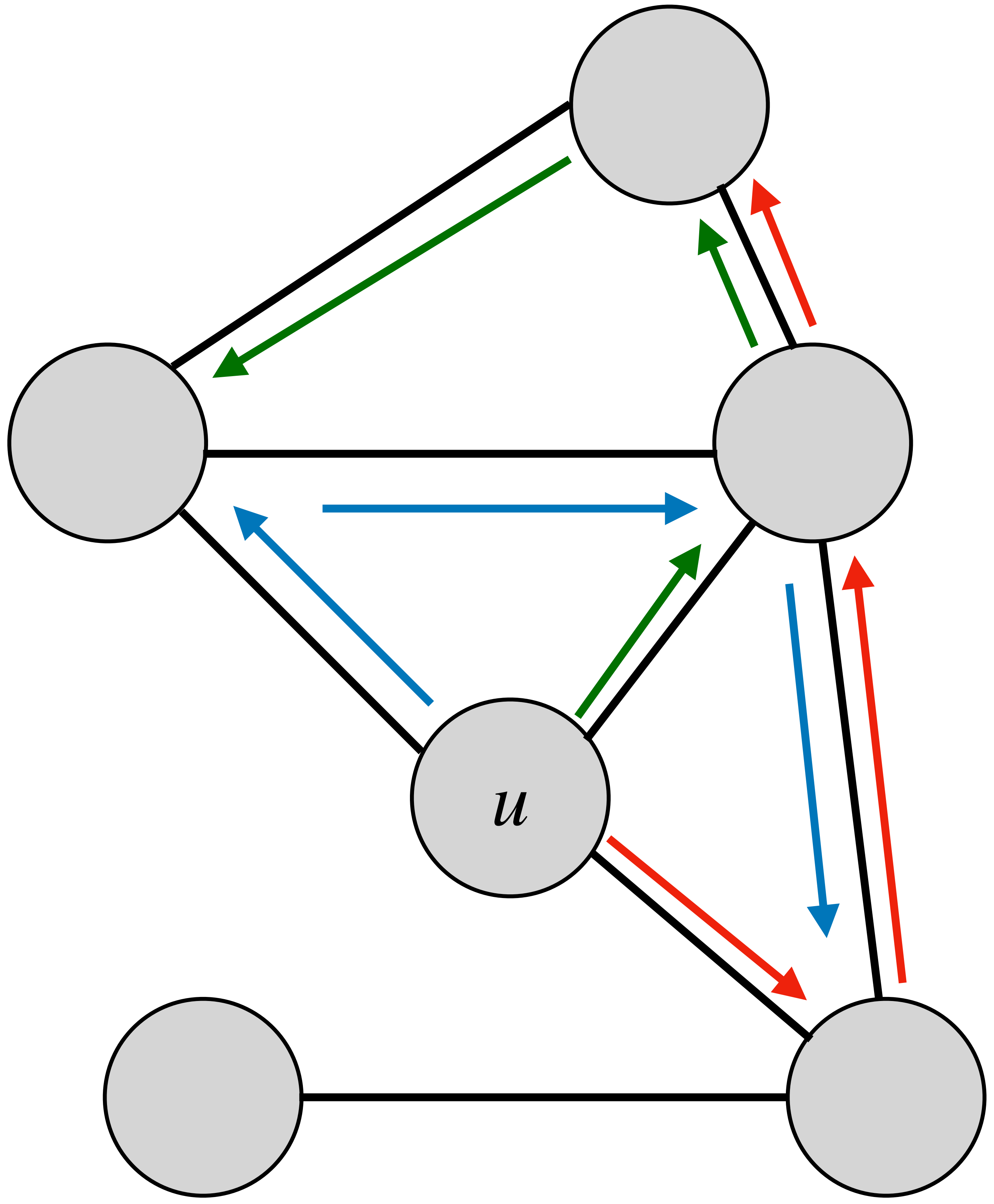

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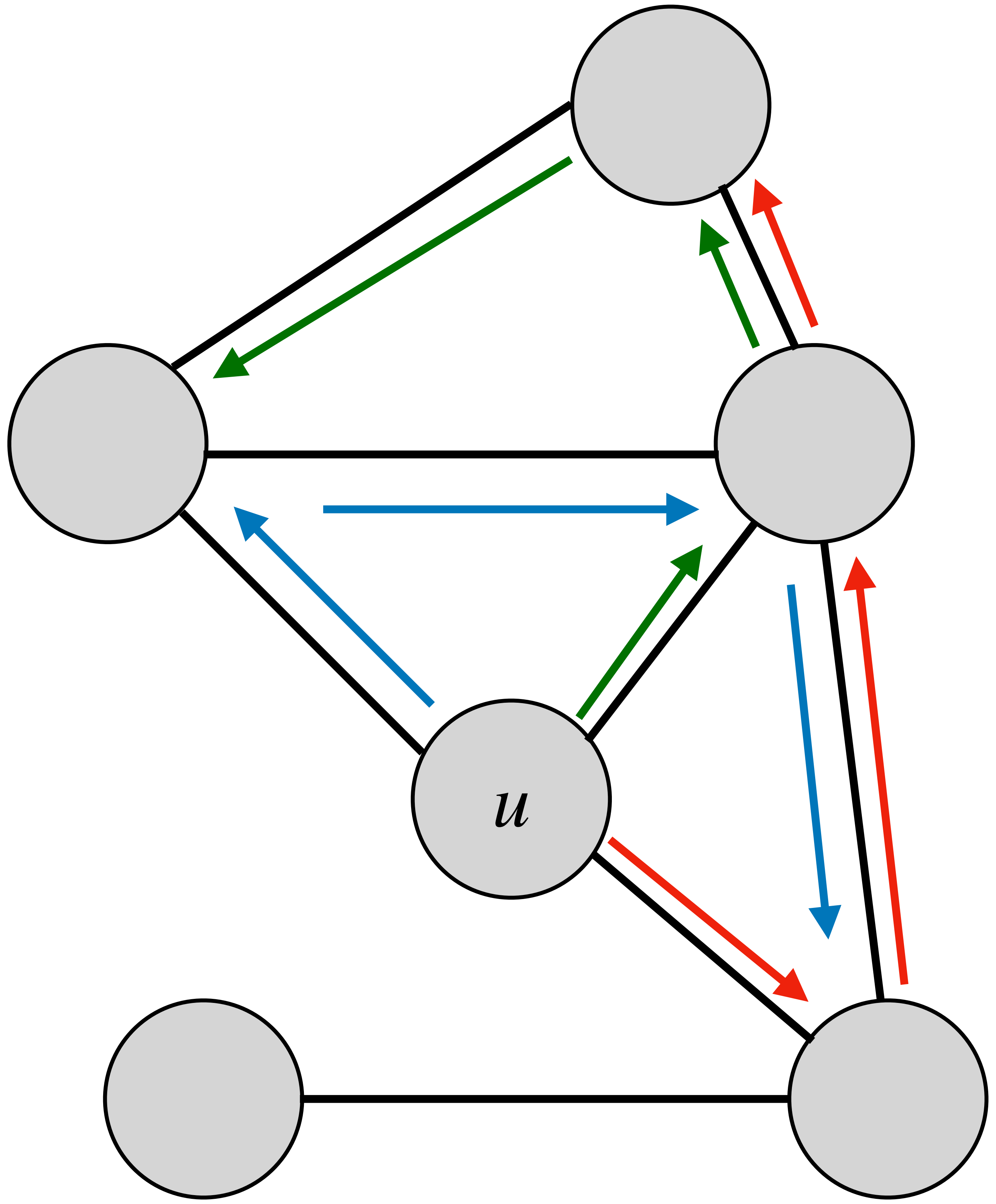

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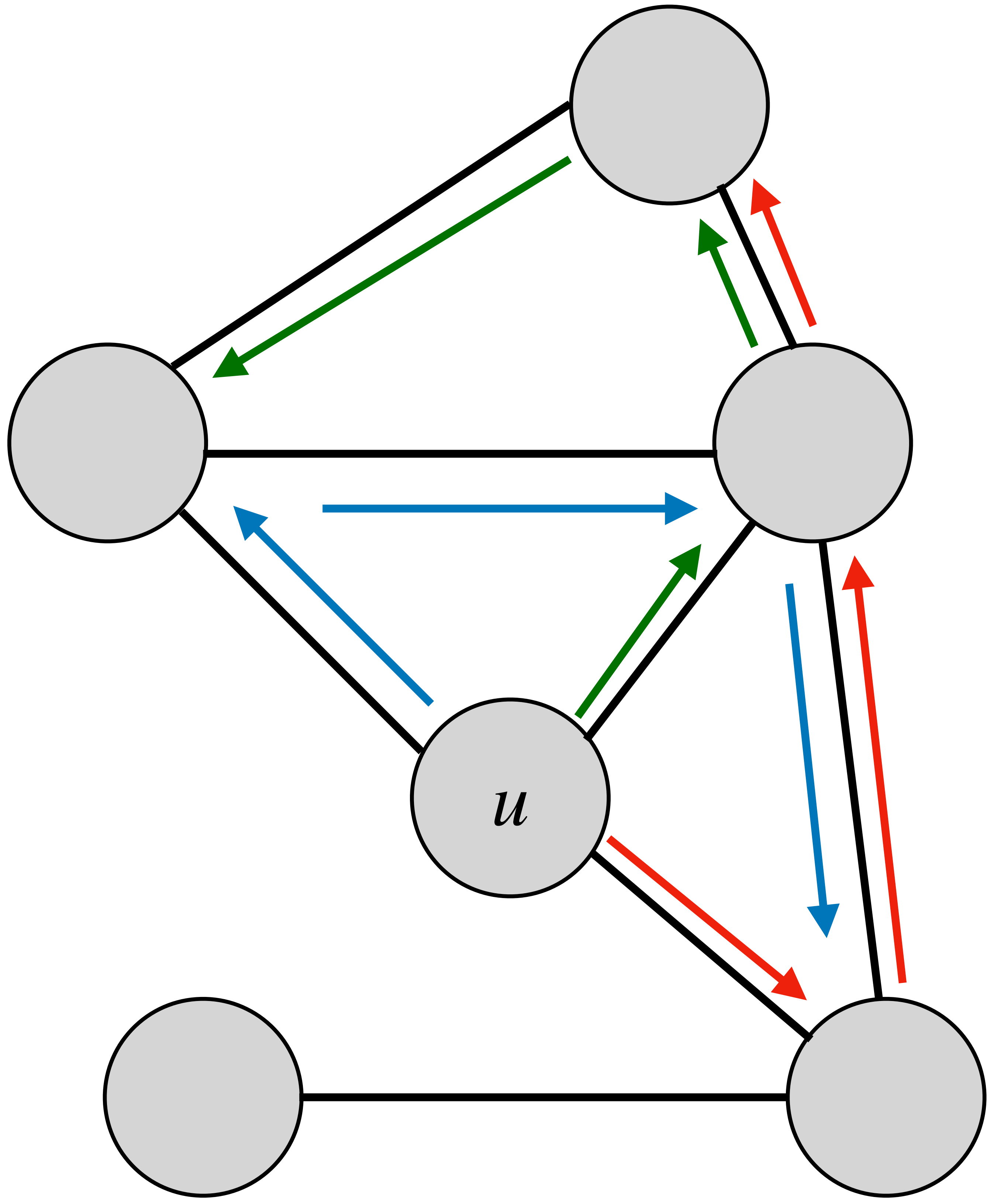

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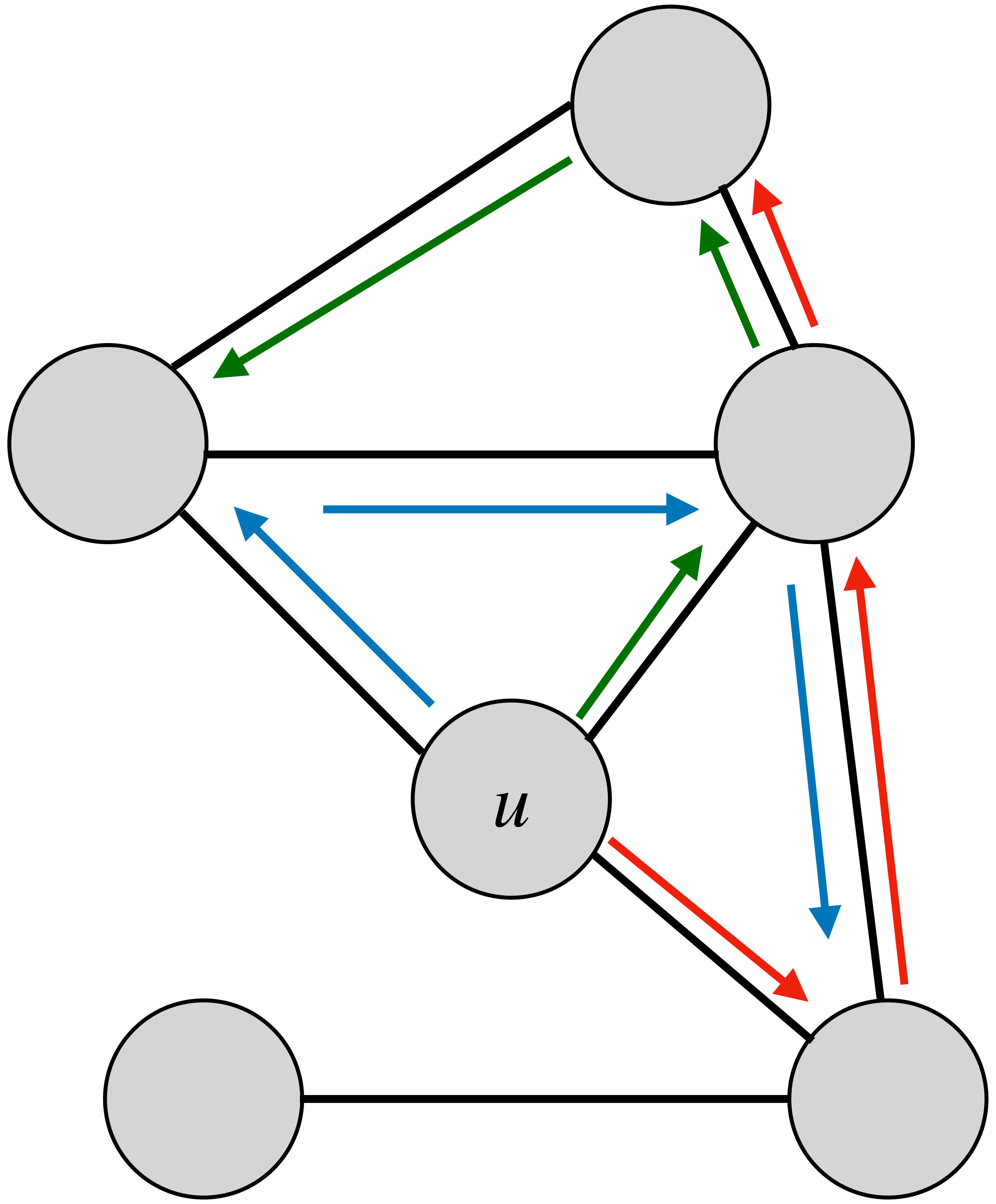

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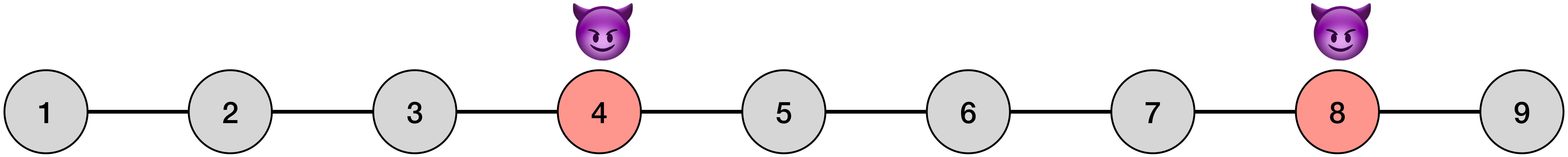
- Maybe we can design protocols just for paths now?

Locally Simulatable MPC on a Path



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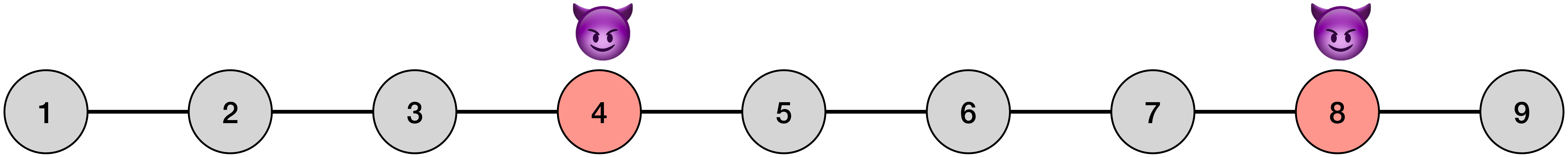

- New Requirement: Can simulate views of 4 and 8 independently --

$$\{\mathsf{Sim}(x_4, y; r_4), \mathsf{Sim}(x_8, y; r_8)\} \approx \mathsf{View}(\Pi)_{4,8}$$

- Namely, r_4, r_8 independent

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- Broadcast == OR function where broadcaster inputs broadcast bit, everyone else inputs 0

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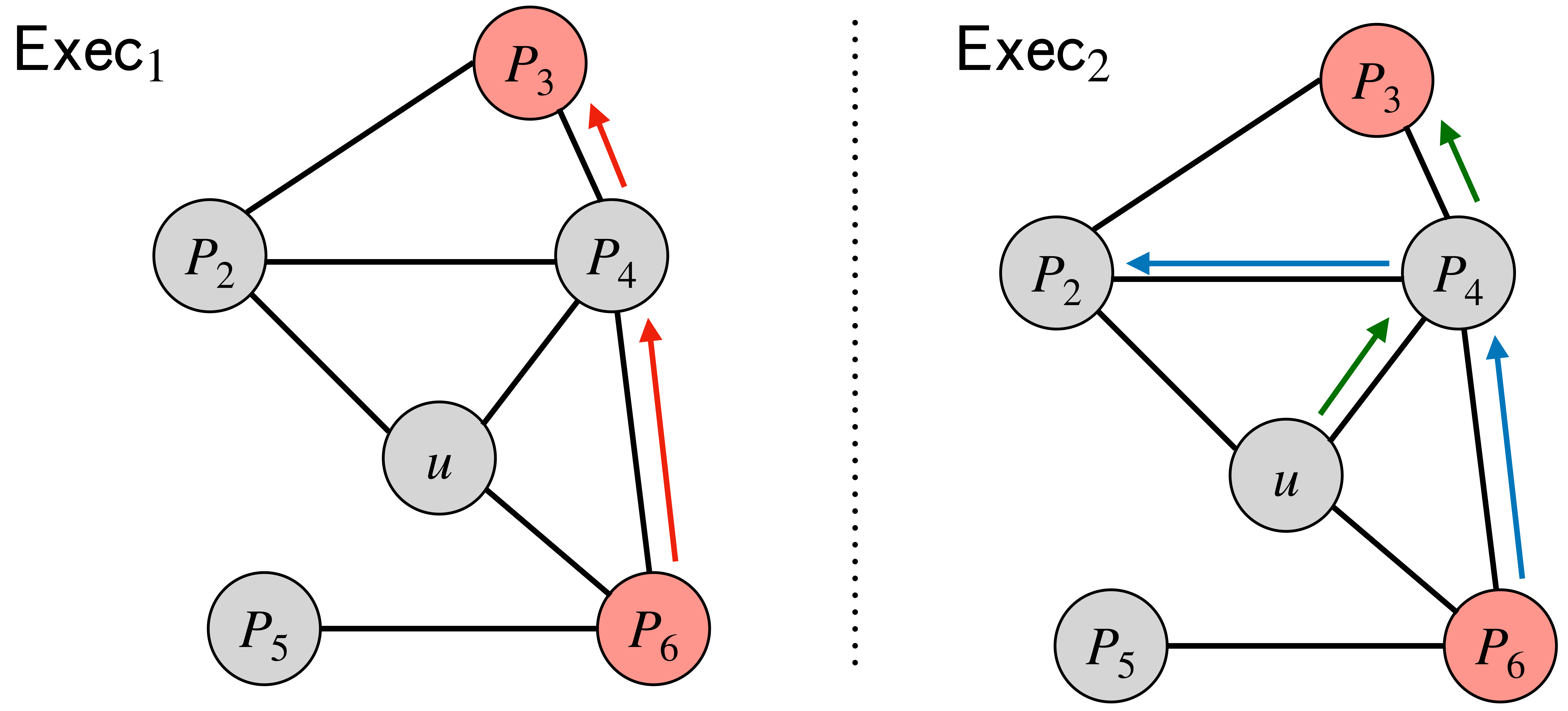

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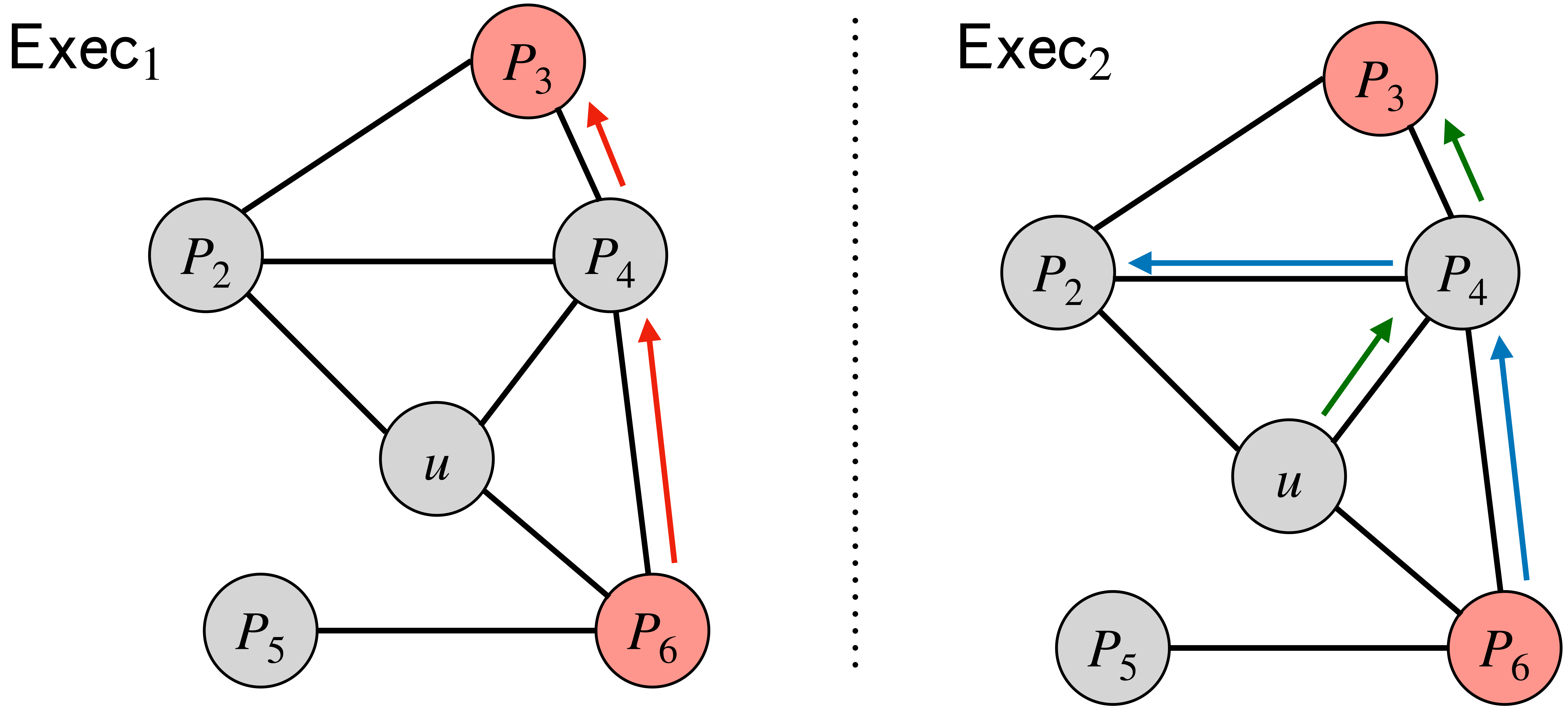
- From view of P_3,P_6, these look the same due to locally simulatability



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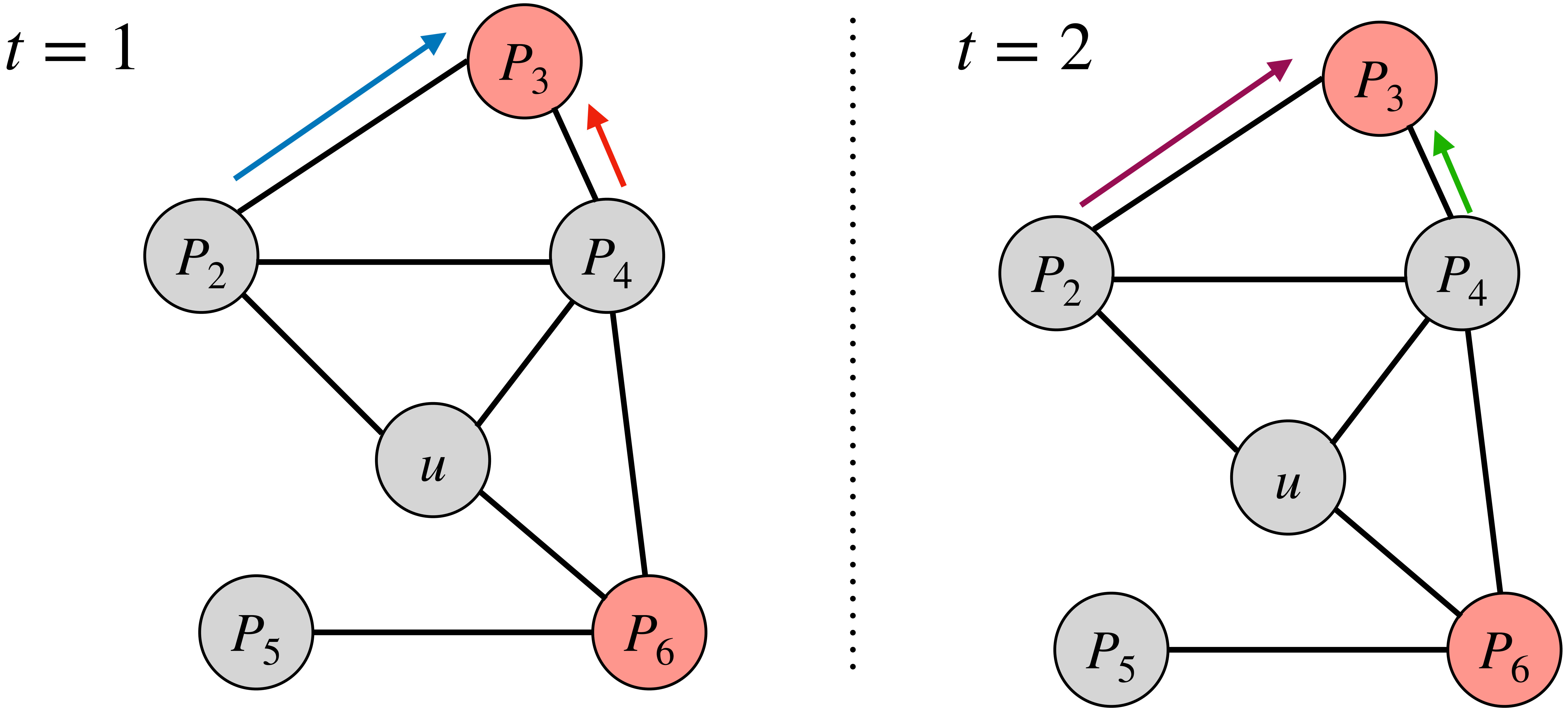

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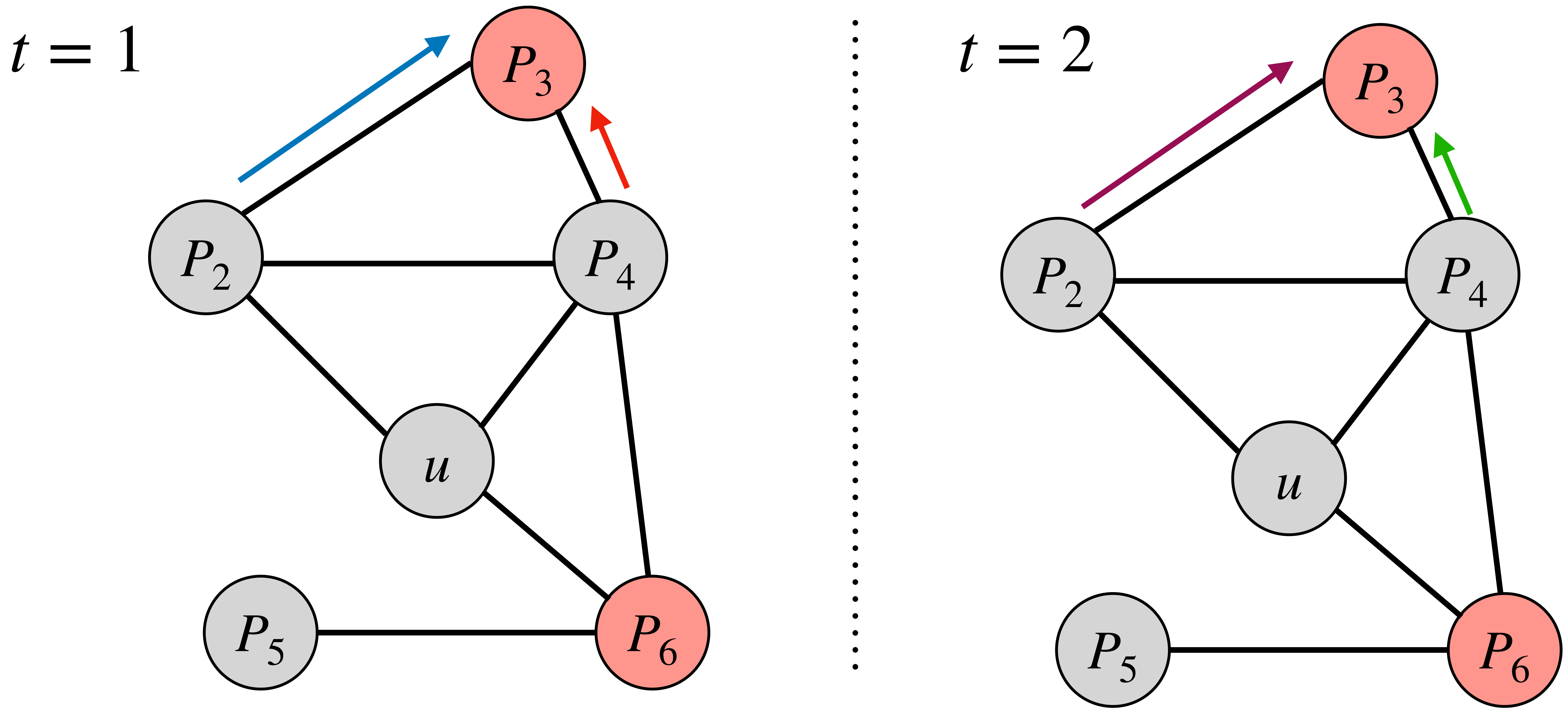
  - Can be done using local simulation



Locally Simulatable OR on a Path

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2PC$(x_{0,0} \vee x_{0,1})$

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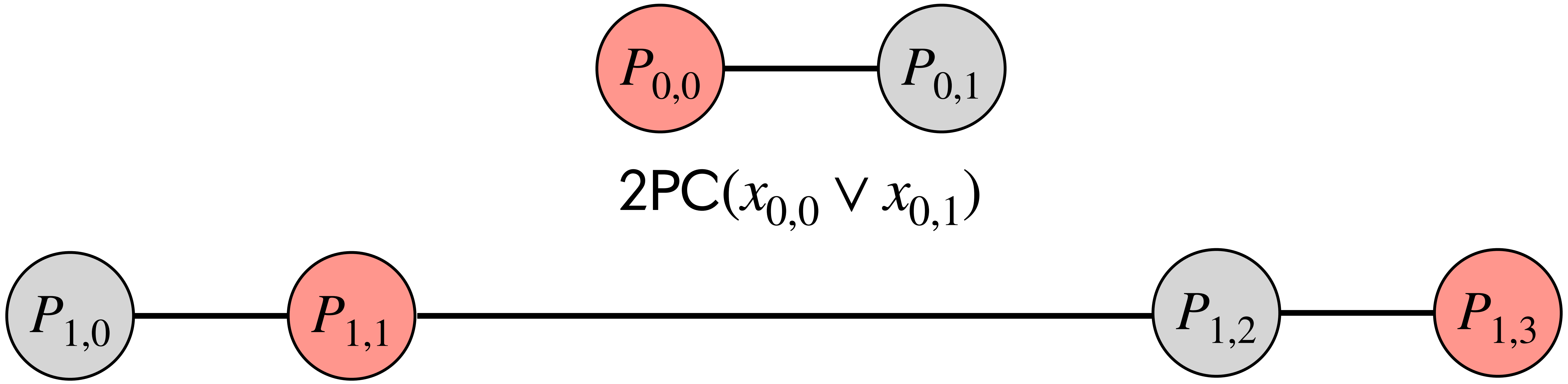

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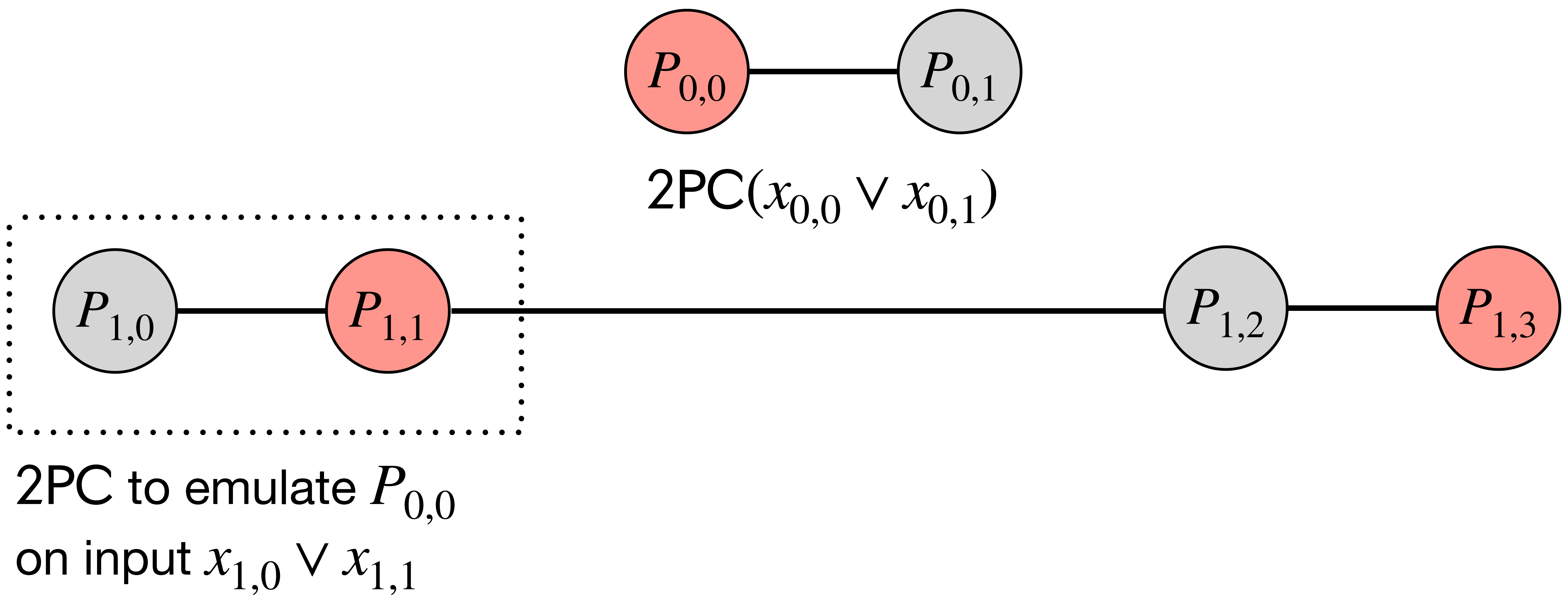

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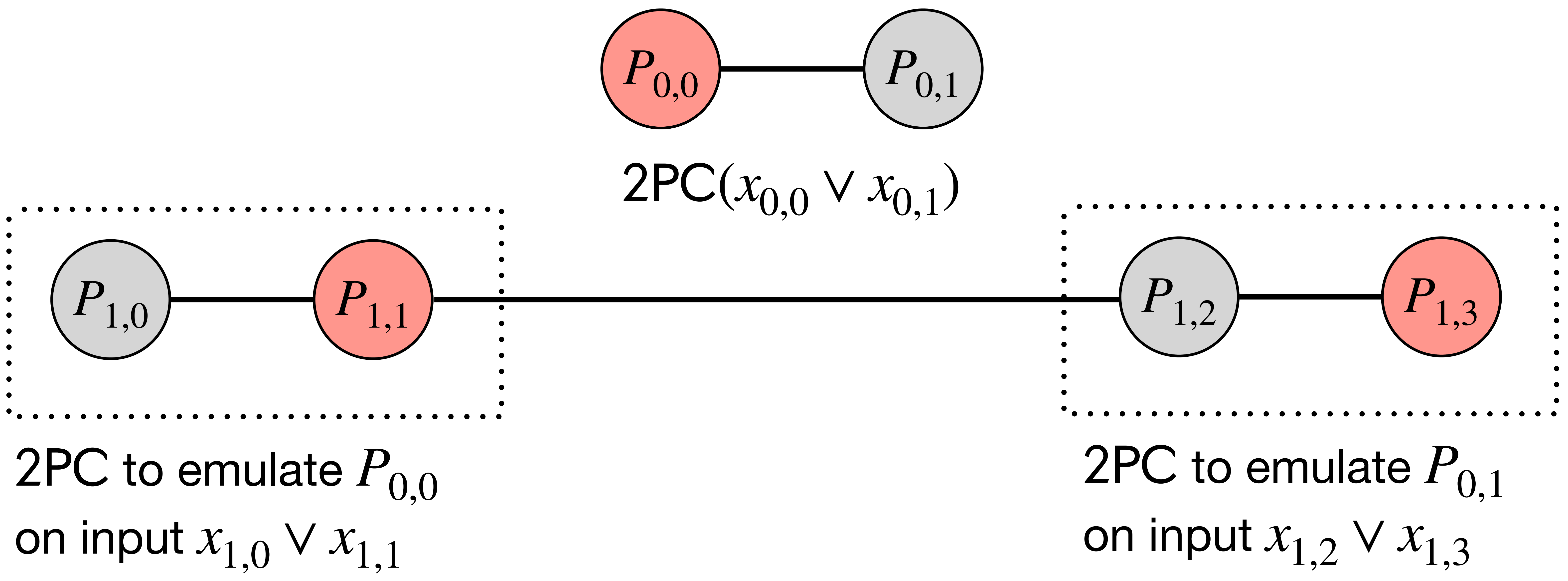

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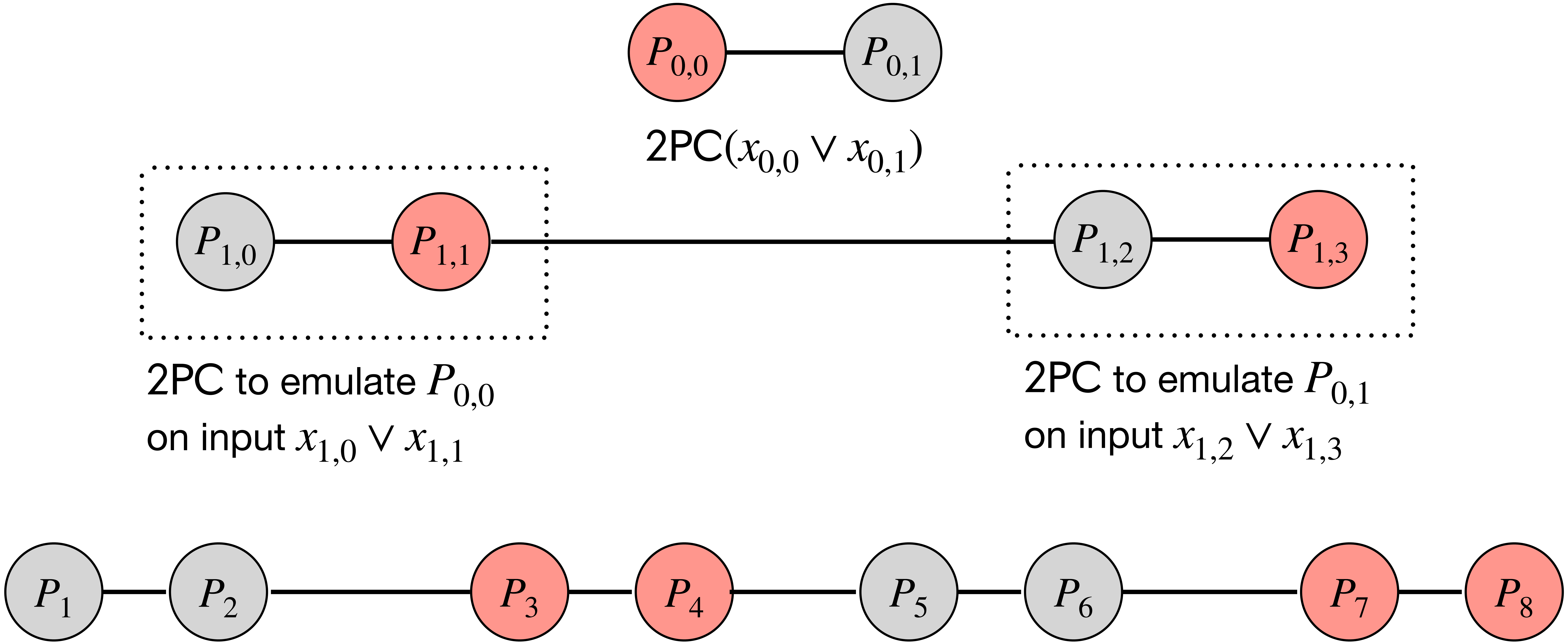

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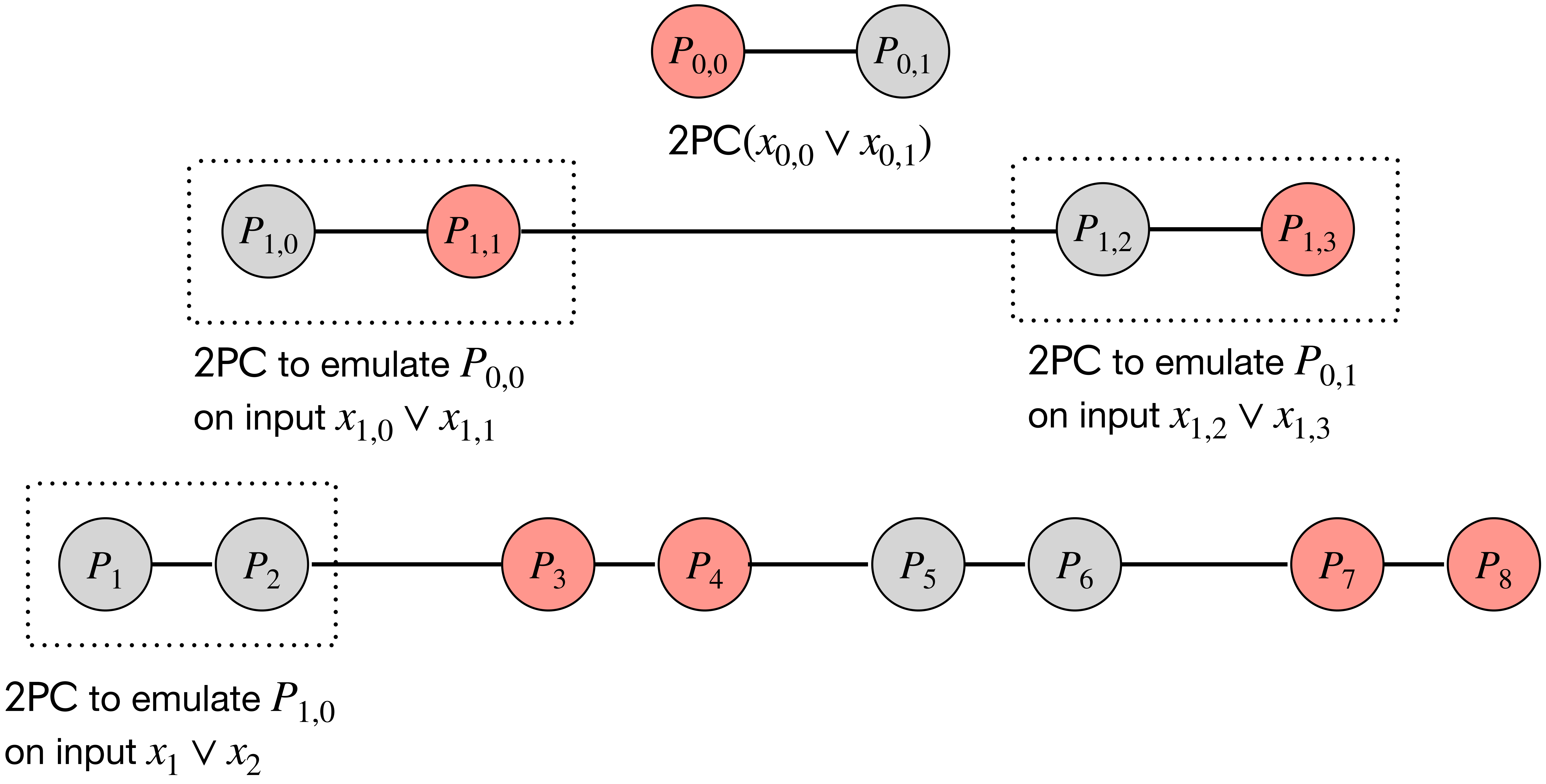

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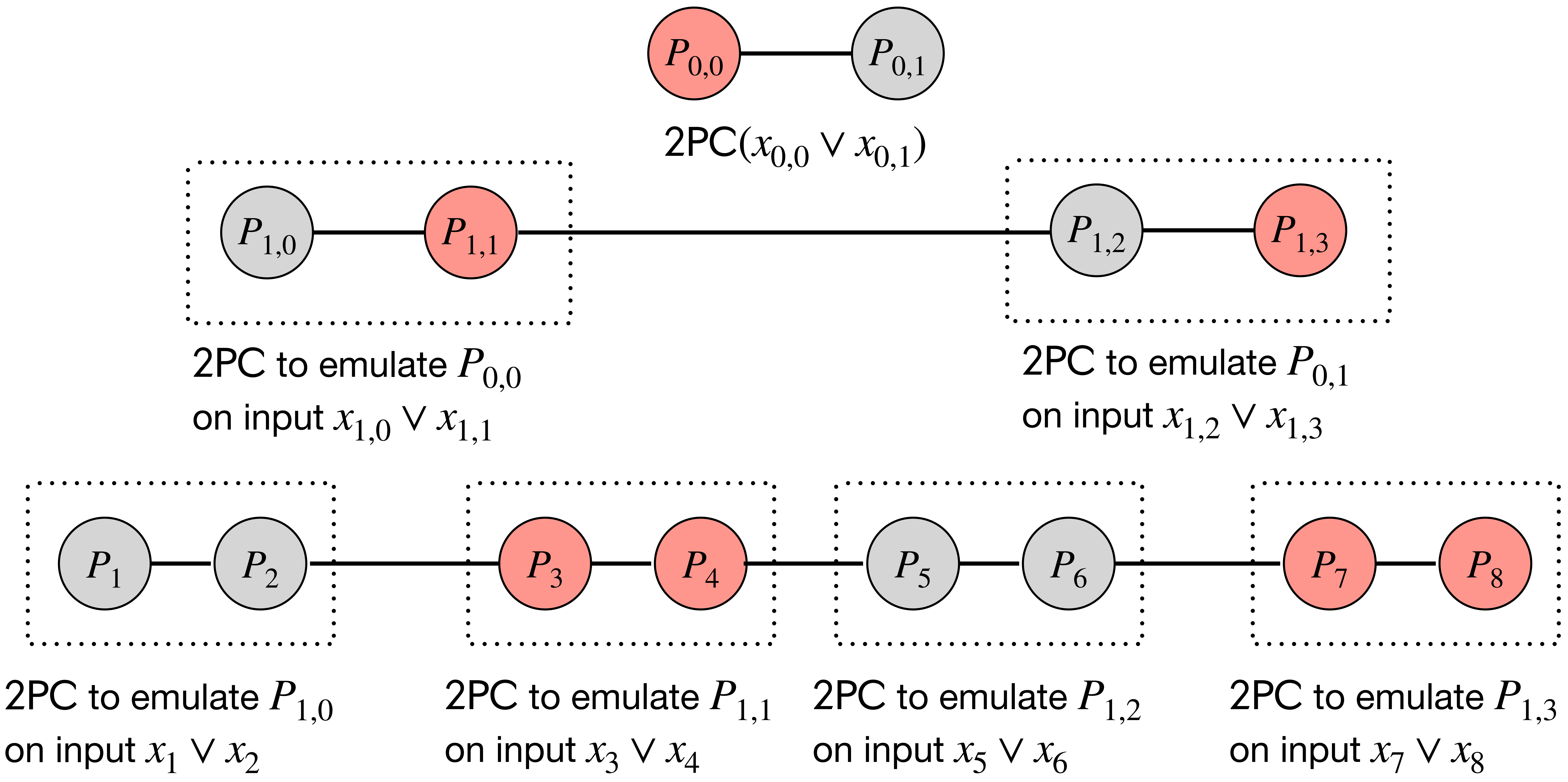

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  - Need to simulate messages for virtual party without entire input (not an issue for [MOR15])
    - If OR = 0, Sim knows all honest inputs = 0 (easy)
    - If OR = 1, Sim pretends all honest inputs = 1 -- OK since simulated messages should be independent of virtual party's input

Conclusion

- We build THC from constant round 2PC with constant overhead
    - First such result for all graph classes (even with constant round/overhead)
- We define Locally Simulatable MPC (may be of independent interest)
- **Still Open:**
    - THC for all graph classes with *only* (polynomial-round) Oblivious Transfer?

Thanks!

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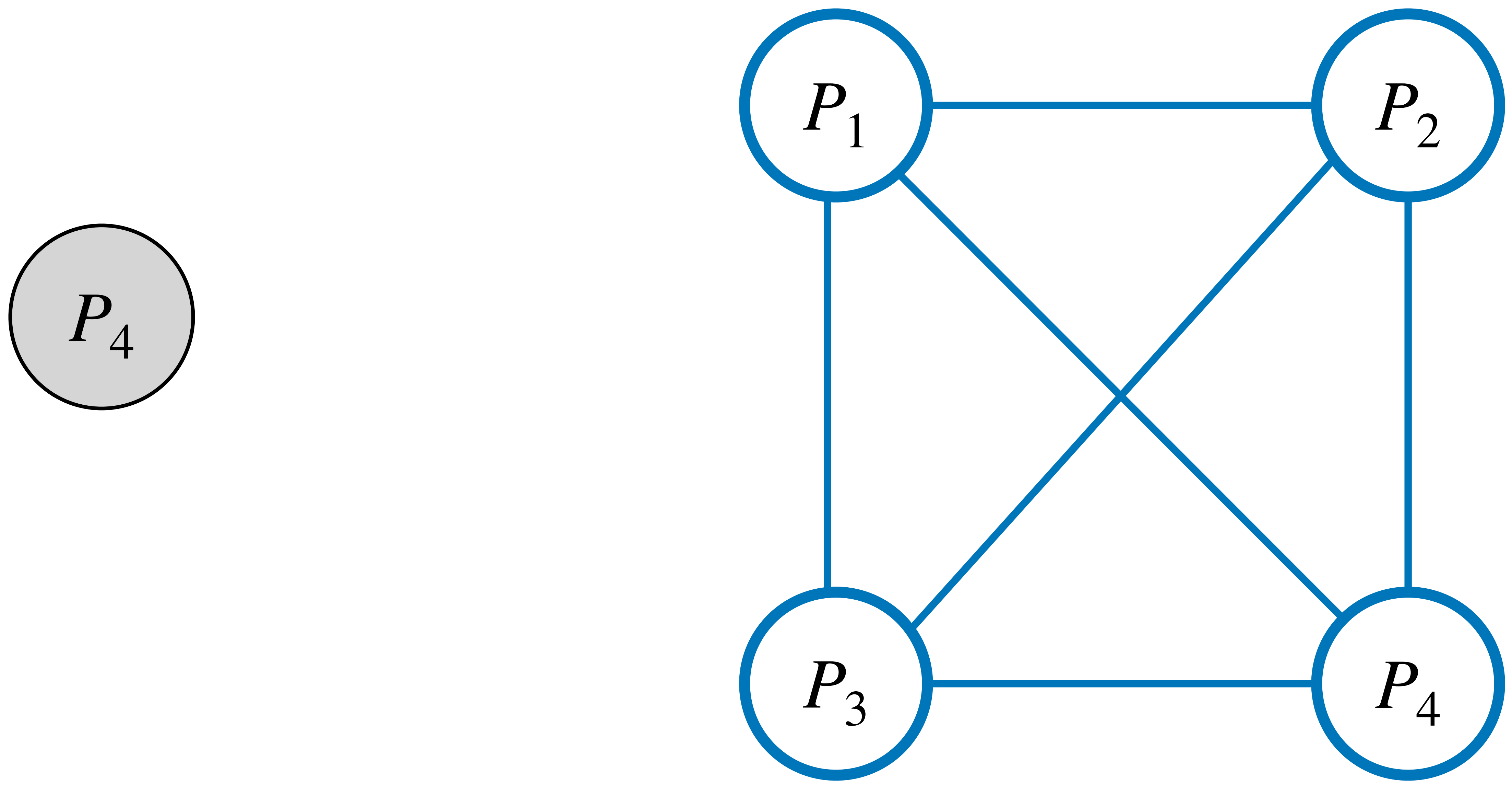
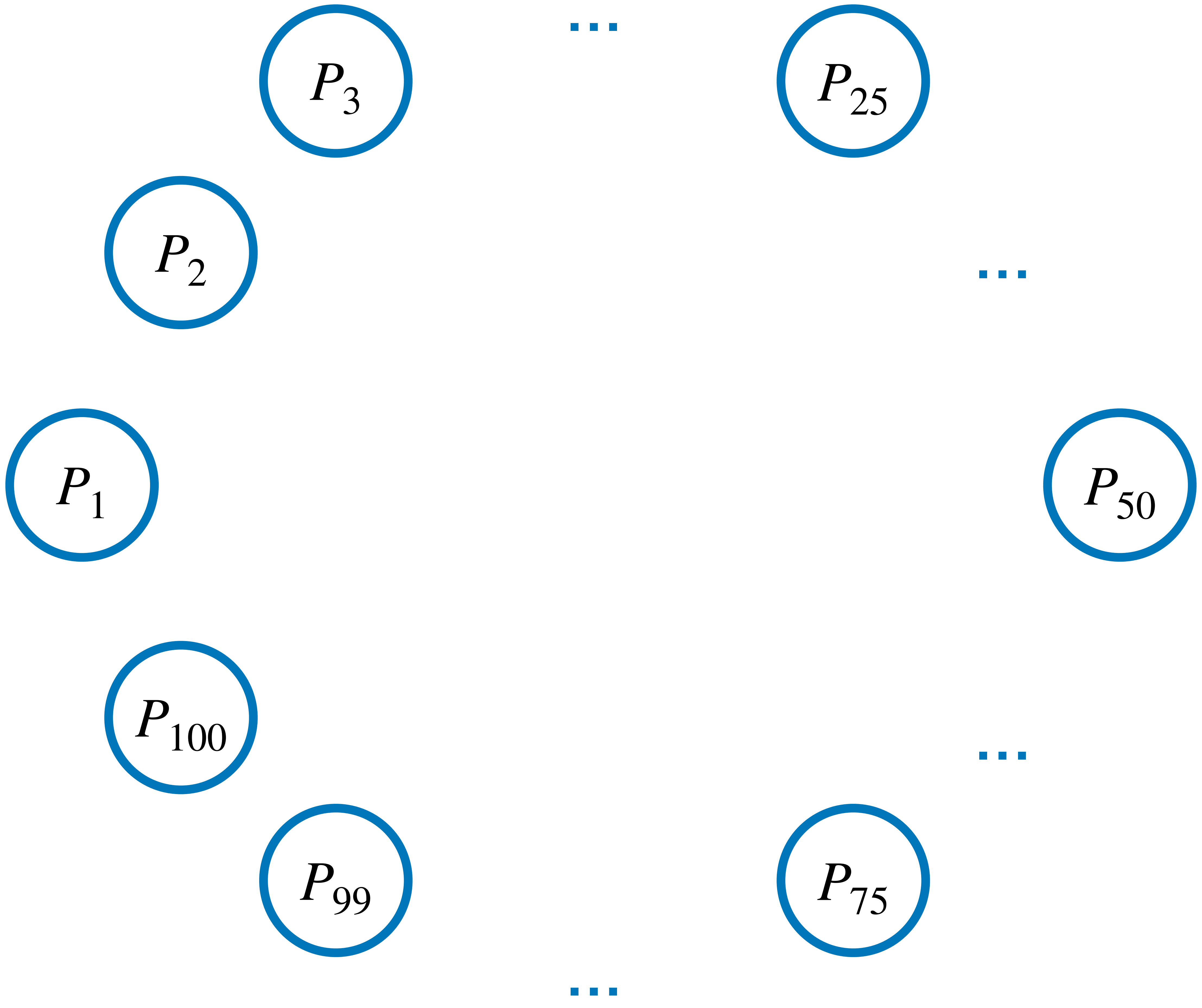
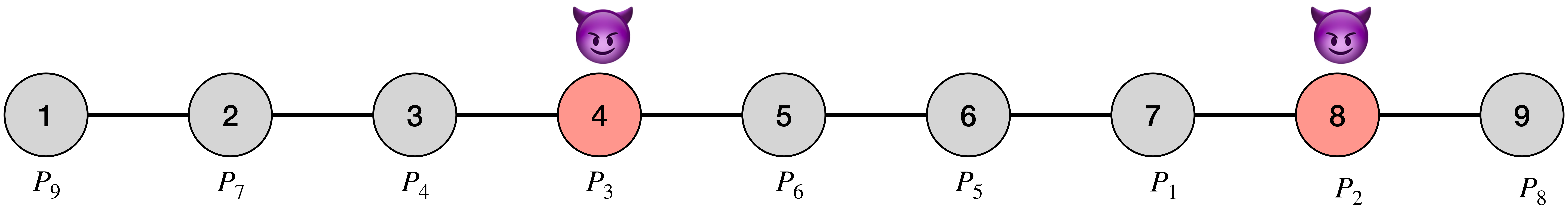


What is Topology-Hiding Computation (THC)?



Local Simulation on a Path



- Requirement: Can simulate views of P_3 and P_2 independently --

$$\{\mathsf{Sim}(x_3, y, (4\text{-}3\text{-}6)), \mathsf{Sim}(x_2, y, (1\text{-}2\text{-}8))\} \approx \mathsf{View}(\Pi)_{3,2}$$

- No Topology-Hiding (Directly) Needed!!

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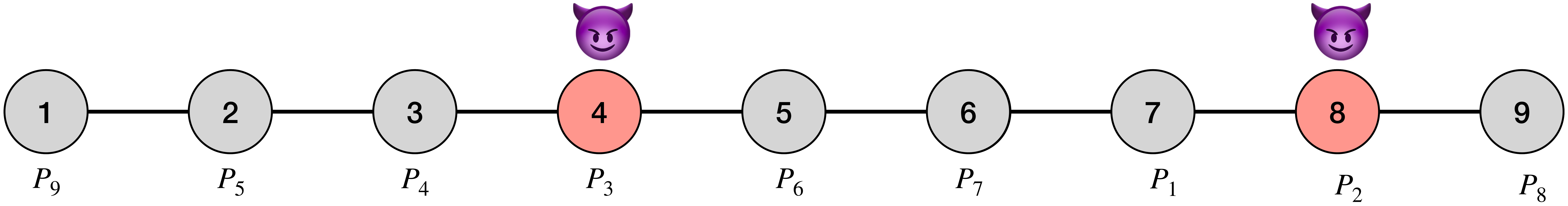

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