# Weakening Assumptions for Publicly-Verifiable Deletion

James Bartusek, Dakshita Khurana, Giulio Malavolta, Alexander Poremba, Michael Walter

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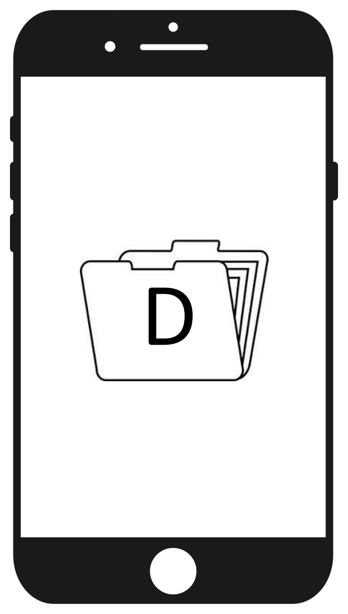
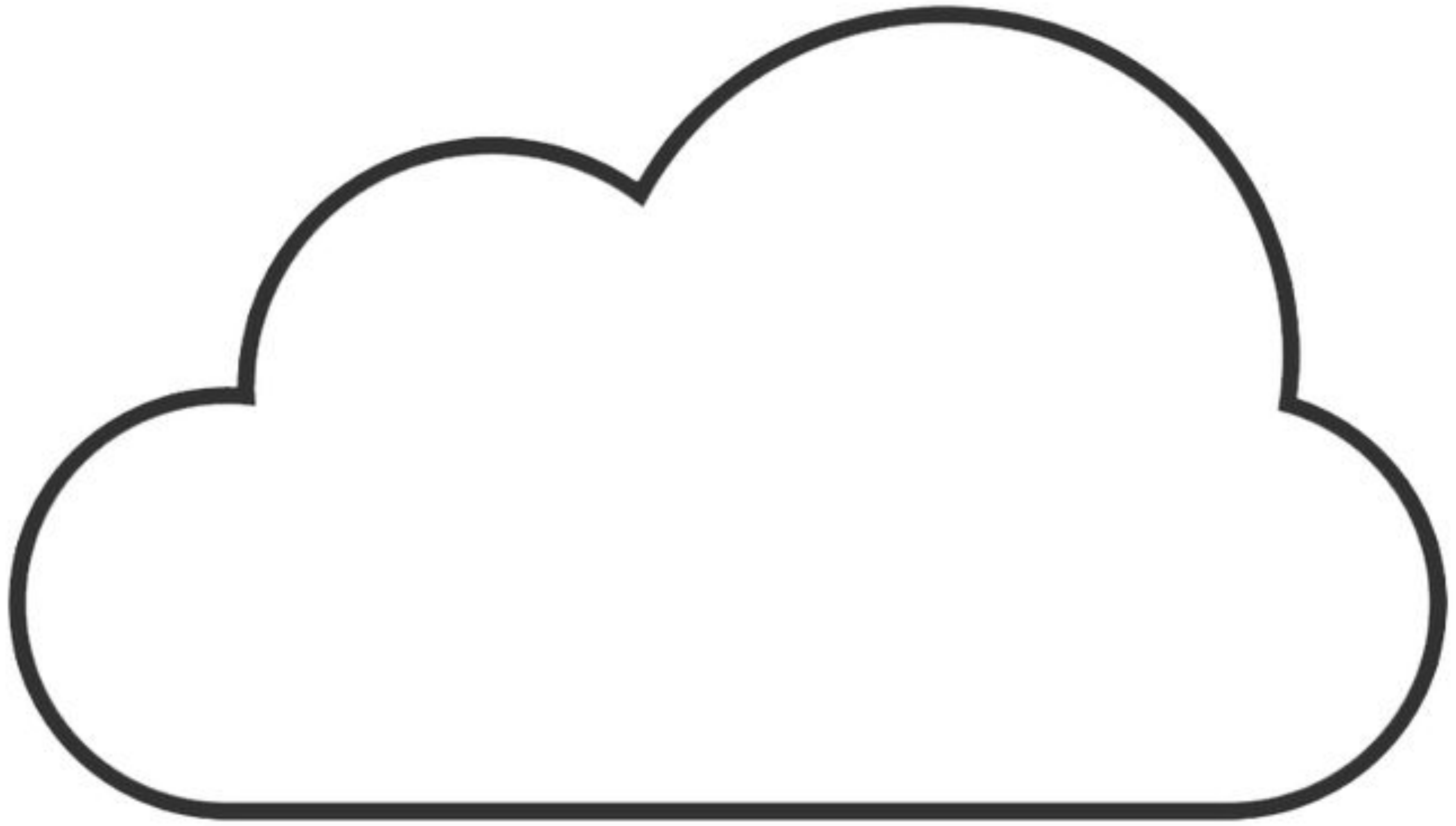
# How to Delete Data



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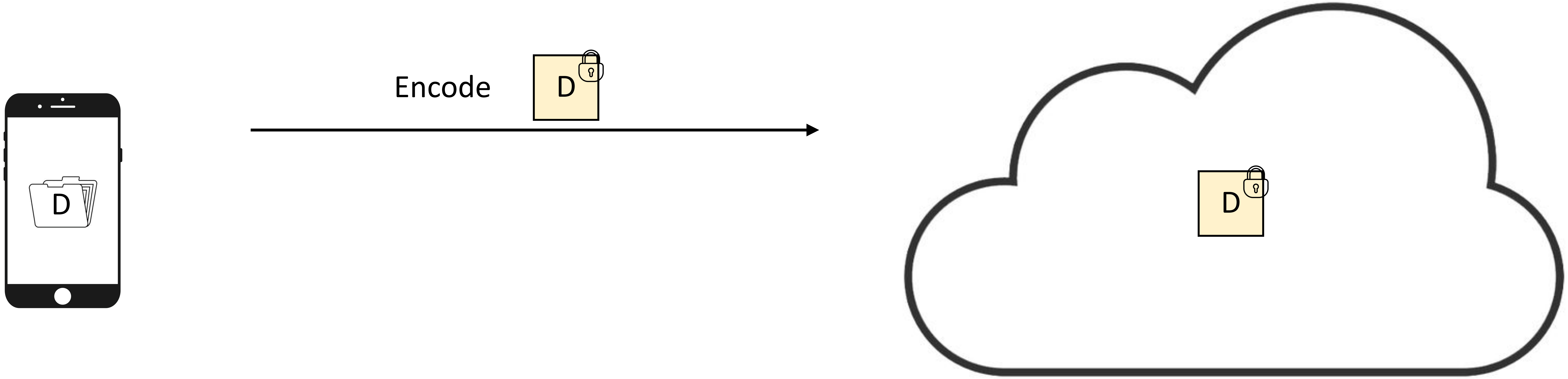

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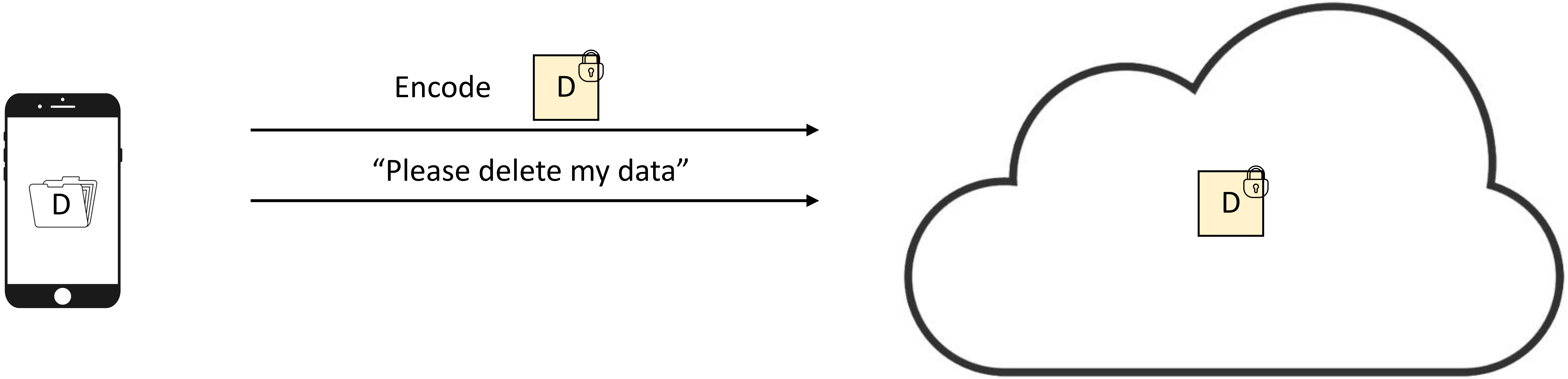

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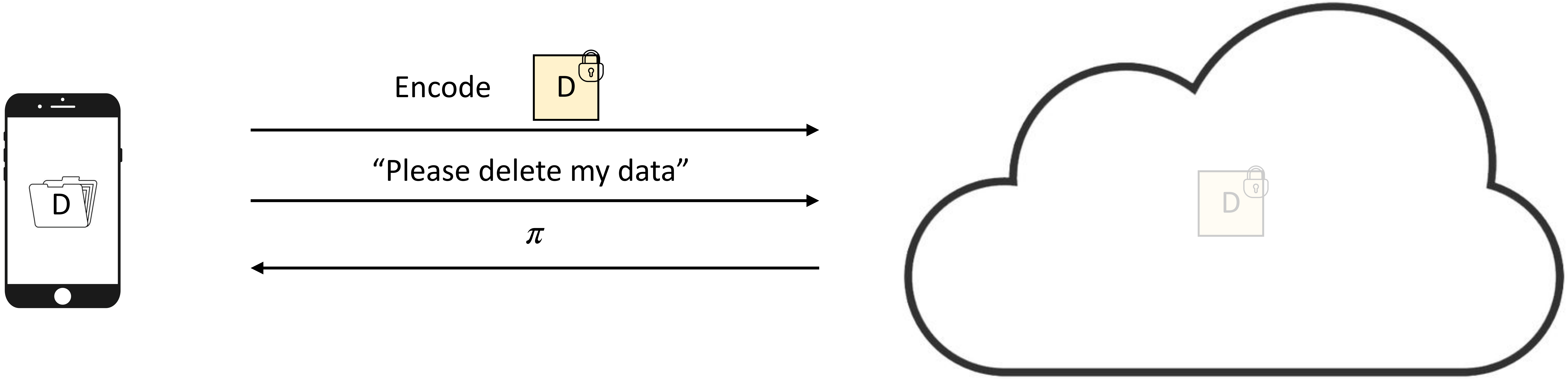

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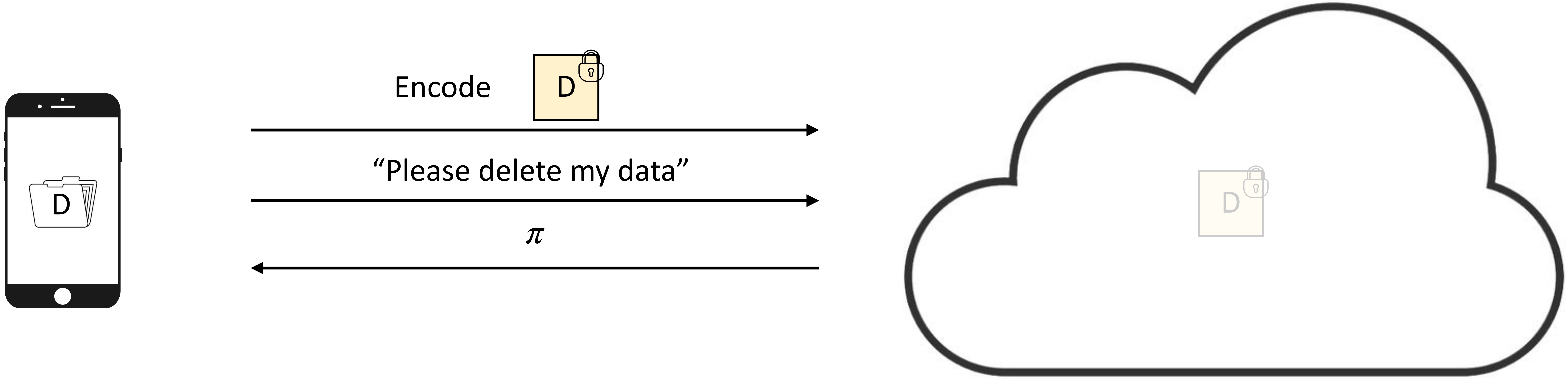


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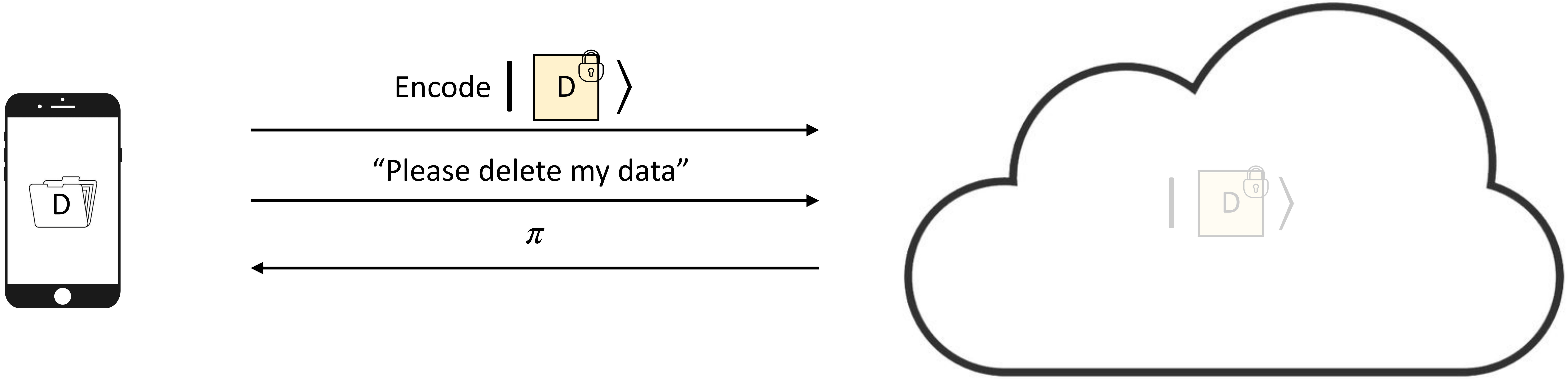


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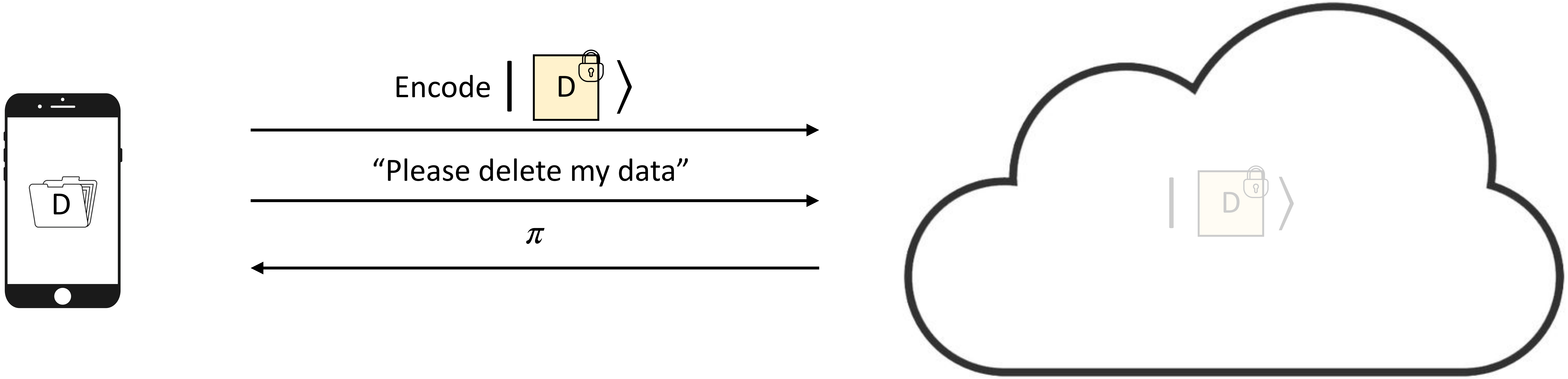


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$\{$ Commitments, SKE, PKE, ABE, WE, (Q)FHE, TimedE, … $\}$ with certified deletion

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A general compiler for X with certified deletion assuming:

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Technique inspired by recent works on Quantum PKE [BGH**M**SVW] [HKNY23] [**M**W23]

# Main Theorem

b = 0

$$\mathscr{A}\left\{ \begin{matrix} (y_0, y_1) \quad \mathsf{Enc}(x_0 \oplus x_1) \\ \frac{1}{\sqrt{2}}\left( |x_0> + |x_1> \right) \end{matrix} \right\}$$

$$\downarrow$$

$$x^* : \mathsf{OWF}(x^*) = y_0 \text{ OR } \mathsf{OWF}(x^*) = y_1$$

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$$\mathscr{A}\left\{ \begin{matrix} (y_0, y_1) \quad \mathsf{Enc}(x_0 \oplus x_1) \\ \frac{1}{\sqrt{2}}\left( |x_0> - |x_1> \right) \end{matrix} \right\}$$

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$$\boxed{\text{Claim: TD}\left(\mathrm{out}_0, \mathrm{out}_1\right) = \mathrm{negl}(\lambda)}$$

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$$\approx_c \text{(by semantic security)}$$

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- Open problems:
  - Construction using product states?
  - Even weaker assumptions?
  - More crypto with certified deletion?