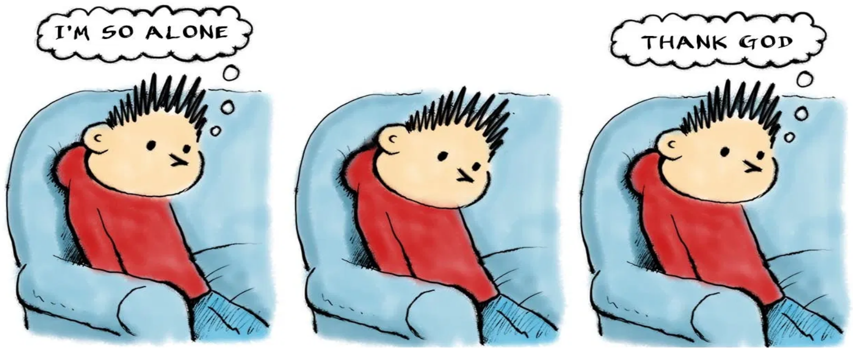
On the Round Complexity of Fully Secure Solitary MPC with Honest Majority

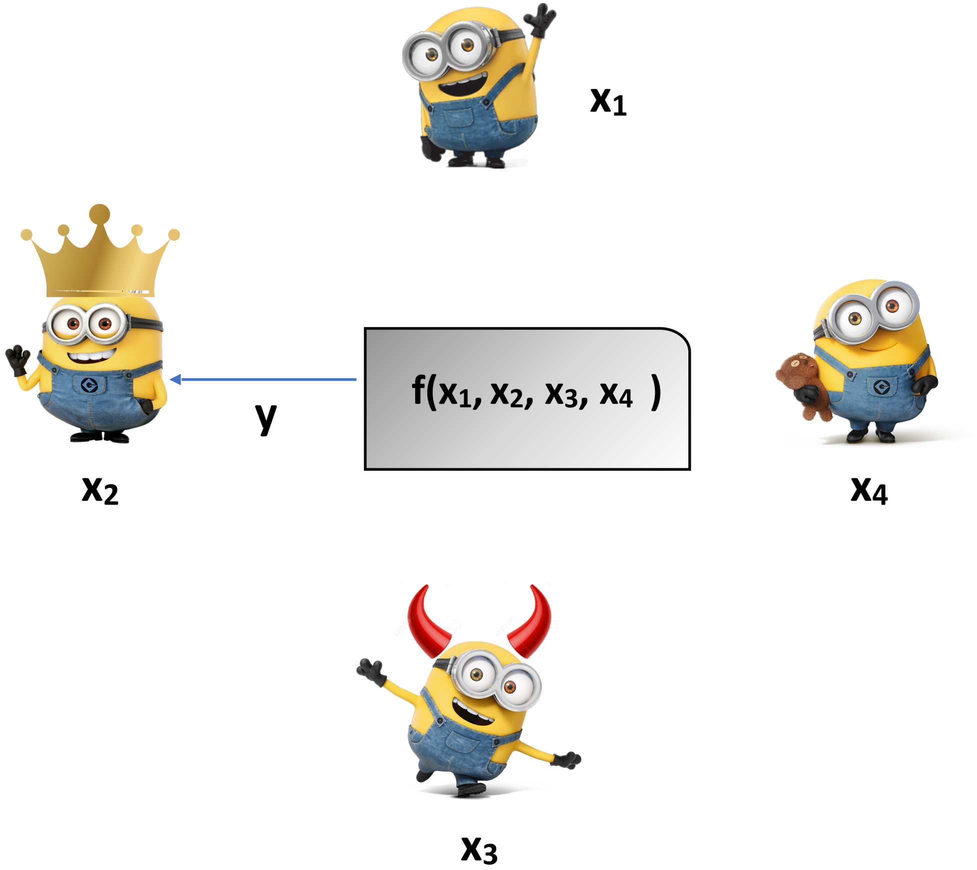
Saikrishna Badrinarayanan, Peihan Miao, Pratyay Mukherjee and **Divya Ravi**

TCC 2023

in BROWN SUPRA ORACLES UNIVERSITY OF AMSTERDAM

Solitary MPC





x_1

x_2

y

$f(x_1, x_2, x_3, x_4\ )$

x_4

x_3

MPC: allows group of parties to securely compute joint function on private inputs

Solitary MPC: only a *single* designated party obtains output

Standard MPC vs Solitary MPC

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Broadcast is necessary in first and second round

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t >= 3

Study special cases
t = 1, 2

5-Round upper bound with PKI (no broadcast)

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[GLS15] 2-round (non-solitary) G.O.D protocol using PKI and Broadcast

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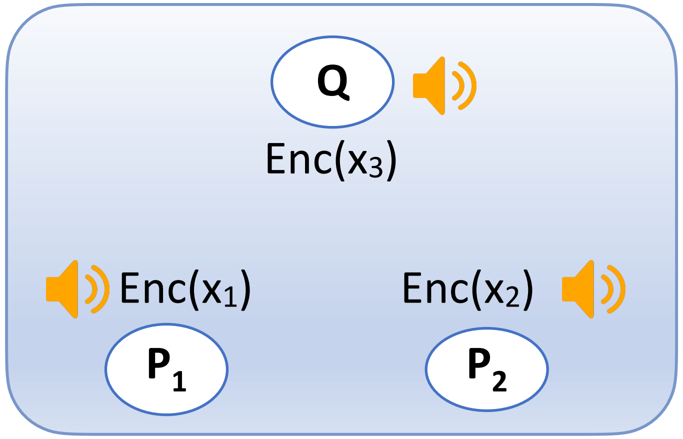

Q

Enc(x_3)

Enc(x_1)

Enc(x_2)

P_1

P_2

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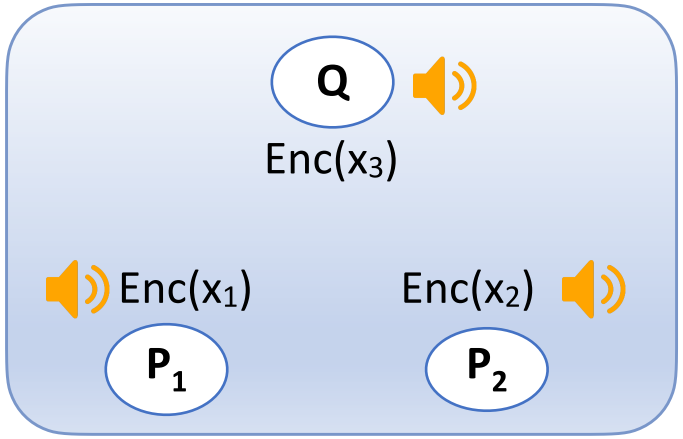

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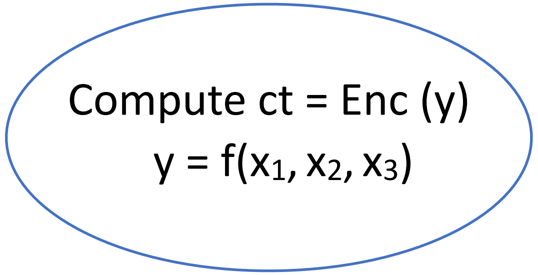

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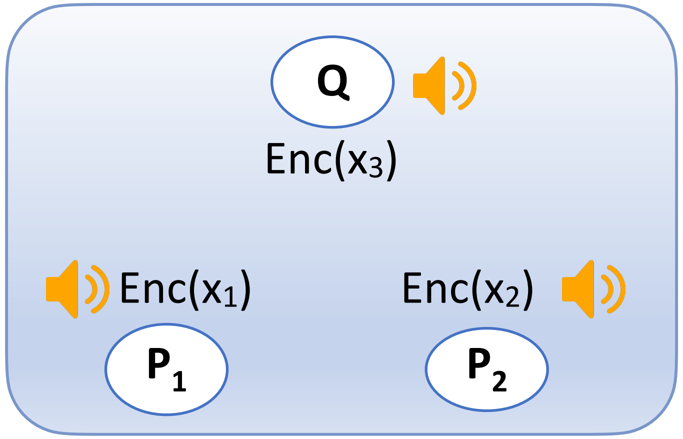

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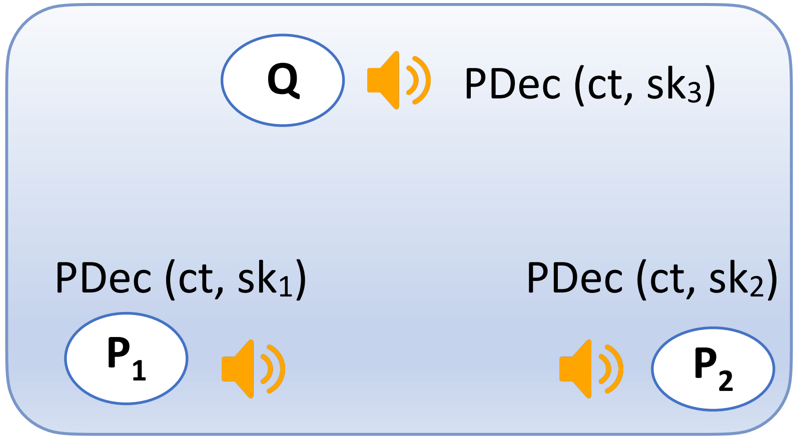
Compute ct = Enc (y)

y = f(x_1, x_2, x_3)



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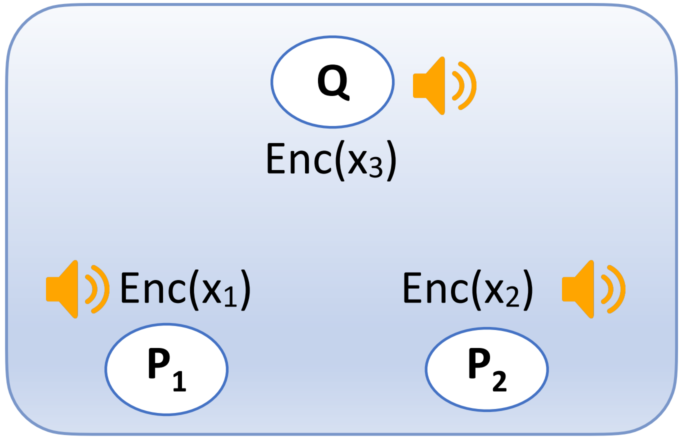

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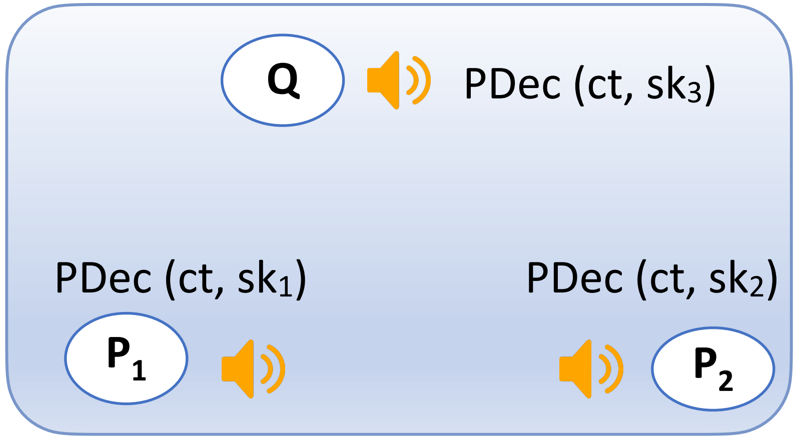

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$(t + 1)$ partial decryptions can be combined to get output

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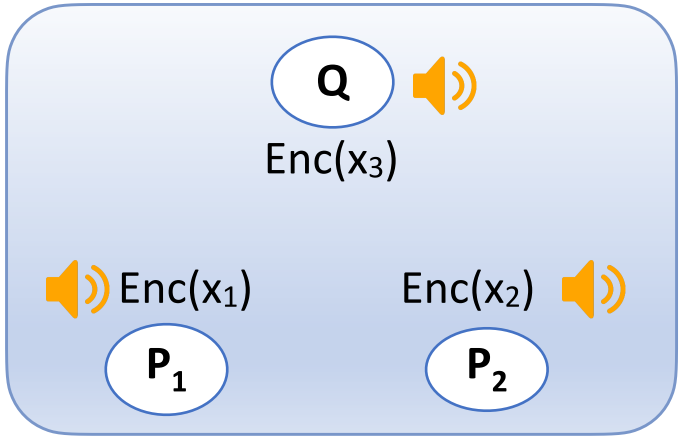

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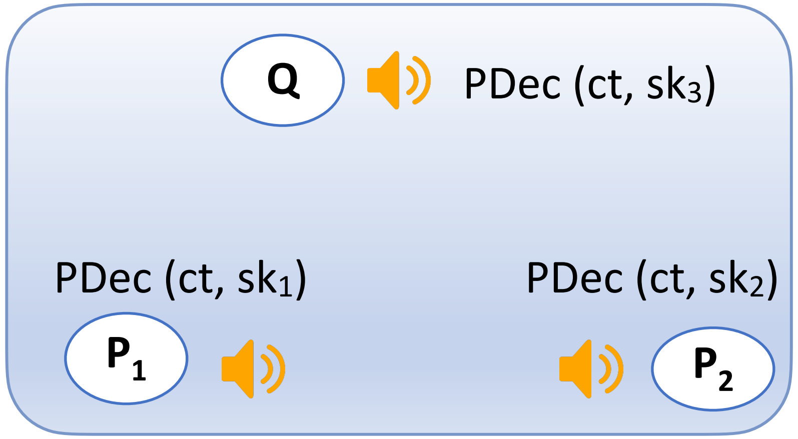

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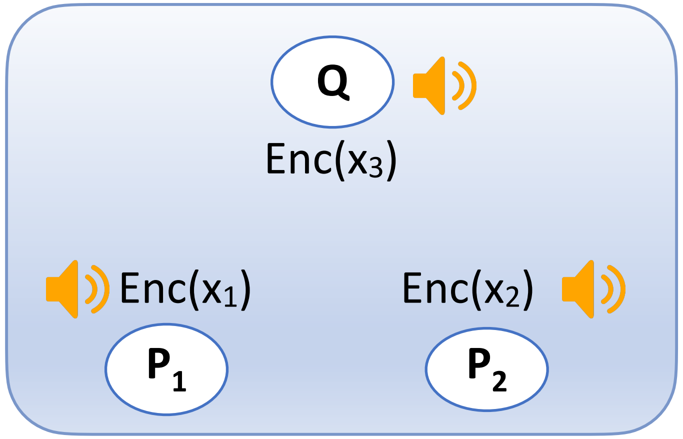

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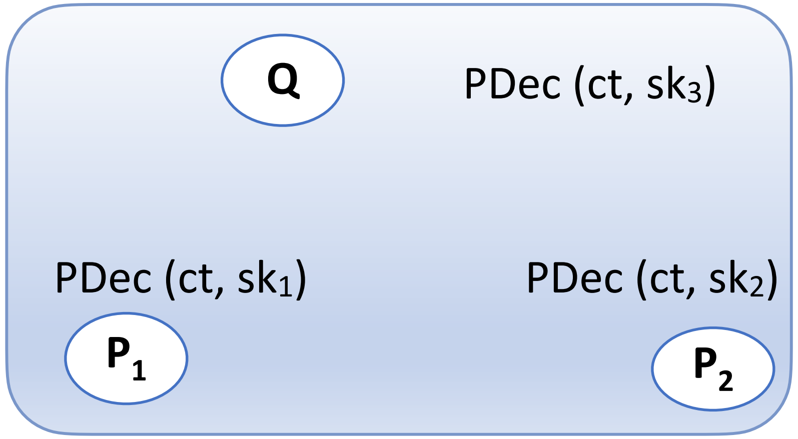

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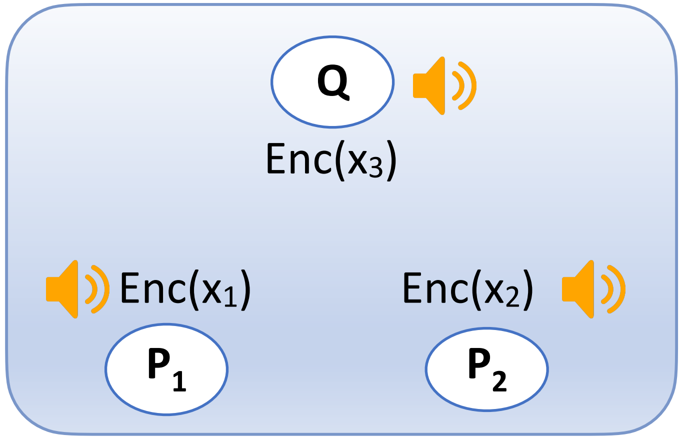

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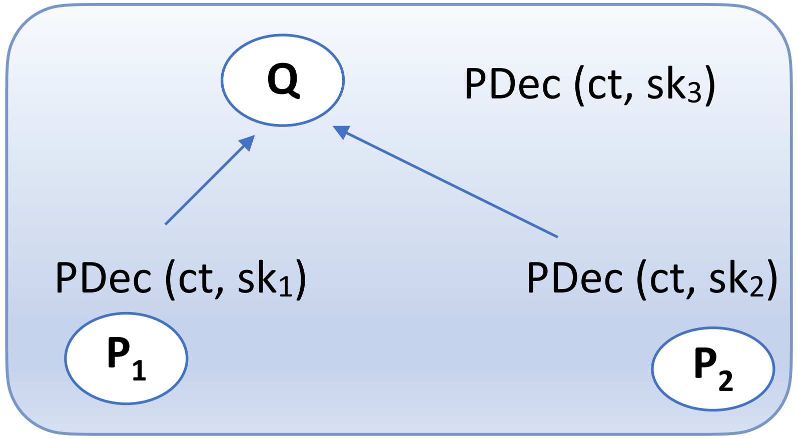

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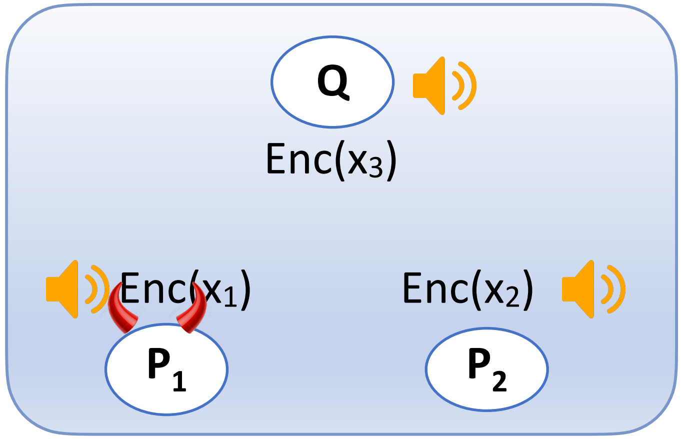

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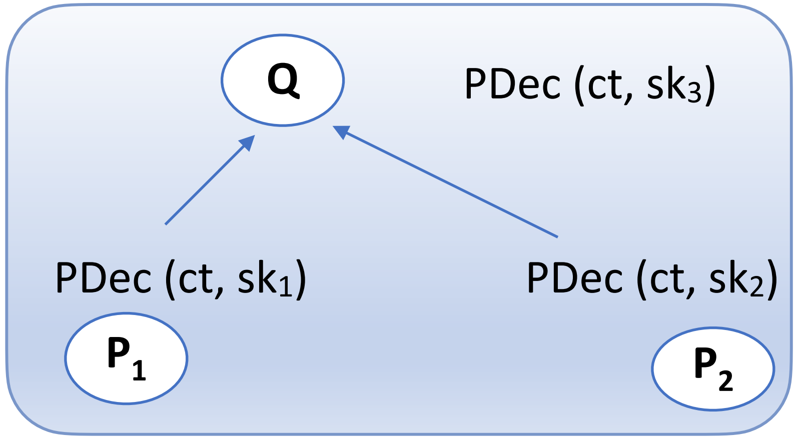

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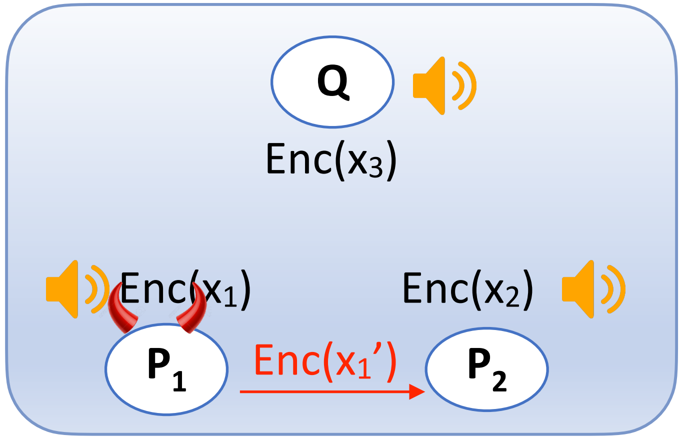

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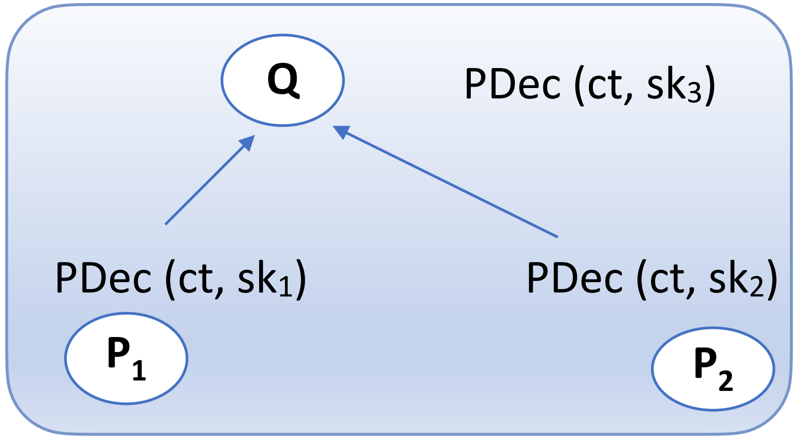

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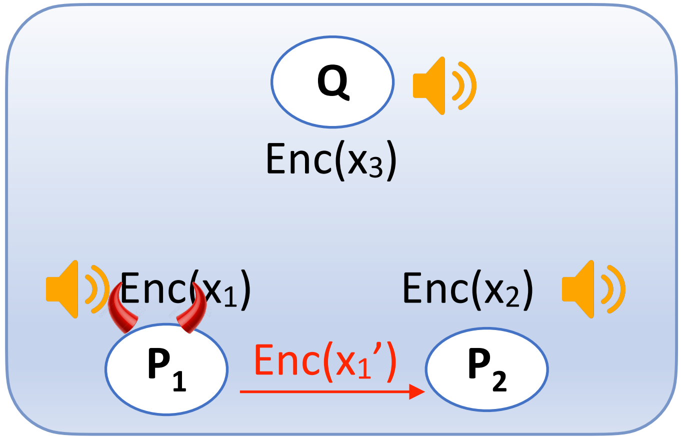

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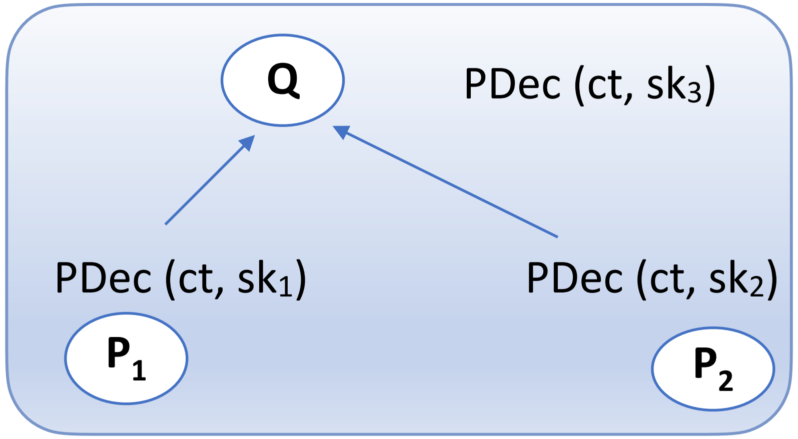

Need to agree!

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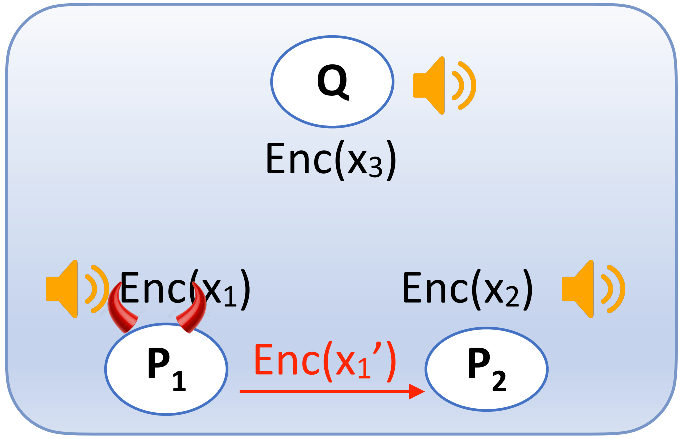

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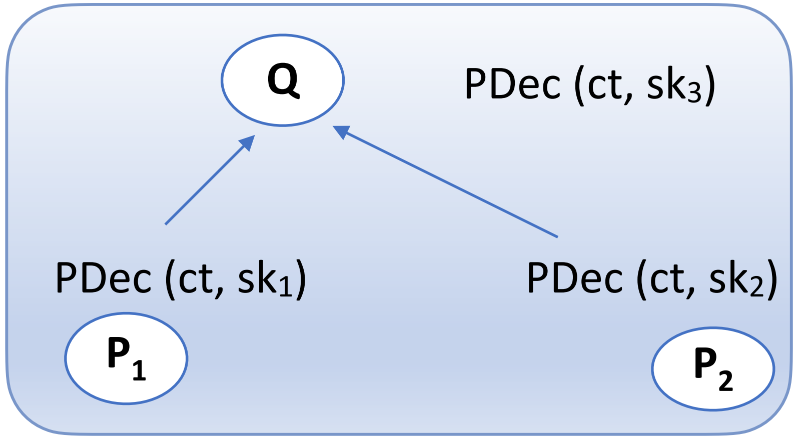

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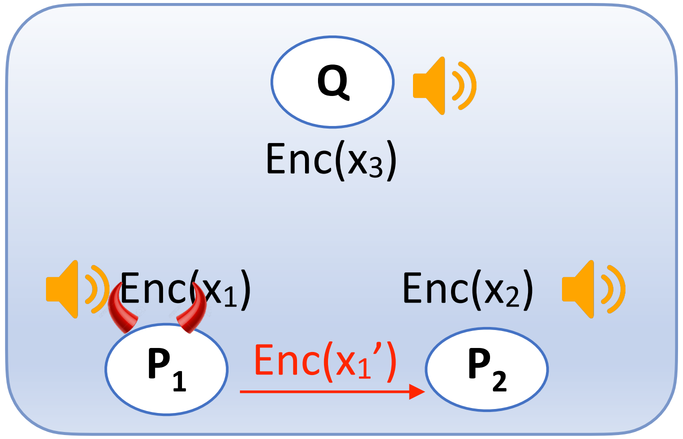

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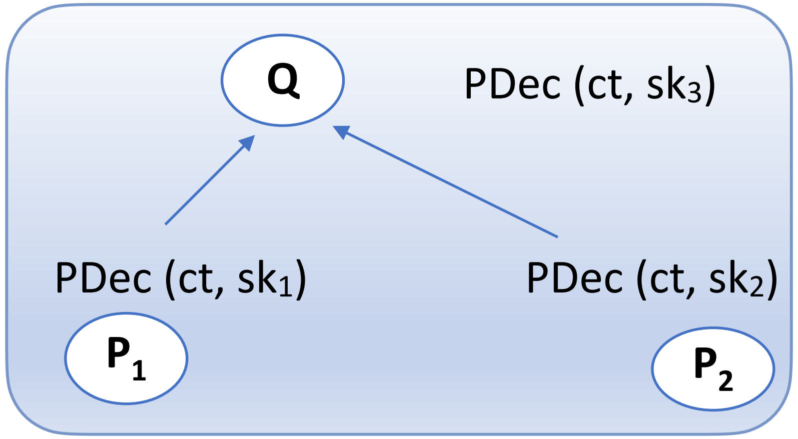

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Need to agree!

Broadcast would need t + 1 rounds

We only need to agree if Q is honest

5-Round upper bound with PKI (no broadcast)

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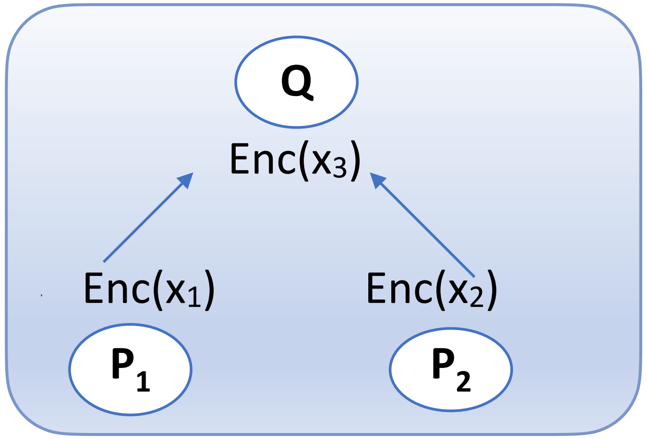

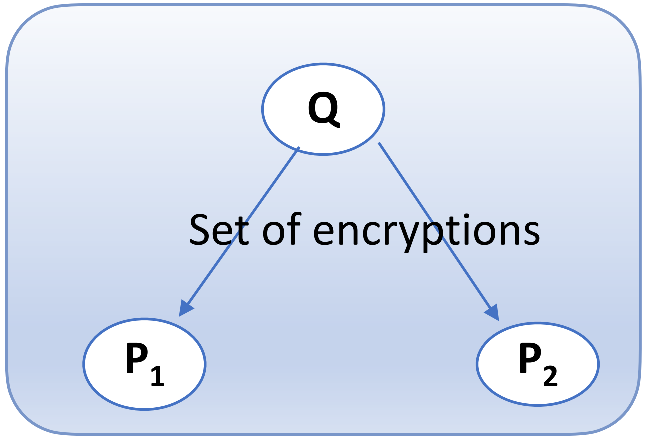

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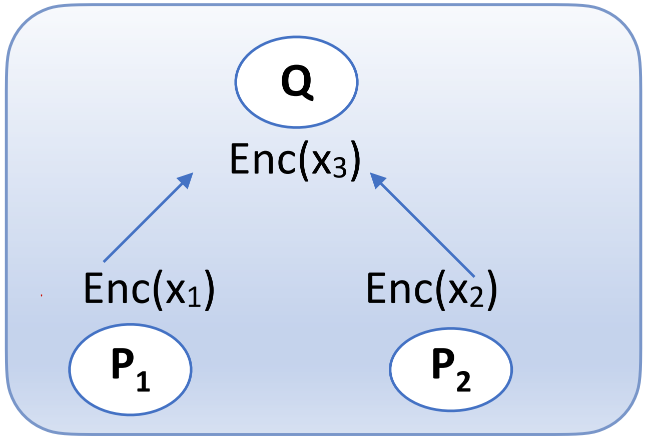

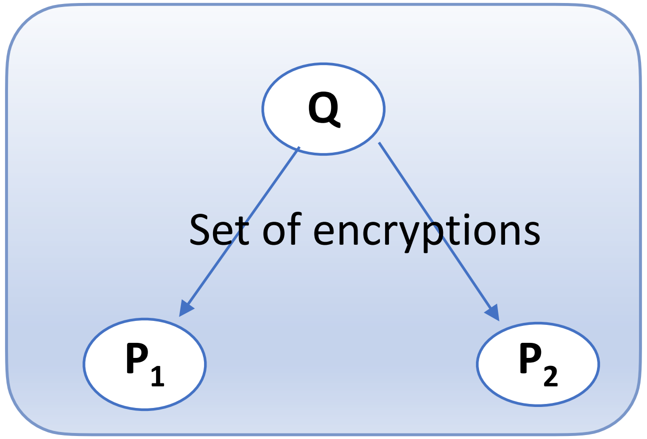

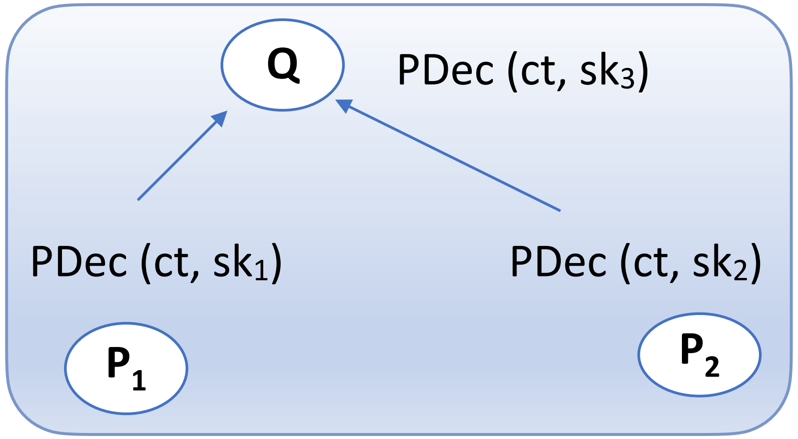

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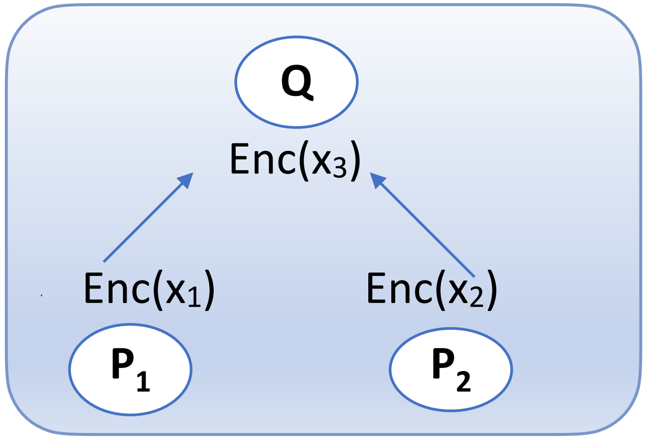

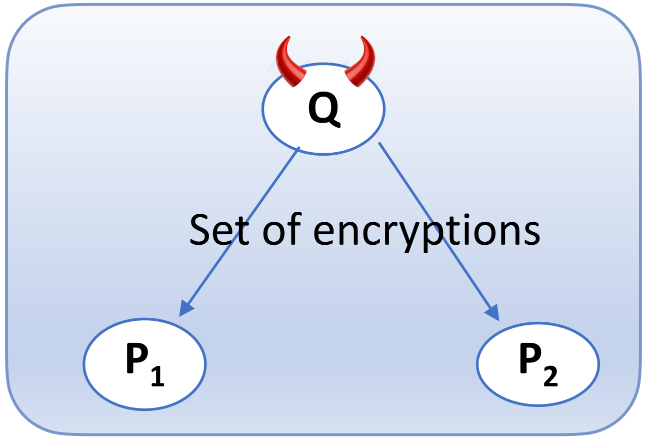

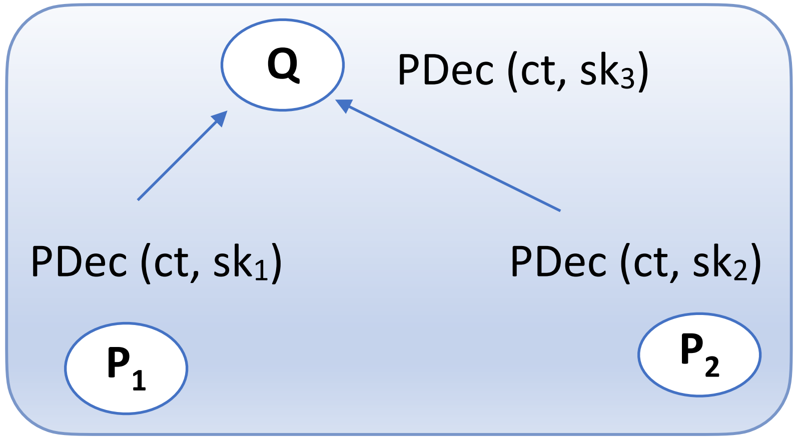

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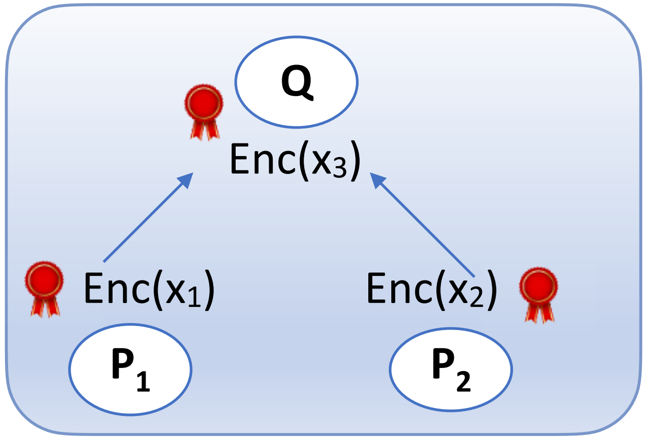
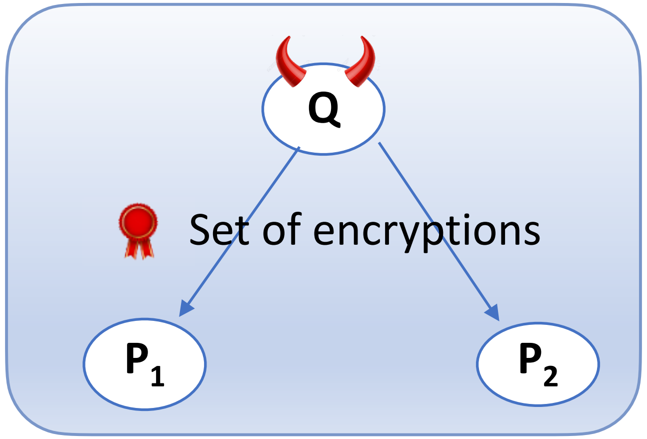
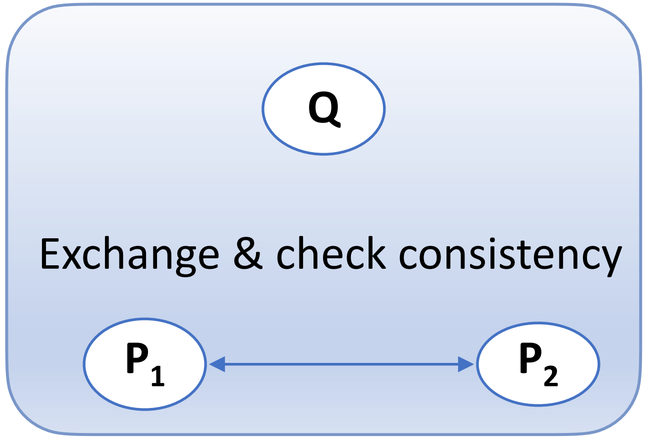
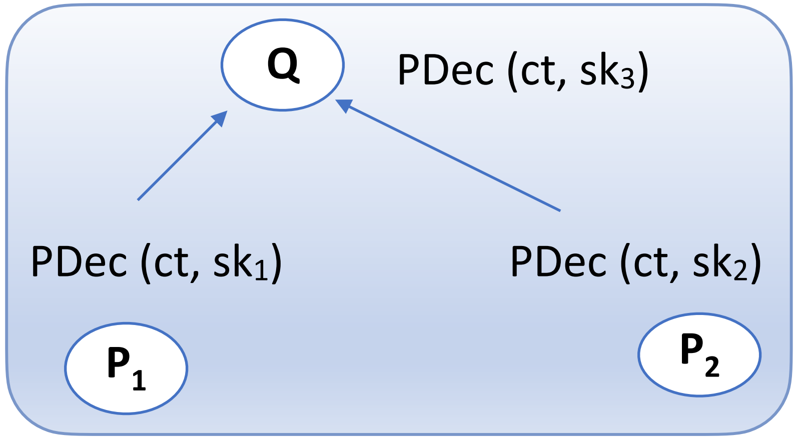




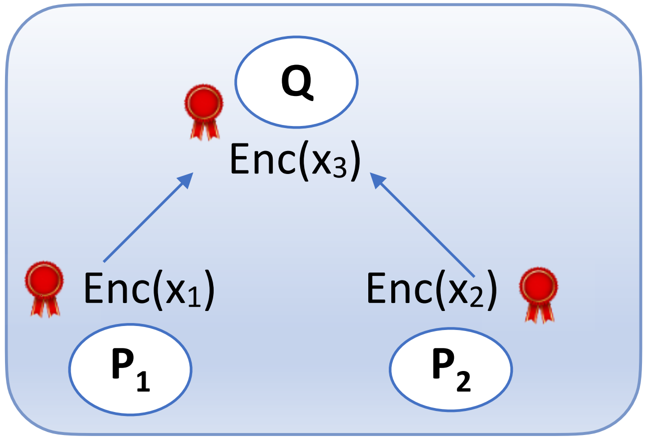


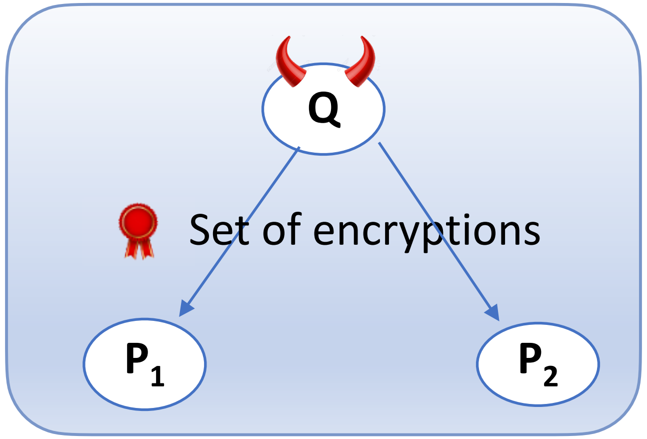


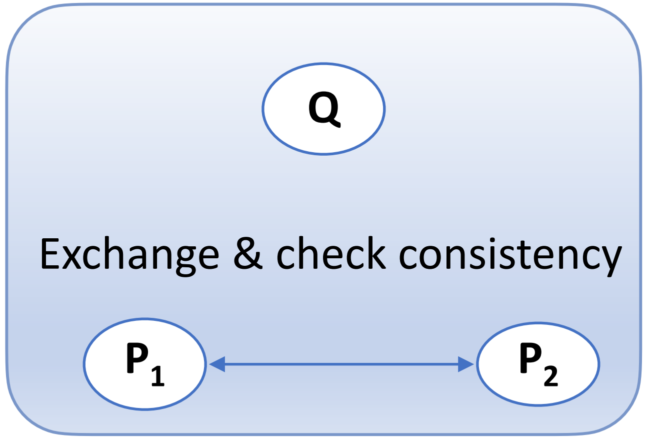
5-Round upper bound with PKI (no broadcast)

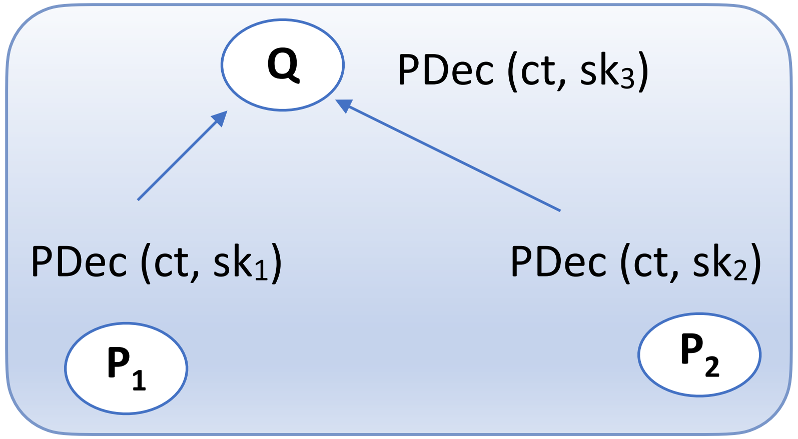
Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

n = 3, t = 1









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Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

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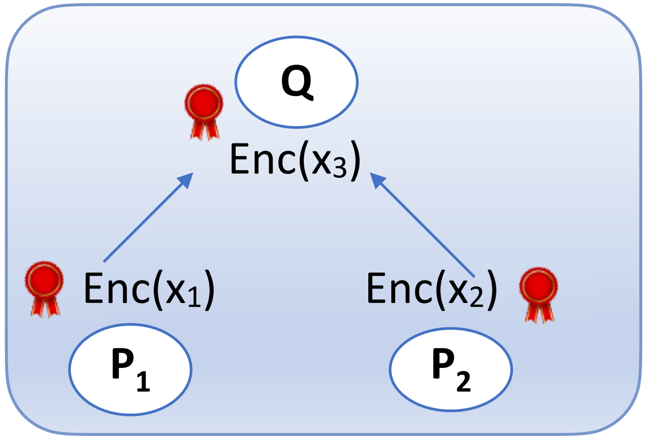


Q

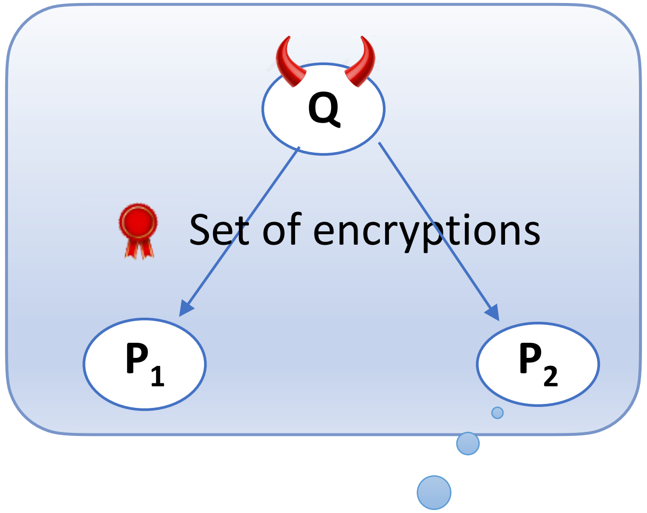
Enc(x_3)

Enc(x_1)

Enc(x_2)

P_1

P_2



Q

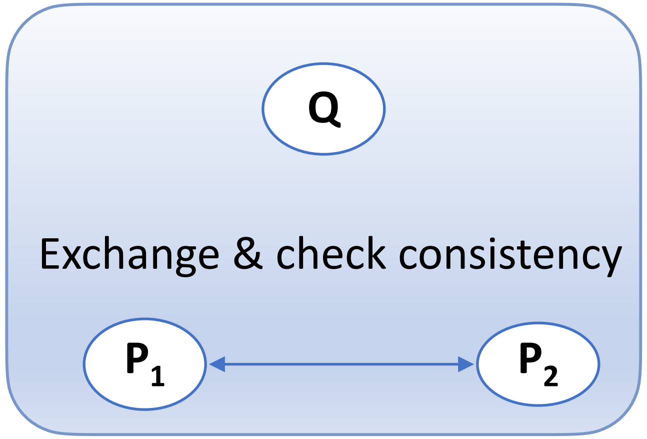
Set of encryptions

P_1

P_2

Q says P_1 did not send anything.

Is Q corrupt or P_1?

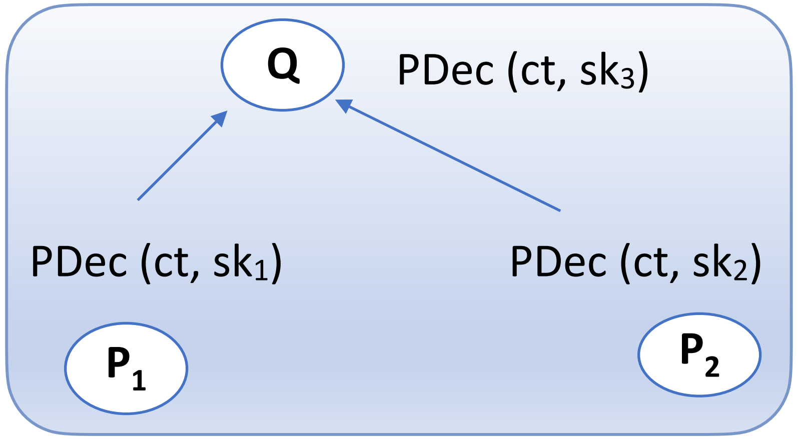


Q

Exchange & check consistency

P_1

P_2



Q

PDec (ct, sk_3)

PDec (ct, sk_1)

PDec (ct, sk_2)

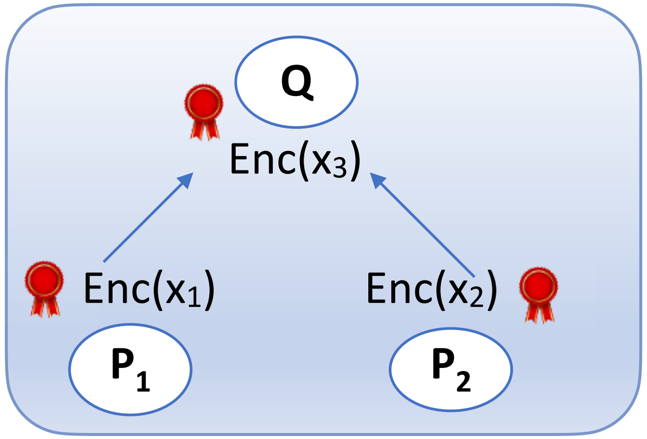
P_1

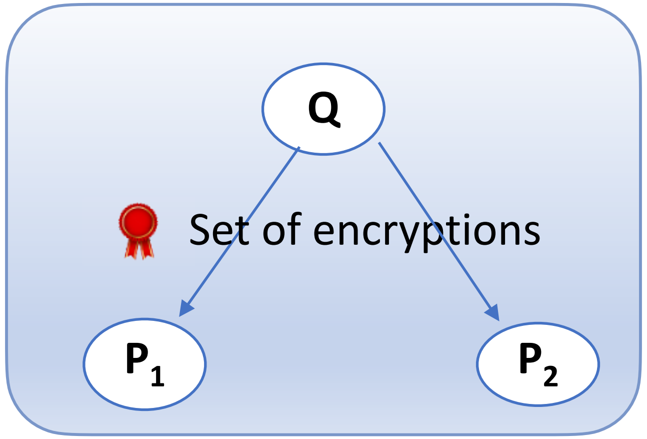
P_2

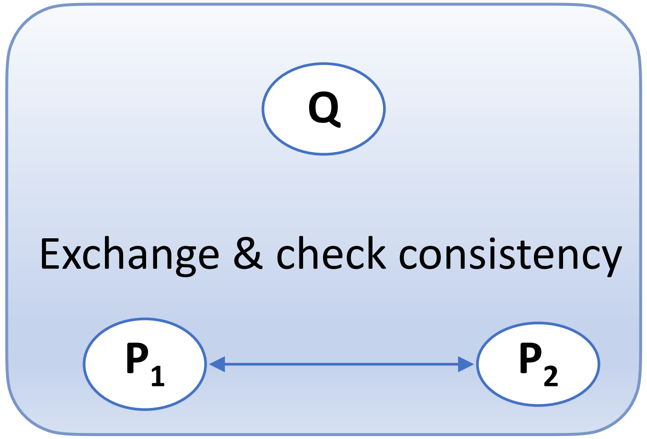
5-Round upper bound with PKI (no broadcast)

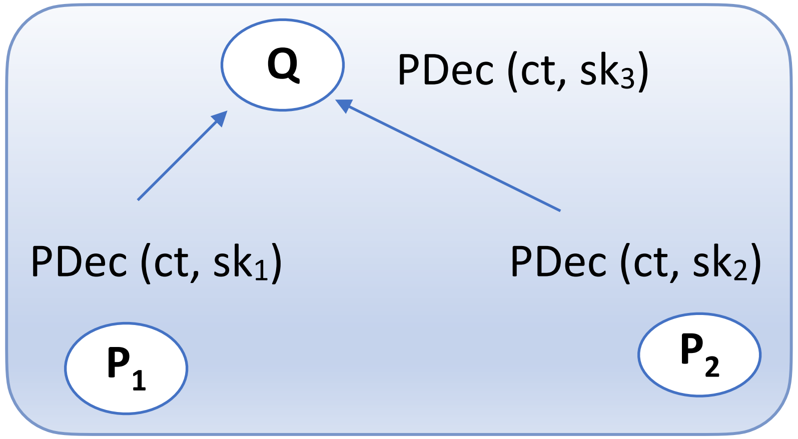
Decentralized threshold FHE Setup: pk, sk_1, sk_2, sk_3

n = 3, t = 1





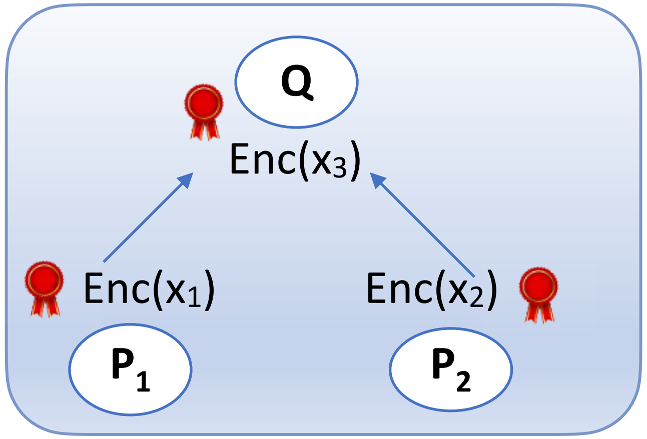


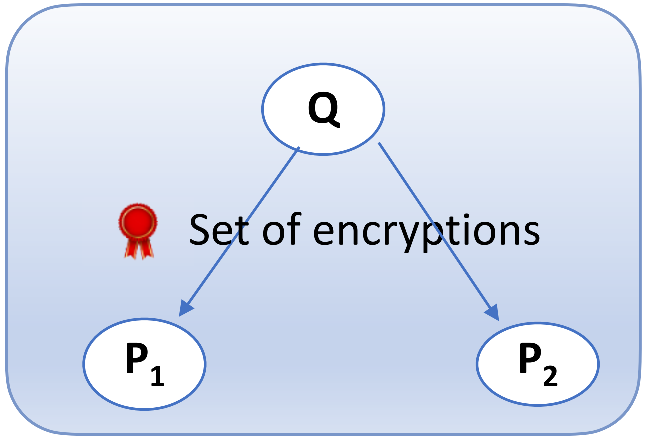


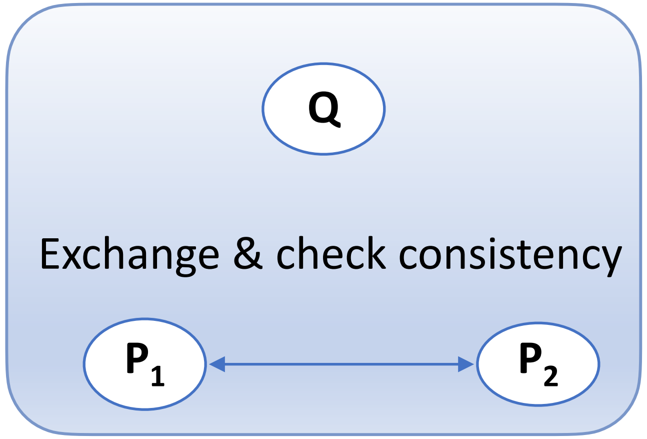
5-Round upper bound with PKI (no broadcast)

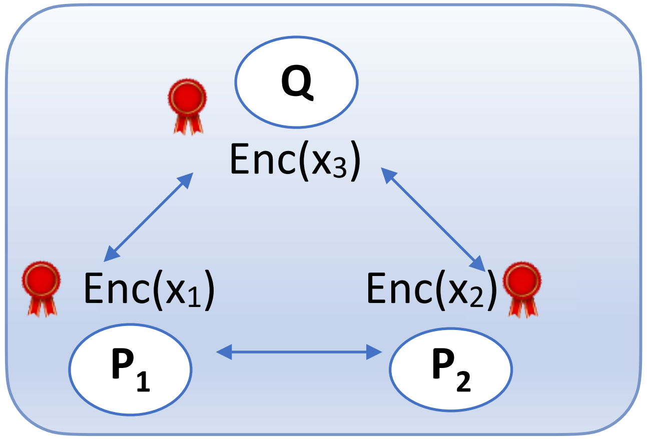
Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

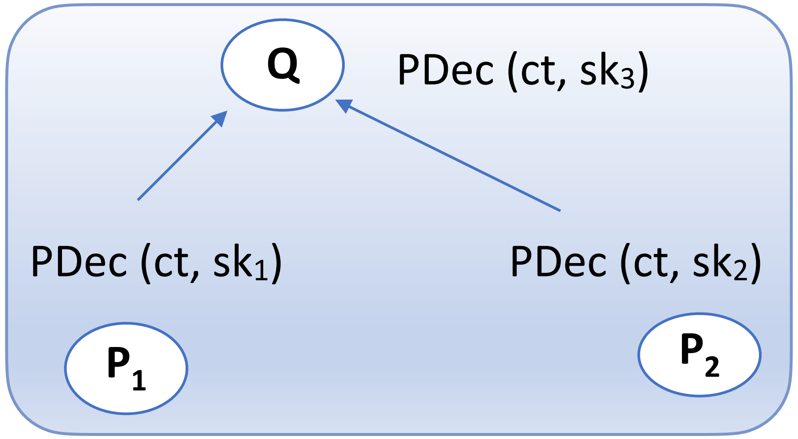
n = 3, t = 1







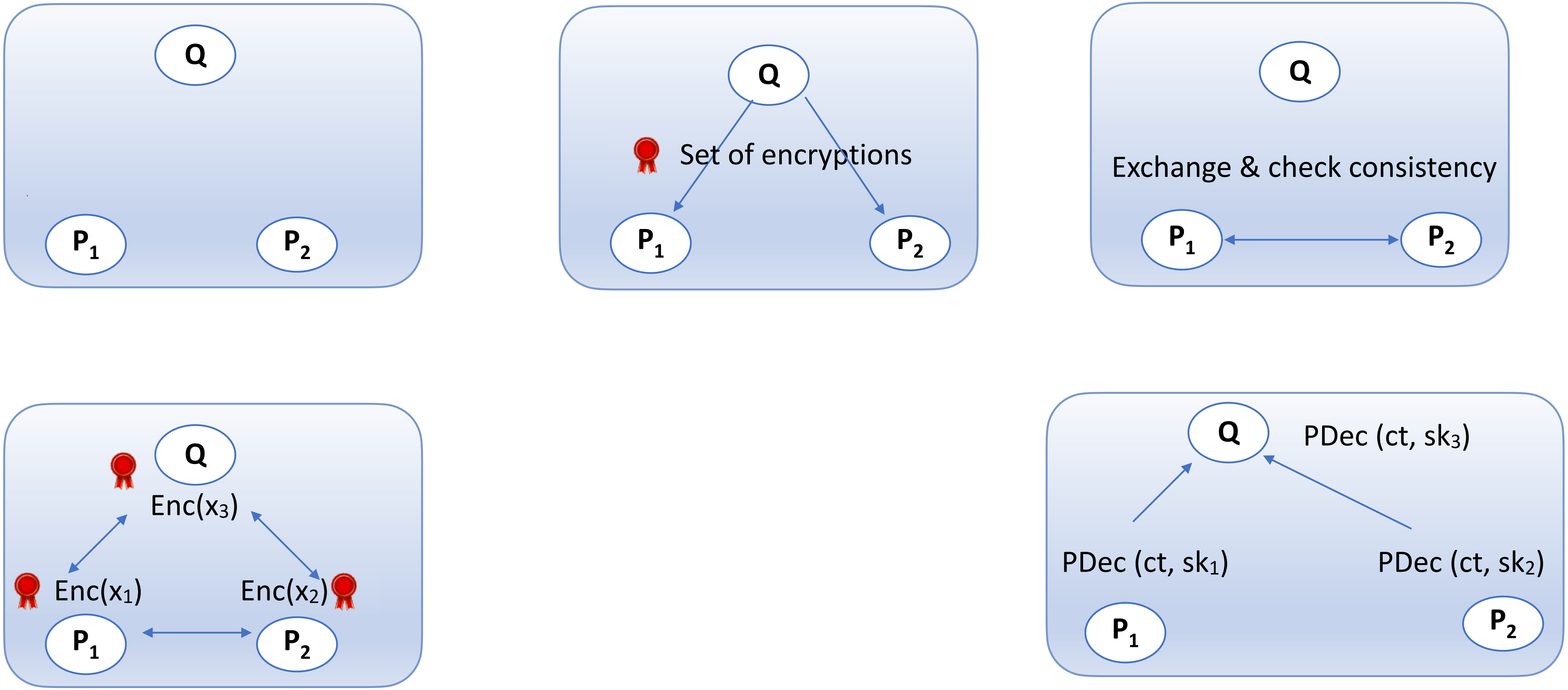




5-Round upper bound with PKI (no broadcast)

Decentralized threshold FHE Setup: pk, sk_1, sk_2, sk_3

n = 3, t = 1



Set of encryptions

Exchange & check consistency

Enc(x_3)

Enc(x_1)

Enc(x_2)

PDec (ct, sk_3)

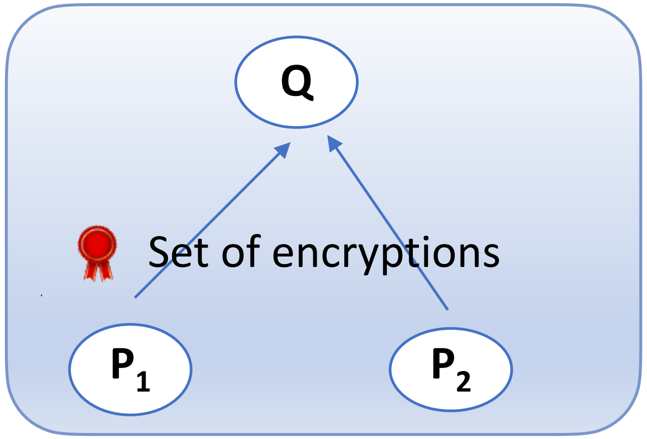
PDec (ct, sk_1)

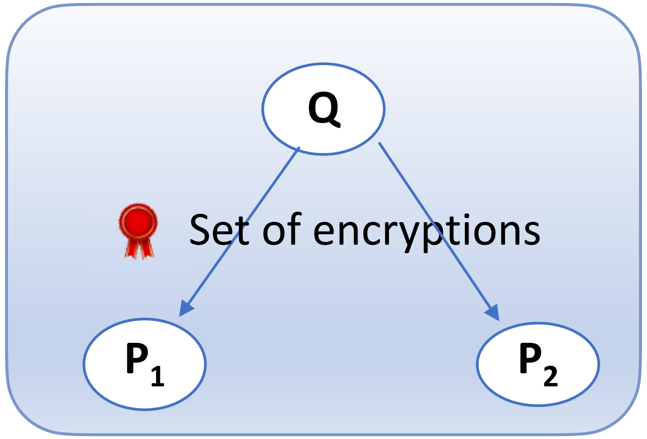
PDec (ct, sk_2)

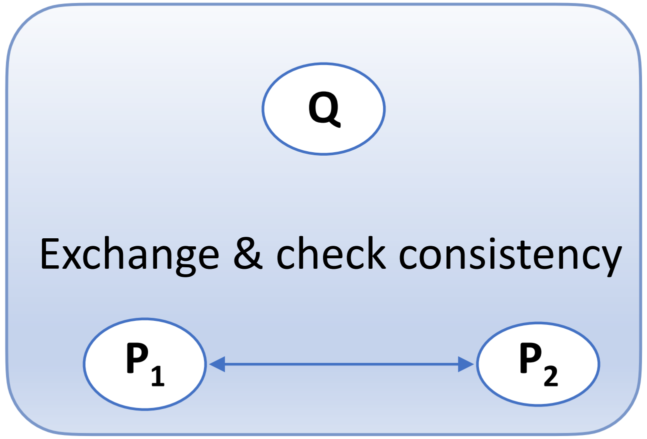
5-Round upper bound with PKI (no broadcast)

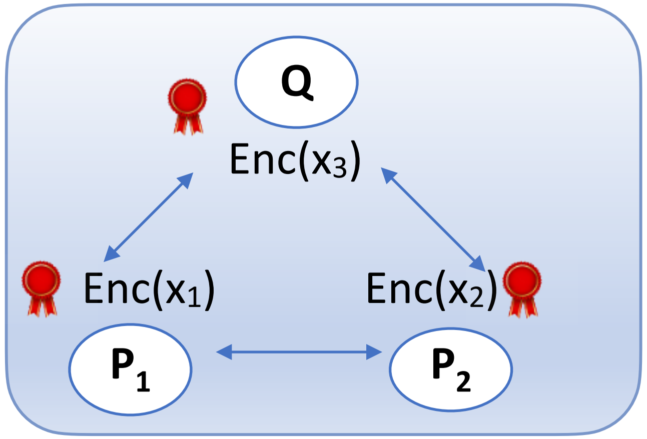
Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

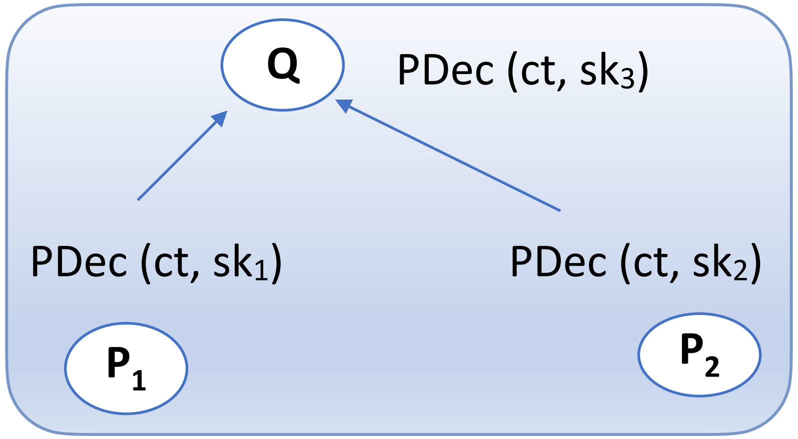
n = 3, t = 1







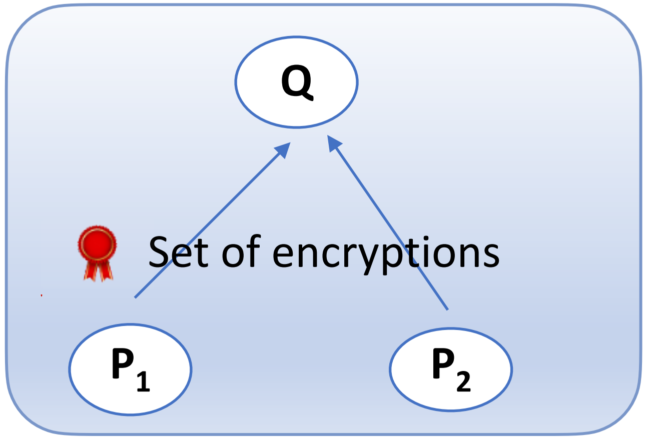


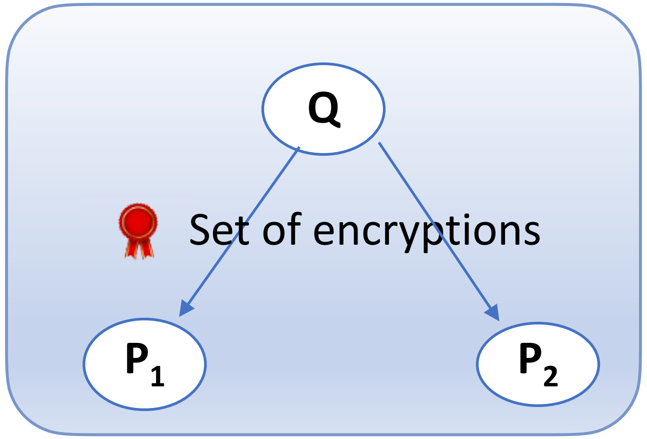


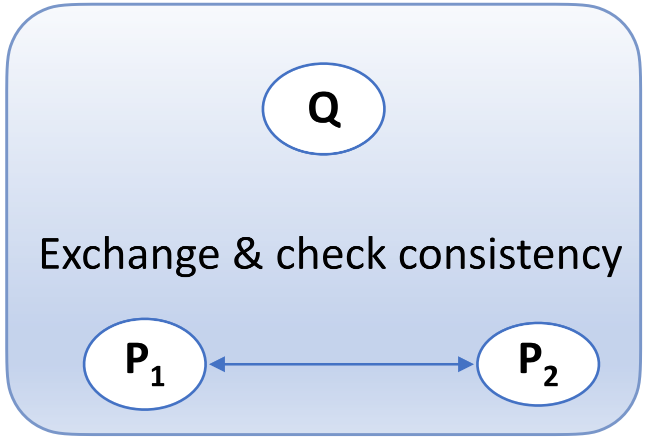
5-Round upper bound with PKI (no broadcast)

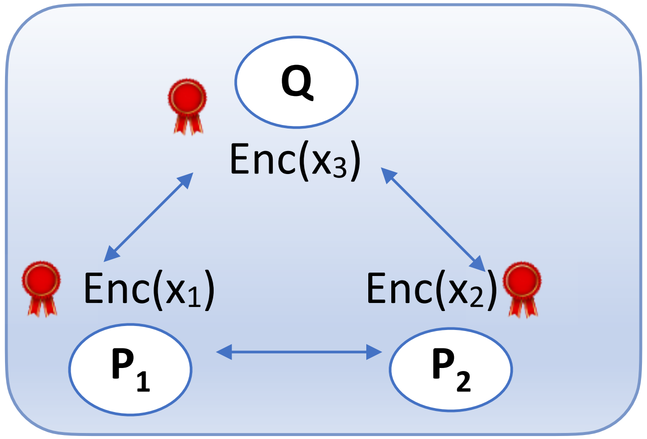
Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

n = 3, t = 1



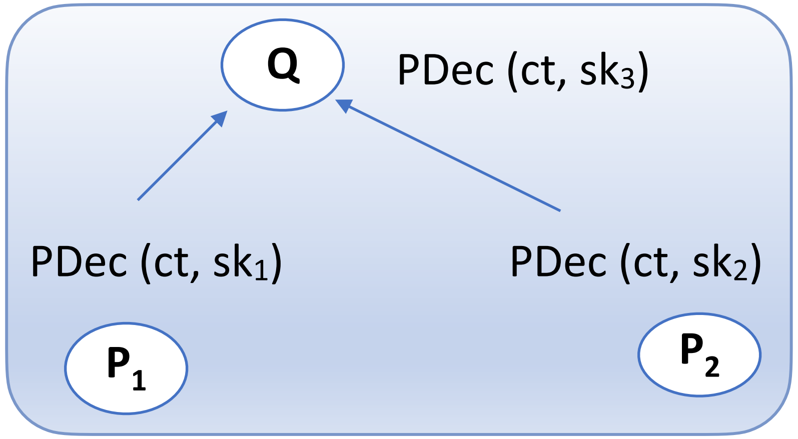






Q must include a cipher text from party

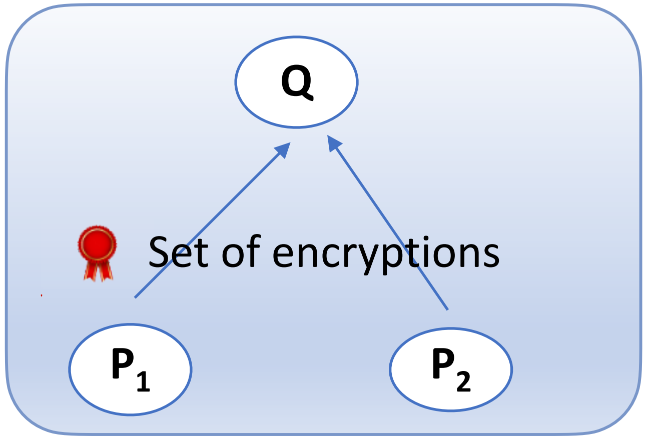
- received directly in round 1
- Via different party in round 2



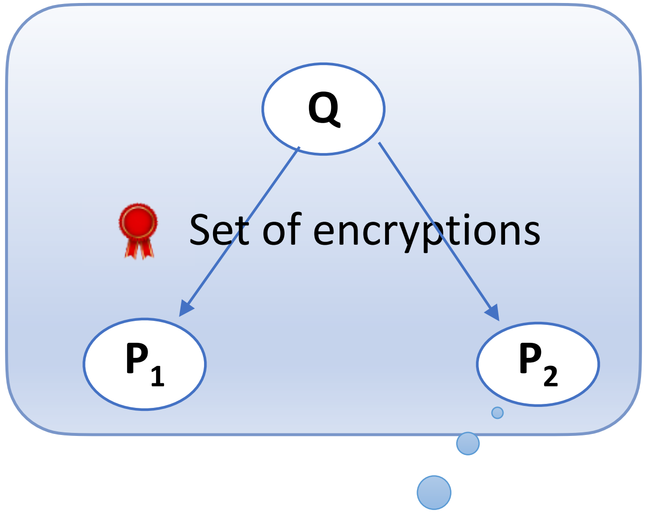
5-Round upper bound with PKI (no broadcast)

Decentralized threshold FHE Setup: pk, sk_1 , sk_2 , sk_3

n = 3, t = 1

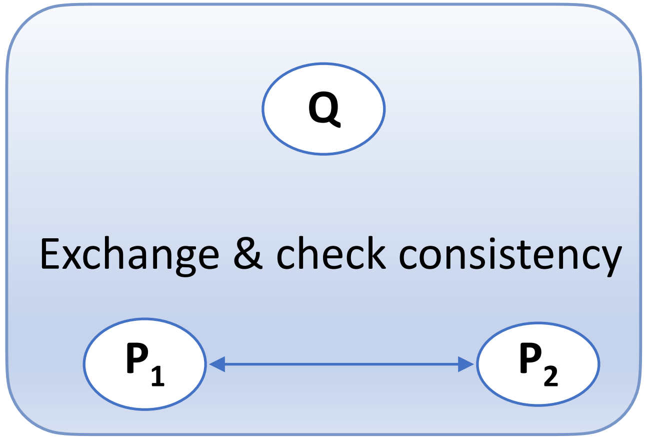


Set of encryptions

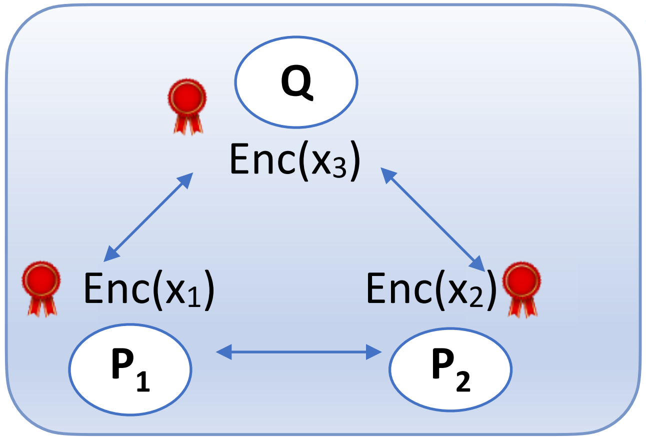


Set of encryptions

Q must include a cipher text for P_1 since I sent him one!



Exchange & check consistency



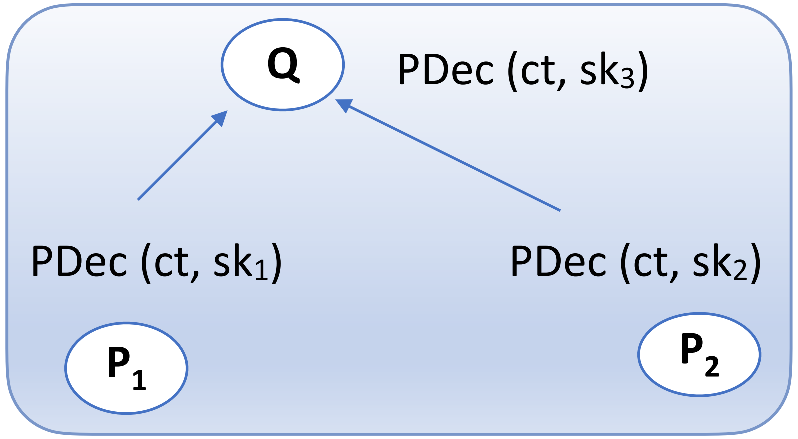
$Enc(x_3)$

$Enc(x_1)$

$Enc(x_2)$

Q must include a cipher text from party

- received directly in round 1
- Via different party in round 2



PDec (ct, sk_3)

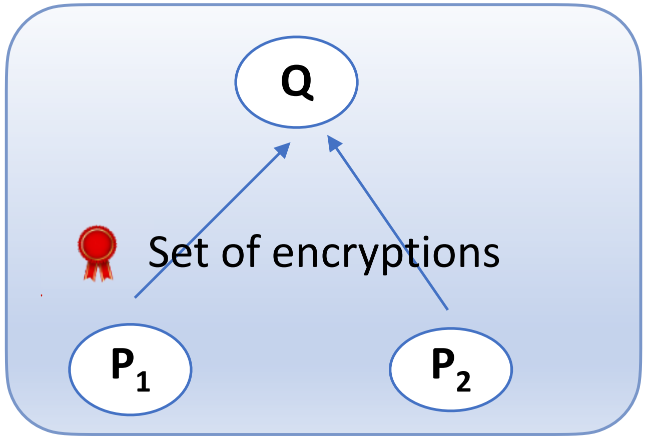
PDec (ct, sk_1)

PDec (ct, sk_2)

5-Round upper bound with PKI (no broadcast)

Decentralized threshold FHE Setup: pk, sk_1, sk_2, sk_3

n = 3, t = 1



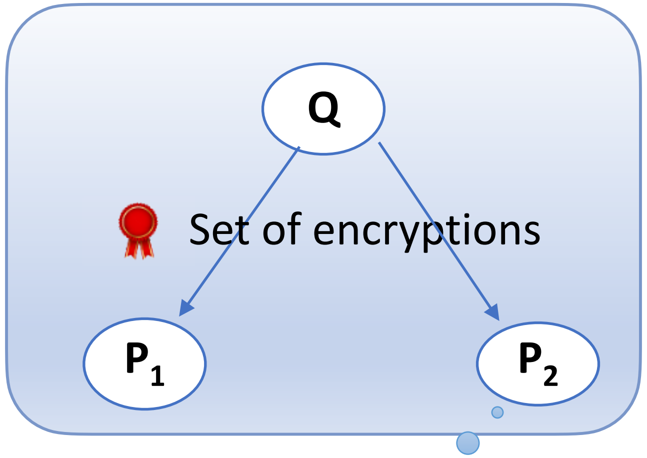
Q

Set of encryptions

P_1

P_2

Round 2



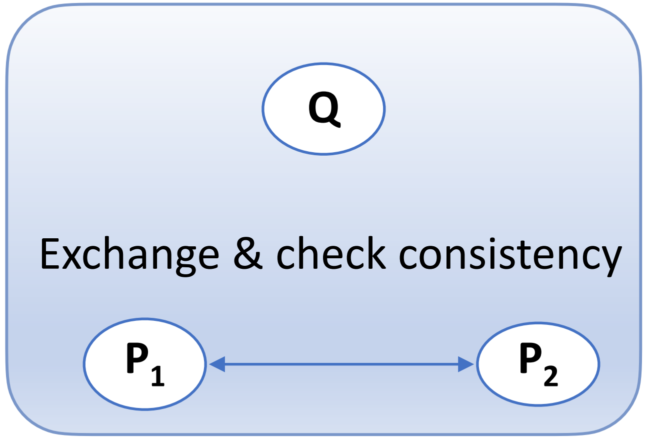
Q

Set of encryptions

P_1

P_2

Round 3



Q

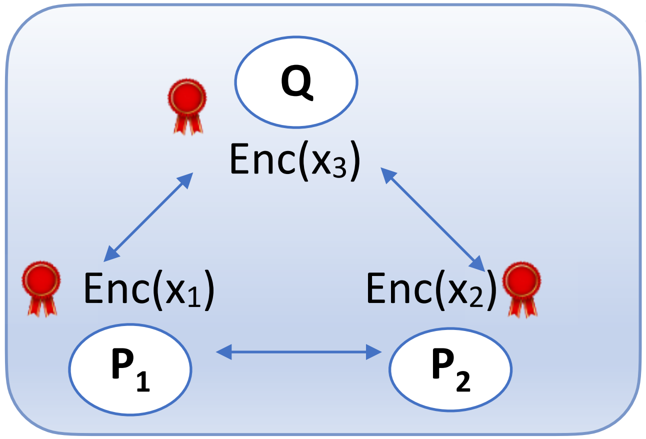
Exchange & check consistency

P_1

P_2

Round 4

Q must include a cipher text for P_1 since I sent him one!



Q

Enc(x_3)

Enc(x_1)

Enc(x_2)

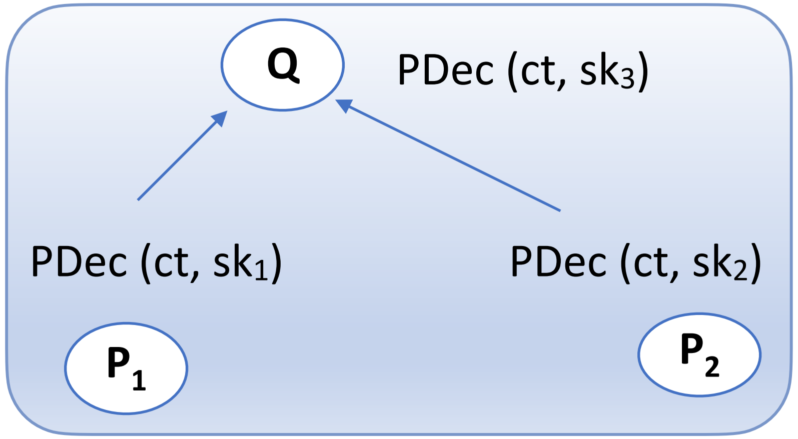
P_1

P_2

Round 1

Q must include a cipher text from party

- received directly in round 1
- Via different party in round 2



Q

PDec (ct, sk_3)

PDec (ct, sk_1)

PDec (ct, sk_2)

P_1

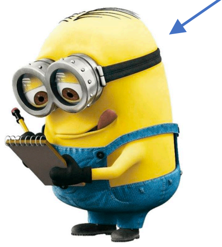
P_2

Round 5

Round Complexity of Solitary MPC with G.O.D

- Need to assume either broadcast or PKI [HIKMR19, FGMO05]

Broadcast	PKI	Threshold	Lower Bound	Upper Bound
✔	✔	t < n/2	2 [HLP11]	2 [GLS15]
✔	✖	t < n/2	3 [This work]	3 [ACGJ18, BJMS18]
✖	✔	n/2 > t >=3	4 [This work]	5 [This Work]



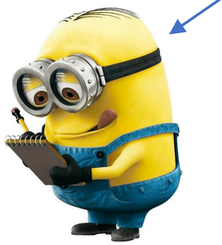
Additional Results:

- Optimal use of broadcast
- Special cases t = 1 & t = 2

Round Complexity of Solitary MPC with G.O.D

- Need to assume either broadcast or PKI [HIKMR19, FGMO05]

Broadcast	PKI	Threshold	Lower Bound	Upper Bound
✓	✓	t < n/2	2 [HLP11]	2 [GLS15]
✓	✖	t < n/2	3 [This work]	3 [ACGJ18, BJMS18]
✖	✓	n/2 > t >=3	4 [This work]	5 [This Work]



Additional Results:
- Optimal use of broadcast
- Special cases t = 1 & t = 2