Limits in the Provable Security of ECDSA Signatures

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Motivation

- (EC)DSA signatures is a standardized signature scheme and used *everywhere*
  - SSL/TLS
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- Comparatively few security results
- Existing results require strong idealization

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<u>Ver$(\mathsf{vk} = X, m, \sigma = (s, t))$**:**</u>

if $(t \stackrel{?}{=} 0) \vee (s \stackrel{?}{=} 0)$ **then**
  return $\perp$
$h = H(m)$
$t' = f\left(\left(g^h X^t\right)^{\frac{1}{s}}\right)$
return $t \stackrel{?}{=} t'$

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Modeling the conversion function [FKP16]

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$$\begin{tikzcd}
\mathbb{G} \arrow[r, "\varphi"] \arrow[rrr, bend left, "f"] & \{0,1\}^L \arrow[r, "\Pi"] & [0{:}2^L - 1] \arrow[r, "\psi"] & \mathbb{Z}_p
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- φ: 2-to-1 function
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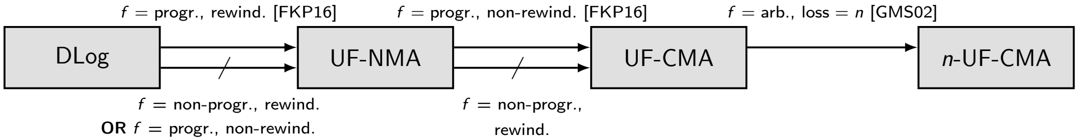
Results

$$\text{DLog} \xrightarrow{f\,=\,\text{progr., rewind. [FKP16]}} \text{UF-NMA} \xrightarrow{f\,=\,\text{progr., non-rewind. [FKP16]}} \text{UF-CMA} \xrightarrow{f\,=\,\text{arb., loss}\,=\,n\ \text{[GMS02]}} n\text{-UF-CMA}$$

Results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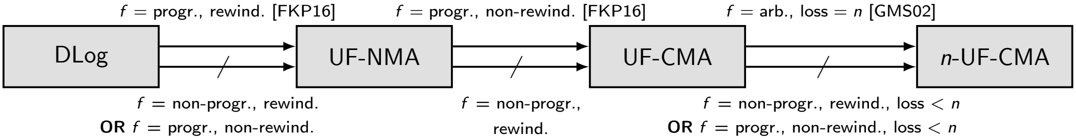

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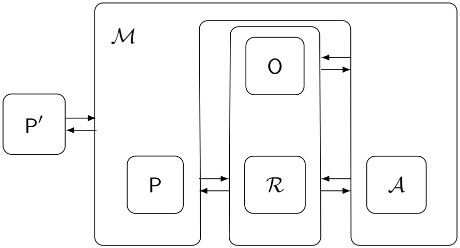
Meta Reduction

- "Reduction against the reduction"



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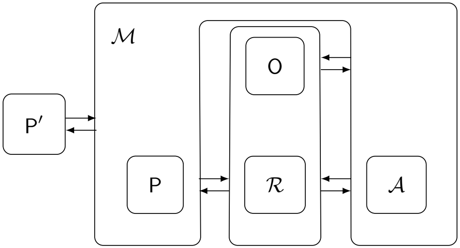

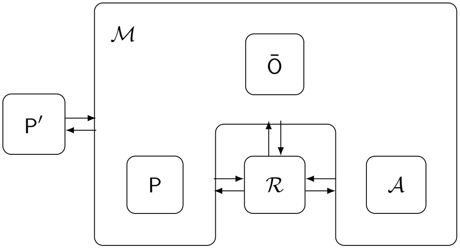

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- Problem 2: How to extract all solutions?
- Solution: AGM and clever simulation of $(\bar{\Pi}, \bar{\Pi}^{-1})$

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  - Find non-algebraic/non-black-box reductions
  - Use stronger assumptions

Thank you!

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