Public-key Encryption with Quantum Keys

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joint work with:

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Impagliazzo's Cryptographic Worlds

Cryptomania: public-key cryptography exists

Public-key encryption | Oblivious transfer (MPC) | Key-Agreement

Minicrypt: one-way functions exist

OWF | PRG | PRF | Signature | Private-key encryption | Zero-knowledge proofs | Commitment

Impagliazzo's Cryptographic Worlds in Quantumania

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↑ ✗ Public-key encryption

Mini Q crypt [GLSV'20, BCKM'20]

Minicrypt Q-oblivious transfer QSMPC

Q: quantum computation & quantum communication

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? Can we go even lower?

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Pseudorandom State Generator*

No quantum poly-time algorithm can distinguish (copies of) output of G from (copies of) an n-qubit state sampled from the Haar distribution

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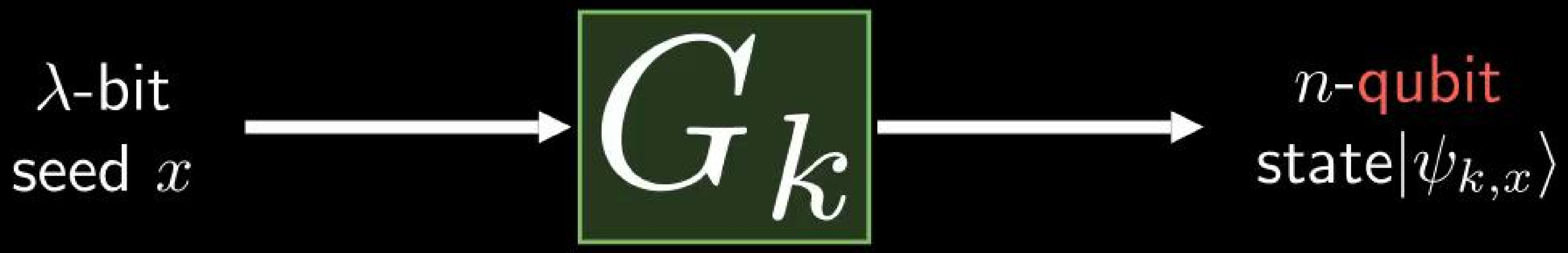

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Where to put (quantum) public-key encryption?

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Our Work: Quantum Public-Key Encryption (qPKE)

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Impossibility

unconditional security of qPKE

- uses quantum shadow tomography to learn the secret-key from public-keys

*another proof using quantum Shanon's bound [MY22]

qPKE: Definitions

Key generation

$\mathsf{sk} \leftarrow \mathsf{Gen}(1^\lambda)$

*sk is classical

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- $|\mathsf{pk}\rangle$ is pure implying a non-trivial way to distribute quantum public keys
    - $|\mathsf{pk}\rangle$ is sent to different CAs and use SWAP-test for validation [Gottesman'05]
    - Only achieve inverse-poly soundness error

qPKE: A Simple Construction from OWFs

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2. $|\mathsf{pk}\rangle \leftarrow \sum_x |x, \mathsf{PRF}(\mathsf{sk}, x)\rangle$

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2. Sample a uniformly random r
3. Encrypt $m \in \{0,1\}$ as $c = (r, x, \mathsf{PRF}(y, r\|m))$

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Decryption

1. Compute $y \leftarrow \mathsf{PRF}(\mathsf{sk}, x)$
2. Return 0 if $c = \mathsf{PRF}(y, r\|0)$, 1 otherwise.

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1. Measure $|\mathsf{pk}\rangle$ to get $(x, |\psi_{\mathsf{sk},x}\rangle)$ // only measure the first register
2. If $m = 0$, return $(x, |\psi_{\mathsf{sk},x}\rangle)$
3. Else, return $(x, \frac{\mathcal{I}}{2^n})$ // i.e., a maximally mixed state

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Decryption

1. Interpret c as $(x, |\phi\rangle)$
1. Return 0 if $|\phi\rangle = |\psi_{\mathsf{sk},x}\rangle$, 1 otherwise

Impossibility

Step 1. Let $|\mathsf{pk}^*\rangle$ be the honestly generated public key.
Our first observation is that, for any other $|\mathsf{pk}'\rangle$, if $|\mathsf{pk}^*\rangle$ and $|\mathsf{pk}'\rangle$ are close, then we can use the corresponding secret key of $|\mathsf{pk}'\rangle$ to decrypt the ciphertext encrypted with $|\mathsf{pk}^*\rangle$

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This is done using the shadow tomography technique [HKP'20]

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- definitions of quantum public-key encryption
- (im)possibility of constructing qPKE

Caveats:

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