University of Stuttgart
Institute of
Information Security

Actively Secure Polynomial Evaluation from Shared Polynomial Encodings

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MPC Setup

1

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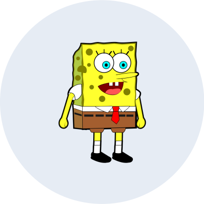


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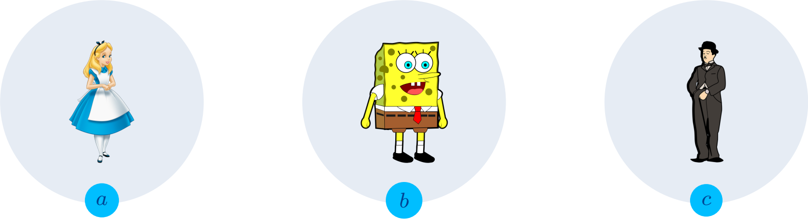

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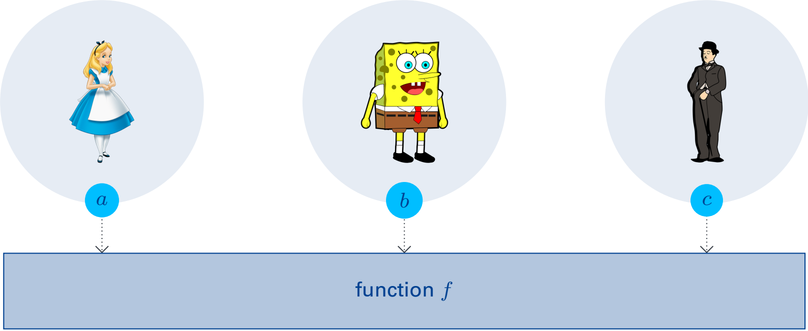

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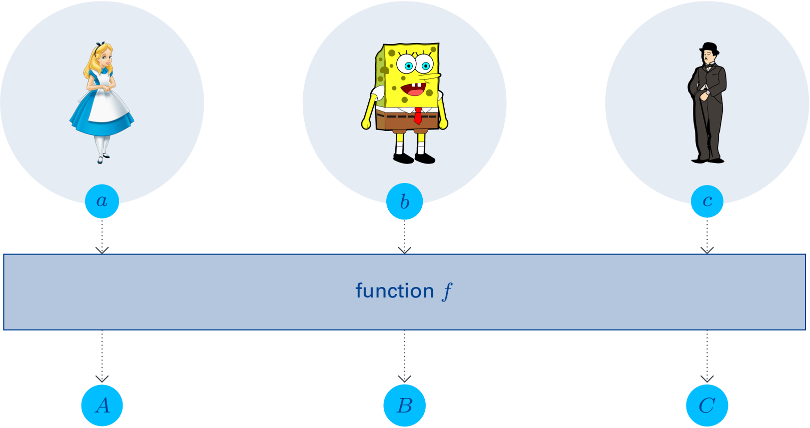

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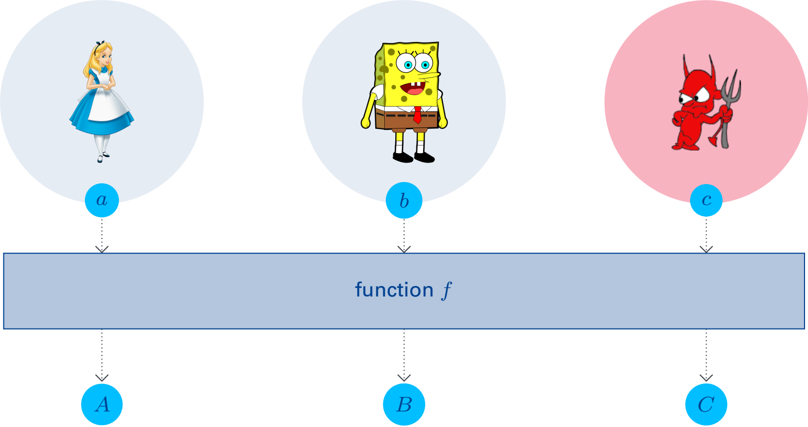

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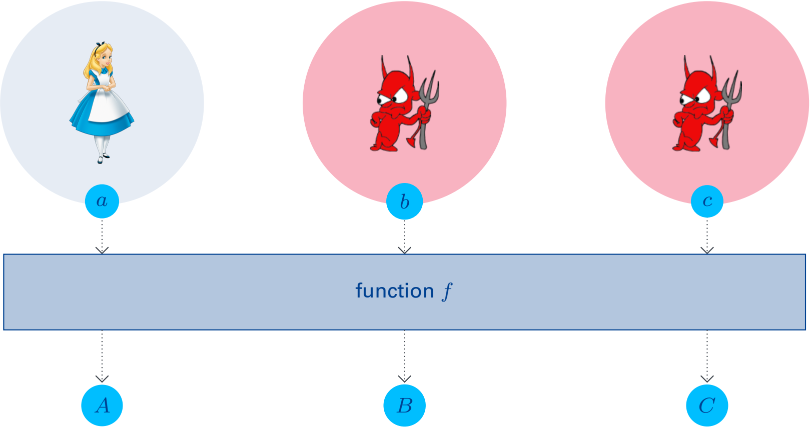

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$\Rightarrow$ *Our Approach:* Replace Beaver triples by a new form of structured randomness to reduce communication down to **one** (amortized) round.

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- We address this problem:

  New form of (moderately-sized) structured randomness (called **polytuples**) to evaluate *multivariate polynomials* and *comparisons* in one (amortized) round

Randomized Encodings

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- *Definition.* Let $X, Y, \hat{Y}, A$ be finite sets and let $f : X \to Y$. A function $\hat{f} : X \times A \to \hat{Y}$ is called *randomized encoding* of f if the following holds:
  - **Correctness.** There exists a reconstruction algorithm $\mathrm{Rec} : \hat{Y} \to Y$ such that

$$\begin{tikzcd} X \times A \arrow[r, "\hat{f}"] \arrow[d, "\mathrm{pr}_1"'] & \hat{Y} \arrow[d, "\mathrm{Rec}"] \\ X \arrow[r, "f"'] & Y \end{tikzcd}$$

    commutes, where $\mathrm{pr}_1 : X \times A \to X, (x, a) \mapsto x$ is the projection.
  - **Privacy.** There exists a simulator Sim such that $\mathrm{Sim}(f(x))$ and $\hat{f}(x, a)$ are identically distributed for all $x \in X$ if a is sampled uniformly from A.

Randomized Encodings

Example. For $f(x_0, x_1) = x_0 x_1$ take

$$\hat{f} = (y_0, y_1, y_2) = (x_0 - a_0, x_1 - a_1, a_1(x_0 - a_0) + a_0(x_1 - a_1) + a_0 a_1)$$

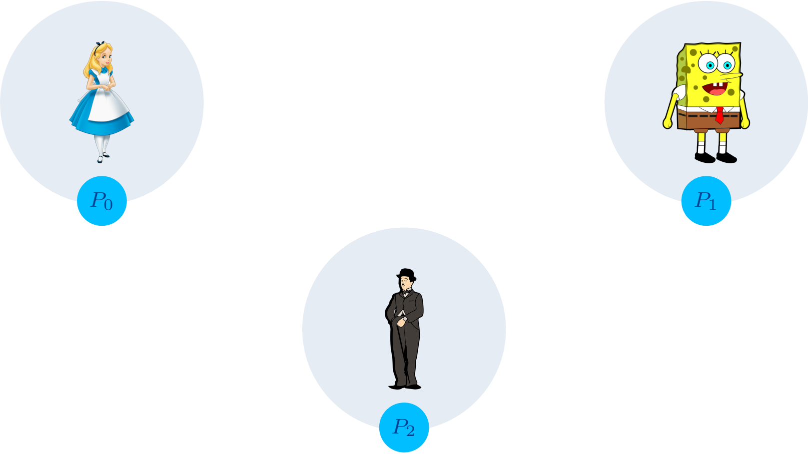
for $a_0, a_1 \in A$ and reconstruct by

$$\mathrm{Rec}(y_0, y_1, y_2) = y_0 y_1 + y_2 = x_0 x_1$$

.

Randomized Encodings as MPC Protocols

- For $f(x_0, \ldots, x_{m-1})$: Compute randomized encoding $\hat{f} = (y_l)_{0 \leq l < k}$ on shares of inputs $[x_j]$ and randomness $[a_t]$.



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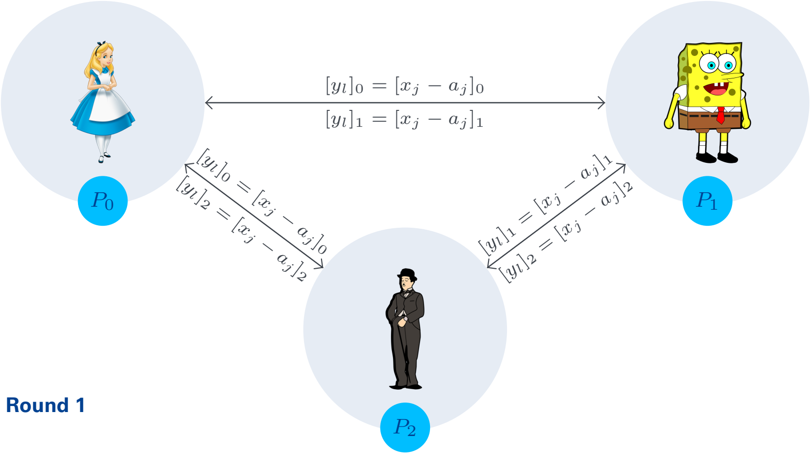

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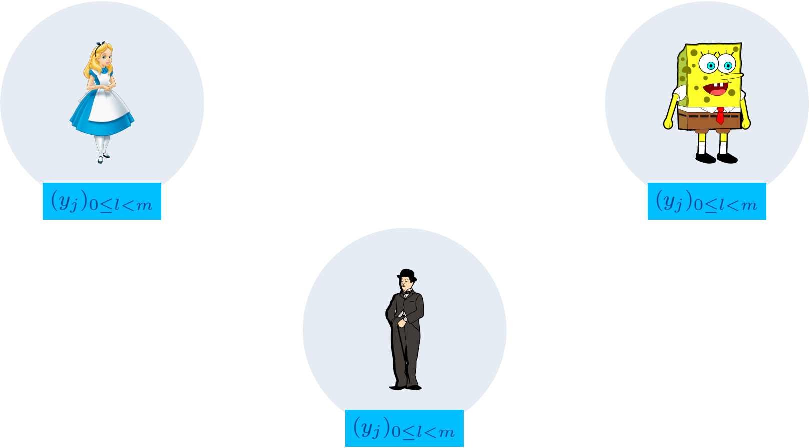

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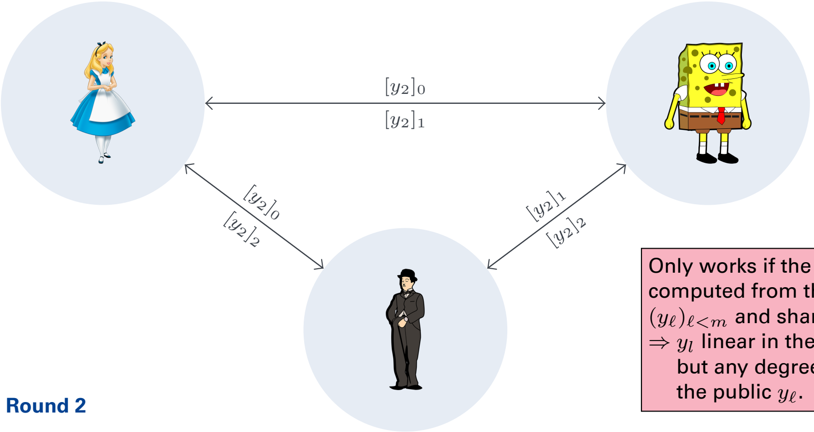
  They exchange the shares and reconstruct with $\mathrm{Rec}(y_0, y_1, y_2) = y_0 y_1 + y_2$.



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$\Rightarrow$ y_l linear in the shares, but any degree in the public y_ℓ.

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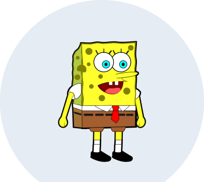

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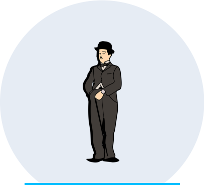

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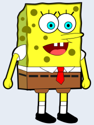

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- The approach extends to degree $m = 2^n$, i.e., we split $x_0 \cdots x_{m-1}$ in 4 degree $\frac{m}{2}$ terms and then each of these terms again in 4 $\frac{m}{4}$ terms, and so on.

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$\Rightarrow$ Output size of the encoding is in $\mathcal{O}(4^n) = \mathcal{O}(m^2)$ (cf. [Cra+03] for a similar result).

$\Rightarrow$ All terms are of total degree at most 3 (counting inputs and randomness to the degree), which is the theoretical minimum established in [Cra+03].

Our Randomized Encodings

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- We can improve over the abovementioned generic approach with standard composition and concatenation:

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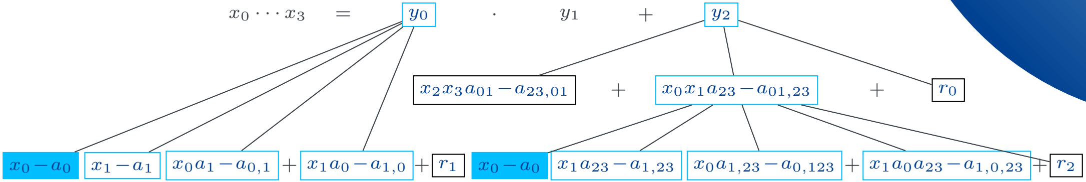

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- Note: generally for two randomized encodings $(y_l)_{l\geq 0}$ of f and $(y'_l)_{l\geq 0}$ of g with $y_0 = y'_0$, the reduced concatenation $(y_0, y_l, y'_l)_{l\geq 1}$ is **not a secure** randomized encoding of $f \times g$.

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- We are then able to compute a product $x_0 \cdots x_{m-1}$ with a randomized encoding of output size $\mathcal{O}(m \log(m))$.

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- We further extend the approach to other tree structures, where we can multiply **any number** of inputs in one multiplication node (instead of 2 in the example above), e.g.:

$$x_0 \cdots x_8 = (x_0 x_1 x_2 - a_{012})(x_3 x_4 x_5 - a_{345})(x_6 x_7 x_8 - a_{678}) + (\text{degree } \leq 3 \text{ in the } x_j)$$

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- We present a recursive formula to compute the exact output size and randomness size for all resulting randomized encodings in our paper.

Comparison and Benchmarks

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- Our protocol needs one round to exchange the masks and one opening round (which in the malicious setup includes a MAC check).
- Our protocol can be used in a multi-round fashion where $f_1 \circ \cdots \circ f_n$ is evaluated in n rounds plus one opening round.

Theoretical Comparison

- We get the following comparison for the computation of $x_1^{d/m} \cdots x_{m-1}^{d/m}$ of degree d with $d/m \in \mathbb{N}$ per party:

Approach	Rounds	Bandwidth	Tuple Size
Beaver Triples e.g. for $d = m = 16$	$\lceil \log d \rceil$ 4	$2(m-1)\lceil \log \frac{d}{m} \rceil$ 30	$3(m-1)\lceil \log d/m \rceil$ 45
Tuples from [CWB18] e.g. for $d = m = 16$	1 1	m 16	$(\frac{d}{m} + 1)^m - 1$ 65535
Example of a Polytuple e.g. for $d = m = 16$	1 1	$\mathcal{O}(m \log(m))$ 41	$\mathcal{O}(d(\log m)^2)$ 149

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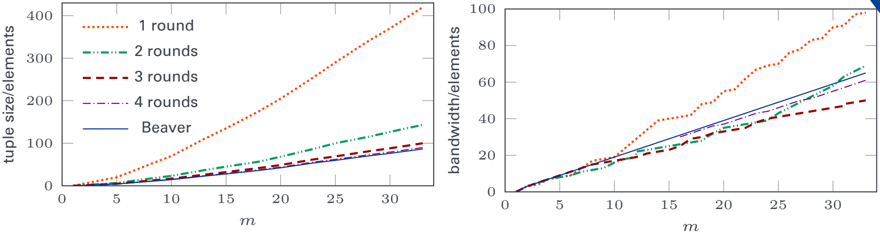
- The table includes only one example of a polytuple; depending on the setup, we can get different trade-offs.

Multi-Round Use and Flexibility

- For example, we can use more communication rounds:



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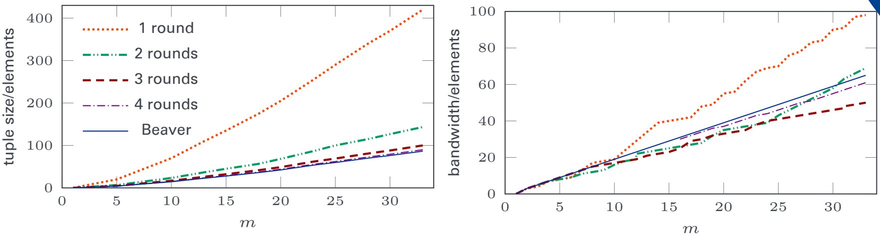

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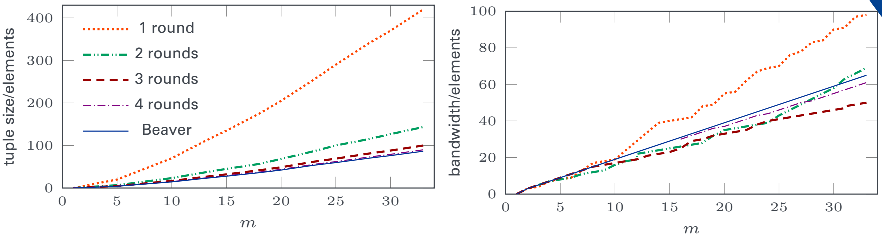

- We can also use different variants of our randomized encoding to trade reduced bandwidth against larger tuple size (while keeping the same round complexity).

⇒ Our protocols can be adapted to different setups, i.e., to bandwidth rate restrictions, network delay, or local computational power.

Implementation

- We have implemented our protocols as an extension of MP-SPDZ [Kel20], the state-of-the-art implementation of SPDZ-like protocols.

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- We have implemented our protocols as an extension of MP-SPDZ [Kel20], the state-of-the-art implementation of SPDZ-like protocols.
- Our implementation has been submitted as an artifact to Asiacrypt 2024 and has been accepted. It will be published with the paper.

Evaluation of Polynomials

- Evaluation of a Gaussian function in 32 variables with different network delays.

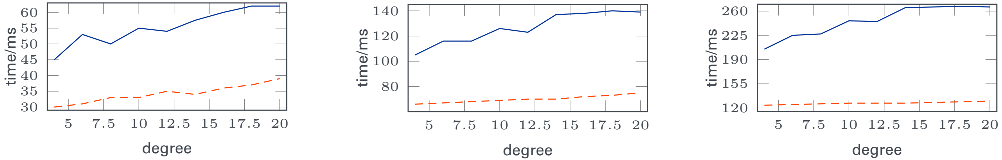

Figure: Benchmarks for Gaussian with 32 variables with 2ms (left), 5ms (middle), 10ms (right) delay; (blue: default MP-SPDZ implementation, orange/dashed: ours).

Comparisons and Rankings

- We can also employ polytuples to (bit-wise) compare values (similar to [Dam+06]).

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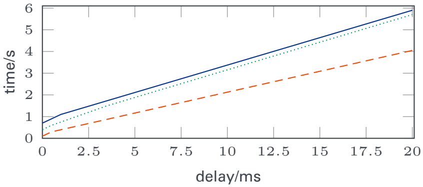
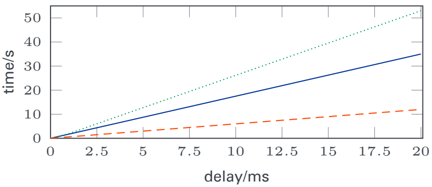
- For sorting 40 items we then get:



(a) Using pairwise inequality tests.



(b) Using inequality and equality tests.

Figure: Benchmarks for sorting (blue: default MP-SPDZ implementation, orange/dashed: ours, green/dotted: MP-SPDZ with edabits [Esc+20]).

Application to Machine Learning Networks

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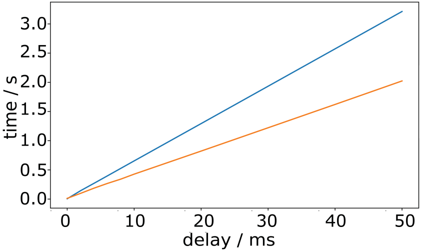
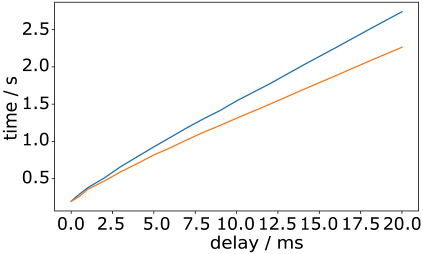
- For example, if we only compute the Argmax layer (left) with our new protocol, we get for a toy ML network (right):



(a) ArgMax Layer, unlimited rate.



(b) Network A [MZ17].

Figure: Benchmarks for an ArgMax layer and the evaluation of a sample neural network included in MP-SPDZ [Kel20] as network A (cf. [Ria+18]; blue: default MP-SPDZ, orange: ours) both without bandwidth restriction.

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- Our randomized encodings have the smallest known output size for arbitrary monomials.
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- Our tuple size is significantly lower than for existing single-round approaches, and also multi-round computations yield improvements (e.g., lower bandwidth and round complexity than Beaver multiplication).
- We evaluated the performance of our protocols for sample applications (evaluation of polynomials, comparisons of secret-shared values, simple machine learning algorithms) and show that polytuples speed up these computations compared to Beaver multiplication.

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Thank you!

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