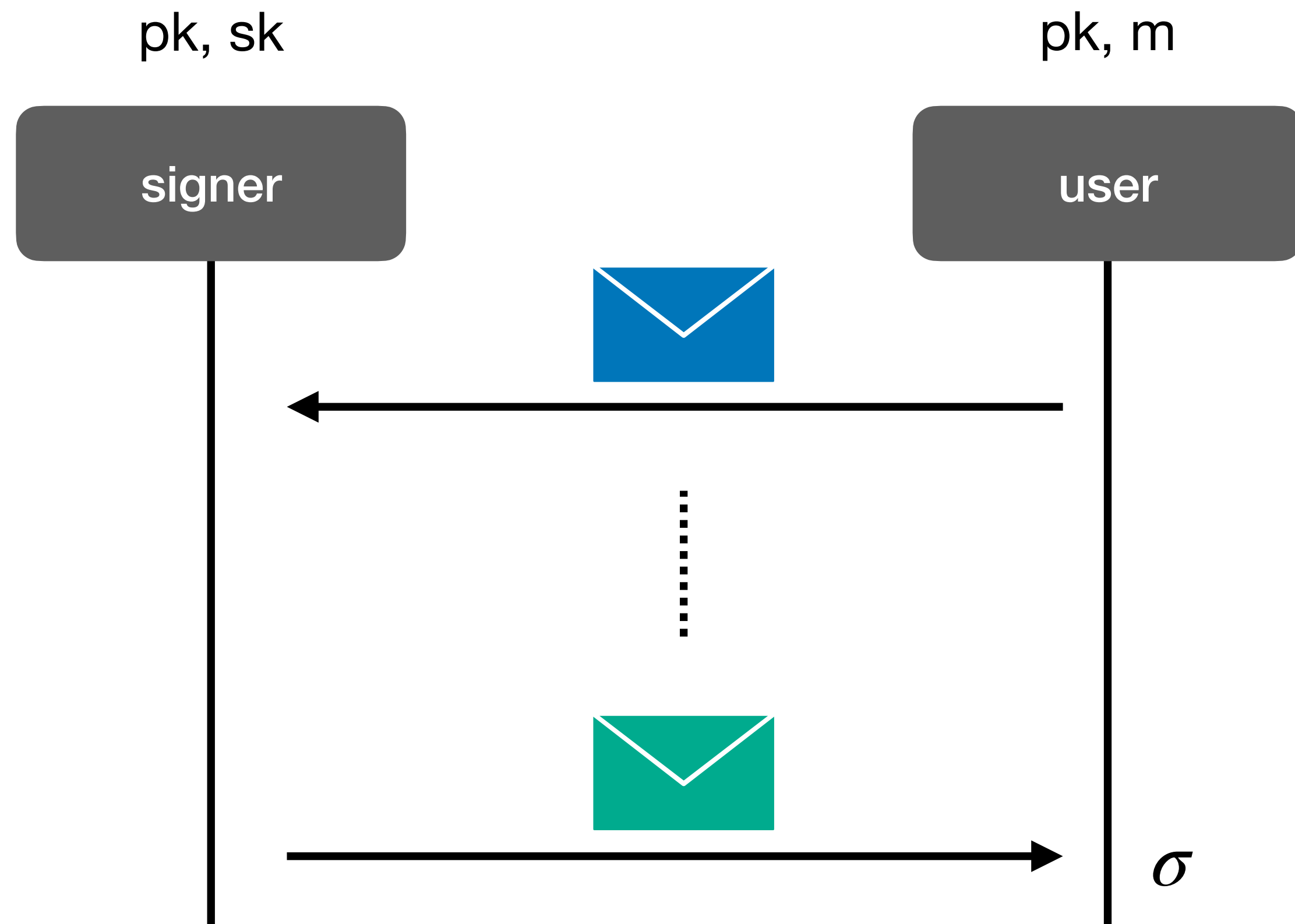


# Practical Blind Signatures in Pairing-free Groups

- Michael Klooß                      ETH Zurich
- Michael Reichle                      ETH Zurich
- Benedikt Wagner                      Ethereum Foundation

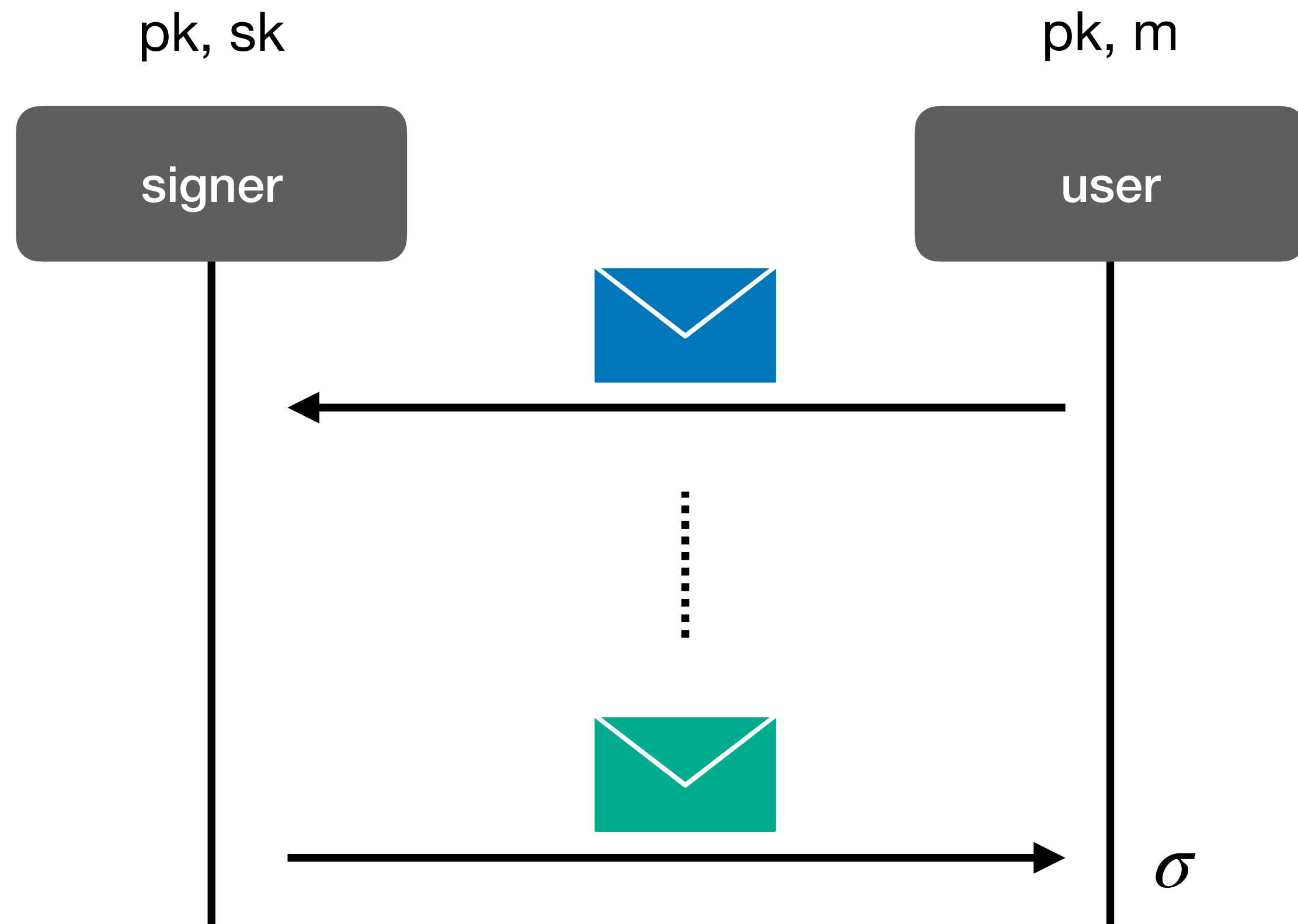
# Blind Signatures



## Correctness:

- honest signatures verify

# Blind Signatures



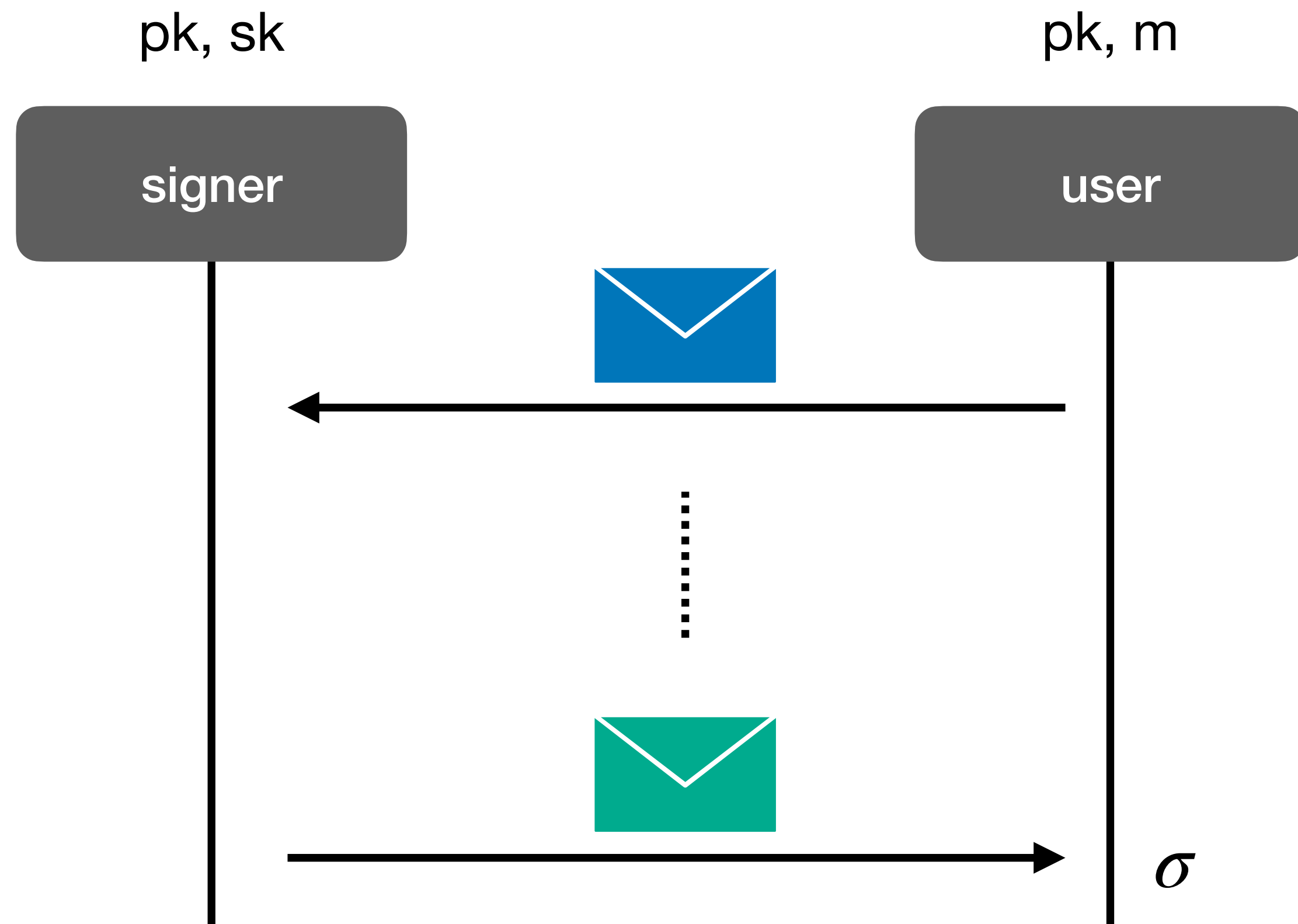
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# Blind Signatures



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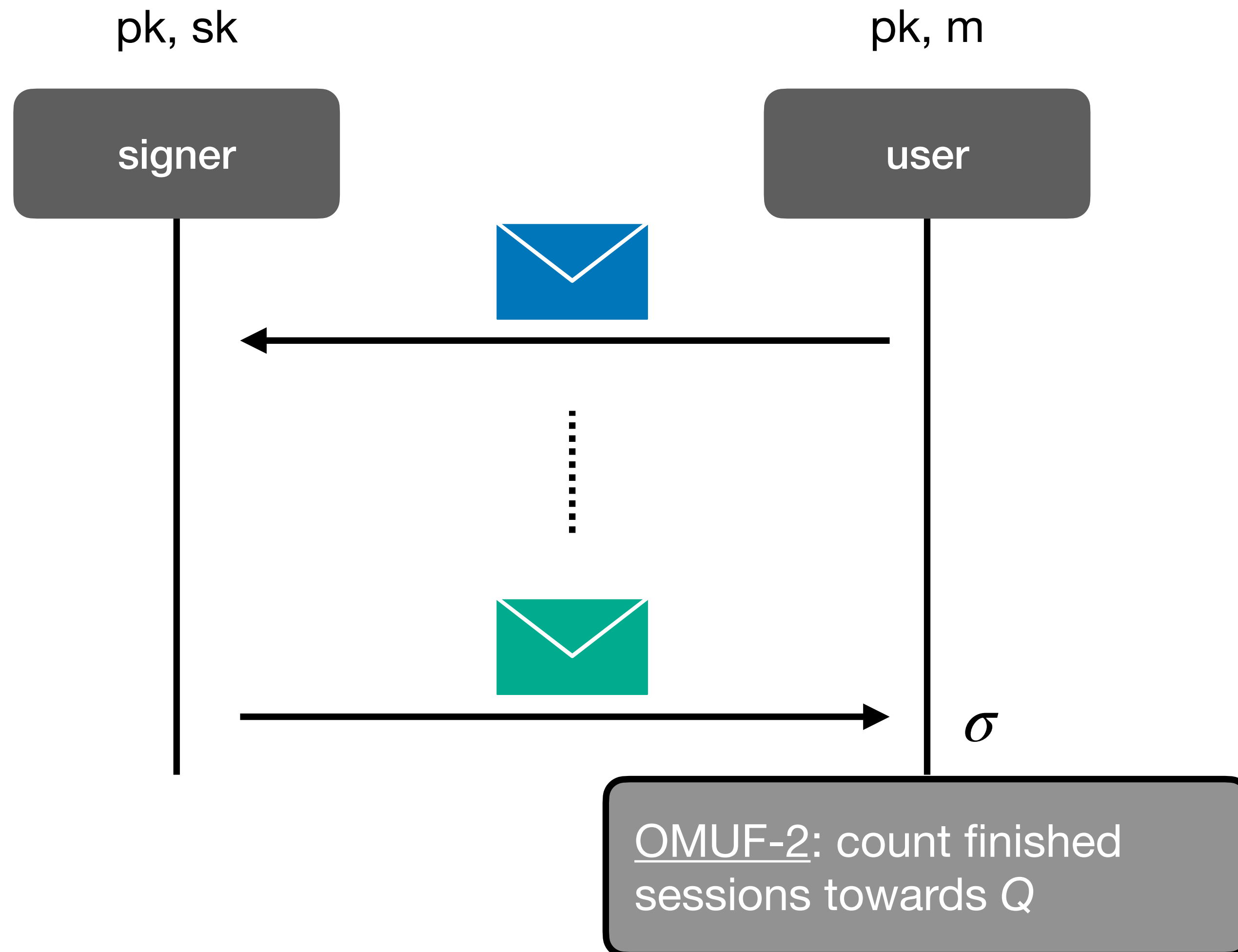
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## One-more Unforgeability:

- user can obtain at most  $Q$  signatures from  $Q$  sessions with distinct messages

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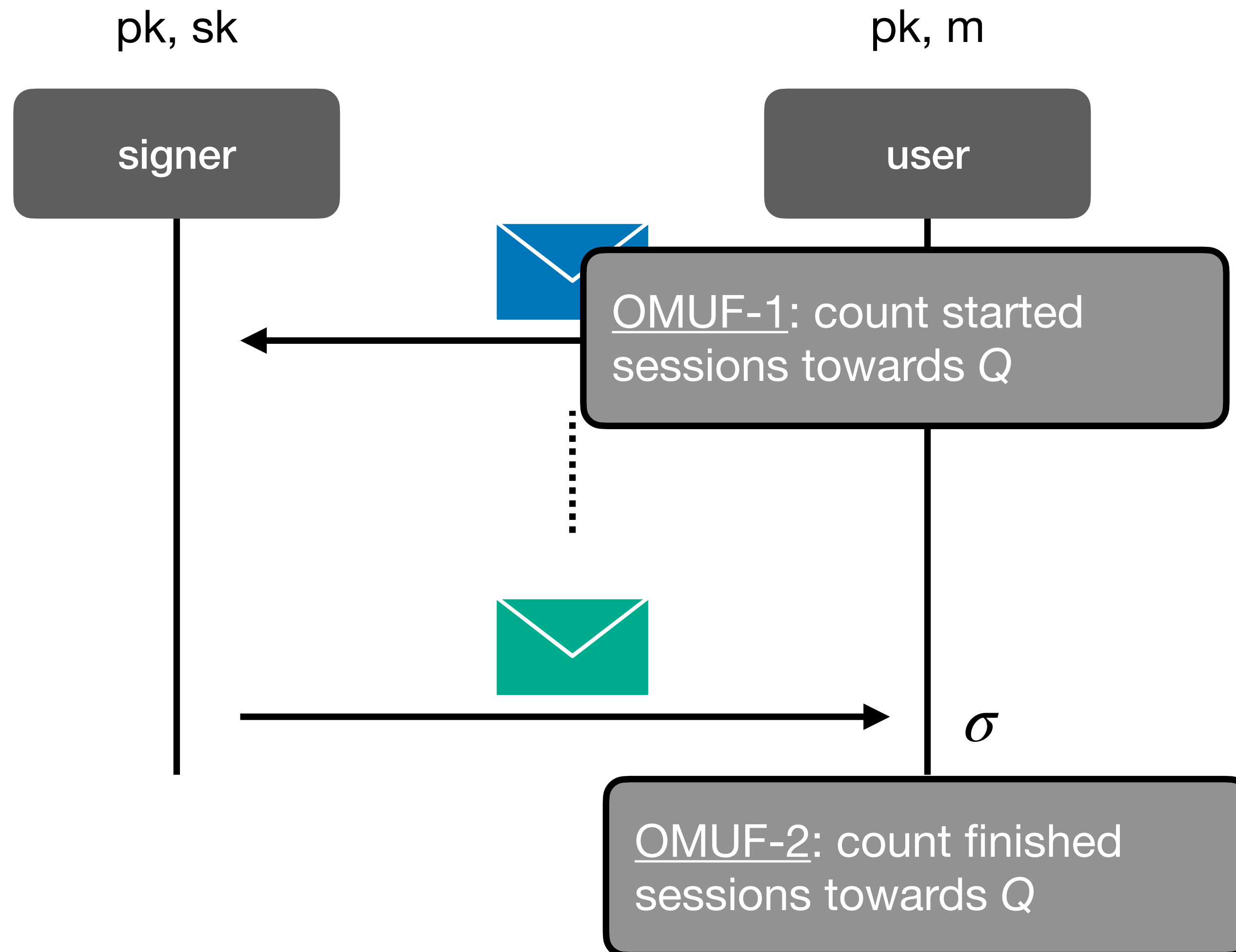
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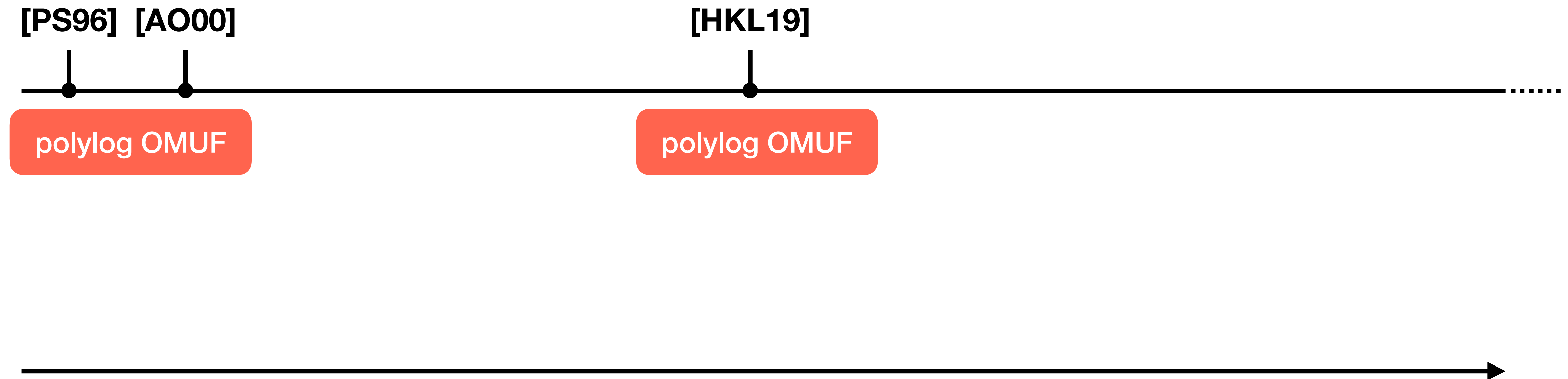
# Blind Signatures in Pairing-free Curves

## Selective Overview



# Blind Signatures in Pairing-free Curves

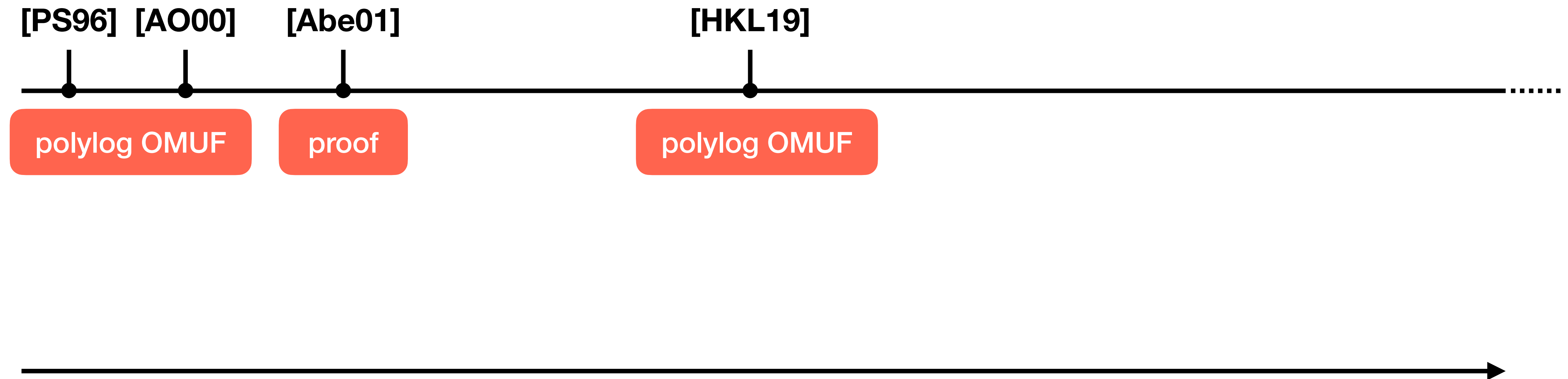
## Selective Overview





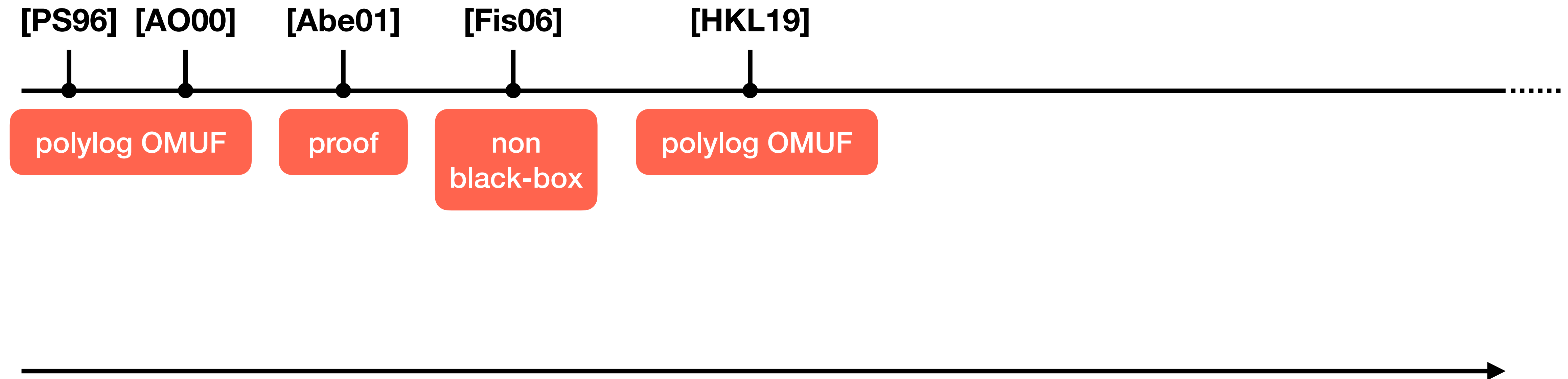
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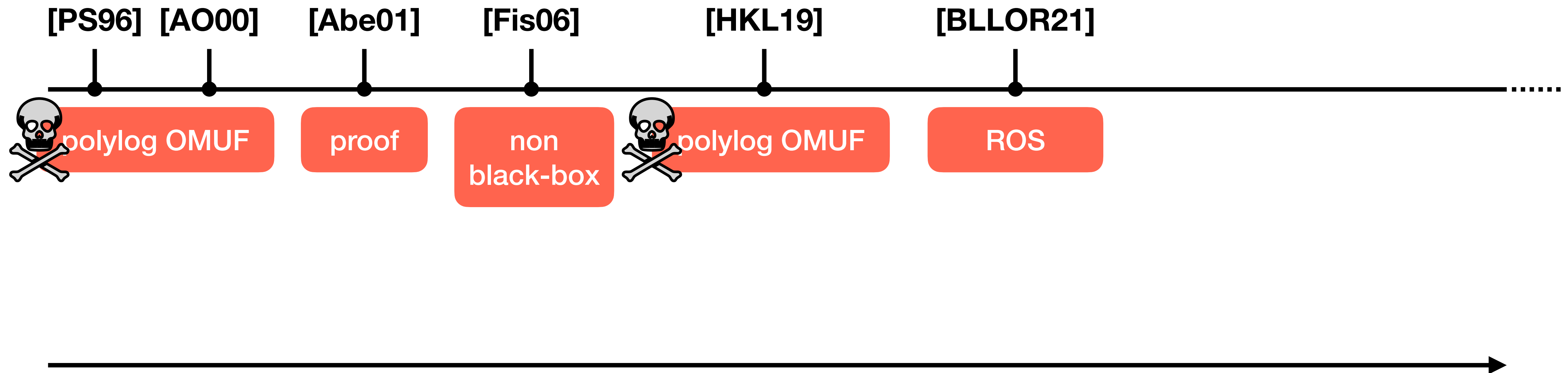
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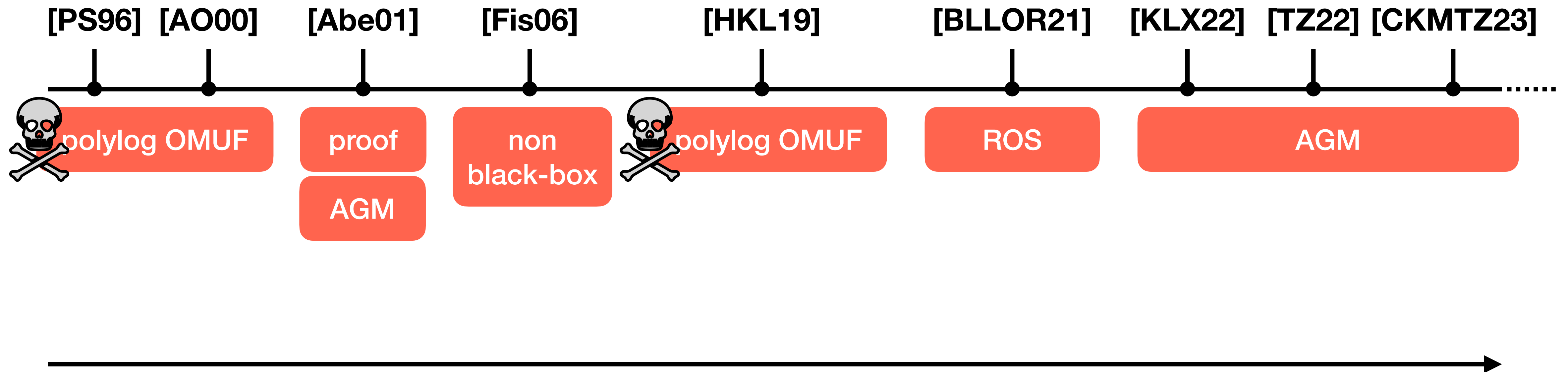
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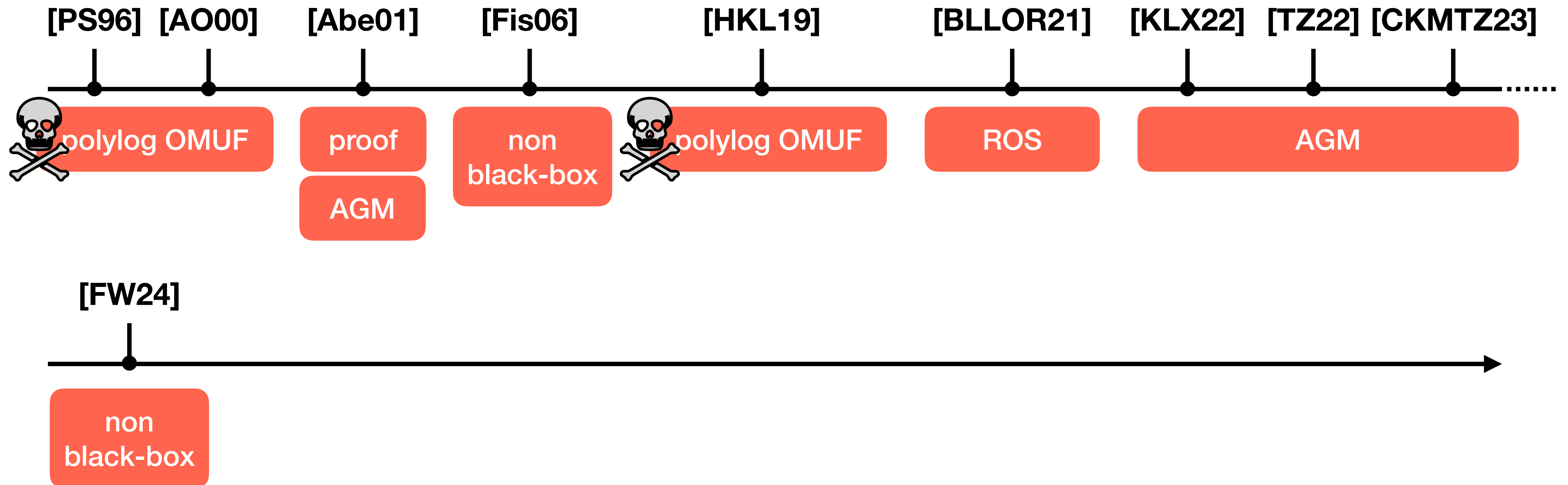
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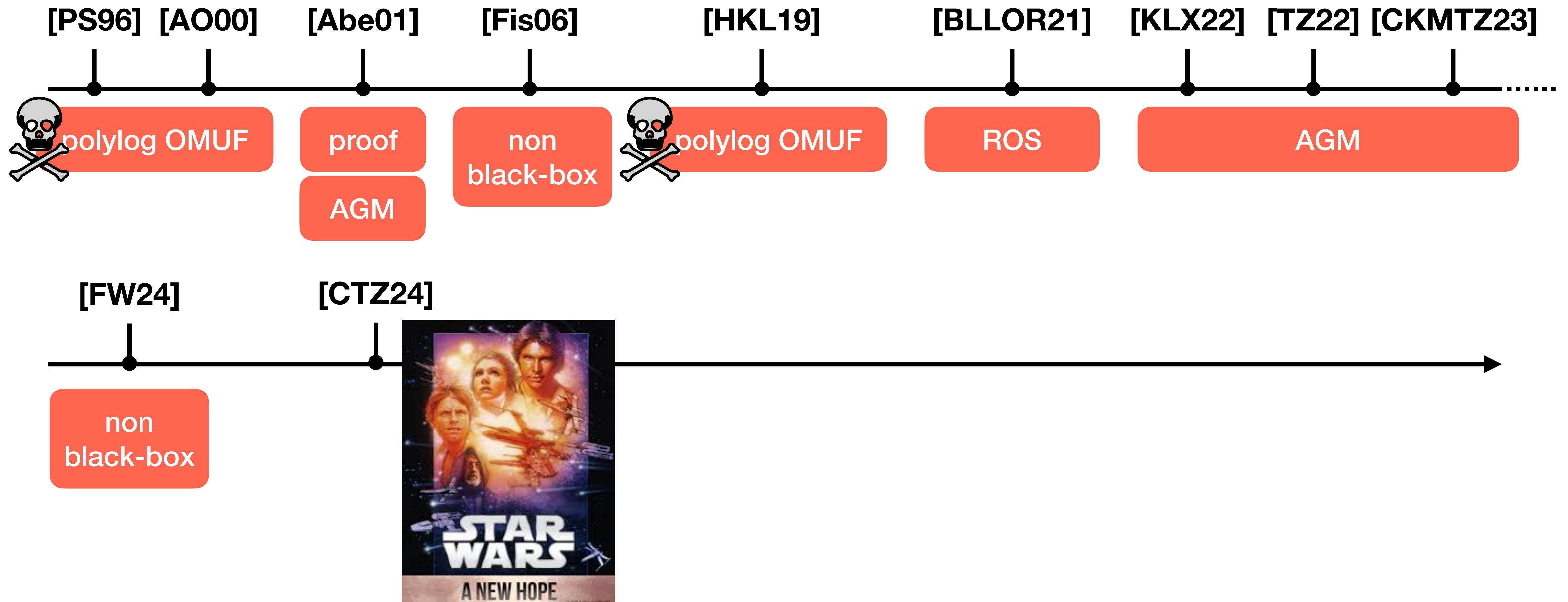
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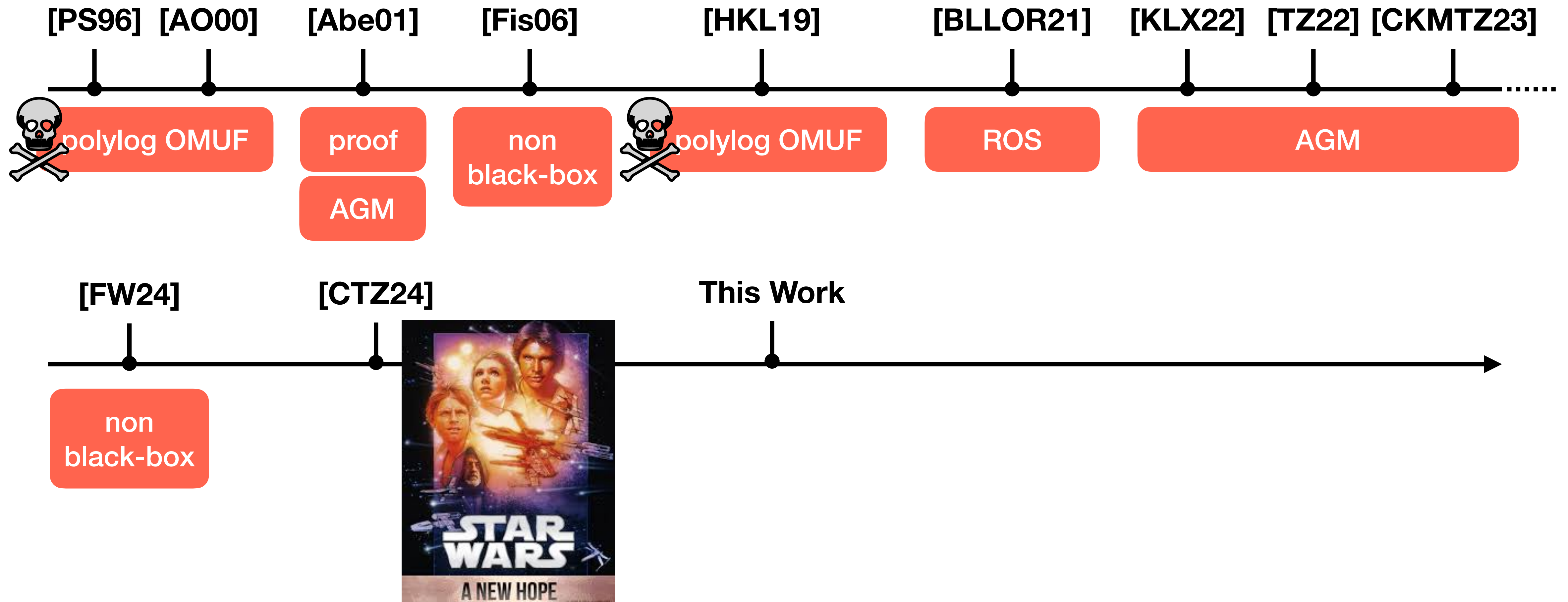
# Blind Signatures in Pairing-free Curves

## Selective Overview



# Blind Signatures in Pairing-free Curves

## Selective Overview



# Efficiency

## Pairing-free blind signature without the AGM

Scheme	Signature Size	Communication Size	Security	Assumption
<b>BS<sub>1</sub> + BS<sub>2</sub></b> <b>[CTZ24]</b>	$1G + 4Z_p$	$5G + 5Z_p$	OMUF-1	OMCDH
<b>BS<sub>3</sub></b> [CTZ24]	$\text{poly}(\lambda)$	$\text{poly}(\lambda)$	OMUF-2	CDH



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Scheme	Signature Size	Communication Size	Security	Assumption
<b>BS<sub>1</sub> + BS<sub>2</sub></b> [CTZ24]	$1\mathbb{G} + 4\mathbb{Z}_p$	$5\mathbb{G} + 5\mathbb{Z}_p$	OMUF-1	OMCDH
<b>BS<sub>3</sub></b> [CTZ24]	$\text{poly}(\lambda)$	$\text{poly}(\lambda)$	OMUF-2	CDH
<b>Our Work</b>	$2\mathbb{G} + 5\mathbb{Z}_p$	$\text{poly}(\lambda)$	OMUF-2	DDH

# CTZ'24

## High-level Overview



replace pairing-based verification of blind BLS  
via FS-compiled  $\Sigma$ -protocol

$pk = X, sk = x$

signer

$pk = X, m$

user

# CTZ'24

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$C$

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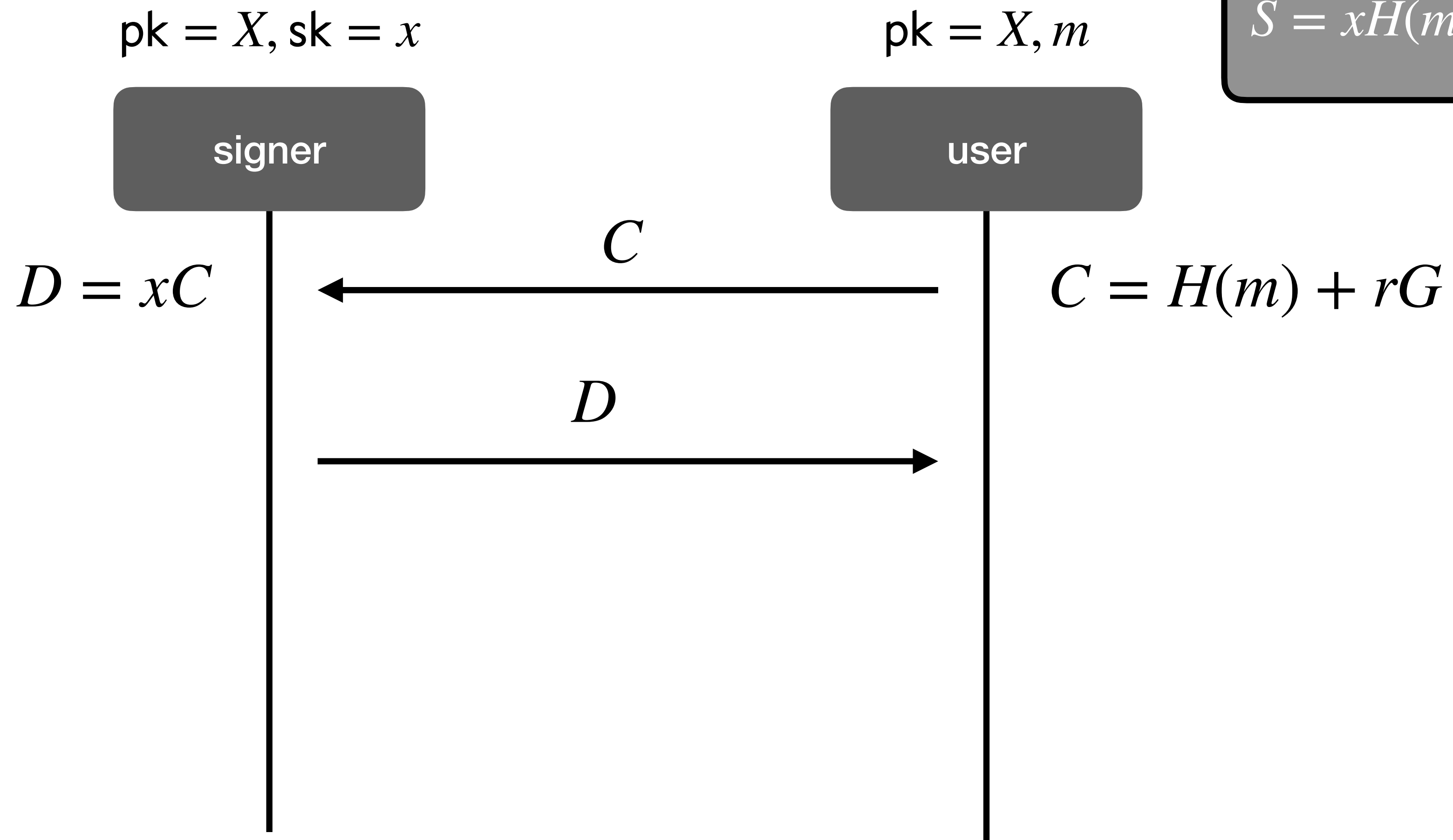
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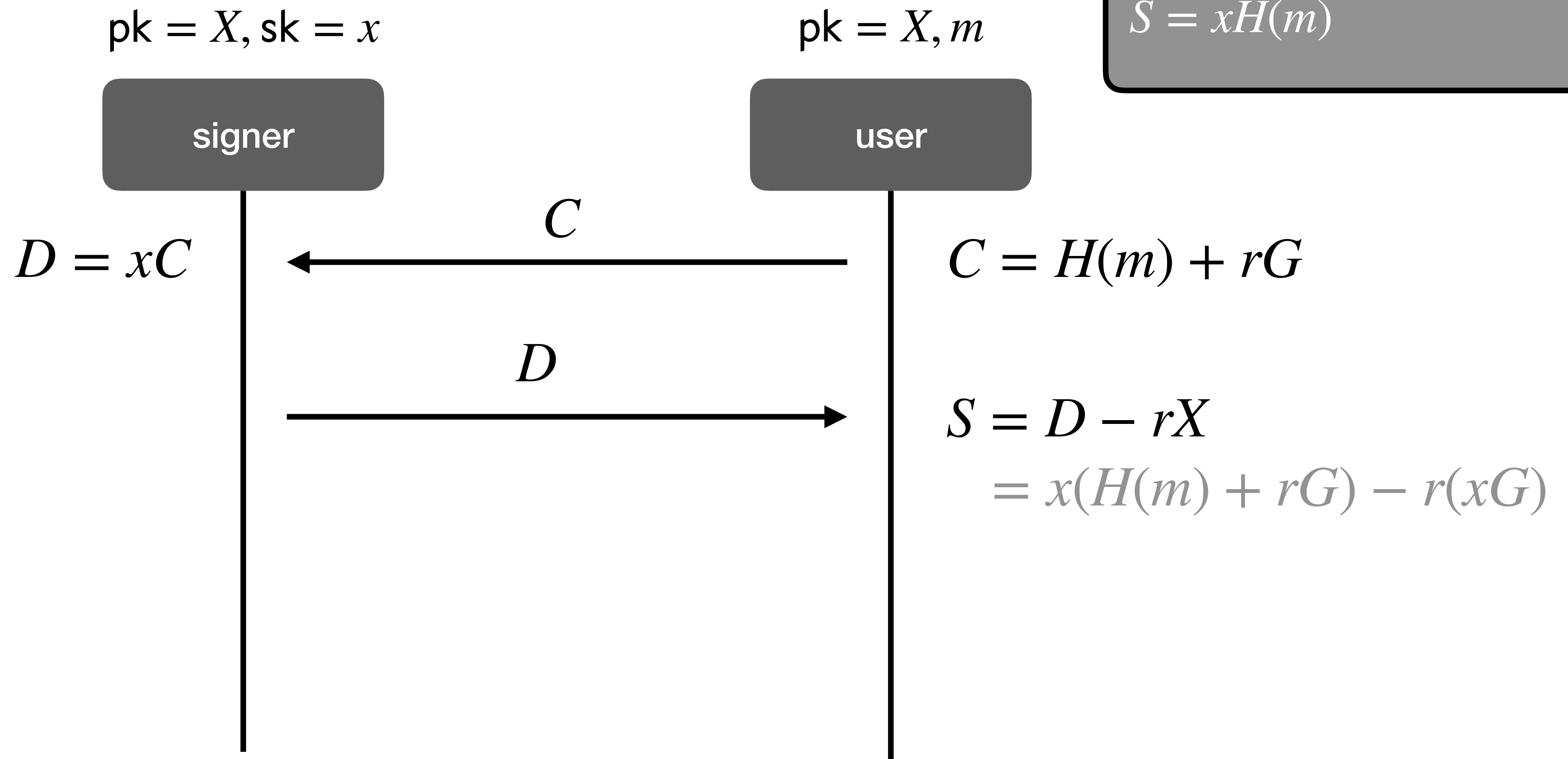


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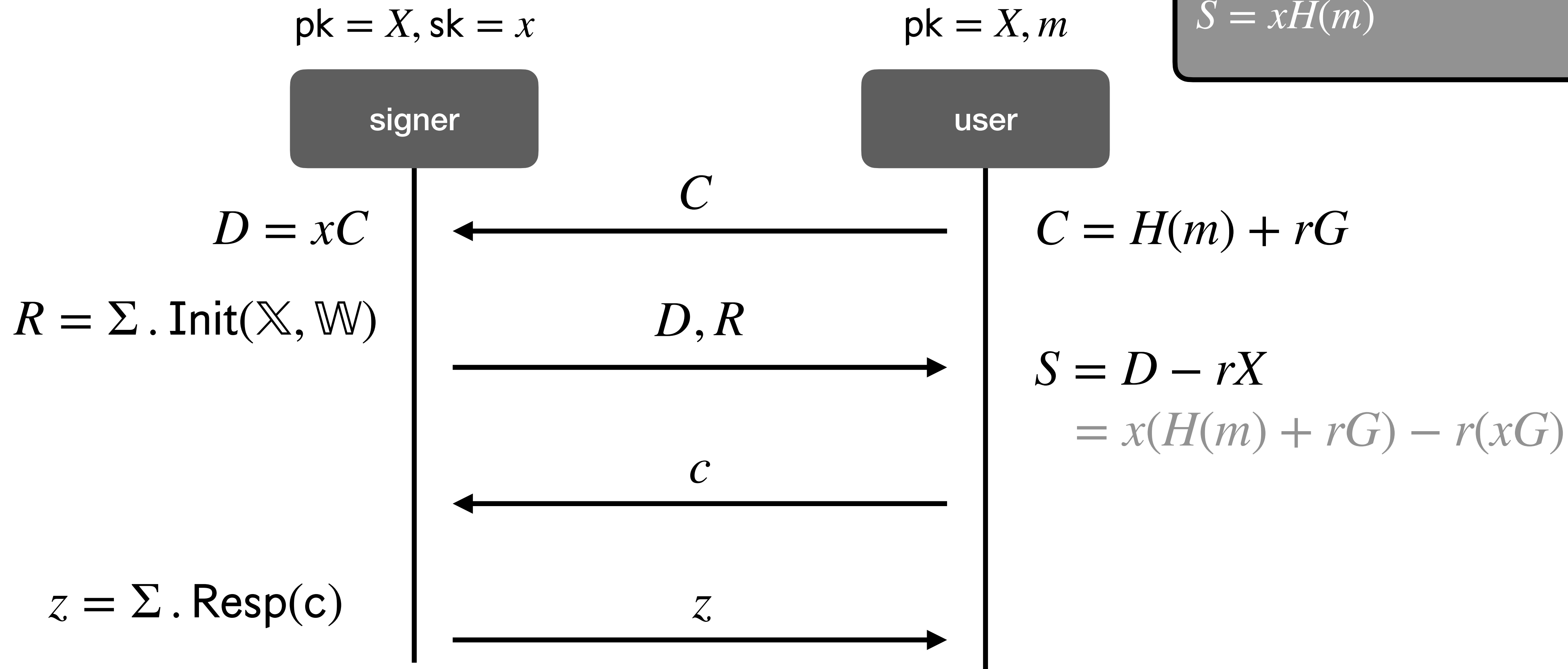
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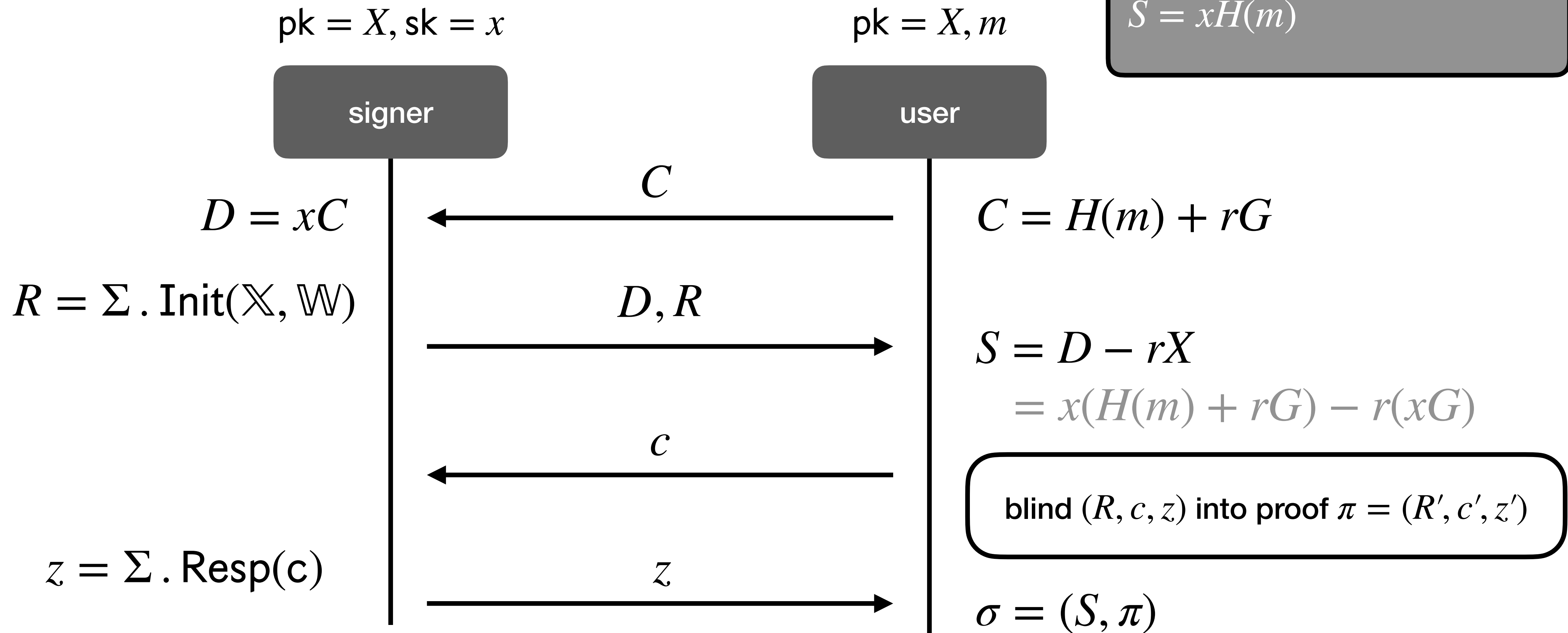
# CTZ'24

## High-level Overview



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# Our Approach



replace pairing-based verification of [KRS23]  
via FS-compiled  $\Sigma$ -protocol

$pk = (U, V, H), sk = u$

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$pk = (U, V, H), m$

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# Our Approach



replace pairing-based verification of [KRS23]  
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$\text{pk} = (U, V, H), \text{sk} = u$

signer

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user

$$\begin{aligned} S_1 &= uV + s(H(m)U + H) \\ S_2 &= sG \end{aligned}$$

# Our Approach



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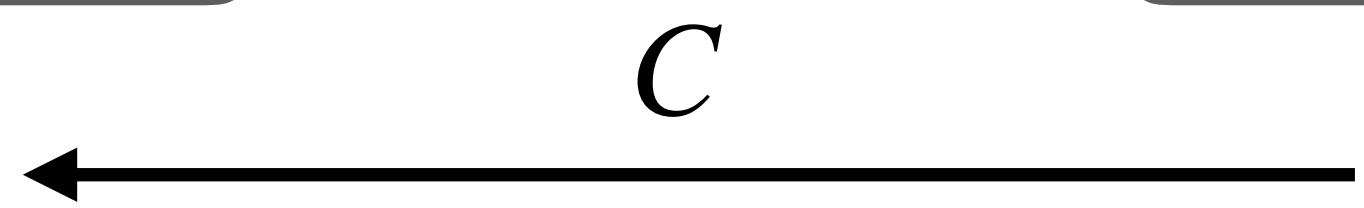
signer

pk =  $(U, V, H)$ ,  $m$

user

$$S_1 = uV + s(H(m)U + H)$$
$$S_2 = sG$$

$C$



$$C = H(m)U + rG$$

# Our Approach



replace pairing-based verification of [KRS23]  
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pk = (U, V, H), sk = u

pk = (U, V, H), m

signer

user

$$D_2 = sG$$

$$D_1 = uV + s(C + H)$$

C



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D



# Our Approach



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C

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D

$$S_2 = D_2 + s'G$$

$$S_1 = D_1 - tS_2$$

# Our Approach

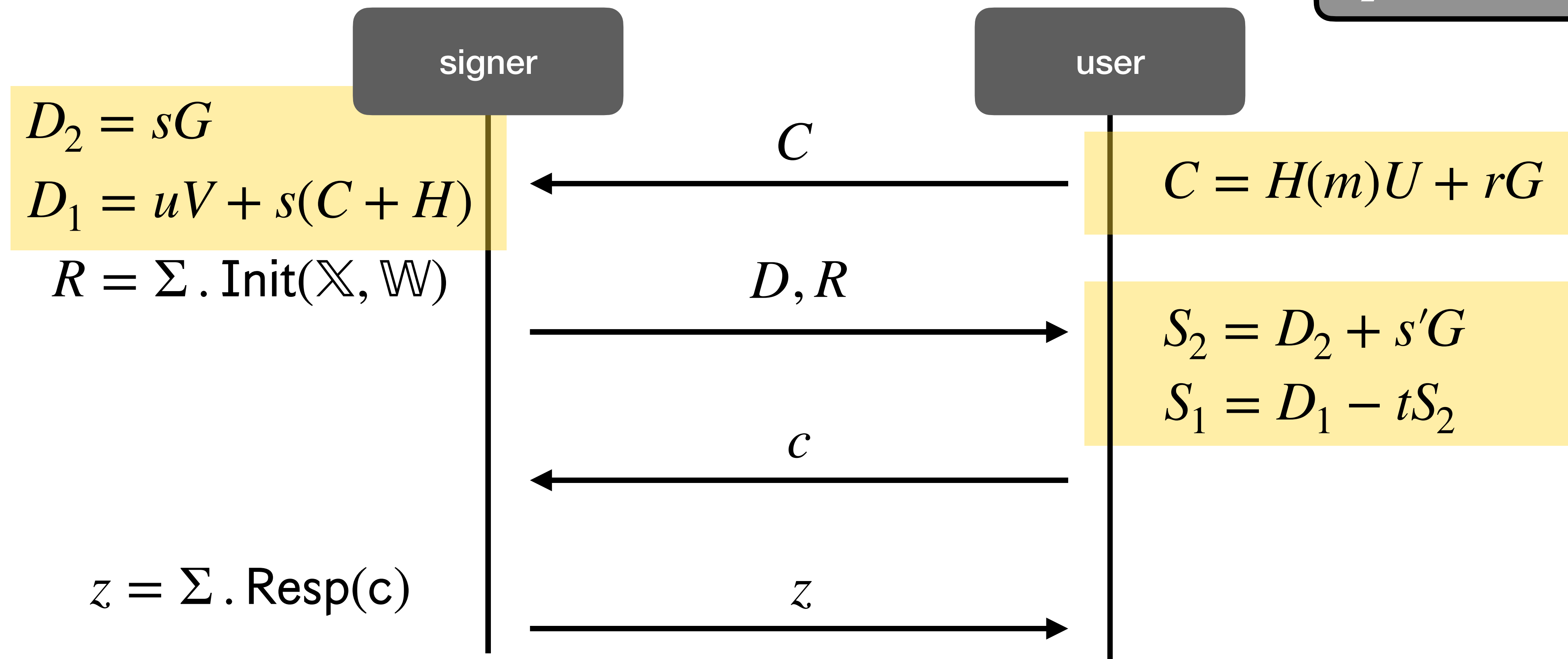


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signer

user

$$D_2 = sG$$

$$D_1 = uV + s(C + H)$$

$$R = \Sigma . \text{Init}(\mathbb{X}, \mathbb{W})$$

C

$$C = H(m)U + rG$$

D, R

$$S_2 = D_2 + s'G$$

$$S_1 = D_1 - tS_2$$

c

blind (R, c, z) into proof  $\pi = (R', c', z')$

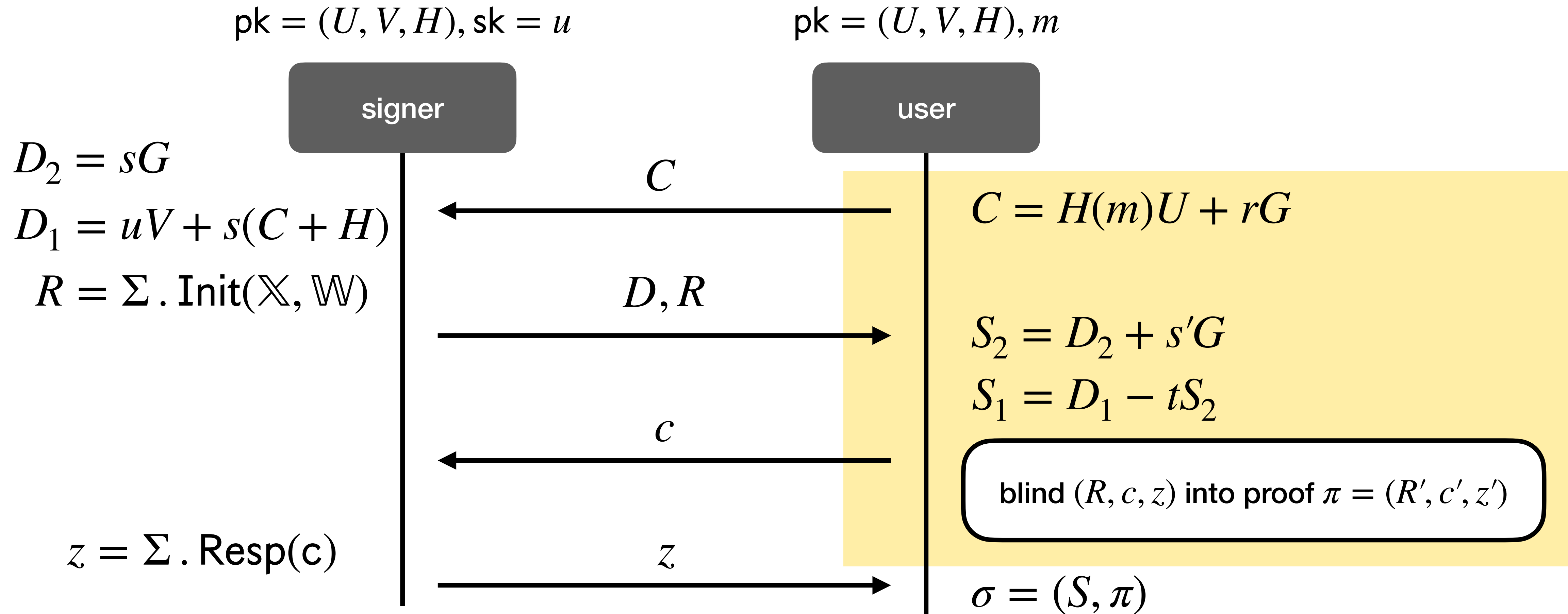
$$z = \Sigma . \text{Resp}(c)$$

z

$$\sigma = (S, \pi)$$

# Blindness

Similar to [CTZ24] and [KRS23]





# One-more Unforgeability

## Approach of [CTZ24]

- Instantiate FS-compiled NIZK  $\pi$  with an OR-proof:
  - **either** signature  $S$  is well-formed
  - **or** know DLog of  $Y = H(0)$

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- Strategy:
  1. under DLog,  $S$  is of the correct form
  2. DLog of  $Y$  is used to simulate without knowing  $sk$

# One-more Unforgeability

## Approach of [CTZ24]

- The argument is subtle

# One-more Unforgeability

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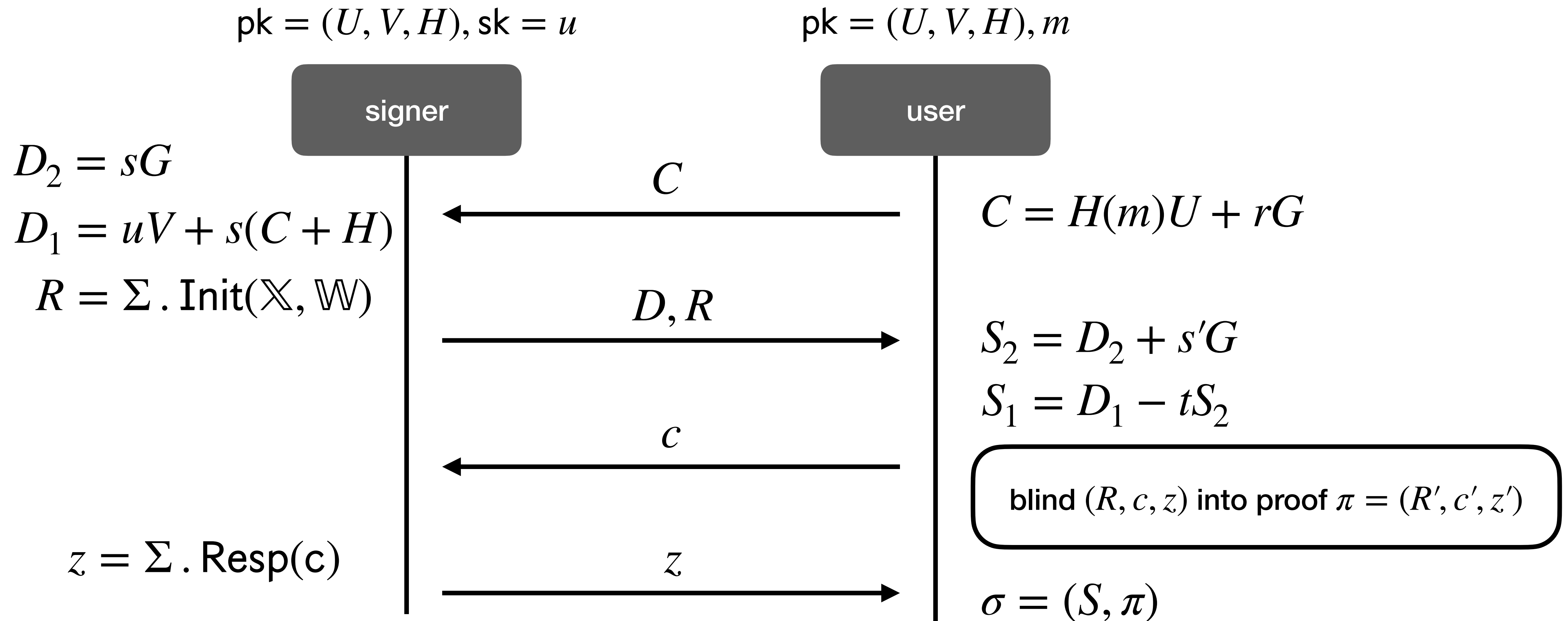
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    - can only argue  $Q$ -OMUF for  $Q$  opened sessions (OMUF-1)
  - $BS_3$ : send commitment instead of  $S$ 
    - OMUF-2 at cost of signature and communication size

# One-more Unforgeability

## OMUF-2 for Free



# One-more Unforgeability

## OMUF-2 for Free

$$pk = (U, V, H), sk = u$$

$$pk = (U, V, H), m$$

signer

user

$$D_2 = sG$$

$$D_1 = uV + s(C + H)$$

$$R = \Sigma . \text{Init}(\mathbb{X}, \mathbb{W})$$

💡  $sH$  is uniform under DDH

$$z = \Sigma . \text{Resp}(c)$$

$C$

$D, R$

$c$

$z$

$$C = H(m)U + rG$$

$$S_2 = D_2 + s'G$$

$$S_1 = D_1 - tS_2$$

blind  $(R, c, z)$  into proof  $\pi = (R', c', z')$

$$\sigma = (S, \pi)$$

# One-more Unforgeability

## OMUF-2 for Free

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$C$

$\$, R$

$c$

$z$

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# One-more Unforgeability

## Avoiding Rewinding

- Instantiate NIZK with an OR-proof:
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requires rewinding to argue that  $S$  is well-formed

# One-more Unforgeability

## Avoiding Rewinding

- Instantiate NIZK with an OR-proof:
  - **either** signature  $S$  is well-formed
  - **or**  $(X, Y, Z) = H(0)$  is a DDH tuple



we can argue that  $S$  is well-formed without rewinding

# Recap

## Pairing-free blind signature without the AGM

Scheme	Signature Size	Communication Size	Security	Assumption
<b>BS<sub>1</sub> + BS<sub>2</sub></b> [CTZ24]	$1G + 4Z_p$	$5G + 5Z_p$	OMUF-1	OMCDH
<b>BS<sub>3</sub></b> [CTZ24]	$\text{poly}(\lambda)$	$\text{poly}(\lambda)$	OMUF-2	CDH
<b>Our Work</b>	$2G + 5Z_p$	$\text{poly}(\lambda)$	OMUF-2	DDH



- tighter reduction
- better efficiency
- partial blindness