Faster Signatures from MPC-in-the-Head

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MPC-in-the-Head Paradigm (MPCitH)

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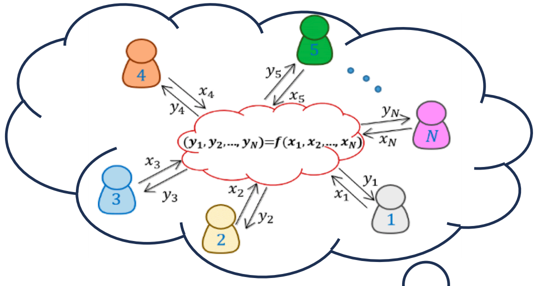
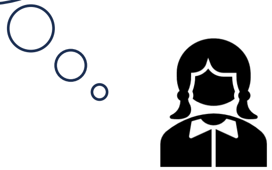
A compiler that transfers MPC protocol into HVZK proof for arbitrary circuits





$\mathrm{w},\ C(\mathrm{w}) = 1$

Prover:
- Additive share w
- Execute MPC protocol in the Head
- Get the views of N parties

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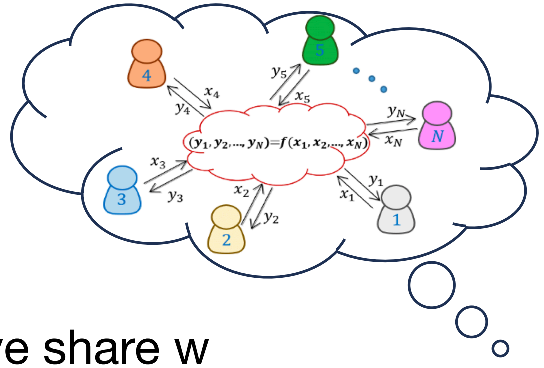

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Commit to all $(\mathrm{view_i})_{i \in [N]}$ →

$\mathrm{P} \in [\mathrm{N}]$ ←

$(\mathrm{view_i})_{i \neq P}$ →

Verifier:
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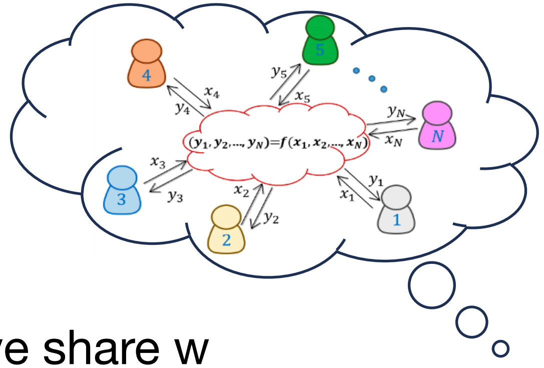

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Soundness error: 1/N

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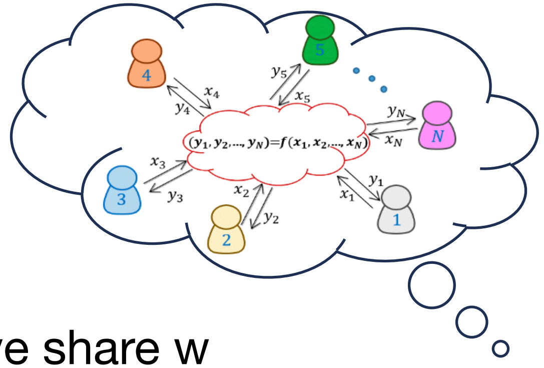

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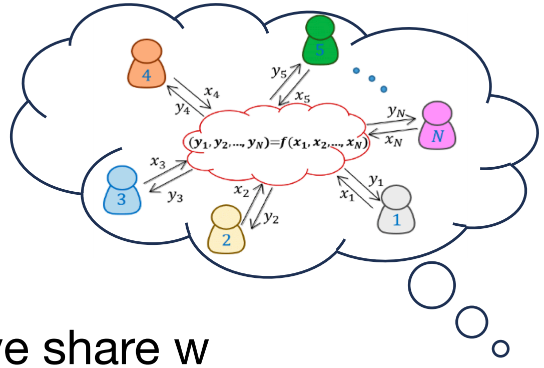

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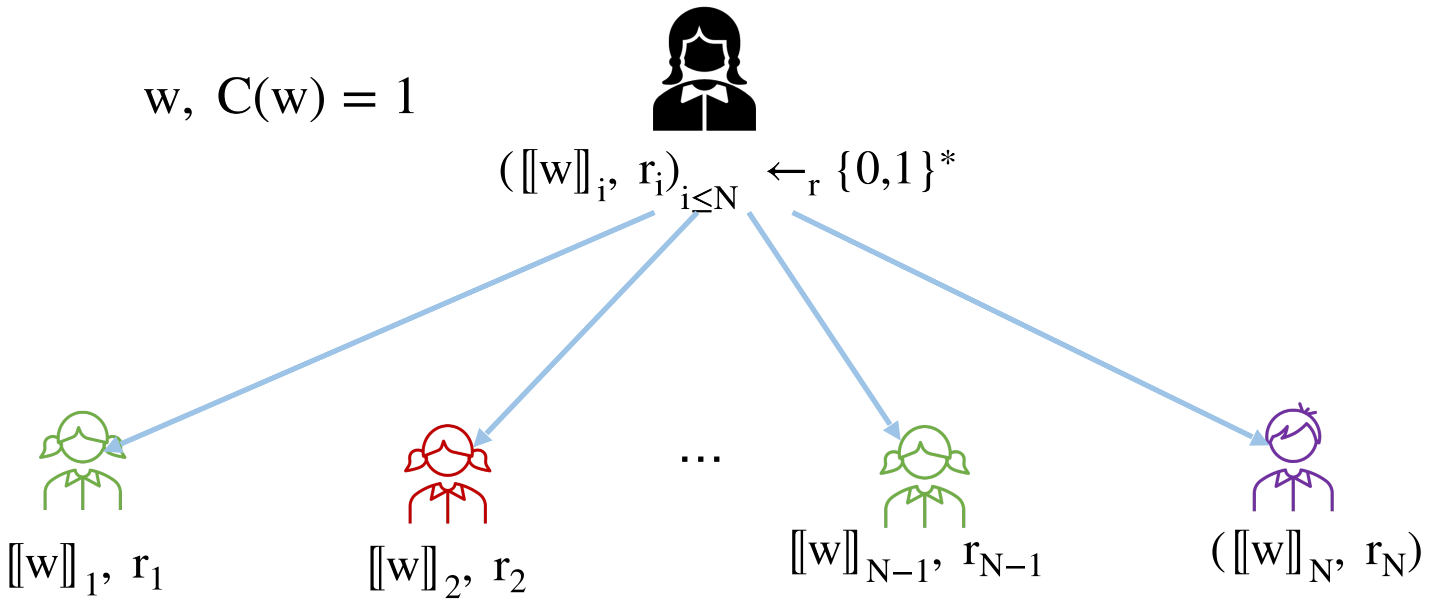
Get a signature from any OWF using Fiat-Shamir

Open all-but-one Views in MPCitH

MPCitH protocol:

- Generate shares of witness with shares of preprocessing material
- All shares can be considered as **random values**



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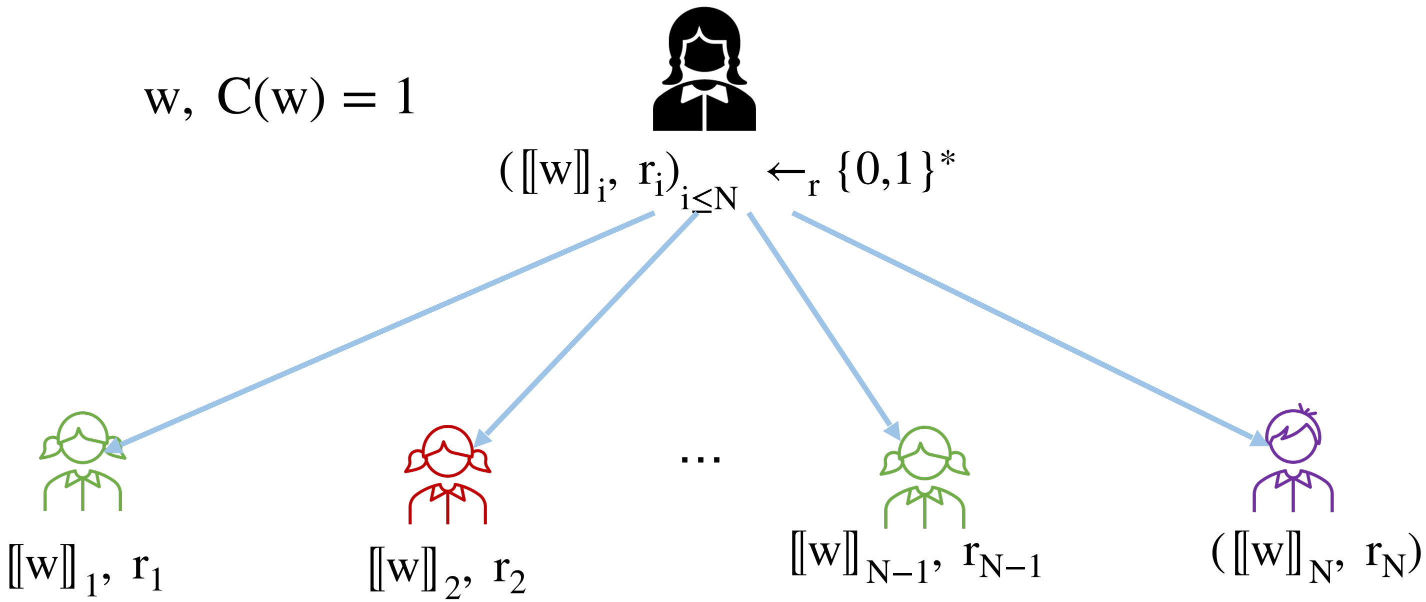

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Requirements in MPCitH-based signatures:

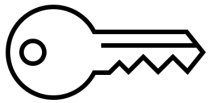
Unforgeability security

Parallel repetitions

Puncturable PRF (PPRF)

Puncturable PRF (PPRF): PRF **F** with master key K: $\mathbf{F}(K, x) \rightarrow y$

Punctured point: $P \leftarrow_r \{0,1\}^{\lambda}$



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Punctured key K{P}

- K{P} is succinct
- Compute **F**(K, x) for all x except P
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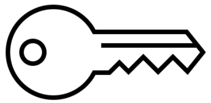

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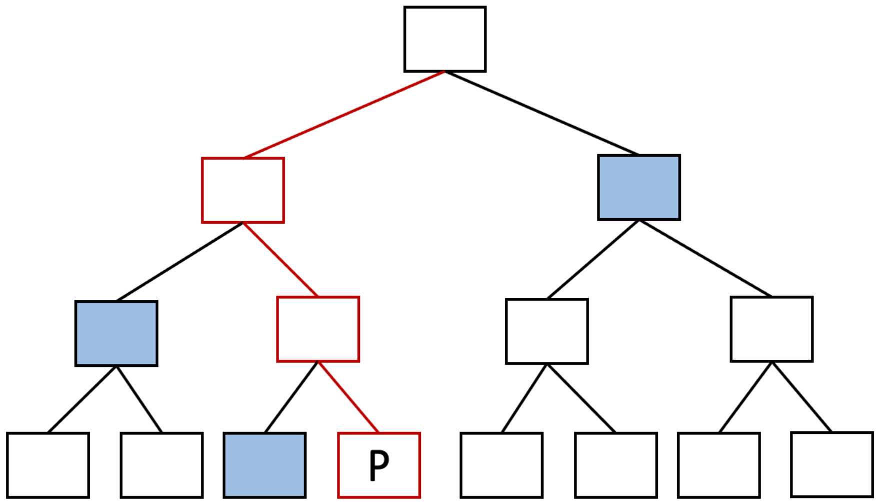
Instantiation PPRF: GGM tree

Length-doubling PRG: $F(x) \rightarrow \left(x_0, x_1\right) \in \{0,1\}^{2\lambda}$



Communication is reduced from $(N-1)\cdot\lambda$ to $\log N\cdot\lambda$

N leaves

Contribution 1: Multi-instance PPRF

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(Q, τ)-multi-instance PPRF:

- Handle the security related to τ-repetitions in each signature
- Drop-in replacement of all PPRF in MPCitH-based signatures as considering Q the is number of queries from AdvA to signing oracle in unforgeability game

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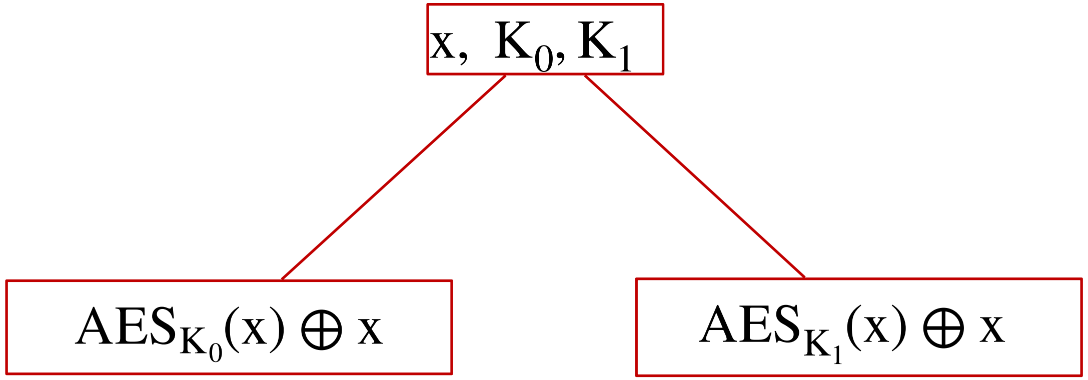
New construction of PRG:

Davies-Meyer function $$F(x, salt) = \left(F_0(x,\ salt),\ F_1(x, salt)\right)$$



For τ-repetitions, (K_0, K_1) is used across all PPRF trees

Instantiate by fixed-key AES-NI (takes only 1.3 cycles per Byte)

Efficient, 12x to 55x speed improvement when plugging in the state of art ([C:JouHut24])

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Construction of PRG:

$$F(x, K_0, K_1) = (AES_{K_0}(x) \oplus x, AES_{K_1}(x) \oplus x)$$

Security:

is proved in the **ideal cipher** using H-coefficient technique

(Q, τ)-instance (t, ϵ)-secure PRG

$(x_i, K_{0,i}, K_{1,i})_{i \le Q}$

$b \leftarrow_r \{0,1\}$

$(K_{0,i}, K_{1,i})_{i \le Q}$ →

$b = 0, F(x_i,\ K_i)$ →

$b = 1$, random

Guess b

Q instances, each instance repeats τ-times using **same salt**

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$(\mathrm{x}_\mathrm{i}, \mathrm{K}_{0,\mathrm{i}}, \mathrm{K}_{1,\mathrm{i}})_{\mathrm{i} \leq \mathrm{Q}}$

$\mathrm{b} \leftarrow_r \{0,1\}$

$(\mathrm{K}_{0,\mathrm{i}}, \mathrm{K}_{1,\mathrm{i}})_{\mathrm{i} \leq \mathrm{Q}}$ →

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Security of PPRF: for a GGM tree of N = 2^D leaves

(t, ϵ)-secure PRG → $\left(Q, \tau\right)$-instance $(t, D.\epsilon)$-secure PPRF

Security loss 5 bits
$(\mathrm{D} = 16,\ \tau = 8, \lambda = 128)$

Contribution 2: New MPCitH-based signature

Assumption: Regular syndrome decoding (RSD),

Sample a matrix $\mathrm{H} \in \mathbb{F}_2^{k \times K}$, $\mathbf{x} \in \mathbb{F}_2^{K}$ such that **x** is w-regular noise vector, set $\mathbf{y} = \mathrm{H} \cdot \mathbf{x}$

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Intuition:

Denote bs = K/w,

	bs				
Vector **x**:	001..000	000...100	.....	100...000	010...000
Position of noises:	i_1	i_2	.....	i_{w-1}	i_w

Compressed vector of **x**: **comp(x)** = $(i_1, i_2, \ldots, i_{w-1}, i_w) \in (\mathbb{Z}_{bs})^w$

Instead of sharing over $\mathbb{F}_2^K$, we share over $\mathbb{Z}_{bs}$ → $\mathbf{w} \cdot \mathbf{log}(\mathbf{bs})$ bits

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Key idea:

$\mathbf{x} \in \mathbb{F}_2^K$ is w-regular noise: sample $\mathbf{r} \leftarrow_r \mathbb{F}_2^K$ is w-regular noise vector

$\mathbf{z}$ = comp($\mathbf{x}$) – comp($\mathbf{r}$) $\in (\mathbb{Z}_{bs})^w$ (positions of noise)

→ $\mathbf{x}$ = $\mathbf{r}$ shifted by $\mathbf{z}$

$\mathbf{r}$ can be considered as a mask of $\mathbf{x}$

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Parties holds shares of **r** $\in \mathbb{F}_2^K$ (w-regular noise); comp(**x**), comp(**r**) $\in (\mathbb{Z}_{bs})^w$:

- All parties broadcast their shares of **z** = comp(**x**) – comp(**r**) and reconstruct **z**
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Soundness:

Prover can cheat in generating a preprocessing

→ verifier adds a permutation π on **r**, i.e., **z** = comp(**x**) – π(comp(**r**))

Takeaway

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- Efficient based on AES-NI
- Used to bootstrap any MPCitH-based signatures
- Benchmark by plugging into [C:JouHut24]

New MPCitH-based signature
- Based on RSD
- Outperform [EC:CCJ23], competitive efficiency

https://ia.cr/2024/252

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