HELIOPOLIS: Verifiable Computation over Homomorphically Encrypted Data from Interactive Oracle Proofs is Practical

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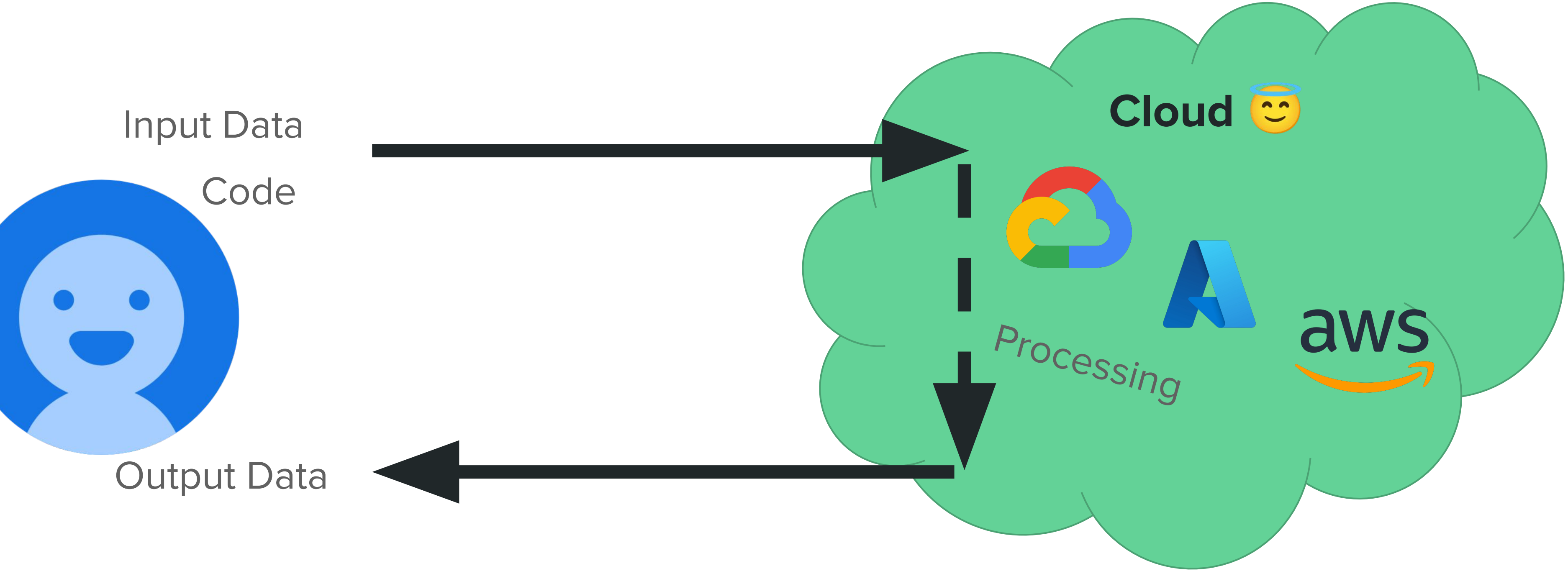
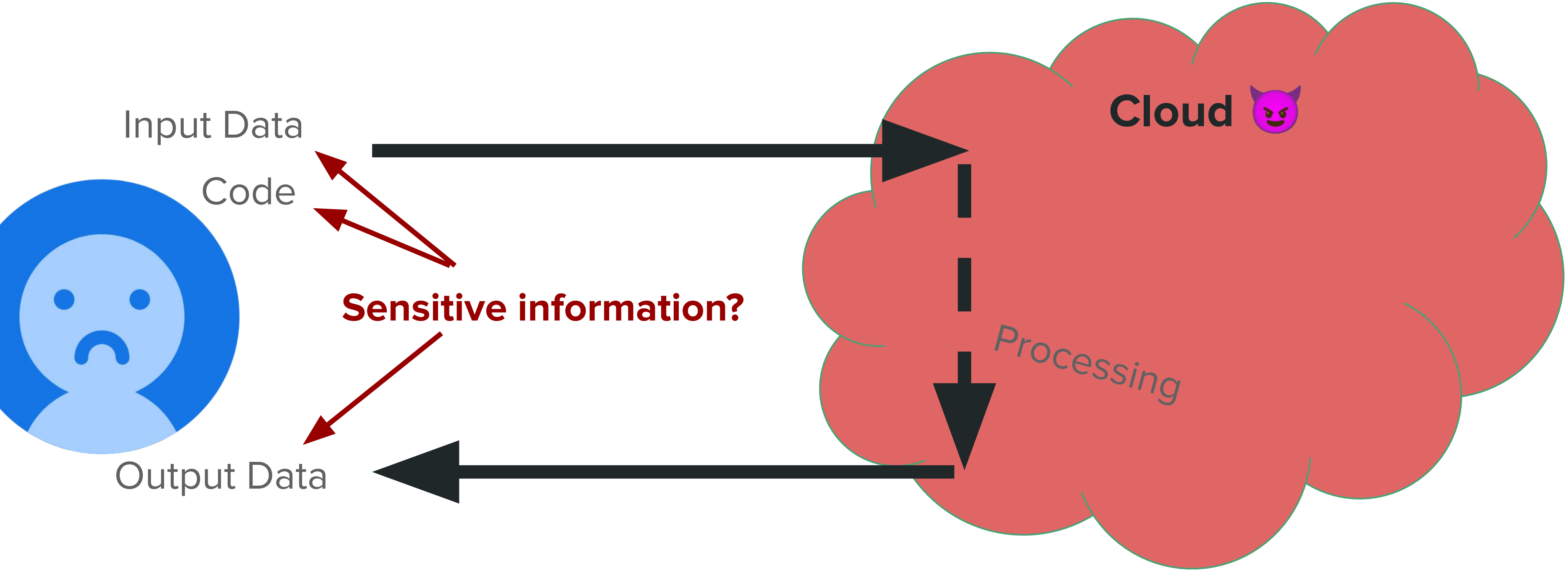
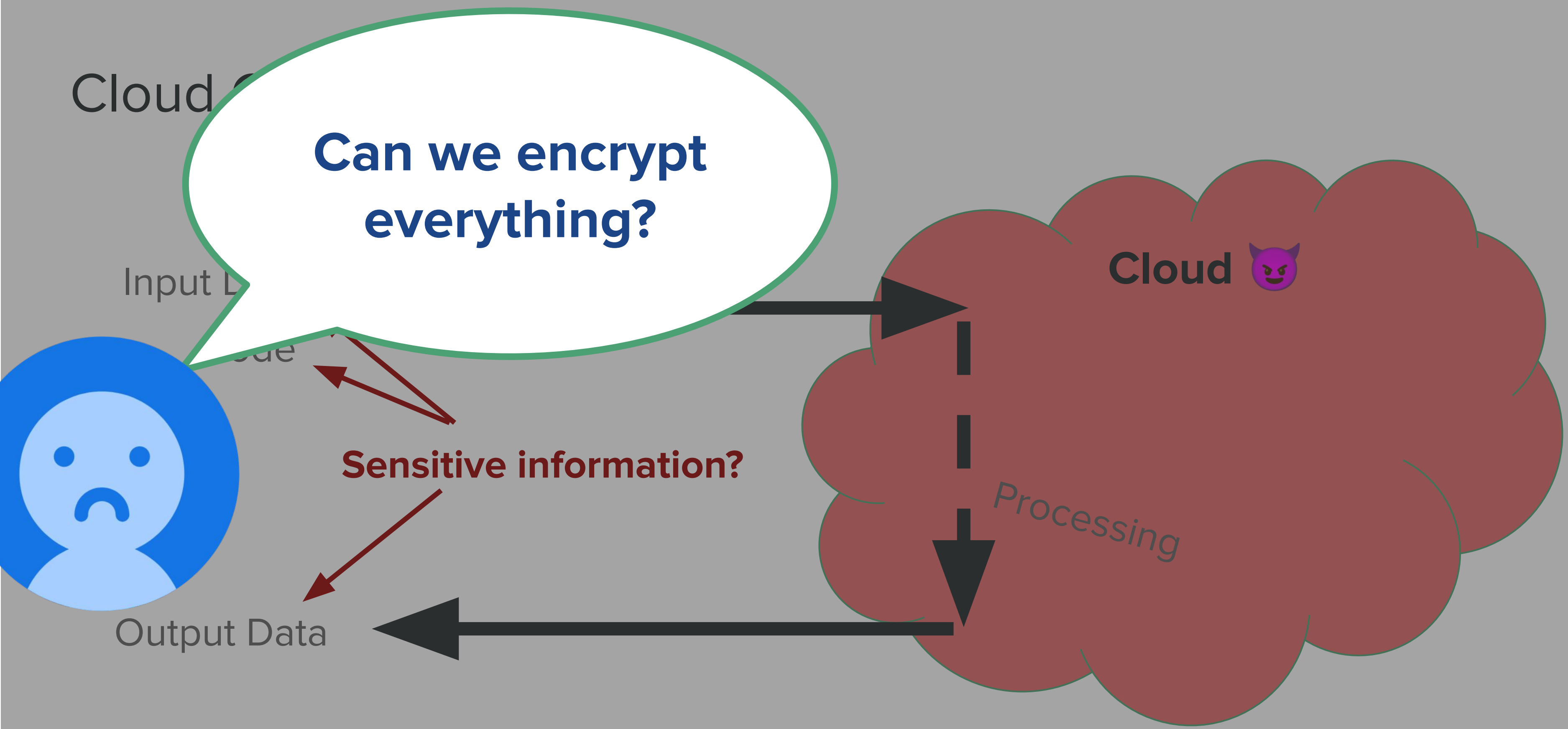
Context

Cloud Computing

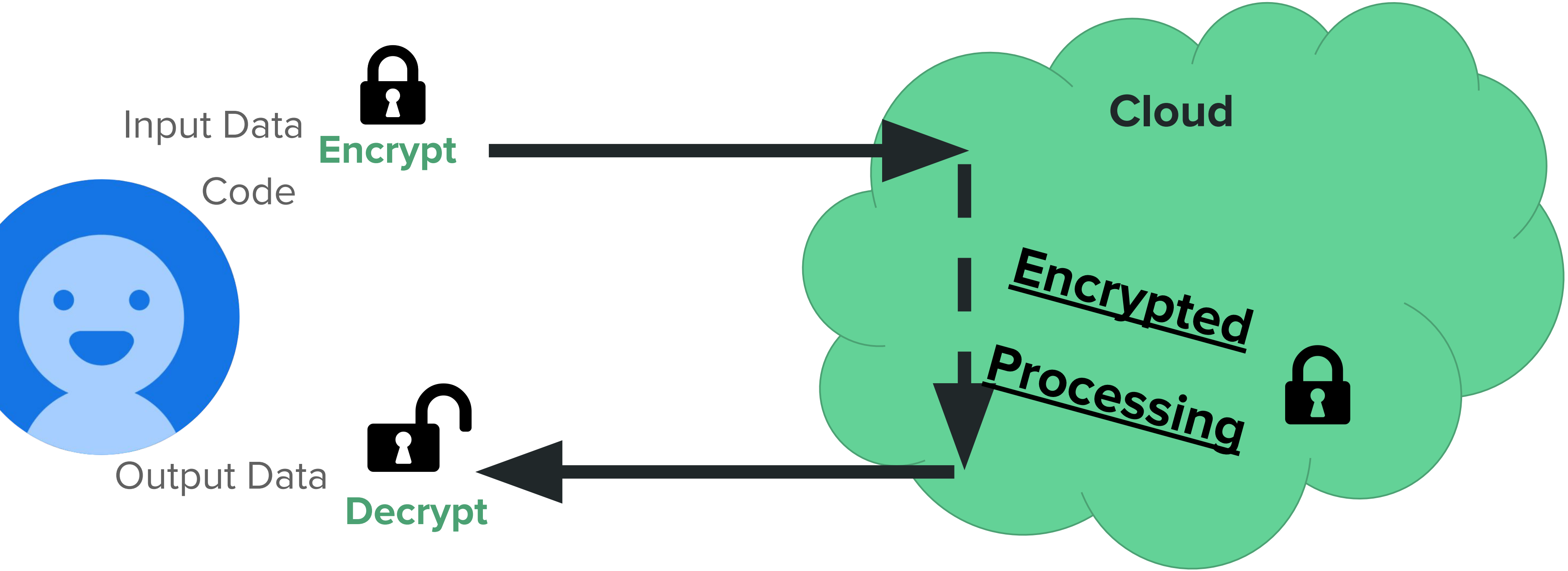


Cloud Computing



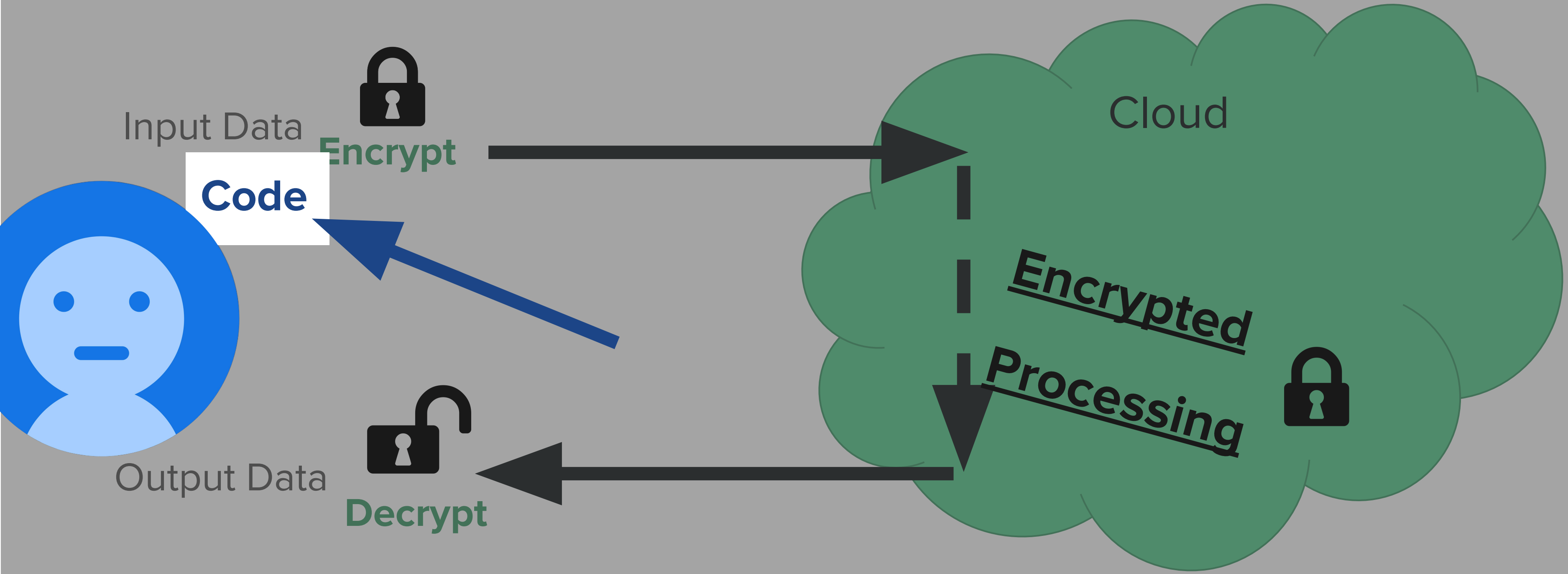

Cloud C
Can we encrypt everything?
Input D
Code
Sensitive information?
Output Data
Cloud 😈
Processing

Homomorphic Encryption (HE)



- **Secure**
- **Functionally Complete**
- **(reasonably) practical**

Homomorphic Encryption (HE)



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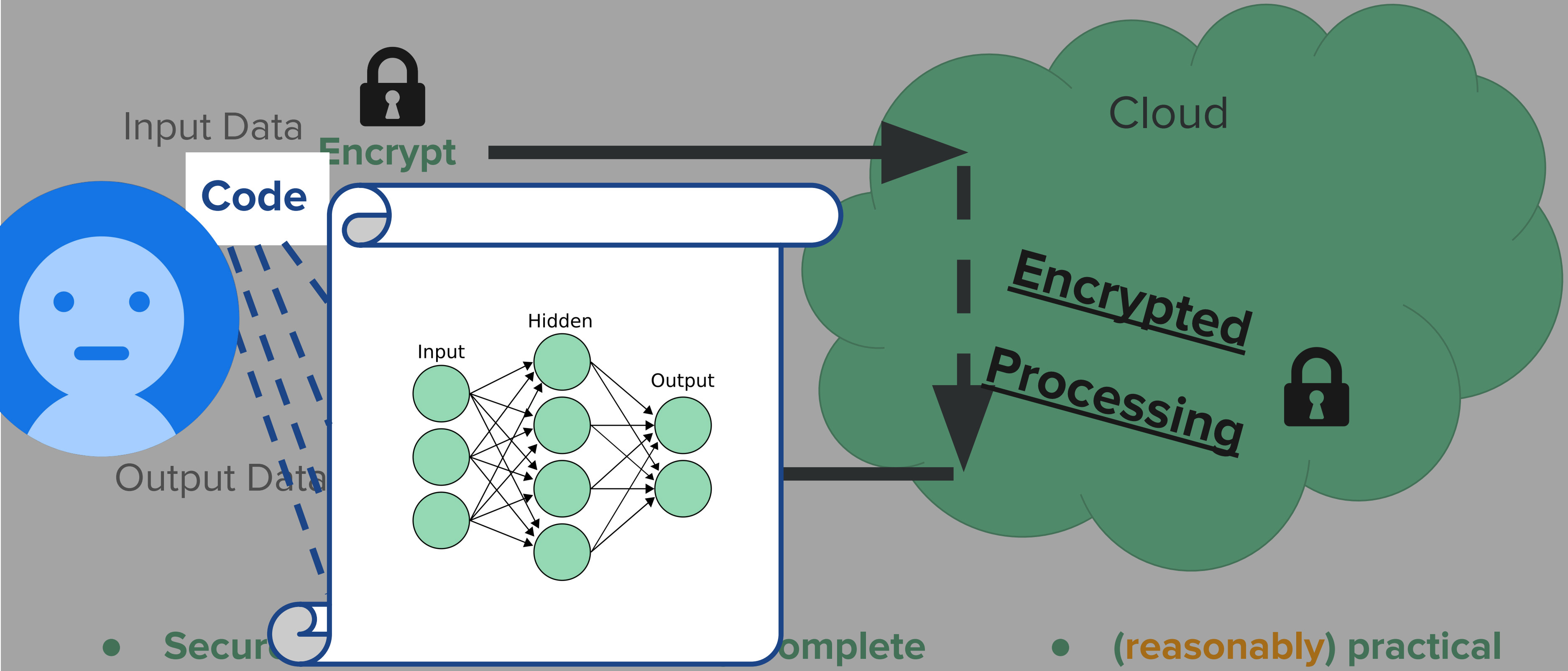

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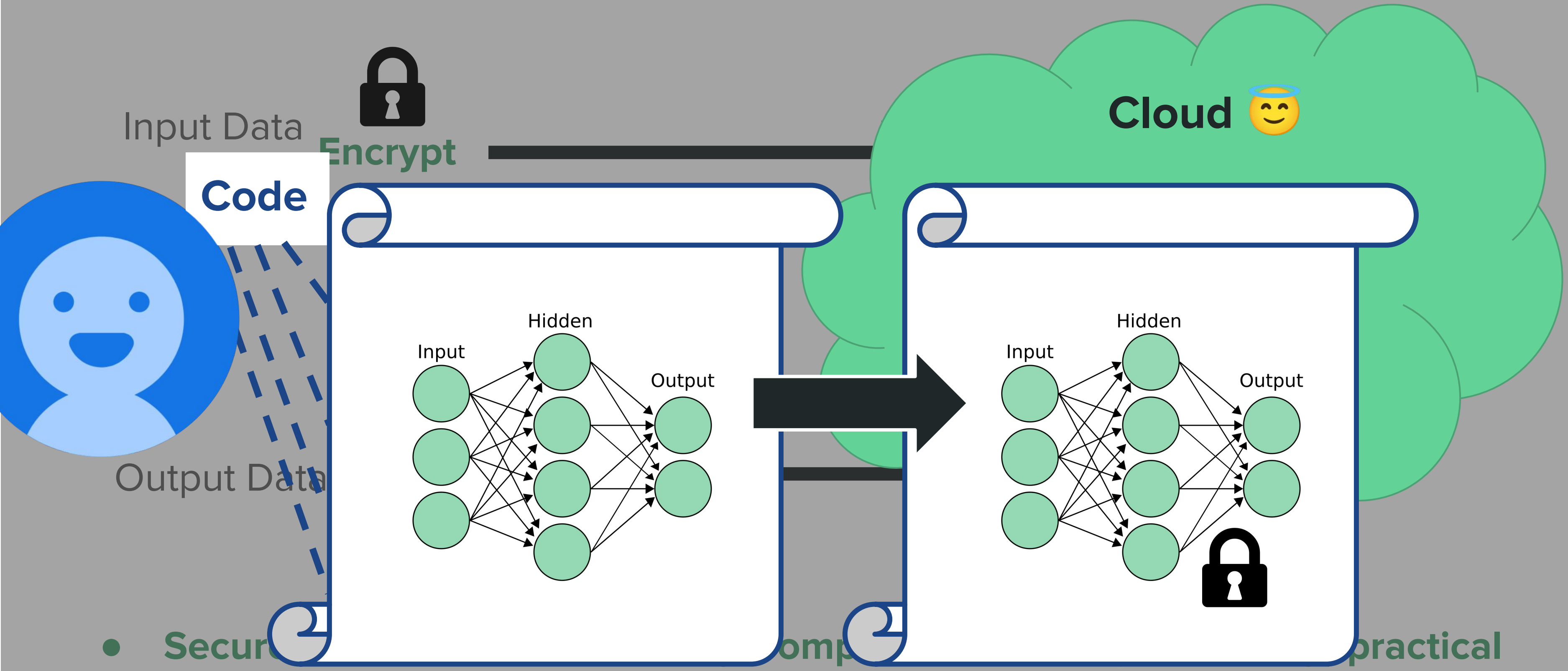
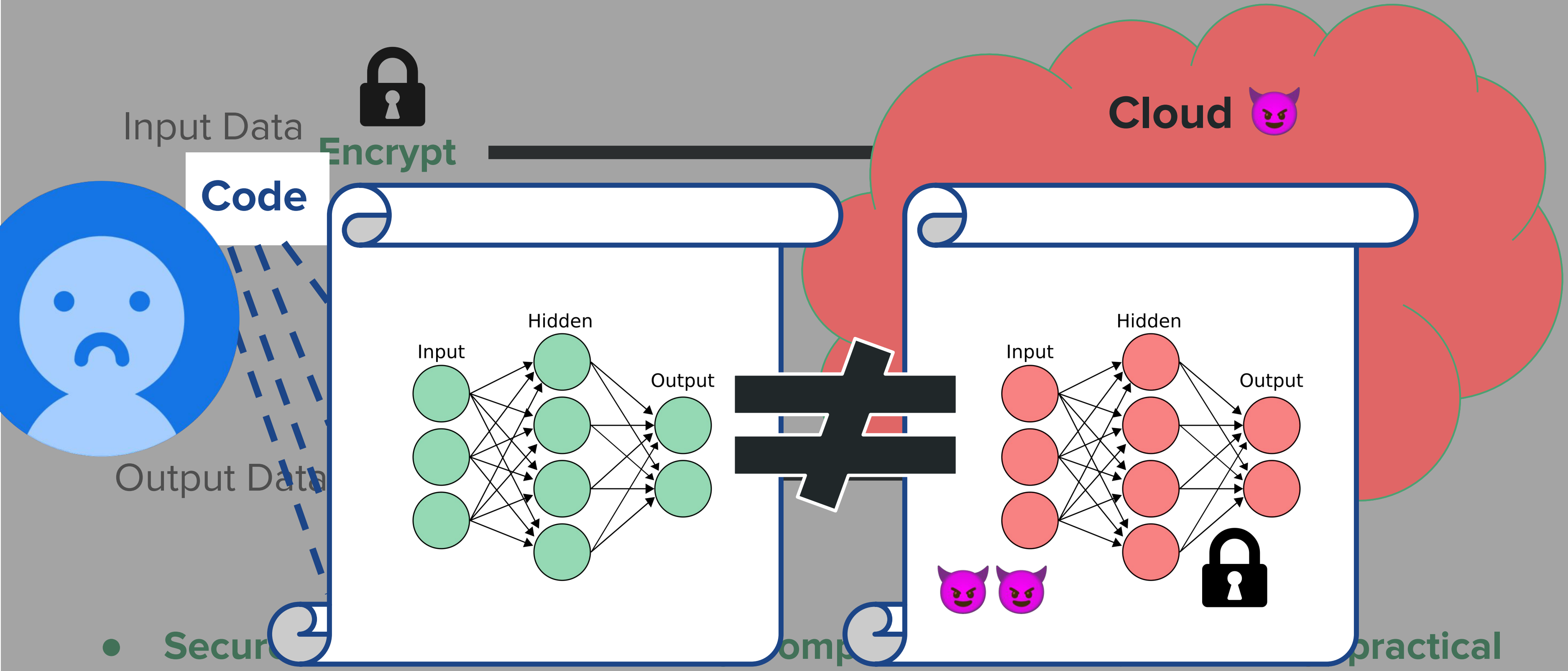
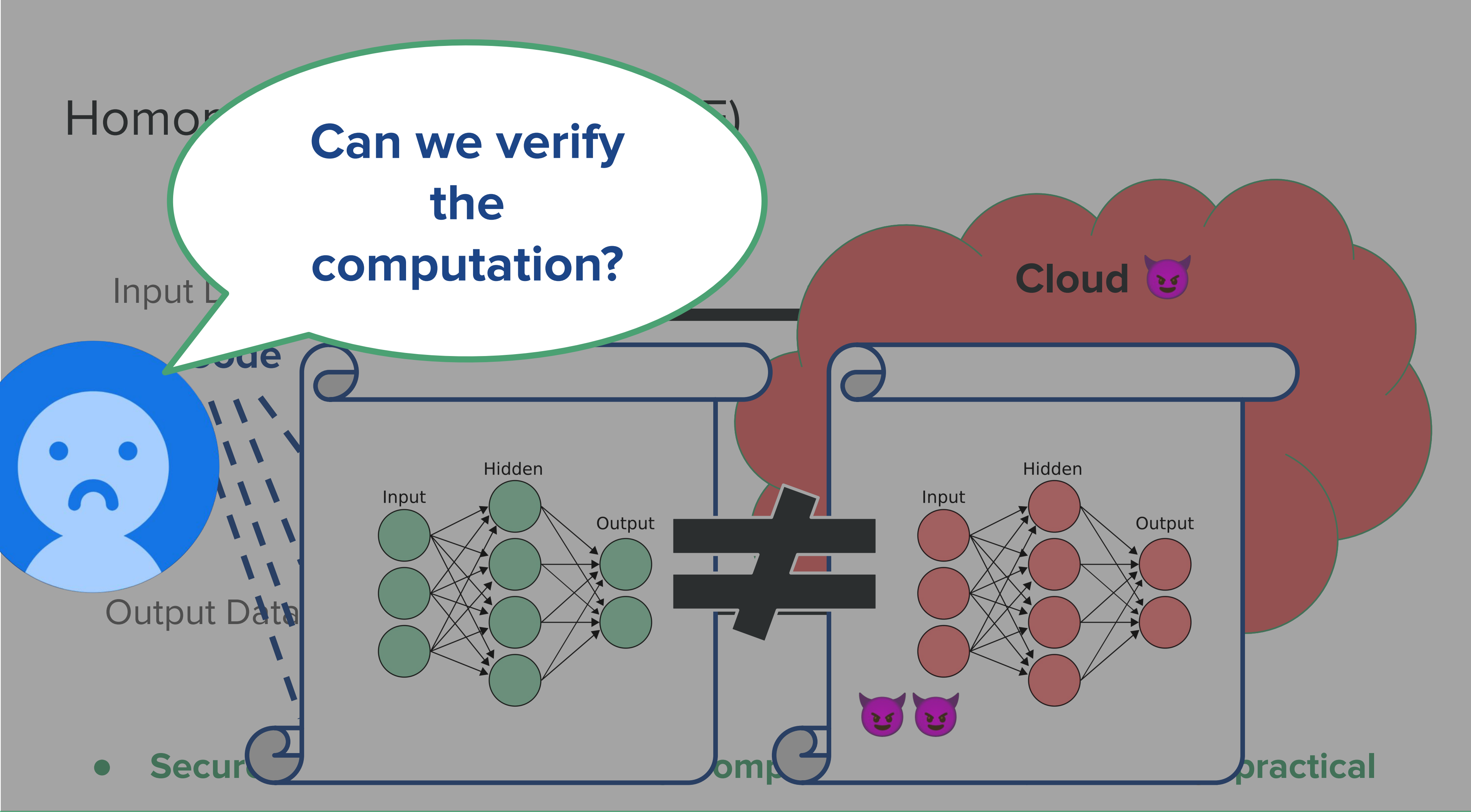


- **Secur… comp… practical**

Homomorphic Encryption (HE)



- **Secur** … **omp** … **practical**


Homo
E)
Can we verify the computation?
Input D
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Output Data
Cloud
Hidden
Input
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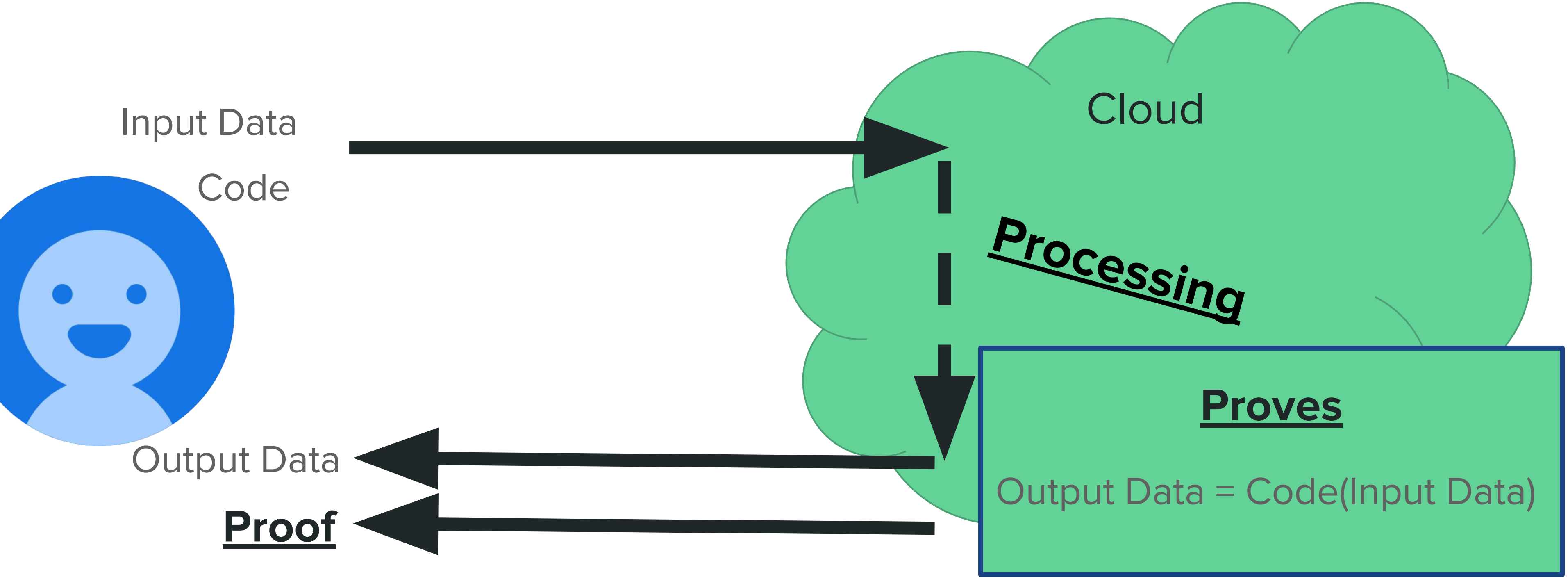
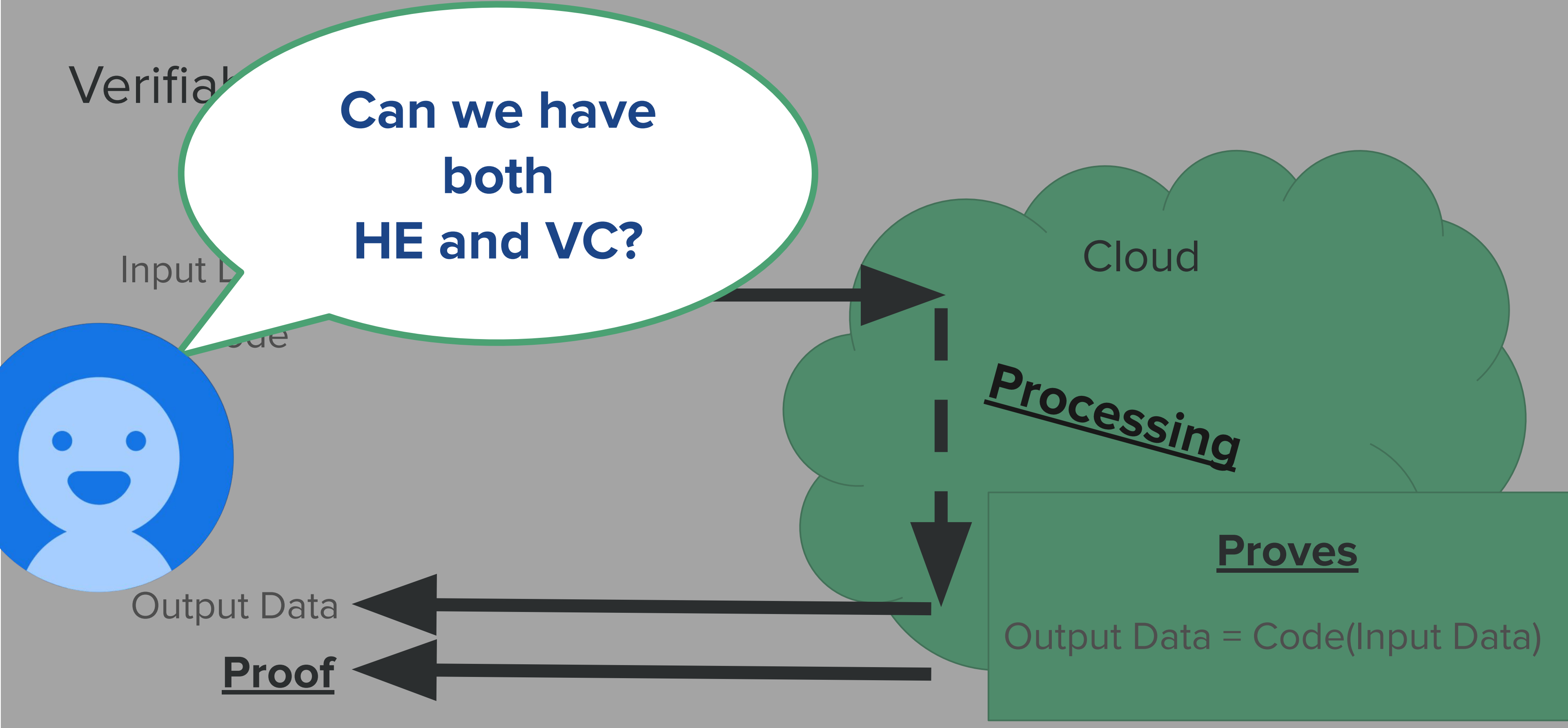
practical

Verifiable computation (VC)



- **Sound**
- **Functionally Complete**
- **(reasonably) practical**



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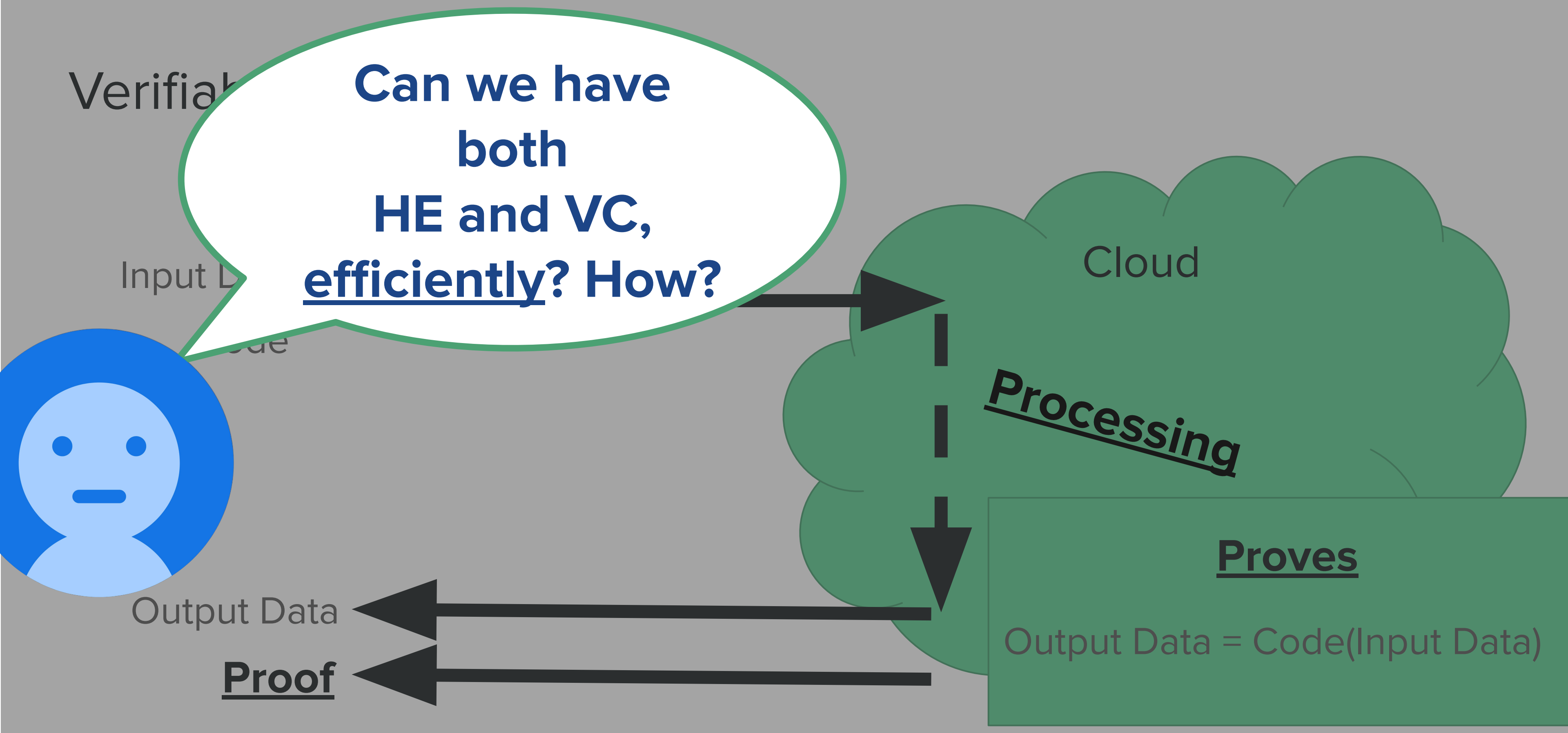

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Main VC-HE approach so far

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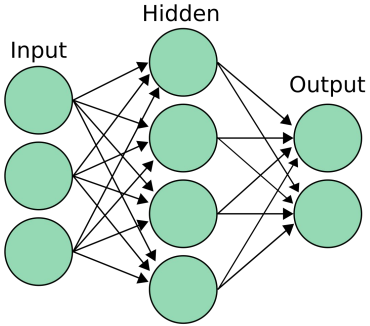
`Prove( HE(`  `) )`

⚠️ **<u>Problem:</u> VC and HE are not friendly**

Verifiable Computation

Efficient if working with:

- Fields
- Algebraic operations

Homomorphic Encryption

Efficient if working with*:

- Huge rings with composite moduli
- Rounding and modular reductions

* considering ciphertext operations

Cleartext operation

A = 35 B = 62

A*B = 2170

1. Linear, algebraic operation
2. Easy to embed in a Field
3. Takes 2 bytes of memory
4. Takes picoseconds

Homomorphic Operation

A = Encrypt(35) B = Encrypt(62)

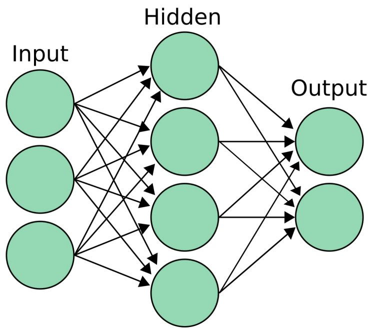
A*B = Mod-Switching(

Key-Switching(

Tensor_Multiplication(A,B)))

1. Not algebraic
2. Efficiency requires amortization
3. Takes kilobytes of memory
4. Takes microseconds

VC-HE so far

Prove(HE(

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⚠️ **<u>Problem:</u> VC and HE are not friendly**

VC-HE so far

Prove(

The first intuition:
Instead of proving HE,
can we HE the proof?

⚠️ **Problem:** VC and HE are not friendly

Verifiable Computation

Proof systems typically require:

- Hash functions
- Large fields

Homomorphic Encryption

Most efficient if working with:

- Rings or small fields
- Algebraic operations

* considering plaintext operations

Our approach (HE-IOPs)

The first intuition:
Instead of proving HE,
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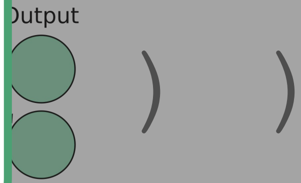


⚠️ **Problem:** VC and HE are not friendly

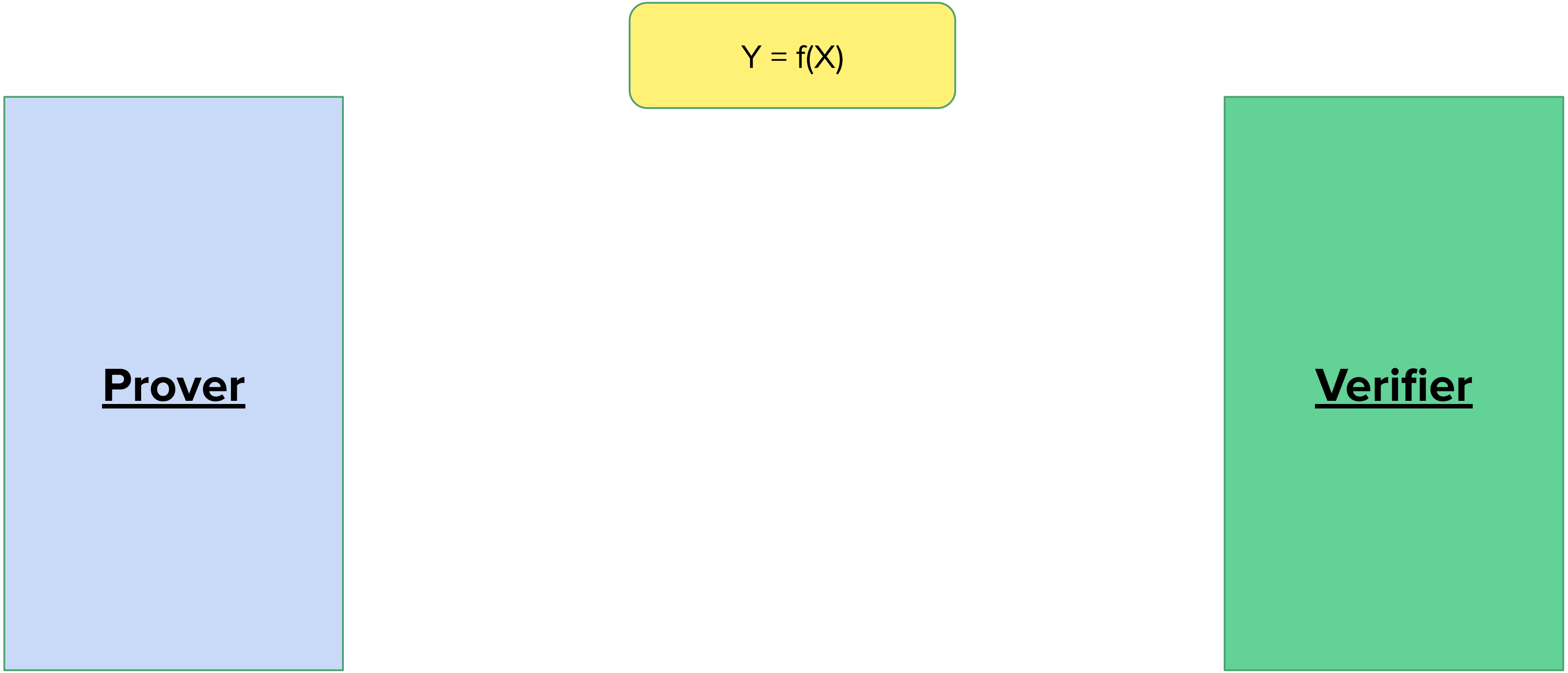
The first intuition:
Instead of proving HE,
can we HE the proof?

utput

⚠️ Problem: VC and HE ar

Our method: HE the information theoretic component of the proof system

Interactive Oracle Proof (IOP)



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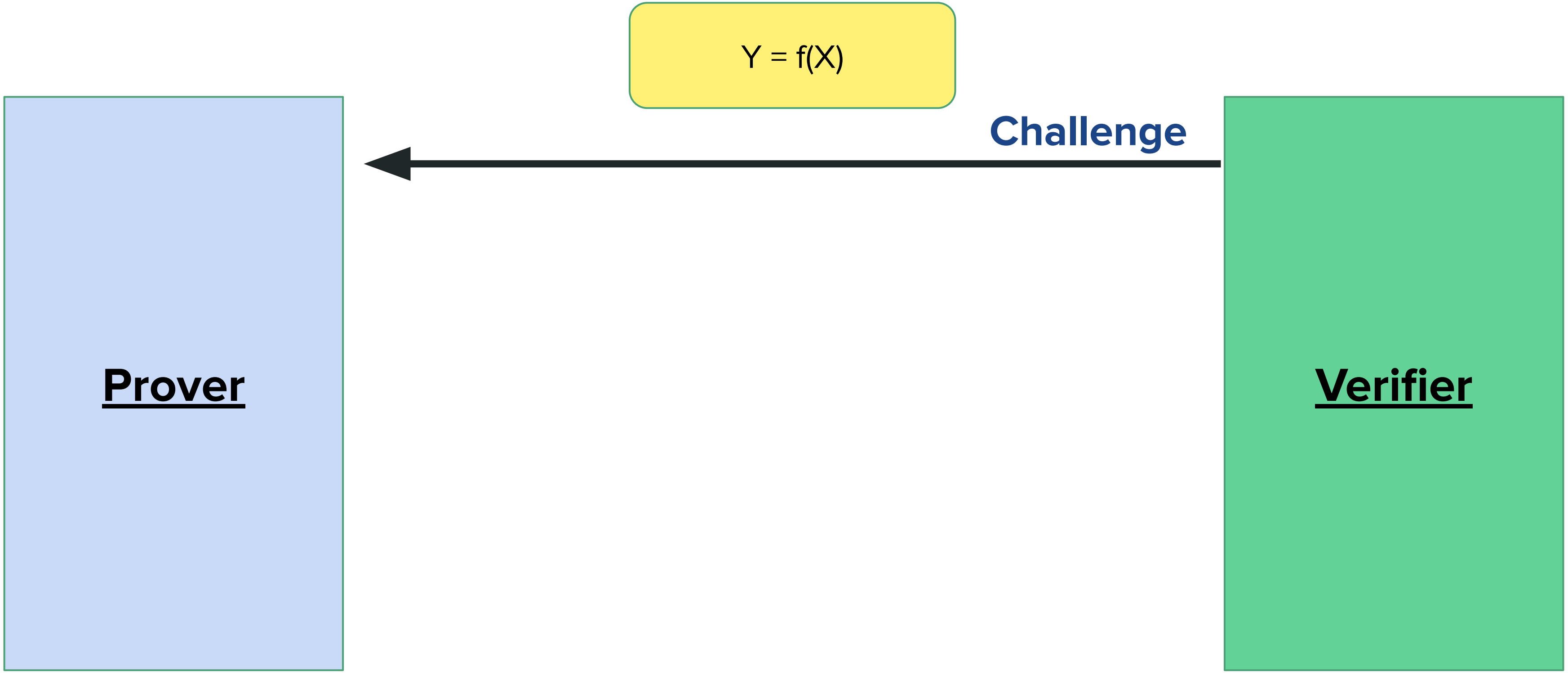

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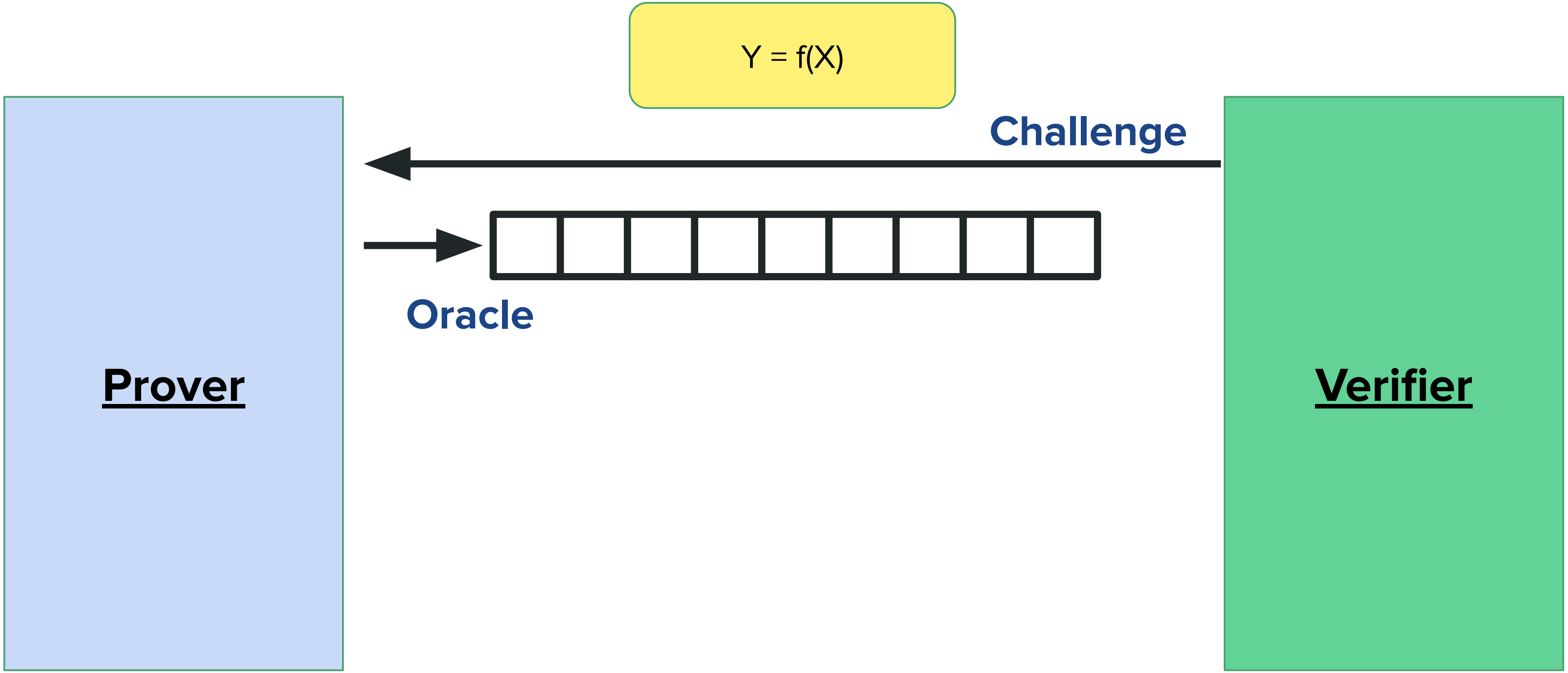

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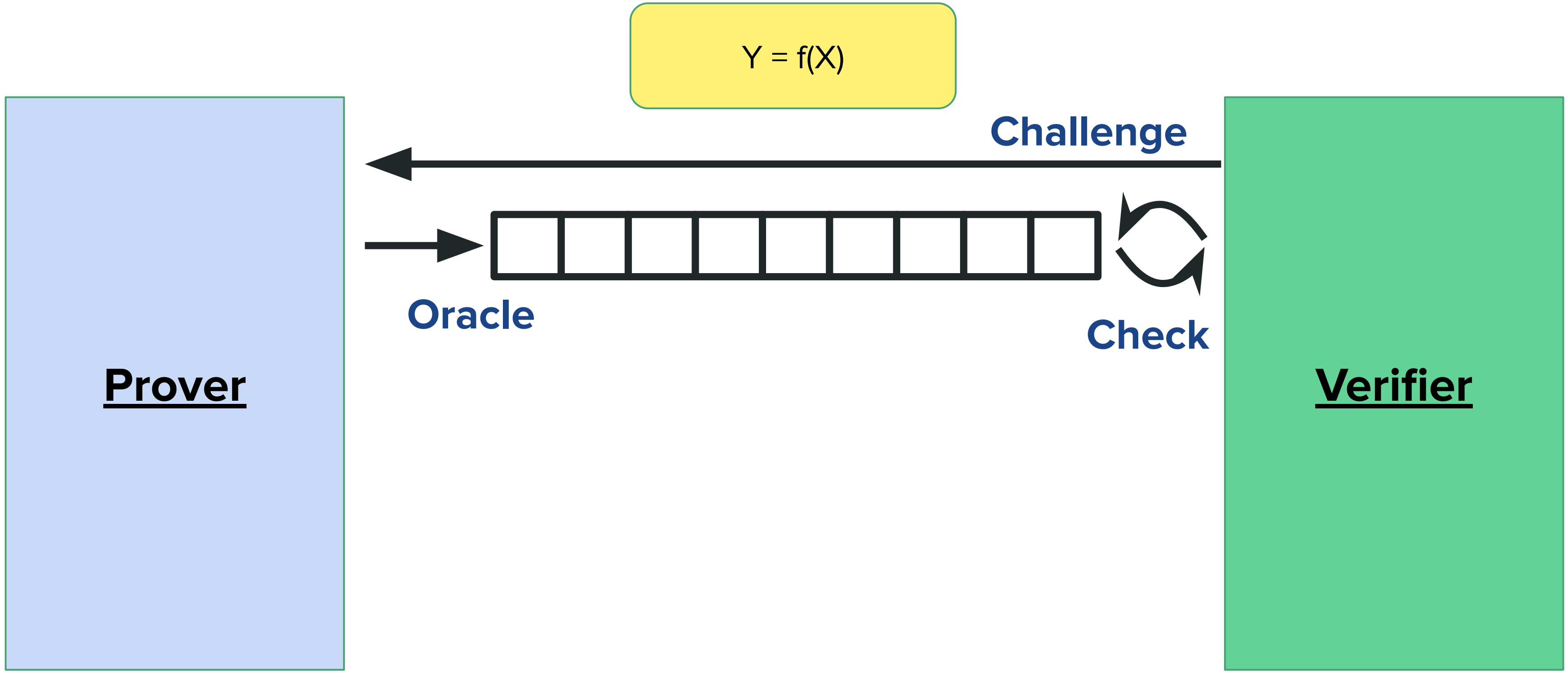

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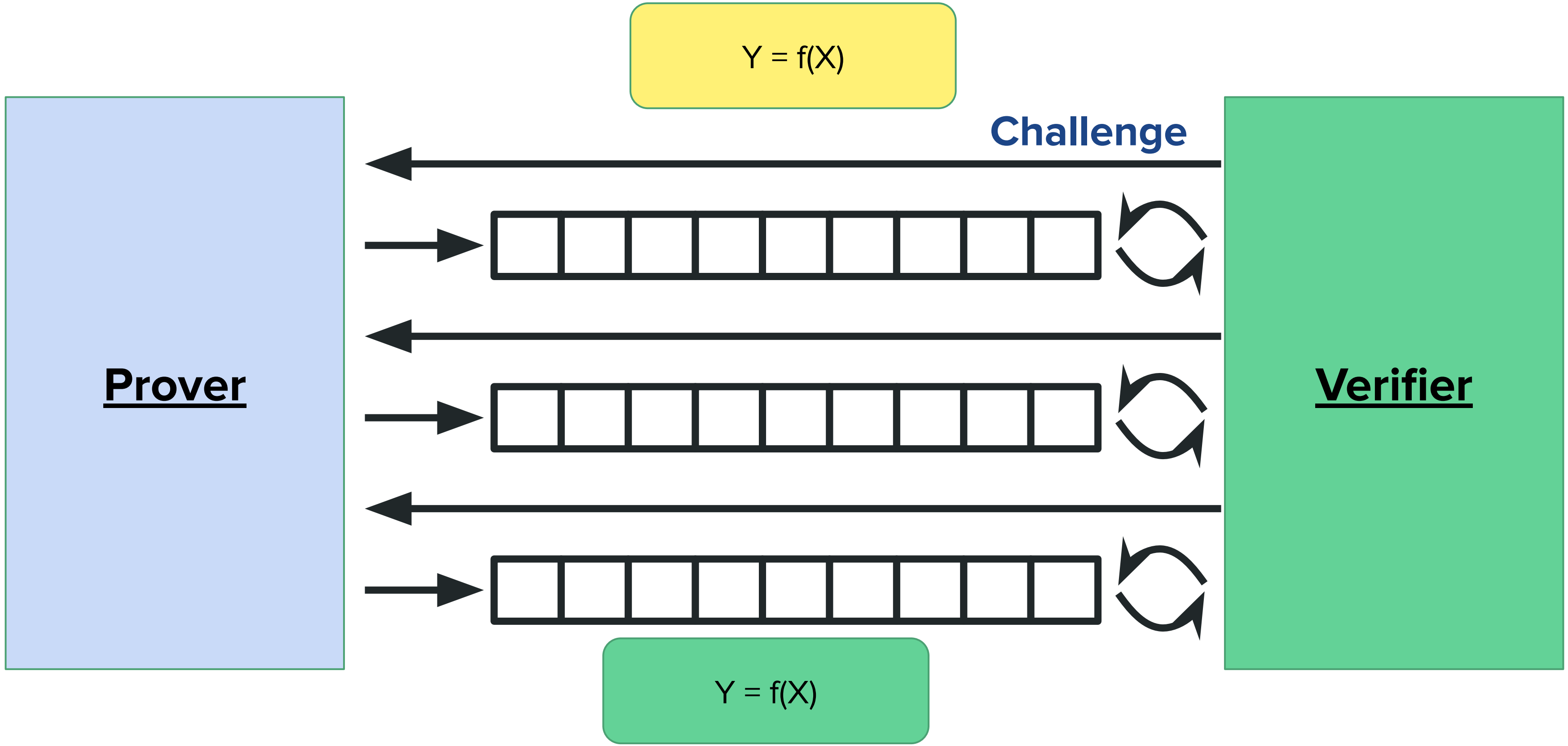
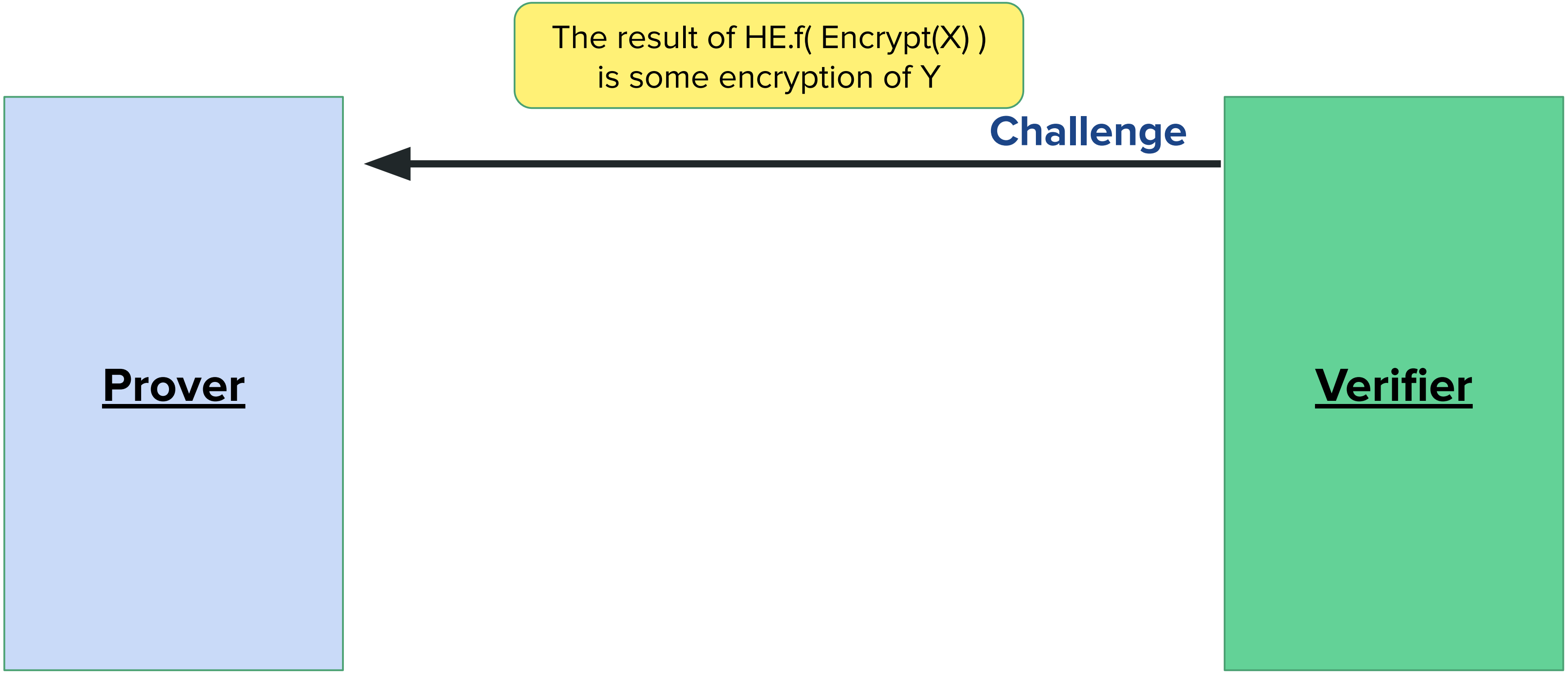


HE Interactive Oracle Proof (HE-IOP)



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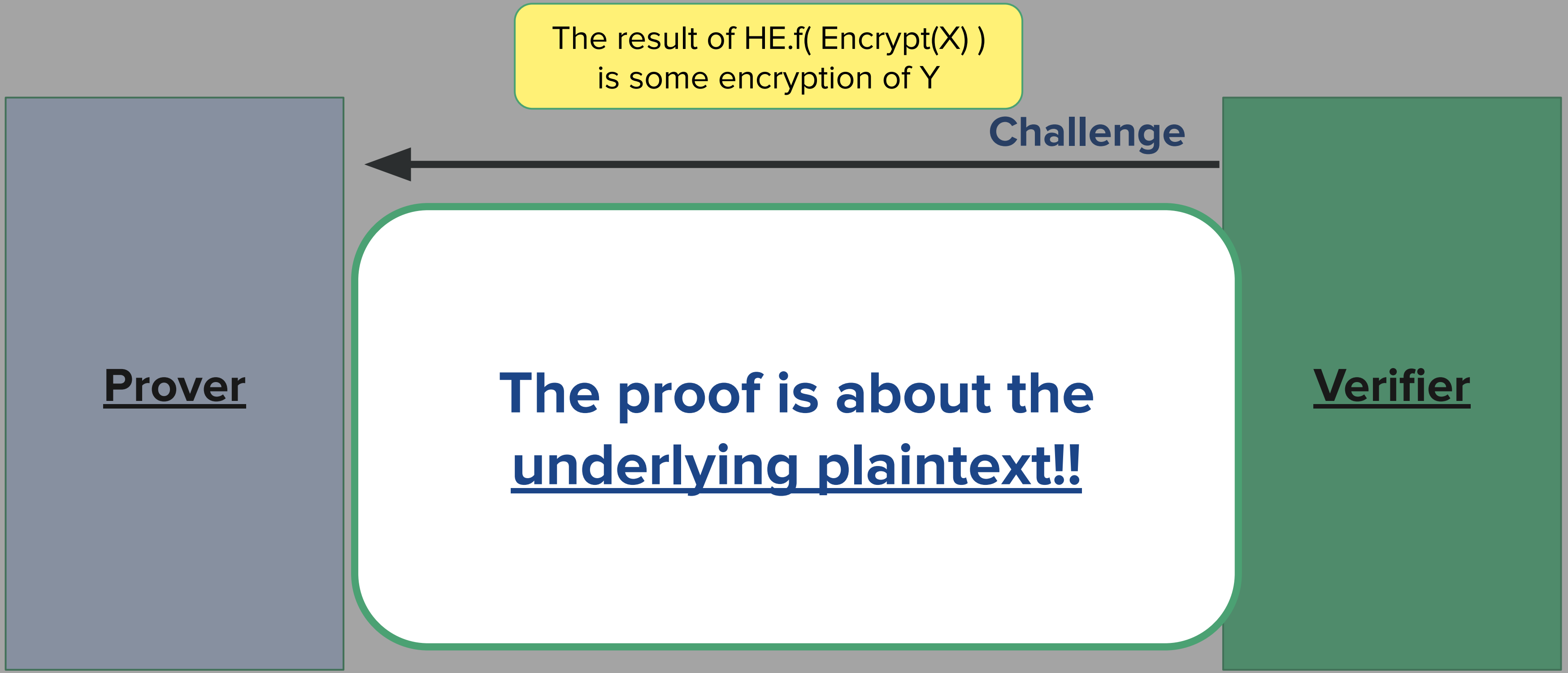

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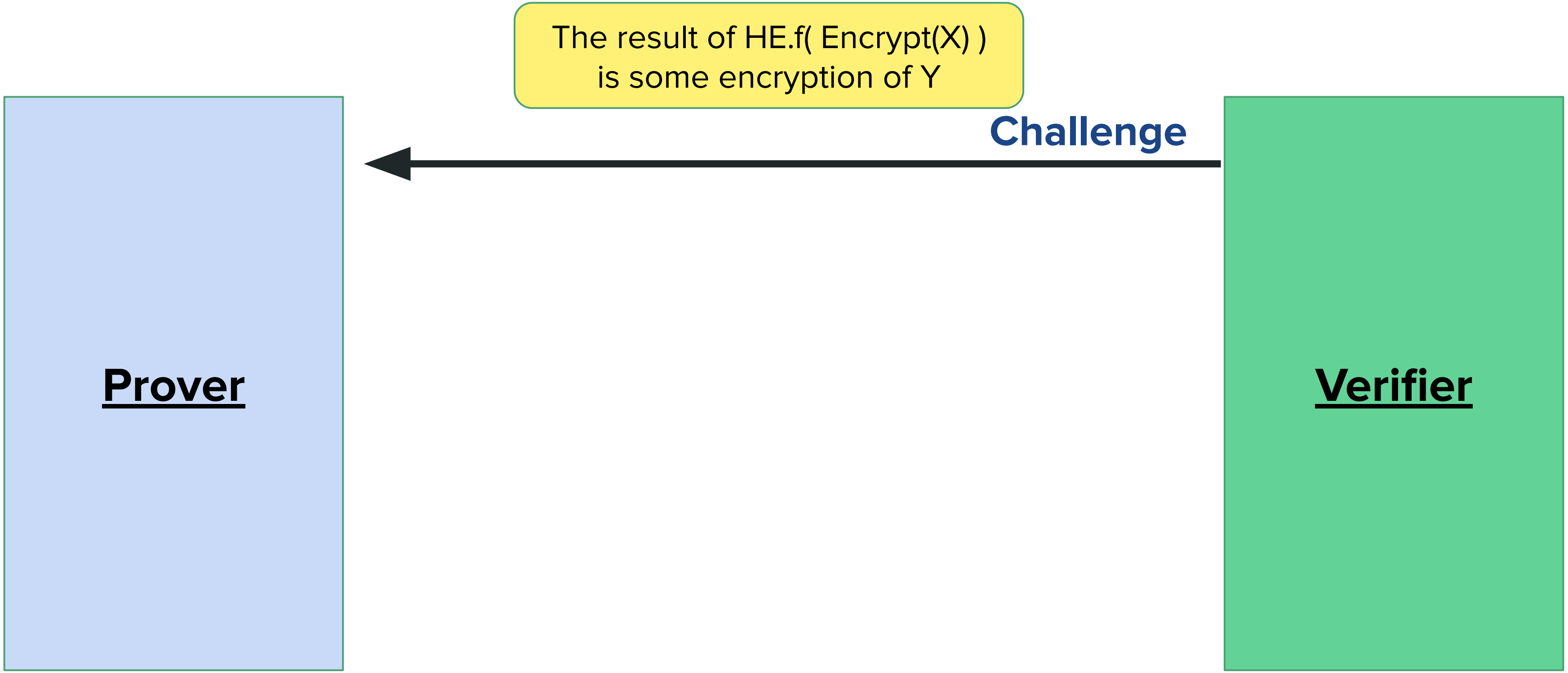

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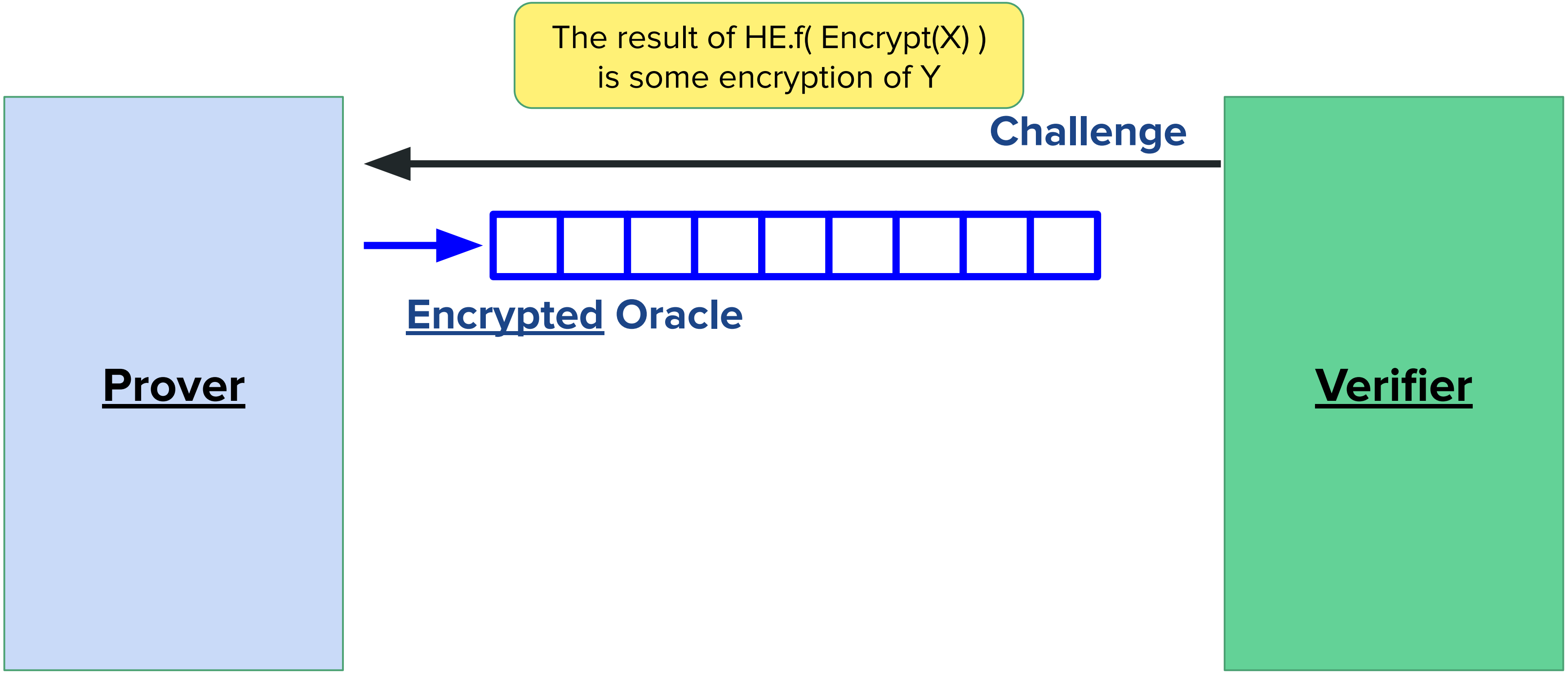

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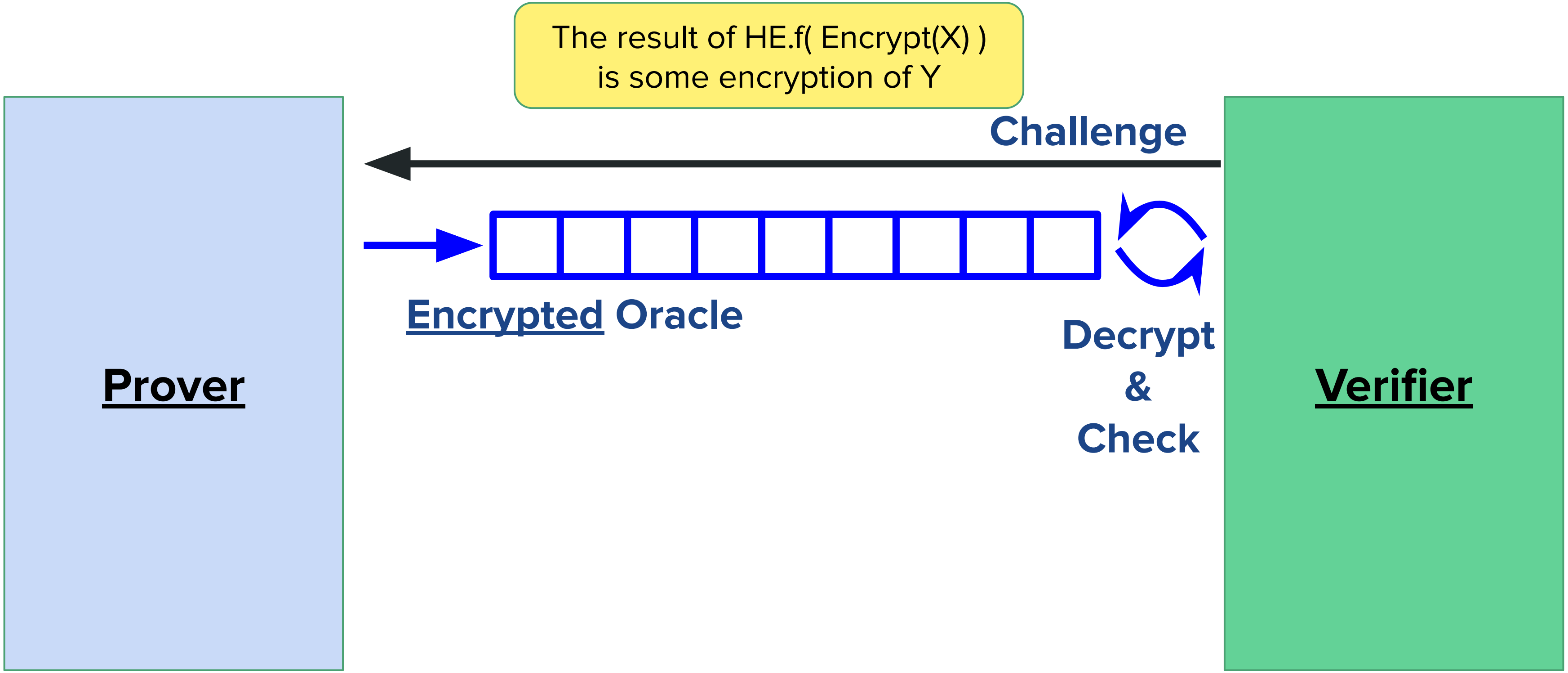

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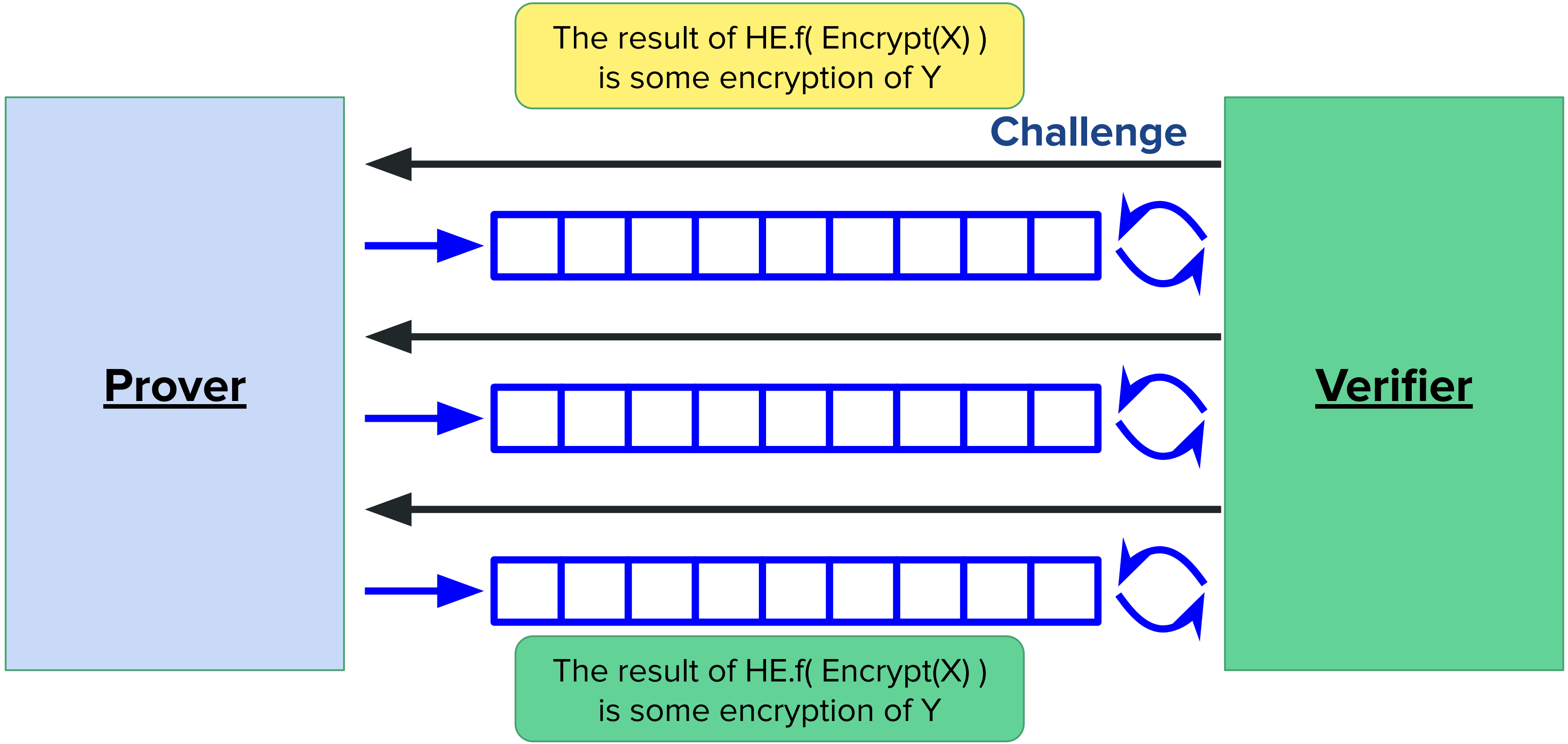

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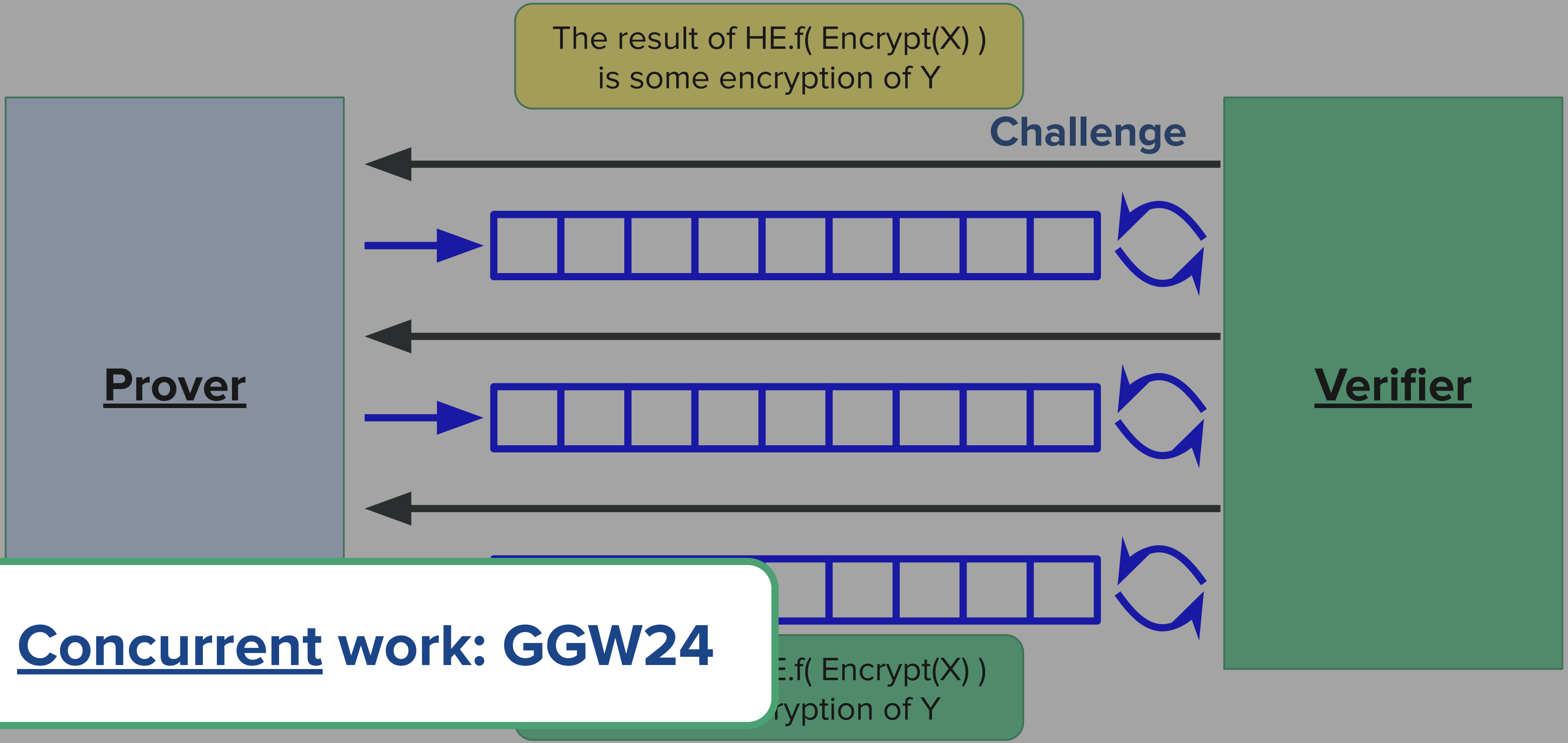

Concurrent work: GGW24

HE-IOPs

- We present a **generic reduction** from **HE-IOP** to the underlying **IOP**
- An **adversary** against the **HE-IOP** can be used against the underlying **IOP**
- Most parameters of the **IOP** are preserved
- We provide **zero-knowledge** (*requires circuit privacy)

HE-IOPs

- We present a … lying **IOP**
- An **adversary** … underlying **IOP**
- Most paramet…
- We provide **ze**…

Why is this better than "HE the proof"?

Verifiable Computation

Proof systems typically require:

- Hash functions
- Large fields

Homomorphic Encryption

Most efficient if working with:

- Rings or small fields
- Algebraic operations

* considering plaintext operations

Verifiable Computation

Proof systems typically require:

- ~~Hash functions~~ ✔
- Large fields ✖

Homomorphic Encryption

Most efficient if working with:

- Rings or small fields ✖
- <u>Algebraic operations</u> ✔

* considering <u>plaintext</u> operations

In practice

We implement **HE-batched-FRI**: an **HE-IOP version** of

(batched) **FRI** (**F**ast **R**eed-Solomon **I**OP of proximity)

We imple... rsion of

(batche... ximity)

HE-FRI is not only an instance of an HE-IOP!
FRI is often used to compile other IOPs!

Practical challenge 1: The field

FRI

- **Typically works with:**

$$|\mathbb{F}_p| \approx 2^{256}$$

HE-FRI

1. **Extension field:**

$$|\mathbb{F}_{p^d}| \approx 2^{256}$$

2. **Efficiently implement it with a tower of extensions:**

$$|\mathbb{F}_{p^{2^{2^{2\cdots}}}}| \approx 2^{256}$$

3. **Tensoring:**

- **Each $\mathbb{F}_{p^k}$ component in a different ciphertext**

Table 3: Practical parameters for FRI based on the maximum size of the input polynomial d.

Maximum input size $\log_2(d)$	D	p	$\log_2(p)$	$\log_2(\|\mathbb{F}_{p^D}\|)$
15	16	65537	16.0	256.0
20	11	23068673	24.5	269.1
25	9	469762049	28.8	259.3
30	7	75161927681	36.1	252.9
35	7	206158430209	37.6	263.1
40	6	6597069766657	42.6	255.5
45	5	1337006139375617	50.2	251.2

HE schemes:

BGV/BFV ✓✓ FHEW/TFHE ✓ ⚠️ CKKS ❌❌

Verifiable Computation

Proof systems typically require:

- ~~Hash functions~~ ✔
- Large fields ✔

Homomorphic Encryption

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Verifiable Computation

Proof systems typically require:

- ~~Hash functio~~
- Large fields

Homomorphic Encryption

Most efficient if working with:

- ...all fields ✔
- ...perations ✔
- ...aintext operations

All problems solved?

FRI

- ~~Hash functions~~ ✔
- Large fields ✔
- Deep
- Requirements for ZK

Homomorphic Encryption

Most efficient if working with:

- Rings or small fields ✔
- Algebraic operations ✔
- Small depth
- Batched computation

* considering plaintext operations

Practical challenge 2: The depth

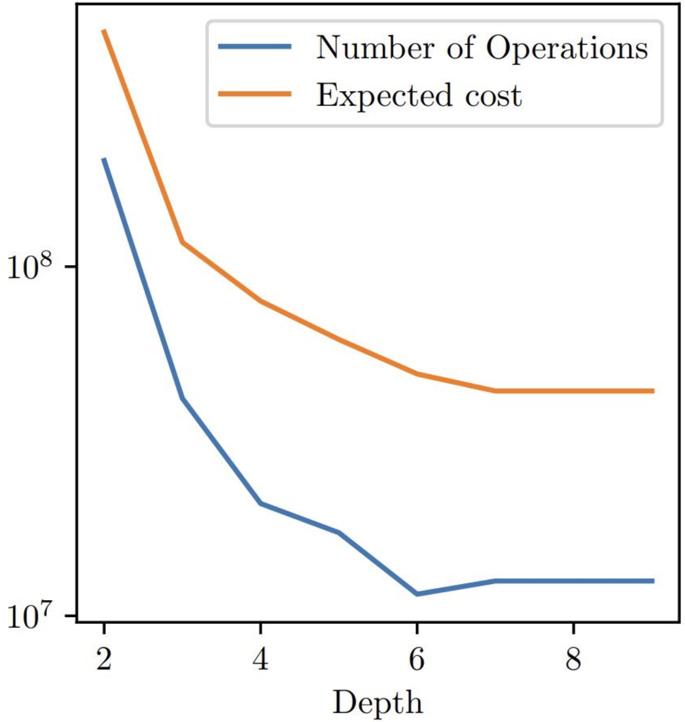
Shallow RS Encoding

- Low-depths NTTs are broadly used in HE
- **Depth:** from **$O(\log(n))$** to **2**
- **Cost:** from **$O(n \log n)$** to **$O(n\sqrt{n})$**

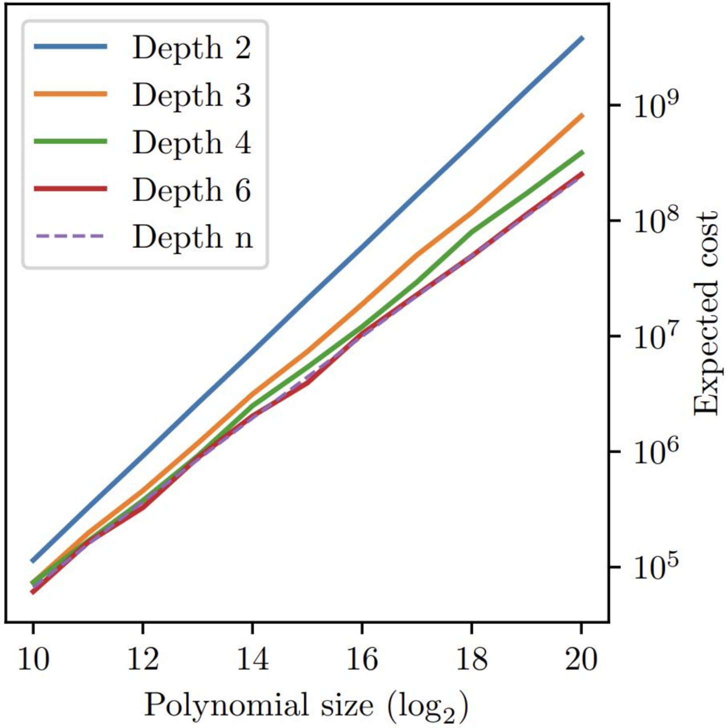
Shallow Folding

- **Does not** change overall complexity!
- **Depth:** from **$O(\log(n))$** to **1**
- **Cost:** from **$O(n)$** to **$O(n \log n)$**

Everything is **configurable**! Cost and depth are trade-offs.



(a) NTT for an input of size 2^{18}



(b) Optimal expected cost

FRI

- ~~Hash functions~~ ✔
- Large fields ✔
- Deep ✔
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Homomorphic Encryption

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- <u>Algebraic operations</u> ✔
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* considering <u>plaintext</u> operations

Practical challenge 3: ZK and HE overhead

HE Packing

Plaintext space: $\mathcal{R}_p \mapsto \mathbb{F}_p \times \mathbb{F}_p \times \cdots \times \mathbb{F}_p$

Problem - On each check:

- The verifier **wants** to learn just **2 points** (performance)
- The prover **doesn't want** the verifier to learn more than **2 points** (ZK)
- HE packing provides at least **N = 2^{12} points**

Repack and (optionally) decompose

Parameter Set	k	N	$\log_2(q)$	Size (bytes)	Decryption Cost
$\mathfrak{P}_0$	1	512	12	8192	5120
$\mathfrak{P}_1$	2	512	25	12288	5632
$\mathfrak{P}_2$	1	1024		16384	11264
$\mathfrak{P}_3$	4	512	52	20480	6656
$\mathfrak{P}_4$	2	1024		24576	12288
$\mathfrak{P}_5$	1	2048		32768	24576

Solves HE overhead: The verifier can have HE parameters **independent** of the circuit (in practice)

HE Packing

Plaintext space: $\mathcal{R}_p \mapsto \mathbb{F}_p \times \mathbb{F}_p \times \cdots \times \mathbb{F}_p$

Problem - On each check:

- The verifier **wants** to learn just **2 points** (performance)
- The prover **doesn't want** the verifier to learn more than **2 points** (ZK)
- ~~HE packing provides at least **N = 2^{12} points**~~
- (repacked) HE packing provides **2 points**

FRI

- ~~Hash functions~~ ✔
- Large fields ✔
- Deep ✔
- Requirements for ZK ✔

Homomorphic Encryption

Most efficient if working with:

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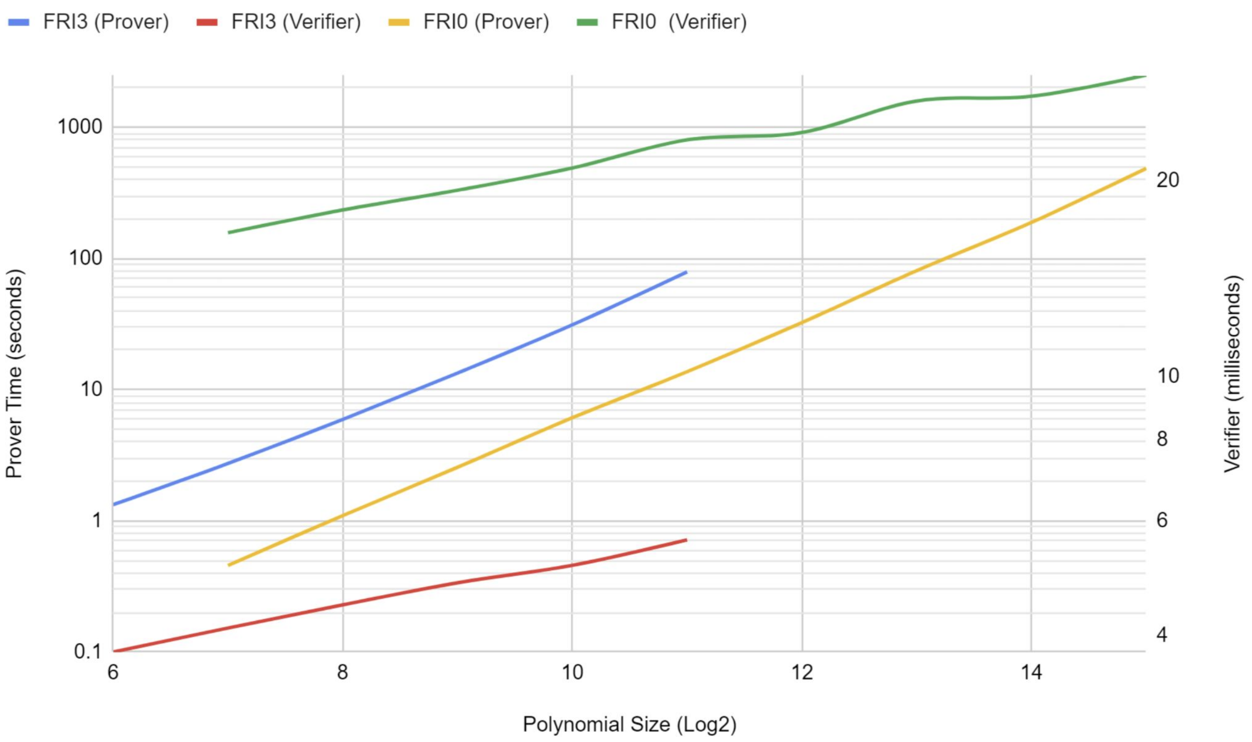
* considering plaintext operations

Results

Results for 4096 batched polynomials



Prover: up to 32 threads - Verifier: single-threaded

For up to 2^{11}:

FRI0 (optimized for prover):

- P time: 0.2 – 5.45s
- V time: 7.08 – 12.29 ms
- Memory: 0.5 – 3.7 GB

FRI3 (optimized for verifier):

- P time: 2.74 – 78.98 s
- V time: 4.10 – 5.61 ms
- Memory: 2.0 – 23.7 GB

Implementation

- Batched for 4096 or 8192 polynomials
- **Non-interactive** (Fiat-Shamir using BLAKE3)
- **Python** with optimizations in **C/CPP**
- **Publicly available**: https://github.com/antoniocgj/HELIOPOLIS
- Artifact accepted: **IACR Results Reproduced**

https://eprint.iacr.org/2023/1949

https://github.com/antoniocgj/HELIOPOLIS

Thank you!

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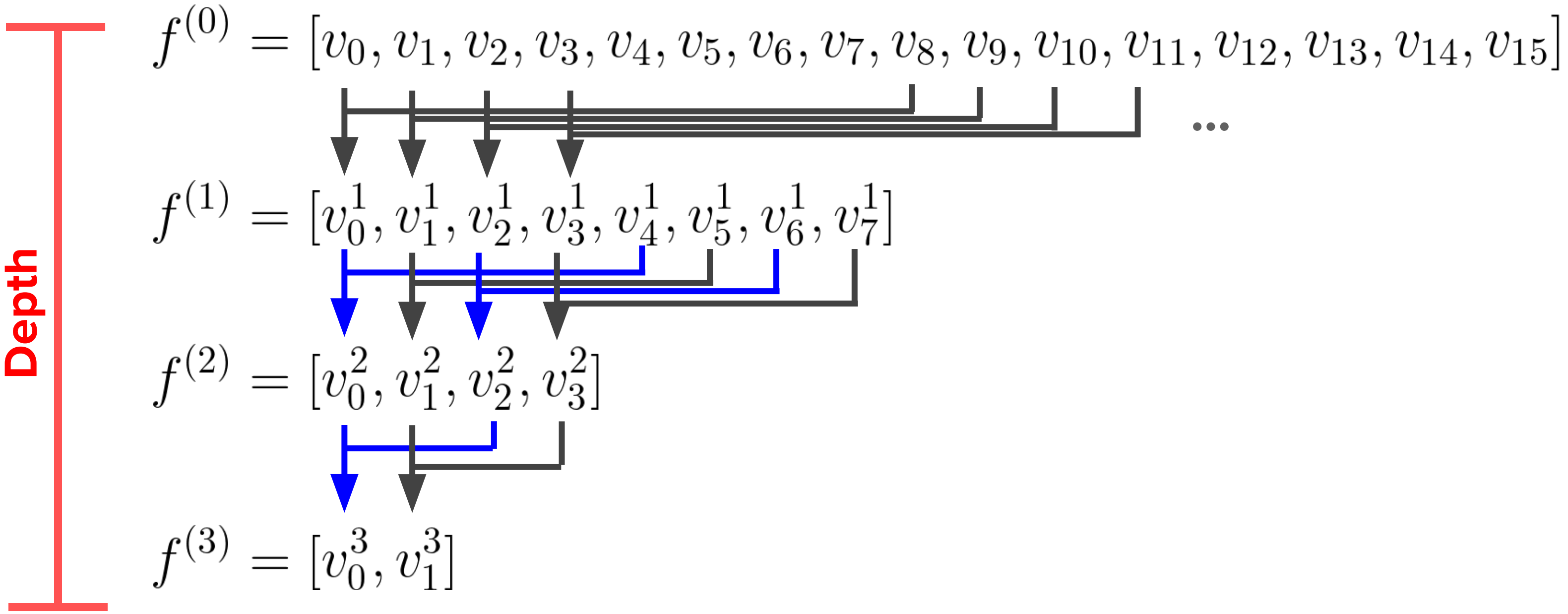
FRI Folding



$$f^{(0)} = [v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}]$$

$$f^{(1)} = [v_0^1, v_1^1, v_2^1, v_3^1, v_4^1, v_5^1, v_6^1, v_7^1]$$

$$f^{(2)} = [v_0^2, v_1^2, v_2^2, v_3^2]$$

$$f^{(3)} = [v_0^3, v_1^3]$$

Reed-Solomon encoding

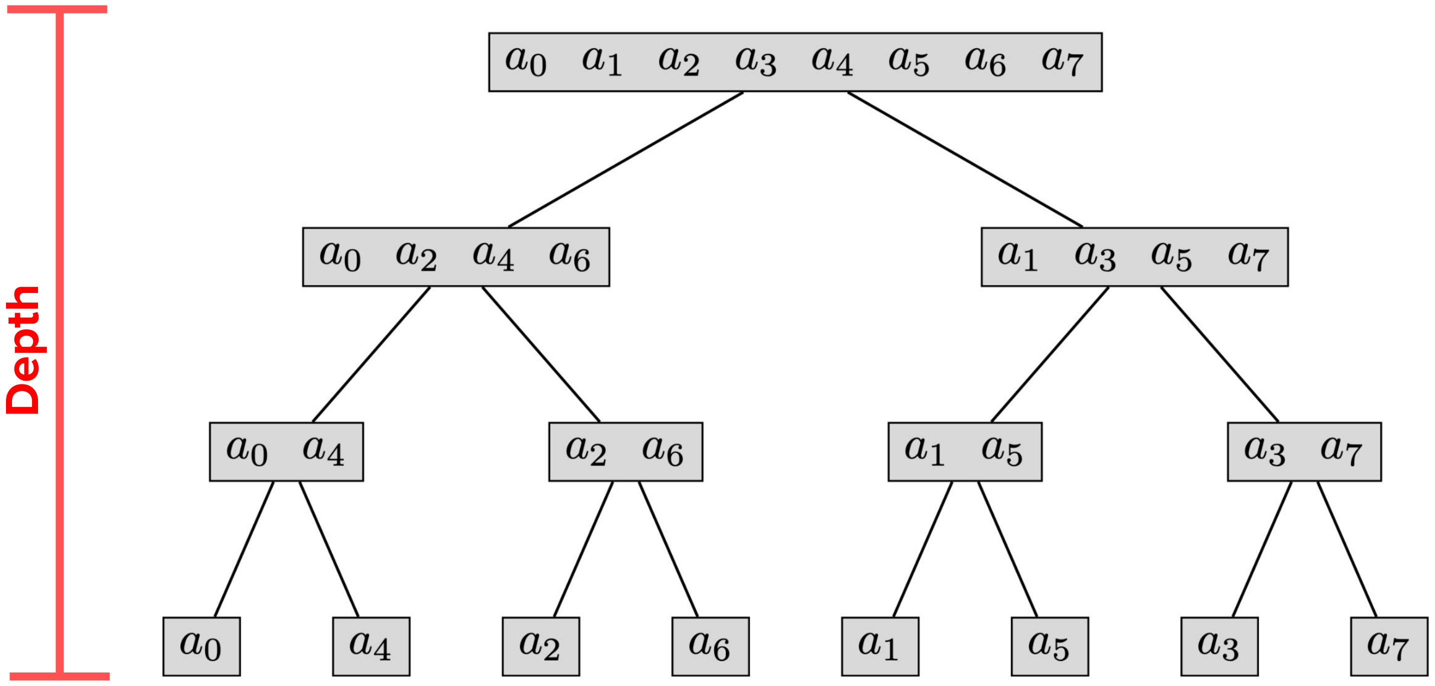
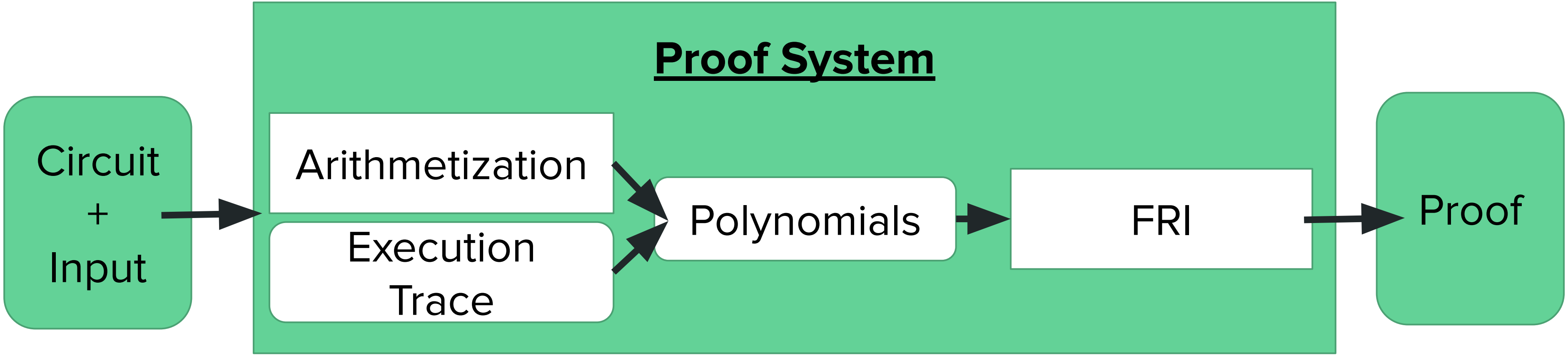


Image from: https://tikz.net/fft-algorithm-analysis/, by Tikz - Alexandros Tsagkaropoulos,

FRI


Proof System
Circuit
+
Input
Arithmetization
Execution
Trace
Polynomials
FRI
Proof