Count Corruptions, Not Users:

Improved Tightness for Signatures, Encryption and Authenticated Key Exchange

Mihir Bellare, Doreen Riepel, Stefano Tessaro, Yizhao Zhang

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Motivation

Modern applications ask for security in the presence of powerful adversaries who may **adaptively corrupt** parties.

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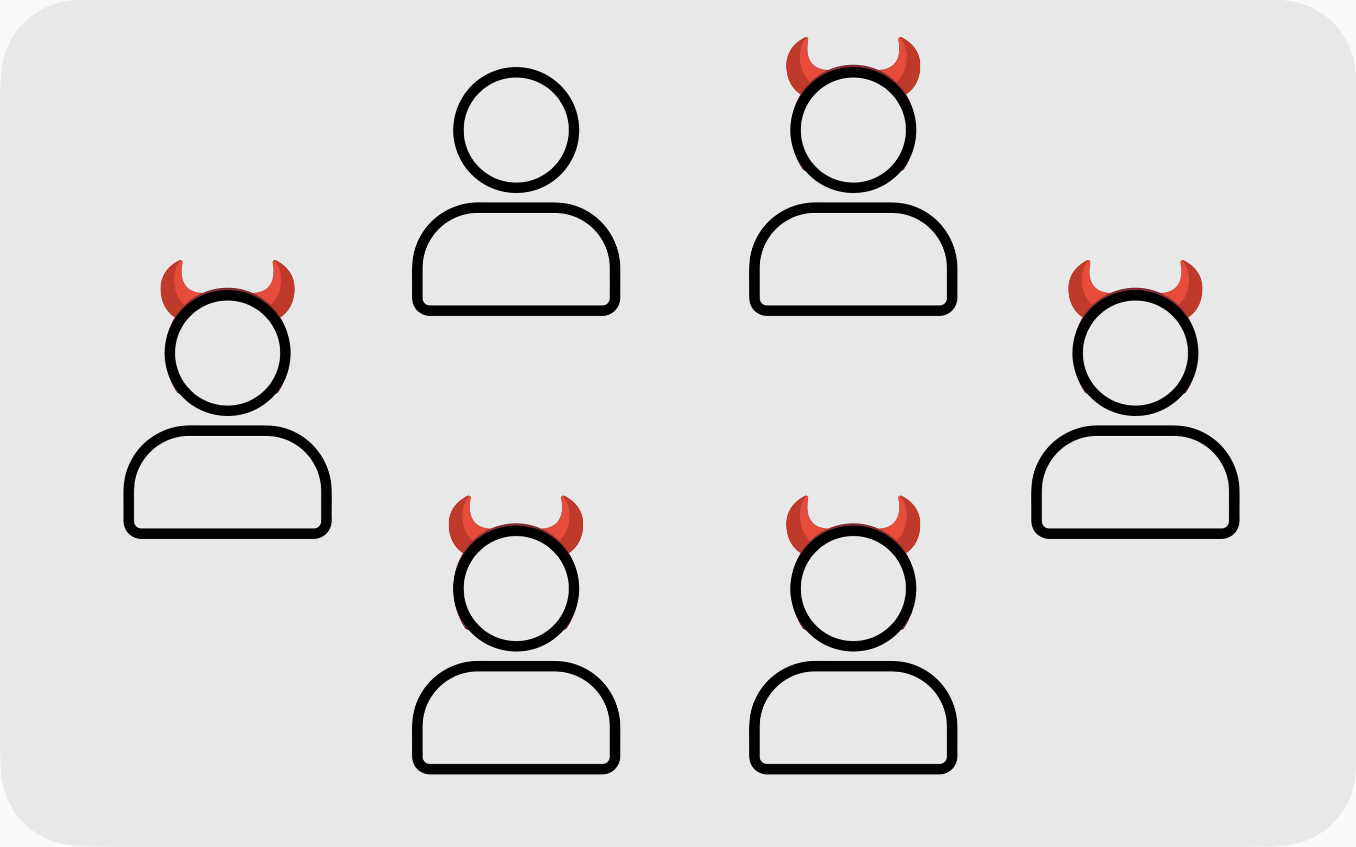

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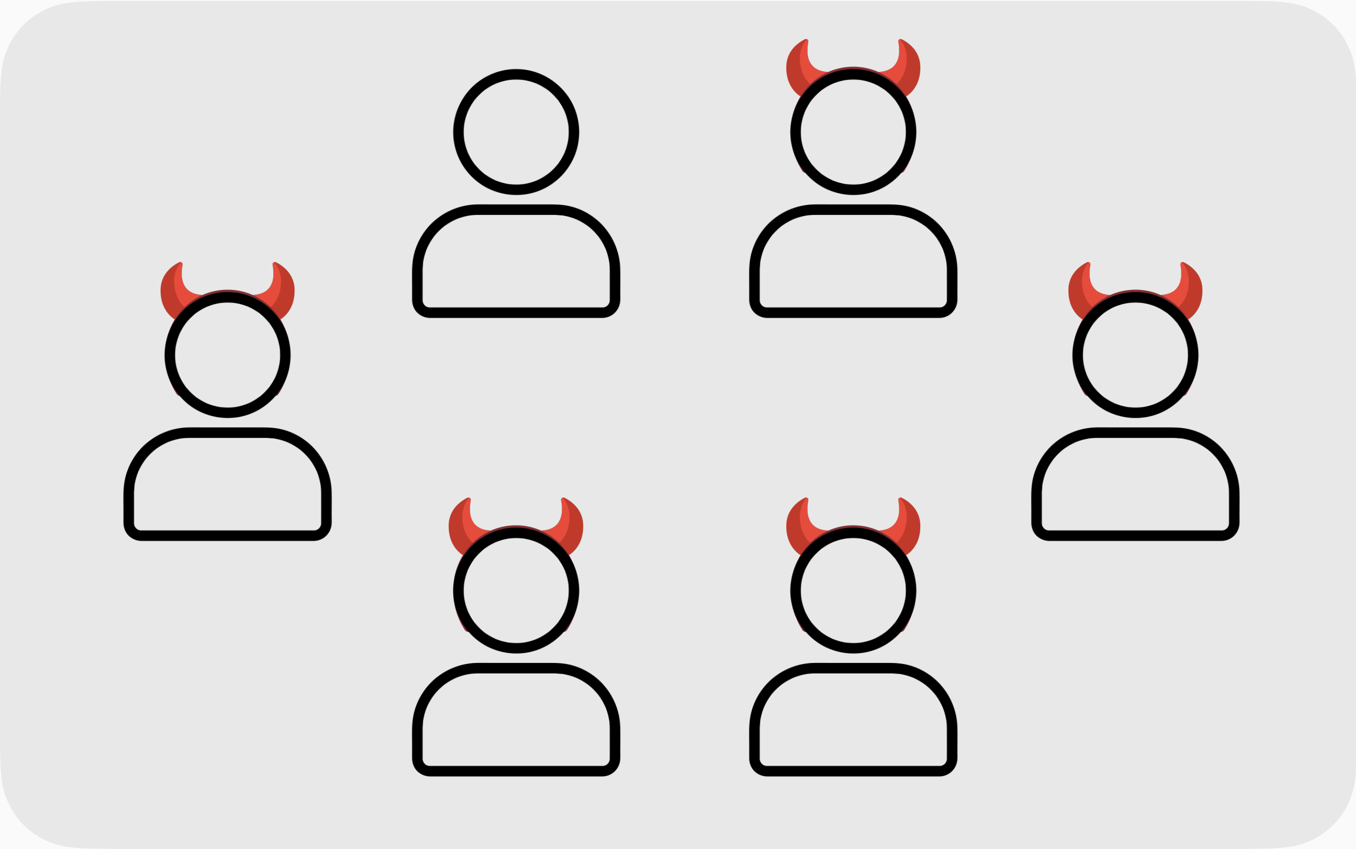

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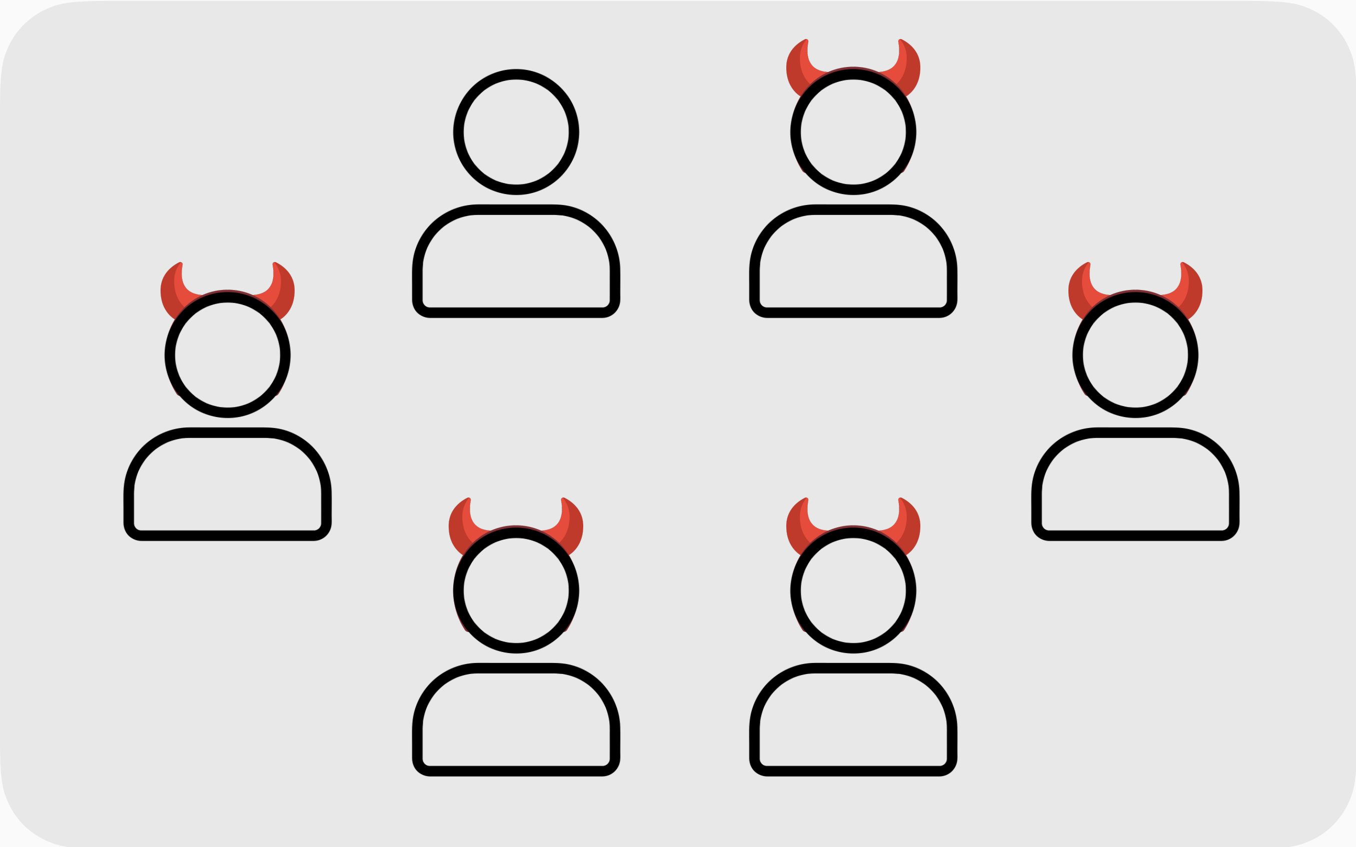

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Microsoft Storm-0885 attack (2023)[1]

- Attackers acquired a Microsoft account (MSA) consumer signing key used to authenticate tokens
- Affected were email accounts of 22 organizations and 500 individuals globally (e.g. top-tier US government officials)

[1] https://www.microsoft.com/en-us/security/blog/2023/07/14/analysis-of-storm-0558-techniques-for-unauthorized-email-access/

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Our model: cp-muc security ("corruption-parametrized")

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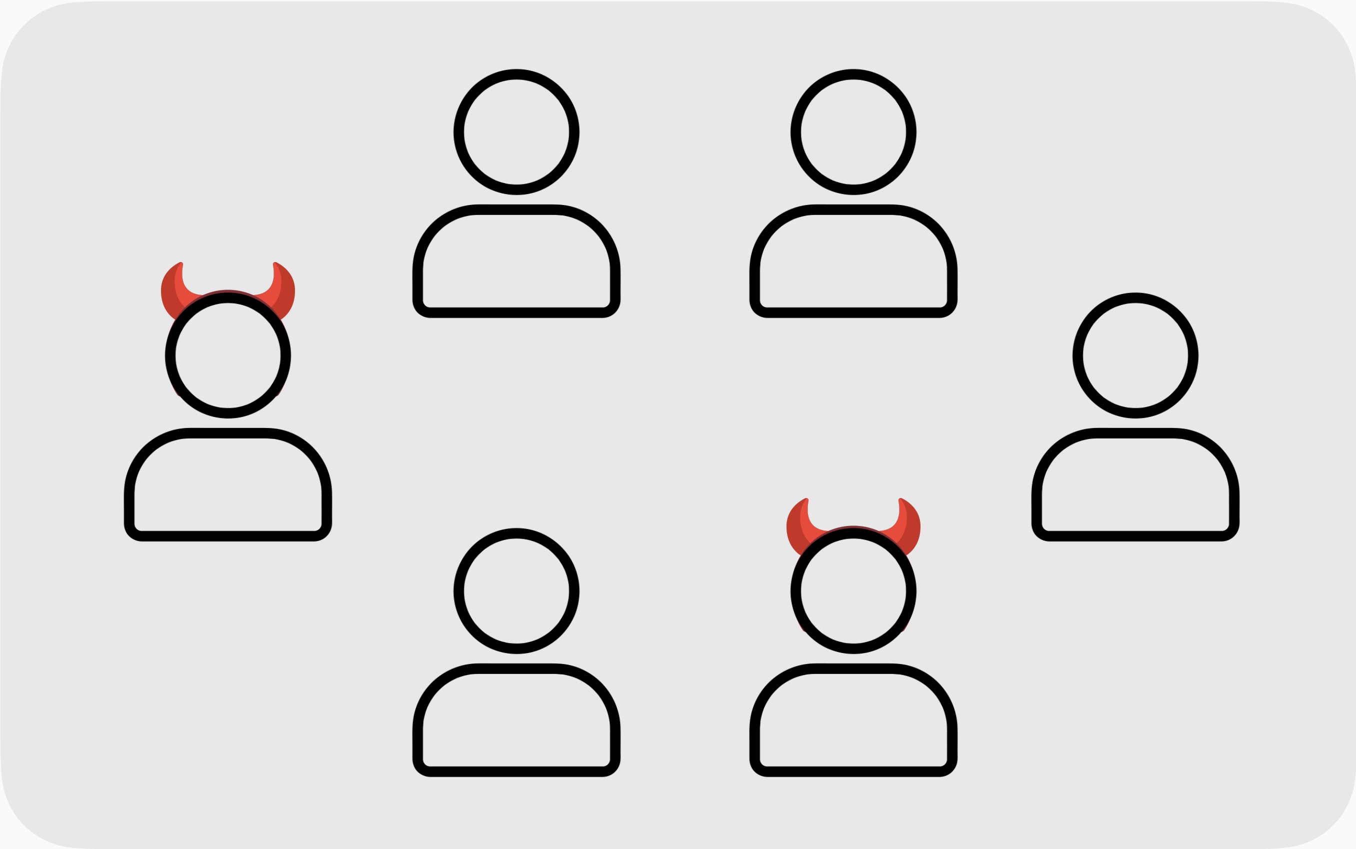
- c corruptions for $c \ll n$



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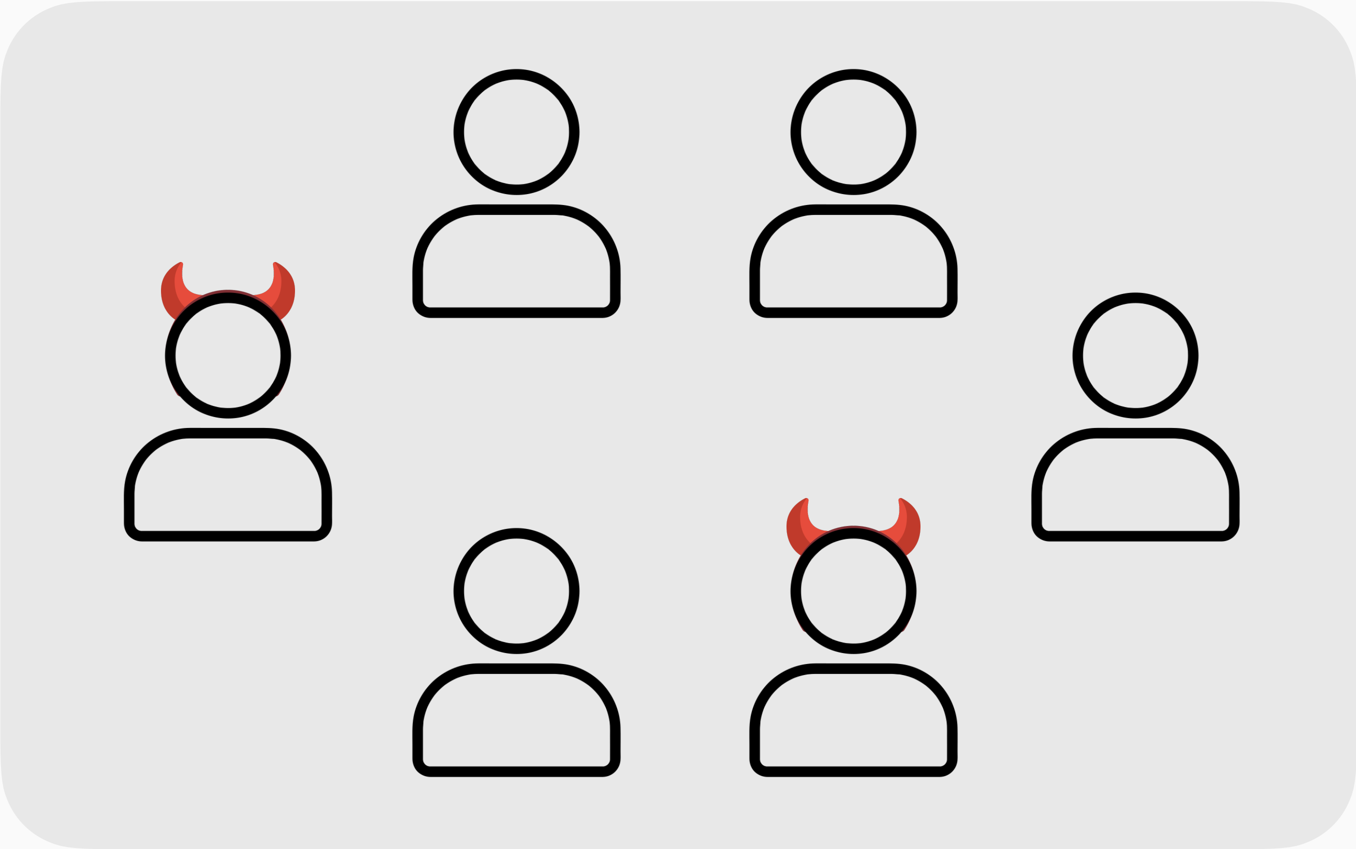

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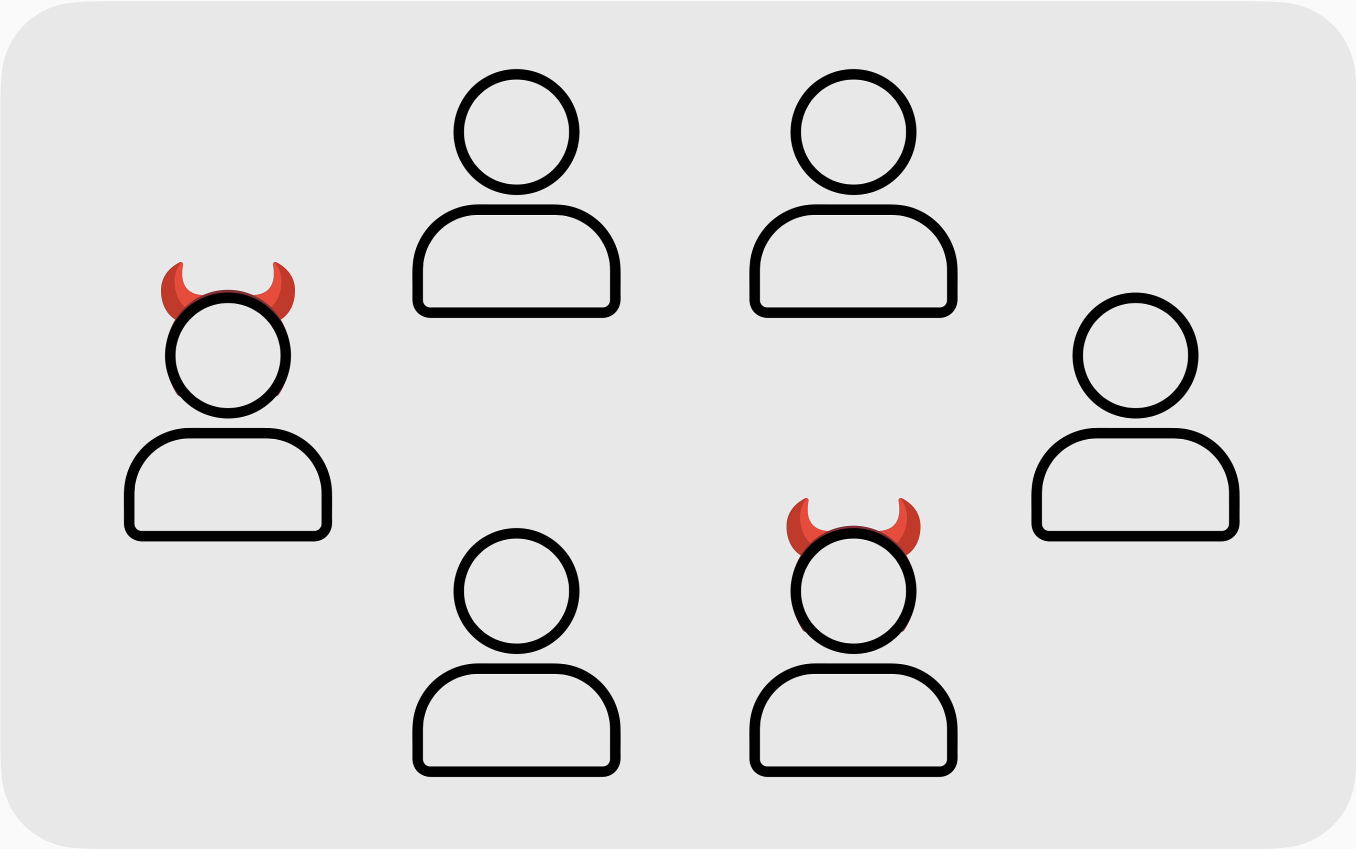

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Goal

- Better concrete security guarantees for protocols deployed in practice, where otherwise tight(er) bounds are unknown or impossible

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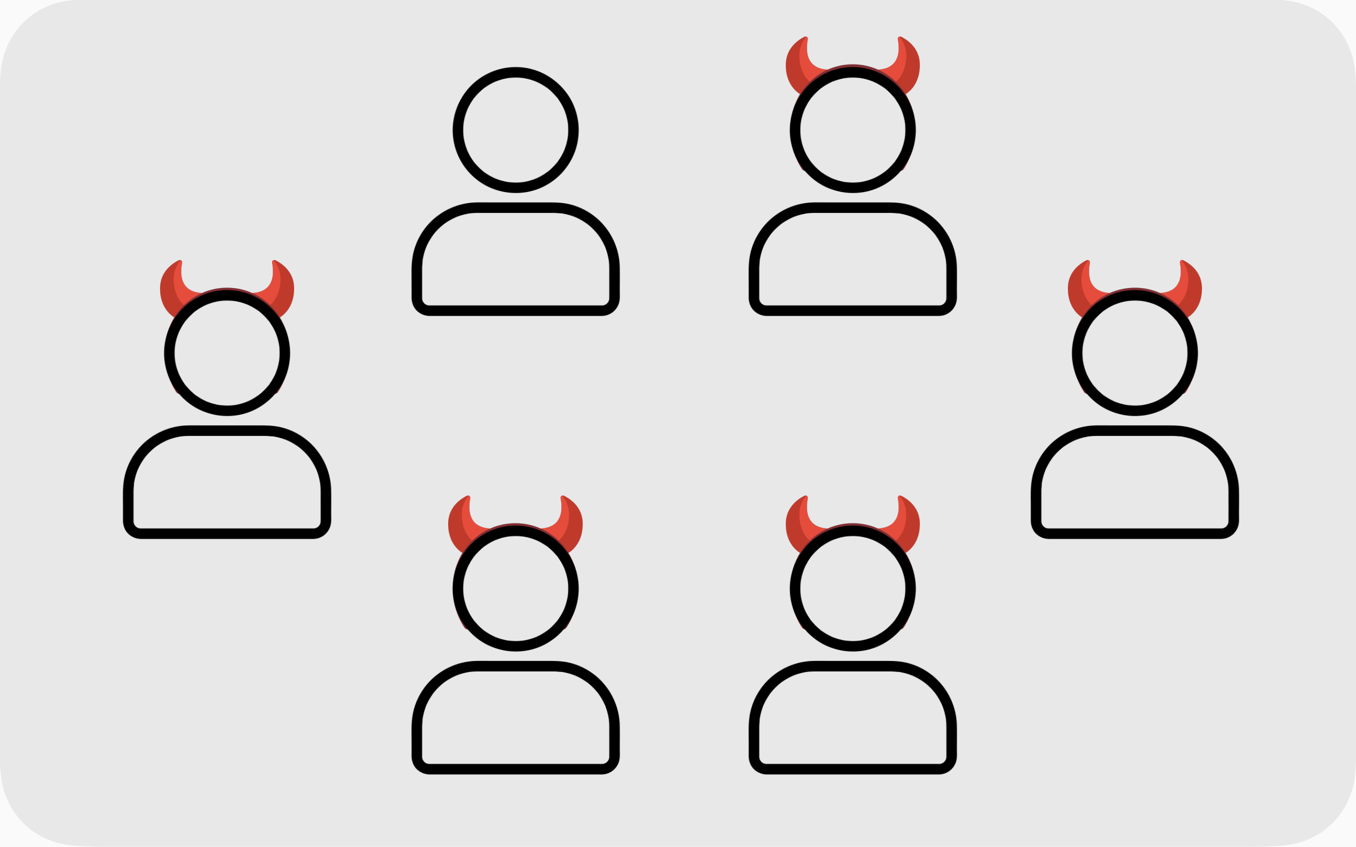
muc security



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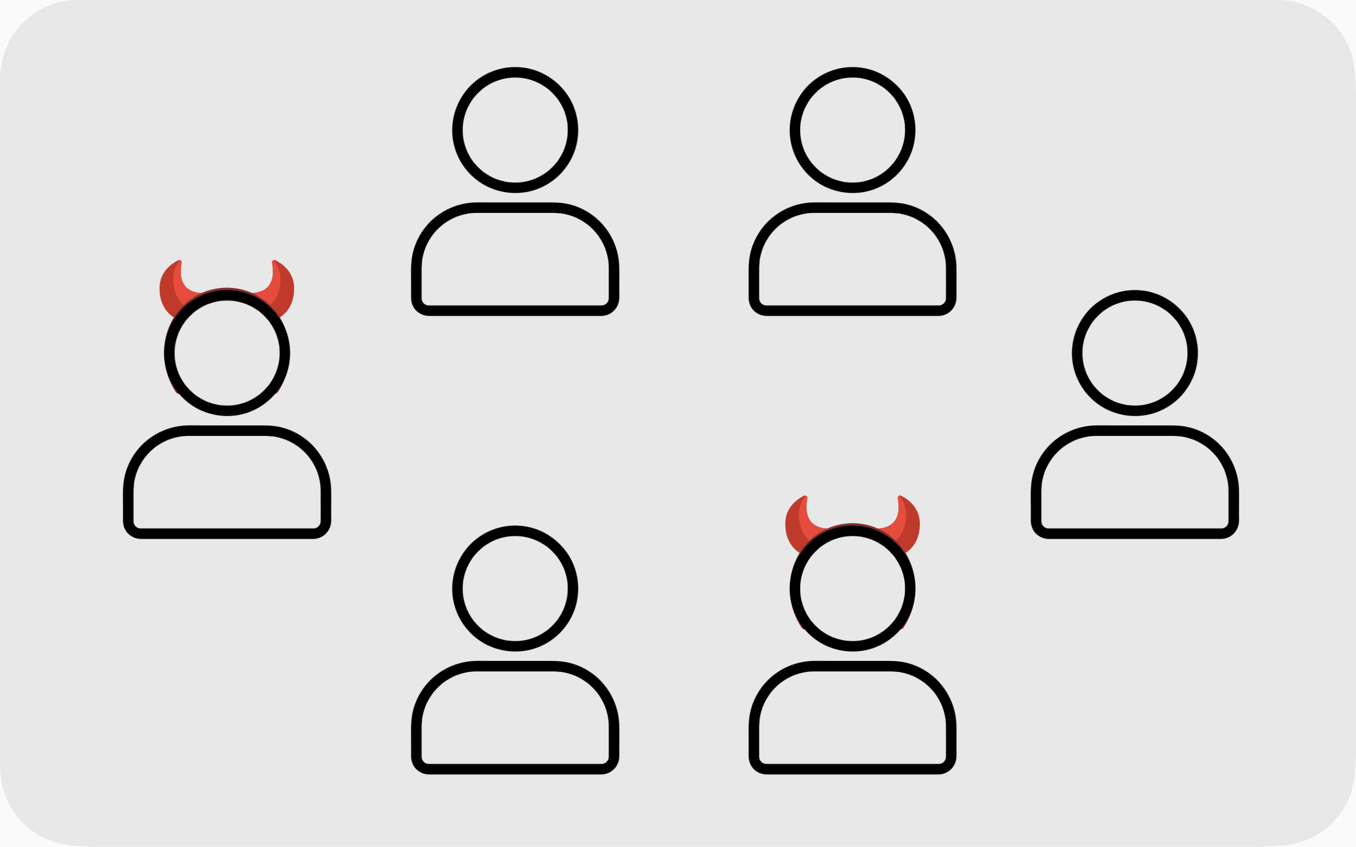

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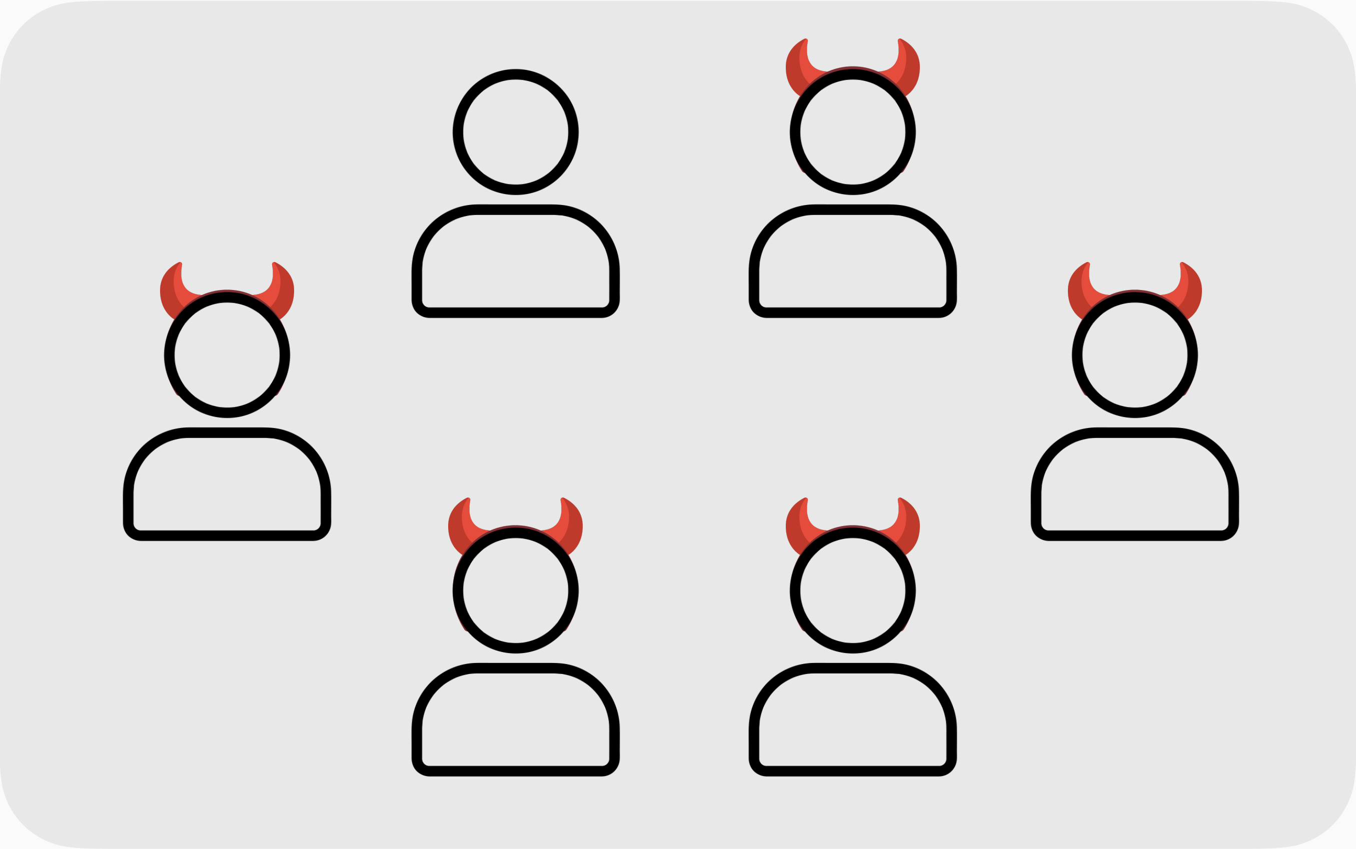

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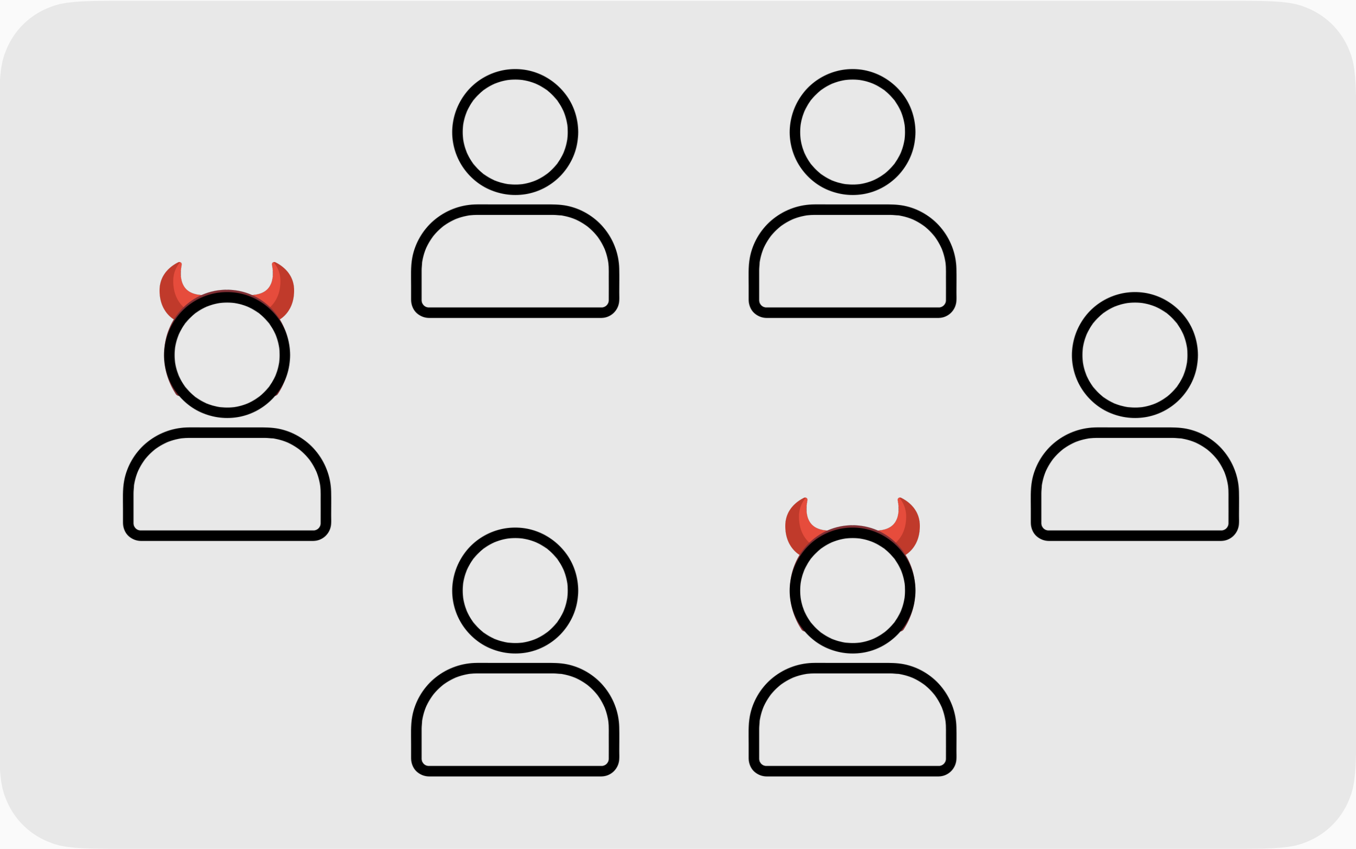

Standard hybrid argument:

- Reduces to single-user (su) security
- Security loss linear in the number of users

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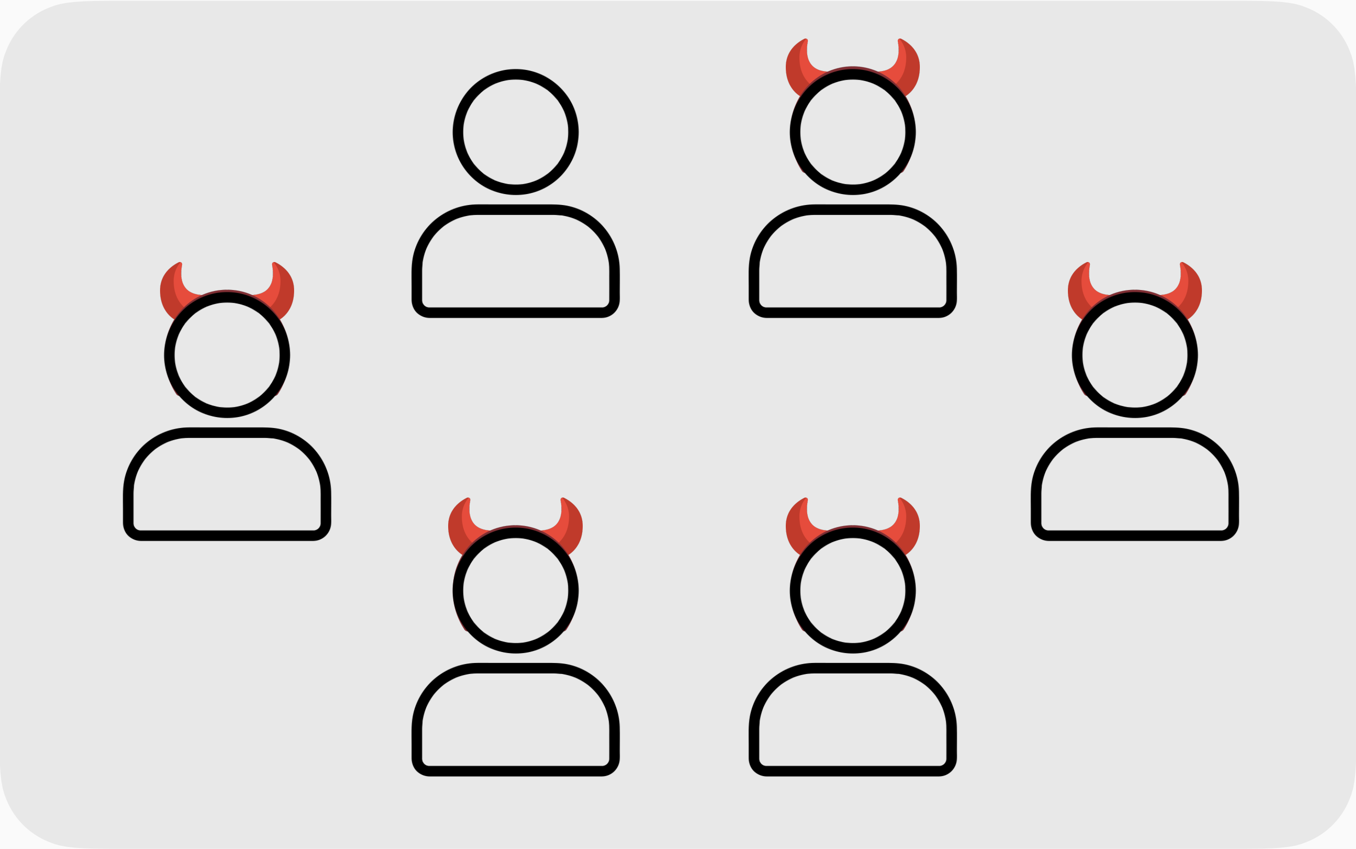

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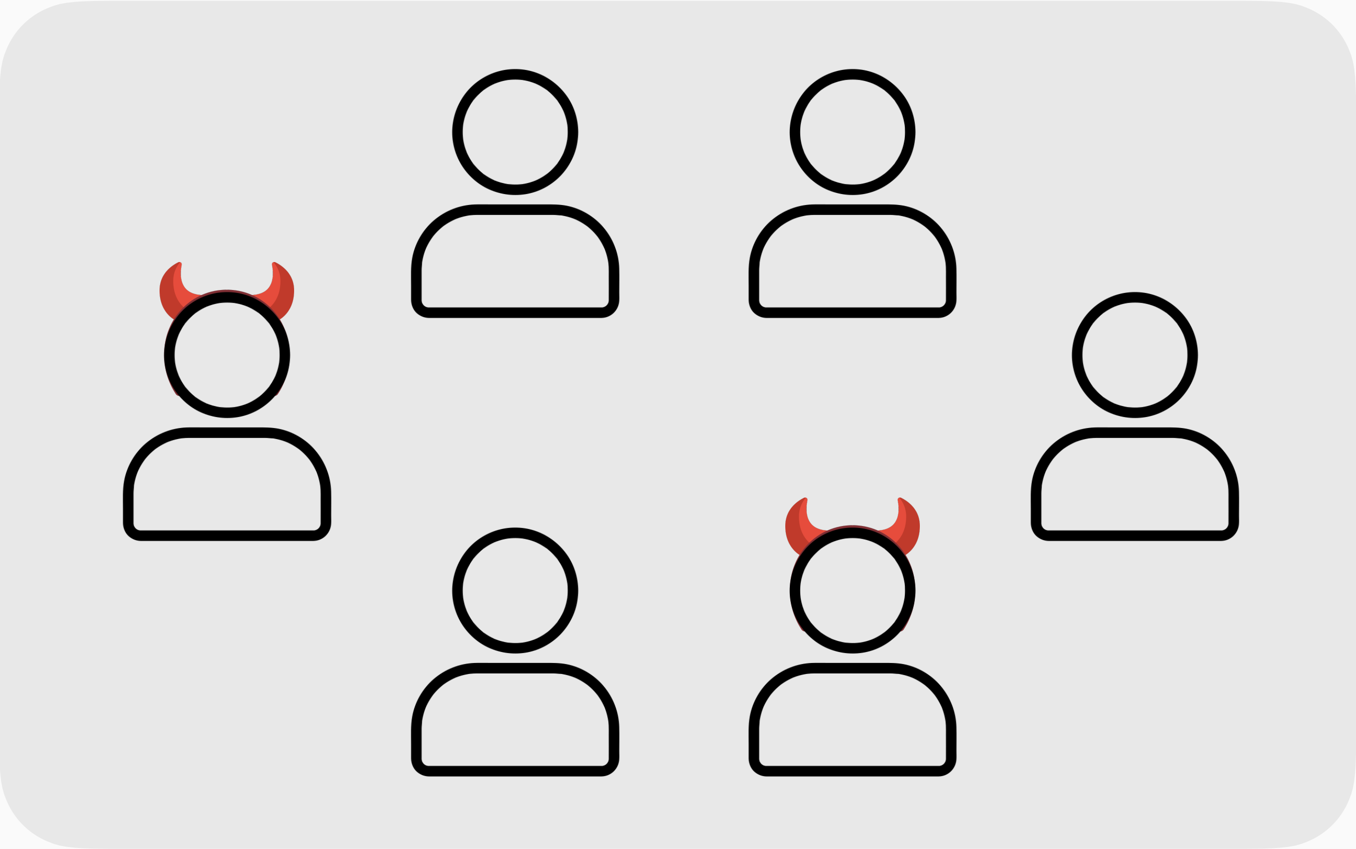

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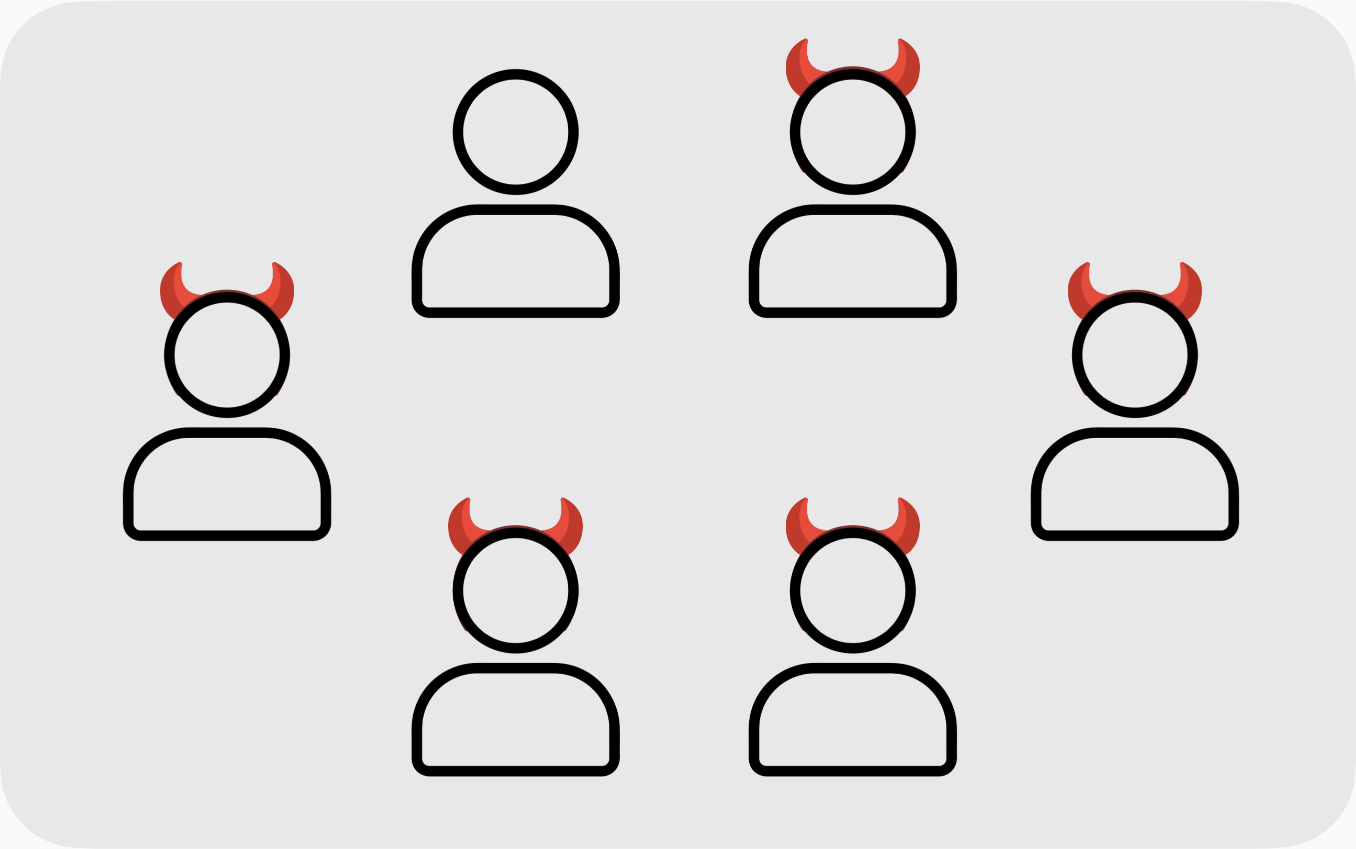

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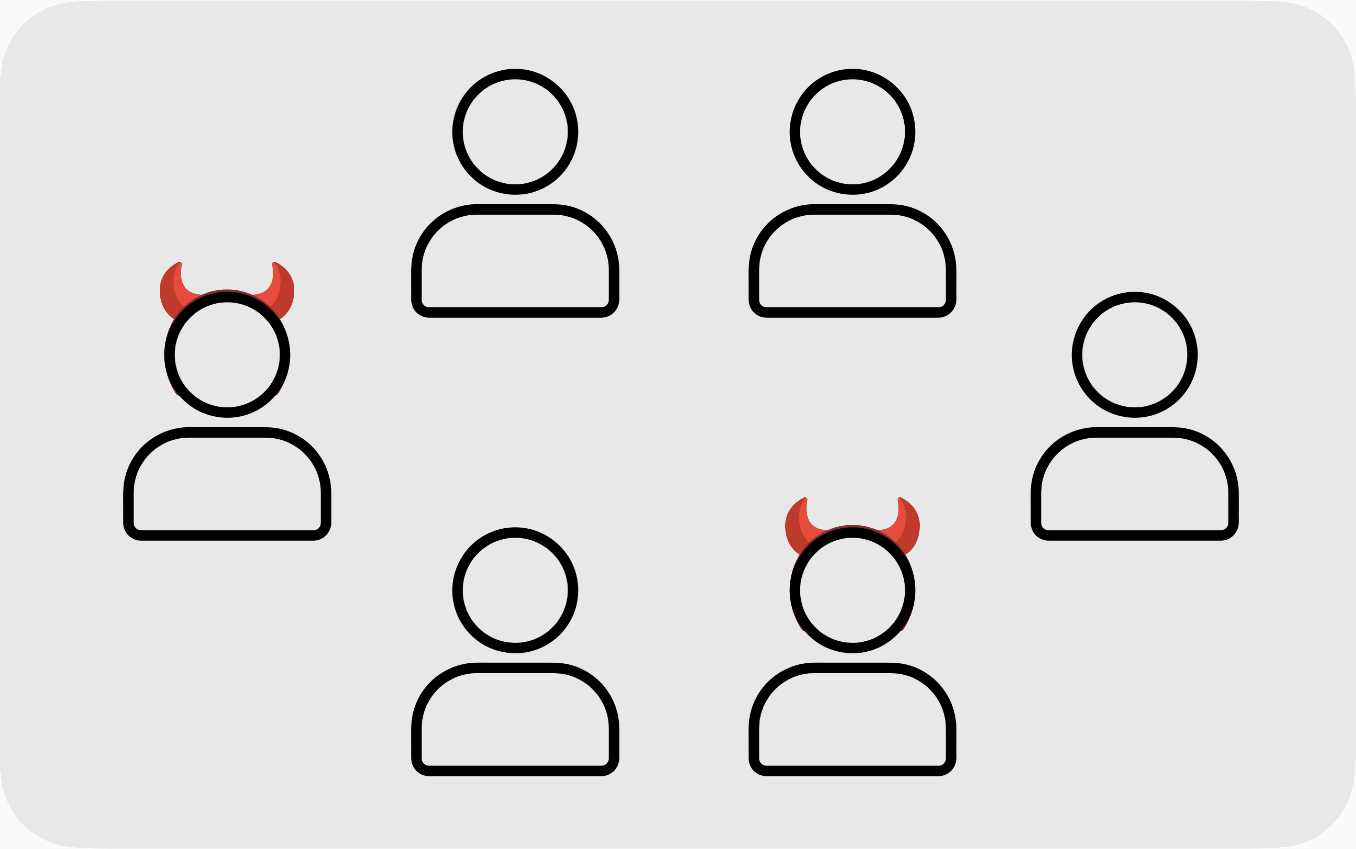

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Main question:

Can we give a general theorem? Under which conditions?

Overview of our Results

Formal security specifications

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Main focus of this talk (using the example of UF-CMA secure signatures)

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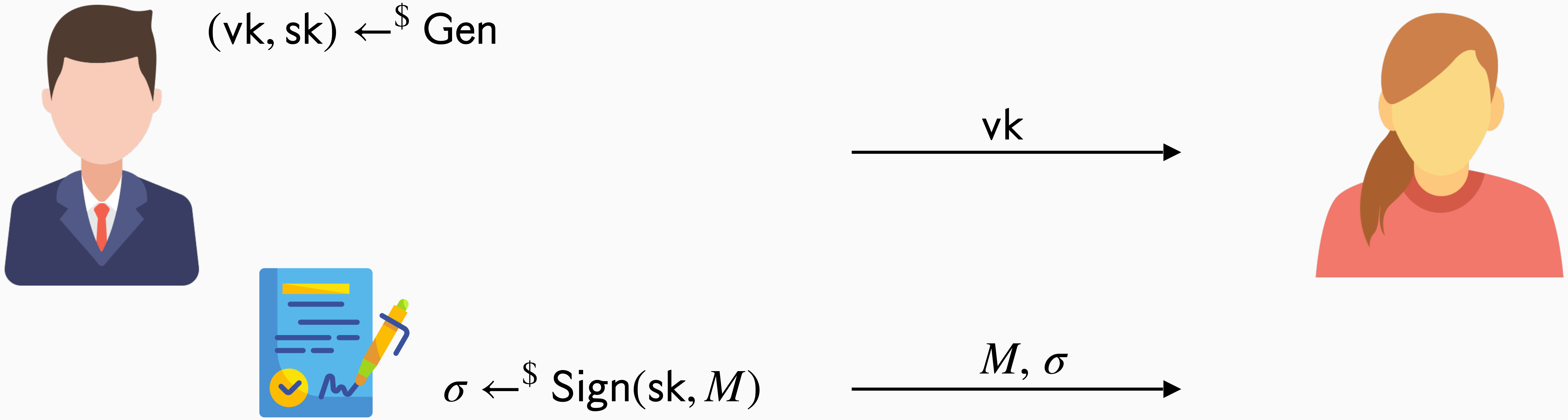

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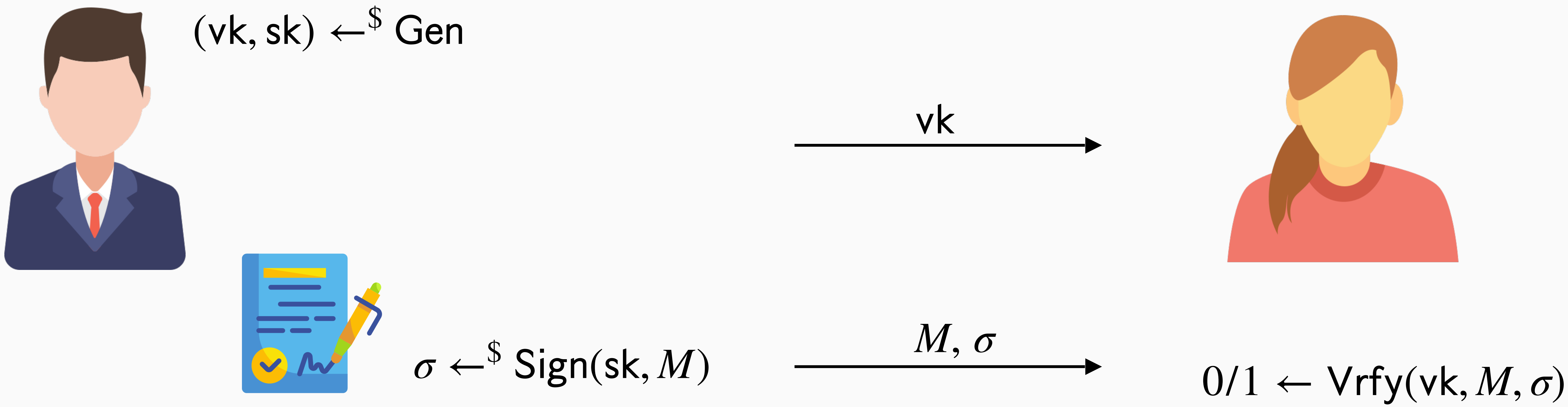

Unforgeability (Single-User)

Game $\mathbf{G}^{\mathsf{uf\text{-}su}}_{\mathsf{Sig}}$

Adversary $\mathcal{A}$

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vk →

← $\textsc{Sign}$: M

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q queries

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mu-tight schemes seem to offer no advantage in the muc setting

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Always need to expect $n - 1$ corruptions

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More fine-grained view
(cp-muc, "corruption-parametrized")

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Theorem (from su/mu to cp-muc):

Let n, c be integers s.t. $0 \leq c < n$. For any adversary $\mathcal{A}$ against uf-muc-(n, c) security of Sig, there exists an adversary $\mathcal{B}$ against uf-mu-m security of Sig s.t.

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main benefit for Type-II schemes

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$n = 100$ Million
$c = 100$ Thousand
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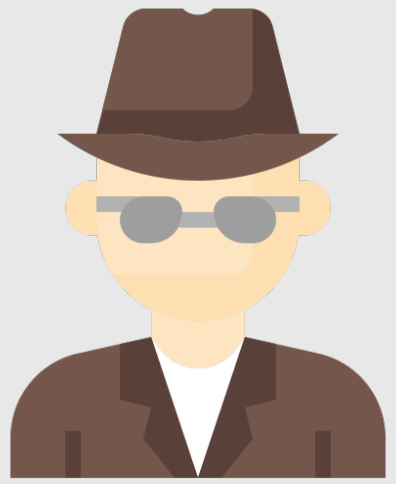
main benefit for Type-II schemes

Inspiration: Optimal bounds for FDH signatures [C:Coron01]

- Instead of losing a factor linear in the number of hash queries, reduction loses number of signing queries

cp-muc Theorem

Refining and generalizing [C:Coron01]



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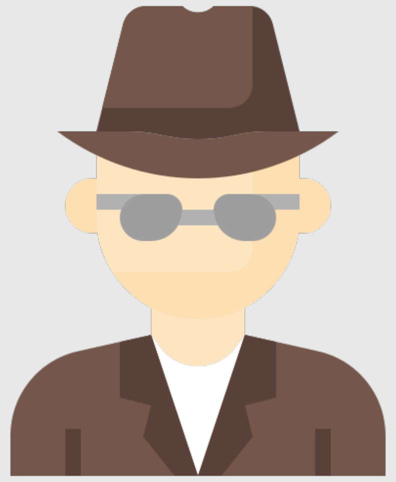
$\mathsf{vk}'_1, \ldots, \mathsf{vk}'_n$ →



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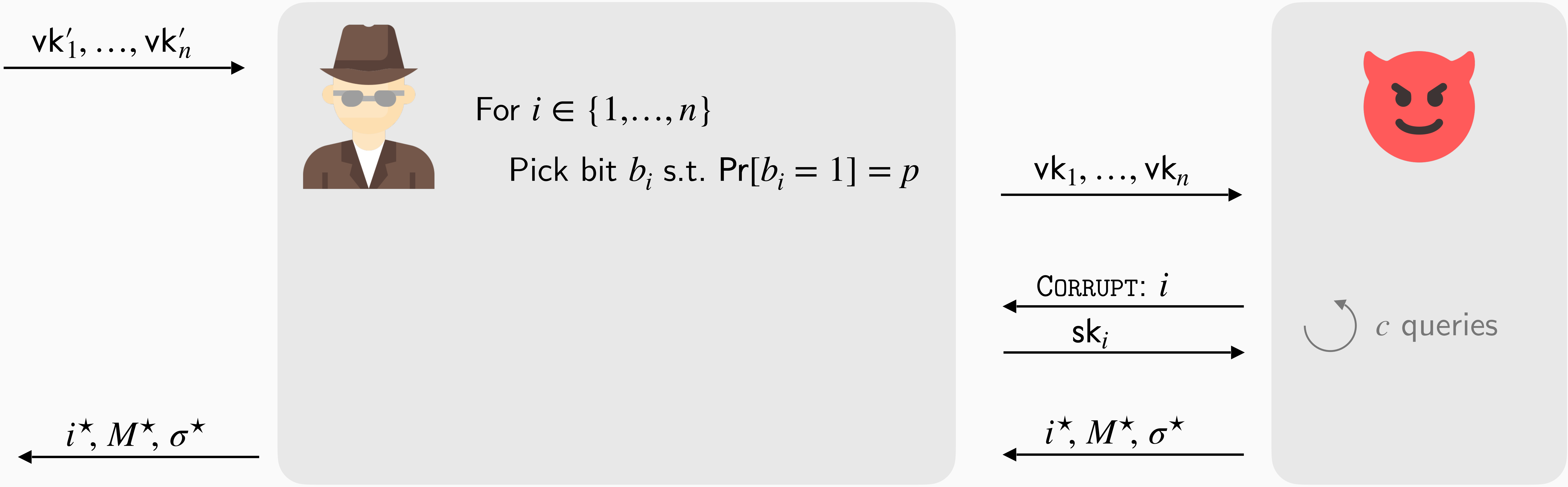

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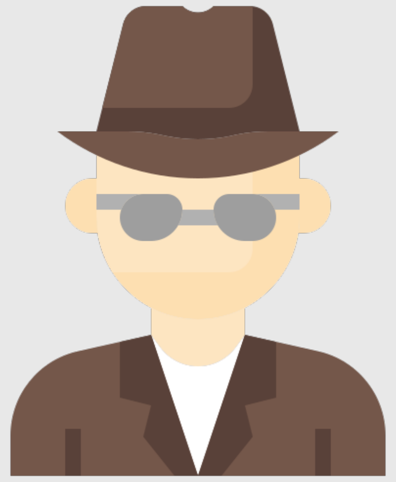

For $i \in \{1, \ldots, n\}$

Pick bit b_i s.t. $\Pr[b_i = 1] = p$

If $b_i = 0$: $(\mathsf{vk}_i, \mathsf{sk}_i) \xleftarrow{\$} \mathsf{Gen}$

$\mathsf{vk}_1, \ldots, \mathsf{vk}_n$ →

← CORRUPT: i

sk_i →

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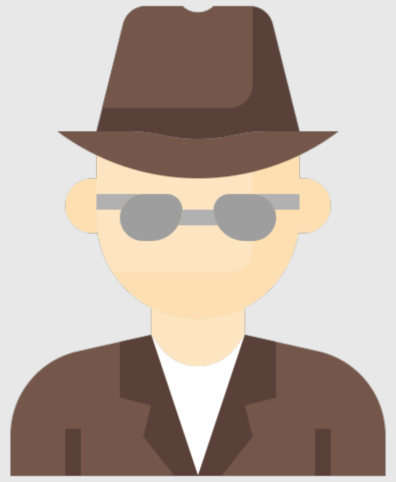

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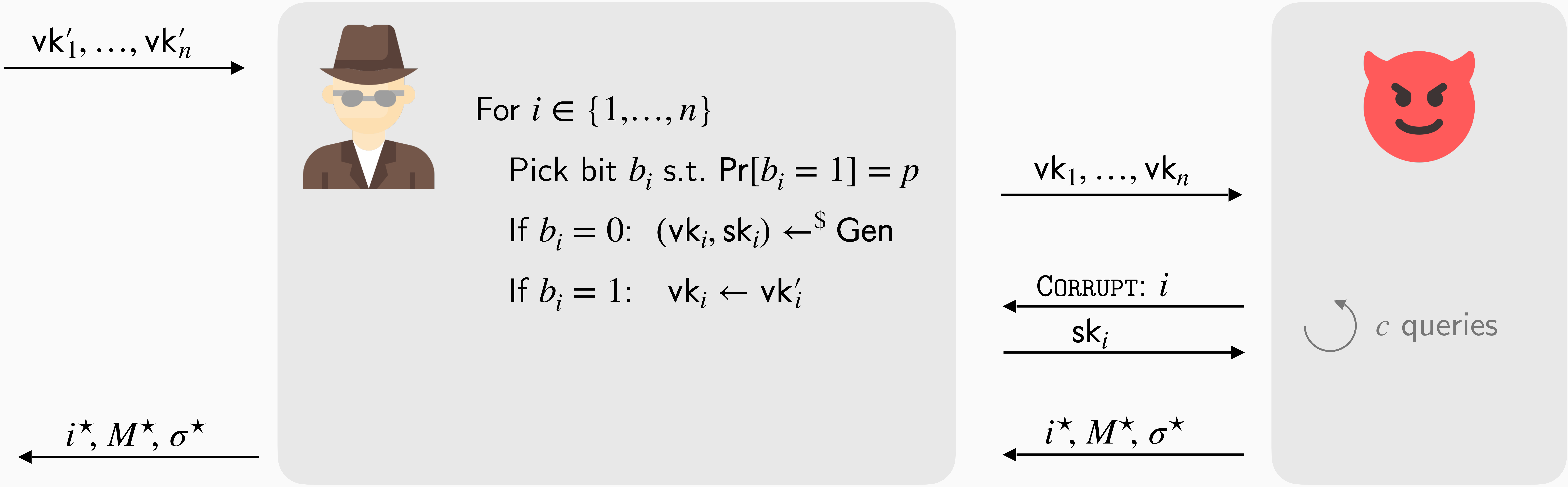

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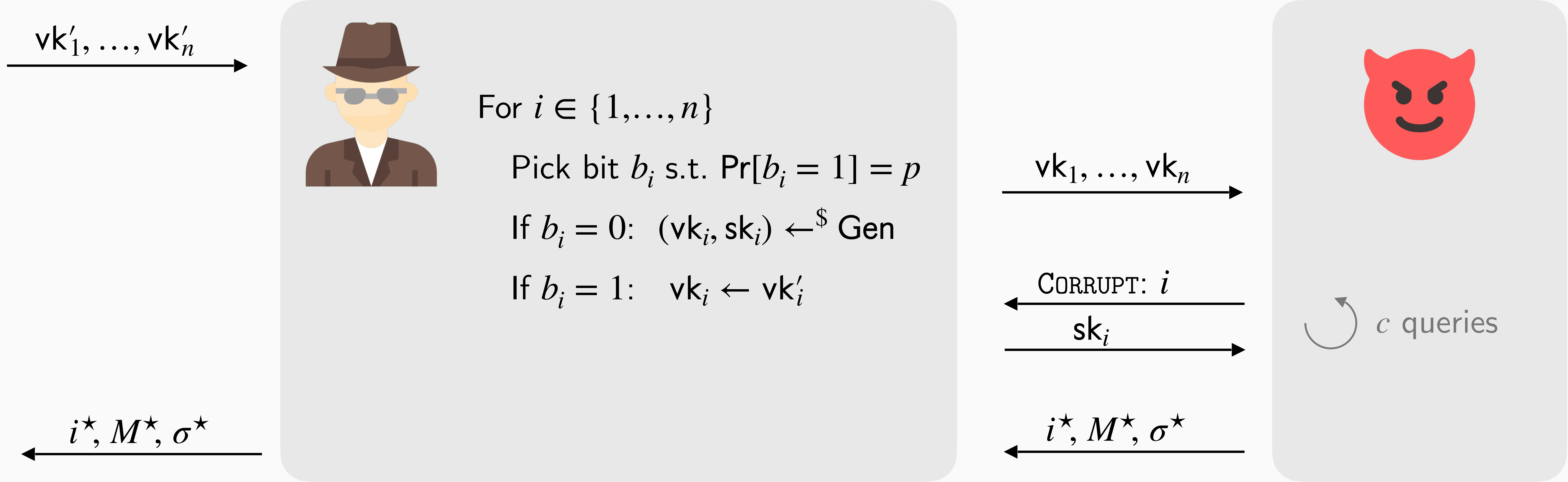

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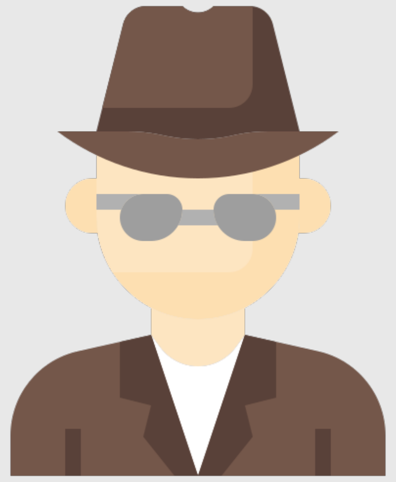
We don't need n public keys!



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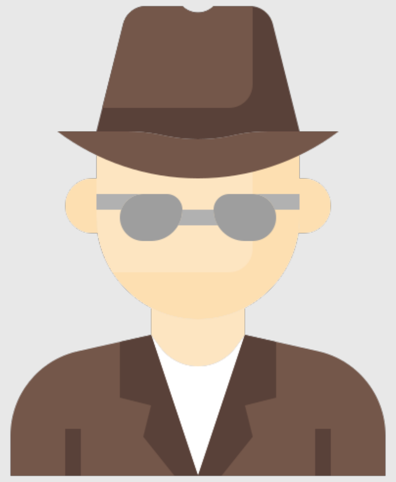

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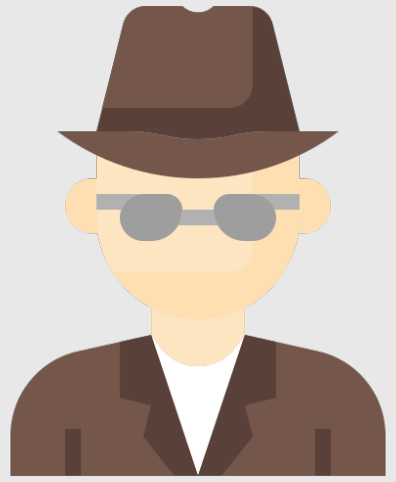

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This may fail for small m!

$\mathsf{vk}_1, \ldots, \mathsf{vk}_n \rightarrow$

$\leftarrow$ CORRUPT: i

$\mathsf{sk}_i \rightarrow$

c queries

$\leftarrow i^\star, M^\star, \sigma^\star$

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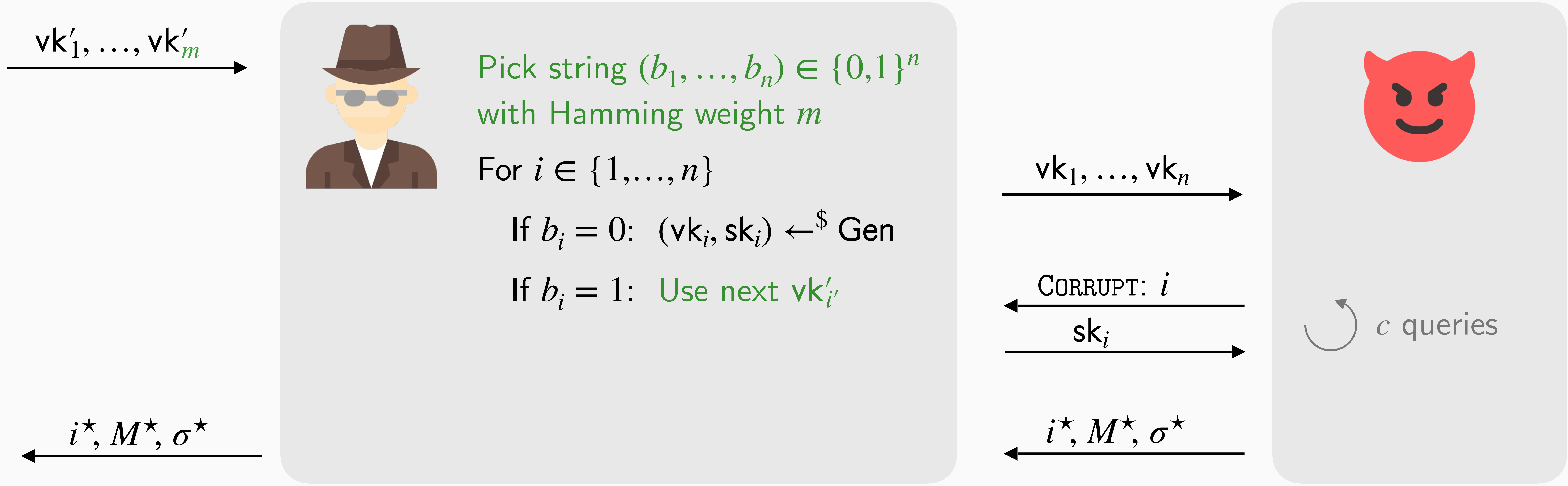

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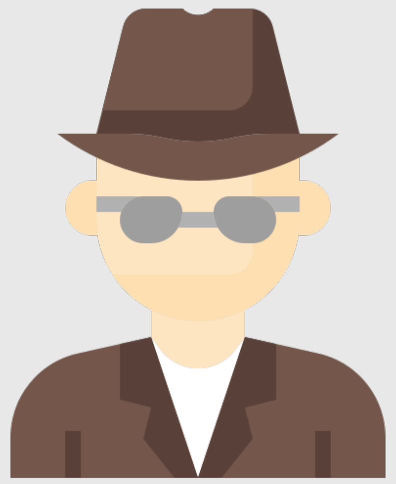

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What is the optimal m?

$\mathsf{vk}_1, \ldots, \mathsf{vk}_n$ →

CORRUPT: i ←

sk_i →

c queries

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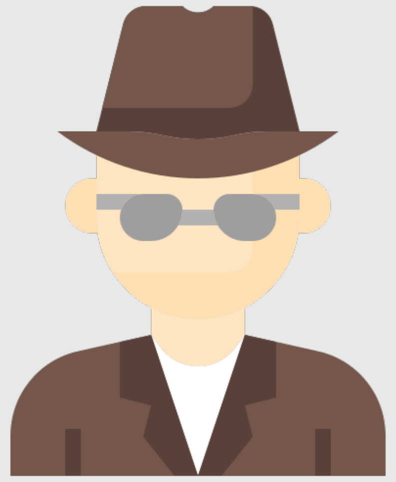
cp-muc Theorem

Refining and generalizing [C:Coron01]

This is captured by our abstraction of Hamming-weight determined samplers (via their success and error probability)

$\mathsf{vk}'_1, \ldots, \mathsf{vk}'_m$ →



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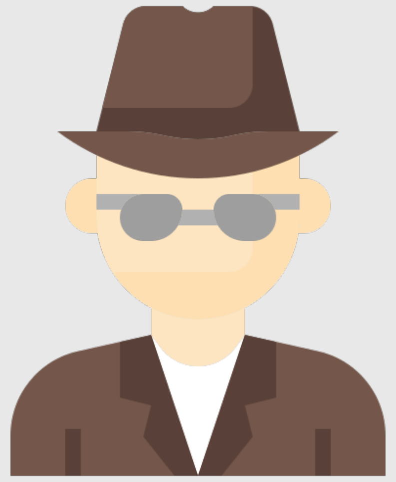

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for $m \approx n/c$

Relations

	Multi-user $\mathsf{Adv}^{\text{uf-mu-}n}_{\mathsf{Sig}}$	**With corruptions** $\mathsf{Adv}^{\text{uf-muc-}n}_{\mathsf{Sig}}$
Type-I no better relations known than the general ones (e.g. RSA)	$\leq n \cdot \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$	$\leq n \cdot \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$
Type-II mu-tight, but not under corruptions (e.g. Schnorr)	$\approx \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$	$\leq n \cdot \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$
Type-III muc-tight (“special” constructions, e.g. [PKC:DGJL21])	$\approx \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$	$\approx \mathsf{Adv}^{\text{uf-su}}_{\mathsf{Sig}}$

Relations

	Multi-user	With corruptions	Parametrized
	$\mathsf{Adv}^{\text{uf-mu-}n}_{\mathsf{Sig}}$	$\mathsf{Adv}^{\text{uf-muc-}n}_{\mathsf{Sig}}$	$\mathsf{Adv}^{\text{uf-muc-}(n,c)}_{\mathsf{Sig}}$
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Relations

	Multi-user $\mathsf{Adv}^{\mathsf{uf\text{-}mu\text{-}}n}_{\mathsf{Sig}}$	With corruptions $\mathsf{Adv}^{\mathsf{uf\text{-}muc\text{-}}n}_{\mathsf{Sig}}$	Parametrized $\mathsf{Adv}^{\mathsf{uf\text{-}muc\text{-}}(n,c)}_{\mathsf{Sig}}$
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Overview of our Results

Formal security specifications

- Syntax that translates into a single-user (su), multi-user (mu) and corruptions (muc) game

Hamming-weight determined samplers

- Technical tool that we introduce
- Essentially it determines how a (suitable) subset of users is picked

General cp-muc theorem (applies to all games which satisfy "locality" property)

- Basically all one-way (OW) games
- Indistinguishability (IND) games with independent challenge bits

Indirect applications of the cp-muc theorem (specialized results for "non-local" and "more advanced" games)

- IND-CCA with a single challenge bit across users (via FO, Hashed ElGamal)
- AKE protocols
- Selective opening security

We also give matching optimality (impossibility) results for a large class of games and schemes.

Conclusion

- In practice the number of corruptions is expected to be much smaller than the number of users.
- This was not reflected in models and thus concrete bounds for signing, encryption and key exchange.
- Our cp-muc framework gives a more fine-grained view and justifies standard parameter choices for many schemes.
  - It applies to Schnorr signatures, ElGamal-type encryption, and more.
- Tight muc security (Type-III schemes) is notoriously hard to achieve and we therefore suggest to focus on tight mu security (Type-II schemes).

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Thank you!