FLI: Folding Lookup Instances

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Joint work with Ignacio Manzur

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Folding

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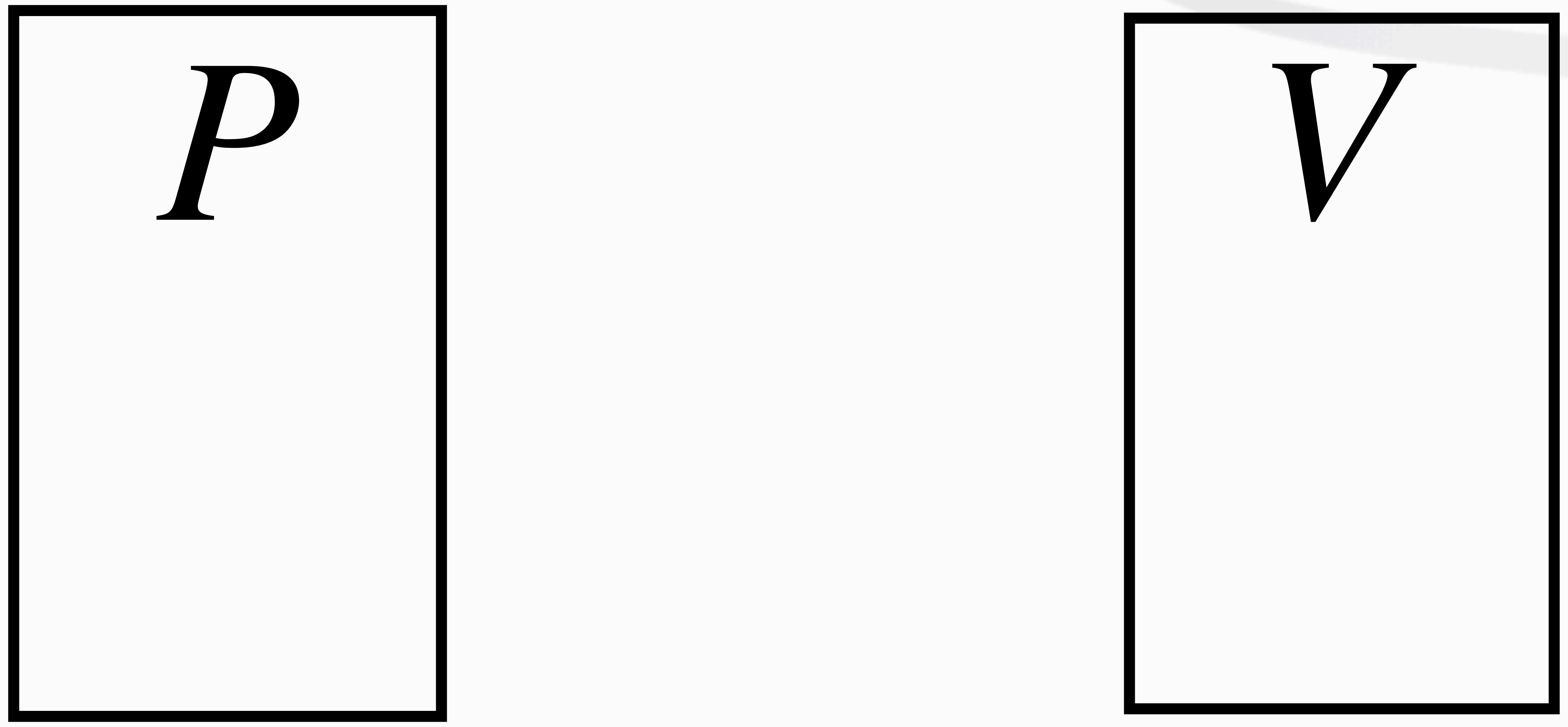

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- P also has witnesses w_1, w_2 such that $(x_1; w_1), (x_2; w_2) \in R$.

P	V
$(x_1; w_1)$	(x_1)
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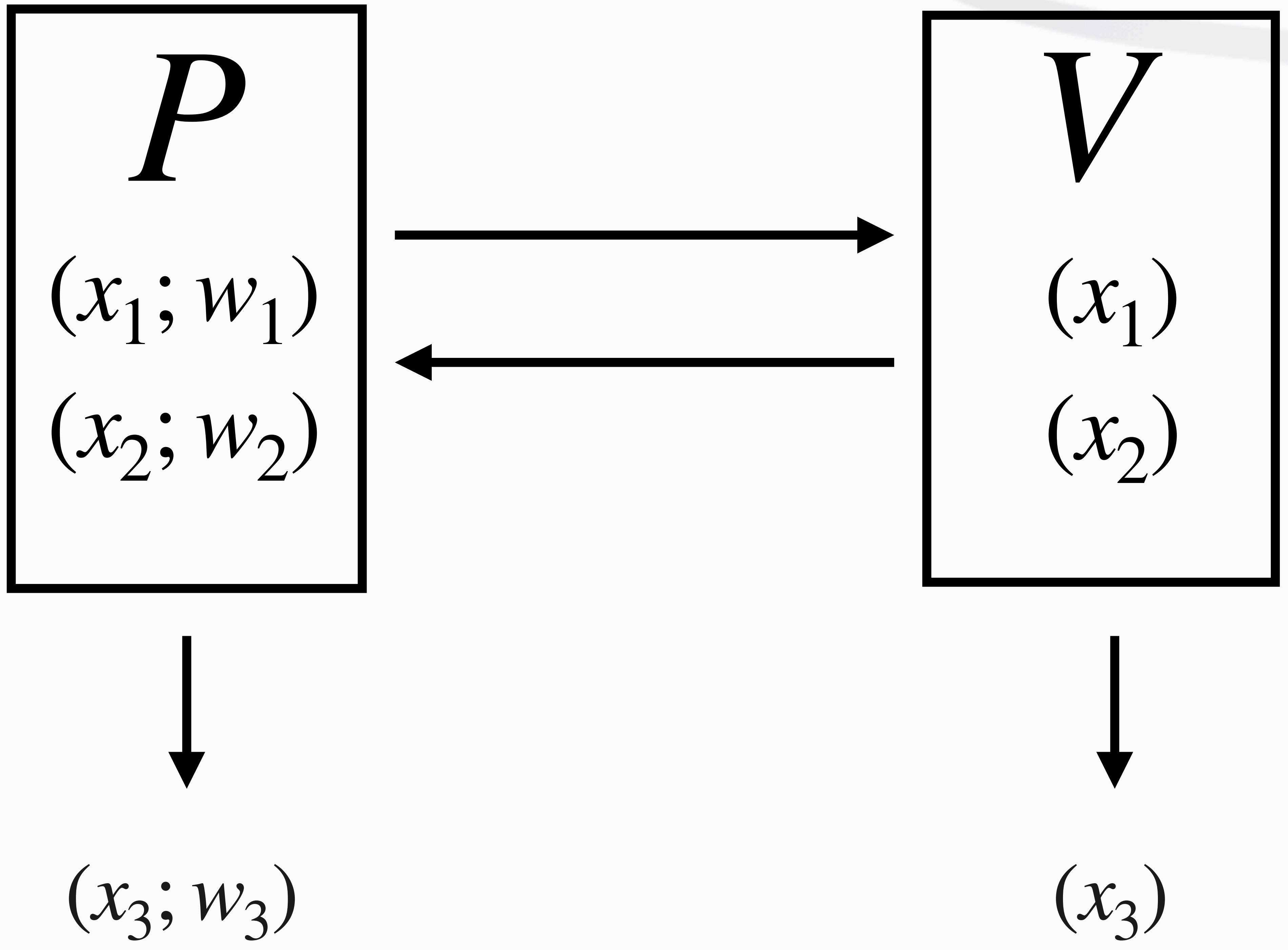

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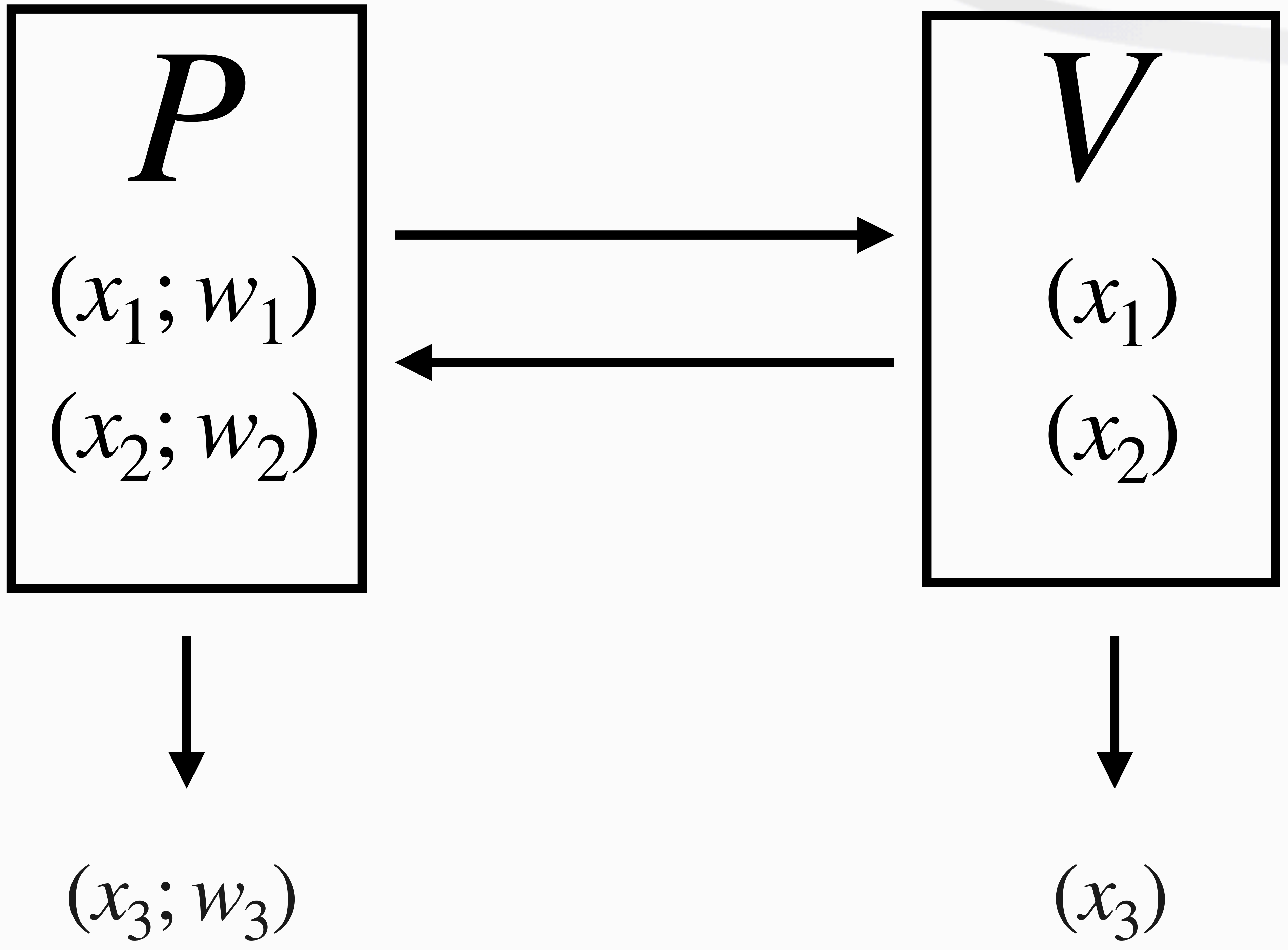

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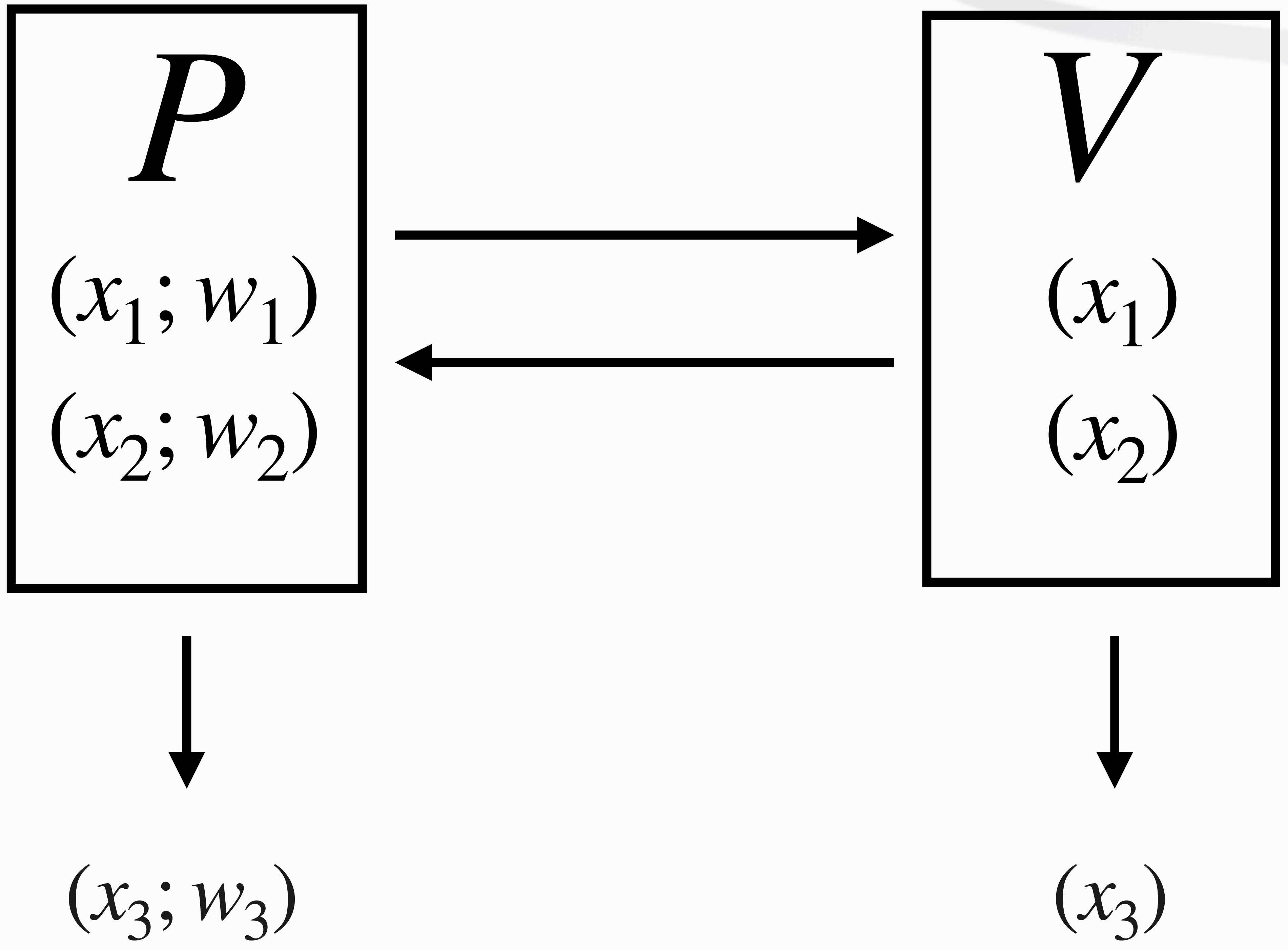

$$(x_3; w_3) \in R \quad \Leftrightarrow_{(e.w.n.p)} \quad \begin{cases} (x_1; w_1) & \in R, \\ (x_2; w_2) & \in R \end{cases}$$

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- Usually the commitment is homomorphic: $\text{cm}_{w_1+w_2} = \text{cm}_{w_1} + \text{cm}_{w_2}$

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Exchange messages ⟷

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- (Disclaimer: this is an extremely simplified, technically incorrect, blueprint)

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    - Here "expensive" means that a huge circuit is required. E.g. SHA-256 requires $\approx 2^{20}$ constraints as R1CS.

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- Simply, the $i-th$ row of M indicates a position of S that equals v_i

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The FLI scheme 🛬

$$R_{elem} = \{\,|v| \times |S| \text{ matrices all whose rows are elementary vectors}\}$$

- We can replace the claim $v \subseteq S$ by "there exists $M \in R_{elem}$ s.t. $M \cdot S^T = v^T$.
- Then to fold two claims

$$v_1 \subseteq S,\ v_2 \subseteq S$$
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- It can be seen that, with high probability (over the choice of α), (*) holds if and only if (**) holds.

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  - $M \cdot \mathbf{1}^T = \mathbf{1}^T$. With the above, this ensures each row contains exactly one 1.

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- Leveraging the sparseness of M_i the overall cost for P and V is similar to Nova on witnesses of size $|v_i|$

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- Note: Any other scheme working with huge SOS-dec. tables needs also to increase the number of folds per step by c (though c can be taken smaller), and FLI can make this step with less commitment costs.

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- We can actually make the S' as small as wanted by making (*) longer.
- Jolt (Arun et al. 2023): Many S's of interest are SOS-dec. E.g. RISC-V instructions

Thanks!

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- These conditions are equivalent to $v = (M_1 \cdot S') + 2^{16}(M_2 \cdot S')$ and $M_1, M_2 \in R_{elem}$
- We now use a Hypernova-style sumcheck to reduce the equality to two linear equalities, plus $M_1, M_2 \in R_{elem}$. Then we perform folding similarly as before.

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  - Then when proving a folded instance, we work with $|v| \cdot |S|^{1/c}$-sized witness.
- FLI can leverage SOS decomposability of S with much less field operations and commitments than other schemes: Protostar (Bünz, Biny Chen, 2023) , Proofs for Deep Thougth (Bünz, Jessica Chen, 2024), NeutronNova (Kothapally, Setty, 2024)

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- Example: An instruction could be bitwise XOR of 64-bit strings. Then $|S| = 2^{3\cdot 64} = 2^{192}$.

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        - Fold the 2^5 lookups and then prove the folded claim.