

# Interval Key-Encapsulation Mechanism

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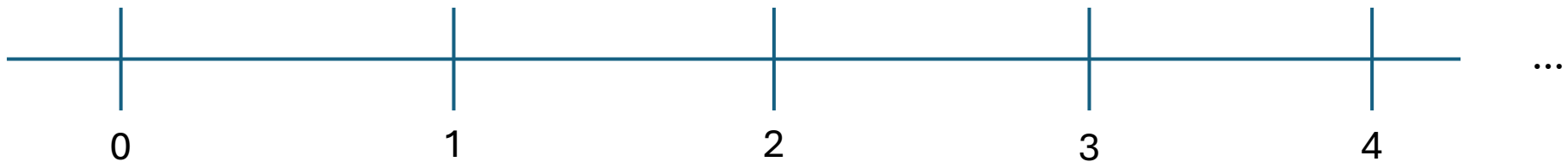
# Starting Point: Forward Secure KEM [CHK03]



$(sk_0, pk) \leftarrow Gen()$



Time



# Starting Point: Forward Secure KEM [CHK03]

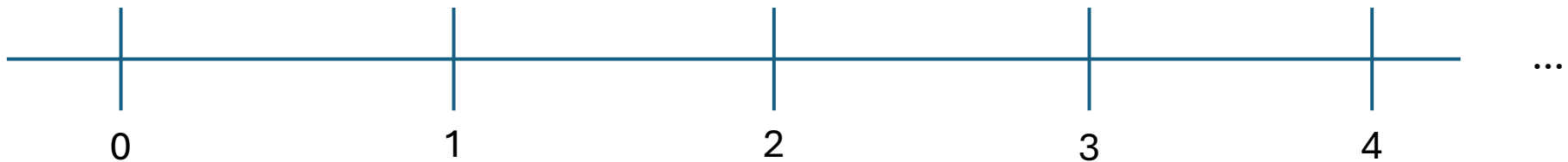


$pk$


$sk_0$

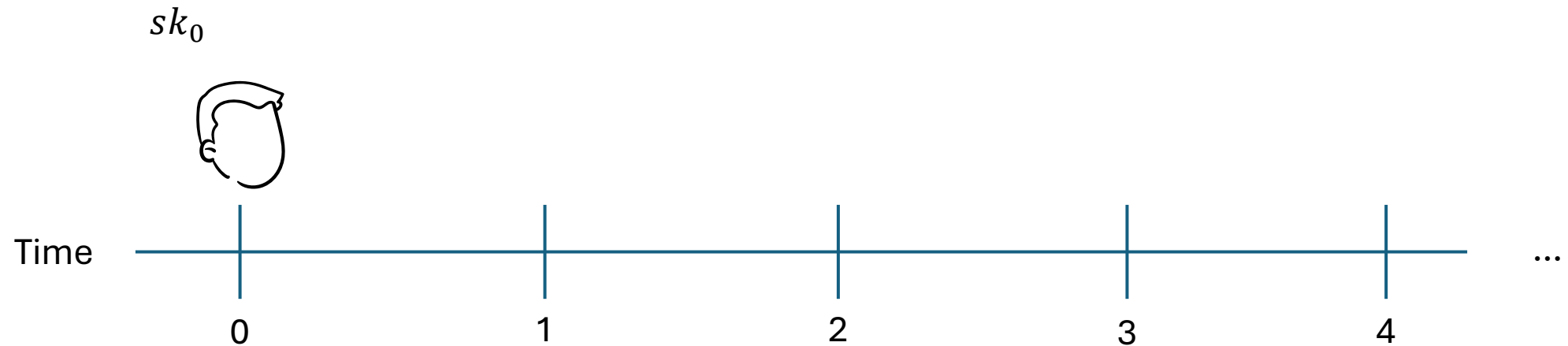


Time



# Starting Point: Forward Secure KEM [CHK03]

  $pk$   
 $(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$



# Starting Point: Forward Secure KEM [CHK03]



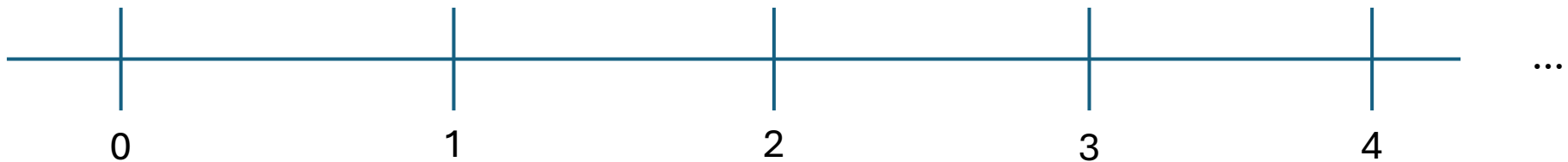
$pk$

$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$sk_0$



Time



# Starting Point: Forward Secure KEM [CHK03]



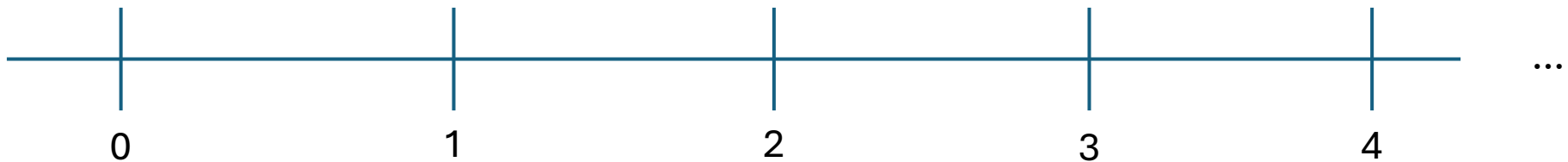
$pk$

$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$$sk_0 \quad k_0 \leftarrow Dec(sk_0, 0, ct_0)$$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

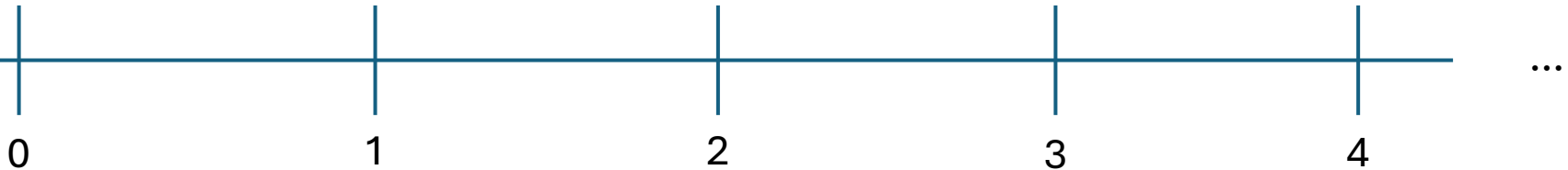
$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$$(k_2, ct_2) \leftarrow Enc(pk, 2)$$

$sk_0$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

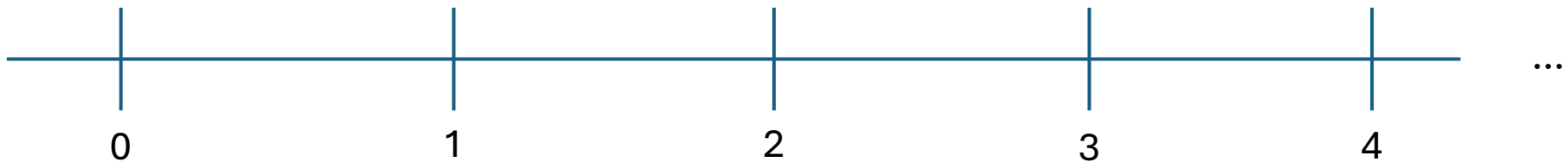
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$sk_0$



Time





# Starting Point: Forward Secure KEM [CHK03]



$pk$

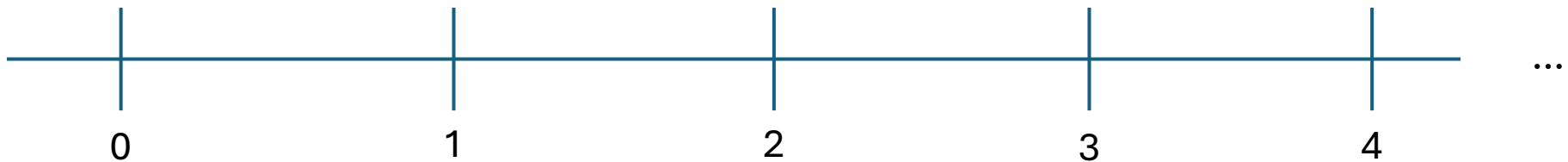
$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$$(k_2, ct_2) \leftarrow_{\$} Enc(pk, 2)$$

$$sk_0 \quad k_2 \leftarrow Dec(sk_0, 2, ct_2)$$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$

$(k_2, ct_2) \leftarrow_{\$} Enc(pk, 2)$

$sk_0$



Time

0

1

2

3

4

...

# Starting Point: Forward Secure KEM [CHK03]



$pk$

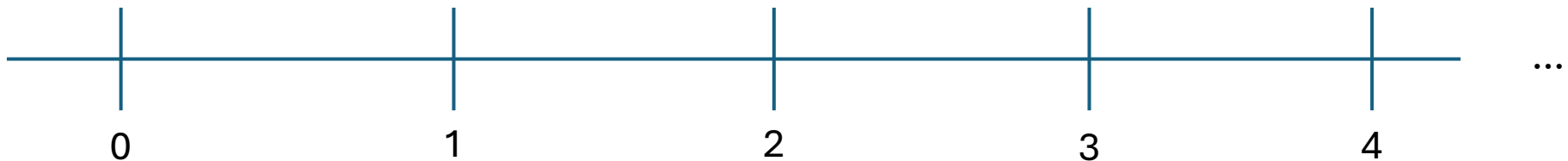
$$(k_0, ct_0) \leftarrow_{\$} \text{Enc}(pk, 0)$$

$$(k_2, ct_2) \leftarrow_{\$} \text{Enc}(pk, 2)$$

$$sk_1 \leftarrow \text{Up}(sk_0)$$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

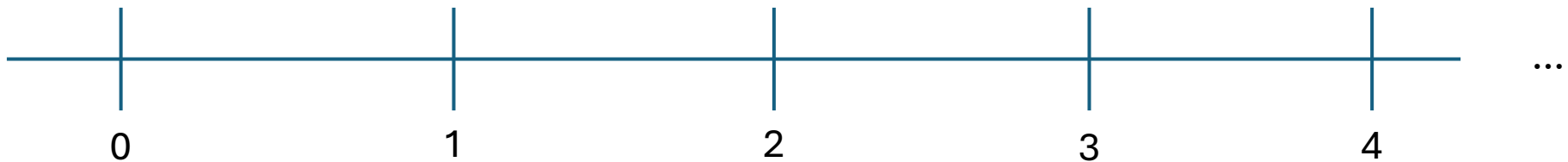
$$(k_0, ct_0) \leftarrow_{\$} \text{Enc}(pk, 0)$$

$$(k_2, ct_2) \leftarrow_{\$} \text{Enc}(pk, 2)$$

$$sk_2 \leftarrow \text{Up}(sk_1)$$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

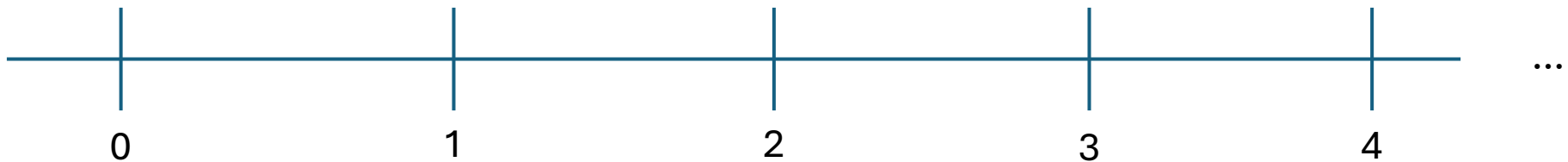
$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$$(k_2, ct_2) \leftarrow_{\$} Enc(pk, 2)$$

$sk_2$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

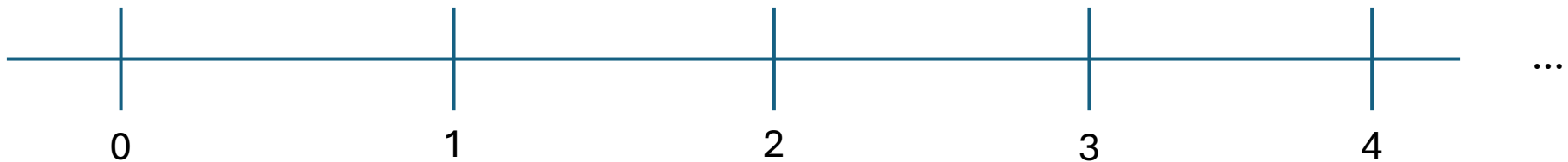
$$(k_2, ct_2) \leftarrow_{\$} Enc(pk, 2)$$



$sk_2$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

$$(k_0, ct_0) \leftarrow_{\$} Enc(pk, 0)$$

$$(k_2, ct_2) \leftarrow_{\$} Enc(pk, 2)$$

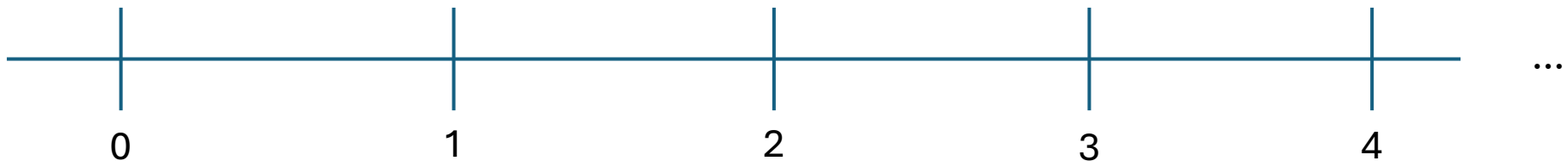
Insecure



$sk_2$



Time



# Starting Point: Forward Secure KEM [CHK03]



$pk$

$$(k_0, ct_0) \leftarrow_{\$} \text{Enc}(pk, 0)$$

Secure – FS!

$$(k_2, ct_2) \leftarrow_{\$} \text{Enc}(pk, 2)$$

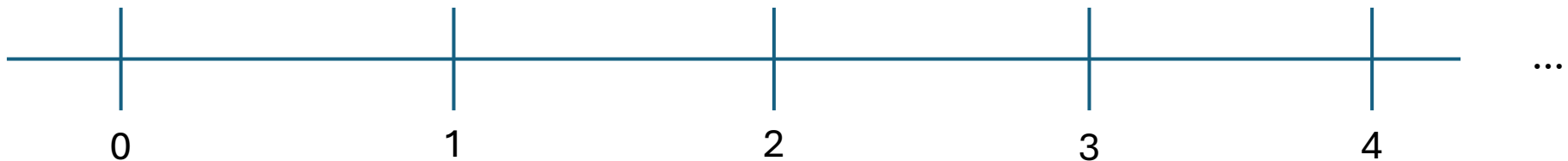
Insecure



$sk_2$



Time





# Starting Point: Forward Secure KEM [CHK03]



$pk$

$$(k_0, ct_0) \leftarrow_{\$} \text{Enc}(pk, 0)$$

Secure – FS!

$$(k_2, ct_2) \leftarrow_{\$} \text{Enc}(pk, 2)$$

Insecure

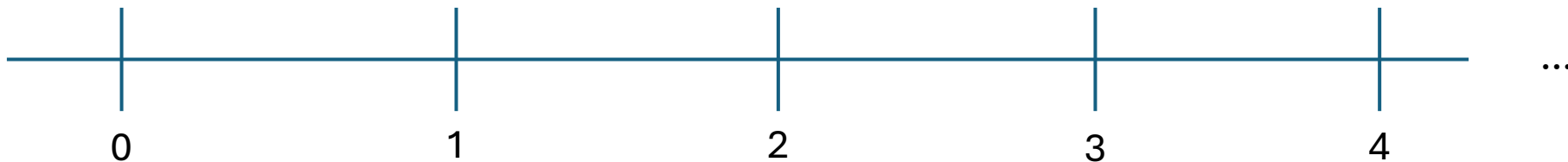


$sk_2$



[CHK03] construct FS-KEM using HIBE-like techniques

Time



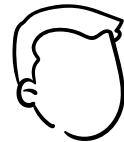
# Disadvantages of FS-KEM



$pk$



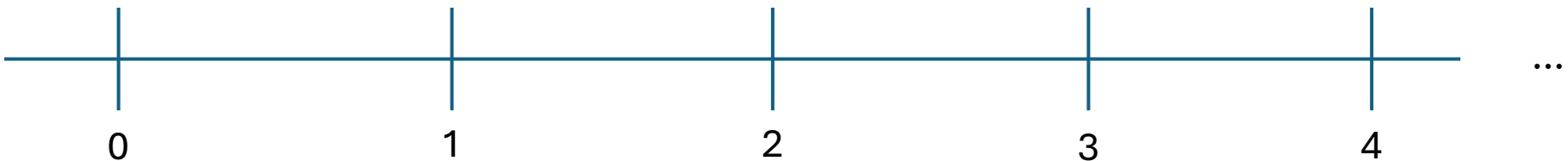
$sk_2$



[CHK03] construct  
FS-KEM using HIBE-  
like techniques

**INEFFICIENT!**

Time



# Disadvantages of FS-KEM



$pk$



$sk_2$



ct's to all future times insecure!



Time

0

1

2

3

4

...

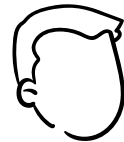
# Interval KEM



$pk$



$sk_2$



Time

0

1

2

3

4

...

# Interval KEM



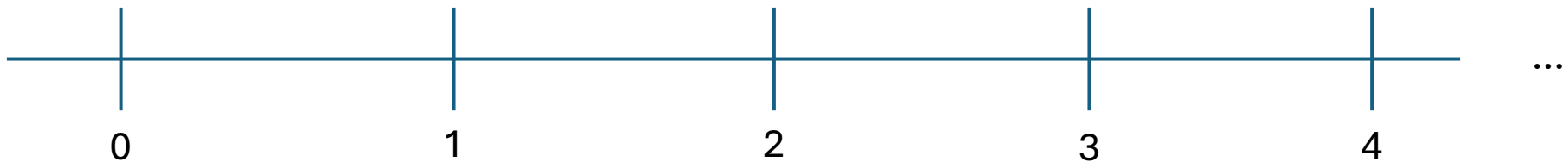
$pk$



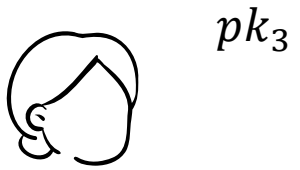
$sk_2$   $(sk_3, pk_3) \leftarrow \text{Update}(sk_2)$



Time



# Interval KEM

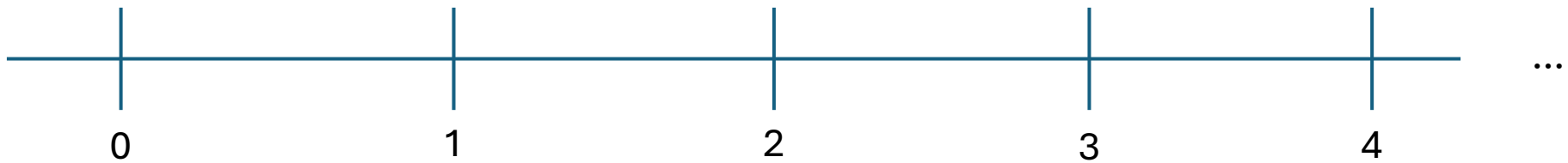


$sk_2$

$sk_3$



Time



# Interval KEM



$pk_3$

$(k_3, ct_3) \leftarrow_{\$} Enc(pk_3, 3)$

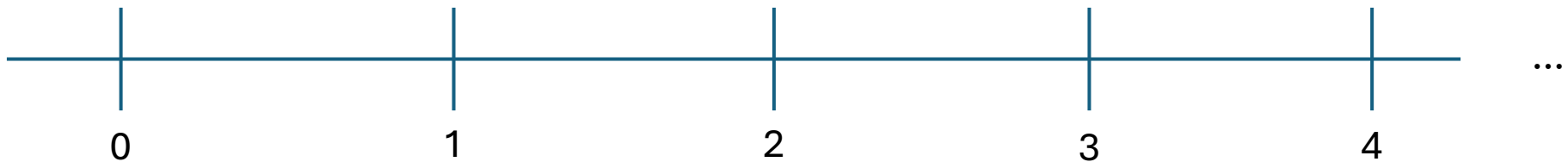


$sk_2$

$sk_3$



Time



# Interval KEM



$$(k_3, ct_3) \leftarrow_{\$} Enc(pk_3, 3)$$

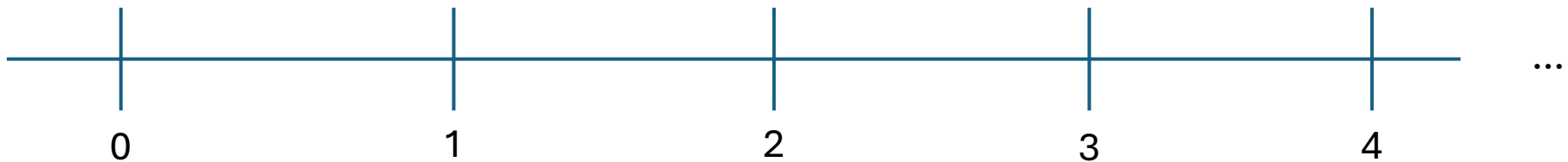


$sk_2$

$sk_3$



Time





# Interval KEM



$$(k_3, ct_3) \leftarrow_{\$} Enc(pk_3, 3)$$

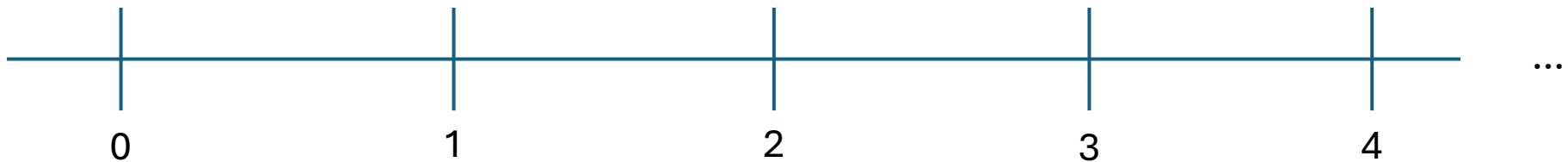


$sk_2$

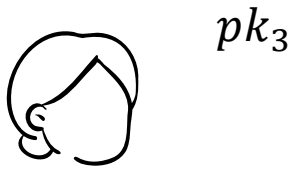
$$sk_3 \quad k_3 \leftarrow Decrypt(sk_3, 3, ct_3)$$



Time



# Interval KEM



$$(k_3, ct_3) \leftarrow_{\$} Enc(pk_3, 3)$$

Secure – PCS!

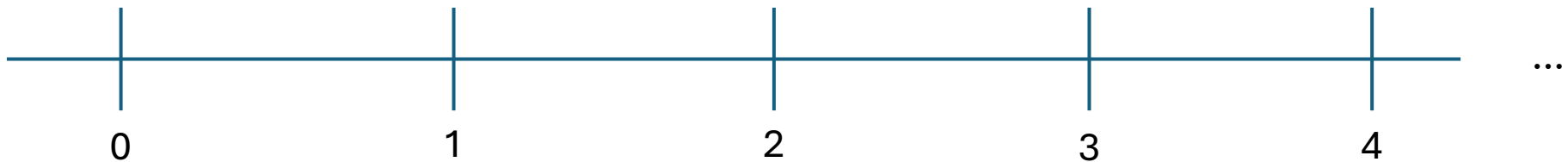


$sk_2$

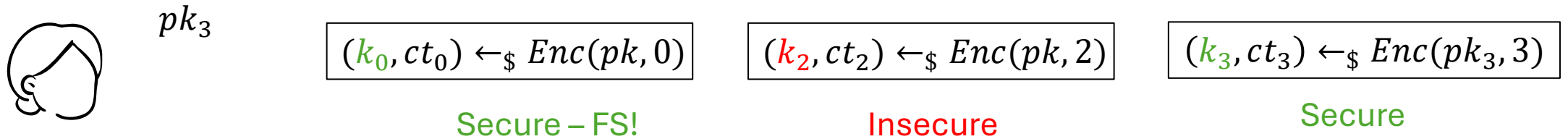
$$sk_3 \quad k_3 \leftarrow Decrypt(sk_3, 3, ct_3)$$



Time

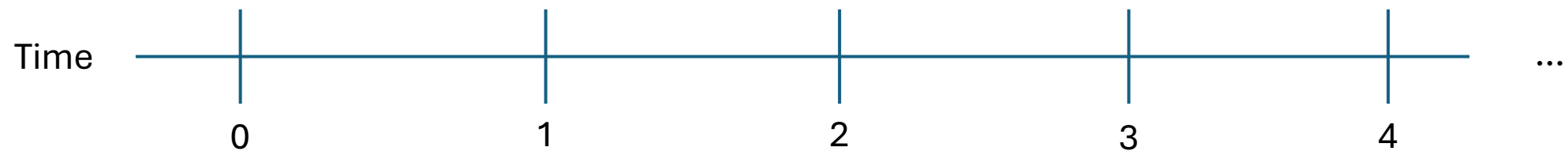


# Interval KEM



$sk_2$

$sk_3 \quad k_3 \leftarrow Decrypt(sk_3, 3, ct_3)$



# Comparison to Session-Based Messaging

- There are many session-based messaging protocols with FS & PCS
  - Double Ratchet Algorithm [PM16]
- **Requires Key Management for each session!**
  - **Each update requires communication linear in # sessions**
- With IKEM, just maintain **one** total secret key for every session
  - **Updates are global!**
  - Everyone encrypts to corresponding public key!

# KEM-based IKEM

- Given  $KM.gen()$ ,  $KM.enc()$ ,  $KM.dec()$

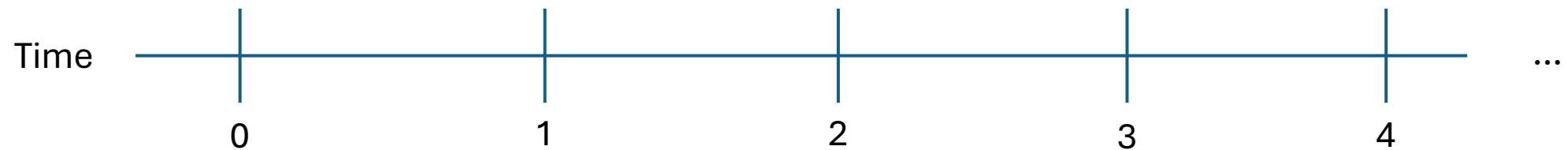
# KEM-based IKEM

- Given  $KM.gen()$ ,  $KM.enc()$ ,  $KM.dec()$
- $Gen(): KM.gen() \rightarrow (sk_0, pk_0)$
- $Up(sk_t): KM.gen \rightarrow (sk_{t+1}, pk_{t+1})$
- $Enc(pk_t): KM.enc(pk_t) \rightarrow_{\$} (k, ct)$
- $Dec(sk_t, ct): KM.dec(sk_t, ct) \rightarrow k$

# KEM-based IKEM

- Given  $KM.gen()$ ,  $KM.enc()$ ,  $KM.dec()$
- $Gen(): KM.gen() \rightarrow (sk_0, pk_0)$
- $Up(sk_t): KM.gen \rightarrow (sk_{t+1}, pk_{t+1})$
- $Enc(pk_t): KM.enc(pk_t) \rightarrow_{\$} (k, ct)$
- $Dec(sk_t, ct): KM.dec(sk_t, ct) \rightarrow k$
- Each  $(sk_t, pk_t)$  indep  $\Rightarrow$  FS and PCS

# Communication in Mesh Networks





# Communication in Mesh Networks



$pk_2$



$pk_2$



$pk_2$



$pk_2$

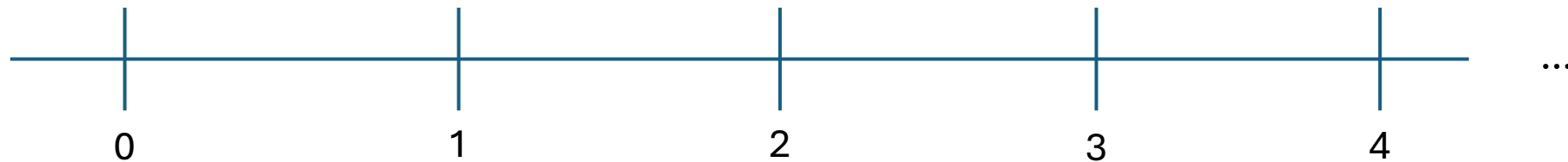


$pk_2$

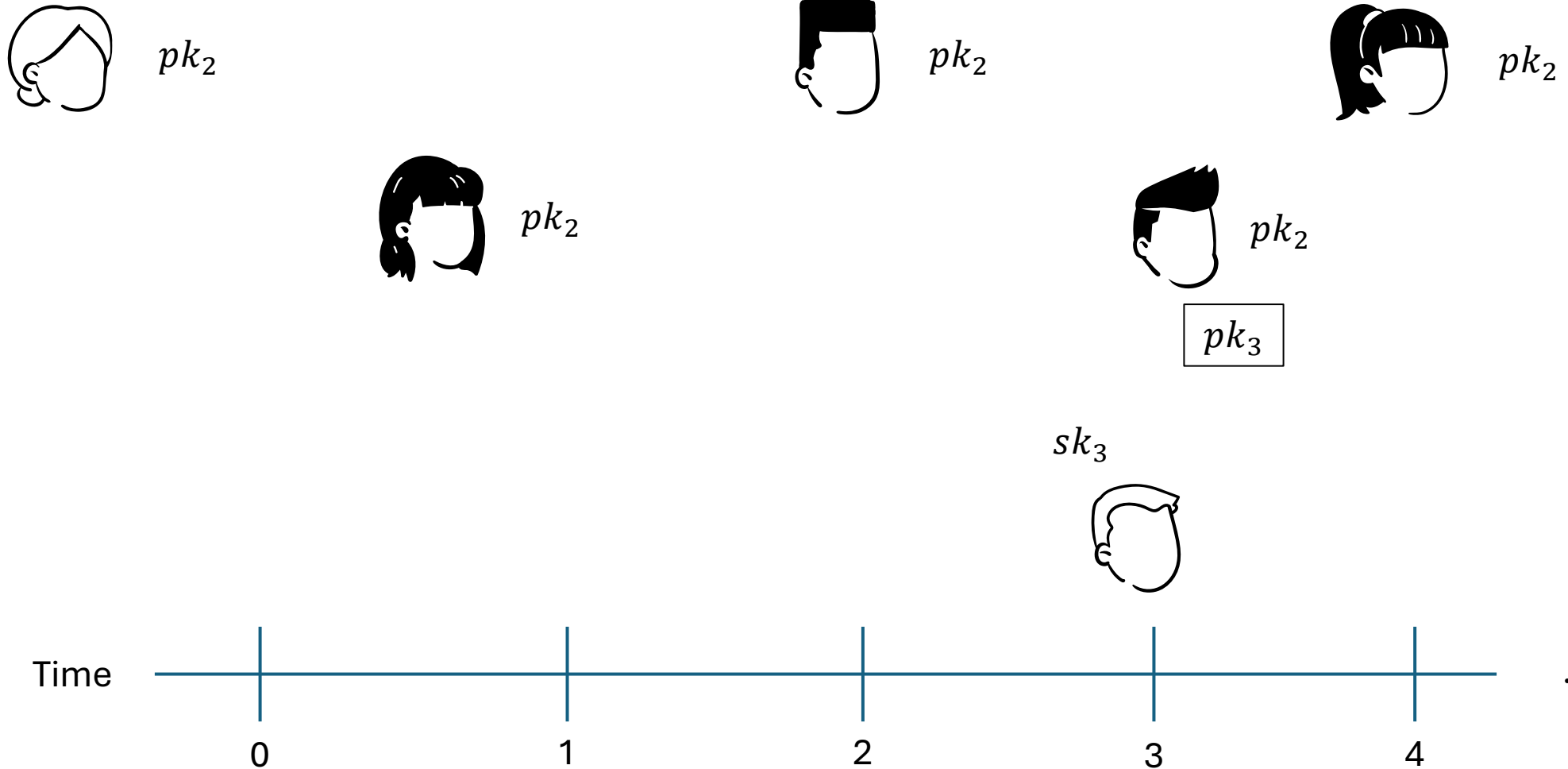
$(sk_3, pk_3) \leftarrow Up(sk_3)$



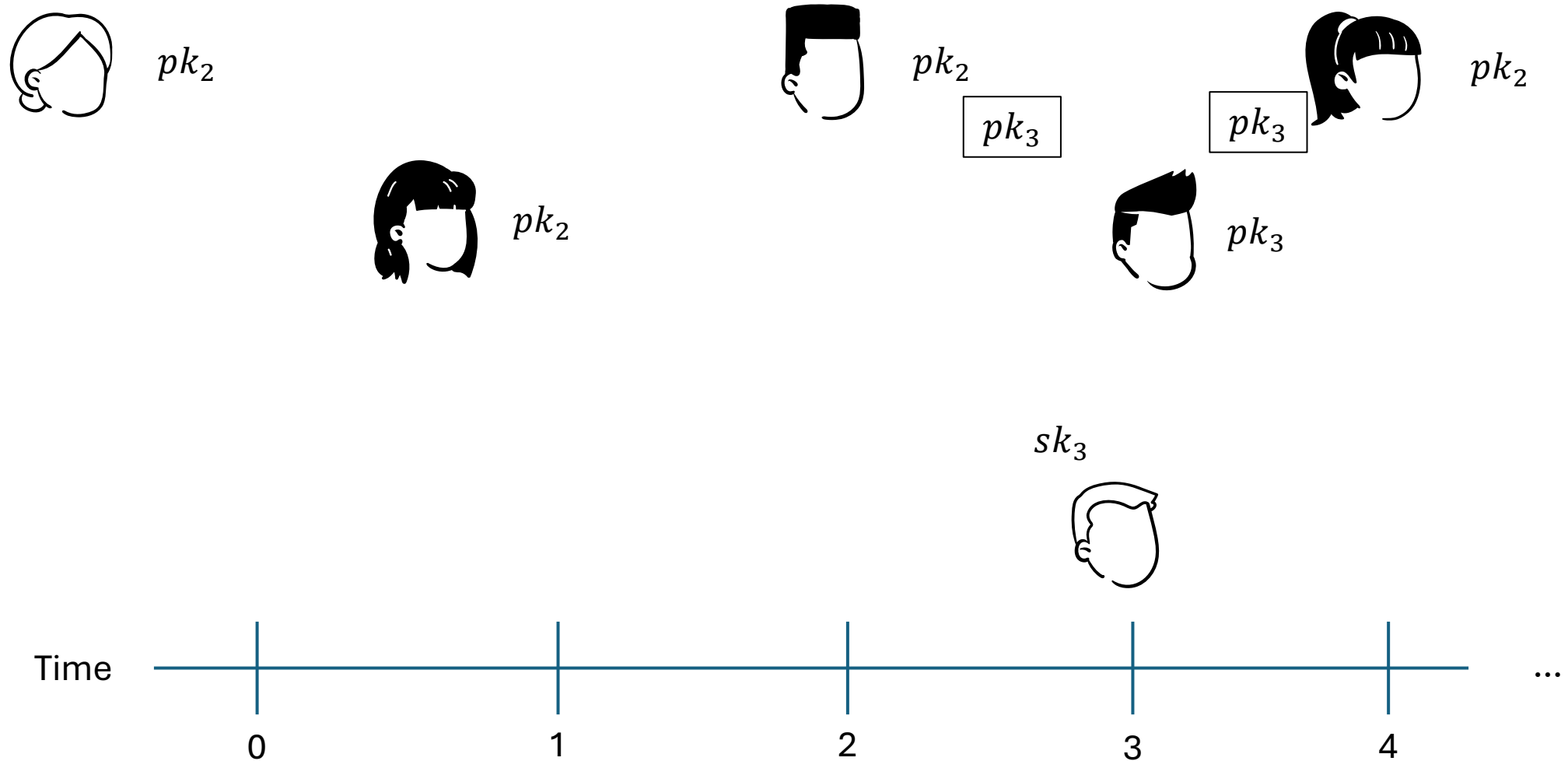
Time



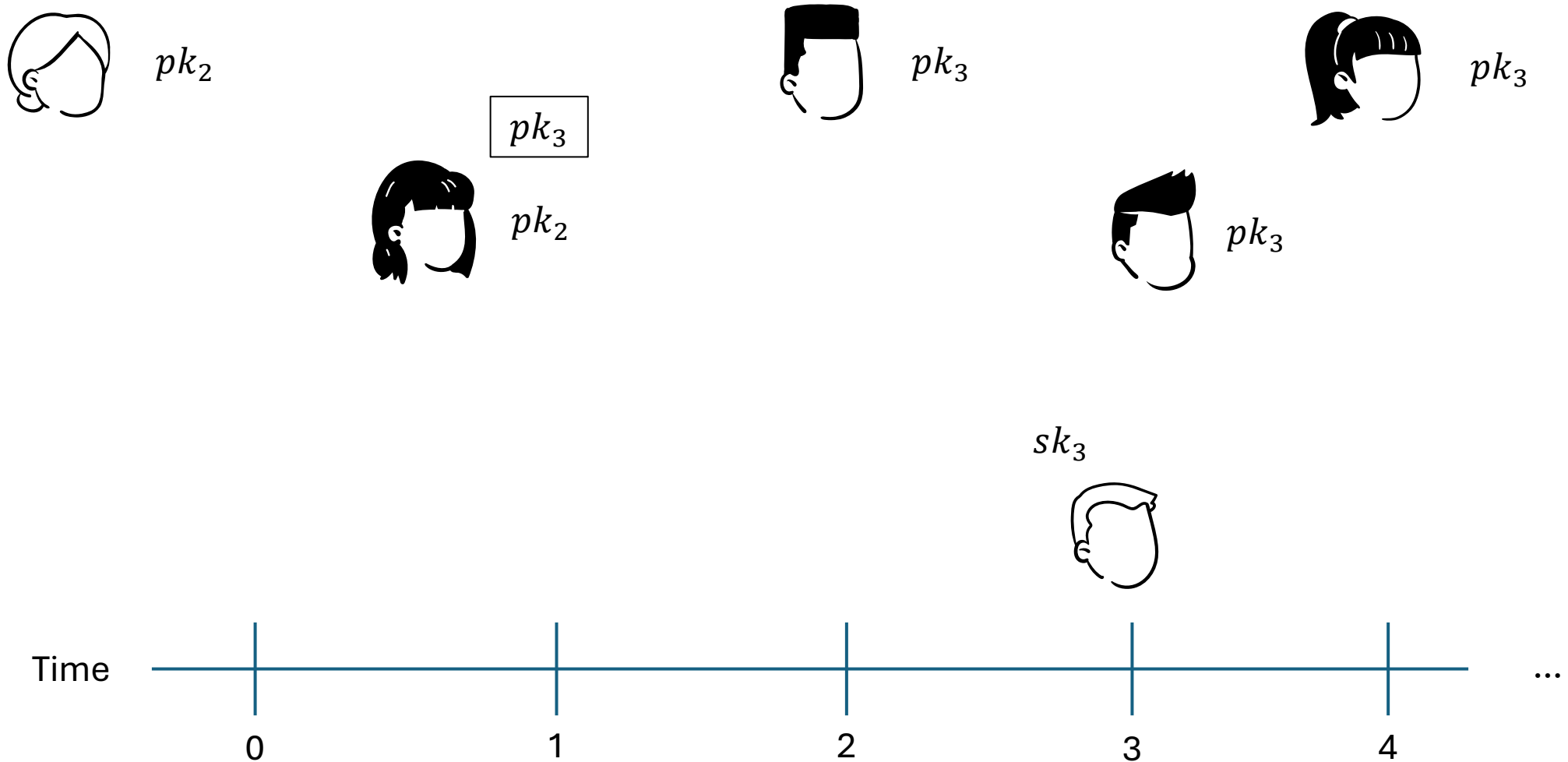
# Communication in Mesh Networks



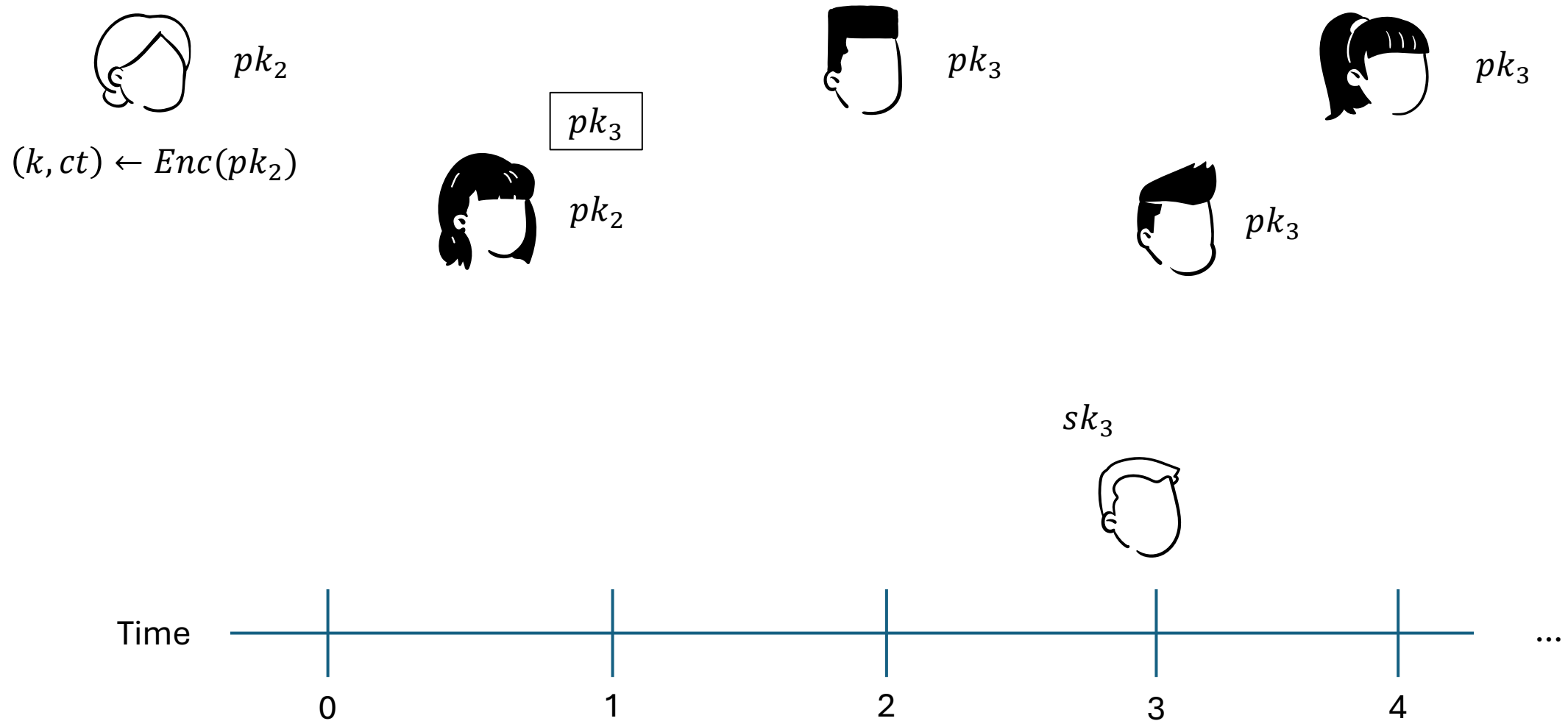
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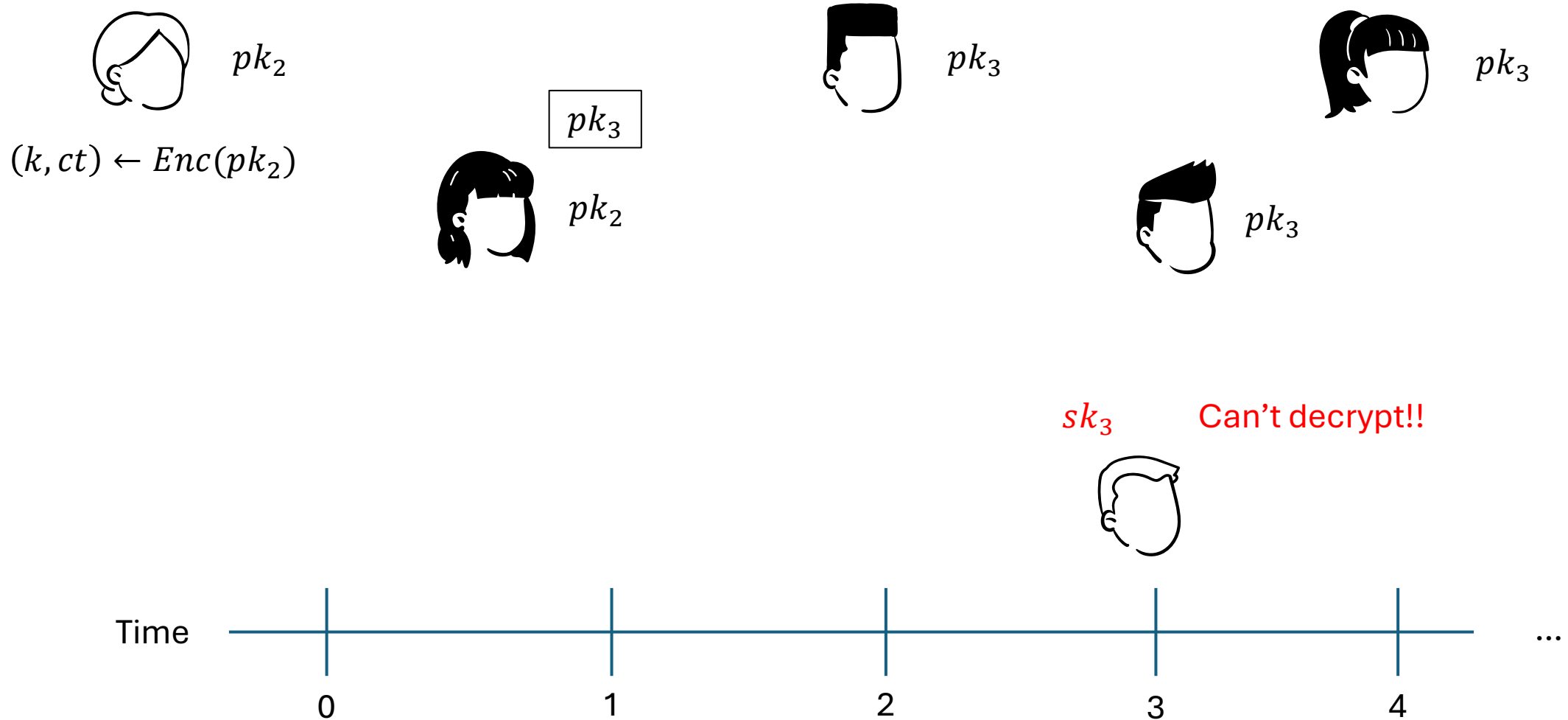
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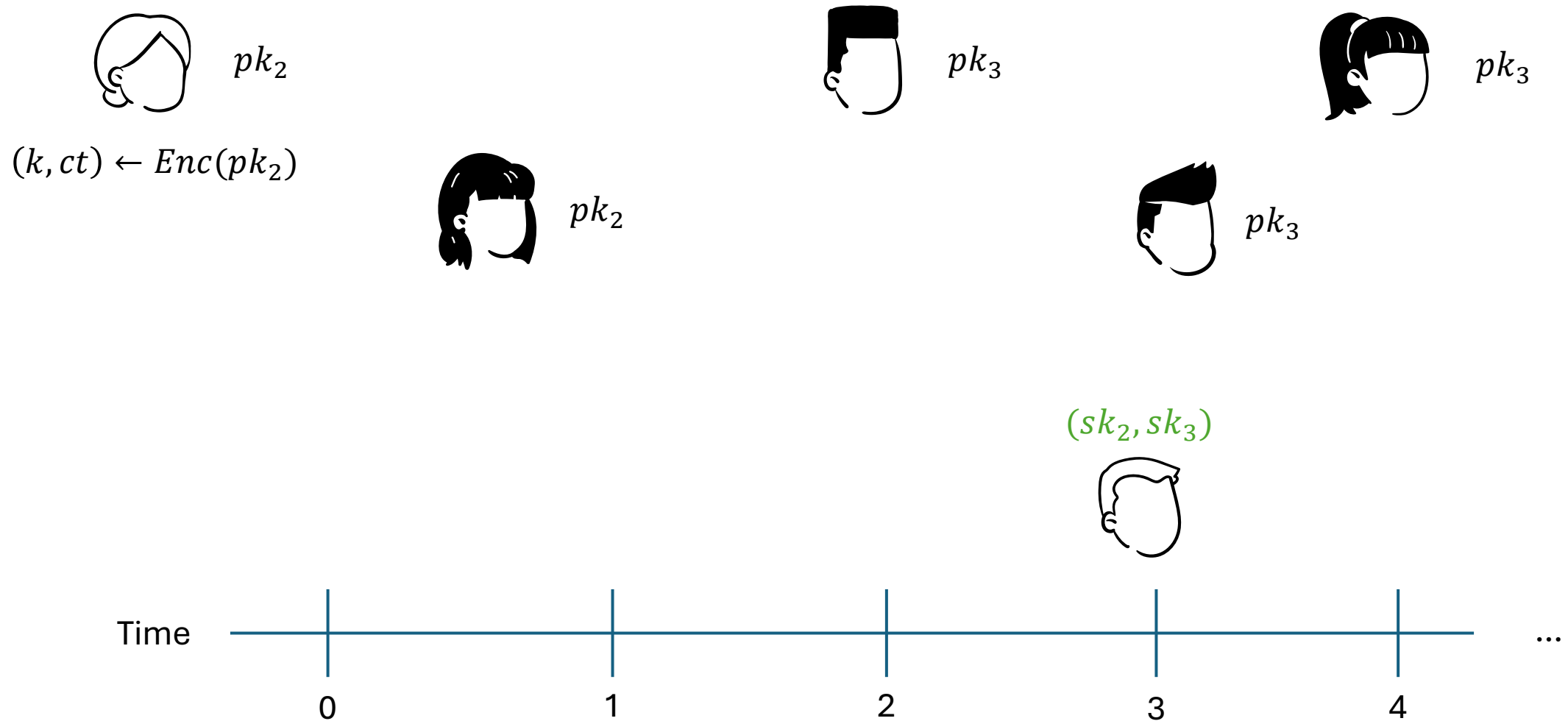
# Communication in Mesh Networks



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# Communication in Mesh Networks



# Communication in Mesh Networks



$pk_2$



$pk_3$



$pk_3$

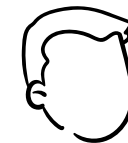


$pk_2$



$pk_3$

$(sk_2, sk_3, sk_4)$



Time

0

1

2

3

4

...



# Communication in Mesh Networks



$pk_4$



$pk_4$



$pk_4$



$pk_4$



$pk_4$

$(sk_2, sk_3, sk_4)$



Time

0

1

2

3

4

...

# Communication in Mesh Networks



$pk_4$



$pk_4$



$pk_4$



$pk_4$



$pk_4$

$Del((sk_2, sk_3, sk_4), (2,3))$



Time

0

1

2

3

4

...

# Communication in Mesh Networks



$pk_4$



$pk_4$



$pk_4$



$pk_4$



$pk_4$

$(sk_4)$



Time

0

1

2

3

4

...

# KEM-based IKEM

- Given  $KM.gen()$ ,  $KM.enc()$ ,  $KM.dec()$
- $Gen(): KM.gen() \rightarrow (sk_0, pk_0)$
- $Up((sk_{t_0}, \dots, sk_{t_1})):$ 
  - $KM.gen \rightarrow (sk_{t_1+1}, pk_{t_1+1})$
  - Output  $((sk_{t_0}, \dots, sk_t, sk_{t_1+1}), pk_{t_1+1})$
- $Enc(pk_t): KM.enc(pk_t) \rightarrow_{\$} (k, (t, ct))$
- $Dec((sk_{t_0}, \dots, sk_{t_1}), (t, ct)): KM.dec(sk_t, ct) \rightarrow k$ 
  - Only works if  $t \in [t_0, t_1]$

# KEM-based IKEM

- Given  $KM.gen(), KM.enc(), KM.dec()$
- $Gen(): KM.gen() \rightarrow (sk_0, pk_0)$
- $Up((sk_{t_0}, \dots, sk_{t_1})):$ 
  - $KM.gen \rightarrow (sk_{t_1+1}, pk_{t_1+1})$
  - Output  $((sk_{t_0}, \dots, sk_t, sk_{t_1+1}), pk_{t_1+1})$
- $Enc(pk_t): KM.enc(pk_t) \rightarrow_{\$} (k, (t, ct))$
- $Dec((sk_{t_0}, \dots, sk_{t_1}), (t, ct)): KM.dec(sk_t, ct) \rightarrow k$ 
  - Only works if  $t \in [t_0, t_1]$
- $Del((sk_{t_0}, \dots, sk_{t_1}), \ell): \text{Output } (sk_{t_0+\ell}, \dots, sk_{t_1})$

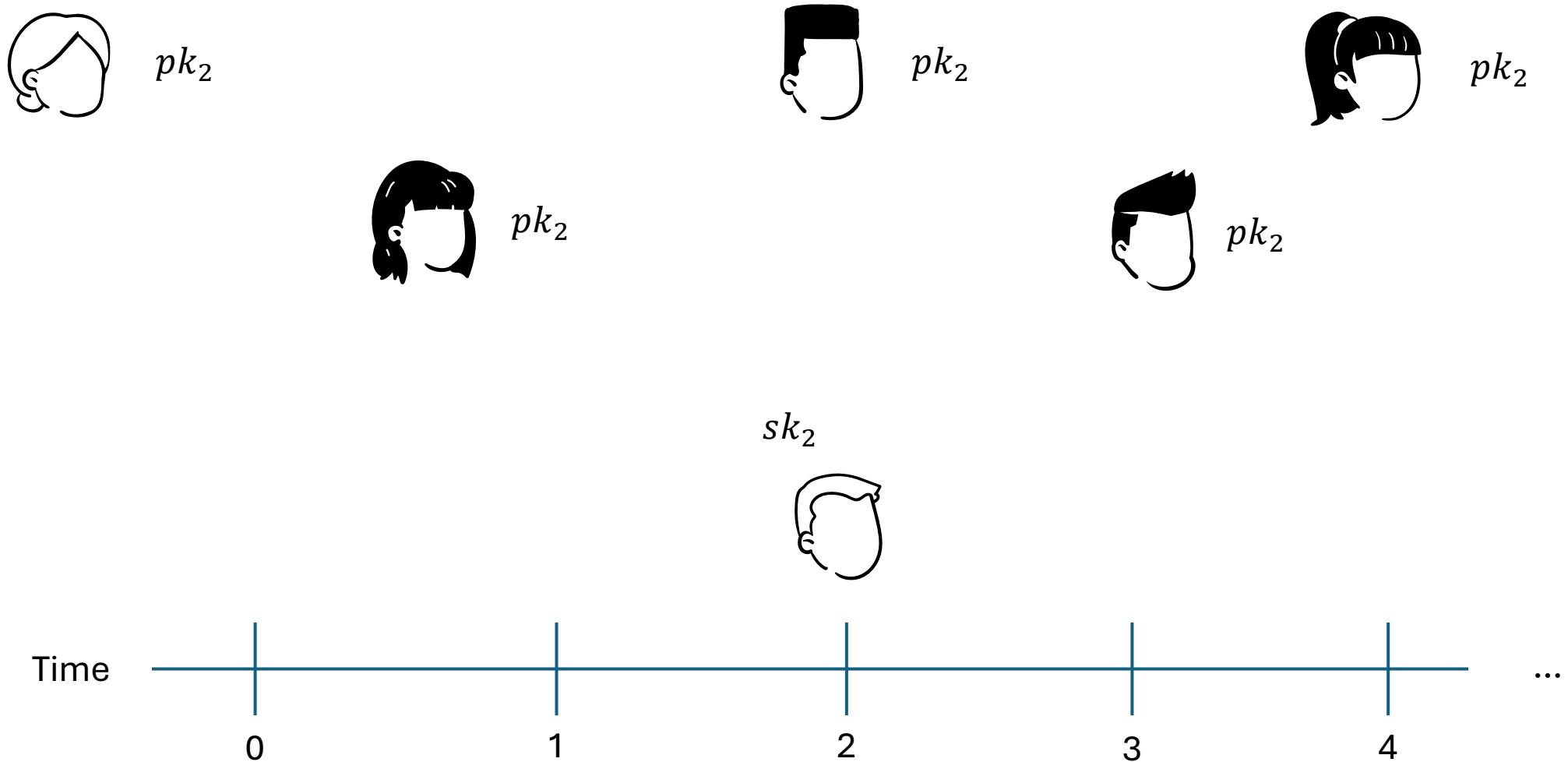
# Secret Key Size Lower Bound

- We want to decapsulate  $ct$  created at any time  $t \in [t_0, t_1]$
- Secret state in  $O(t_1 - t_0)$  inevitable?
  - Yes
- Intuition:
  - Don't know when compromises happen
    - ⇒ Each  $Up()$  needs independent randomness
  - Cannot compress independent randomness

# Secret Key Size Lower Bound

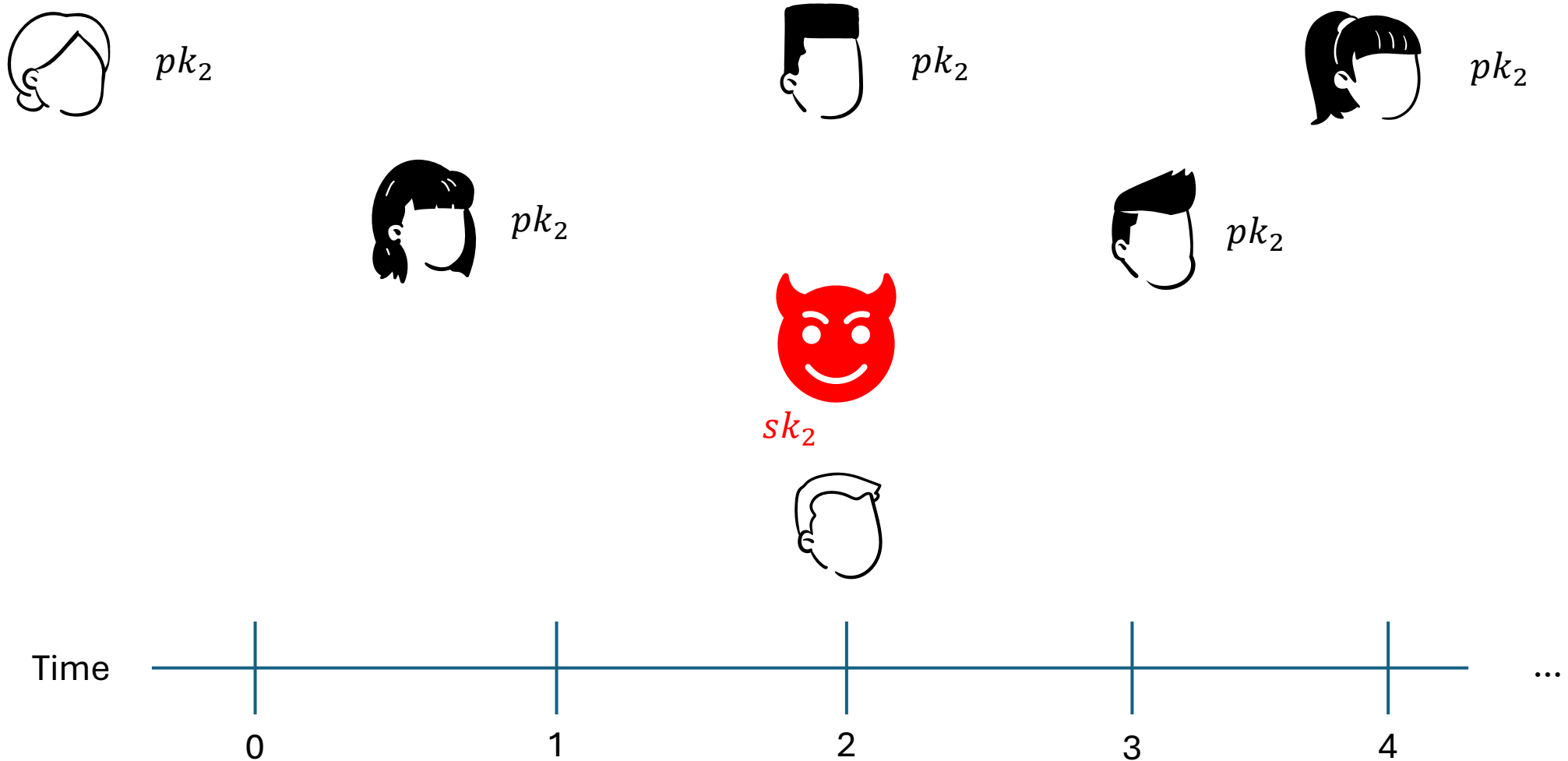
- We want to decapsulate  $ct$  created at any time  $t \in [t_0, t_1]$
- Secret state in  $O(t_1 - t_0)$  inevitable?
  - Yes
- Intuition:
  - Don't know when compromises happen
    - ⇒ Each  $Up()$  needs independent randomness
  - Cannot compress independent randomness
- Circumvent lower bound via external (public) storage

# Stronger Security for Mesh Networks [BRT23]

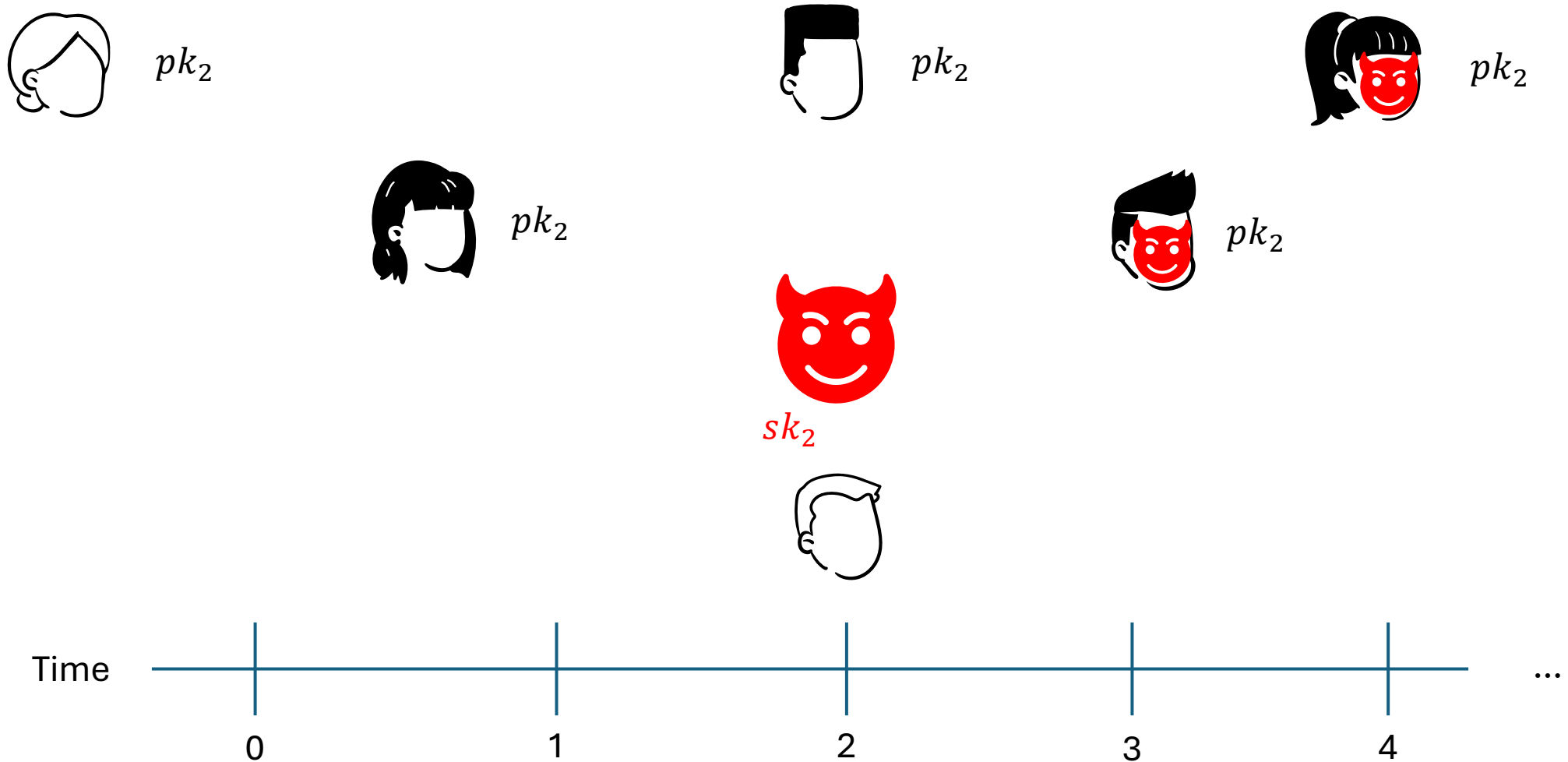




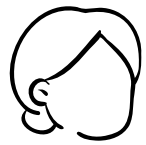
# Stronger Security for Mesh Networks [BRT23]



# Stronger Security for Mesh Networks [BRT23]



# Stronger Security for Mesh Networks [BRT23]



$pk_2$



$pk_2$



$pk_2$



$pk_2$



$pk_2$

$((sk_2, sk_3), pk_3) \leftarrow Up(sk_2)$



Time

0

1

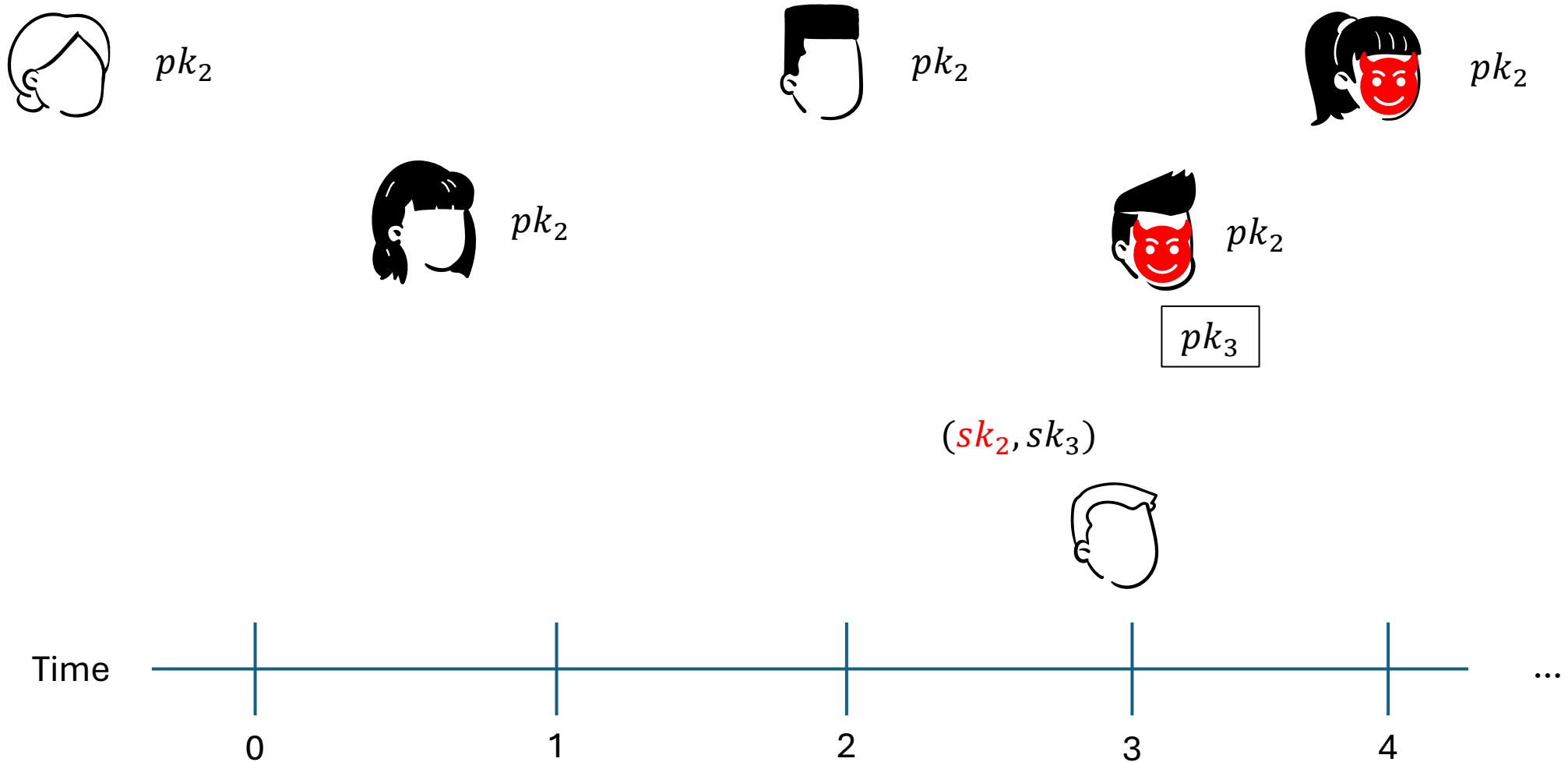
2

3

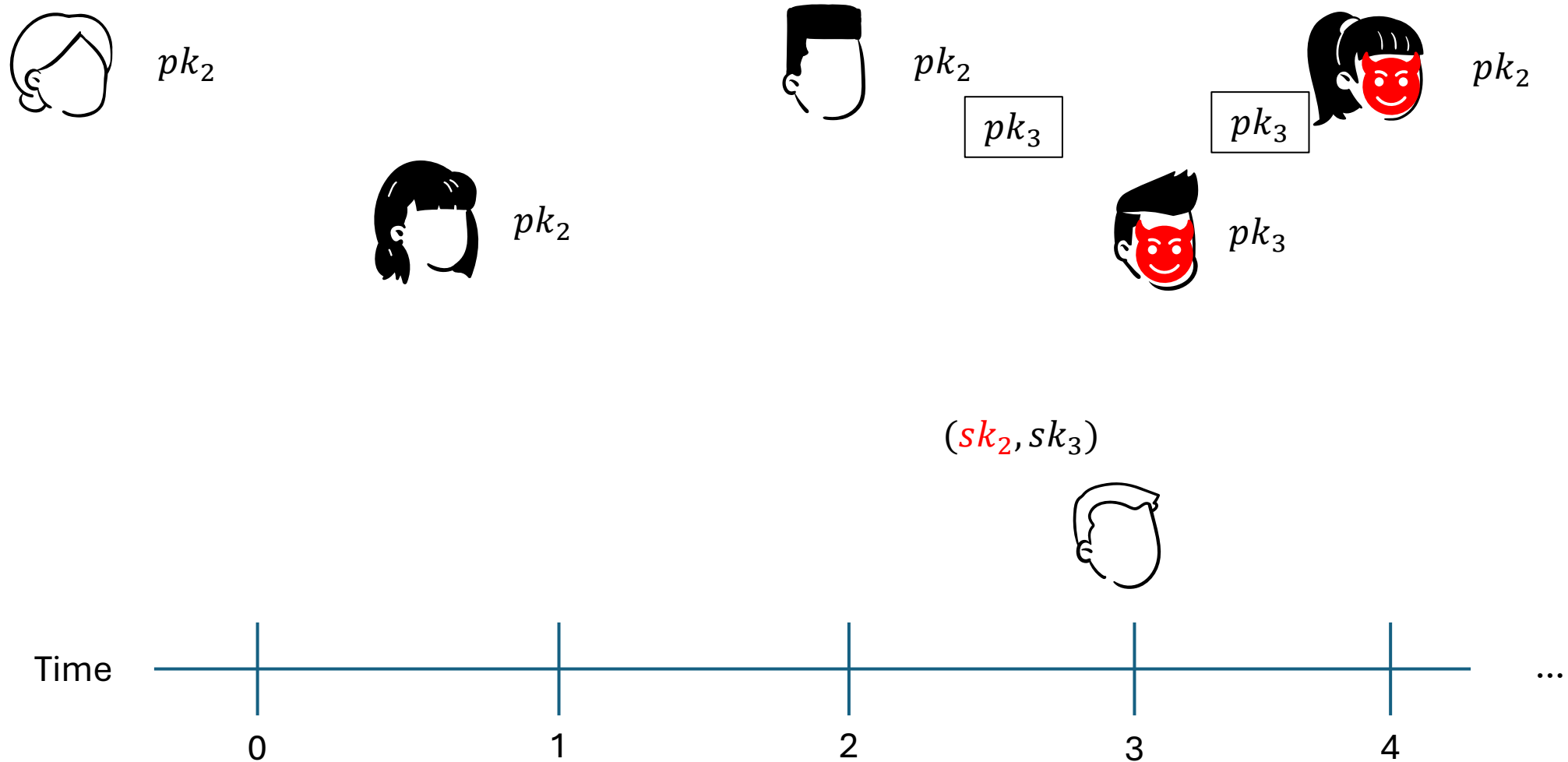
4

...

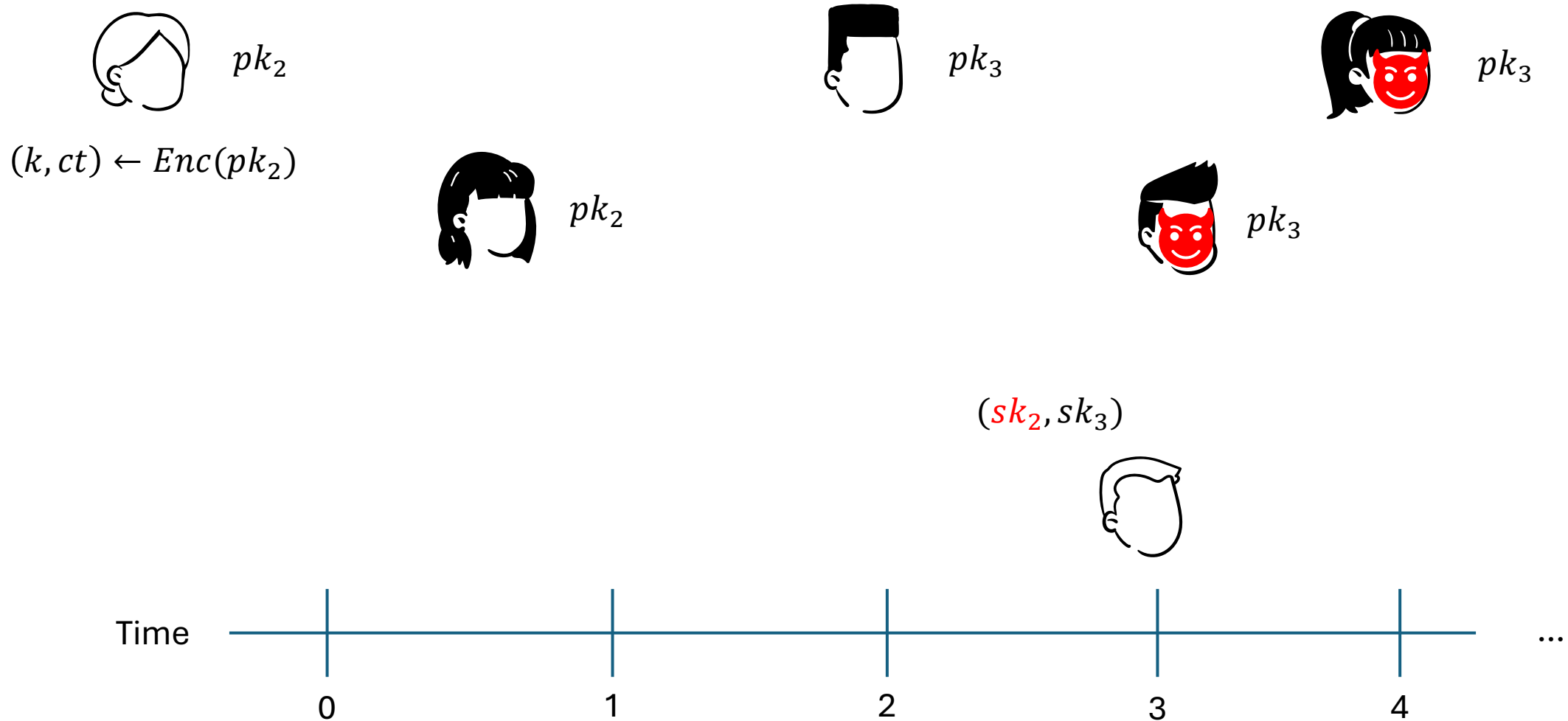
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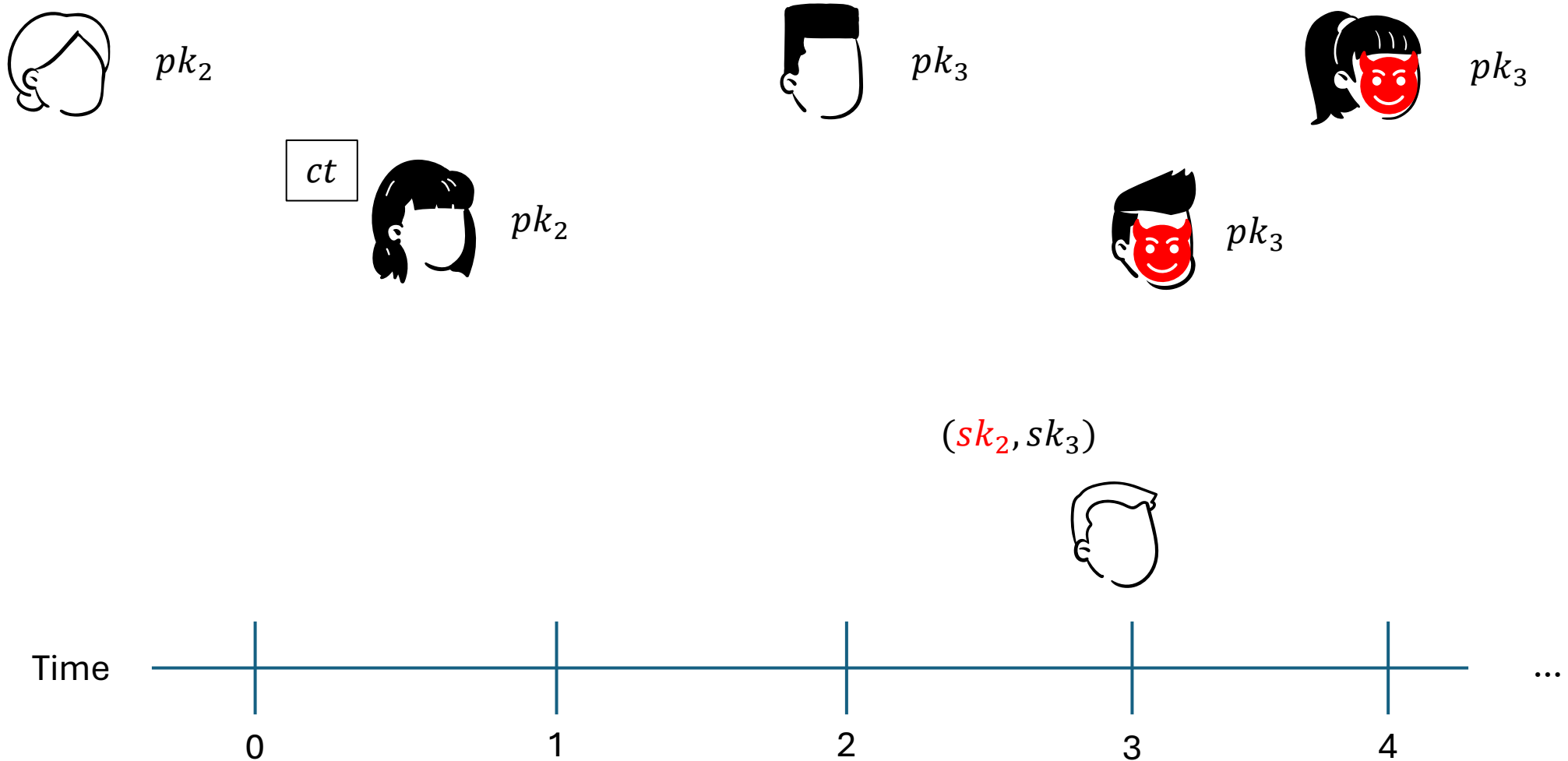
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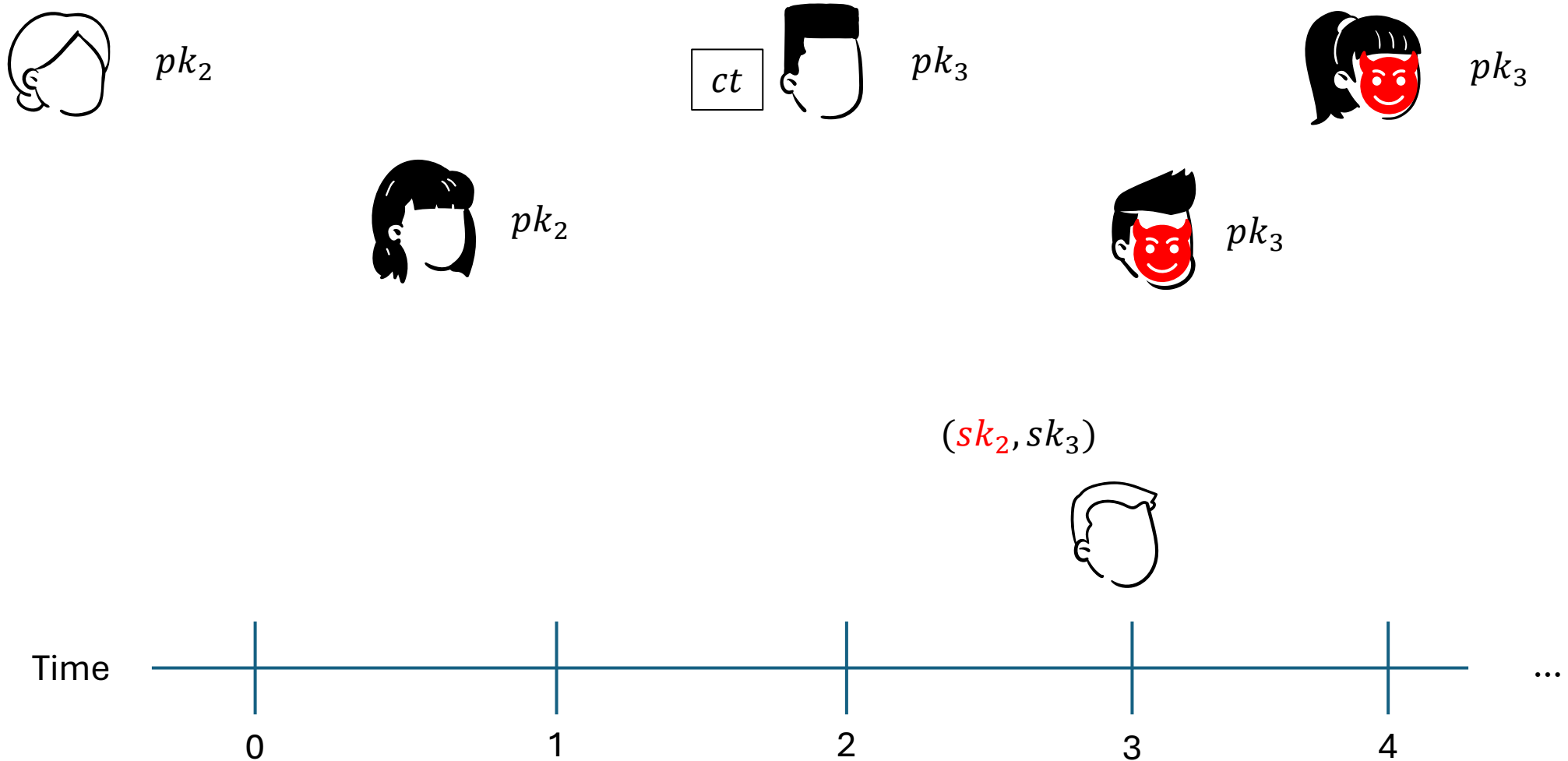
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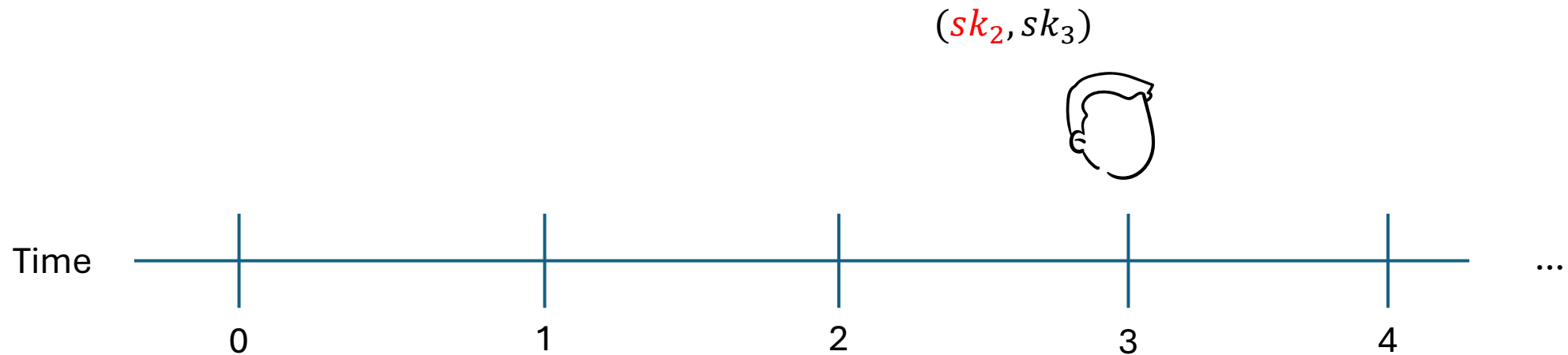
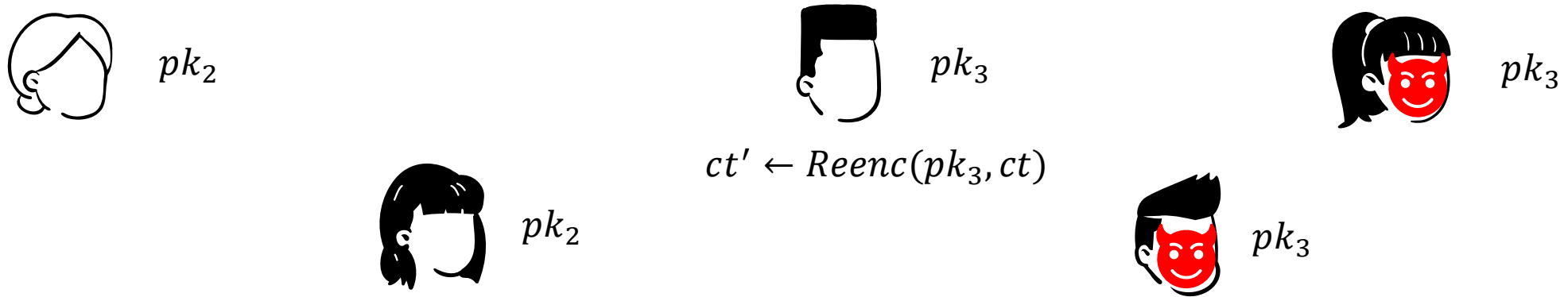


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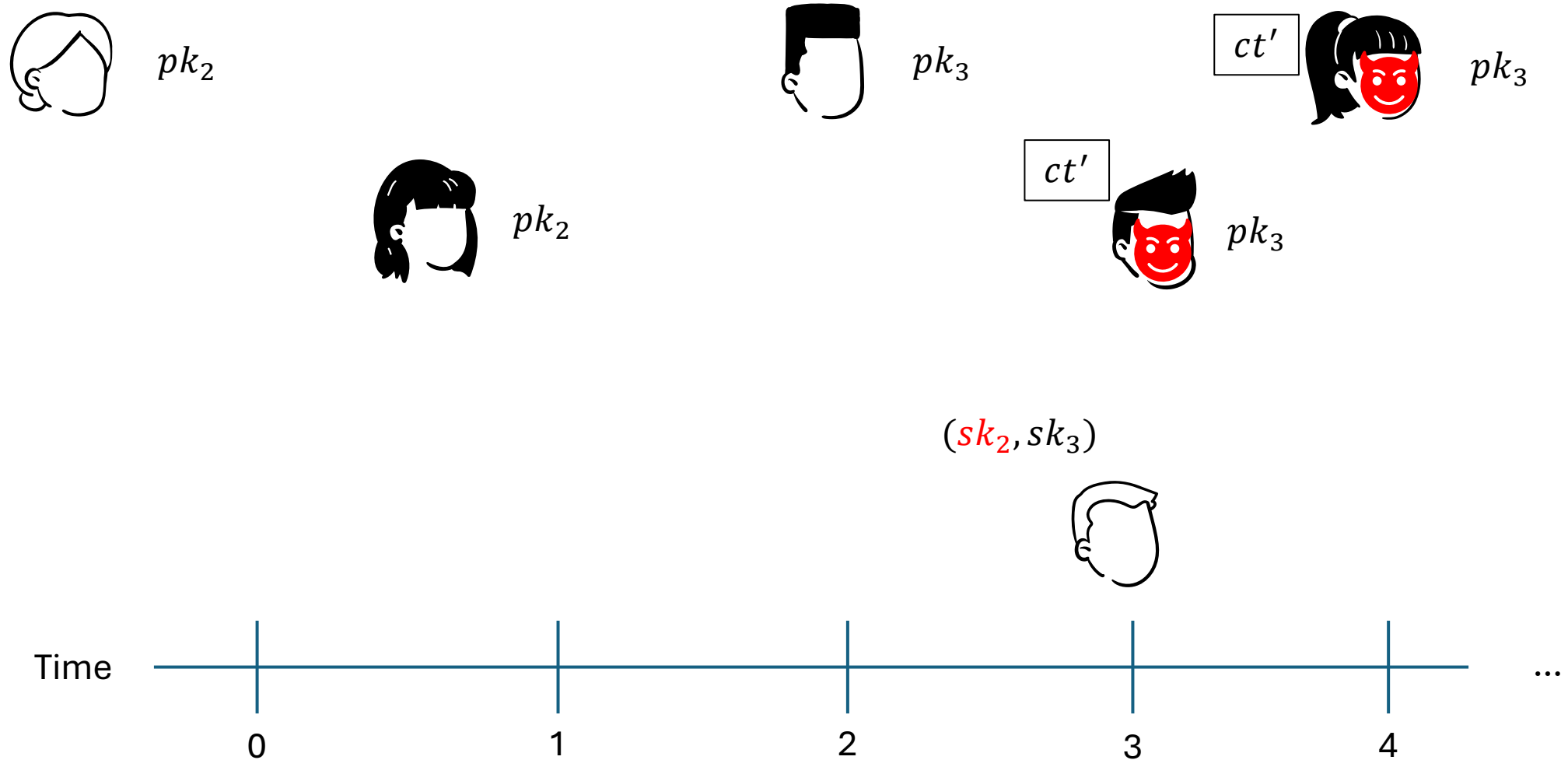




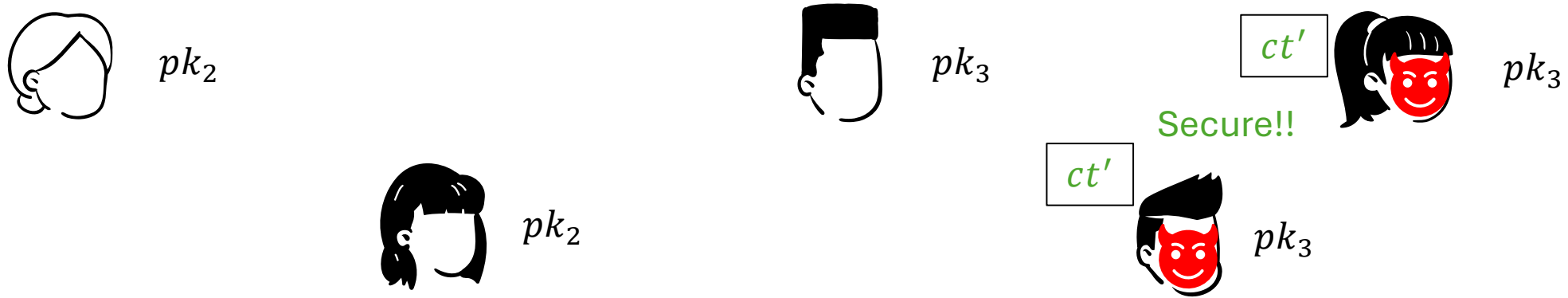
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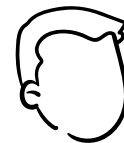
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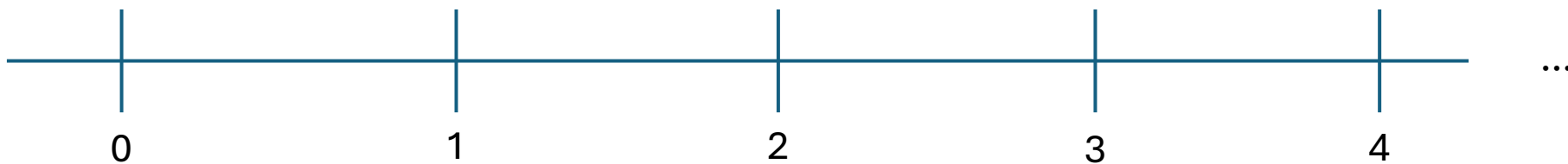
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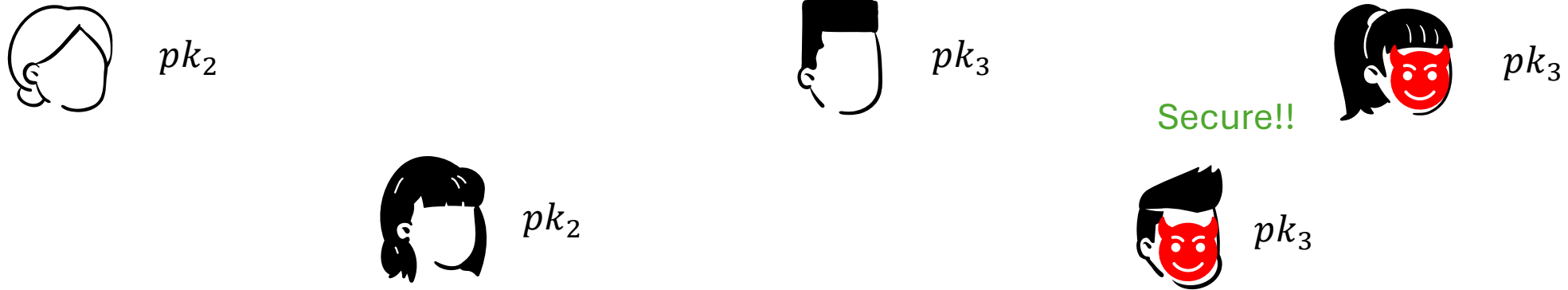
$(sk_2, sk_3)$



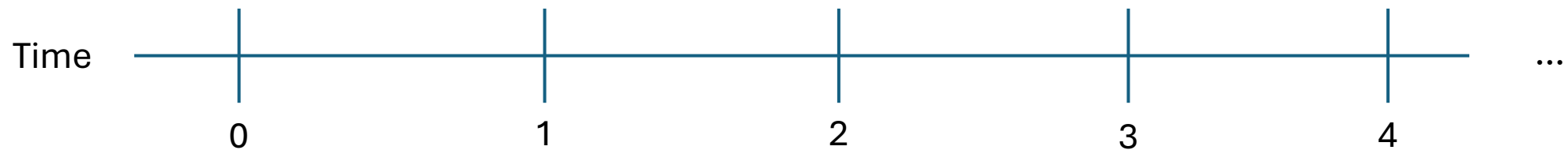
Time



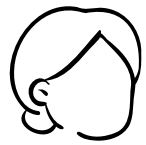
# Stronger Security for Mesh Networks [BRT23]



$(sk_2, sk_3)$   $ct'$



# Stronger Security for Mesh Networks [BRT23]



$pk_2$



$pk_3$



$pk_3$



$pk_2$



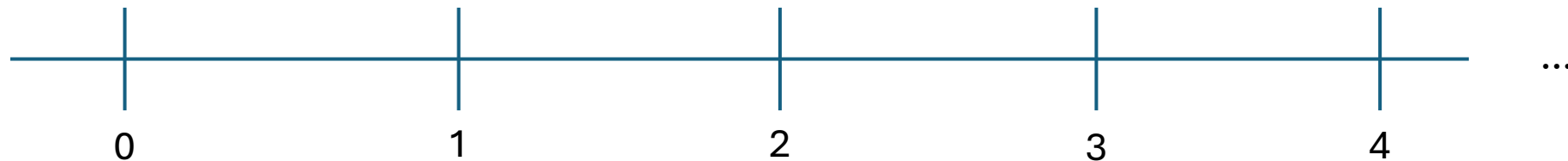
$pk_3$

Secure!!

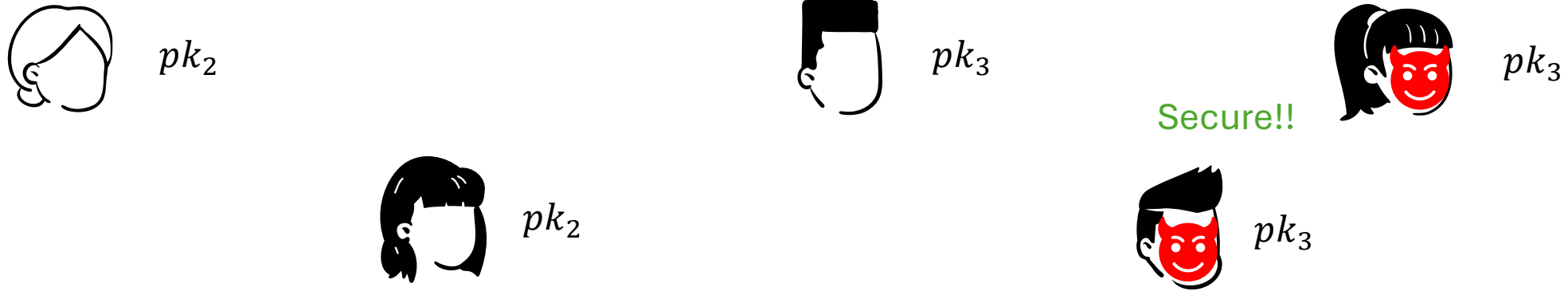
$(sk_2, sk_3) \quad k \leftarrow Dec((sk_2, sk_3), ct)$



Time

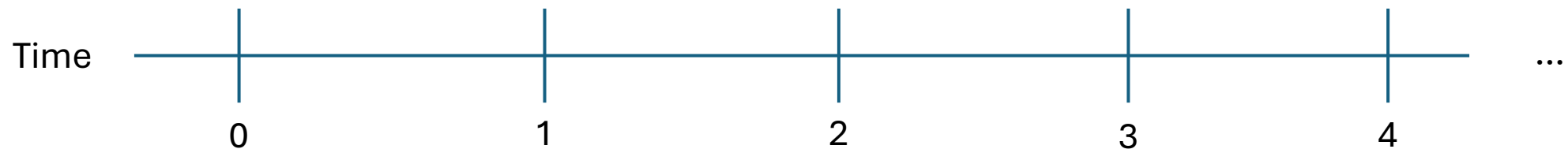


# Stronger Security for Mesh Networks [BRT23]



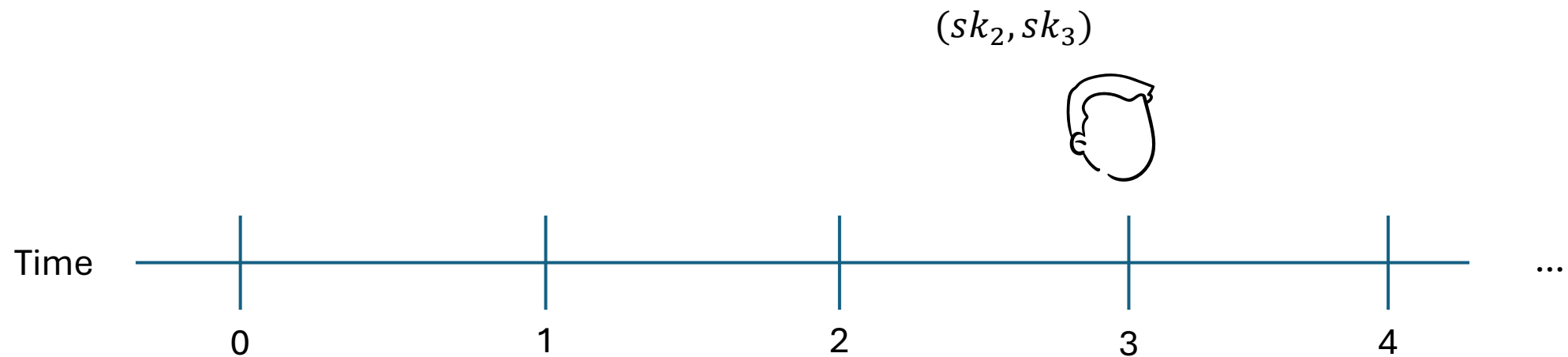
We call this extended notion **IKEMR**

$$(sk_2, sk_3) \quad k \leftarrow Dec((sk_2, sk_3), ct)$$



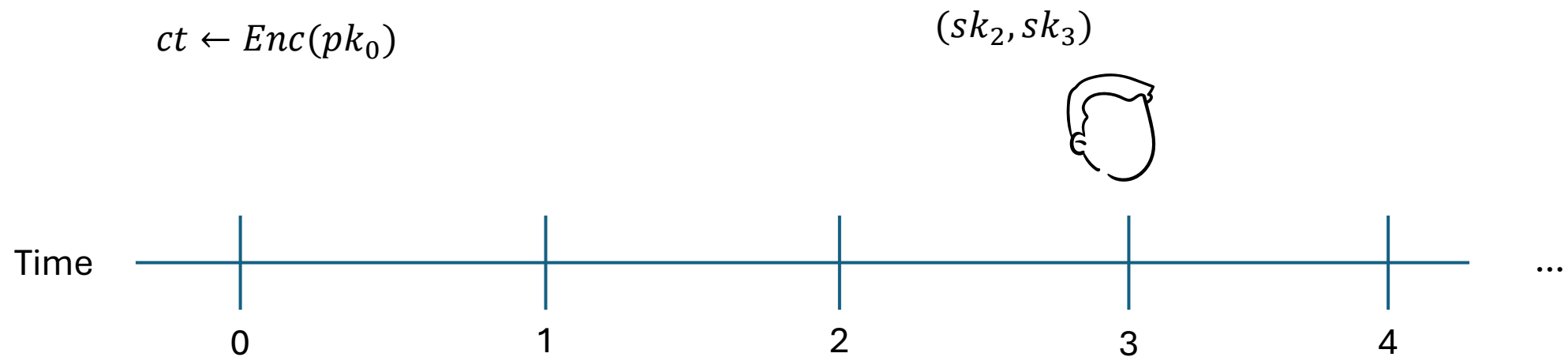
# Security & Correctness for IKEMR

- Security & correctness wrt. time of **first** encapsulation



# Security & Correctness for IKEMR

- Security & correctness wrt. time of **first** encapsulation





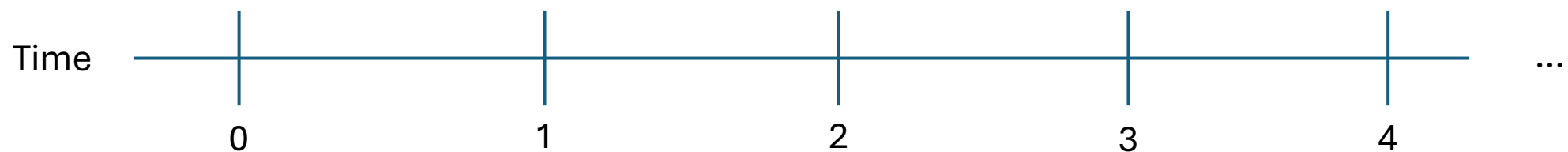
# Security & Correctness for IKEMR

- Security & correctness wrt. time of **first** encapsulation

Cannot be re-encapsulated for Bob!

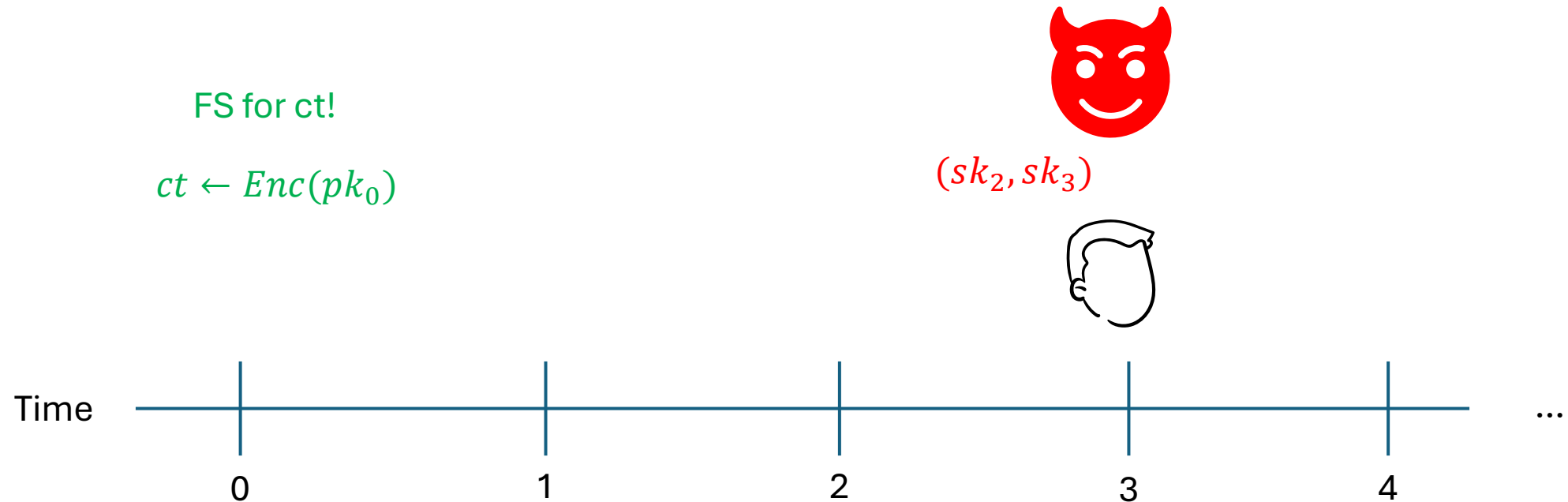
$ct \leftarrow Enc(pk_0)$

$(sk_2, sk_3)$



# Security & Correctness for IKEMR

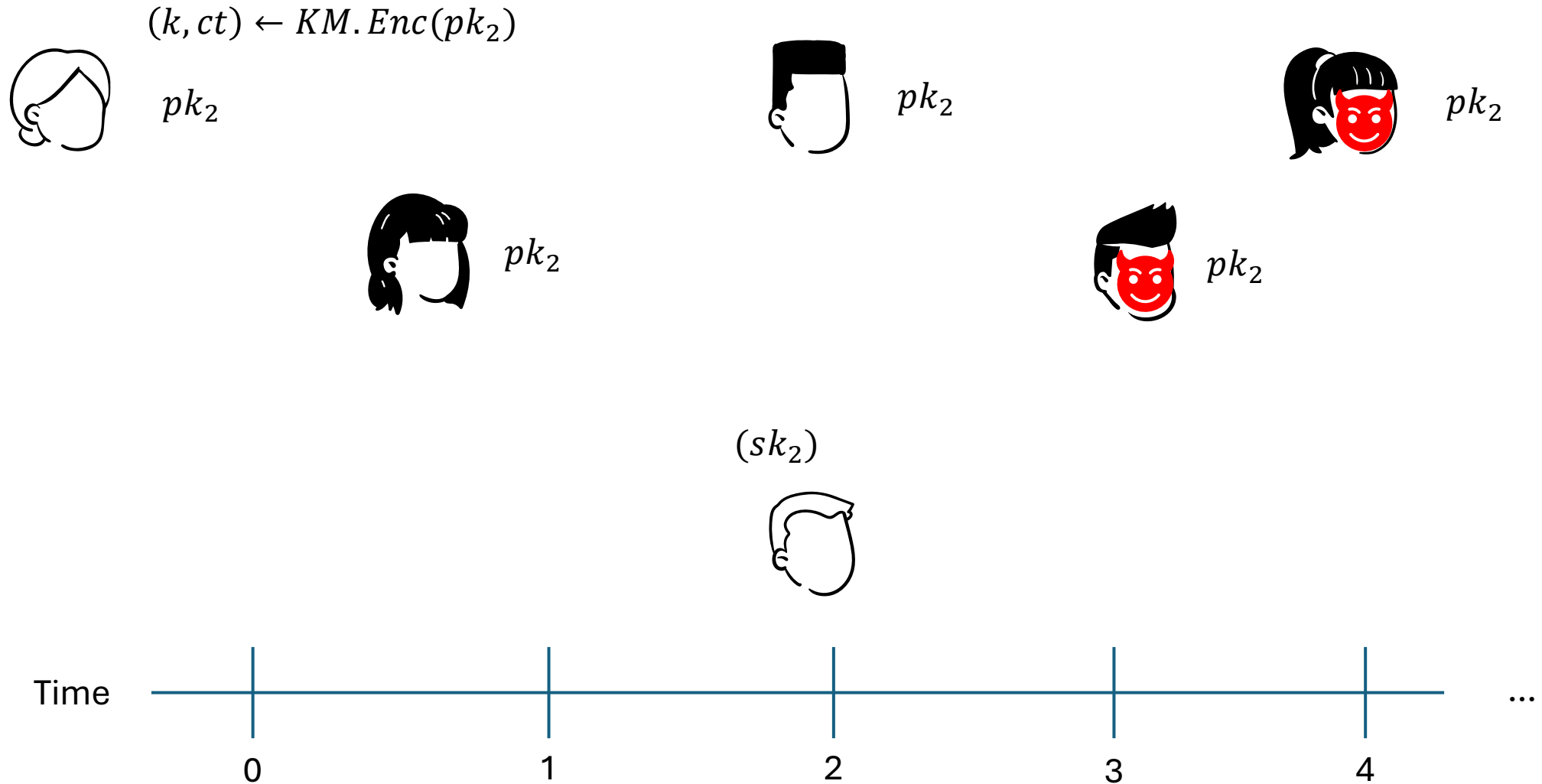
- Security & correctness wrt. time of **first** encapsulation



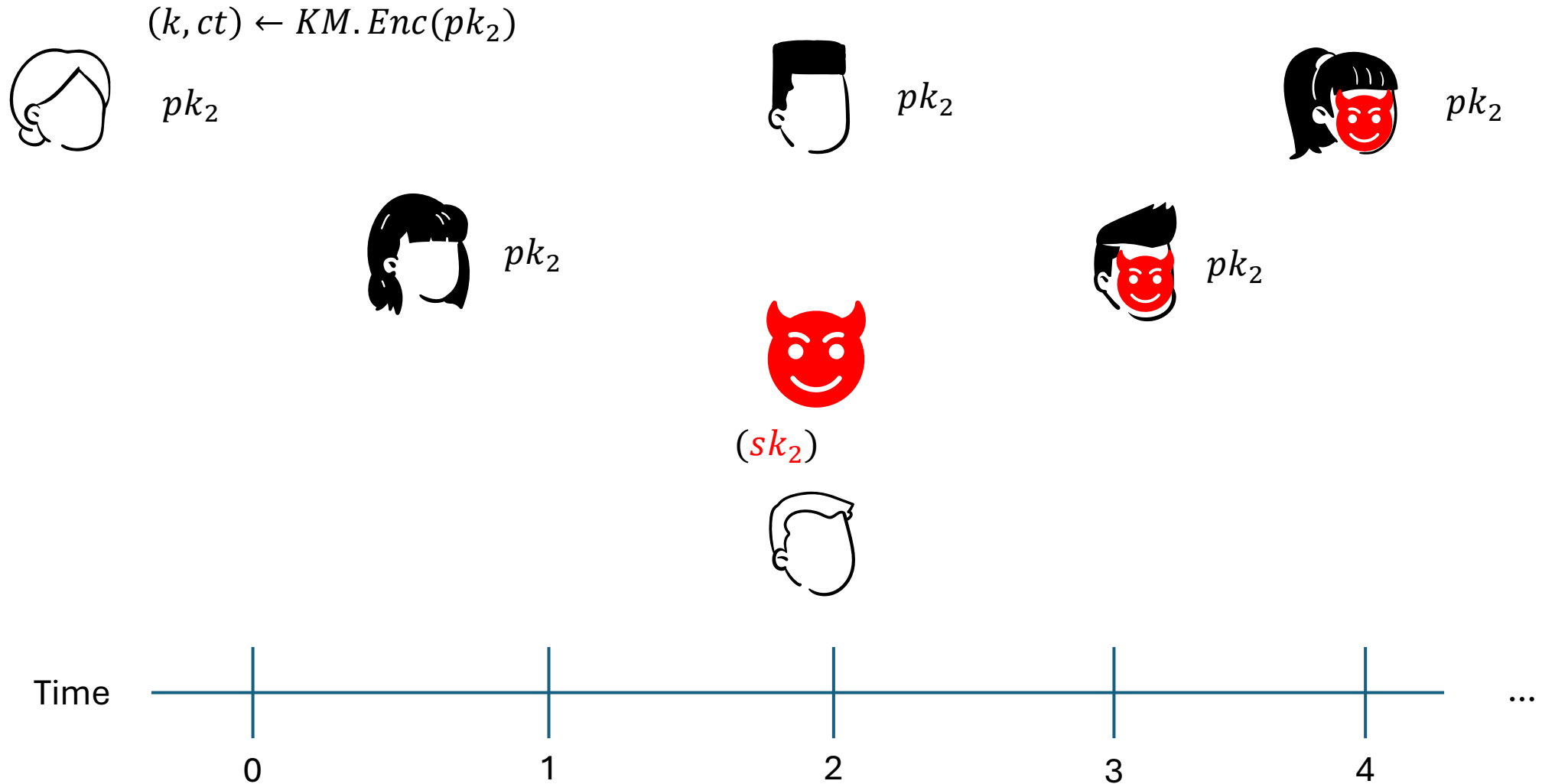
# Comparison to Proxy Re-Encryption (PRE)

- PRE ciphertexts encrypted under  $pk_A$  can be re-encrypted to  $pk_B$
- Key Derivation
  - PRE: *Re-encryption* key  $rk_{A \rightarrow B}$  from  $sk_A, pk_B$
  - IKEMR:  $(sk_B, pk_B)$  from  $sk_A$
- Path
  - PRE: Cycles of re-encryption
  - IKEMR: Only to future
- Assumptions
  - Unidirectional PRE with  $O(1)$  size ct's from  $\geq$  FHE
  - IKEMR with  $O(1)$  size ct's from TDPs! (see next)
- PRE seemingly useless for IKEMR
  - Continuous re-enc to future  $\Rightarrow$  **no FS!**

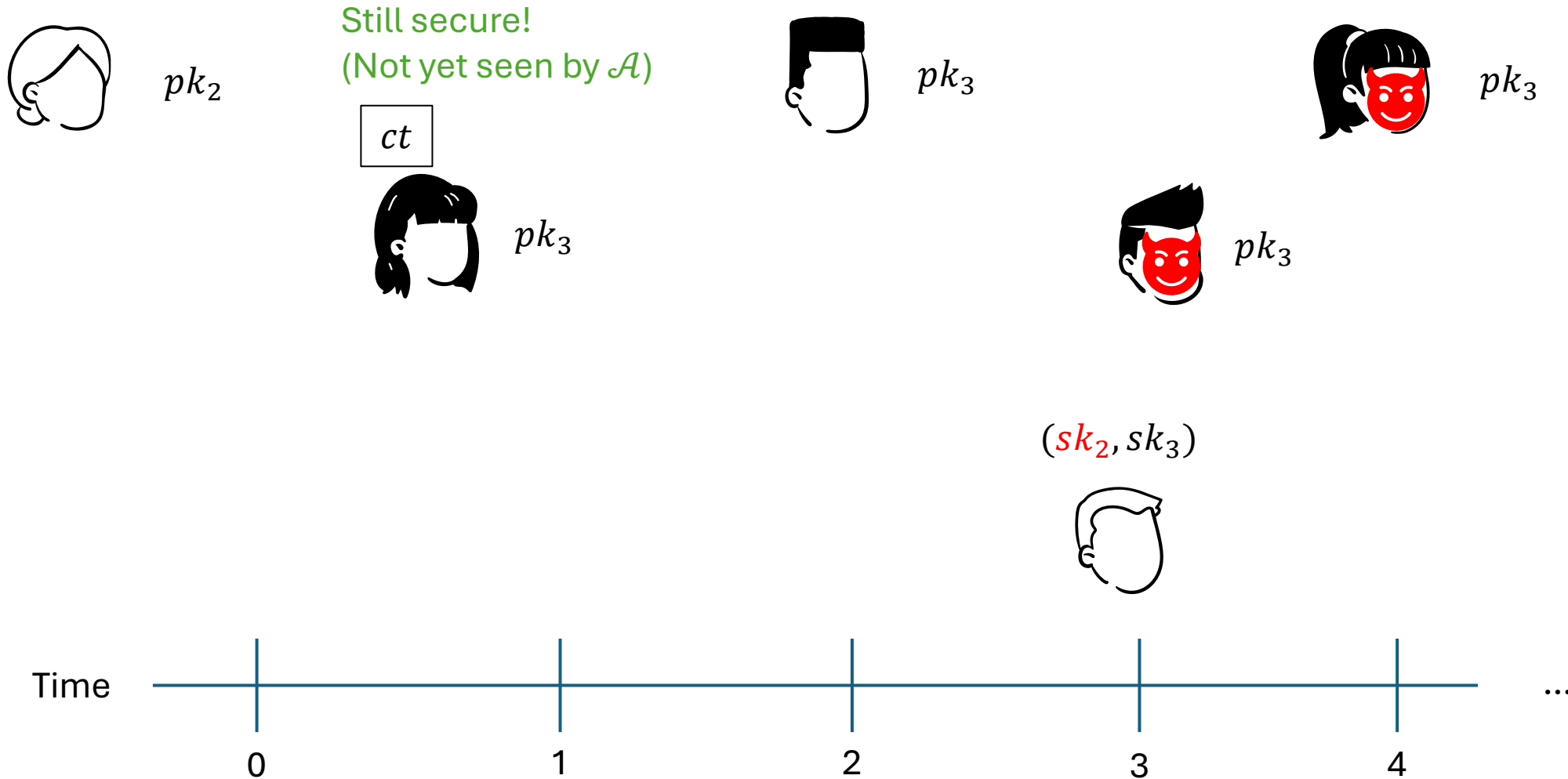
# Simple Extension of Basic IKEM Fails!



# Simple Extension of Basic IKEM Fails!



# Simple Extension of Basic IKEM Fails!



# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$



$pk_3$

Compute  $(k', ct'_1) \leftarrow KM.Enc(pk_3)$   
and  $ct'_2 \leftarrow k' \oplus ct$

$(sk_2, sk_3)$



Time

0

1

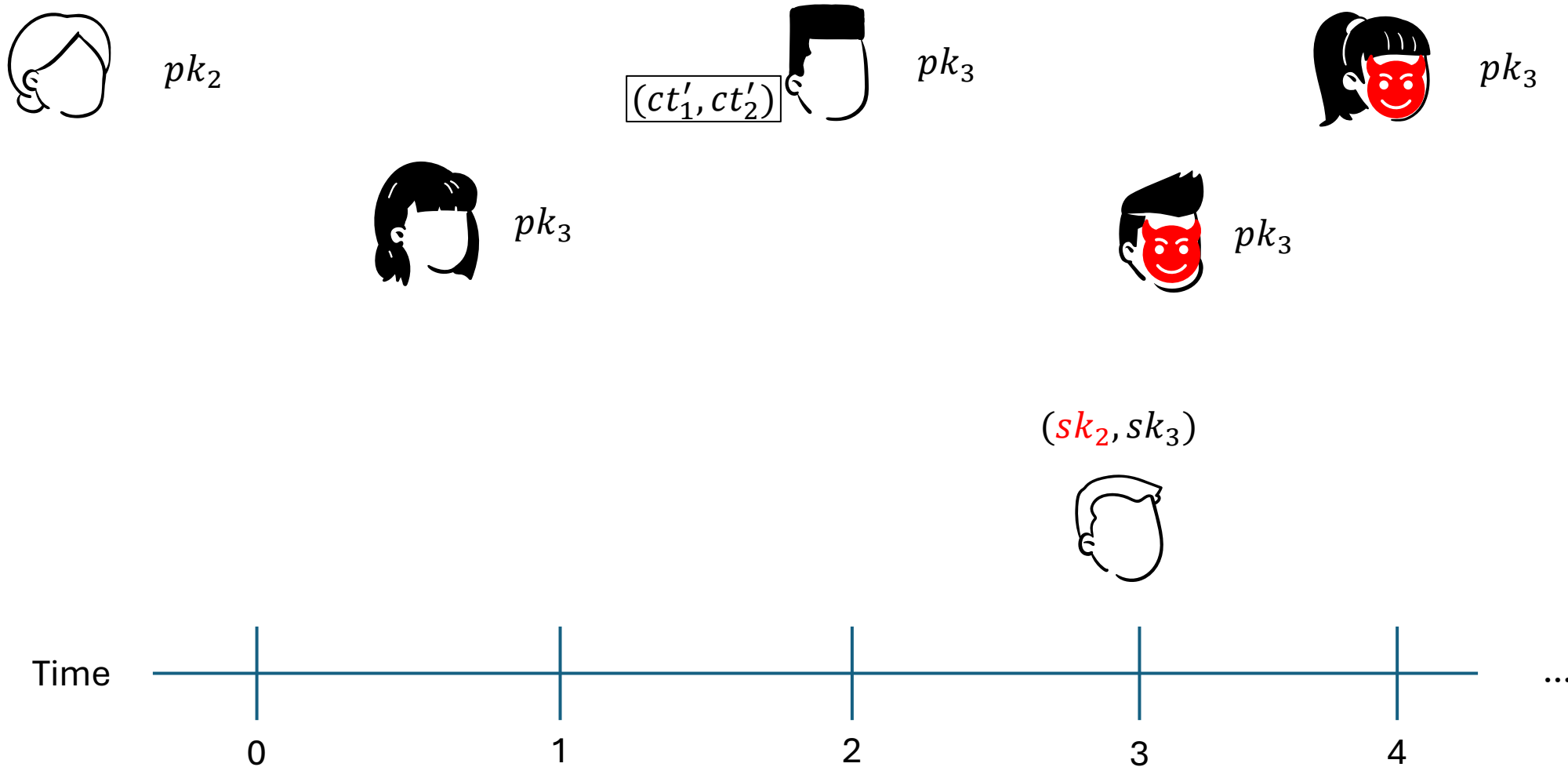
2

3

4

...

# Simple Extension of Basic IKEM Fails!





# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$

$(ct'_1, ct'_2)$



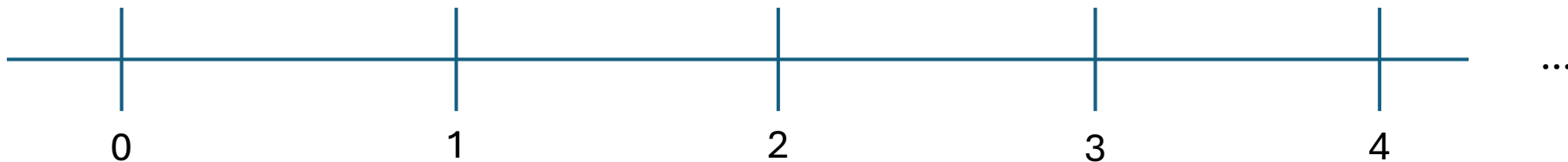
$pk_3$

Still secure!  
(re-enc using  $pk_3$ )

$(sk_2, sk_3)$



Time



# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



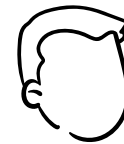
$pk_3$

$(ct'_1, ct'_2)$

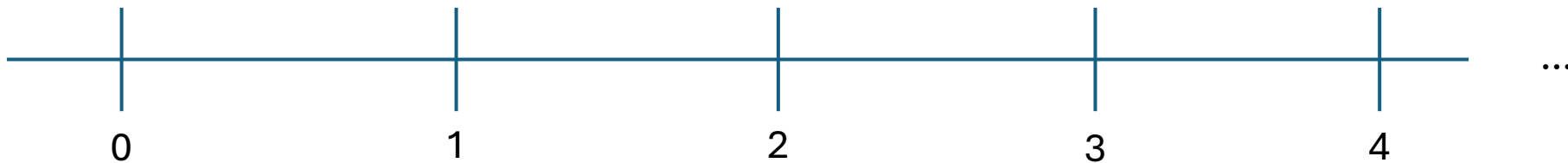


$pk_3$

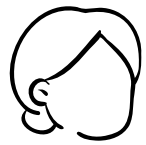
$(sk_3)$



Time



# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$

$(ct'_1, ct'_2)$



$pk_3$



$(sk_3)$



Time

0

1

2

3

4

...

# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$

$(ct'_1, ct'_2)$



$pk_3$

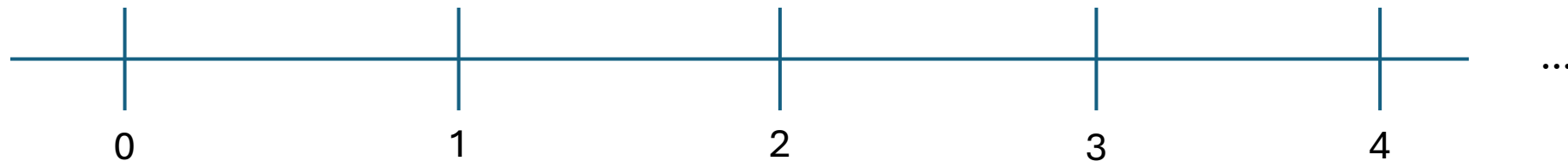
Want this to still be secure!  
(Interval shortened to  
exclude  $t = 2$ )



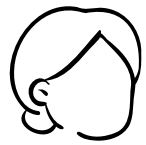
$(sk_3)$



Time



# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$

$(ct'_1, ct'_2)$



$pk_3$

Want this to still be secure!  
(Interval shortened to  
exclude  $t = 2$ )

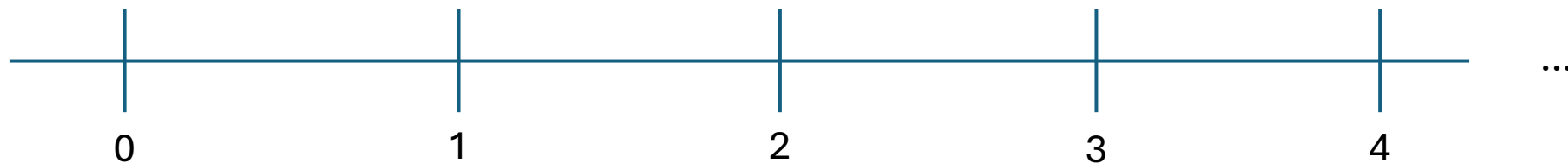


$(sk_3)$

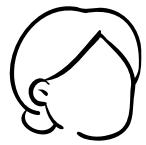
But both  $sk_2$  and  $sk_3$   
leaked! => insecure!



Time



# Simple Extension of Basic IKEM Fails!



$pk_2$



$pk_3$



$pk_3$



$pk_3$

$(ct'_1, ct'_2)$



$pk_3$

Want this to still be secure!  
(Interval shortened to  
exclude  $t = 2$ )

But both  $sk_2$  and  $sk_3$   
leaked! => insecure!

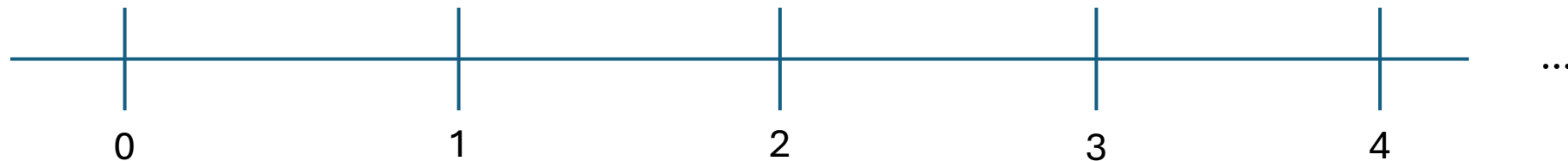


$(sk_3)$

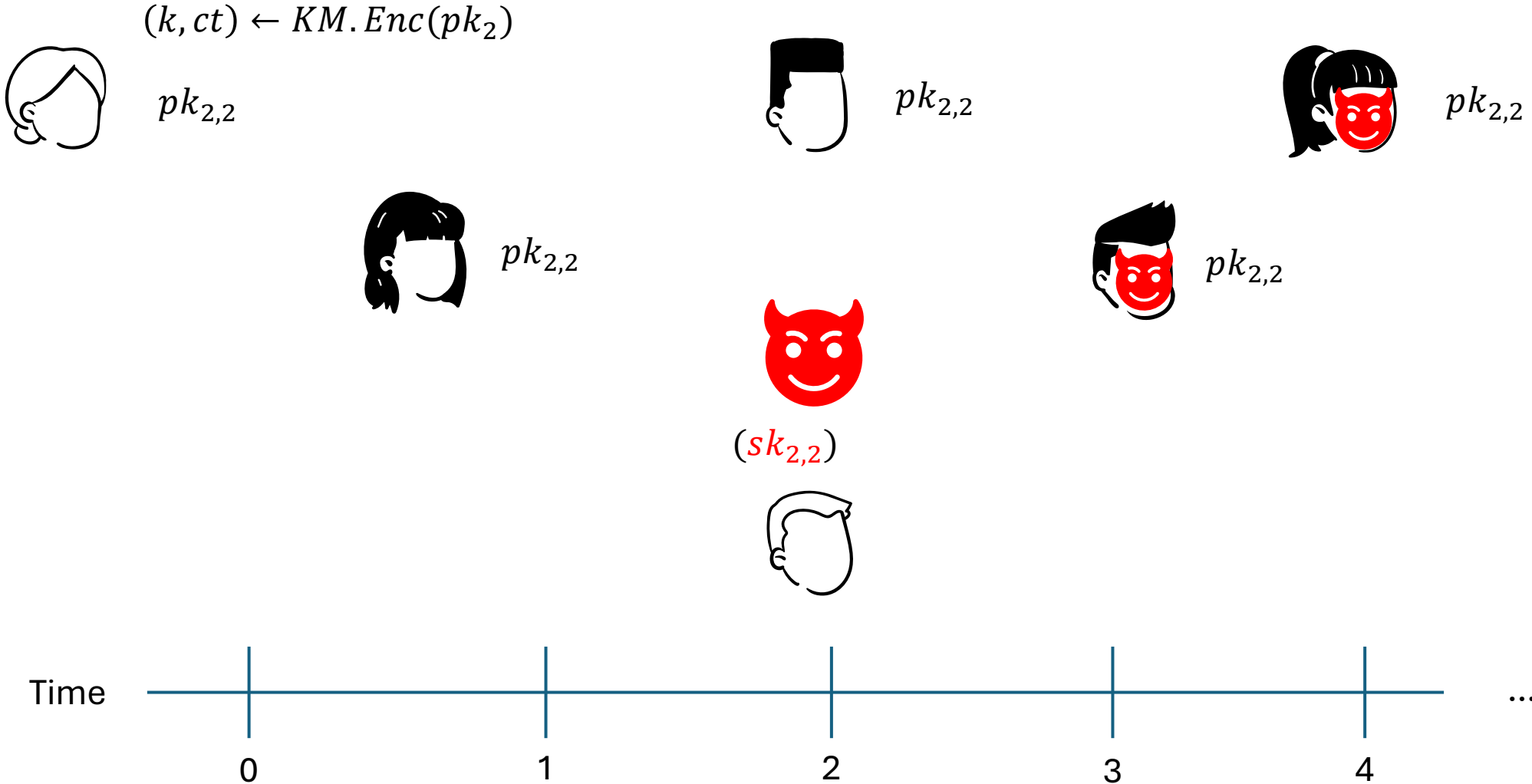


Note: can't just  
delete  $sk_3$ !

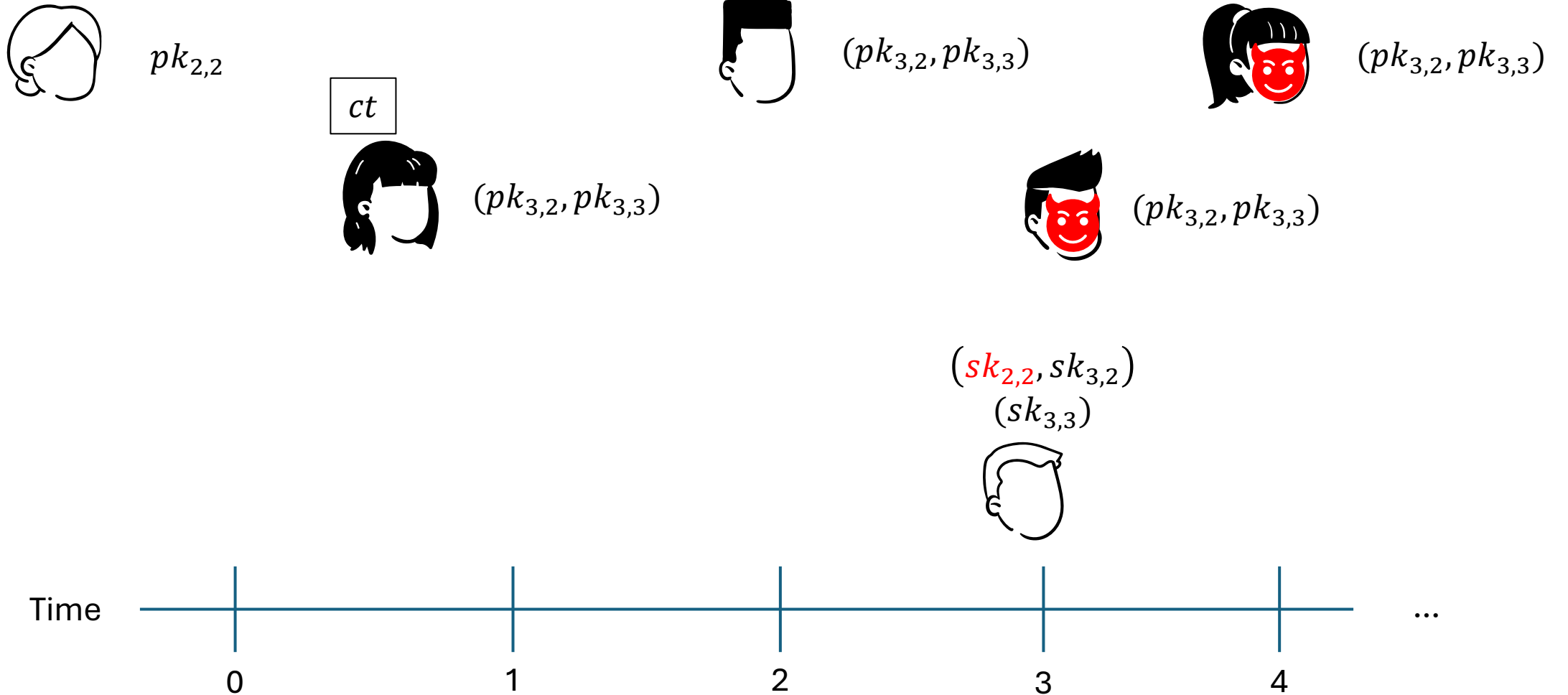
Time



# The Fix



# The Fix





# The Fix



$pk_{2,2}$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



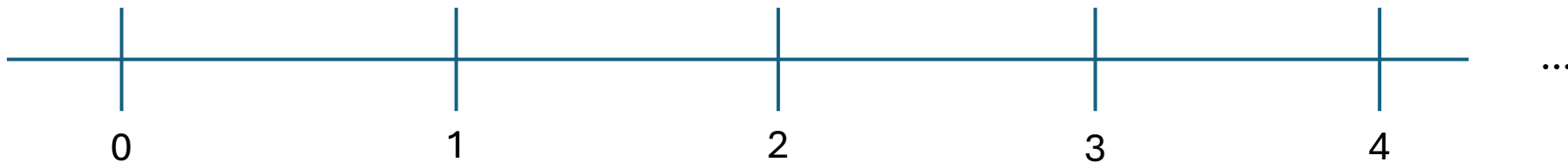
$(pk_{3,2}, pk_{3,3})$

Compute  $(k', ct'_1) \leftarrow KM.Enc(pk_{3,2})$   
and  $ct'_2 \leftarrow k' \oplus ct$

$(sk_{2,2}, sk_{3,2})$   
 $(sk_{3,3})$



Time



# The Fix



$pk_{2,2}$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$

$(ct'_1, ct'_2)$



$(pk_{3,2}, pk_{3,3})$

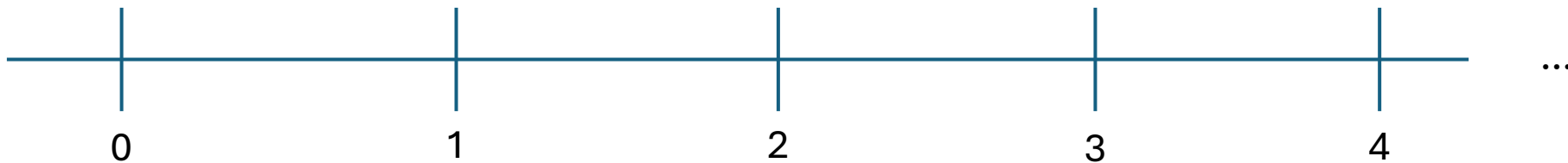
Still secure!  
(re-enc using  $pk_{3,2}$ )

$(sk_{2,2}, sk_{3,2})$

$(sk_{3,3})$



Time



# The Fix



$pk_{2,2}$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$

$(ct'_1, ct'_2)$

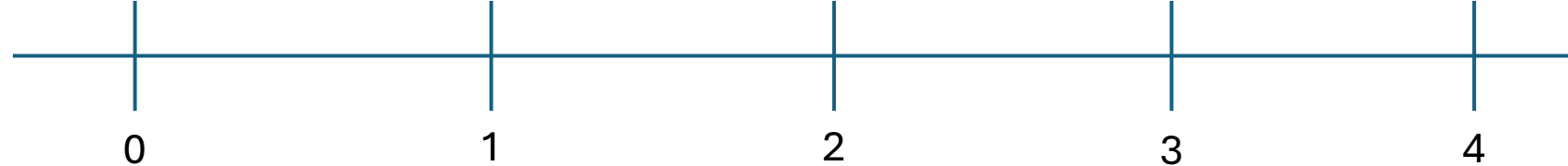


$(pk_{3,2}, pk_{3,3})$

$(sk_{3,3})$



Time



# The Fix



$pk_{2,2}$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$

$(ct'_1, ct'_2)$



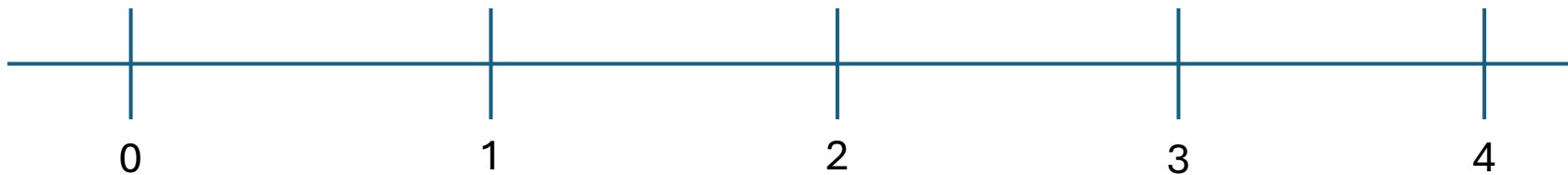
$(pk_{3,2}, pk_{3,3})$



$(sk_{3,3})$

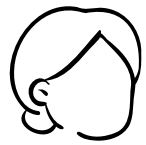


Time



...

# The Fix



$pk_{2,2}$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$



$(pk_{3,2}, pk_{3,3})$

$(ct'_1, ct'_2)$



$(pk_{3,2}, pk_{3,3})$

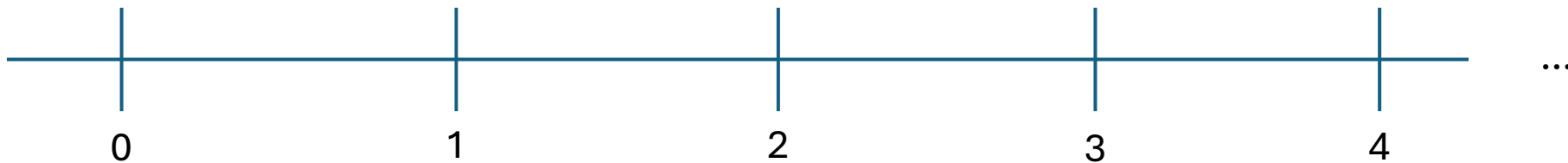
Still secure!  $sk_{3,2}$  gone!



$(sk_{3,3})$



Time



# The Fix – Efficiency

- Secret state in  $O(\ell)$  = “decryption interval”
  - PRG chain: start at seed  $s_{t,t-\ell}$  and derive  $(sk_{t,t-\ell}, \dots, sk_{t,t})$  iteratively
  - To remove time  $t - \ell$ : eval  $\text{PRG}(s_{t,t-\ell}) \rightarrow (s_{t,t-\ell+1}, sk_{t,t-\ell+t})$
- Ciphertext grows with # of re-encs
- Public key at each time  $t$  in  $O(\ell)$

# IKEMR with Small Ciphertexts

- Trapdoor Permutation (TDP) instead of KEM
  - (Cryptographic) ct stays the same size after each re-enc
  - Decryptor: which keys to use to decrypt?
    - ct only grows by  $+\log \lambda$  (time label) with each re-enc
    - (Instead of  $+\lambda$  factor)
- ct always  $O(1)$ ;  $pk_t$  grows to  $O(\ell^2)$

# Additional Properties

- Alternatively make each  $pk_t$   $O(1)$  size
  - Uses FS-PKE
  - Cts grow additive  $\lambda$  factor with each re-enc
  - Sk size now  $\ell \log \ell$
- Replayable CCA Security! (Like PRE; hard to define CCA-Security)
  - Some subtleties for definition; constructions use standard techniques



# Thanks!

<https://ia.cr/2024/1454>