

# Deletions and Dishonesty

## PDS in Adversarial Settings

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**Joint work with**

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# Probabilistic Data Structures (PDS)

A way to

**compactly represent**  
**(tons of) data**

and

provide approximate  
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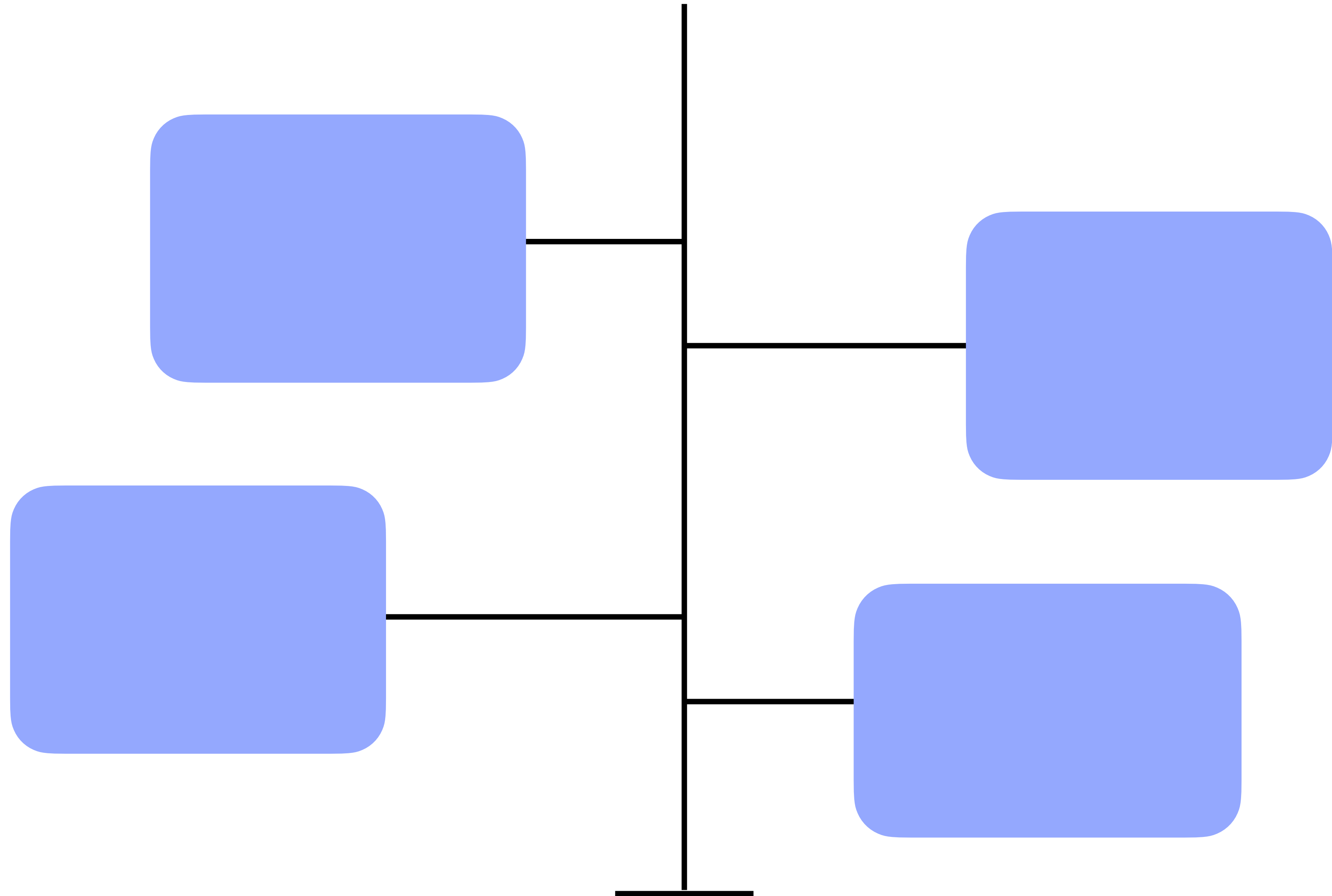
- Frequency estimation  
How many times does  $x$  appear in the data?  
**Count-min sketch, HeavyKeeper**
- Membership queries  
Is  $x$  in the set?  
**Bloom filter, Cuckoo filter, Counting filter**

# Probabilistic Data Structures (PDS)

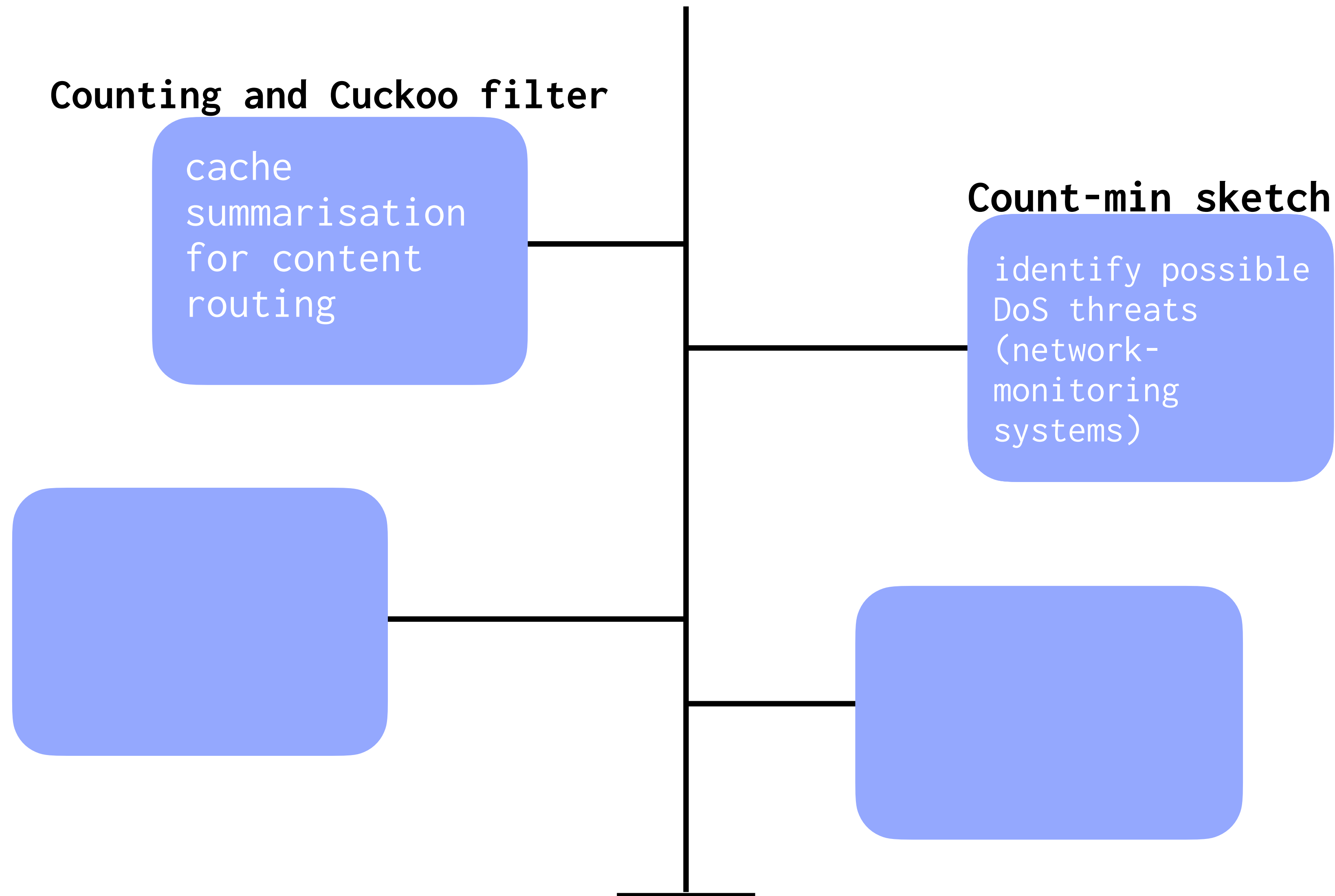
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- Membership queries  
Is  $x$  in the set?  
**Bloom filter, Cuckoo filter, Counting filter**
- Cardinality estimation  
How many distinct elements are in the set?  
**HyperLogLog, KMV estimator**

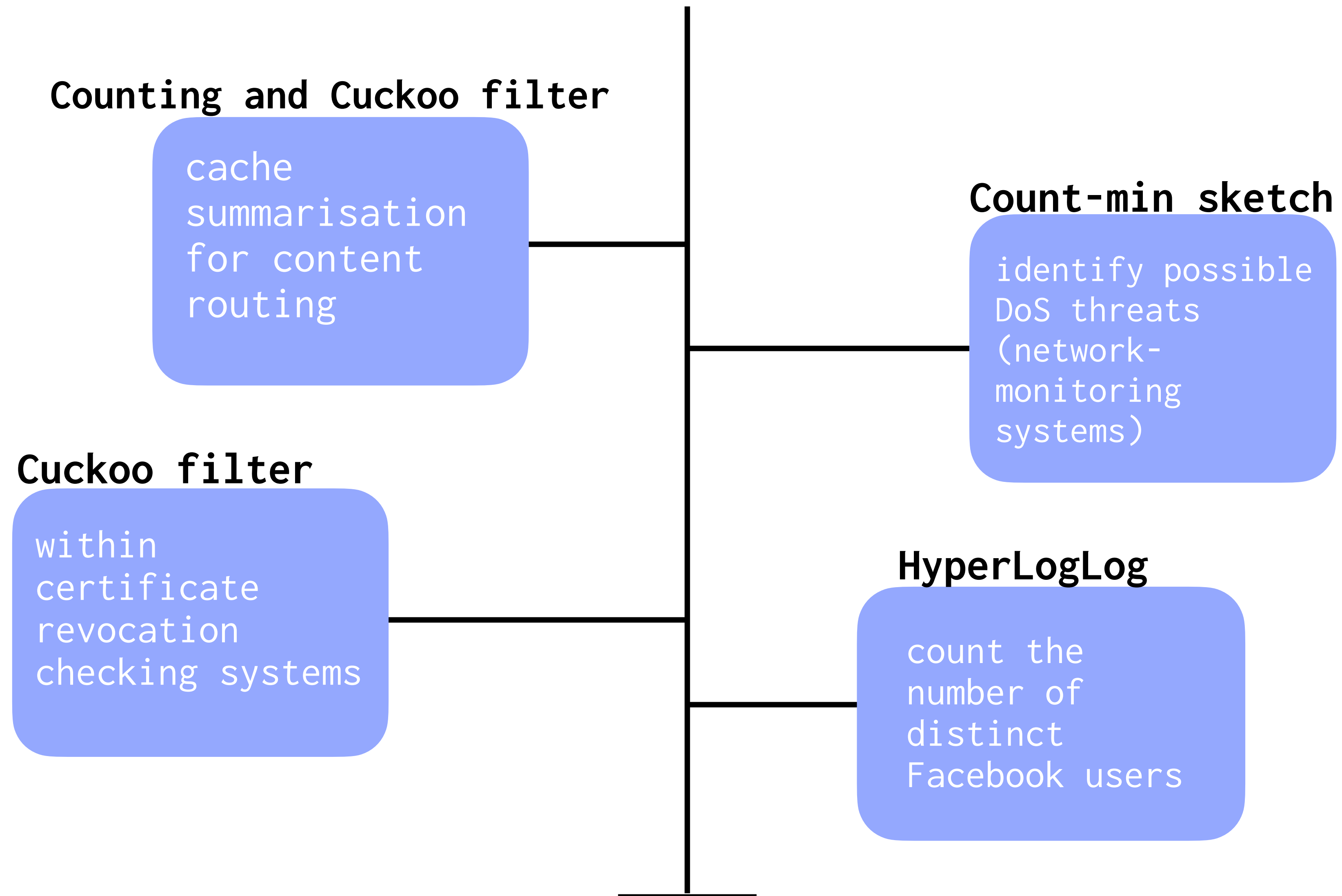
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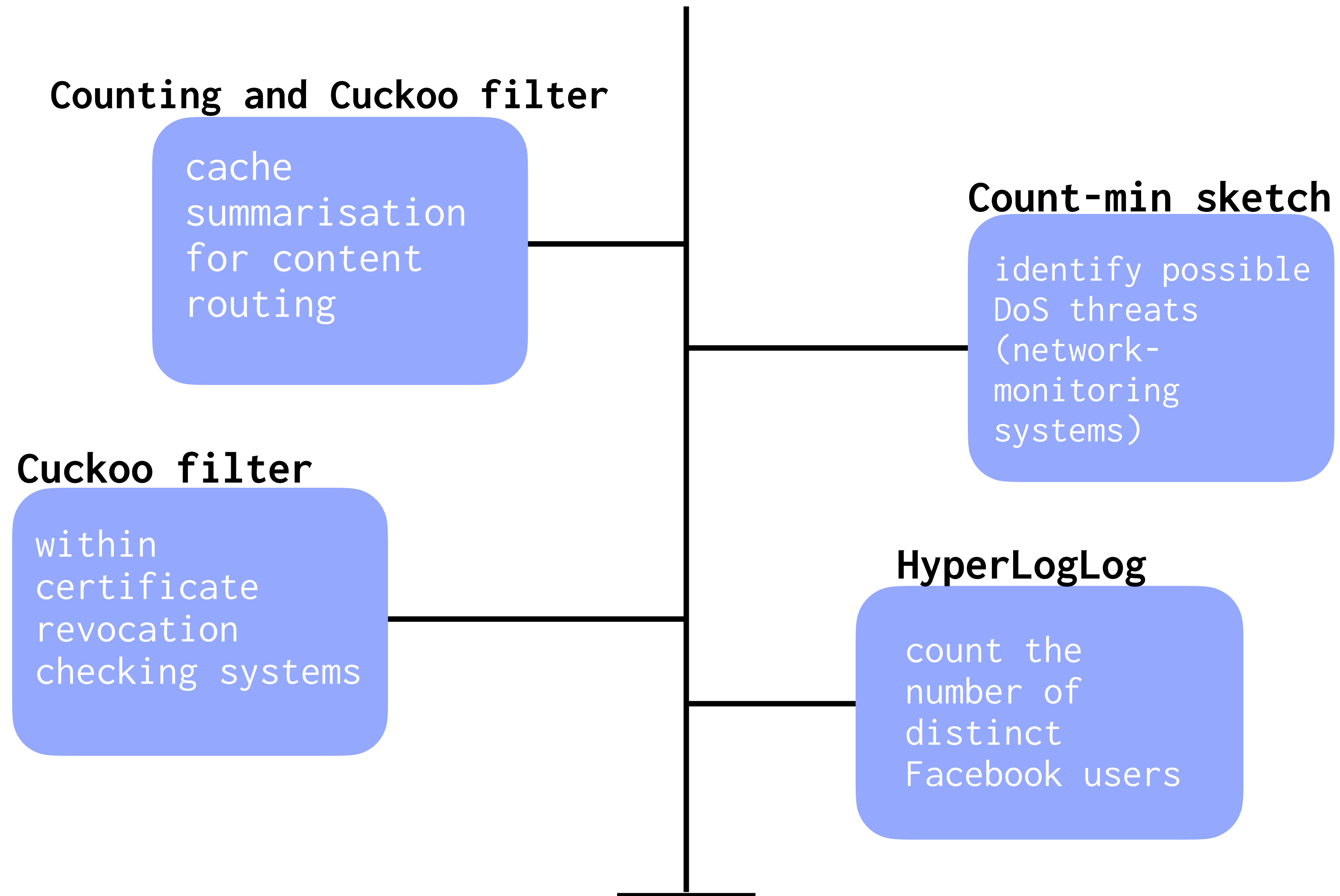


# Where are PDS used?





# What can go wrong?



# PDS in adversarial settings

Adversarial  
correctness

- Adversary can **interfere** with the correct functionality of the PDS

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Secure PDS

- How can we **provably protect** PDS in adversarial settings?

# This work

Adversarial  
correctness

Privacy

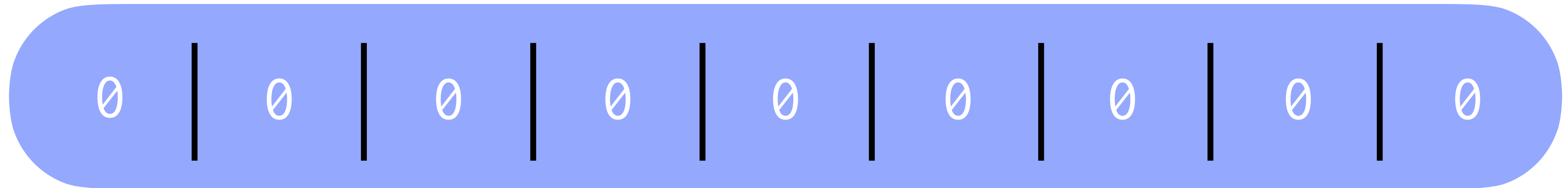
Secure AMQ-PDS  
W/ deletions

- Adversary can interfere with the correct functionality of a class of AMQ-PDS w/ deletions, e.g., Counting and Cuckoo Filters
- Adversary could try to **learn** about the elements represented by the PDS (e.g., [FPUV22])
- How can we provably protect e.g. Counting and Cuckoo Filters in adversarial settings?

# Counting filter

m counters

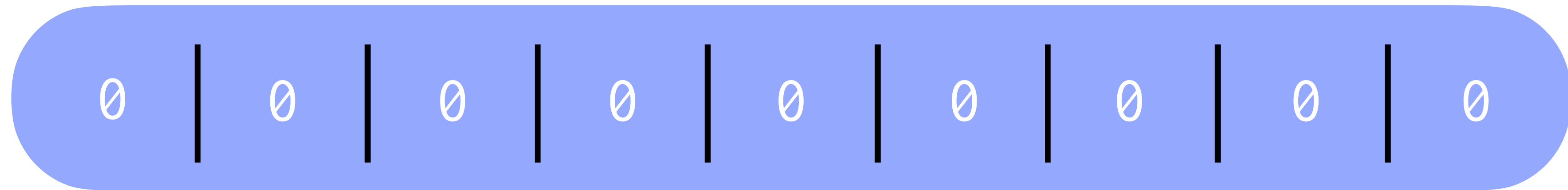
1 row



# Counting filter

m counters

1 row



hash(x) = 2 | 5 | 9 | 1 | 3

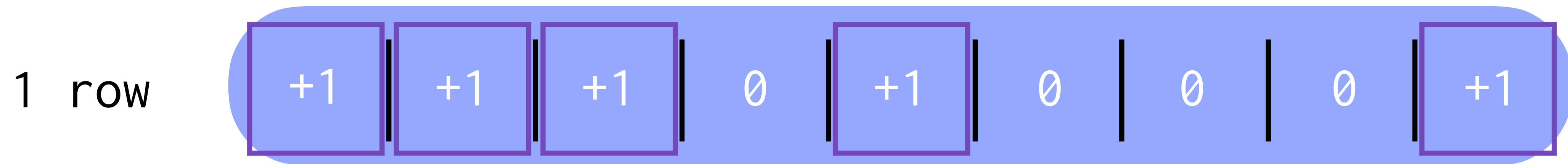


k counters



# Counting filter: insert(x)

m counters



hash(x) = 2 | 5 | 9 | 1 | 3



k counters

# Counting filter: insert(..)

m counters

1 row



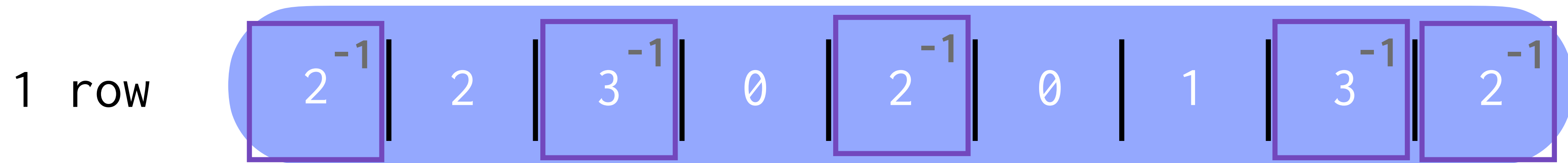
hash(x) = 2 | 5 | 9 | 1 | 3

hash(y) = 8 | 9 | 3 | 1 | 5

hash(z) = 7 | 8 | 3 | 2 | 8

# Counting filter: delete(y)

m counters



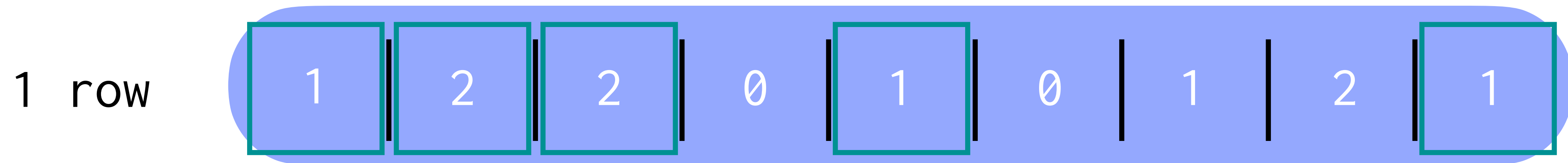
hash(y) = 8 | 9 | 3 | 1 | 5



k counters

# Counting filter: query(x)

m counters



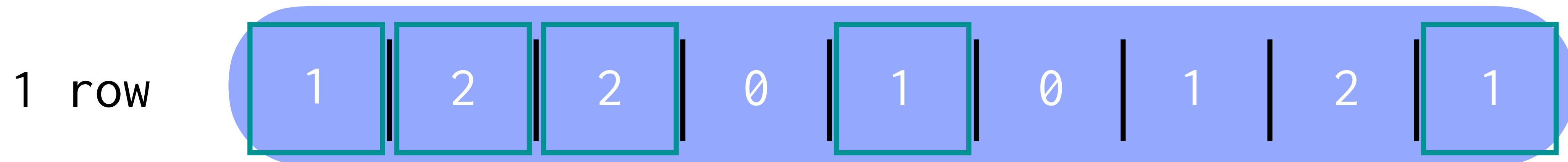
hash(x) = 2 | 5 | 9 | 1 | 3



k counters

# Counting filter: query(x)

m counters

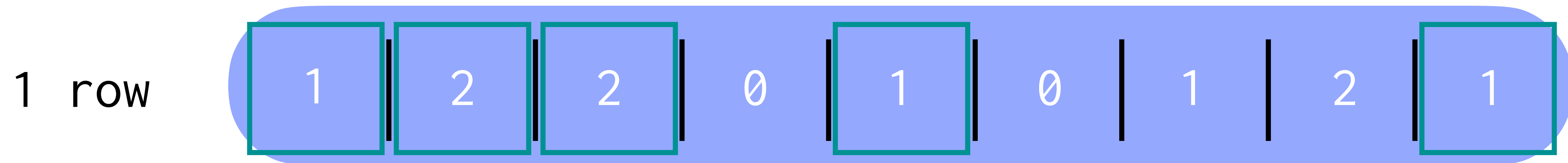


hash(x) = 2 | 5 | 9 | 1 | 3

CF(x) = [all cnt(x) > 0]

# Counting filter: query(x)

m counters



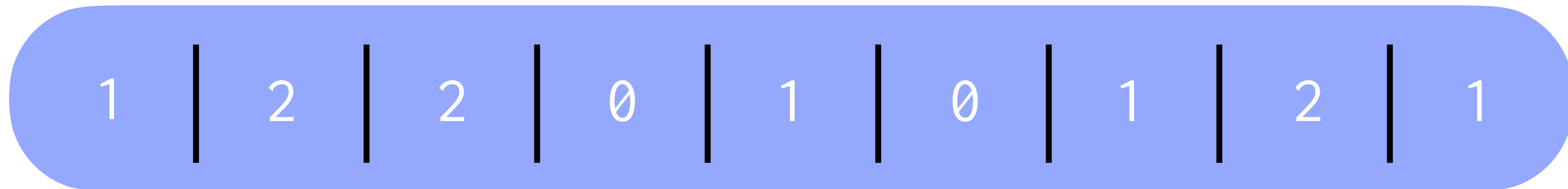
hash(x) = 2 | 5 | 9 | 1 | 3

CF(x) = T

# Counting filter: query(x)

m counters

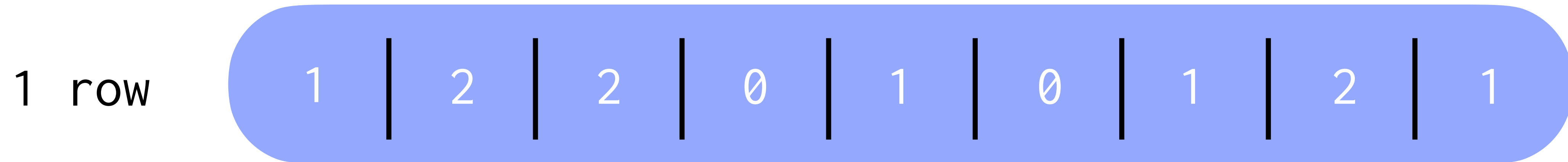
1 row



False positives and negatives ?

# Counting filter: query(x)

m counters



False positives:  $\Pr[\text{FP}] = f(m, k, n)$

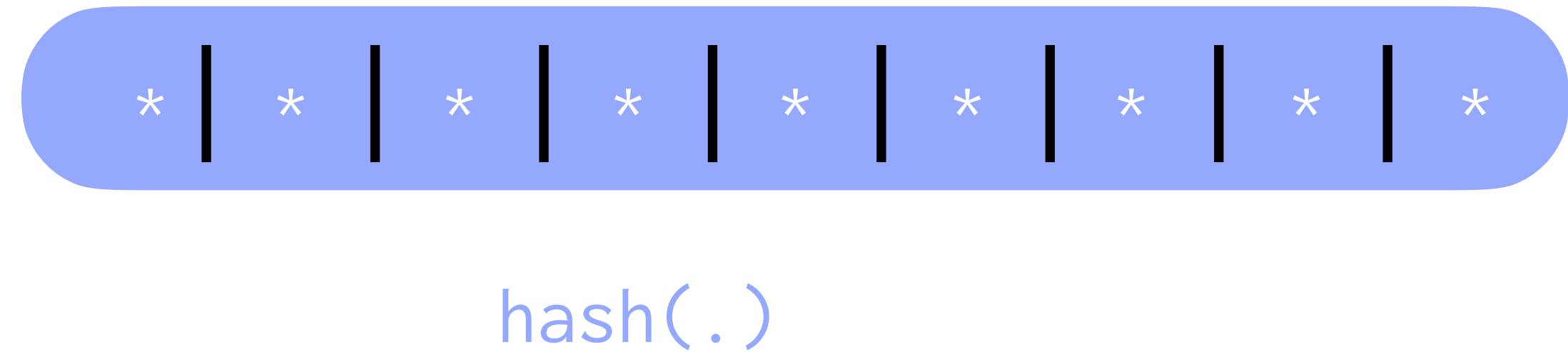
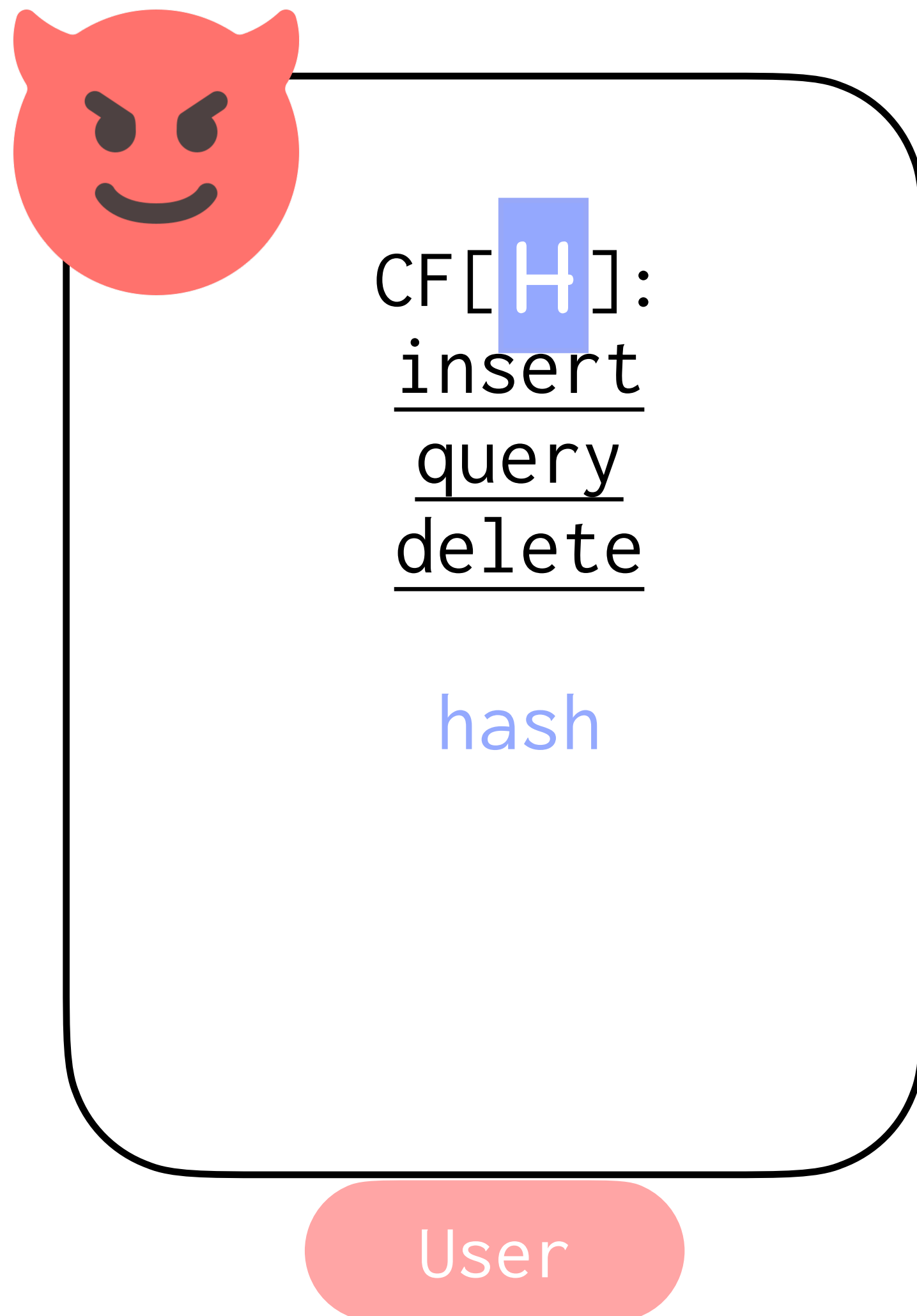
Annotations for the function  $f(m, k, n)$ :

- size of filter (points to  $m$ )
- number of hash functions (points to  $k$ )
- number of elements in filter (points to  $n$ )

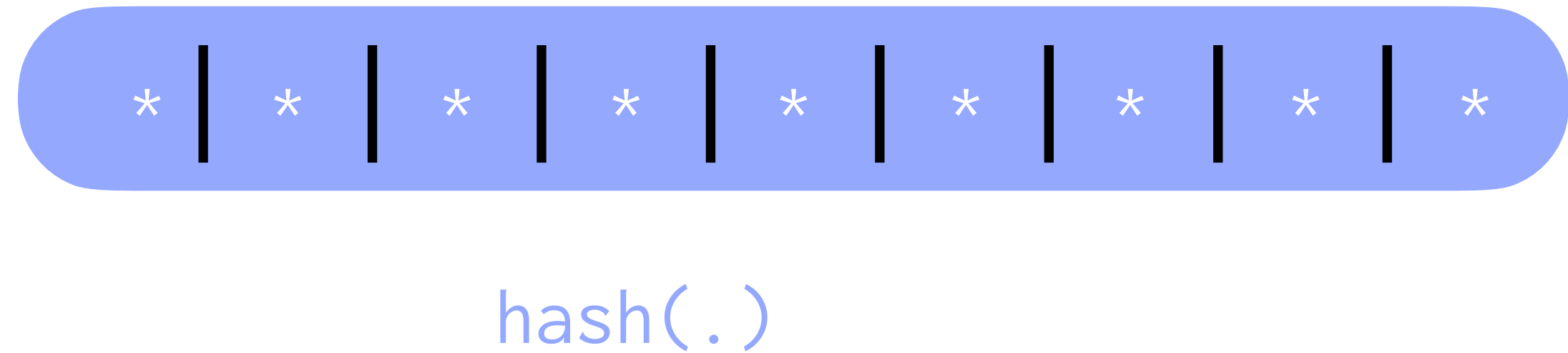
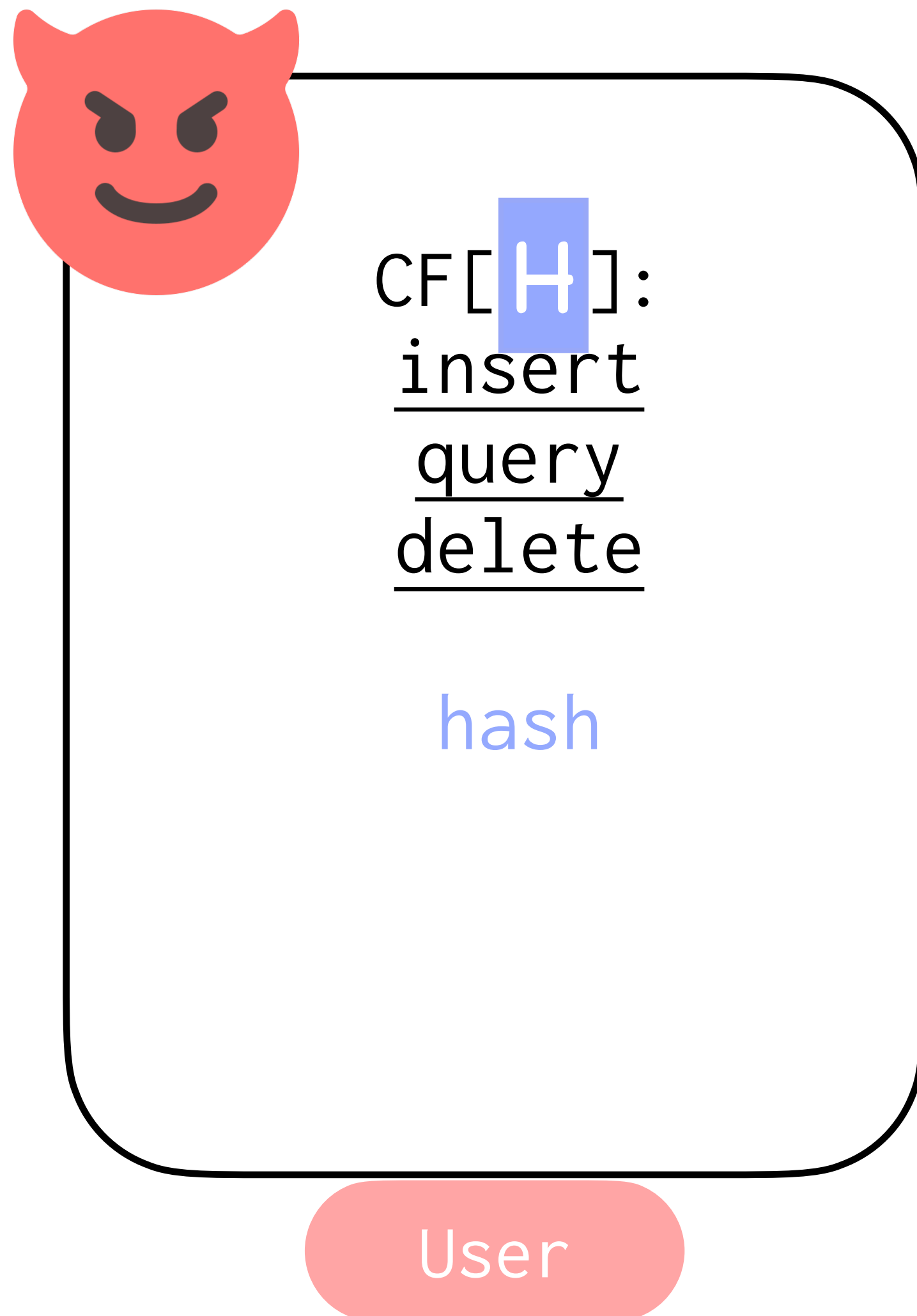
False negatives: often assumed not to occur



# What can go wrong in adversarial settings?



# What can go wrong in adversarial settings?



Public hash functions = precomputation attacks

# What can go wrong? [CPS19]

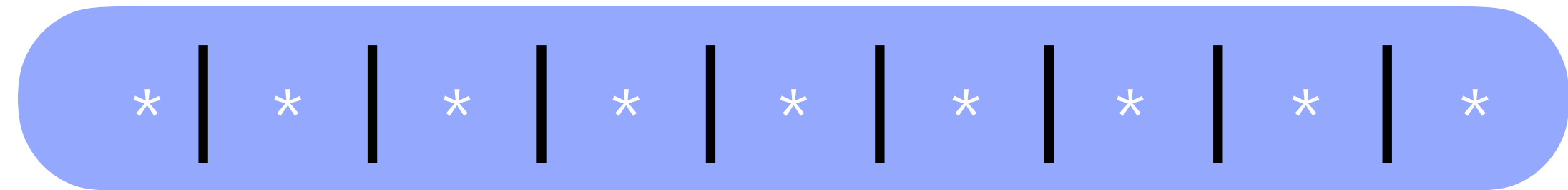


CF[H]:  
insert  
query  
delete

hash

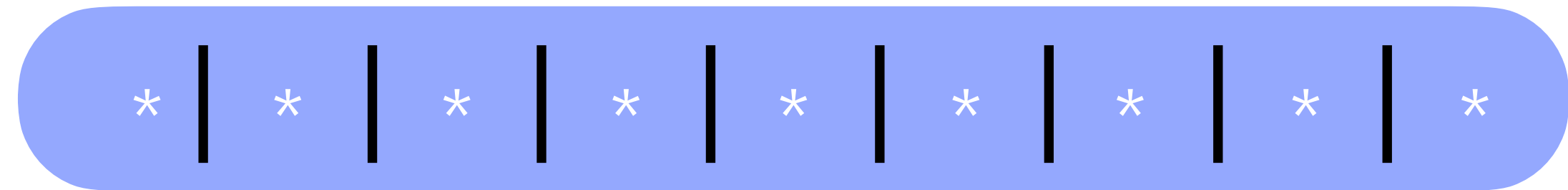
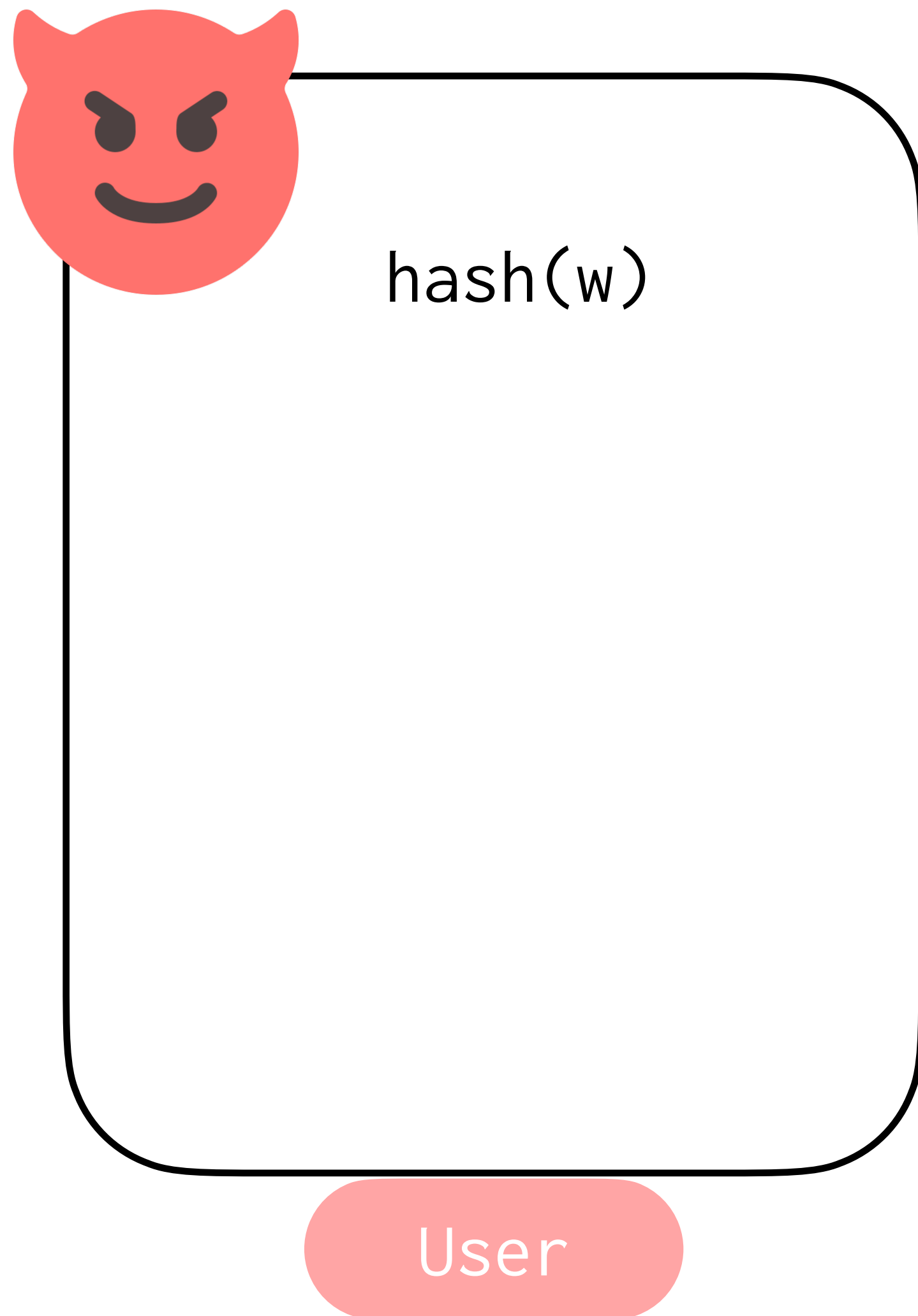
**goal:** make  
target element w  
a false positive

User



hash(.)

# What can go wrong?



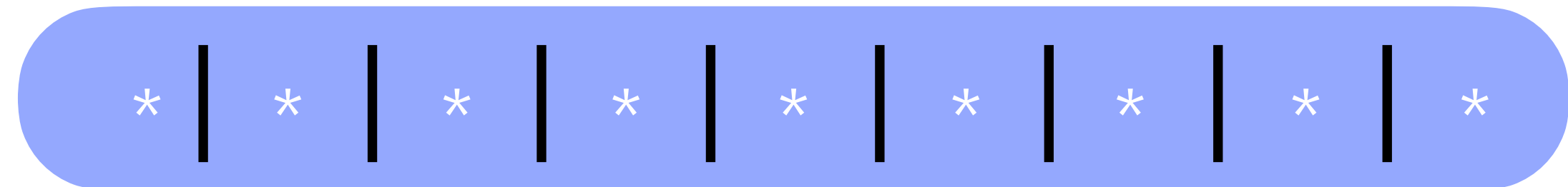
hash(w) = 1 | 3 | 9 | 2 | 8

# What can go wrong?



hash(w)  
hash(x)  
hash(y)  
hash(z)

User



hash(w) = 1 | 3 | 9 | 2 | 8

hash(x) = 2 | 5 | 9 | 1 | 3

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# What can go wrong?



hash(w)  
hash(x)  
hash(y)  
hash(z)

insert x  
insert y

query(w) = T

User

> 0 | > 0 | > 0 | \* | \* | \* | \* | > 0 | > 0

hash(w) = 1 | 3 | 9 | 2 | 8

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What could go wrong, beyond the paper focus?

### **Bloom filter**

tamper with  
the false  
positive  
probability

### **HyperLogLog**

inflate the  
cardinality  
estimate  
[PR22]

### **Count-min sketch**

cause  
frequency of  
an element to  
be  
unexpectedly  
overestimated  
[MFS23]

How do we define secure PDS?

AMQ-PDS[H]

insert

query

delete



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AMQ-PDS[  ]

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# How do we define secure PDS?

Game-based notions  
[NY15, CPS19]

Specific adversarial  
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# How do we define secure PDS?

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Simulation-based notions  
[PR22, FPUV22]

Any adversarial goal

# Simulation-based framework

**Real world**



**Ideal world**



# Simulation-based framework

## Real world

AMQ-PDS[  ]

insert  
query  
delete

adversary interacts  
with a concrete  
AMQ-PDS

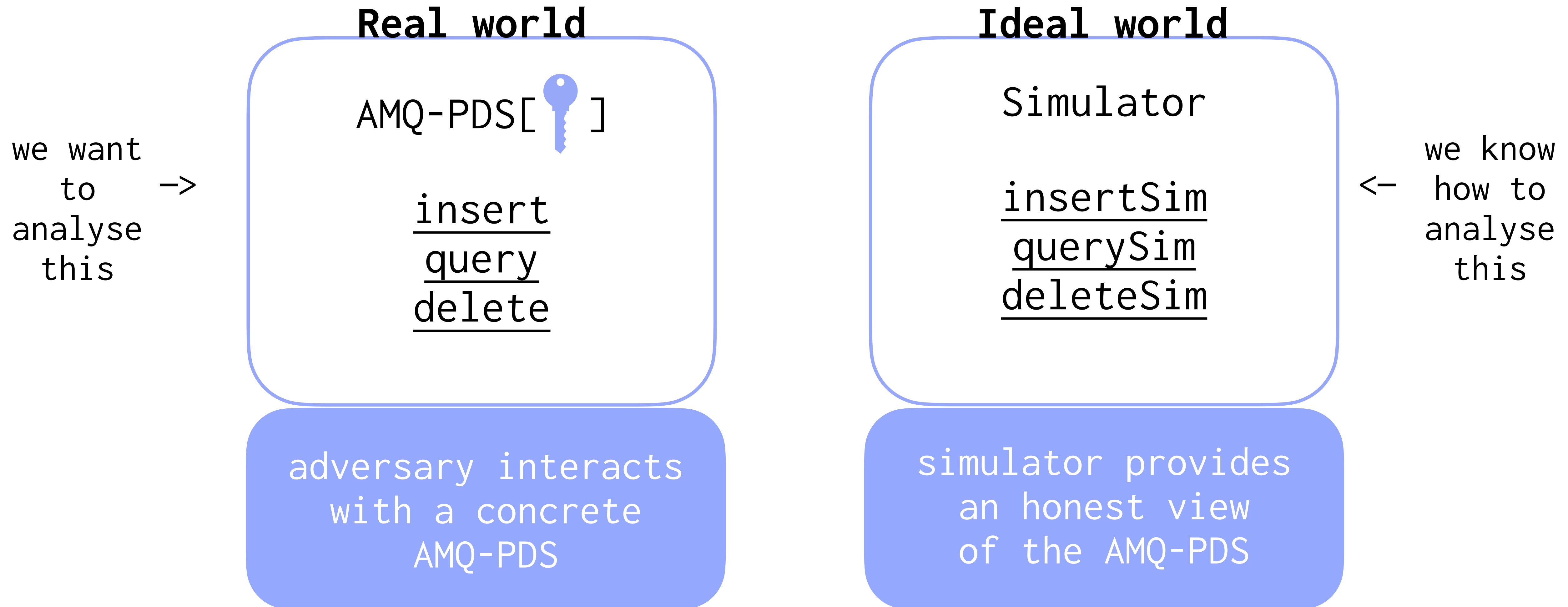
## Ideal world

Simulator

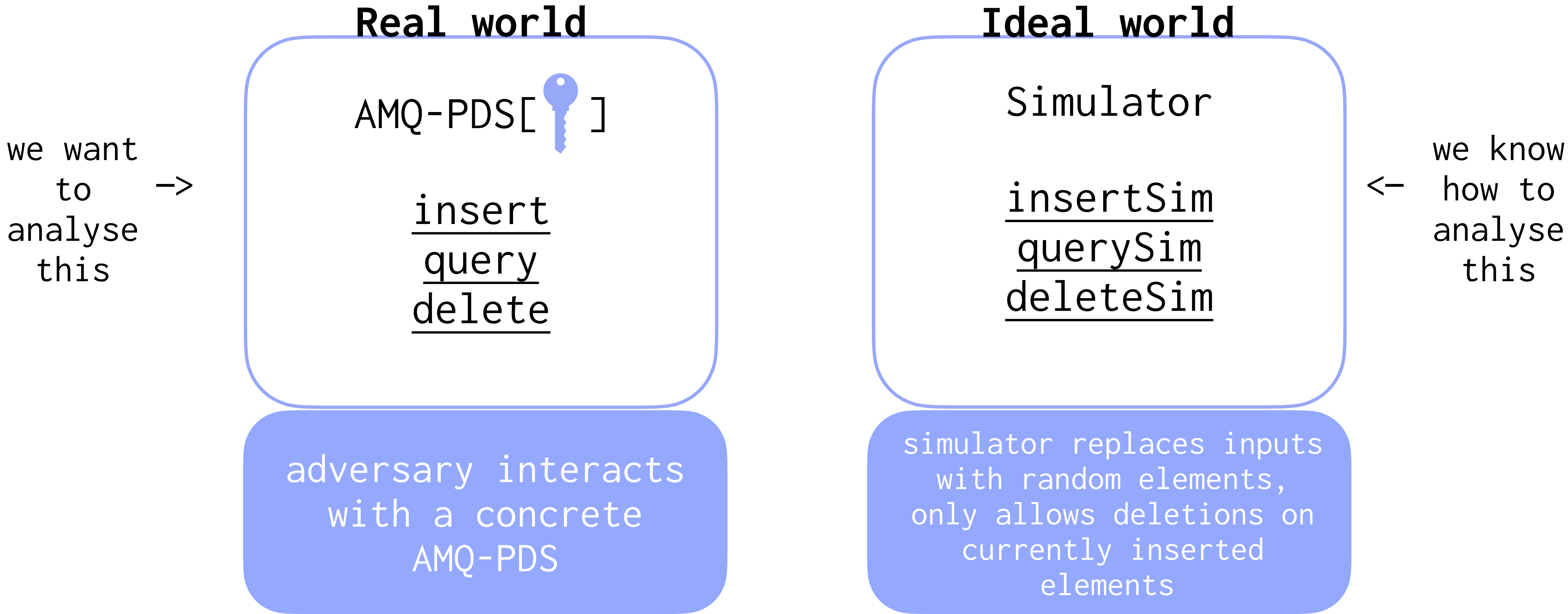
insertSim  
querySim  
deleteSim

adversary interacts  
with a simulator

# Simulation-based framework



# Adversarial correctness



# Adversarial correctness

## Real world

AMQ-PDS[  ]

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insertSim  
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simulator provides  
an honest view  
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—

function of  
the number of  
different  
oracle calls

maximal  
false positive  
probability  
given  $q_{ins}$   
elements



$$\leq (q_{ins} + 2q_{qry} + q_{del}) \cdot \Pr[FP | q_{ins}] + \dots$$



term(FP, q)

# Adversarial correctness

## Real world

AMQ-PDS[  ]

insert  
query  
delete

adversary interacts  
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AMQ-PDS

## Ideal world

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deleteSim

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$$\leq \text{term}(\text{FP}, q) + 2\Pr[IF \mid q_{ins}] + \epsilon$$

PRF  
advantage



maximal  
insertion  
failure  
probability  
given  $q_{ins}$   
elements

# Adversarial correctness

**Real world**

any  
adversarial  
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$\leq$

**Ideal world**

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# Adversarial correctness

## Real world

finding a false  
positive given  
q\_ins  
insertions,  
q\_del deletions,  
q\_qry queries

adversary interacts  
with a concrete  
AMQ-PDS

$\leq$

## Ideal world

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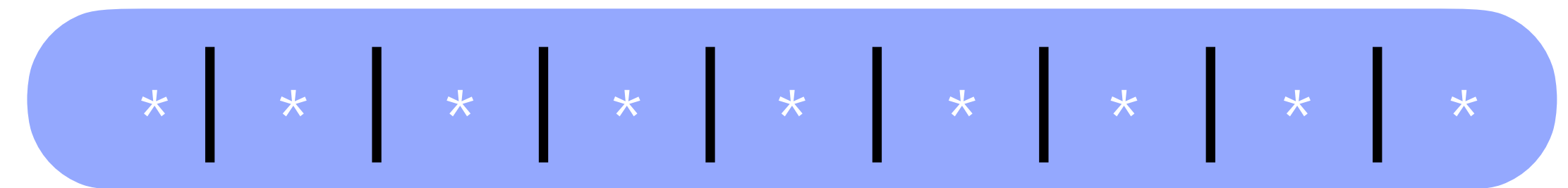
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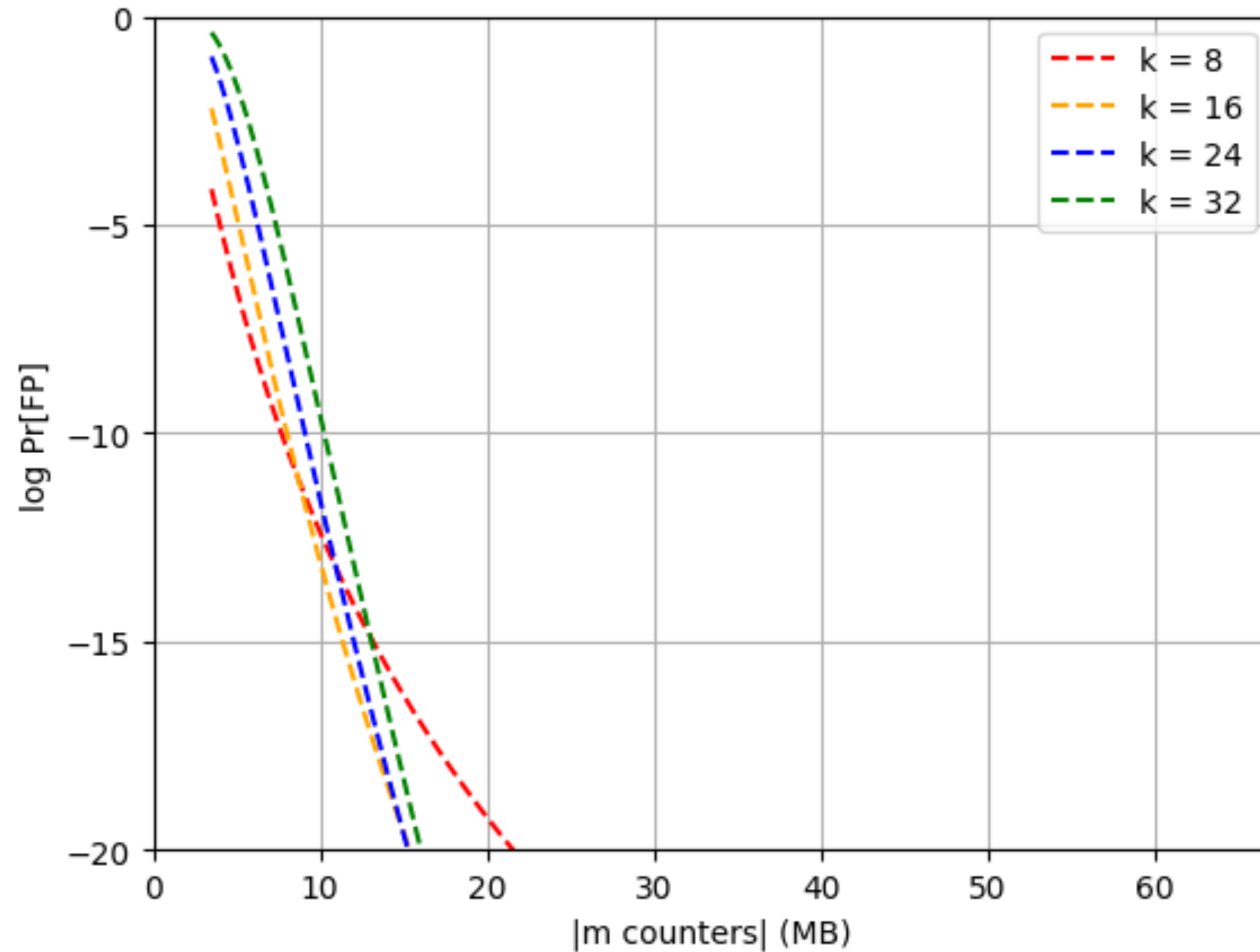
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Counting filter



PRF(.)

# Securing Counting filters in practice

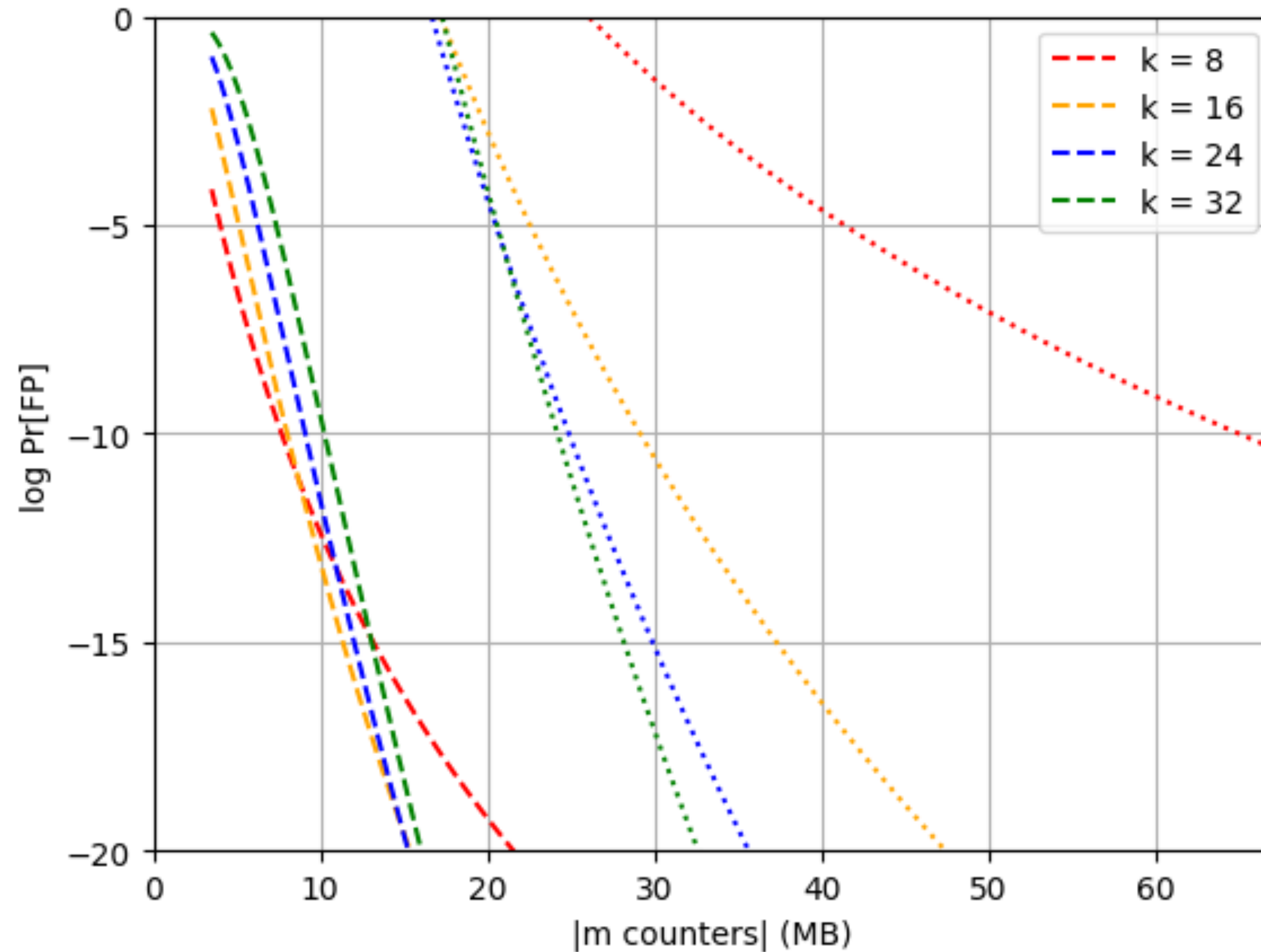


----- honest setting

Maximum counter value is 15

User makes  $2^{20}$  insertions,  
deletions and queries

# Securing Counting filters in practice



----- honest setting

..... adversarial setting  
w/ deletions

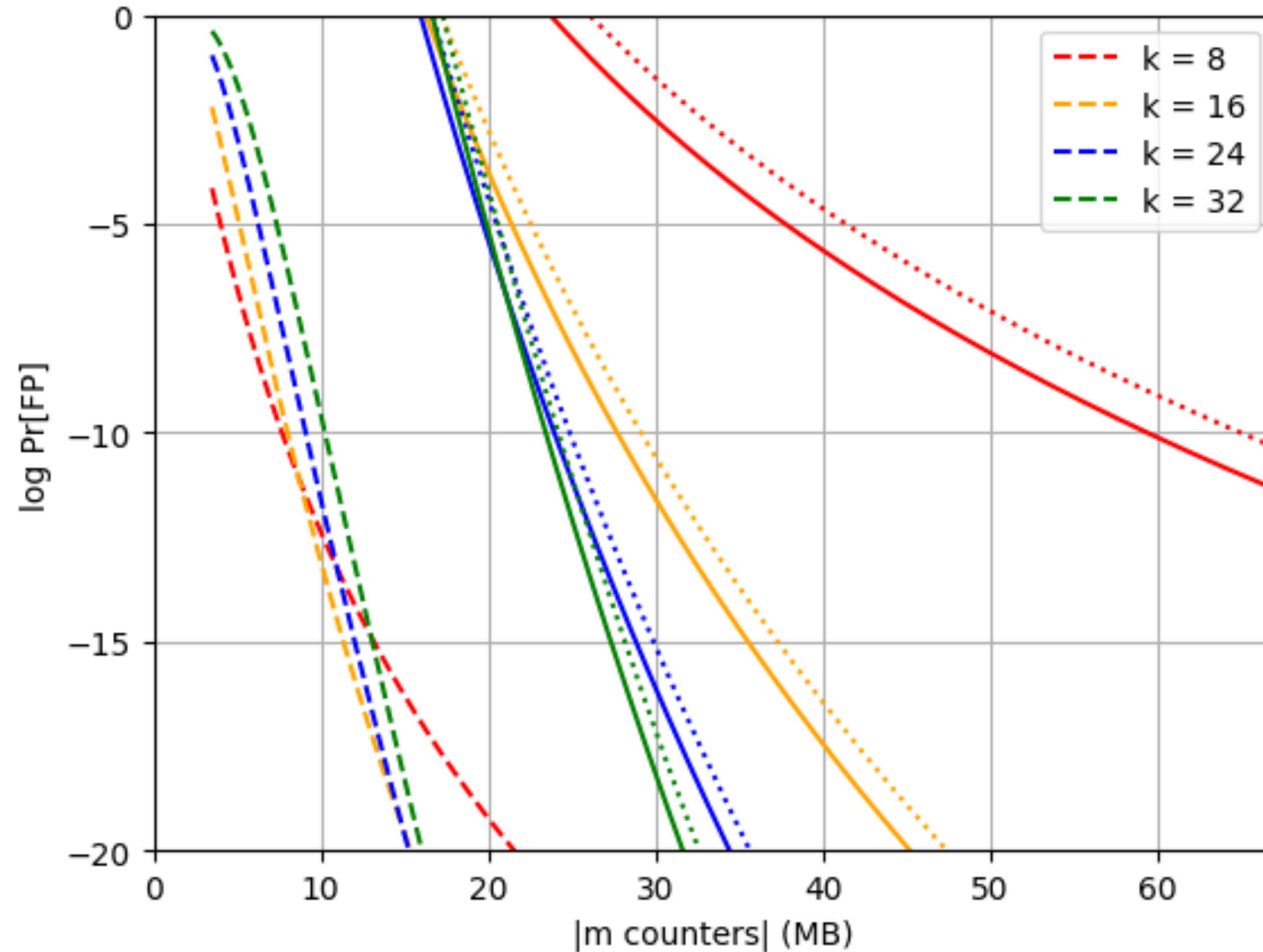
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$$\epsilon = 2^{-128}$$



# Securing Counting filters in practice



----- honest setting

..... adversarial setting  
w/ deletions

———— adversarial setting  
w/o deletions

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$$\epsilon = 2^{-128}$$

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- Our work formalises the honest ‘view’ for AMQ-PDS with deletions, which is distinct from the one for the insertion-only case ([FPUV22]).

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- Deletions notably enhance adversarial power.
- However, with proper parameter selection, claimed correctness under adversaries remains achievable even in the presence of deletions.

# Future works

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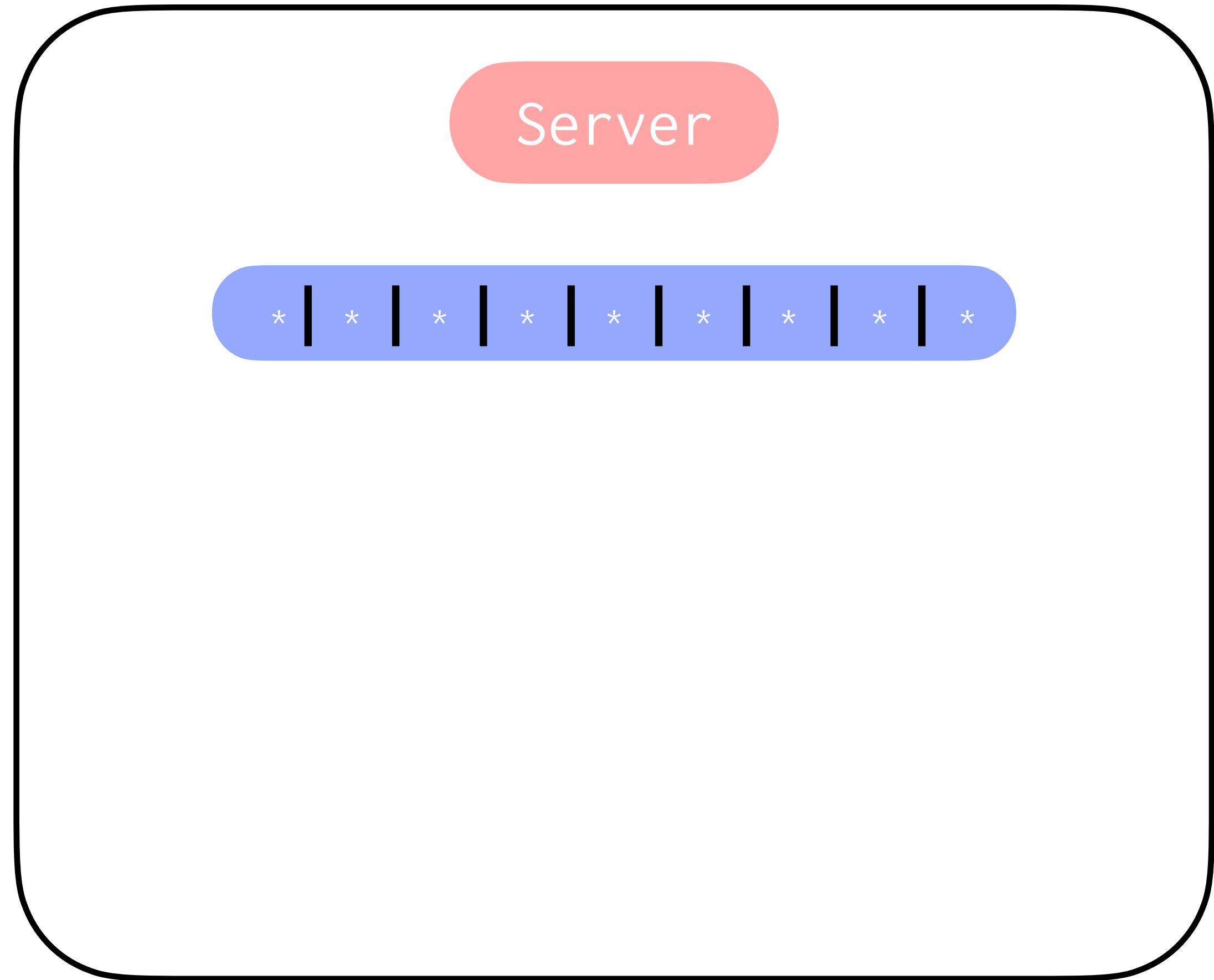
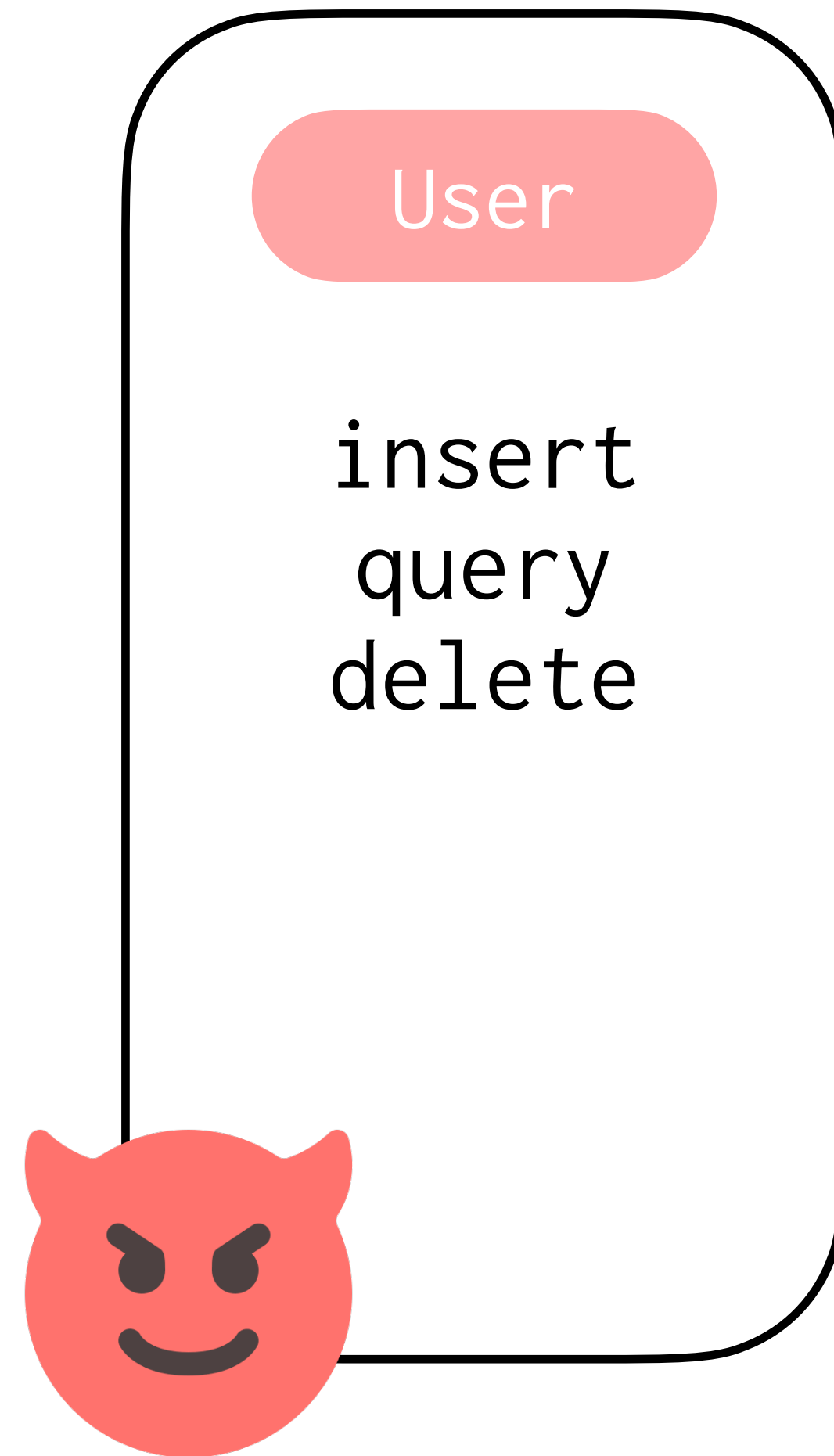
- Privacy of AMQ-PDS w/ deletions (see [FPUV22] for w/o deletions)

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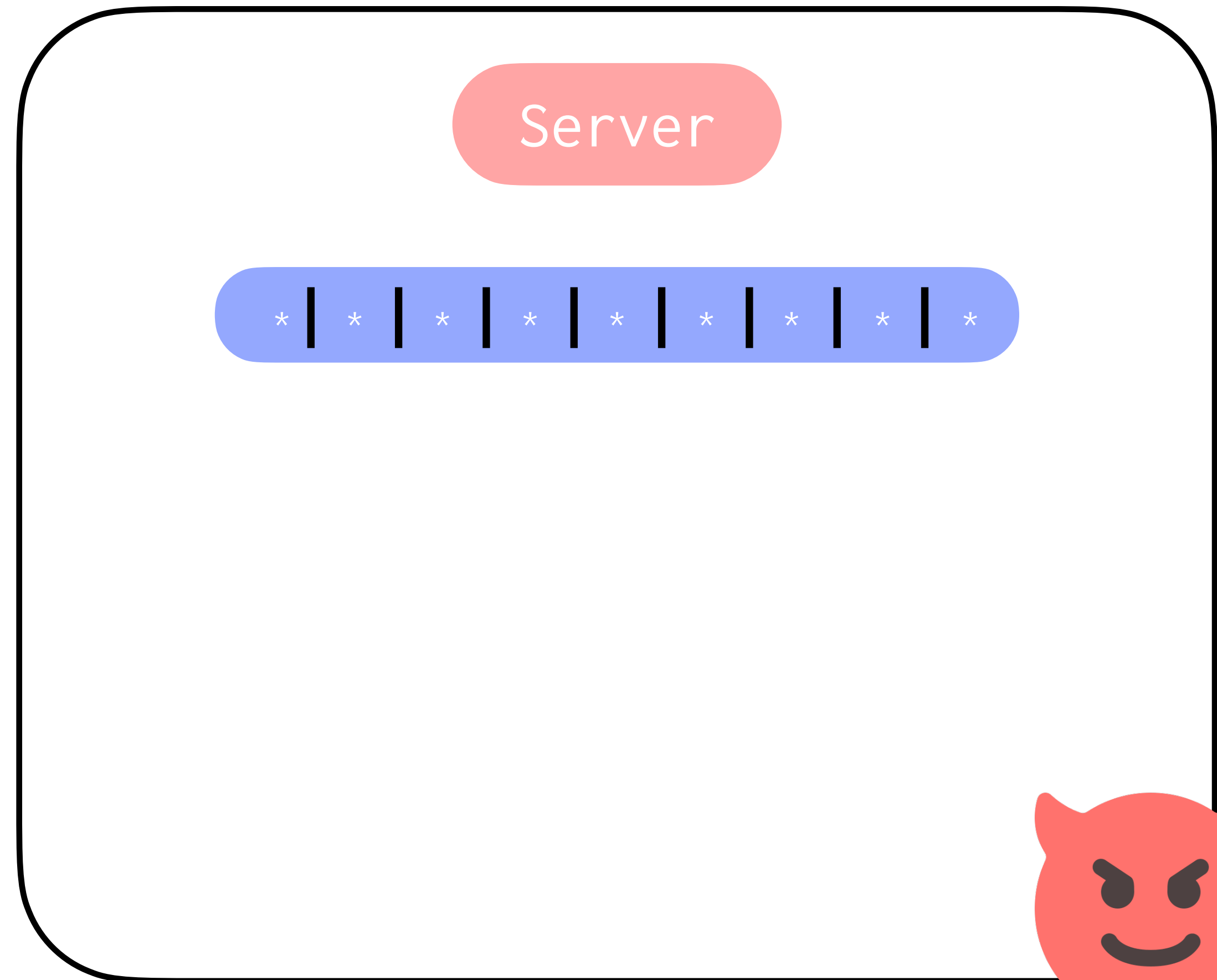
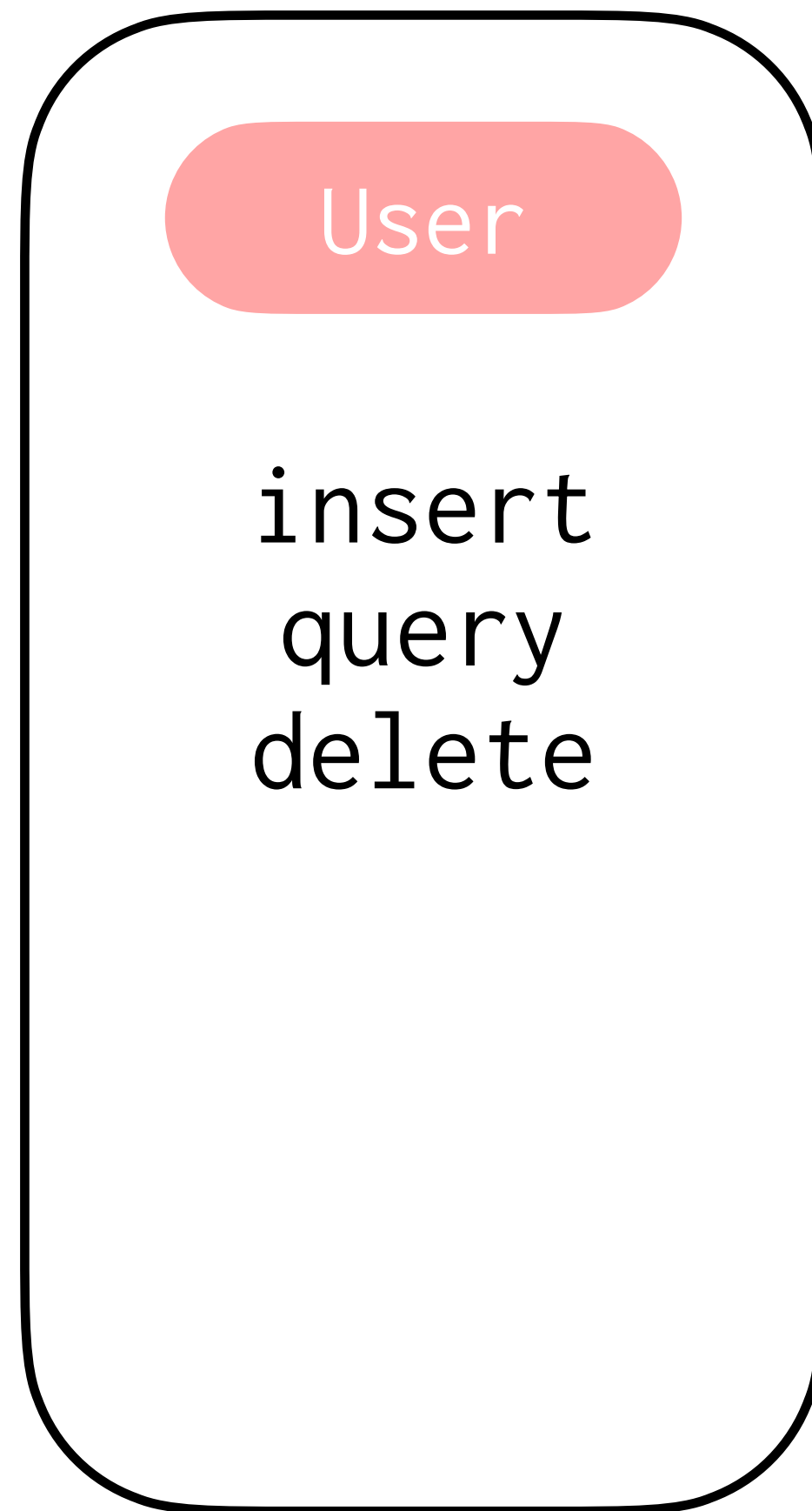
- Privacy of AMQ-PDS w/ deletions (see [FPUV22] for w/o deletions)
- Consider other PDS in adversarial settings (see [MFS23], [CPS19] for frequency estimation PDS and their adversarial correctness)



# Future work



# Future work



What if the server is malicious  
and the user is honest?

# Thank you!

Follow up/parallel work:

- Privacy implications of AMQ-based PQ TLS authentication (CoNEXT24)
  - <https://dl.acm.org/doi/10.1145/3680121.3697813>
- Probabilistic Data Structures in the Wild: A Security Analysis of Redis
  - <https://eprint.iacr.org/2024/1312>
- Scalable Probabilistic Data Structures in Adversarial Environments (Raguso, Masters project)

# Thank you!

Full paper: <https://eprint.iacr.org/2024/1911>

# References

- MFS23
  - <https://eprint.iacr.org/2023/1366>
- FPUV22
  - <https://eprint.iacr.org/2022/1186>
- PR22
  - <https://eprint.iacr.org/2021/1139>
- CPS19
  - <https://eprint.iacr.org/2019/1221>
- NY15
  - <https://eprint.iacr.org/2015/543>
- FN
  - <https://blog.fleek.network/post/bloom-and-cuckoo-filters-for-cache-summarization/>