Traitor Tracing without Trusted Authority from Registered Functional Encryption

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Scenario: Group Messaging

- L users wish to broadcast messages to each other privately, such that:
 - Small ciphertext, e.g. sublinear in L [Efficiency]
 - No information about any message exchanged is revealed [CPA-Security]
 - Trace a user that leaked its own secret key (e.g. device compromised) [Trace]
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 → Allows to exclude traitor from the group
- Desired primitive: Traitor Tracing [CFN94]

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- Key escrow problem: No security if authority is corrupt

Motivation

• This work:

Efficient traitor-tracing without a trusted authority

- Goals:
- Remove trusted authority
- Non-trivial, concrete efficiency (Ciphertext grows sublinear in number of users)
- Security from simple and well-understood objects (e.g. not iO)

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- More applications of our RFEs

Related Works: Prior + Concurrent

- All prior schemes require trusted authority, except [Luo22]:
 - [Luo22]: No Setup + Relies on iO + Non-compact master public key + Non-deterministic decryption

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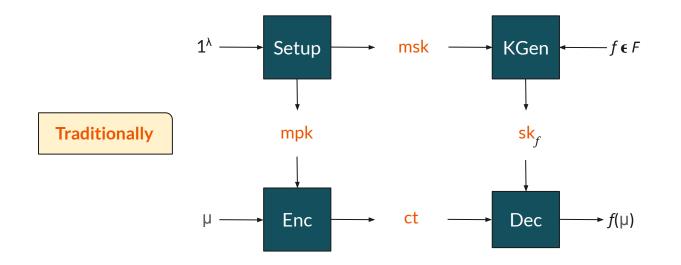
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 - Registration-based Encryption [GHMR18, GHM+19]
 - Removes trusted authority in IBE

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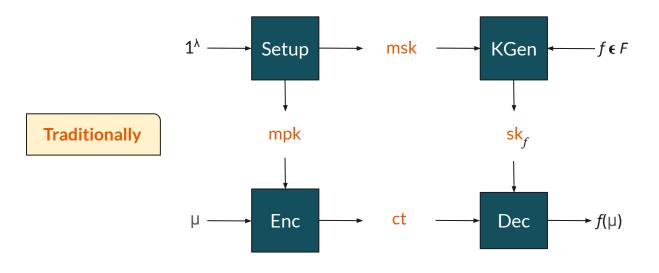
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 - Removes trusted authority in IBE
- Concurrent works on RFE:
 - [DPY23] gets RLFE (with non-transparent setup) in GGM
 - o [ZLZ+24] gets (very selective) RQFE and RLFE from variants of k-Lin, with non-transparent setup

Functional Encryption

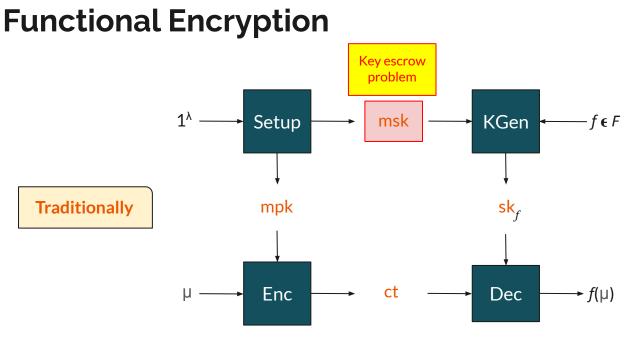
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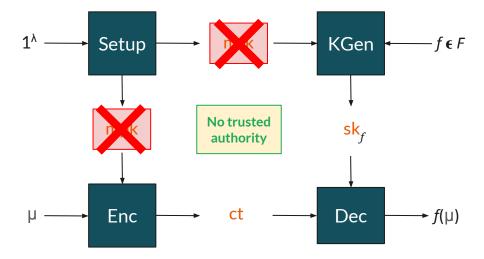


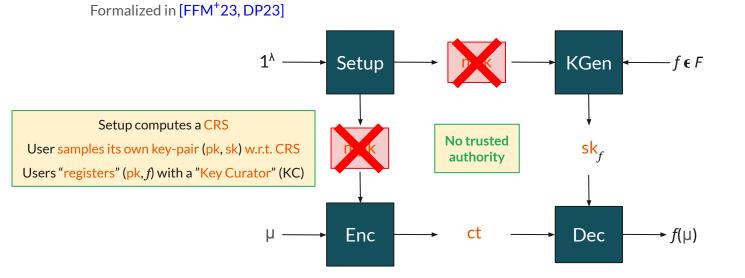
Security (Informally): $[mpk, \{sk_f\}, ct(\mu_0), \{sk_f\}] \approx [mpk, \{sk_f\}, ct(\mu_1), \{sk_f\}]$ provided $f(\mu_0) = f(\mu_1)$

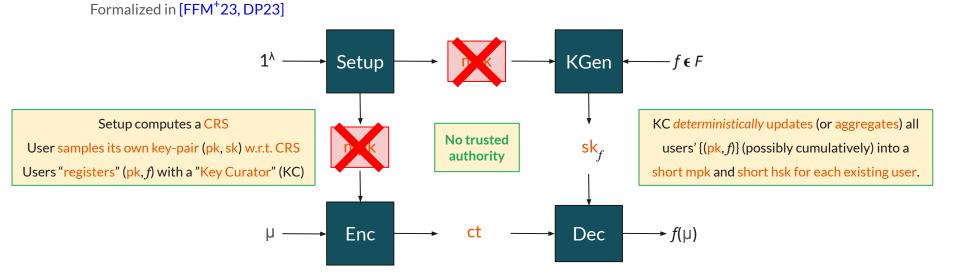


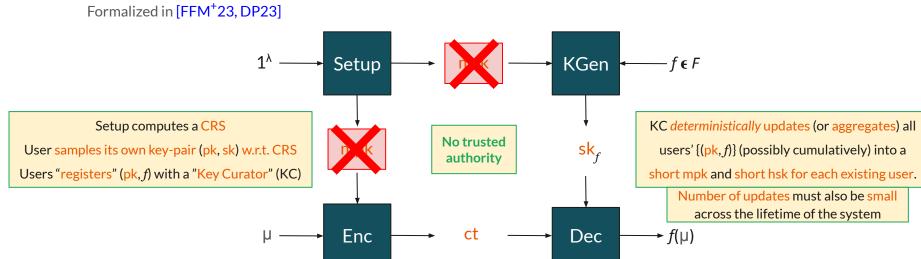
No security if msk gets leaked

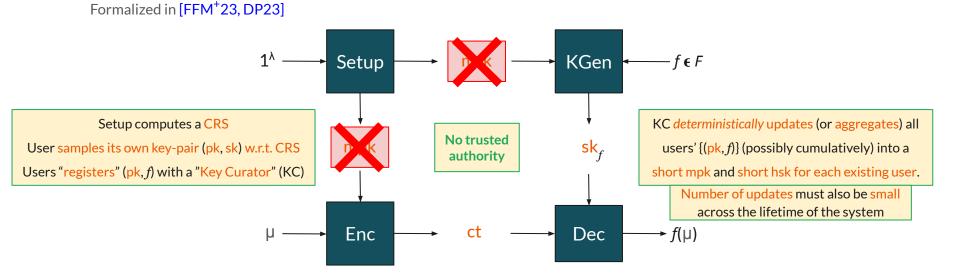
Formalized in [FFM⁺23, DP23]











Security (Informally): Similar to FE, except now registered keys can be generated maliciously

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 - Aggregate users into system + generate helper secret key
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• Linearly-homomorphic KGen for function *f*

$$\underbrace{\mathsf{KGen}(\mathsf{msk}_0, f)}_{\mathsf{sk}_f^{(0)}} * \underbrace{\mathsf{KGen}(\mathsf{msk}_1, f)}_{\mathsf{sk}_f^{(1)}} = \mathsf{KGen}(\mathsf{msk}_0 + \mathsf{msk}_1, f)$$

$$* = \text{Group operation}$$
Secret keys from independent instances can be publicly combined into a secret key for the global instance.

• Given "master secret key homomorphic" QFE, define RQFE "global" master public key:

$$\widetilde{\mathsf{mpk}} = \mathsf{mpk}_1 * \ldots * \mathsf{mpk}_L = \mathsf{Encode}(\mathsf{msk}_1 + \ldots + \mathsf{msk}_L)$$

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- Each user also publishes helper information to help others decrypt their own share

• Each user j provides helper secret keys for each user $i \neq j$ (rely on "weak" setting):

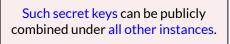
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• Apply homomorphic property to user keys:

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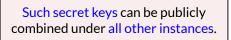
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Each independent QFE instance provide secret keys for functions under all other instances.

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• Each user *i* misses msk for exactly the *i*-th function f_i , which is known to itself

$$\widetilde{\mathsf{sk}}_{f_i} = \mathsf{KGen}(\sum_{j \neq i} \mathsf{msk}_j, f_i) * \mathsf{KGen}(\mathsf{msk}_i, f_i) = \mathsf{KGen}(\widetilde{\mathsf{msk}}, f_i)$$

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- We provide multiple transformations for security against maliciously registered keys
 - NIZK: prove well-formedness of keys
 - Leverage random oracle on our RQFE: Setup remains transparent
 - Modify RQFE scheme (without random oracle, loses transparent Setup)

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• We formalize the full chain of transformations in the registered setting Main observation: Weak RQFE suffices

Implementation: Registered PLBE

Sizes for L = 1024:

> crs: 135KB, mpk: 6.6KB, pk: 102.5KB, sk: 97B,

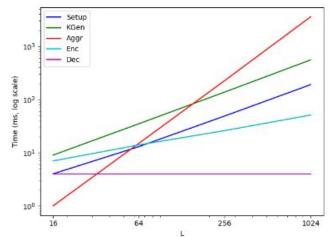
ciphertext: 6.7KB hsk: 194B

Runtimes on PC:

(AMD Ryzen 5 5600X, 3.7GHz CPU, 32GB of RAM)

L	Time (ms)				
	Setup	KGen	Aggr	Enc	Dec
16	3.86	9.04	1.06	7.26	4.04
64	13.31	35.14	14.56	13.53	4.04
256	48.94	138.17	226.93	26.11	4.04
1024	189.57	553.87	3576.37	51.2428	4.04

Table 4: Runtimes of our RPLBE algorithms for different L.



Other Application: Registered Threshold Encryption

- Registered Threshold Encryption (RTE):
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- *t*-out-of-L threshold:
 - User *i* runs RFE.KGen for a linear function $(1, i, ..., i^{t-1})$
 - Encrypt message μ : random degree *t*-1 polynomial P with P(0) = μ
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RLFE → RTE RQFE → RTE (transparent setup)

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- Prototype implementation
- (More) Applications from our work:
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