Adaptive Hardcore Bit and Quantum Key Leasing over Classical Channel from LWE with Polynomial Modulus

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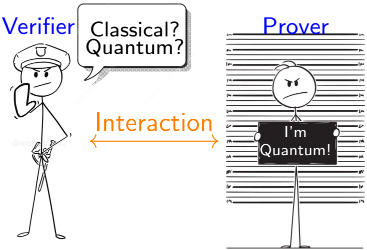
What is Proofs of Quantumness



Schemes	Assumptions
[BCM+18]	LWE
[BKVV20]	Random Oracle & Ring-LWE
[KMCVY22, KLVY23,BGKM+23]	Bell's inequality & (*Ring*-)LWE

What is Secure Key Leasing?

Revoke the key

Schemes	Channels
[APV23,AKN$^+$23]	Quantum Channel
[CGJL24]	Classical Channel

Our Results

Schemes	Assumptions	Modulus
[BCM+18]'s PoQ	LWE	superpoly
[BKVV20]'s PoQ	Random Oracle & Ring-LWE	poly
[KMCVY22,KLVY23 BGKM+23]'s PoQs	Bell's inequality & (*Ring*-)LWE	poly
Our PoQ	LWE	poly
[CGJL23]'s PKE-SKL	LWE	superpoly
Our PKE-SKL	LWE	poly

* PoQ: Proofs of Quantumness;
* PKE-SKL: Public Key Encryption with Secure Key Leasing.

Road Map

1. Proofs of Quantumness
   - Claw-Free Function
   - Noisy Claw-Free Function
   - Proofs of Quantumness
   - Our improvements on Proofs of Quantumness
2. Public Key Encryption with Secure Key Leasing
   - What is PKE-SKL?
   - How to realize?
3. Future works?

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   - What is PKE-SKL?
   - How to realize?
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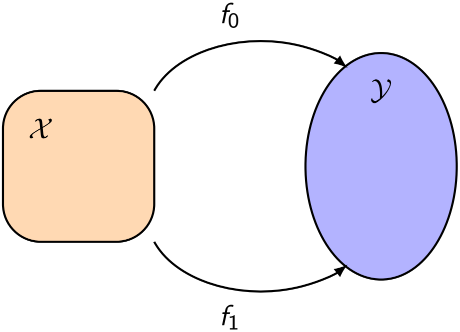
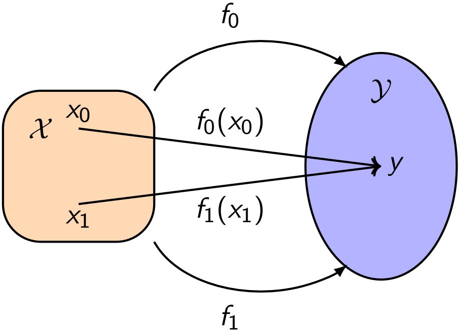
Claw-Free Functions



A pair of public injective functions f_0 and f_1 with the same range. It has the following two essential properties:

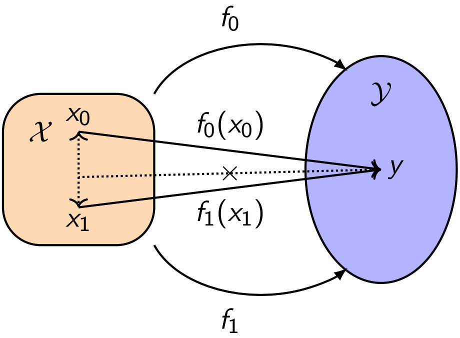
Claw-Free Functions



A pair of public injective functions f_0 and f_1 with the same range. It has the following two essential properties:

- **Claw:**
  For any $\mathbf{y}$ in the range, $\exists \mathbf{x}_0, \mathbf{x}_1$, $f_0(\mathbf{x}_0) = f_1(\mathbf{x}_1) = \mathbf{y}$.

Claw-Free Functions



A pair of public injective functions f_0 and f_1 with the same range. It has the following two essential properties:

- **Claw:**
  For any $\mathbf{y}$ in the range, $\exists \mathbf{x}_0, \mathbf{x}_1$, $f_0(\mathbf{x}_0) = f_1(\mathbf{x}_1) = \mathbf{y}$.
- **Claw-Free:**
  For any $\mathbf{y}$ in the range, hard to find $(\mathbf{x}_0, \mathbf{x}_1)$: $f_0(\mathbf{x}_0) = f_1(\mathbf{x}_1) = \mathbf{y}$.

LWE-based Noisy Claw-Free Function [BCM+18]:

Given $k = (\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0)$, **can we get Claw-Free function?**

-

LWE-based Noisy Claw-Free Function [BCM+18]:

Given $k = (\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0)$,

- $f_{k,0}(\mathbf{x}) = \mathbf{A}\mathbf{x}$
  $f_{k,1}(\mathbf{x}) = \mathbf{A}\mathbf{x} + b(\mathbf{A}\mathbf{s} + \mathbf{e}_0)$**?**

$f_{k,0}(\mathbf{x}) \quad \bullet \bullet \, f_{k,1}(\mathbf{x} - \mathbf{s})$

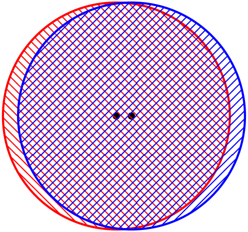
Not claw,

but close!

LWE-based Noisy Claw-Free Function [BCM+18]:

Given $k = (\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0)$, the distribution function:

- $\forall \mathbf{y} \in \mathbb{Z}_q^m$: $(f'_{k,b}(\mathbf{x}))(\mathbf{y}) = D_{\mathbb{Z}^m, r, 2r\sqrt{m}}(\mathbf{y} - \mathbf{A}\mathbf{x} - b \cdot (\mathbf{A}\mathbf{s} + \mathbf{e}_0))$

$f'_{k,0}(\mathbf{x})$  $f'_{k,1}(\mathbf{x} - \mathbf{s})$

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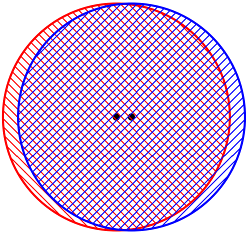

- Claw: $supp(f'_{k,0}(\mathbf{x})) \bigcap supp(f'_{k,1}(\mathbf{x} - \mathbf{s})) \neq \varnothing$.

LWE-based Noisy Claw-Free Function [BCM+18]:

Given $k = (\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0)$,

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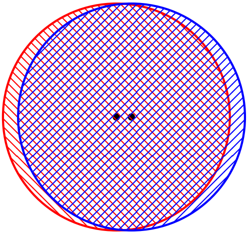
$f'_{k,0}(\mathbf{x})$  $f'_{k,1}(\mathbf{x} - \mathbf{s})$

- Claw: $supp(f'_{k,0}(\mathbf{x})) \bigcap supp(f'_{k,1}(\mathbf{x} - \mathbf{s})) \neq \varnothing$.
- Claw-Free: Finding a claw $\Rightarrow$ breaking LWE.

How to generate claw in superposition? [BCM+18]

- Generate $\sum_{\substack{b\in\{0,1\}\\ \mathbf{x}\in\mathcal{X}}} \sum_{\|\mathbf{e}\|\leq 2r\sqrt{m}} \sqrt{D_{\mathbb{Z}^m,r}(\mathbf{e})}|b\rangle|\mathbf{x}\rangle|\mathbf{e}\rangle$; [GR02]

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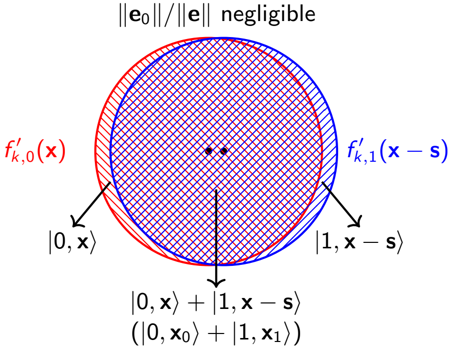
- Given $k = (\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0 \bmod q)$ Compute

$$\sum_{\substack{b\in\{0,1\}\\ \mathbf{x}\in\mathcal{X}}} \sum_{\|\mathbf{e}\|\leq 2r\sqrt{m}} \sqrt{D_{\mathbb{Z}^m,r}(\mathbf{e})}|b\rangle|\mathbf{x}\rangle|\underbrace{\mathbf{A}(\mathbf{x}+b\mathbf{s}) + b\mathbf{e}_0 + \mathbf{e}}_{f'_{k,b}(\mathbf{x})}\rangle$$



Measurement $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}'$

$$\begin{tikzcd}
 & {|0, \mathbf{x}_0\rangle + |1, \mathbf{x}_1\rangle} \arrow[dl, "\text{Standard Measure}"'] \arrow[dr, "\text{Hadamard Measure}"] & \\
(b, \mathbf{x}_b) & & (c, \mathbf{d})
\end{tikzcd}$$

- $\mathbf{d} \sim U(\{0,1\}^{n\lceil \log q \rceil})$, [BCM+18] shows $c = \mathbf{d}^\top \cdot (\mathbf{x}_0 \oplus \mathbf{x}_1) = \langle I_{b,\mathbf{x}_b}(\mathbf{d}), \mathbf{s} \rangle$.

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Can we get both values of measurements?

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Can we get both values of measurements? No!

$$
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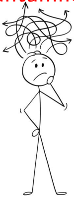
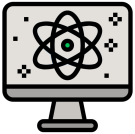
Can we get both values of measurements? No!

- **Adaptive hardcore bit [BCM$^+$18]**: Given $(\mathbf{A}, \mathbf{A}\mathbf{s} + \mathbf{e}_0)$ with superpolynomail modulus and $\mathbf{d} \xleftarrow{\$} \{0,1\}^{n\lceil \log q \rceil}$, the adversary picks $(b, \mathbf{x}_b)$, hard to get $c = \mathbf{d}^\top \cdot (\mathbf{x}_0 \oplus \mathbf{x}_1) = \langle I_{b,\mathbf{x}_b}(\mathbf{d}), \mathbf{s}\rangle$.

Can we prove the quantumness?



Quantum



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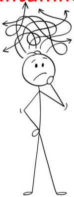
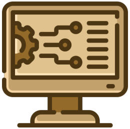


Randomly ask for values under either Hadamard or Standard measurement

Classical



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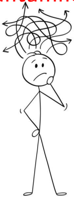

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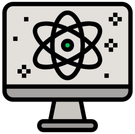
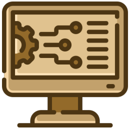


Do either measurement accordingly under quantum computation

$\Longrightarrow$ Completeness

Classical



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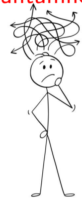

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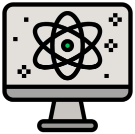

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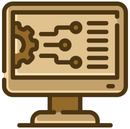


Need to know values under both measurement in advanced, impossible due to **AHB**.

$\Longrightarrow$ Soundness

NTCF-based Proofs of Quantumness[BCM+18]:

For $i \in [N]$, repeat:

1. Generate , $\mathsf{td}_{\mathbf{A}_i}$, $k_i = (\mathbf{A}_i, \mathbf{A}_i\mathbf{s}_i + \mathbf{e}_{0,i})$

$\xrightarrow{k_i}$

2. Generate $|0, \mathbf{x}_{i,0}\rangle + |1, \mathbf{x}_{i,1}\rangle$ measurement $\mathbf{y}_i$

$\xleftarrow{\mathbf{y}_i}$

3. $r_i \leftarrow \{0, 1\}$

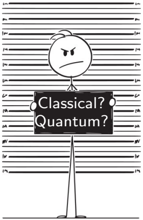
$\xrightarrow{r_i}$

4. 1) $r_i = 0$, Standard measure, $\sigma_{r_i,i} = (b_i, \mathbf{x}_{b_i,i})$
2) $r_i = 1$, Hadamard measure, $\sigma_{r_i,i} = (c, \mathbf{d}_i)$

$\xleftarrow{\sigma_{r_i,i}}$

5. Check validity of the measured values.

Verifier



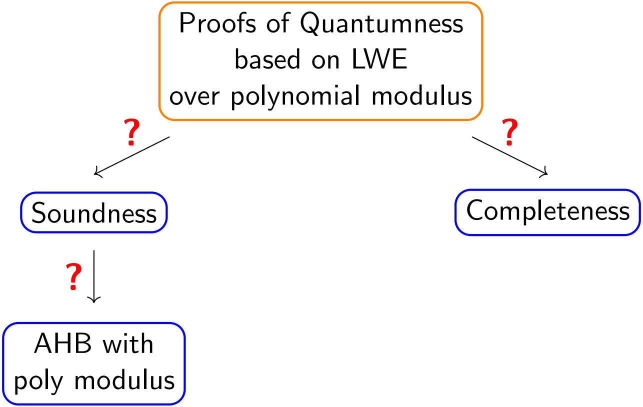
Prover

Proofs of Quantumness
based on LWE
over polynomial modulus

? ?

Soundness

Completeness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Soundness
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AHB with
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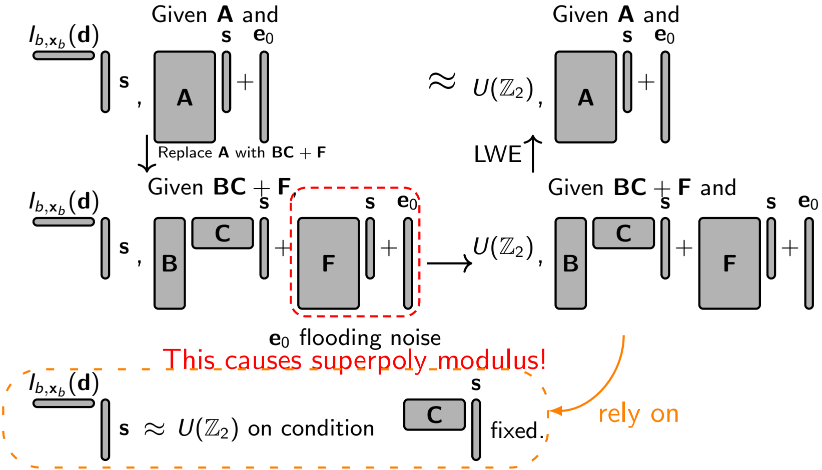
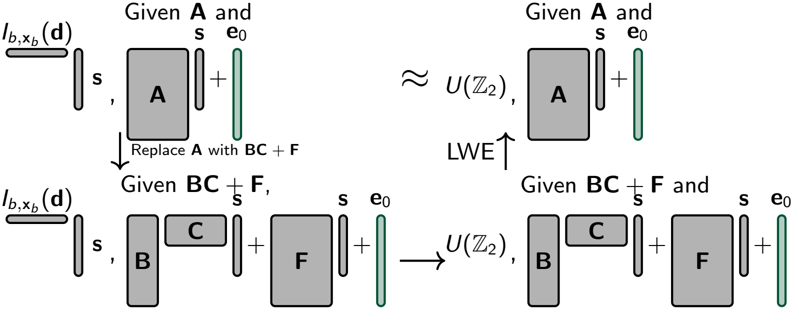
Sketch of proof of AHB in [BCM+18]:



- $\Rightarrow$ Given $(\mathbf{A}, \mathbf{As} + \mathbf{e}_0 \bmod q)$, $\langle I_{b,\mathbf{x}_b}(\mathbf{d}), \mathbf{s}\rangle \approx U(\mathbb{Z}_2) \Rightarrow$ AHB.

Our Proof:



e_0 covers Fs completely, Necessary?

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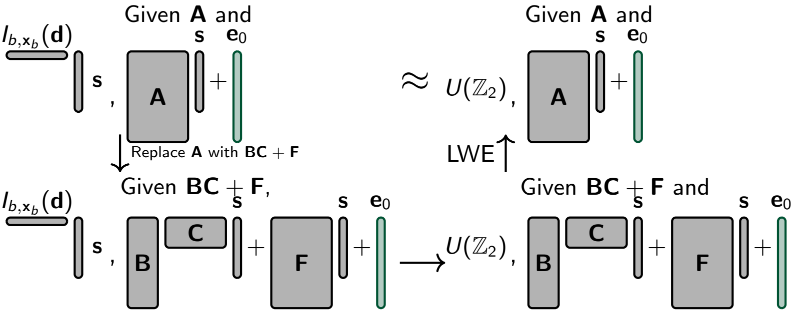
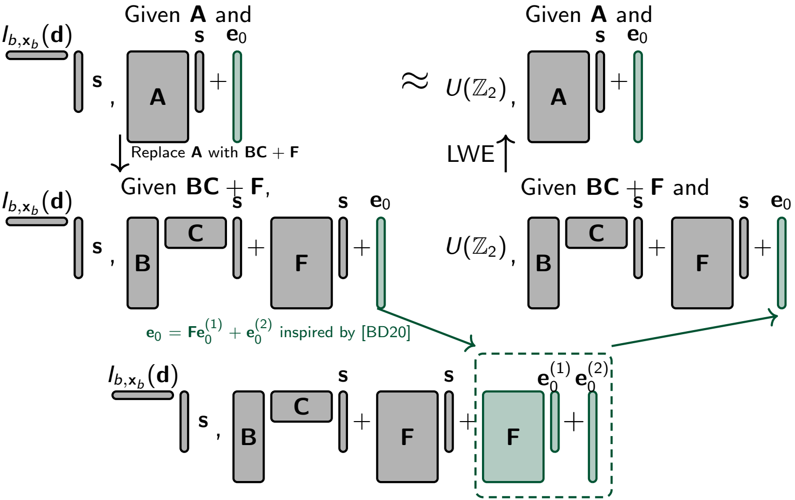
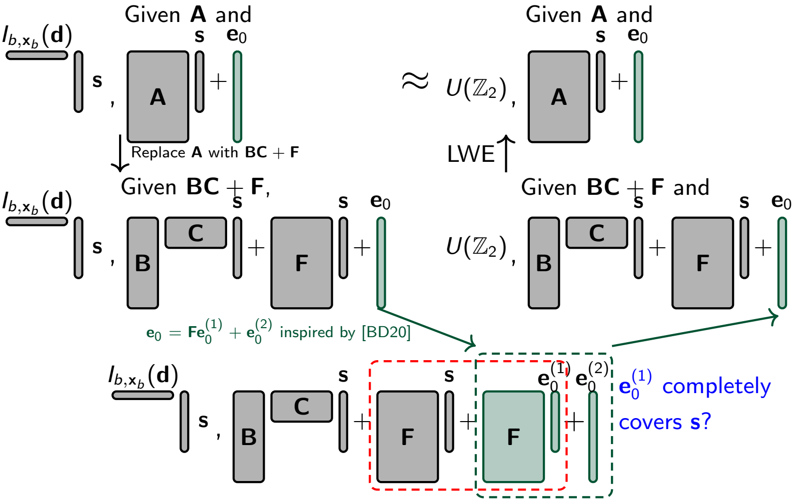


$\mathbf{e}_0$ covers Fs completely, Necessary? **No!**

Our Proof:


Given A and
$I_{b,\mathbf{x}_b}(\mathbf{d})$
s
s e0
A
$\approx$
$U(\mathbb{Z}_2)$,
Replace A with BC + F
LWE
Given BC + F,
Given BC + F and
B
C
F
$\mathbf{e}_0 = \mathbf{F}\mathbf{e}_0^{(1)} + \mathbf{e}_0^{(2)}$ inspired by [BD20]
$\mathbf{e}_0^{(1)}$ $\mathbf{e}_0^{(2)}$

Our Proof: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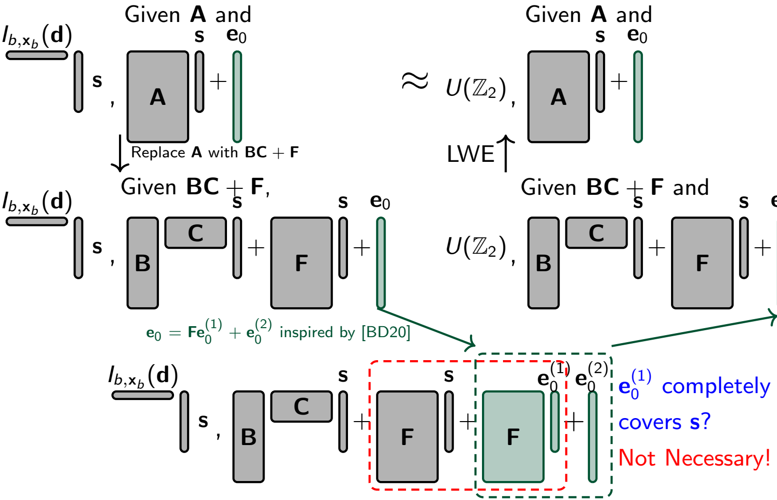
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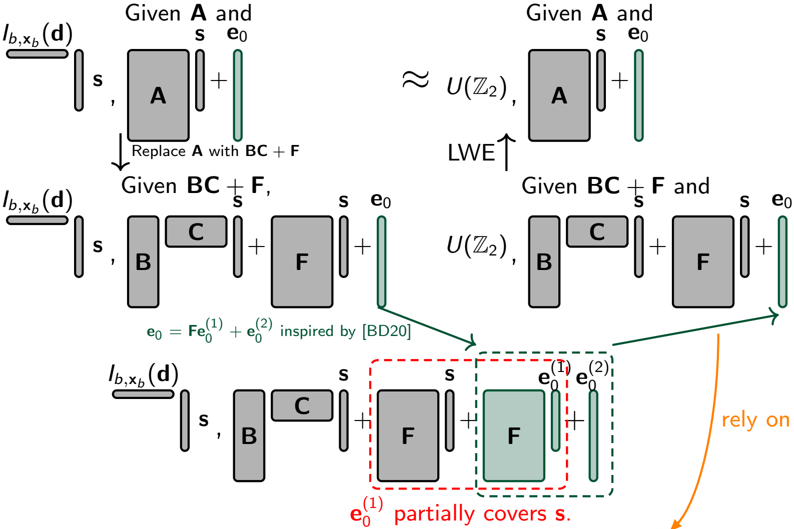
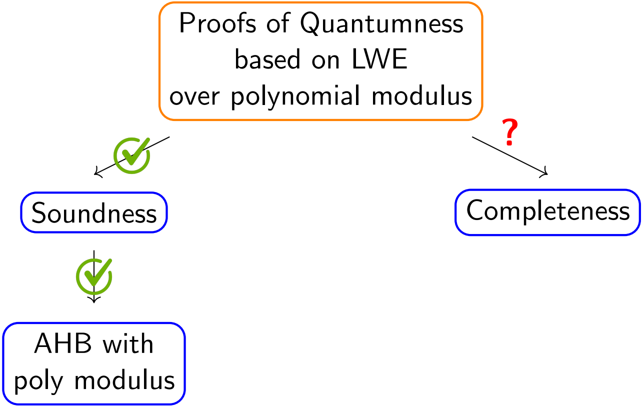
$\mathbf{e}_0^{(1)}$ $\mathbf{e}_0^{(2)}$
$\mathbf{e}_0^{(1)}$ completely covers **s**?
Not Necessary!

Our Proof:



$\langle I_{b,\mathbf{x}_b}(\mathbf{d}), \mathbf{s}\rangle \approx U(\mathbb{Z}_2)$ on condition $\mathbf{Cs}$ and $\mathbf{s} + \mathbf{e}_0^{(1)}$ are fixed, $\mathbf{e}_0^{(1)} \hookleftarrow D_{\mathbb{Z}^n, n, n^{3/2}}$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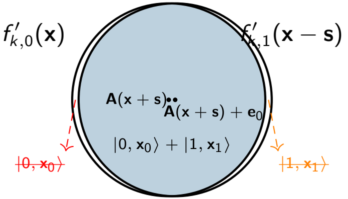
?

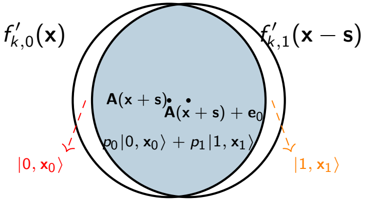
$\|\mathbf{e}_0\|/\|\mathbf{e}\|$ noticeable

$\|\mathbf{e}_0\|/\|\mathbf{e}\|$ negligible [BCM+18]



- Generate $|0,\mathbf{x}\rangle + |1,\mathbf{x}-\mathbf{s}\rangle$
- Do Hadamard measurement, $(c,\mathbf{d})$ satisfies $c = \mathbf{d}^\top(\mathbf{x}_0 \oplus \mathbf{x}_1)$ overwhelmingly.

$\|\mathbf{e}_0\|/\|\mathbf{e}\|$ noticeable [BKVV20]



- Generate $p_0|0,\mathbf{x}\rangle + p_1|1,\mathbf{x}-\mathbf{s}\rangle$, not close to $|0,\mathbf{x}\rangle + |1,\mathbf{x}-\mathbf{s}\rangle$
- Do Hadamard measurement, $(c,\mathbf{d})$ satisfies $c = \mathbf{d}^\top(\mathbf{x}_0 \oplus \mathbf{x}_1)$ with probability at least 0.8.

Can Quantum Computer pass the check?

- Do standard measurement $\Rightarrow$ still $(0, \mathbf{x}_0)$ or $(1, \mathbf{x}_1)$

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Can Quantum Computer pass the check?

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- Do Hadamard measurement for N_1 times,
  * $c = \mathbf{d}^\top \cdot (\mathbf{x}_0 \oplus \mathbf{x}_1) \bmod 2$ with probability at least 0.8.
  * Claim a threshold $0.75N_1$.

Proofs of Quantumness based on LWE over polynomial modulus

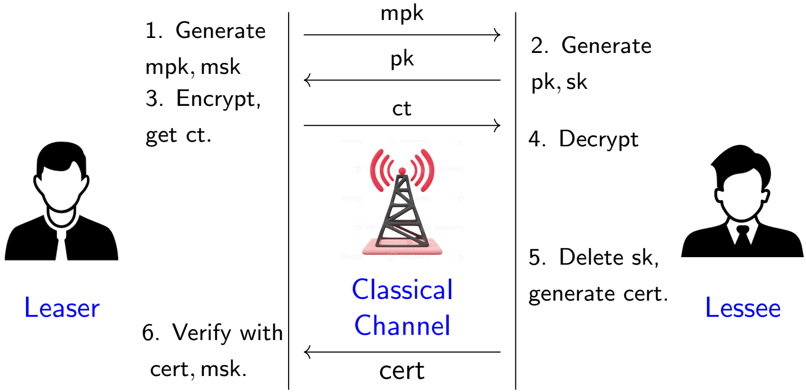
Soundness

AHB with poly modulus

Completeness

$\|\mathbf{e}_0\|/\|\mathbf{e}\|$ noticeable

What is Public Key Encryption with Secure Key Leasing over Classical Channel?[CGJL23]



How to generate key?[CGJL23]

Leaser

1. Generate

$k = (\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}_0)$

$\mathsf{msk} = \mathbf{T}_{\mathbf{A}}$.

$\xrightarrow{k}$

2. Generate

$|0, \mathbf{x}\rangle + |1, \mathbf{x} - \mathbf{s}\rangle$

and $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$.

let $\mathsf{pk} = (\mathbf{A}, \mathbf{b}, \mathbf{y})$,

$\mathsf{sk} = |0, \mathbf{x}\rangle + |1, \mathbf{x} - \mathbf{s}\rangle$

Lessee

How to generate key?[CGJL23]

Leaser

1. Generate

$k = (\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}_0)$

$\mathsf{msk} = \mathbf{T}_{\mathbf{A}}$.

$\xrightarrow{\quad k \quad}$

2. Generate

$|0, \mathbf{x}\rangle + |1, \mathbf{x} - \mathbf{s}\rangle$

and $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$.

let $\mathsf{pk} = (\mathbf{A}, \mathbf{b}, \mathbf{y})$,

$\mathsf{sk} = |0, \mathbf{x}\rangle + |1, \mathbf{x} - \mathbf{s}\rangle$

Lessee

Polynomial?

How to generate key?[CGJL23]

Leaser

1. Generate

$k = (\mathbf{A}, \mathbf{b} = \mathbf{A}\mathbf{s} + \mathbf{e}_0)$

$\mathsf{msk} = \mathbf{T}_{\mathbf{A}}$.

$\xrightarrow{k}$

2. Generate

$p_0|0, \mathbf{x}\rangle + p_1|1, \mathbf{x} - \mathbf{s}\rangle$

and $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$.

let $\mathsf{pk} = (\mathbf{A}, \mathbf{b}, \mathbf{y})$,

$\mathsf{sk} = p_0|0, \mathbf{x}\rangle + p_1|1, \mathbf{x} - \mathbf{s}\rangle$

Lessee

Polynomial?

How to encrypt and decrypt?[CGJL23]

With ciphertext $\mathsf{ct}_1 = \mathbf{r}^\top \mathbf{A}$, $\mathsf{ct}_2 = \mathbf{r}^\top \mathbf{b}$, $\mathsf{ct}_3 = \mathbf{r}^\top \mathbf{y} + \mathbf{e}' + \lceil q/2 \rceil \mu$:

$$\begin{tikzcd}
 & \begin{array}{c}\left(p_0|0,\mathbf{x}\rangle + p_1|1,\mathbf{x}-\mathbf{s}\rangle\right)\\ \otimes|\mathsf{ct}_3 - b\cdot \mathsf{ct}_2\rangle\end{array} \arrow[dl, "b=0"'] \arrow[dr, "b=1"] & \\
|\mathbf{r}^\top \mathbf{A}\mathbf{x} + \mathbf{r}^\top \mathbf{e} + \mathbf{e}' + \lceil q/2 \rceil \mu\rangle \arrow[d, "\text{with } \mathsf{ct}_1,\ \mathbf{x}"'] & & |\mathbf{r}^\top \mathbf{A}(\mathbf{x}-\mathbf{s}) + \mathbf{r}^\top(\mathbf{e}-\mathbf{e}_0) + \mathbf{e}' + \lceil q/2 \rceil \mu\rangle \arrow[d, "\text{with } \mathsf{ct}_1,\ \mathbf{x}-\mathbf{s}"] \\
|\left(\mathbf{r}^\top \mathbf{e} + \mathbf{e}' + \lceil q/2 \rceil \mu\right)/\lceil q/2 \rceil\rangle \arrow[dr, "="] & & |\left(\mathbf{r}^\top(\mathbf{e}-\mathbf{e}_0) + \mathbf{e}' + \lceil q/2 \rceil \mu\right)/\lceil q/2 \rceil\rangle \arrow[dl, "="'] \\
 & |\mu\rangle &
\end{tikzcd}$$

IND-CPA

$$\mathsf{Enc}(\mathsf{pk}, 0) \quad \xleftarrow{\text{indistinguishable}} \!\!\!\!\rightarrow \quad \mathsf{Enc}(\mathsf{pk}, 1)$$

$$\Uparrow$$

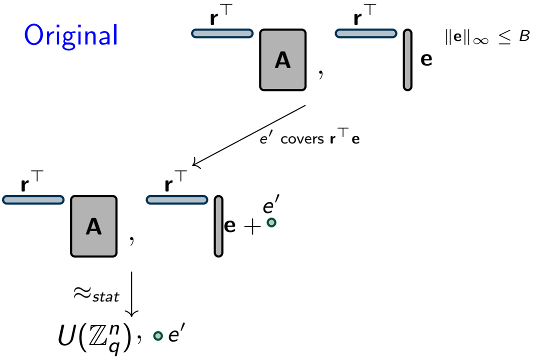
$$\mathsf{ct}_3 = \mathbf{r}^\top \mathbf{y} + \mathbf{e}' = \mathbf{r}^\top \mathbf{A}\mathbf{x} + \mathbf{r}^\top \mathbf{e} + \mathbf{e}' \quad \xleftarrow{\text{indistinguishable}} \!\!\!\!\rightarrow \quad \mathsf{ct}_3 = \mathbf{r}^\top \mathbf{y} + \mathbf{e}' + \lceil q/2 \rceil = \mathbf{r}^\top \mathbf{A}\mathbf{x} + \mathbf{r}^\top \mathbf{e} + \mathbf{e}' + \lceil q/2 \rceil$$

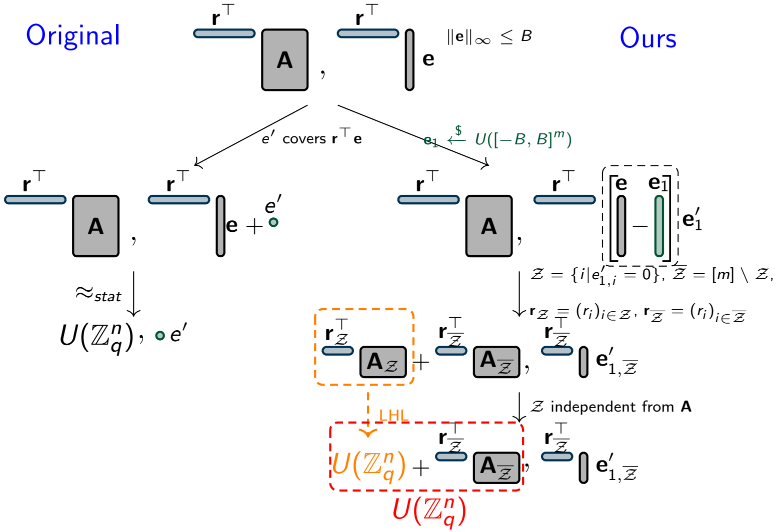
$$\Uparrow \text{ One-time Pad}$$

$$\mathbf{r}^\top \mathbf{A}\mathbf{x} + \mathbf{r}^\top \mathbf{e} + \mathbf{e}' \quad \xleftarrow{\text{indistinguishable}} \!\!\!\!\rightarrow \quad U(\mathbb{Z}_q)$$

$$\Uparrow \text{ [CGJL23]}$$

$$\mathbf{r}^\top \mathbf{A} \approx U(\mathbb{Z}_q^n) \text{ condition on } \mathbf{r}^\top \mathbf{e}$$


Original
$\mathbf{r}^\top$ $\mathbf{A}$, $\mathbf{r}^\top$ $\mathbf{e}$
$\|\mathbf{e}\|_\infty \leq B$
e' covers $\mathbf{r}^\top \mathbf{e}$
$\mathbf{r}^\top$ $\mathbf{A}$, $\mathbf{r}^\top$ $\mathbf{e} + e'$
$\approx_{stat}$
$U(\mathbb{Z}_q^n)$, e'


Original
Ours
$\mathbf{r}^\top \mathbf{A}$, $\mathbf{r}^\top \mathbf{e}$
$\|\mathbf{e}\|_\infty \le B$
e' covers $\mathbf{r}^\top \mathbf{e}$
$\mathbf{e}_1 \xleftarrow{\$} U([-B, B]^m)$
$\mathbf{r}^\top \mathbf{A}$, $\mathbf{r}^\top \mathbf{e} + e'$
$\approx_{stat}$
$U(\mathbb{Z}_q^n)$, e'
$\mathbf{r}^\top \mathbf{A}$, $\mathbf{r}^\top [\mathbf{e} - \mathbf{e}_1]$ $\mathbf{e}'_1$
$\mathcal{Z} = \{i | e'_{1,i} = 0\}$, $\overline{\mathcal{Z}} = [m] \setminus \mathcal{Z}$,
$\mathbf{r}_{\mathcal{Z}} = (r_i)_{i \in \mathcal{Z}}$, $\mathbf{r}_{\overline{\mathcal{Z}}} = (r_i)_{i \in \overline{\mathcal{Z}}}$
$\mathbf{r}_{\mathcal{Z}}^\top \mathbf{A}_{\mathcal{Z}} + \mathbf{r}_{\overline{\mathcal{Z}}}^\top \mathbf{A}_{\overline{\mathcal{Z}}}$, $\mathbf{r}_{\overline{\mathcal{Z}}}^\top \mathbf{e}'_{1,\overline{\mathcal{Z}}}$
$\mathcal{Z}$ independent from $\mathbf{A}$
LHL
$U(\mathbb{Z}_q^n) + \mathbf{r}_{\overline{\mathcal{Z}}}^\top \mathbf{A}_{\overline{\mathcal{Z}}}$, $\mathbf{r}_{\overline{\mathcal{Z}}}^\top \mathbf{e}'_{1,\overline{\mathcal{Z}}}$
$U(\mathbb{Z}_q^n)$

How to delete?[CGJL23]

$$|0, \mathbf{x}\rangle + |1, \mathbf{x} - \mathbf{s}\rangle$$

two keys

Hadamard Measure

$(b, \mathbf{x} - b\mathbf{s})$

$(c, \mathbf{d})$

Due to AHB, one getting $(c, \mathbf{d})$ cannot get either $(b, \mathbf{x} - b\mathbf{s})$, hence cannot decrypt.

Leasor: check $c = \mathbf{d}^\top(\mathbf{x} \oplus (\mathbf{x} - \mathbf{s}))$

How to delete?[CGJL23]

$$\begin{tikzcd} & {|0,\mathbf{x}\rangle + |1,\mathbf{x}-\mathbf{s}\rangle} \arrow[dl, dashed, "\text{two keys}"'] \arrow[dr, "\text{Hadamard Measure}"] & \\ (b,\mathbf{x}-b\mathbf{s}) & & (c,\mathbf{d}) \arrow[ll, "\times"] \arrow[d] \\ & & \text{Leasor: check } c = \mathbf{d}^{\top}(\mathbf{x} \oplus (\mathbf{x}-\mathbf{s})) \end{tikzcd}$$

Due to AHB, one getting $(c, \mathbf{d})$ cannot get either $(b, \mathbf{x} - b\mathbf{s})$, hence cannot decrypt.

Polynomial?

How to delete?[CGJL23]

$$p_0|0, \mathbf{x}\rangle + p_1|1, \mathbf{x} - \mathbf{s}\rangle$$

two keys

Hadamard
Measure

$(b, \mathbf{x} - b\mathbf{s})$ $\xleftarrow{\times}$ $(c, \mathbf{d})$

Due to AHB, one getting $(c, \mathbf{d})$
cannot get either $(b, \mathbf{x} - b\mathbf{s})$,
hence cannot decrypt.

Leasor: check
$c = \mathbf{d}^\top(\mathbf{x} \oplus (\mathbf{x} - \mathbf{s}))$

Polynomial?

How to delete?[CGJL23]

$$p_0|0, \mathbf{x}\rangle + p_1|1, \mathbf{x} - \mathbf{s}\rangle$$

two keys

Hadamard
Measure

$(b, \mathbf{x} - b\mathbf{s})$ $\quad\not\longleftarrow\quad$ $(c, \mathbf{d})$

Due to AHB, one getting $(c, \mathbf{d})$
cannot get either $(b, \mathbf{x} - b\mathbf{s})$,
hence cannot decrypt.

$$c = \mathbf{d}^\top(\mathbf{x} \oplus (\mathbf{x} - \mathbf{s}))$$
with probability at least 0.8.

Polynomial?

How to delete?[CGJL23]

$$p_0|0, \mathbf{x}\rangle + p_1|1, \mathbf{x} - \mathbf{s}\rangle$$

two keys

Hadamard
Measure

$(b, \mathbf{x} - b\mathbf{s})$ $\qquad$ $(c, \mathbf{d})$

Due to AHB, one getting $(c, \mathbf{d})$
cannot get either $(b, \mathbf{x} - b\mathbf{s})$,
hence cannot decrypt.

$c = \mathbf{d}^\top(\mathbf{x} \oplus (\mathbf{x} - \mathbf{s}))$
with probability at least 0.8.

Set a threshold 0.75

Polynomial?

Future works?

- Applications of Noticeable NTCF?
  - Efficient Revocable quantum digital signature based on Noticeable NTCF?
  - Quantum delegated computation based on NTCF?
- Applications of Secure key leasing?
  - Revocable broadcast encryption?
- ......

[AKN+23] S. Agrawal et al. "Public Key Encryption with Secure Key Leasing", Eurocrypt'23.

[AMR23] N. Alamati et al. "Candidate trapdoor claw-free functions from group actions with applications to quantum protocols", TCC'22.

[APV23] P. Ananth et al. "Revocable cryptography from learning with errors", TCC'23.

[BCM+18] Z. Brakerski et al. "A Cryptographic Test of Quantumness and Certifiable Randomness from a Single Quantum Device", FOCS'18.

[BD20] Z. Brakerski et al. "Hardness of LWE on general entropic distributions", Eurocrypt'20.

[BGKM+23] Z. Brakerski et al. "Simple tests of quantumness also certify qubits", Crypto'23.

[BKVV20] Z. Brakerski et al. "Simpler Proofs of Quantumness", TQC'20.

[CGJL23] O. Chardouveils et al. "Quantum key leasing for pke and fhe with a classical lessor", QCrypt'24.

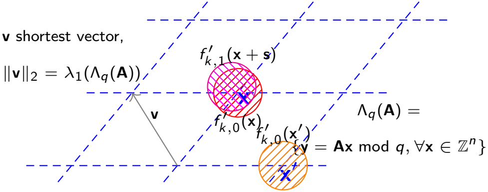
[GR02] L. Grover et al."Creating superpositions that correspond to efficiently integrable probability distributions", https://arxiv.org/pdf/quant-ph/0208112

[KMC+22] G. Kahanamoku-Meyer et al. "Classically verifiable quantum advantage from a computational bell test", Nature Physics'22.

[KLVY23] Y. Kalai et al. "Quantum advantage from any non-local game", STC'23.

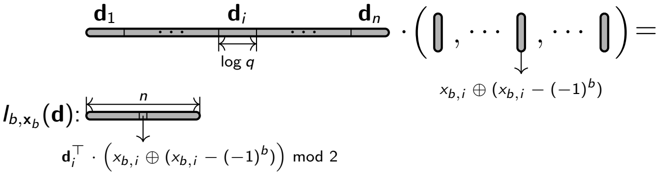
[Reg05] O. Regev "On lattices, learning with errors, random linear codes, and cryptography", STOC'05.

Questions?



- Injection: Since $\|\mathbf{e}\| \ll \lambda_1(\Lambda_q(\mathbf{A}))$, for $\mathbf{x} \neq \mathbf{x}'$, $supp(f'_{k,b}(\mathbf{x})) \bigcap supp(f'_{k,b'}(\mathbf{x}')) = \varnothing,\ b, b' \in \{0,1\}$.

- Given $(b, \mathbf{x}_b, \mathbf{d})$, compute
$I_{b,\mathbf{x}_b}(\mathbf{d}) = \left(\mathbf{d}_i^\top \cdot \left(x_{b,i} \oplus (x_{b,i} - (-1)^b)\right) \bmod 2\right)_{i\in[n]}$



- [BCM+18] shows $c = \mathbf{d}^\top \cdot (\mathbf{x}_0 \oplus \mathbf{x}_1) = \langle I_{b,\mathbf{x}_b}(\mathbf{d}), \mathbf{s}\rangle$.