Constrained Pseudorandom Functions for Inner-Product Predicates from Weaker Assumptions

Sacha Servan-Schreiber

MIT CSAIL

This talk: New ways of building constrained PRFs

Overview

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- **Background on PRFs and constrained PRFs**

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- **A secret sharing perspective on constrained PRFs**

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- **Our framework and instantiations**

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- Evaluation

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Overview

- Background on PRFs and constrained PRFs
- A secret sharing perspective on constrained PRFs
- Our framework and instantiations
- Evaluation
- Open problems

Constrained PRFs

Standard PRF security

A function $F : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ is a PRF if:

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Setup phase (one time)

Challenger

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Query phase (repeatable)

$\xleftarrow{x_i}$

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Query phase (repeatable)

$\xleftarrow{\quad x_i \quad}$ Distinguisher

4. $$y_i := \begin{cases} F\left(k,\ x_i\right) & \text{if } b = 0 \\ R\left(x_i\right) & \text{if } b = 1 \end{cases}$$

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$\xleftarrow{x_i}$

$\xrightarrow{y_i}$

Distinguisher

Constrained Pseudorandom Function (**C**PRF)

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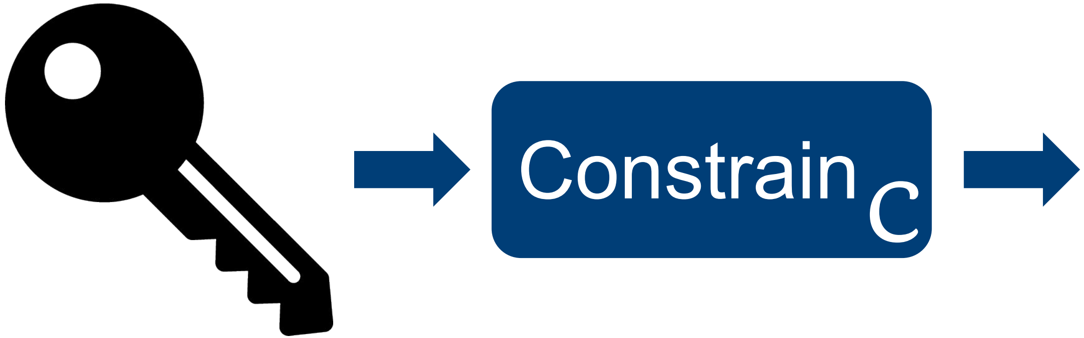

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Constrained Pseudorandom Function (**C**PRF)

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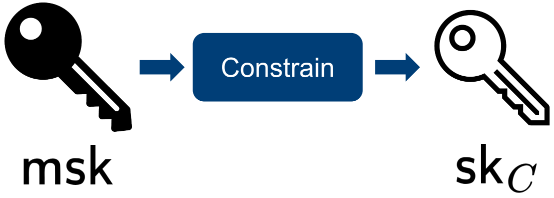
Master PRF Key **Constrained PRF Key**

Constrained key 🔑 can be used to evaluate $F(🔑, x)$ for all $x \in \mathcal{X}$ where $C(x) = 0$

Constrained Pseudorandom Function (CPRF)



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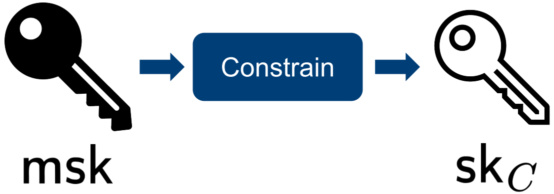

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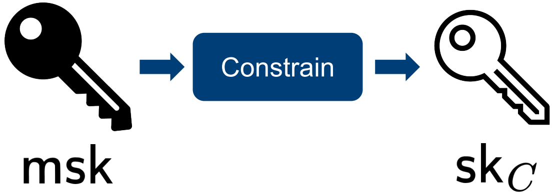

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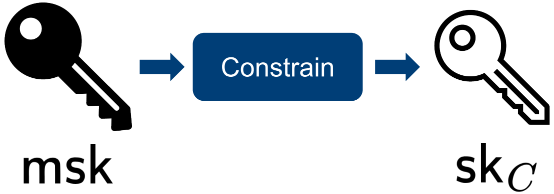

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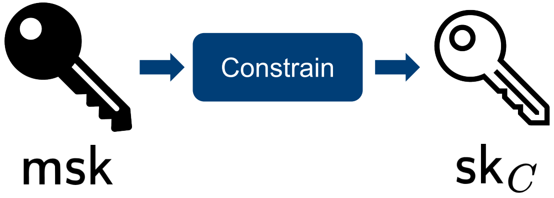

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Hiding (optional): C is hidden given sk_C

Constrained Pseudorandom Function (**C**PRF)

Our focus: Inner-product predicates

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Constrained Pseudorandom Function (**C**PRF)

Our focus: Inn

Predicate satisfied if and only if the inner product is zero

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- **t-CNF** predicates (for constant t) [DKN+20]

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- **Bit-fixing** predicates (special case of t-CNF) [DKN+20]

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Can be used to build other predicates, generically:

- **t-CNF** predicates (for constant t) [DKN+20]
- **Bit-fixing** predicates (special case of t-CNF) [DKN+20]
- **Matrix-product** predicates (folklore & this work)

Security Definitions

(1-key, selective) CPRF security game

Setup phase (one time)

Challenger

Distinguisher

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1. msk $\xleftarrow{R} \mathcal{K}$

Distinguisher

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Challenger

$\xleftarrow{C}$

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1. $\mathsf{msk} \xleftarrow{R} \mathcal{K}$
2. $R \xleftarrow{R} \mathcal{F}uns\,(\mathcal{X}, \mathcal{Y})$
3. $\mathsf{sk}_C \leftarrow \mathsf{Constrain}\,(\mathsf{msk}, C)$

Challenger

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Query phase (repeatable)

C

sk_C

Distinguisher

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Query phase (repeatable)

Challenger

C

sk_C

Distinguisher

x_i

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1. $\mathsf{msk} \xleftarrow{R} \mathcal{K}$
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Query phase (repeatable)

5. $y_i := \begin{cases} F\left(\mathsf{msk},\ x_i\right) & \text{if } b = 0 \\ R\left(x_i\right) & \text{if } b = 1 \end{cases}$

Need: $C\left(x_i\right) \neq 0$ for all $x_i \in \mathcal{X}$.

C ←

sk_C →

x_i ←

Distinguisher

(1-key, selective) CPRF security game

Setup phase (one time)

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$\xleftarrow{\quad C \quad}$

$\xrightarrow{\quad \mathsf{sk}_C \quad}$

Distinguisher

$\xleftarrow{\quad x_i \quad}$

$\xrightarrow{\quad y_i \quad}$

(1-key, adaptive) CPRF security game

Setup phase (one time)

Challenger

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2. $R \xleftarrow{R} \mathcal{F}uns\,(\mathcal{X}, \mathcal{Y})$
3. $\mathsf{sk}_C \leftarrow$ Constrai
4. $b \xleftarrow{R} \{0, 1\}$

Adaptive security lets the adversary query the challenger *before* sending the constraint.

Query phase (repeatable)

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Need: $C\,(x_i) \neq 0$ for all $x_i \in \mathcal{X}$.

$\xleftarrow{C}$

$\xrightarrow{\mathsf{sk}_C}$

$\xleftarrow{x_i}$

$\xrightarrow{y_i}$

Distinguisher

The current landscape

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	Assumptions	Security	Hiding	Comments

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[AMN+18]	L-DDHI + DDH	Selective	✗	For NC^1

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[CMPR23]	DCR	Selective	✓	

Can we build CPRFs from weaker assumptions?

New results

	Assumptions	Security	Hiding	Comments
Generic CPRFs	LWE or iO	Selective	✓	For NC and P/poly
[AMN+18]	L-DDHI + DDH	Selective	✗	For NC^1
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[CMPR23]	DCR	Selective	✓	

Can we build CPRFs for inner-product predicates using random oracles?

New results

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This work	**ROM**	**Adaptive**	**✓**	

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[CMPR23]	DCR	Selective	✓	
This work	**ROM**	**Adaptive**	✓	

Can we build CPRFs for inner-product predicates from DDH?

New results

	Assumptions	Security	Hiding	Comments
Generic CPRFs	LWE or iO	Selective	✓	For NC and P/poly
[AMN+18]	L-DDHI + DDH	Selective	✗	For NC^1
[AMN+18]	L-DDHI + ROM	Adaptive	✗	For NC^1
[CMPR23]	DCR	Selective	✓	
This work	**ROM**	**Adaptive**	✓	
This work	**DDH**	**Selective**	✓	

New results

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[AMN+18]	L-DDHI + DDH	Selective	✗	For NC[1]
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[CMPR23]	DCR	Selective	✓	
This work	**ROM**	**Adaptive**	✓	
This work	**DDH**	**Selective**	✓	

Can we build CPRFs for inner-product predicates from LPN?

New results

	Assumptions	Security	Hiding	Comments
Generic CPRFs	LWE or iO	Selective	✓	For NC and P/poly
[AMN+18]	L-DDHI + DDH	Selective	✗	For NC^1
[AMN+18]	L-DDHI + ROM	Adaptive	✗	For NC^1
[CMPR23]	DCR	Selective	✓	
This work	**ROM**	**Adaptive**	✓	
This work	**DDH**	**Selective**	✓	
This work	**VDLPN**	**Selective**	✓	Weak CPRF (random inputs)

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This work	**ROM**	**Adaptive**	✓	
This work	**DDH**	**Selective**	✓	
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Can we build CPRFs for inner-product predicates from OWF?

New results

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[AMN+18]	L-DDHI + ROM	Adaptive	✗	For NC^1
[CMPR23]	DCR	Selective	✓	
This work	**ROM**	**Adaptive**	✓	
This work	**DDH**	**Selective**	✓	
This work	**VDLPN**	**Selective**	✓	Weak CPRF (random inputs)
This work	**OWF**	**Selective**	✓	Only for a polynomial domain

A secret sharing perspective on constrained PRFs

A secret-sharing perspective

Idea: view msk and $\mathsf{sk}_{\mathbf{z}}$ as being secret shares of the constraint vector **z**:

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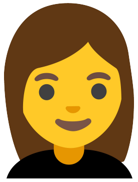
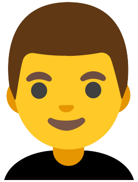


Alice

$$\mathbf{z}_0 - \mathbf{z}_1 = \mathbf{z}$$



Bob

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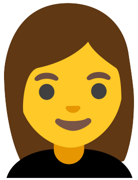
Idea: view msk and $\mathsf{sk}_{\mathbf{z}}$ as being secret shares of the constraint vector **z**:



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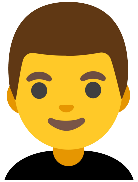

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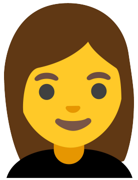

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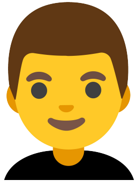

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One share (master share) can be sampled independently of the constraint vector **z**

$$\mathsf{sk}_{\mathbf{z}} = \mathbf{z}_1$$

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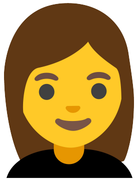

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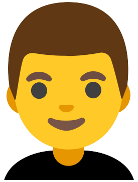

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For an input $\mathbf{x}$:

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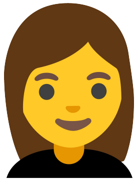

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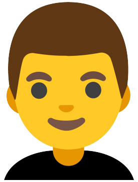
$$\mathbf{z}_0 - \mathbf{z}_1 = \mathbf{z}$$



Bob

$$\mathsf{msk} = \mathbf{z}_0$$

For an input $\mathbf{x}$:

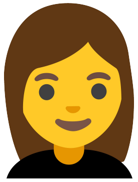
$$\mathsf{sk}_{\mathbf{z}} = \mathbf{z}_1$$

$$k_A := \langle \mathbf{z}_0, \mathbf{x} \rangle$$

$$k_B := \langle \mathbf{z}_1, \mathbf{x} \rangle$$

A secret-sharing perspective

Idea: view msk and $\mathsf{sk}_{\mathbf{z}}$ as being secret shares of the constraint vector **z**:



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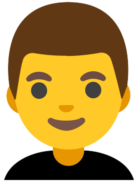

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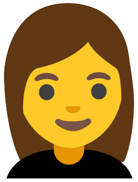

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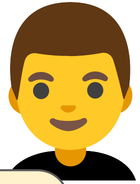

ob

$$k_A = k_B \text{ when } \langle \mathbf{z}, \mathbf{x} \rangle = 0$$

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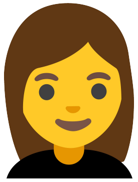

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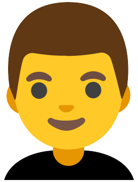

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$$\mathsf{msk} = \mathbf{z}_0$$

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$$F(k_A, \mathbf{x})$$

For an input $\mathbf{x}$:

$$k_A - k_B = \langle \mathbf{z}, \mathbf{x} \rangle$$

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$$k_B := \langle \mathbf{z}_1, \mathbf{x} \rangle$$

$$F(k_B, \mathbf{x})$$

Same PRF output

A first attempt at constructing a CPRF

$$\mathsf{msk} := \boldsymbol{z_0}$$

A first attempt at constructing a CPRF

For a constraint vector $\mathbf{z}$:

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A first attempt at constructing a CPRF

For a constraint vector $\mathbf{z}$:

$$\mathsf{msk} := \mathbf{z_0}$$

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Is this secure?

A first attempt at constructing a CPRF

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Is this secure? No, because $\mathbf{z_0} = \mathbf{z_1} + \mathbf{z}$

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Is this secure? No, because $\mathbf{z_0} = \mathbf{z_1} + \mathbf{z}$; **possible to recover the master key!**

A second attempt at constructing a CPRF

For a constraint vector $\mathbf{z}$:

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A second attempt at constructing a CPRF

For a constraint vector $\mathbf{z}$:

$$\Delta \xleftarrow{R} \mathbb{F}$$

$$\mathsf{msk} := \mathbf{z_0}$$

$$\mathsf{sk}_{\mathbf{z}} := \mathbf{z_1} = \mathbf{z_0} - \Delta\mathbf{z}$$

Eval($\mathsf{msk}, \mathbf{x}$ **):**

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$$\langle \mathbf{z}_0, \mathbf{x} \rangle = \langle \Delta\mathbf{z}, \mathbf{x} \rangle + \langle \mathbf{z}_1, \mathbf{x} \rangle = \Delta\langle \mathbf{z}, \mathbf{x} \rangle + \langle \mathbf{z}_1, \mathbf{x} \rangle = \langle \mathbf{z}_1, \mathbf{x} \rangle$$

Problem: keys are highly correlated

For a constraint vector $\mathbf{z}$:

$$\Delta \xleftarrow{R} \mathbb{F}$$

$$\mathsf{msk} := \mathbf{z_0}$$

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Evaluations are using correlated keys

Problem: generally insecure if keys are correlated

Problem: keys are highly correlated

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Evaluations are using correlated keys

Solution: use a related-key secure PRFs

A general framework

RKA-secure PRFs

Regular security for a PRF

A function $F : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ is a secure PRF if:

Regular security for a PRF

A function $F : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$ is a secure PRF if:

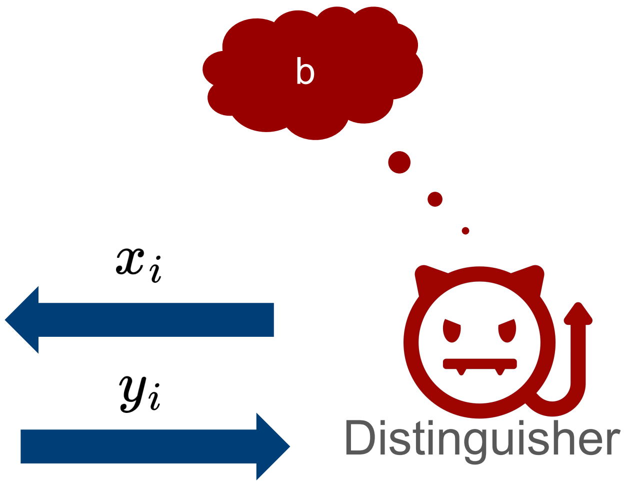
Setup phase (one time)

Challenger

1. $k \overset{R}{\leftarrow} \mathcal{K}$
2. $R \overset{R}{\leftarrow} \mathcal{F}uns(\mathcal{X}, \mathcal{Y})$
3. $b \overset{R}{\leftarrow} \{0, 1\}$

Query phase (repeatable)

5. $y_i := \begin{cases} F(k,\ x_i) & \text{if } b = 0 \\ R(x_i) & \text{if } b = 1 \end{cases}$



Related Key Attack (RKA) security for a PRF

A function $F : \mathcal{K} \times \mathcal{X} \rightarrow \mathcal{Y}$ is an **RKA-secure** PRF if:

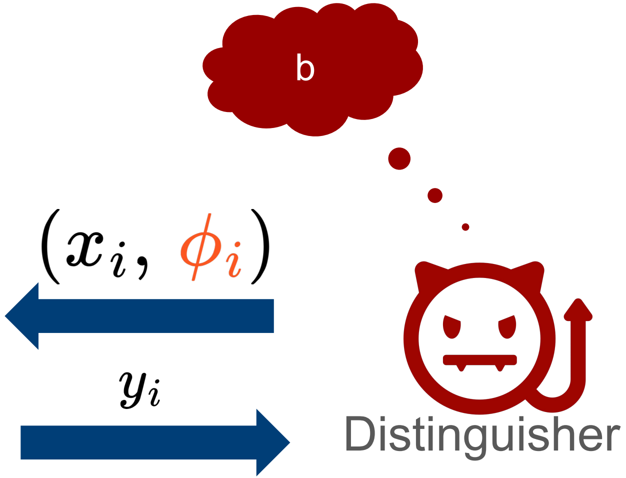
Setup phase (one time)

Challenger

1. $k \xleftarrow{R} \mathcal{K}$
2. $R \xleftarrow{R} \mathcal{F}uns\left(\left(\mathcal{X}, \Phi\right), \mathcal{Y}\right)$
3. $b \xleftarrow{R} \{0, 1\}$

Query phase (repeatable)

5. $y_i := \begin{cases} F\left(\phi_i\left(k\right), x_i\right) & \text{if } b = 0 \\ R\left(x_i, \phi_i\right) & \text{if } b = 1 \end{cases}$



For a class of key derivation functions $\Phi : \mathcal{K} \rightarrow \mathcal{K}$

Solution: Use a PRF with related-key security

The inner product $\langle \mathbf{z_1}, \mathbf{x} \rangle = \langle \mathbf{z_0}, \mathbf{x} \rangle - \mathbf{\Delta} \langle \mathbf{z}, \mathbf{x} \rangle$

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$\mathsf{msk} := \mathbf{z_0}$

Eval($\mathsf{msk}, \mathbf{x}$**):**

1. $k := \langle \mathbf{z_0}, \mathbf{x} \rangle$
2. Return $F(k, \mathbf{x})$

$\mathsf{sk}_{\mathbf{z}} := \mathbf{z_0} - \Delta \mathbf{z} = \mathbf{z_1}$

CEval($\mathsf{sk}_{\mathbf{z}}, \mathbf{x}$**):**

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Need F to be RKA-secure for
Affine Functions

Reduction to RKA security

Step 1: The (1 key, selective) CPRF security game

$\Delta \xleftarrow{R} \mathcal{K}$

$\mathbf{z}_0 \xleftarrow{R} \mathcal{K}^\ell$

$\mathbf{z}_1 := \mathbf{z}_0 - \Delta \mathbf{z}$

$\xleftarrow{\quad \mathbf{z} \quad}$

$\xrightarrow{\quad \mathsf{sk}_{\mathbf{z}} := \mathbf{z}_1 \quad}$

$\xleftarrow{\quad \mathbf{x}_i \quad}$

$\xrightarrow{\quad F\left(\langle \mathbf{z}_0, \mathbf{x}_i \rangle, \mathbf{x}_i\right) \quad}$

Step 2: Change definition of $\mathbf{z}_0$ to be in terms of $\mathbf{z}_1$

$$\Delta \xleftarrow{R} \mathcal{K}$$

$$\mathbf{z}_1 \xleftarrow{R} \mathcal{K}^\ell$$

$$\mathbf{z}_0 := \mathbf{z}_1 + \Delta \mathbf{z}$$

$$\xleftarrow{\quad \mathbf{z} \quad}$$

$$\xrightarrow{\quad \mathsf{sk}_{\mathbf{z}} := \mathbf{z}_1 \quad}$$

$$\xleftarrow{\quad \mathbf{x}_i \quad}$$

$$\xrightarrow{\quad F\left(\langle \mathbf{z}_0, \mathbf{x}_i \rangle, \mathbf{x}_i\right) \quad}$$

Step 3: Define the inner-product as an affine function

$$\Delta \overset{R}{\leftarrow} \mathcal{K}$$

$$\mathbf{z_1} \overset{R}{\leftarrow} \mathcal{K}^{\ell}$$

$$\mathbf{z_0} := \mathbf{z_1} + \Delta \mathbf{z}$$

$$b_i := \sum_{j=1}^{\ell} \left(\mathbf{z_1}[j] \cdot \mathbf{x}_i[j]\right)$$

$$a_i := \sum_{j=1}^{\ell} \left(\mathbf{z}[j] \cdot \mathbf{x}_i[j]\right)$$

$$\xleftarrow{\mathbf{z}}$$

$$\xrightarrow{\mathsf{sk}_{\mathbf{z}} := \mathbf{z_1}}$$

$$\xleftarrow{\mathbf{x}_i}$$

$$\xrightarrow{F\left(a_i \Delta + b_i, \mathbf{x_i}\right)}$$

Step 3: Define the inner-product as an affine function

$\Delta \xleftarrow{R} \mathcal{K}$

$\mathbf{z_1} \xleftarrow{R} \mathcal{K}^{\ell}$

$\mathbf{z_0} := \mathbf{z}$

$\phi_i : \Delta \mapsto a_i \Delta + b_i$

$b_i := \sum_{j=1}^{\ell} \left(\mathbf{z_1}[j] \cdot \mathbf{x}_i[j] \right)$

$a_i := \sum_{j=1}^{\ell} \left(\mathbf{z}[j] \cdot \mathbf{x}_i[j] \right)$

$\xleftarrow{\mathbf{z}}$

$\xrightarrow{\mathsf{sk}_{\mathbf{z}} := \mathbf{z_1}}$

$\xleftarrow{\mathbf{x}_i}$

$\xrightarrow{F(\phi_i(\Delta), \mathbf{x}_i)}$

Step 4: Reduce to RKA security

The key Δ is not sampled anymore…

$$\mathbf{z}_1 \xleftarrow{R} \mathcal{K}^\ell$$

$$a_i := \sum_{j=1}^{\ell} \left(\mathbf{z}_1[j] \cdot \mathbf{x}_i[j]\right)$$

$$b_i := \sum_{j=1}^{\ell} \left(\mathbf{z}[j] \cdot \mathbf{x}_i[j]\right)$$

Query RKA PRF challenger on input:

$$\left(\phi_i := (a_i, b_i),\ \mathbf{x}_\mathrm{i}\right)$$

And get back: $F\left(\phi_i(\Delta), \mathbf{x}_i\right)$

$\mathsf{sk}_\mathbf{z} := \mathbf{z}_1$

$\mathbf{x}_i$

$F\left(\phi_i(\Delta), \mathbf{x}_i\right)$

Constructions from RKA-secure PRFs

Constructions of CPRFs from RKA-secure PRFs

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In the **random oracle model (ROM)**

Easy to construct RKA-secure PRFs in the ROM

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From **DDH**

Directly follows from the affine RKA-secure construction of [ABP+14]

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Practical Constructions

From **Variable Density LPN**

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Almost directly follows from OWF-based RKA secure construction of [AW14]

Evaluation

Artifact Badges: Available, Functional, and Reproduced.

https://github.com/sachaservan/cprf

Evaluation of the **random oracle** based CPRF

ℓ **(length of vector)**	**Evaluation time**
10	2 μs
50	10 μs
100	19 μs
500	98 μs
1000	200 μs

Implemented in Go (v1.20) without any significant optimizations

Bottleneck: inner-product computation in the finite field

Evaluation of the **DDH-based** CPRF

ℓ **(length of vector)**	**Evaluation time**
10	8 ms
50	11 ms
100	16 ms
500	46 ms
1000	85 ms

Implemented in Go (v1.20) without any significant optimizations

Bottleneck: exponentiations in the group

Open Questions

Concrete applications for CPRFs with inner product predicates?

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Extending constructions to NC^1 constraints?

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Instantiating the framework under more assumptions?

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Extending constructions to NC^1 constraints?

Open Questions

Instantiating the framework under more assumptions?

OWF construction with superpolynomial domain?

Thank you!

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ePrint: ia.cr/**2024/058**

Constrained Pseudorandom Functions for Inner-Product Predicates from Weaker Assumptions

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MIT

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