Verifiable Secret Sharing from Symmetric Key Cryptography with Improved Optimistic Complexity

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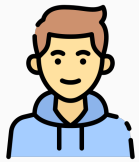
Secret Sharing

dealer



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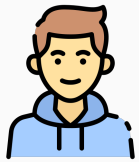

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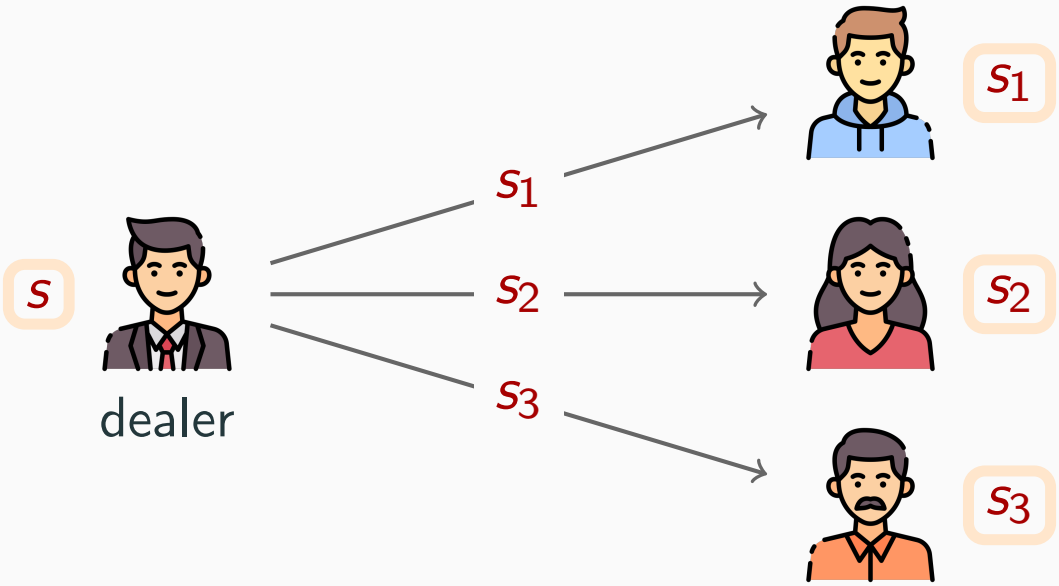

Secret Sharing

s

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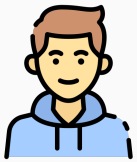

s_1

s_2

s_3

t privacy

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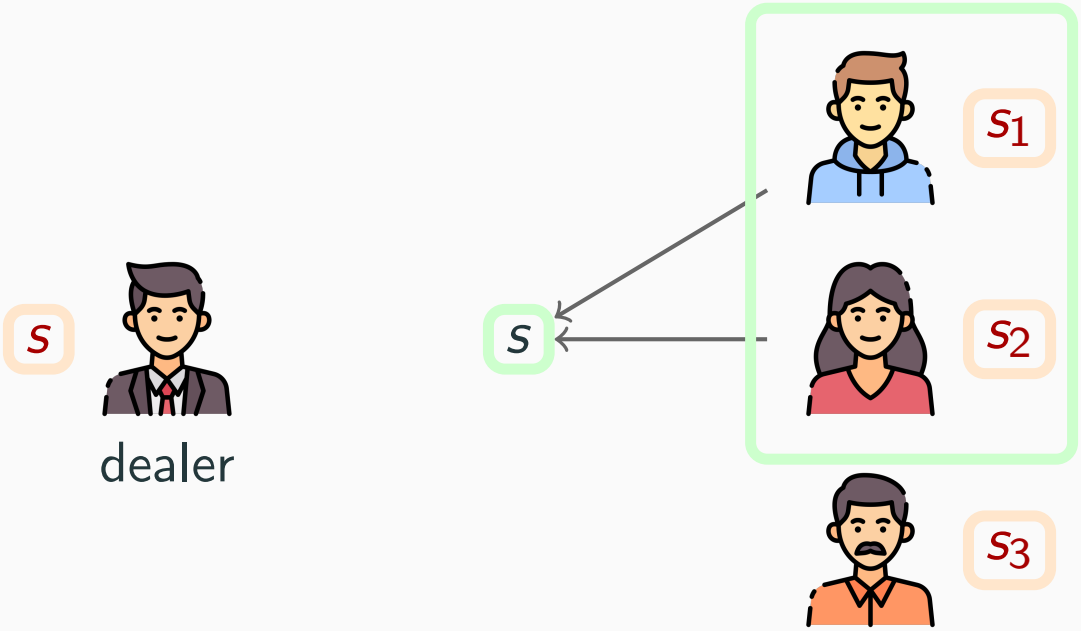

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$t + 1$ reconstruction

Shamir Secret Sharing

$s \in \mathbb{F}$ and let $\alpha_0, \alpha_1, \ldots, \alpha_n \in \mathbb{F}$ be distinct points.

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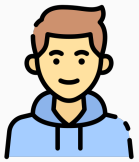

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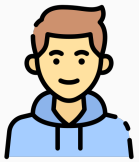

$s \in \mathbb{F}$

$f \leftarrow^{\$} \mathbb{F}[x]_t$

$f(\alpha_0) = s$

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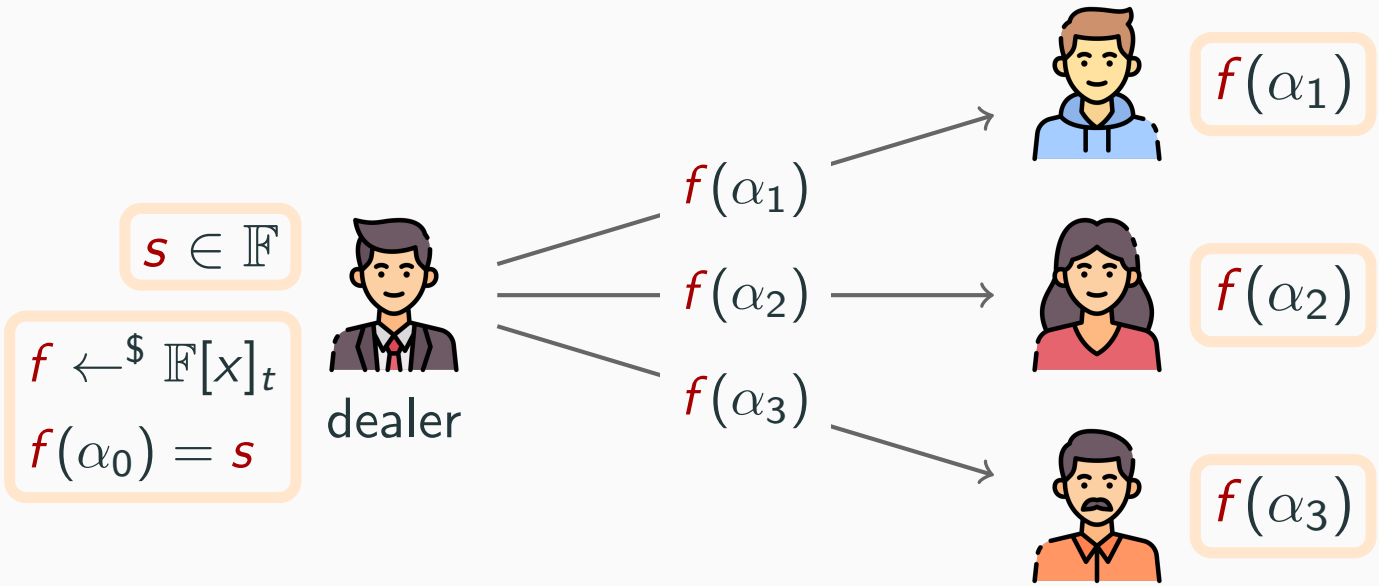

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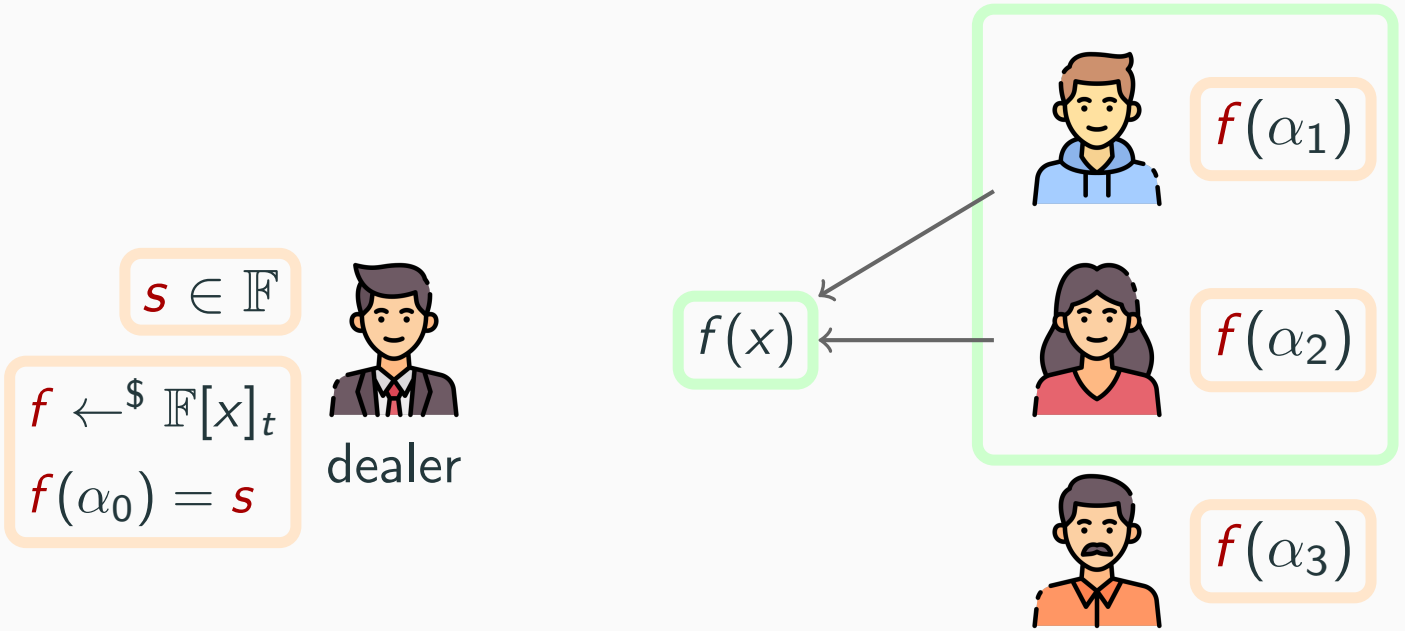

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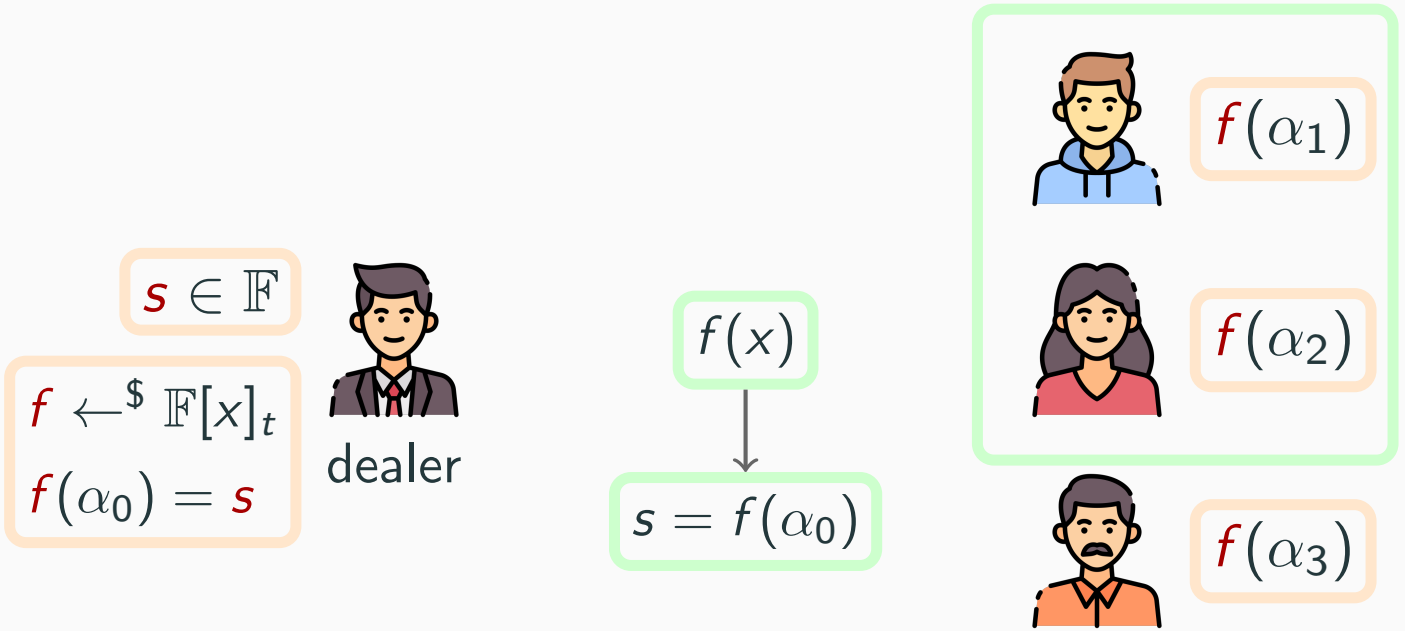

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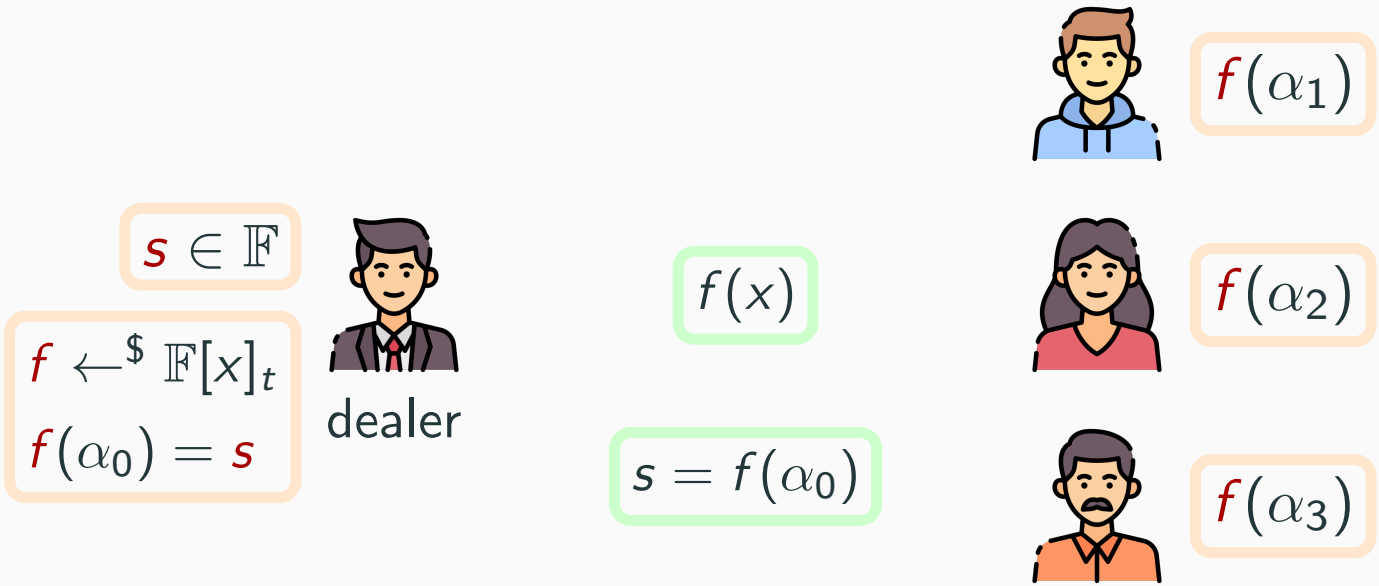

Works over any ring R if $\{\alpha_0, \alpha_1, \ldots, \alpha_n\}$ is an **exceptional set**, i.e. $\alpha_i - \alpha_j$ is invertible.

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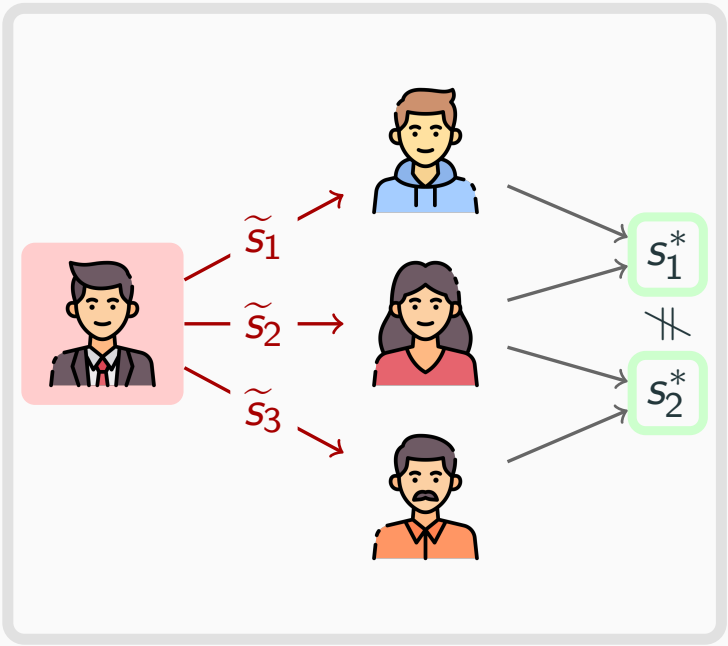

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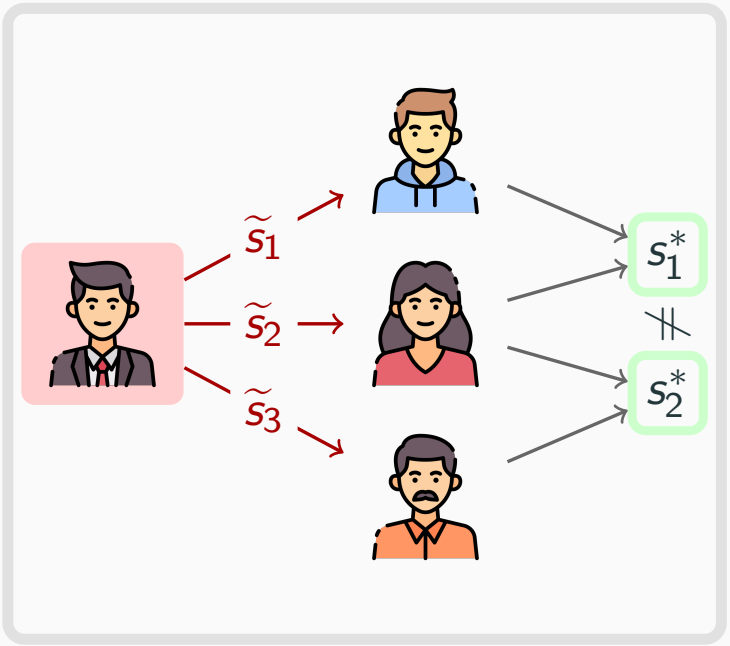

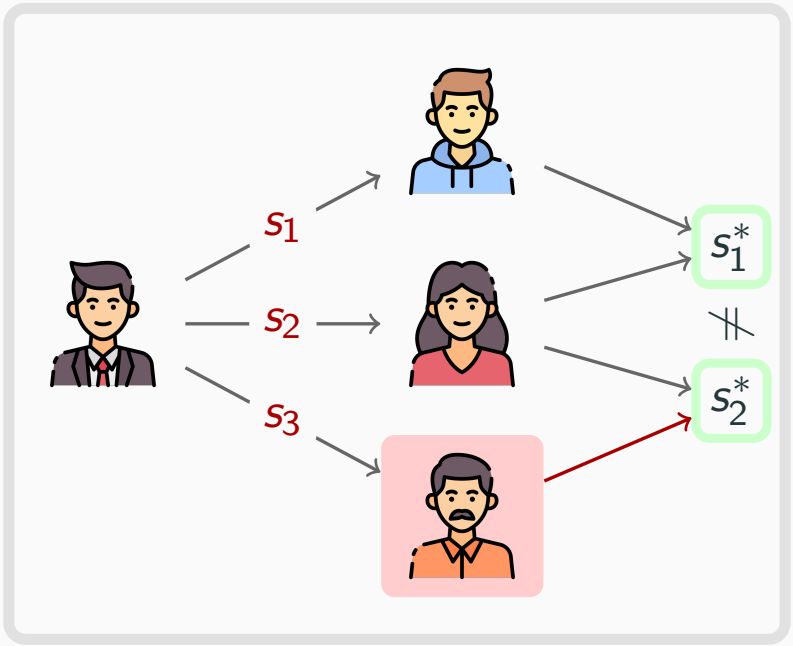

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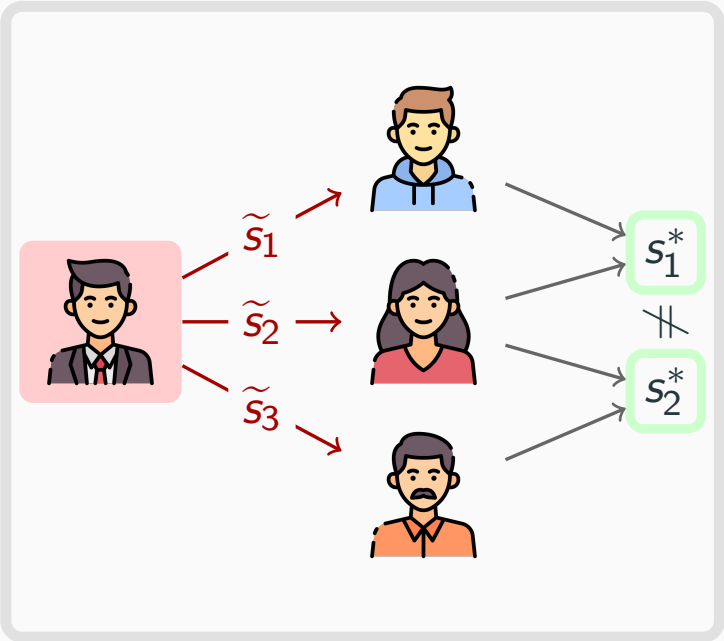

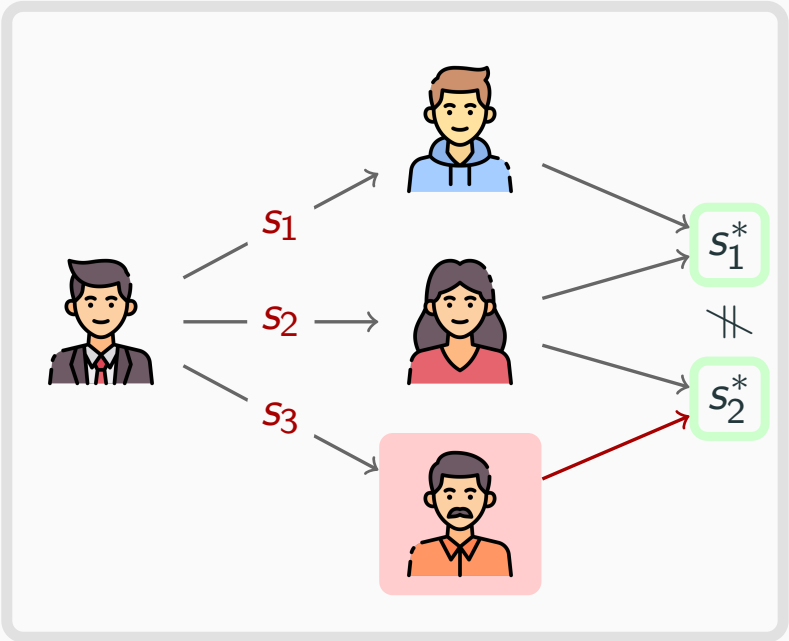

Issue for applications such as **Distributed Key Generation**.

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- **Strong Commitment:** As before, but after `Share` honest users also get private shares consistent with s.

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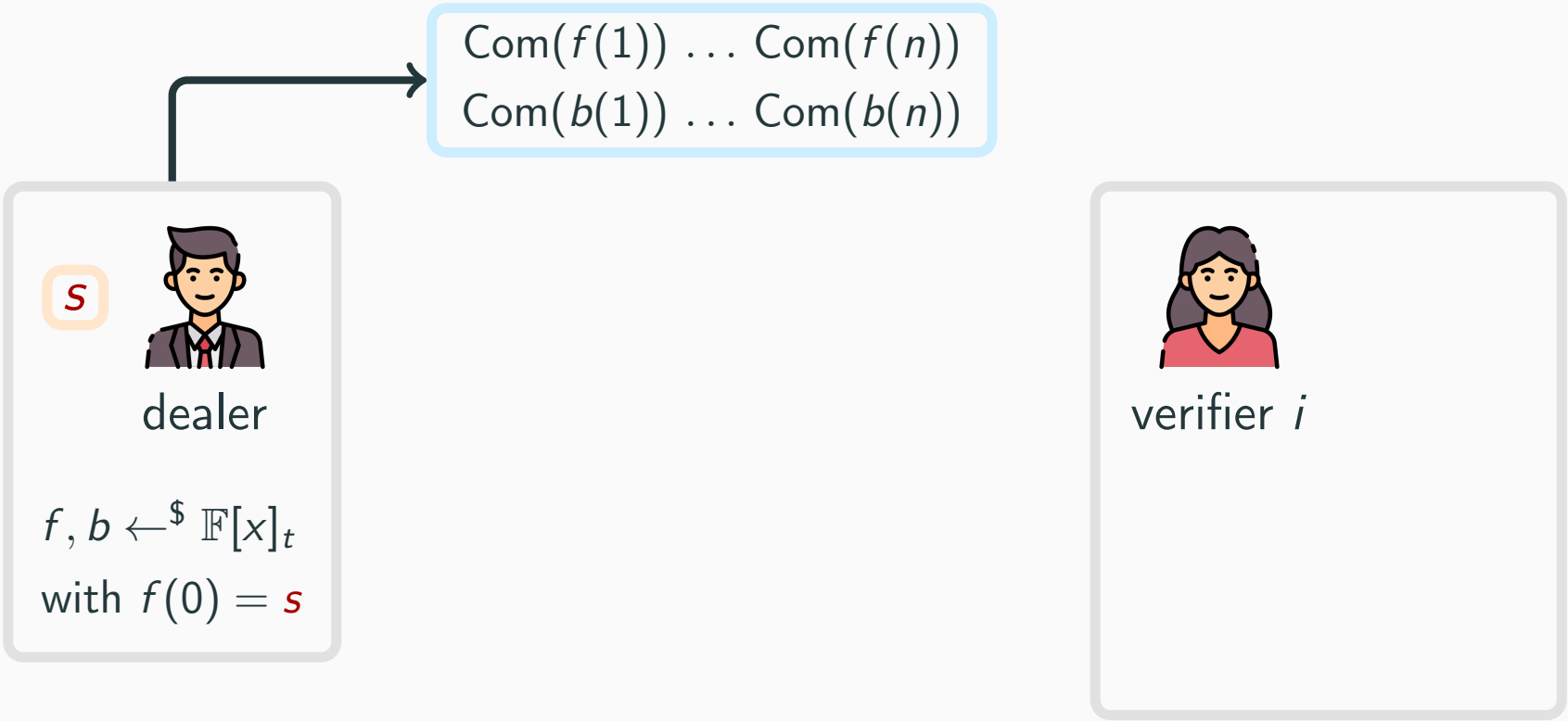

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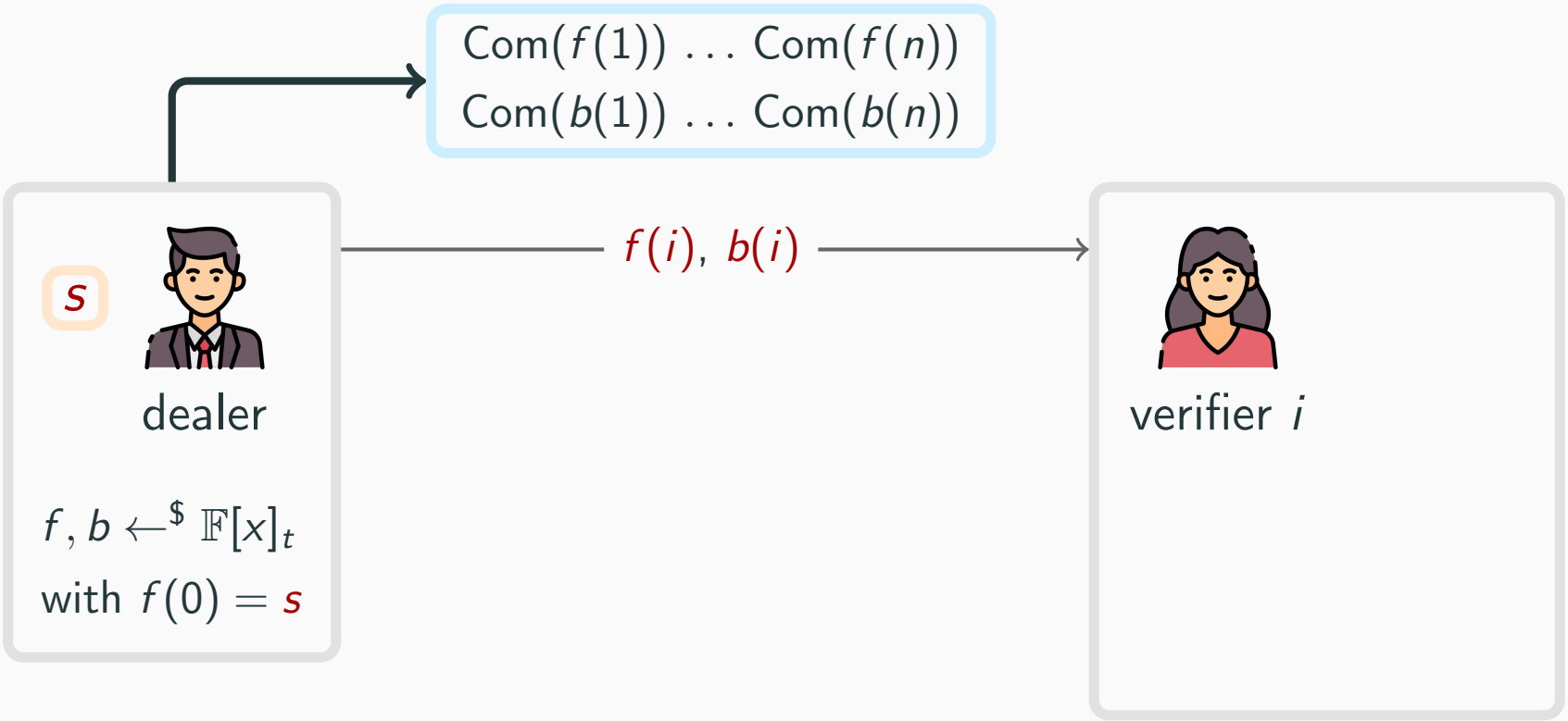

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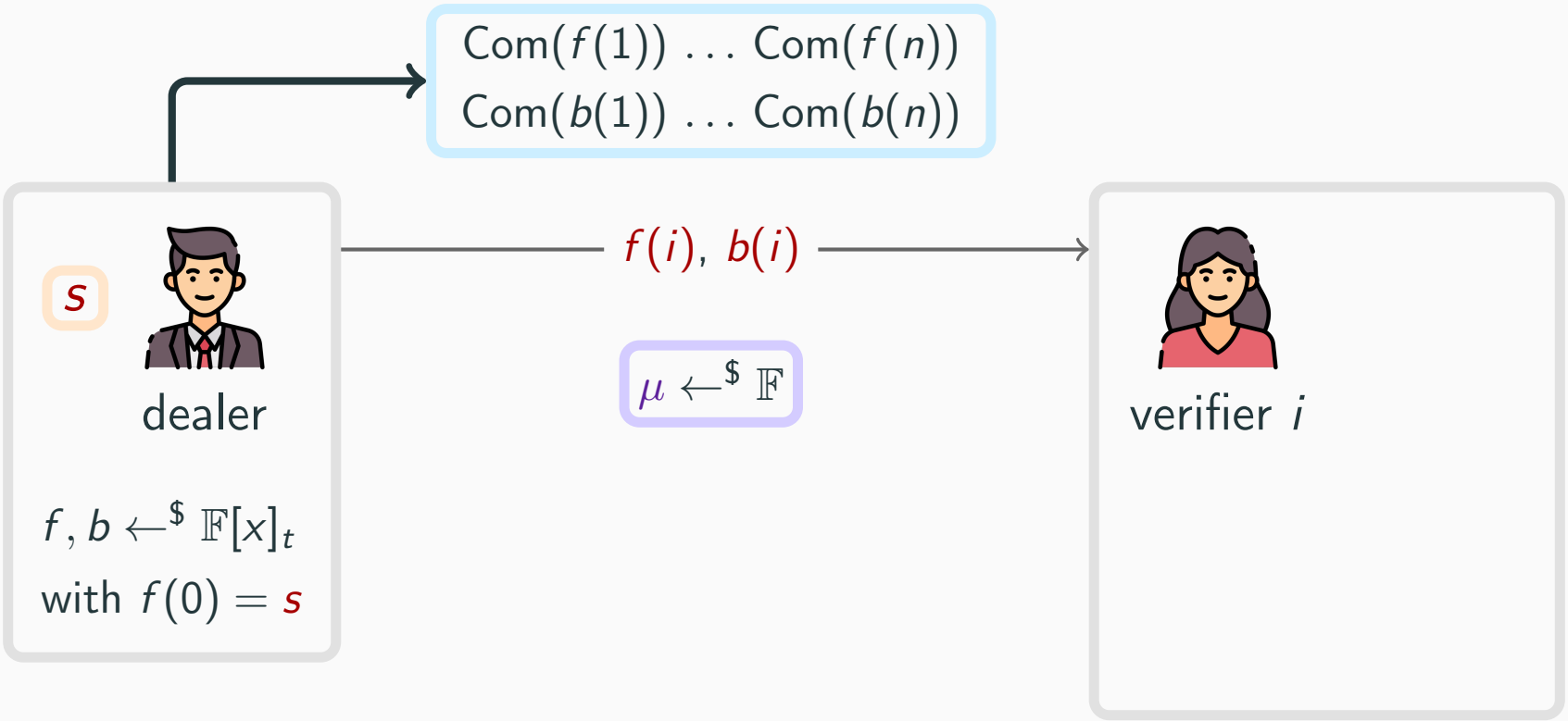

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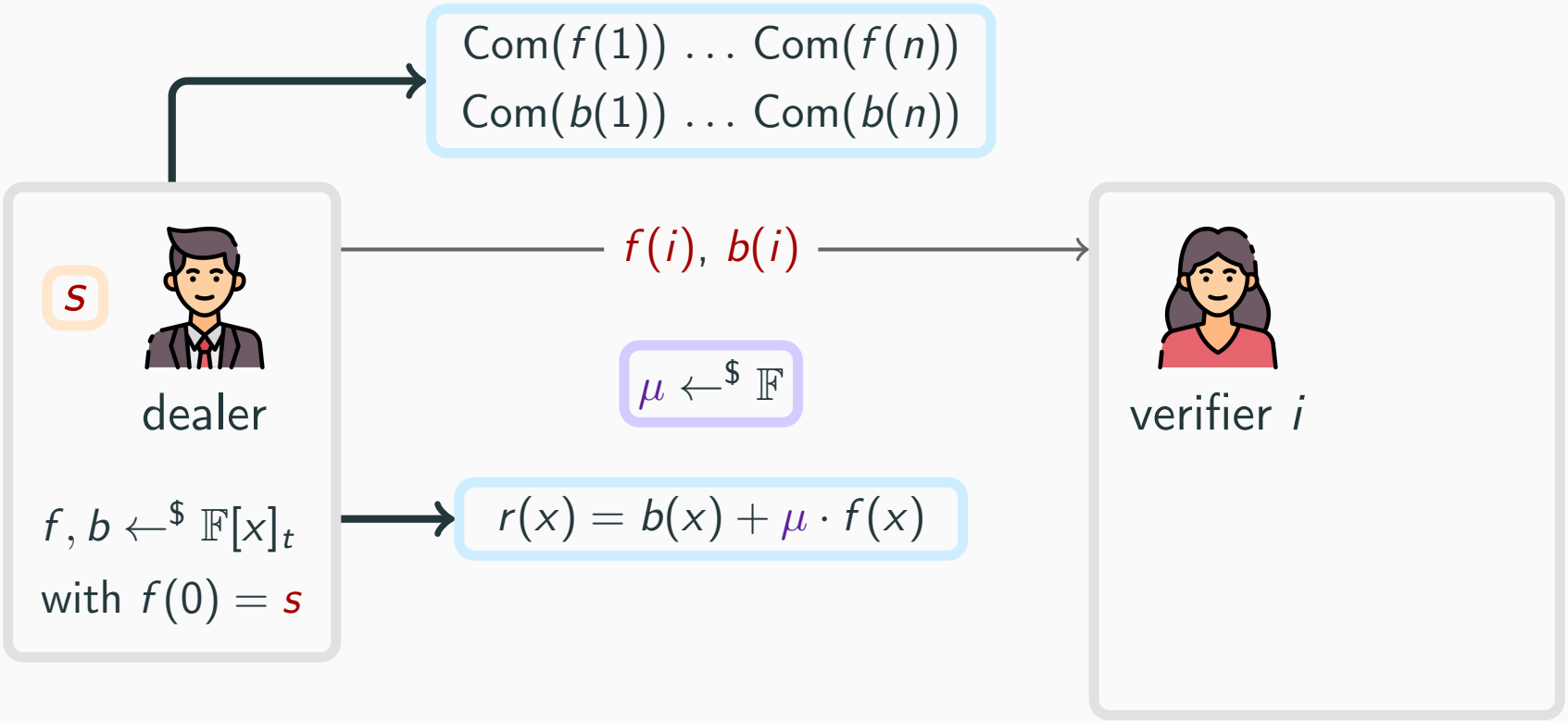

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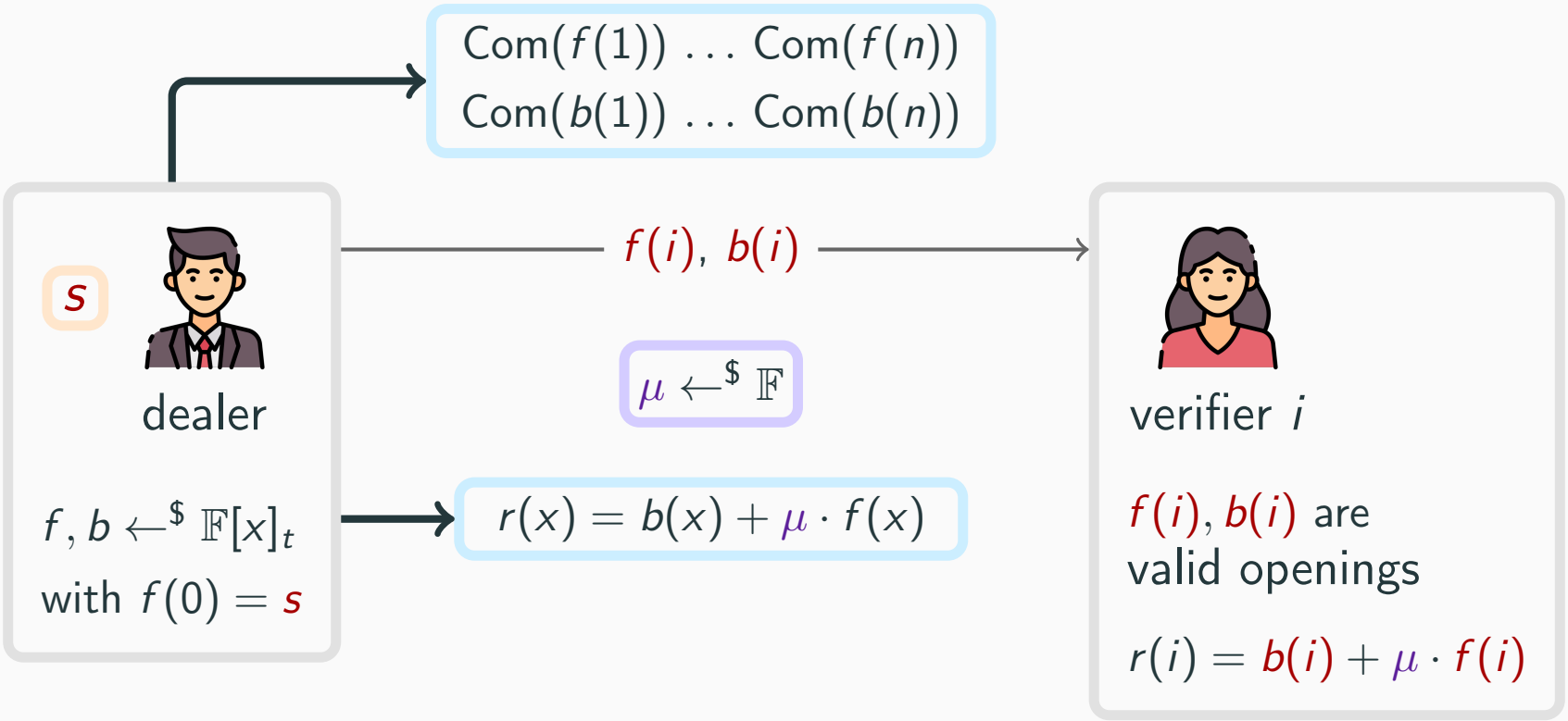

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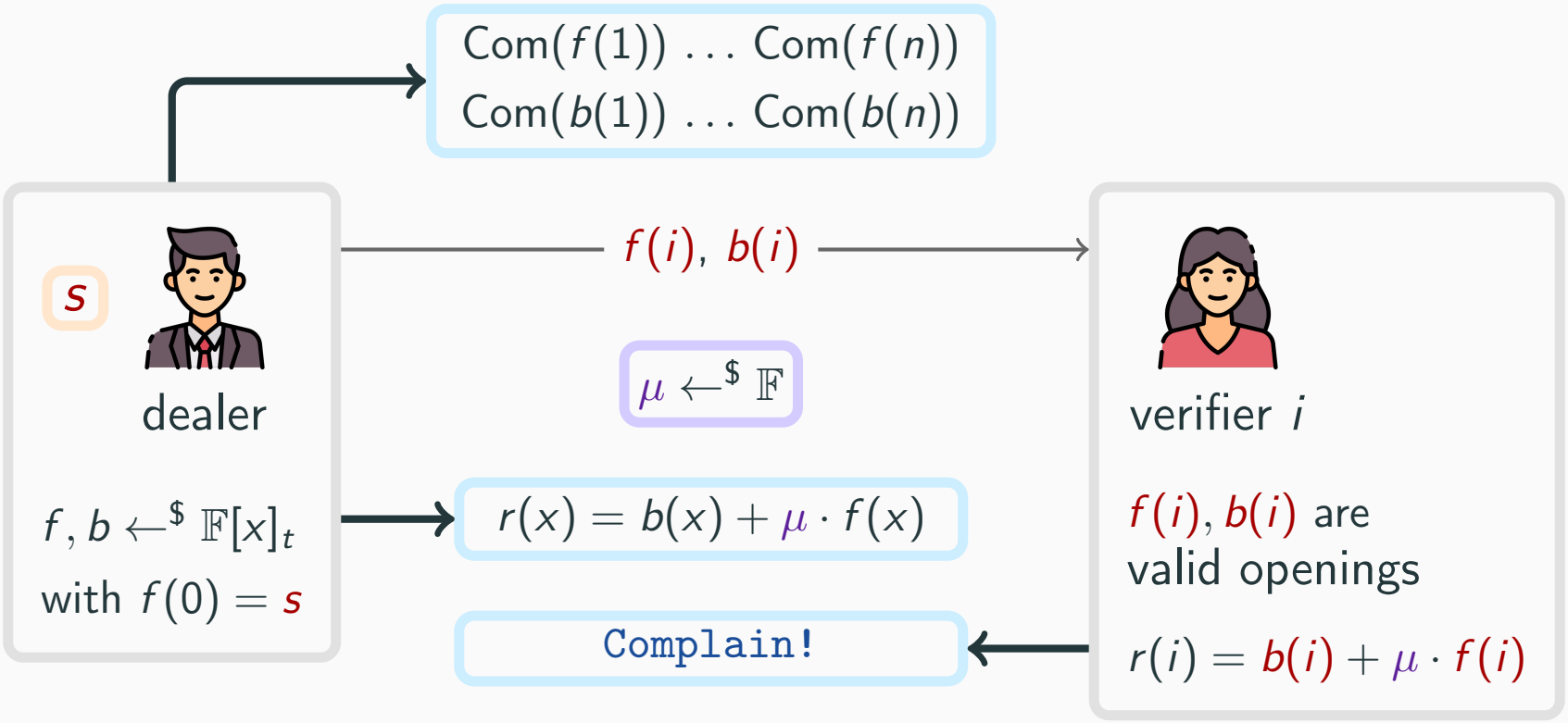

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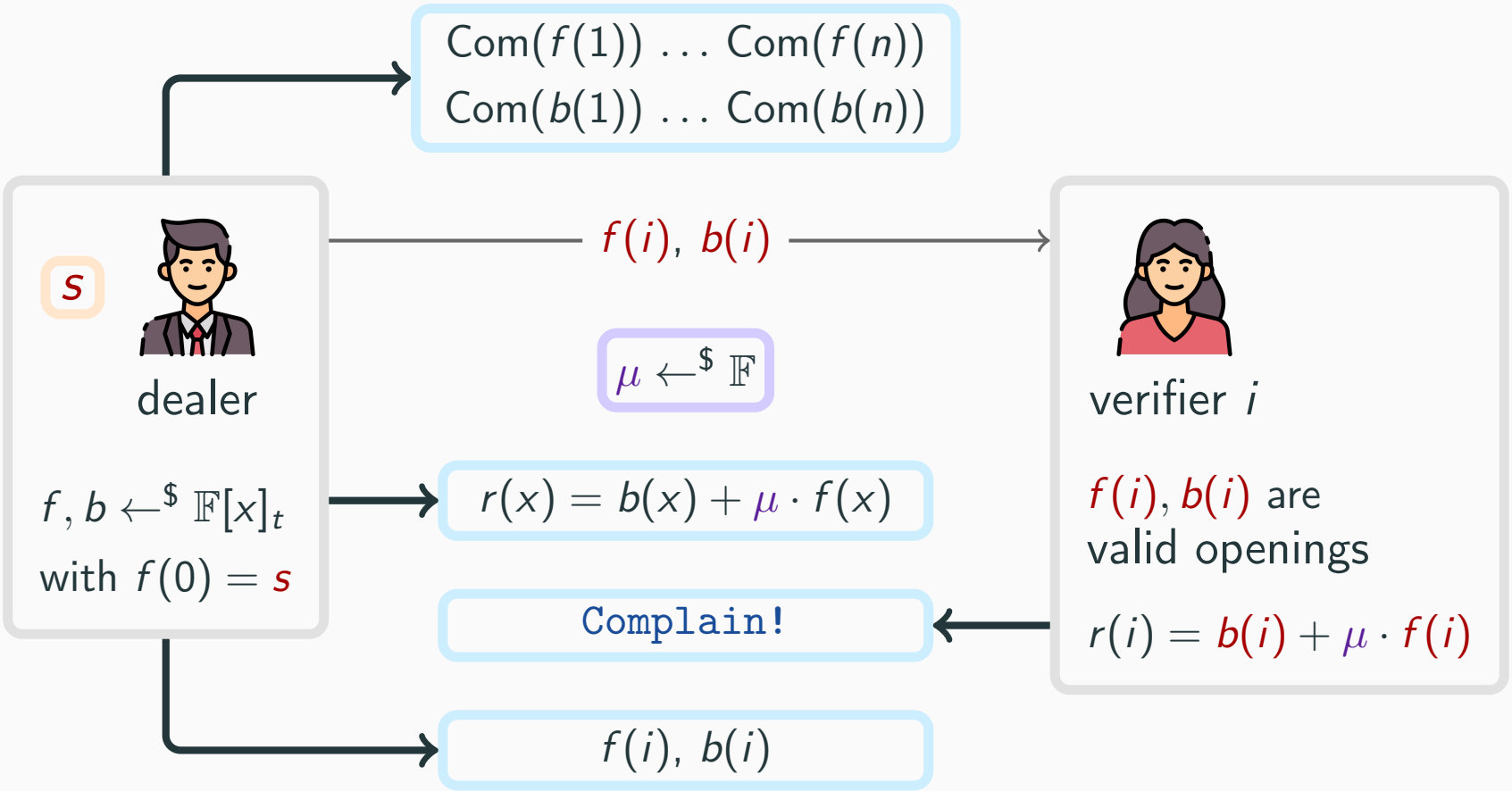

[ABCP23]: Zooming Out

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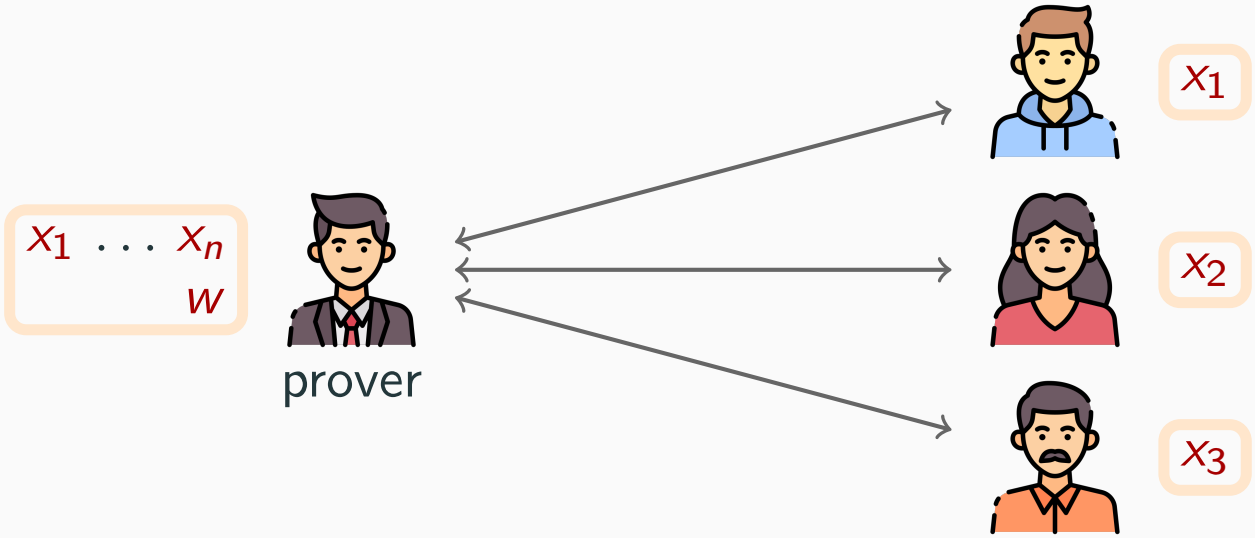

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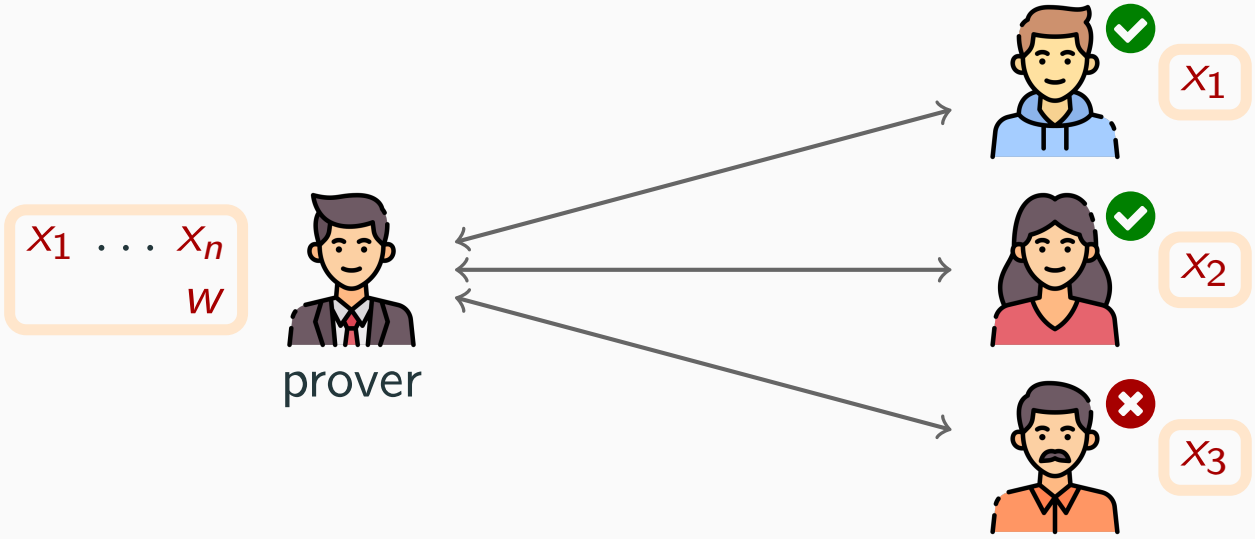

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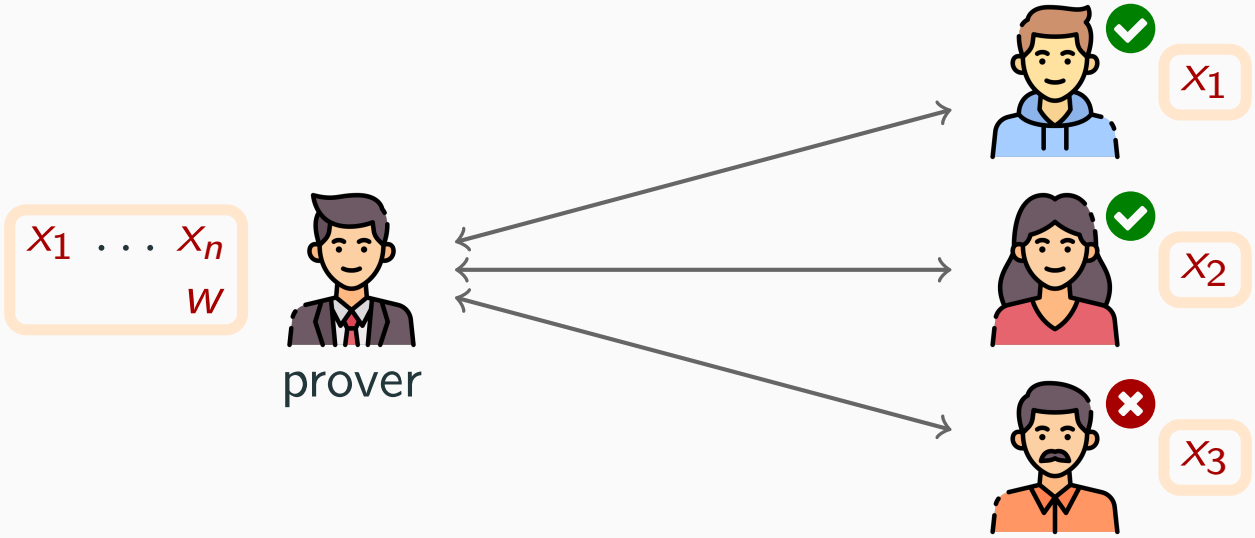

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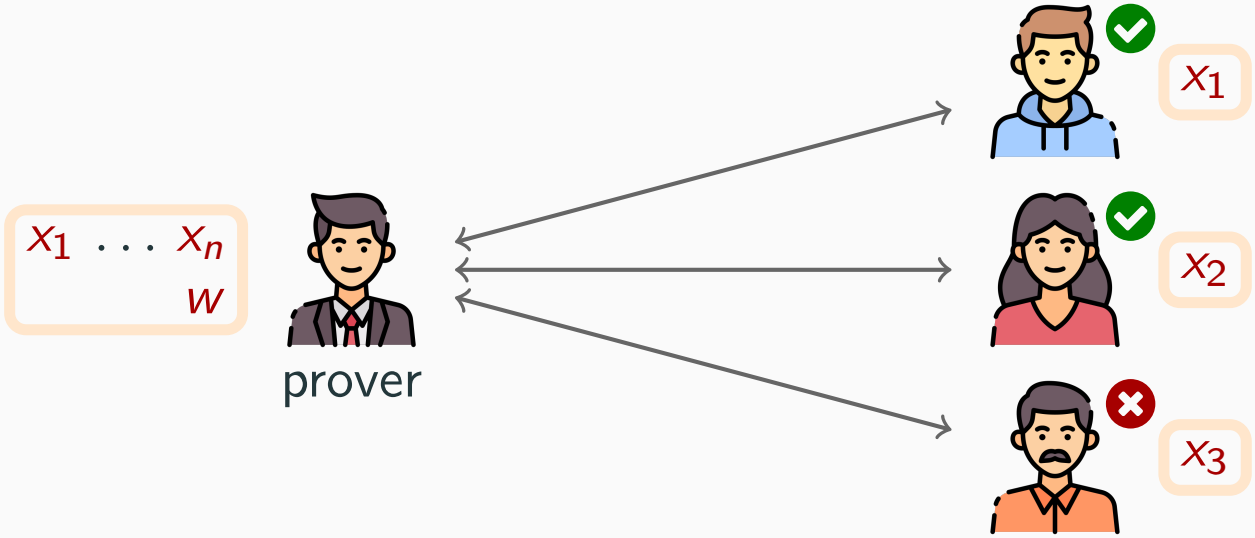

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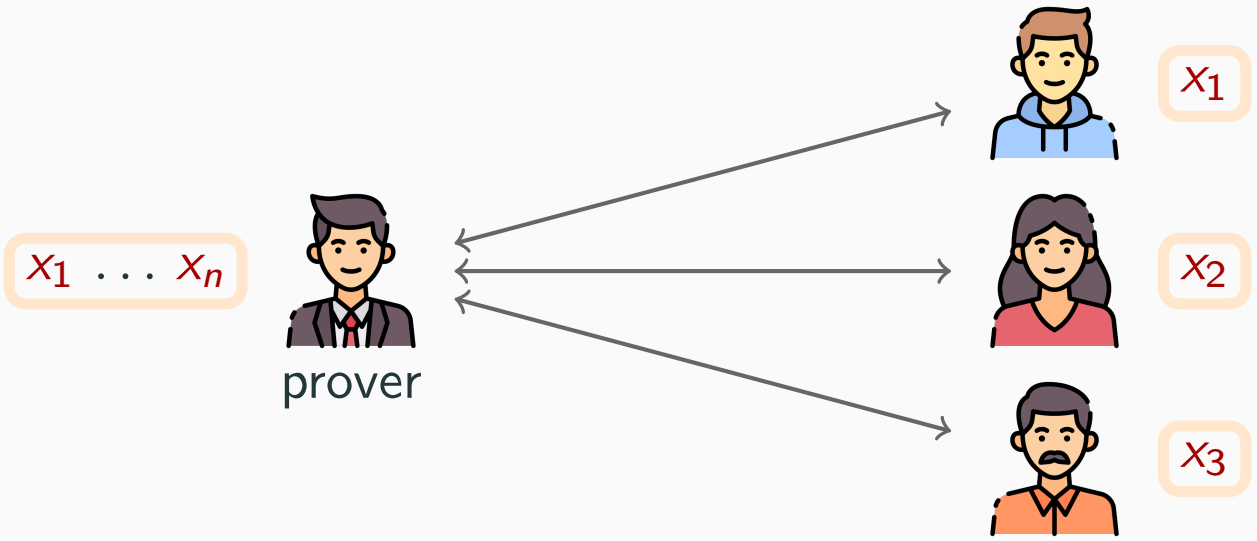
Soundness*: If there exists no z, w with $z_i = x_i$ for all honest V_i and $(z, w) \in \mathcal{R}$, **at least one** honest verifier rejects w.h.p.

Distributed Proofs: Low Degree



Relation: $(x_1 \ldots x_n) \in \mathcal{R}_d$ if there exists $f \in \mathbb{F}[x]$ with $\deg(f) \leq d$ and $f(\alpha_i) = x_i$

Low Degree Proof: Direct Test [ABCP23]

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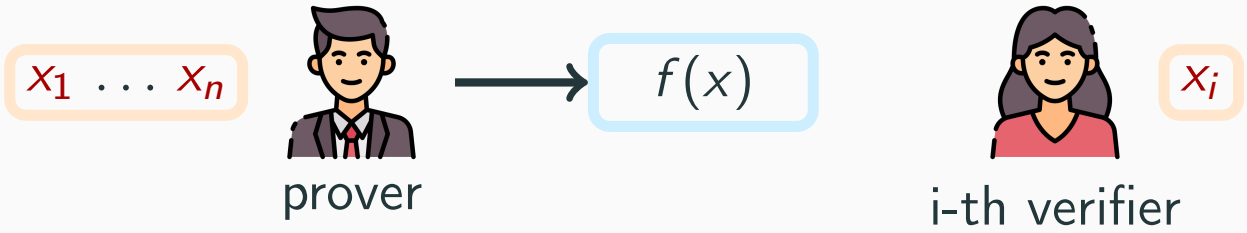

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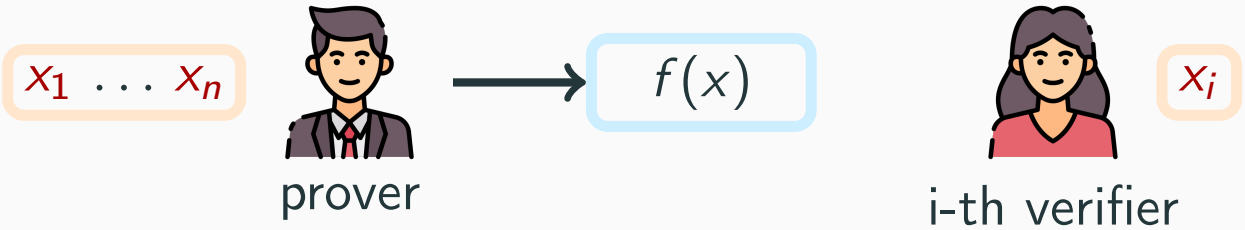

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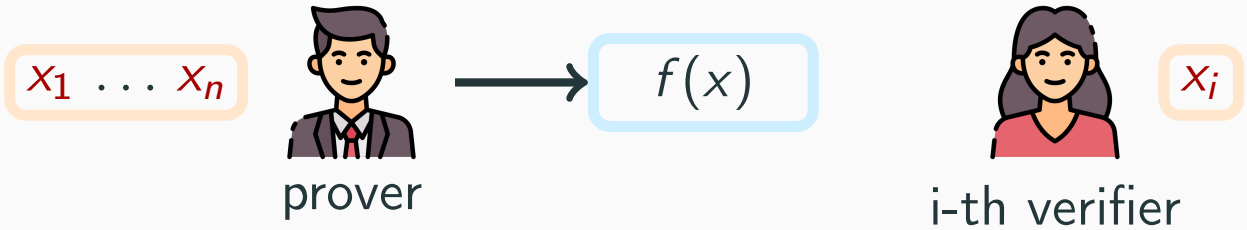

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⚠ $\Omega(d)$ **communication** and **verification**.

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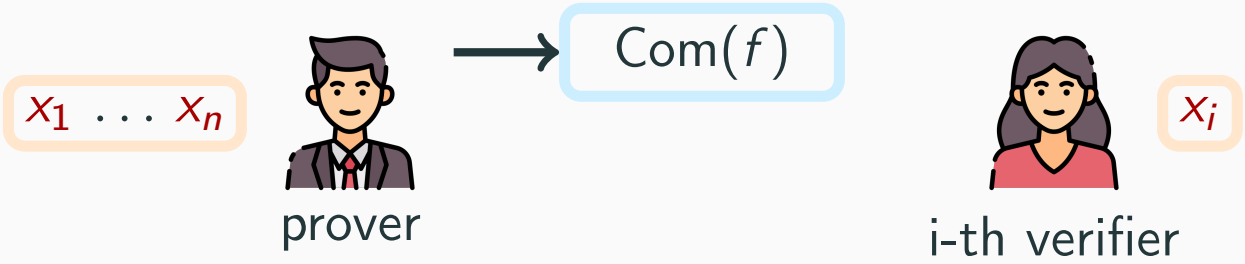

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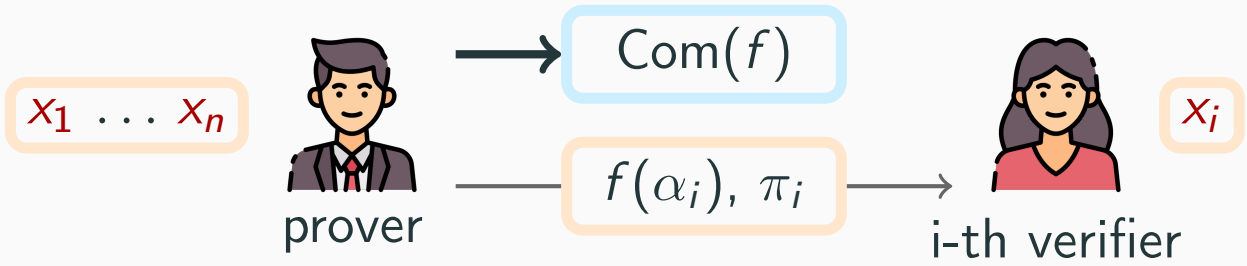

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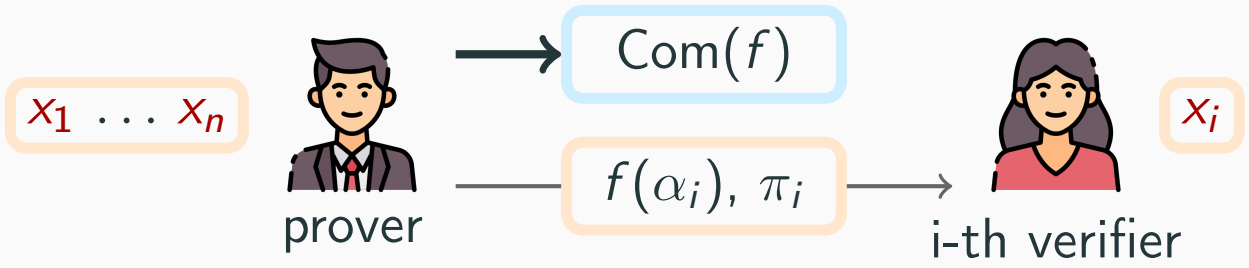

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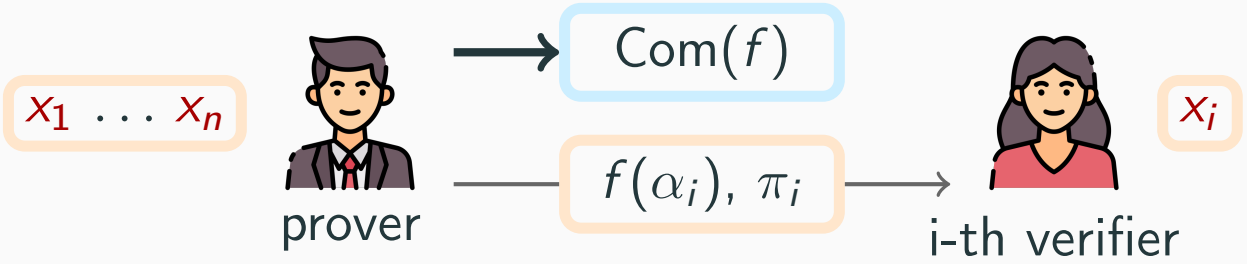

$f \in \mathbb{F}[x]_d : f(\alpha_j) = x_j$ $\quad$ π_i is valid and $f(\alpha_i) = x_i$

⚠ Some PC are **non post-quantum** [KZG10]

⚠ Some PC require $\Omega(n^2)$ **prover time** for multi-point evaluations, such as FRI [BBHR18]

Low Degree Proof: Folding

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prover

x_i

i-th verifier

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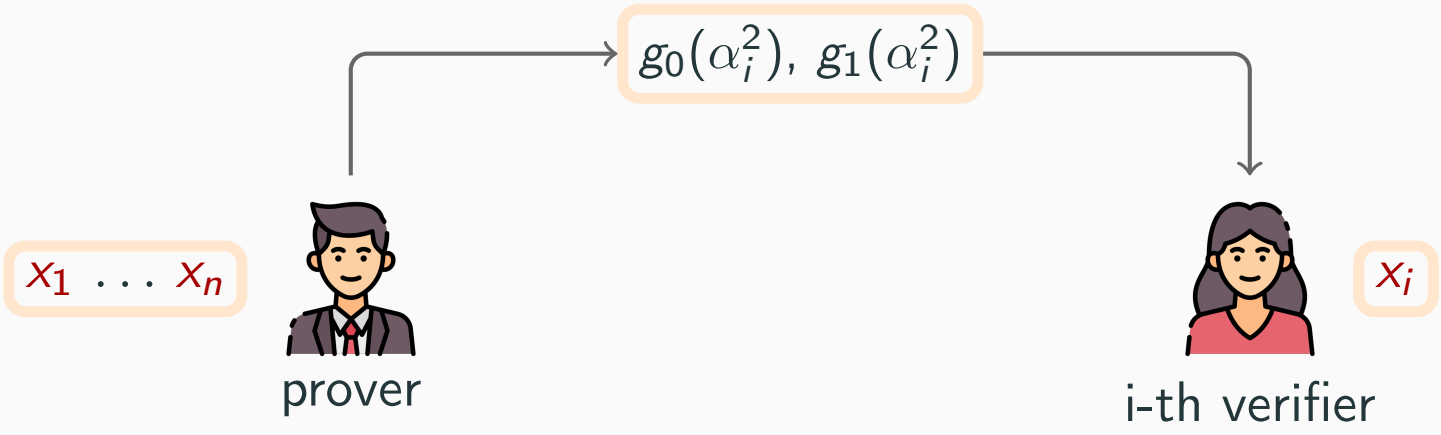
$f(x) = g_0(x^2) + x \cdot g_1(x^2)$

Low Degree Proof: Folding



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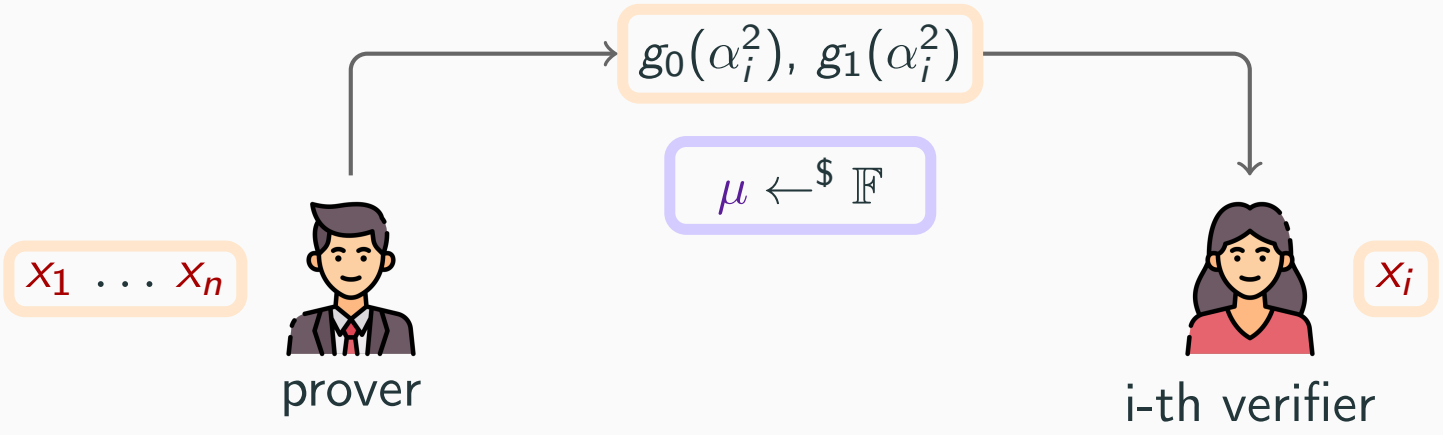

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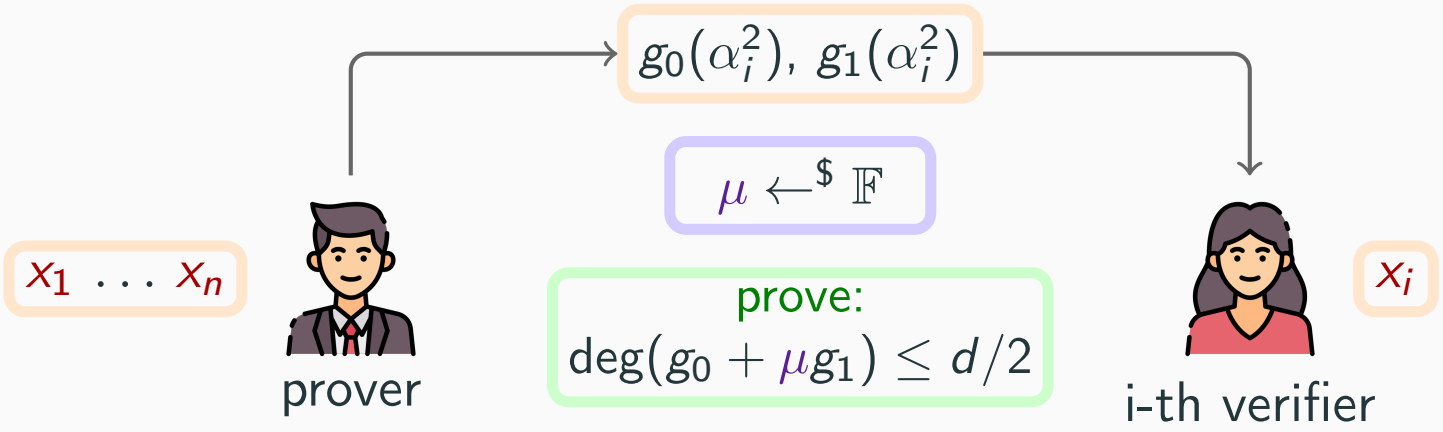

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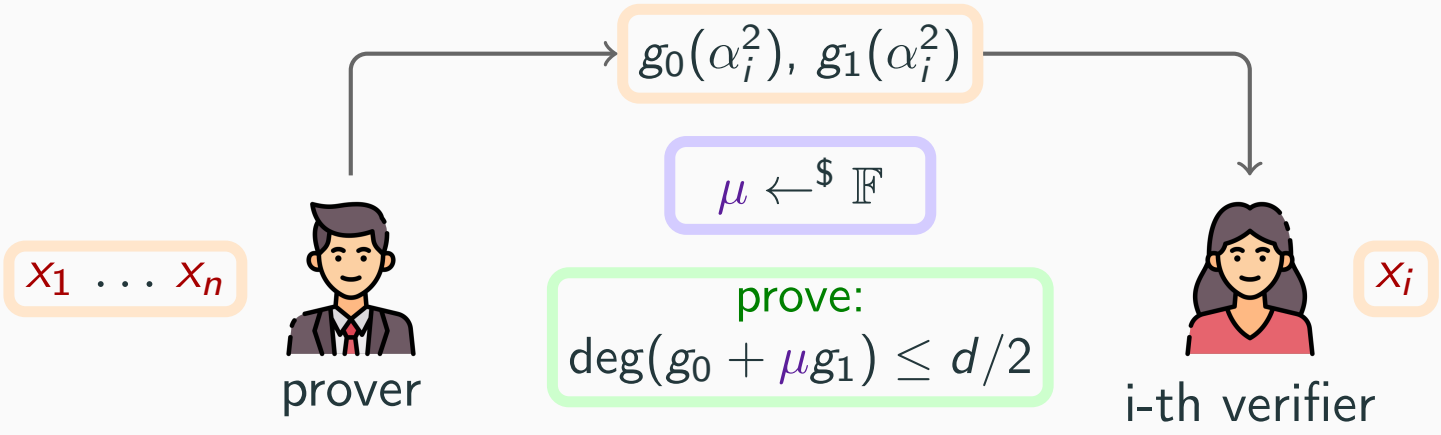

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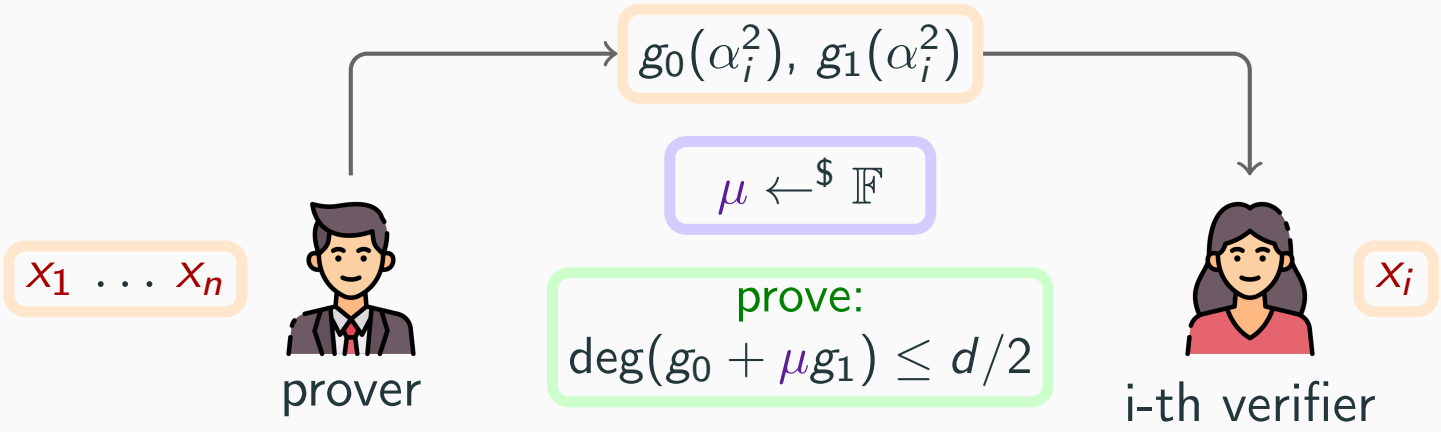

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✅ Sound with **at least** $d + 1$ honest verifiers.

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- **Send** $(m_{j,i}, \pi_{j,i})$ to V_i with $\pi_{j,i}$ opening of R_j in i.

Low Degree Proof in the ROM

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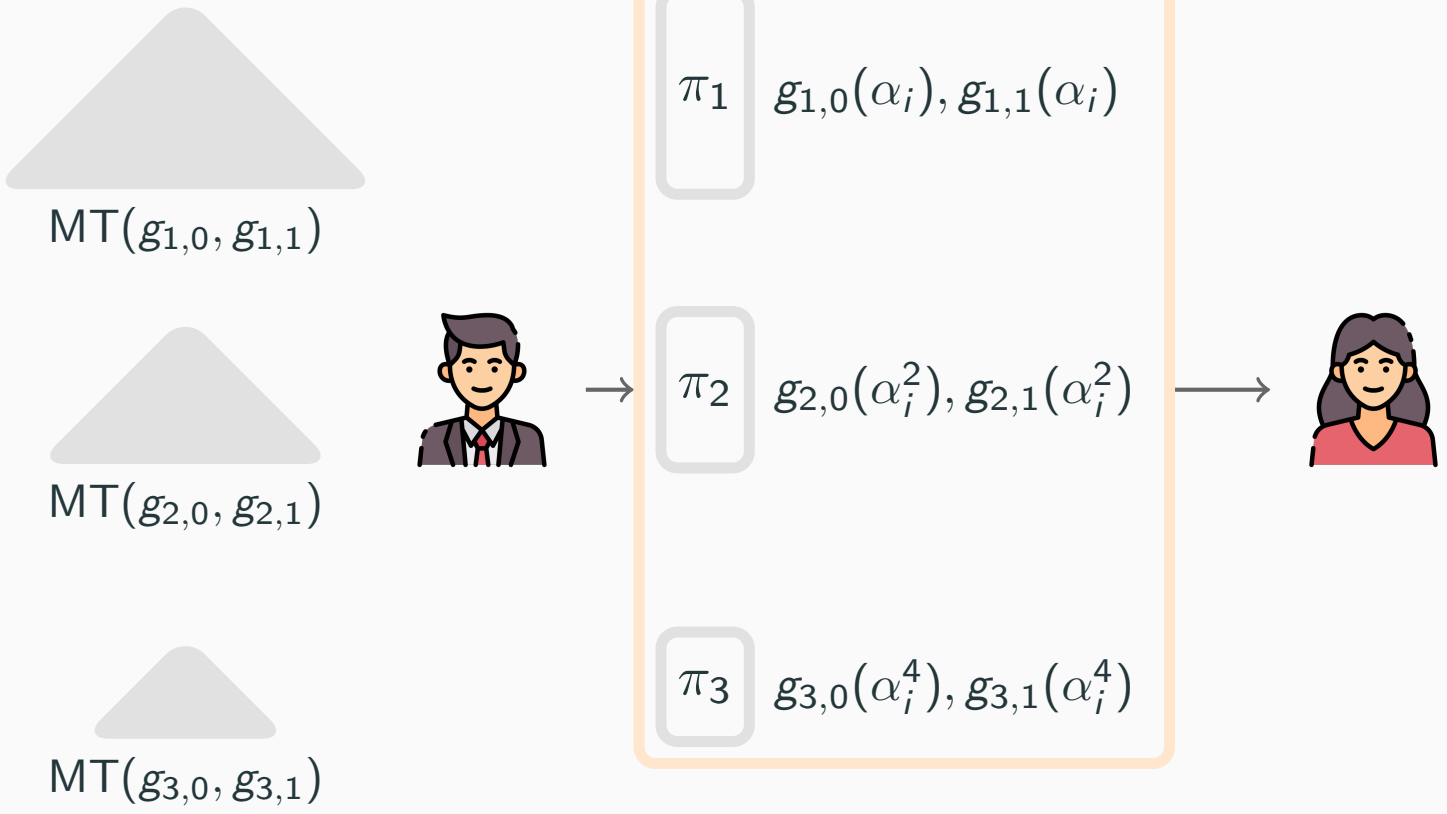
$\mathsf{MT}(g_{1,0}, g_{1,1})$

$\mathsf{MT}(g_{2,0}, g_{2,1})$

$\mathsf{MT}(g_{3,0}, g_{3,1})$

Low Degree Proof in the ROM



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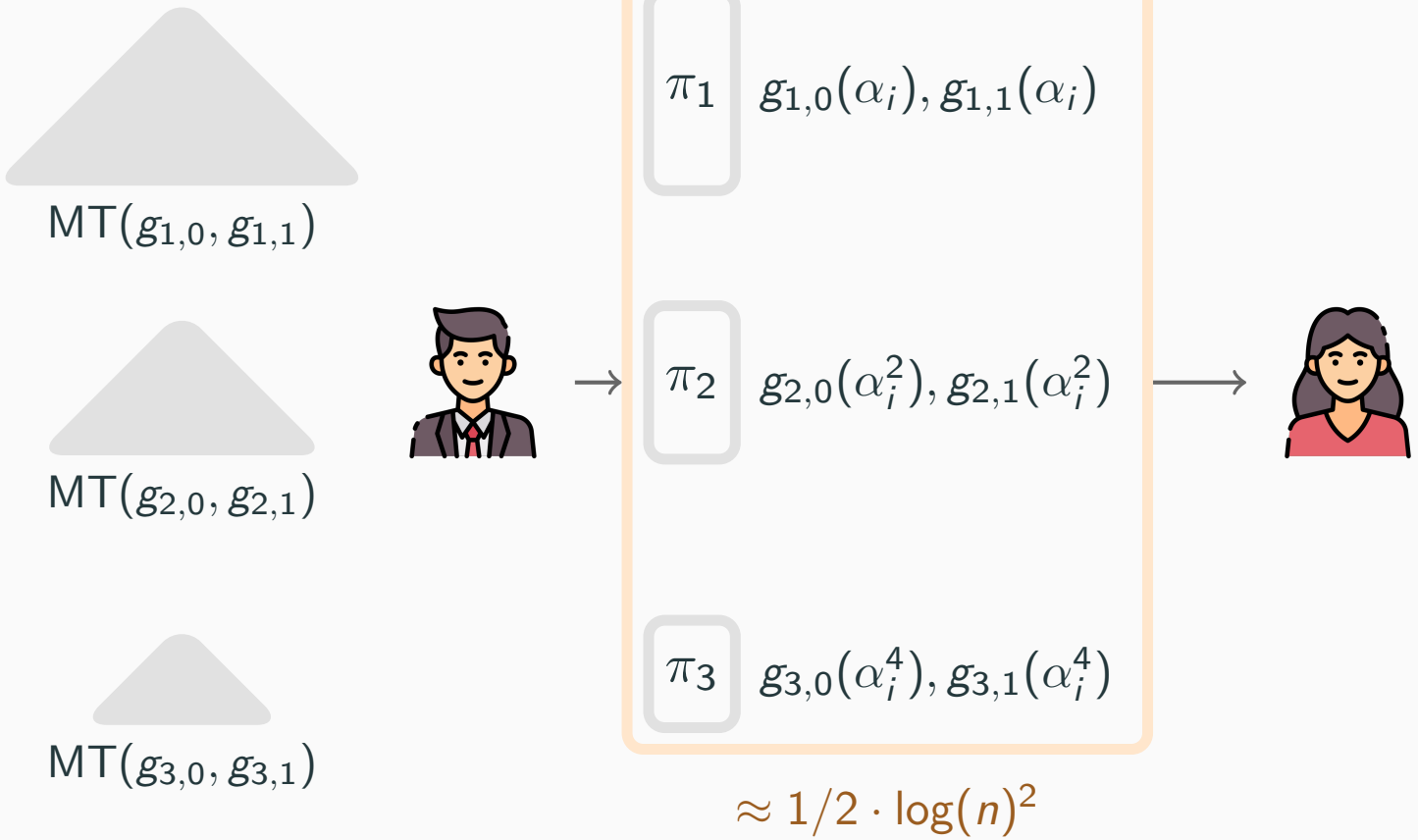

Conclusion

We presented a new (3-round) **VSS** in the **ROM** secure against $t < n/2$ corruptions with:

- **Sublinear** verifier's **download** and **computational** complexity in the best case.
- **Comparable costs** with state of the art VSS [ABCP23] in the worst case.

Thanks for your attention!