

# Interactive Threshold Mercurial Signatures and Applications



KYOTO UNIVERSITY

Masayuki Abe,



KYOTO UNIVERSITY

[Masaya Nanri,](#)



Octavio Perez Kempner



KYOTO UNIVERSITY

Mehdi Tibouchi

## Digital Signatures

→ Assuring information and issuers' integrity with mathematical technique



## Additional Demands

- ▶ For easier development of high level security application
- ▶ For hiding privacy of credentials



Structure-Preserving Signatures (SPS) and successive tools were developed !

# What is SPS and its Extension (SPS-EQ) ?

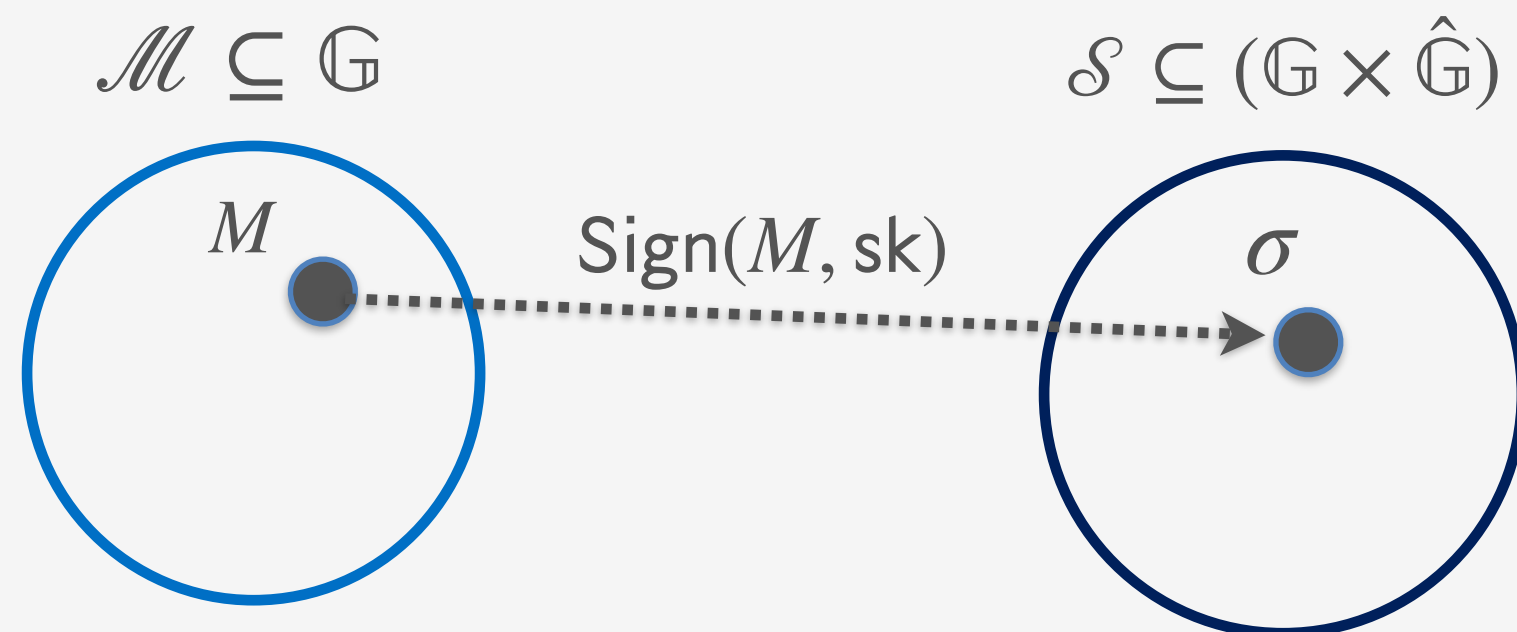
## Structure Preserving Signatures [AFG10]

Messages, signatures and verification keys are included in the same pairing groups

Verification uses pairing operation

$\mathbb{G}$  and  $\hat{\mathbb{G}}$  : pairing group ordered by  $p$ ,  $G$  and  $\hat{G}$  : generators of each group,  $e$  : map from  $(\mathbb{G}, \hat{\mathbb{G}})$  to  $\mathbb{G}_T$

$M \in \mathcal{M} \subseteq \mathbb{G}$  : messages ,  $\sigma \in \mathcal{S} \subseteq (\mathbb{G} \times \hat{\mathbb{G}})$  : signatures ,  $(sk, vk) \in (\mathbb{Z}_p^* \times \hat{\mathbb{G}})$  : signing keys and verification keys



Verify<sub>vk</sub>( $M, \sigma$ )

$$\prod_i \prod_j e(Y_i, X_j)^{c_{ij}} = 1$$

# What is SPS and its Extension (SPS-EQ) ?

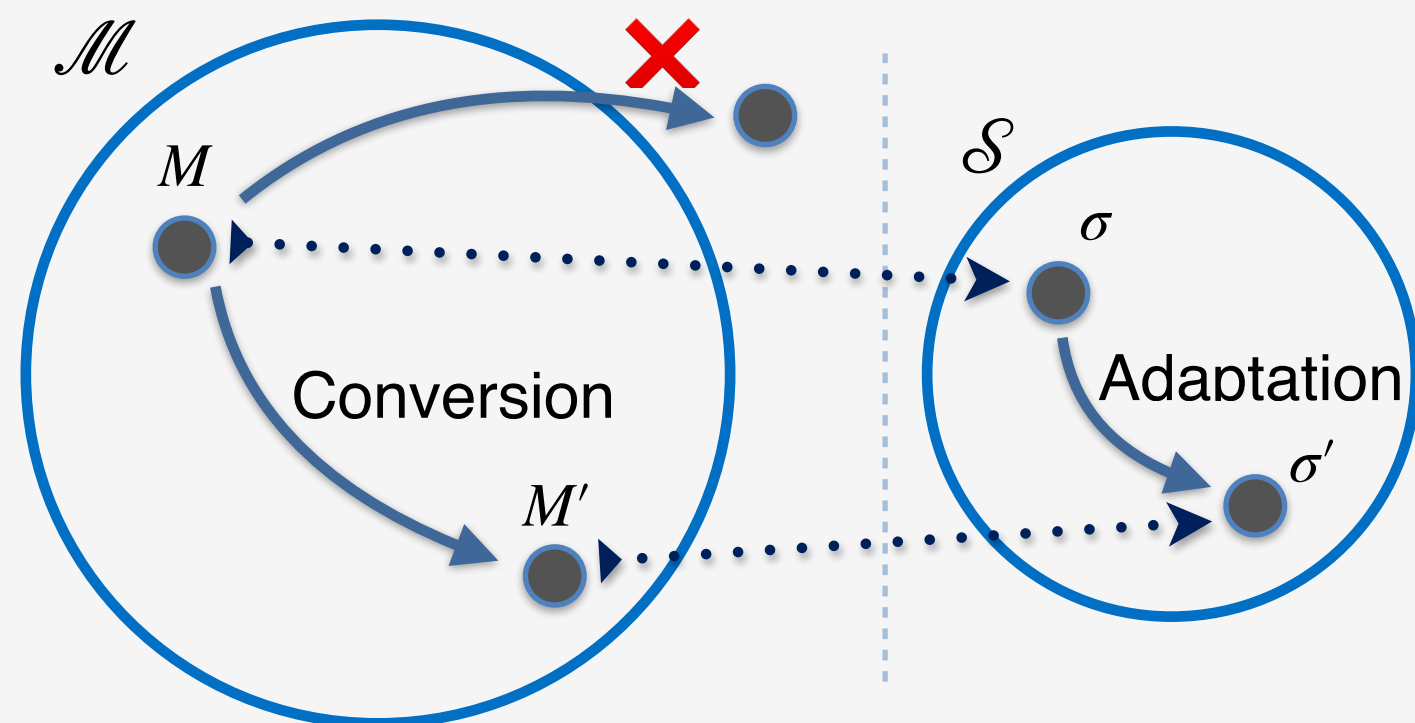
## Structure Preserving Signatures [AFG10]

Messages, signatures and verification keys are included in the same pairing groups

Verification uses pairing operation

## Structure Preserving Signatures on Equivalence Class [FHS14]

Signatures can be issued for a certain equivalence class defined over the message space



Anonymizing signatures leads to

Privacy enhanced credentials

## Methods in [FHS14] + Key Conversion

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Sign( $M, \text{sk}$ )  $\rightarrow \sigma$

$$\sigma = (Z, Y, \hat{Y}) = \left( \left( \prod_{i=1}^{\ell} M_i^{x_i} \right)^y, G^{\frac{1}{y}}, \hat{G}^{\frac{1}{y}} \right)$$

Verify( $M, \sigma, \text{vk}$ )  $\rightarrow 0$  or  $1$

$$\prod_{i=1}^{\ell} e(M_i, \hat{X}_i) = e(Z, \hat{Y}) \wedge e(Y, \hat{G}) = e(G, \hat{Y})$$

ChangeRep( $\text{vk}, M, \sigma, \mu$ )  $\rightarrow (M', \sigma')$

ConvertSig( $\text{vk}, M, \sigma ; \rho$ )  $\rightarrow \tilde{\sigma}$

ConvertVK( $\text{vk} ; \rho$ )  $\rightarrow \tilde{\text{vk}}$

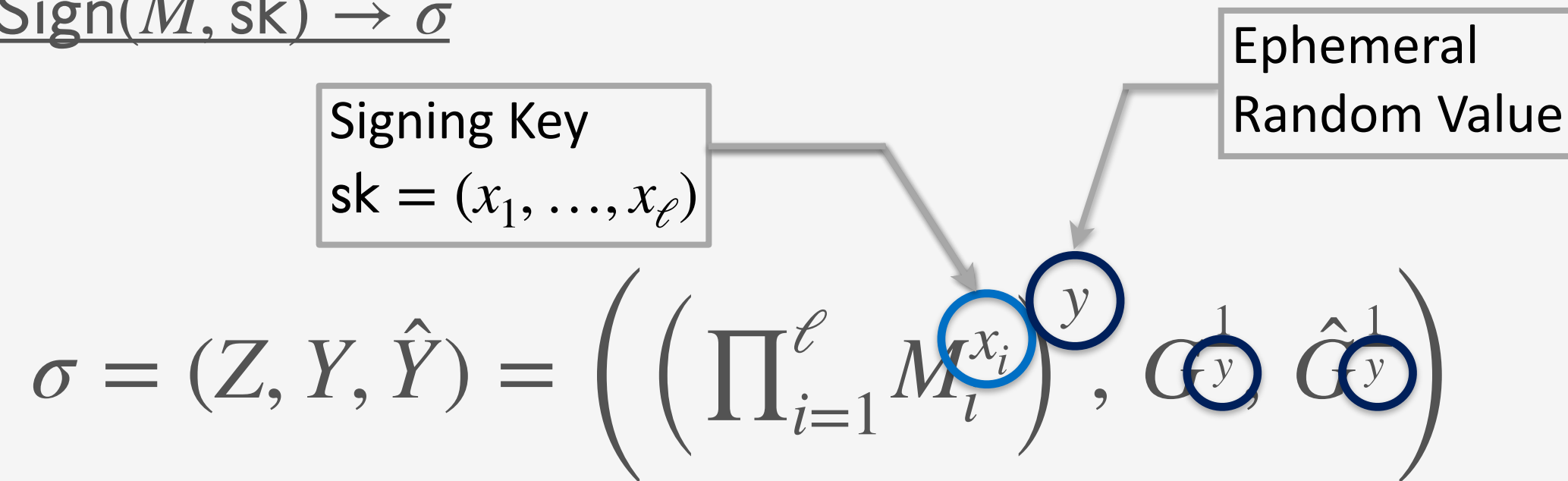
$$\tilde{\text{vk}} = \text{vk}^{\rho} = (\hat{G}^{x_1 \rho}, \dots, \hat{G}^{x_{\ell} \rho})$$

ConvertSK( $\text{sk} ; \rho$ )  $\rightarrow \tilde{\text{sk}}$

$$\tilde{\text{sk}} = \text{sk}^{\rho} = (x_1^{\rho}, \dots, x_{\ell}^{\rho})$$

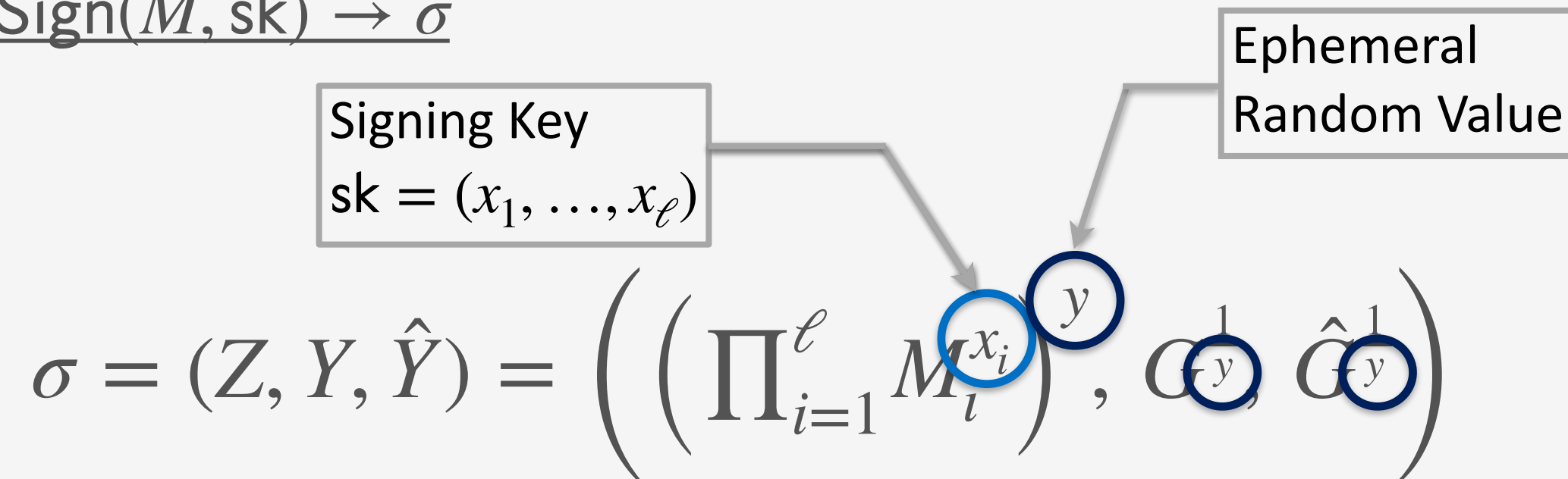
## Methods in [FHS14] + Key Conversion

Sign( $M, sk$ )  $\rightarrow \sigma$



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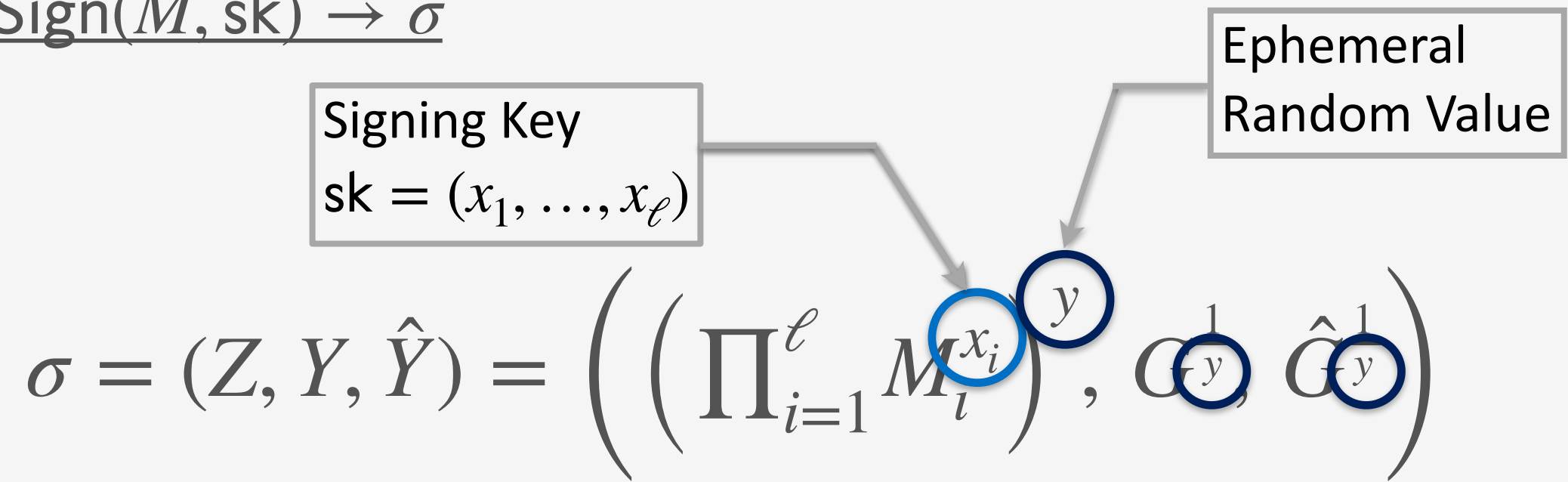
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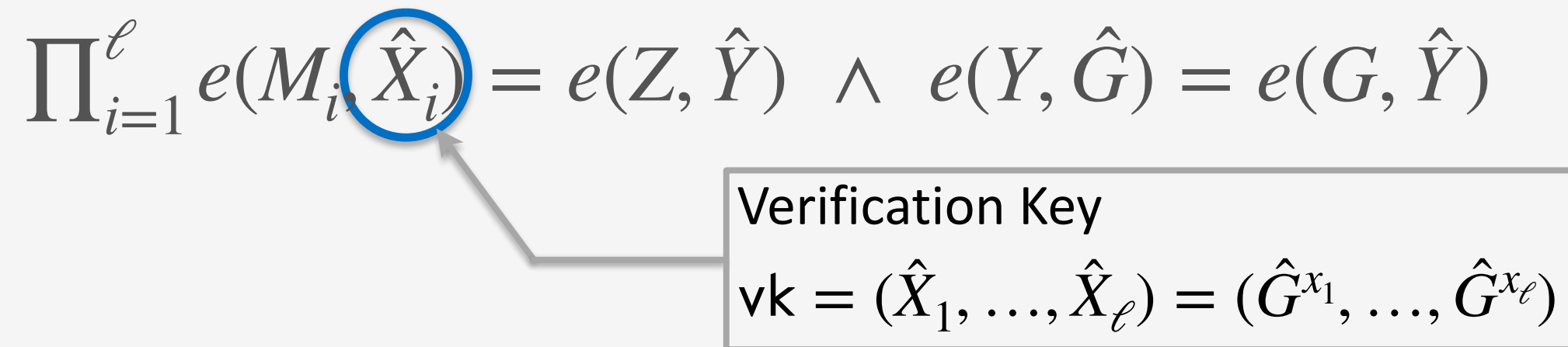
Verification Key  
 $vk = (\hat{X}_1, \dots, \hat{X}_\ell) = (\hat{G}^{x_1}, \dots, \hat{G}^{x_\ell})$

## Methods in [FHS14] + Key Conversion

Sign( $M, sk$ )  $\rightarrow \sigma$



Verify( $M, \sigma, vk$ )  $\rightarrow 0$  or  $1$



Sampled Random Value

ConvertSig( $vk, M, \sigma ; \rho$ )  $\rightarrow \tilde{\sigma}$

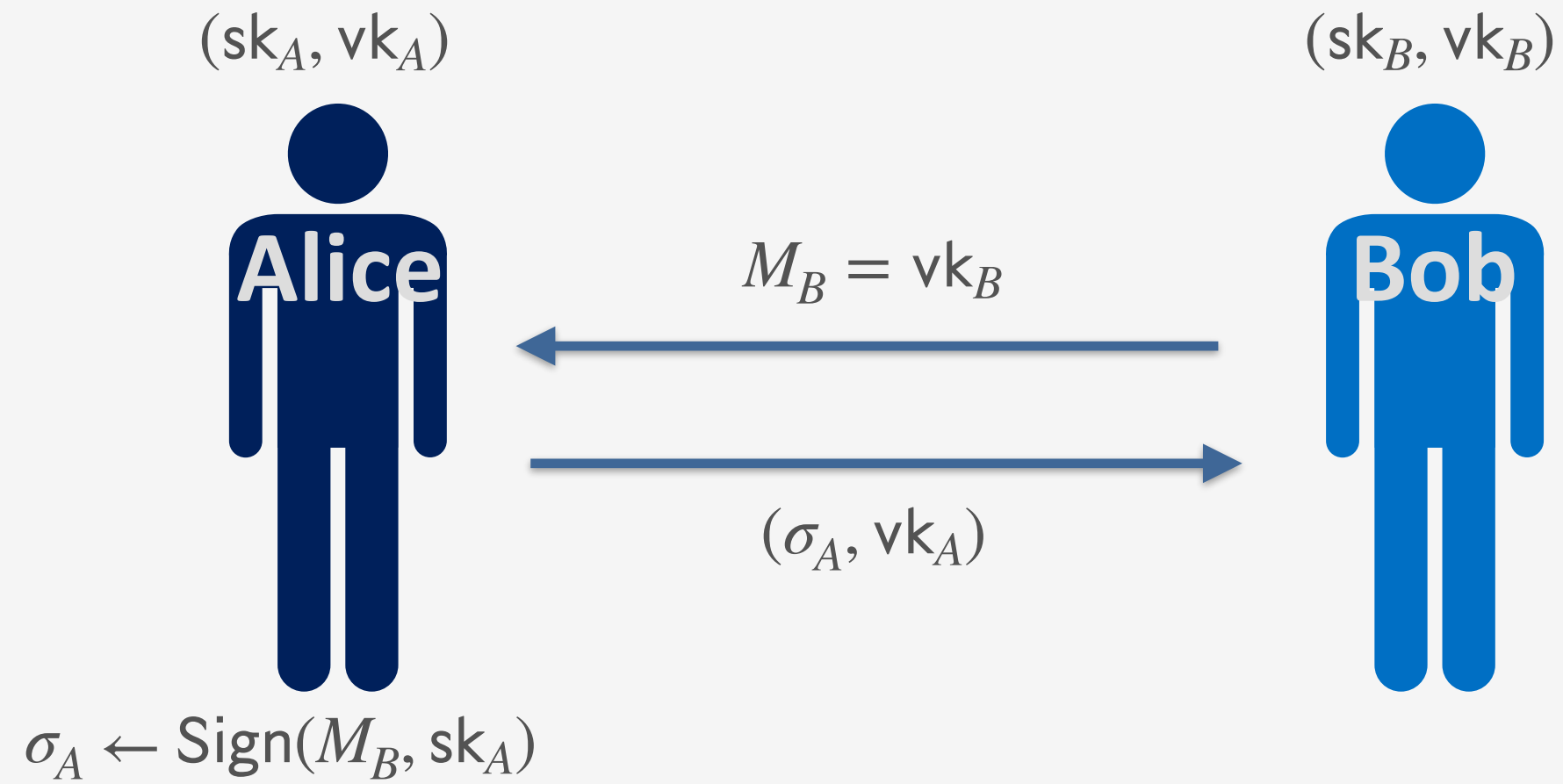
ConvertVK( $vk ; \rho$ )  $\rightarrow \tilde{vk}$

$$\tilde{vk} = (\hat{X}_1^\rho, \dots, \hat{X}_\ell^\rho)$$

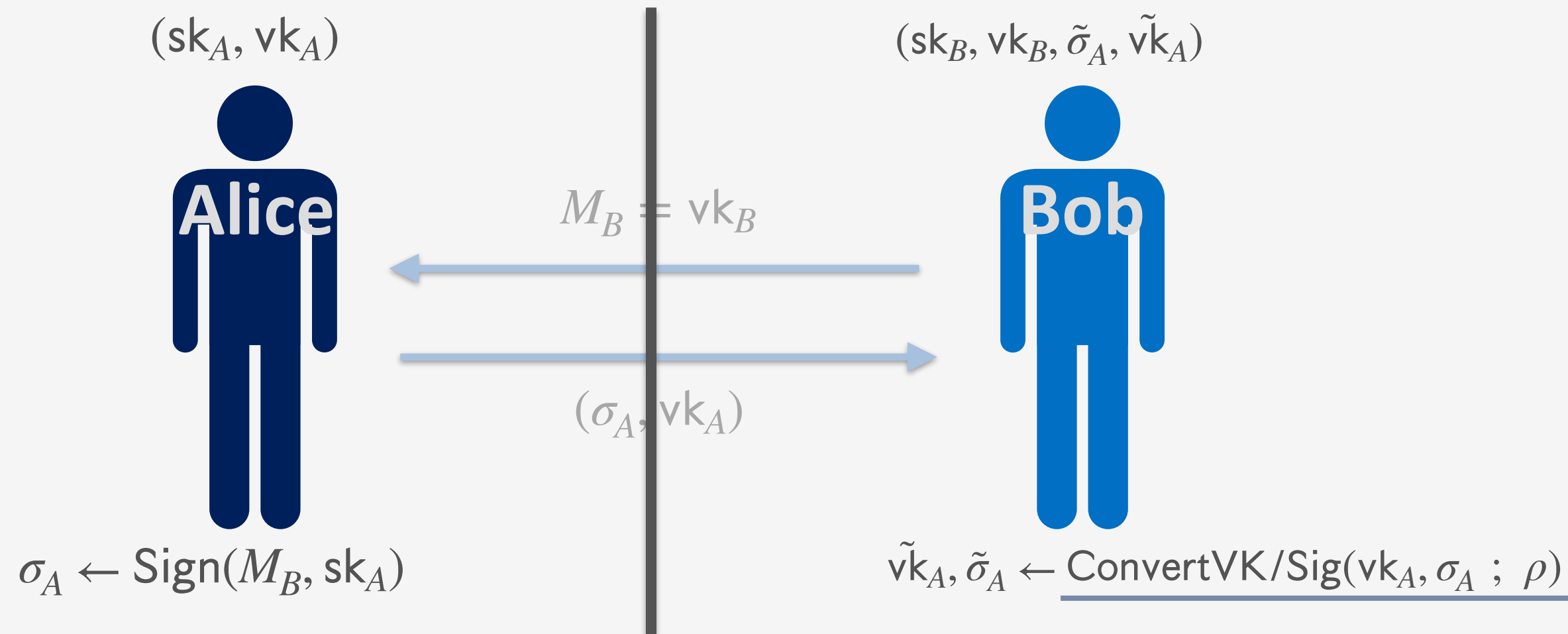


# Anonymize Credentials for Privacy

Example case : Alice gives a credential to Bob like PKI



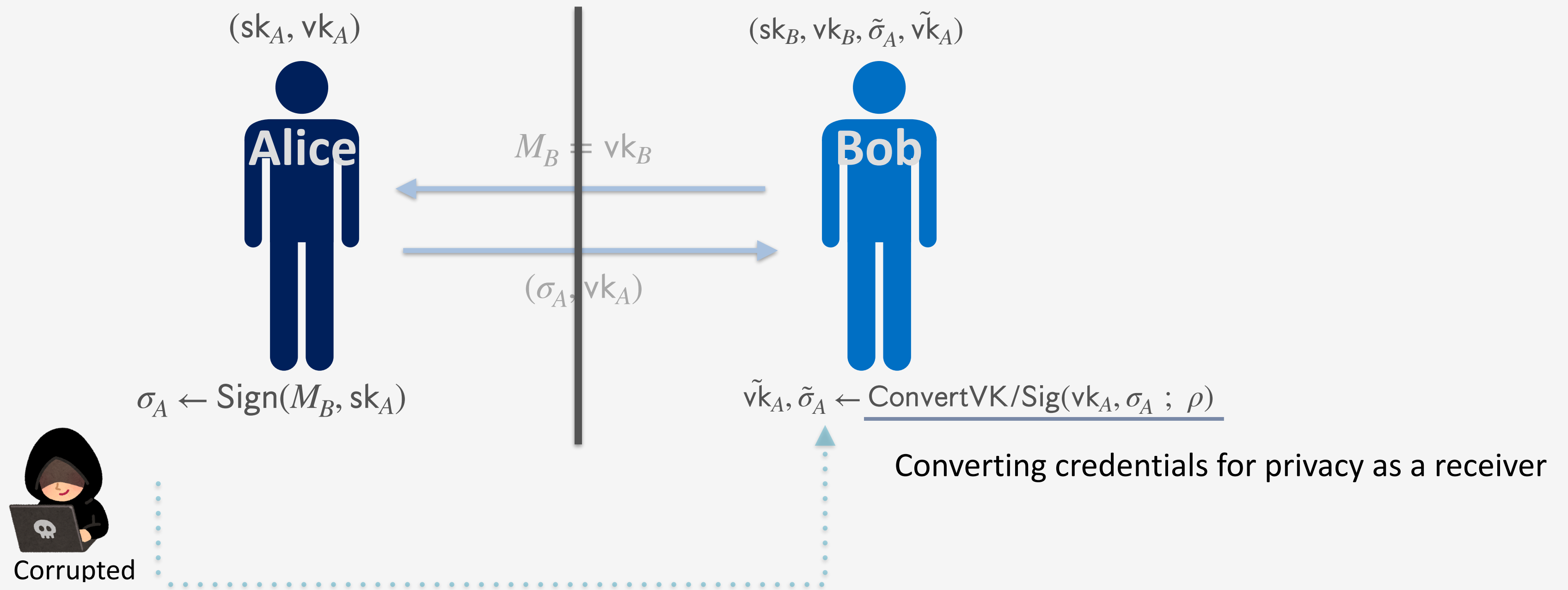
Example case : Alice gives a credential to Bob like PKI



Converting credentials for privacy as a receiver

# Anonymize Credentials for Privacy

Example case : Alice gives a credential to Bob like PKI



Adversary wants to find the relation to the credentials

... Who is the issuer of the anonymized signatures ?

# Problem : Weak unlinkability in Mercurial Signatures

Unlinkability doesn't hold for the corrupted signer

→ The Single malicious signer has chance to trace converted key and signatures

KeyGen(pp,  $\ell(\kappa)$ ) → (vk, sk)

$$\text{sk} = (x_1, \dots, x_\ell)$$

$$\text{vk} = (\hat{X}_1, \dots, \hat{X}_\ell) = (\hat{G}^{x_1}, \dots, \hat{G}^{x_\ell})$$



ConvertVK(vk,  $\rho$ ) →  $\tilde{\text{vk}}$

$$\tilde{\text{vk}} = (\hat{X}_1^\rho, \dots, \hat{X}_\ell^\rho)$$

Obviously, this public values are the trigger

# Problem : Weak unlinkability in Mercurial Signatures

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ConvertVK(vk,  $\rho$ ) →  $\tilde{\text{vk}}$

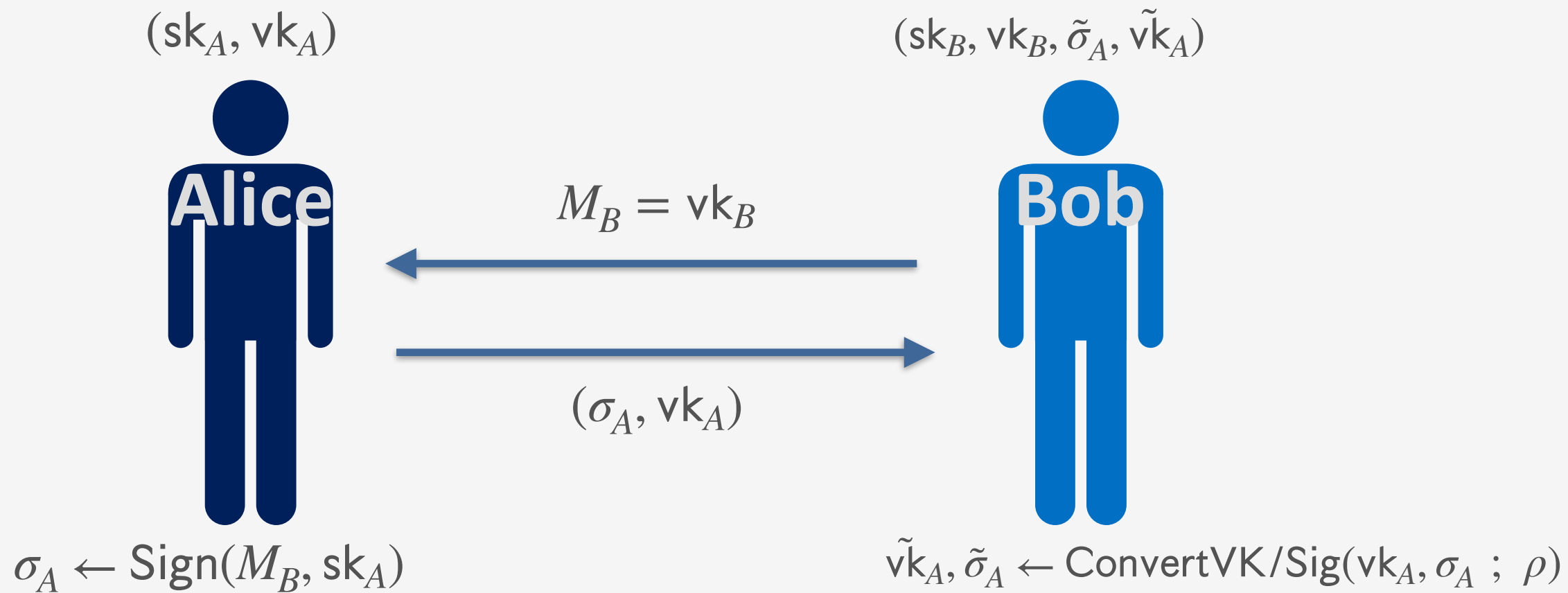
$$\tilde{\text{vk}} = (\hat{X}_1^\rho, \dots, \hat{X}_\ell^\rho)$$

$\tilde{\text{vk}} = (\hat{X}_1^\rho, \dots, \hat{X}_\ell^\rho)$  is in the same class as  $\text{vk} = (\hat{G}^{x_1}, \dots, \hat{G}^{x_\ell})$

If and only if  $\hat{X}_1^{\rho \cdot \frac{1}{x_1}} = \dots = \hat{X}_\ell^{\rho \cdot \frac{1}{x_\ell}}$ , only the single signer has  $\text{sk}^{-1} = \left(\frac{1}{x_1}, \dots, \frac{1}{x_\ell}\right)$

# Issue : Weak unlinkability makes threat for privacy...

Example case : A gives a credential to B like PKI



Corrupted

Even B has  $\tilde{vk}_A$  instead of  $vk_A$ , A can find  $\tilde{vk}_A$  is converted from  $vk_A$ .  
If and only if  $\tilde{\sigma}_A$  is verified by  $\tilde{vk}_A$ , the credential is issued by A.

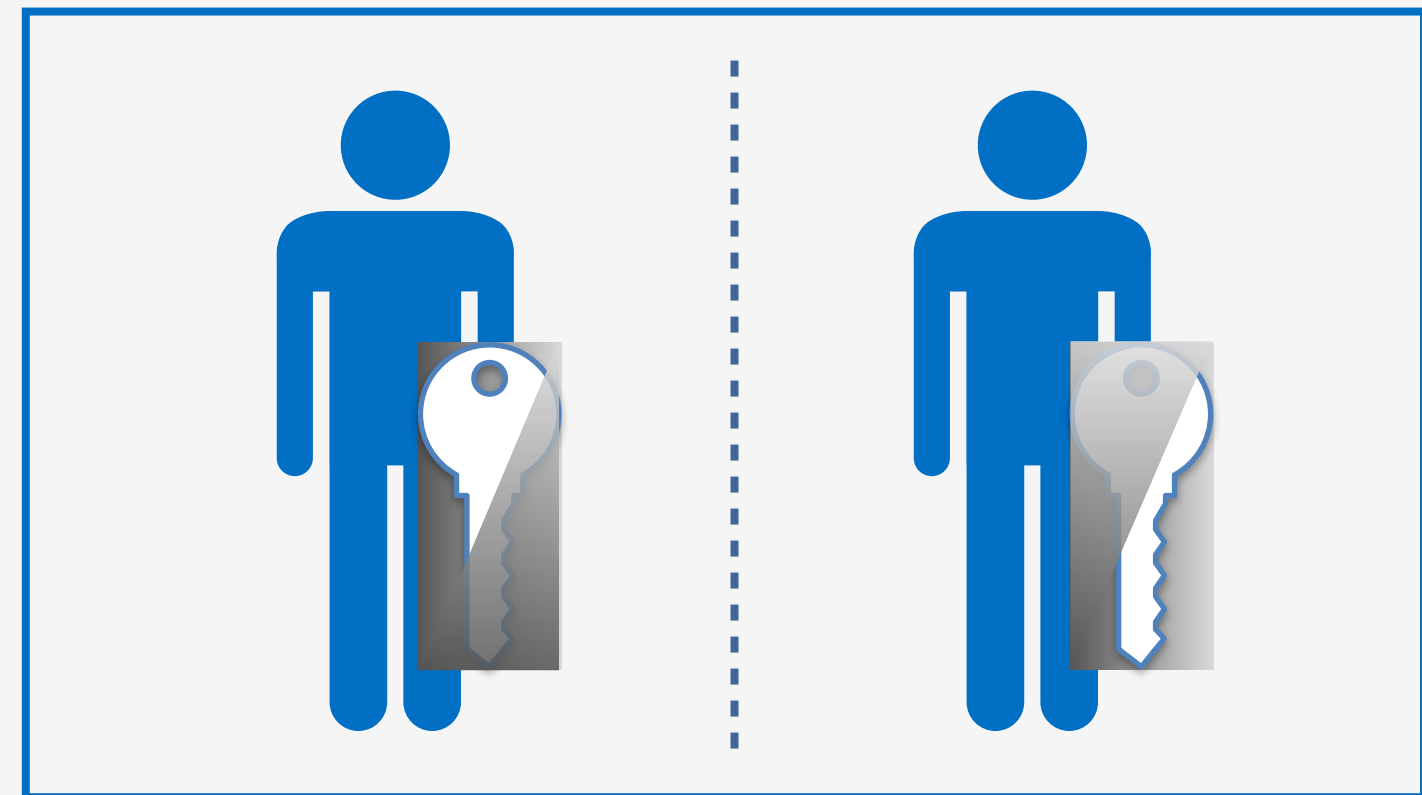
# Our approach : Splitting the Signer

No one can have the full signing key to trace the conversion

[CL19]  
Single Trusted Signer



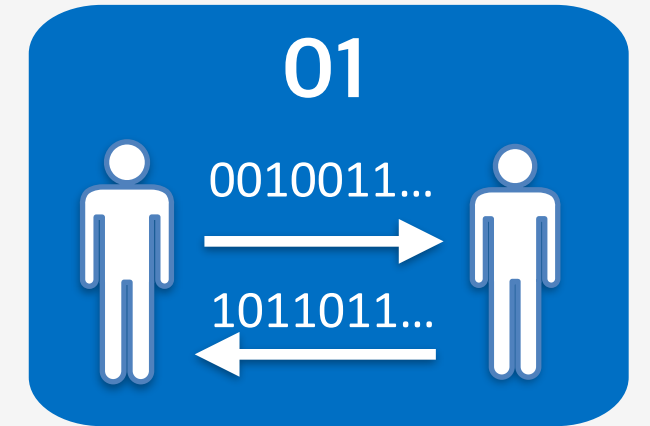
Our Approach :  
Distributed Signers



Each party doesn't trust the opponent.

## 01 | Sequential communication model between Two parties

- It allows 1 party corruption



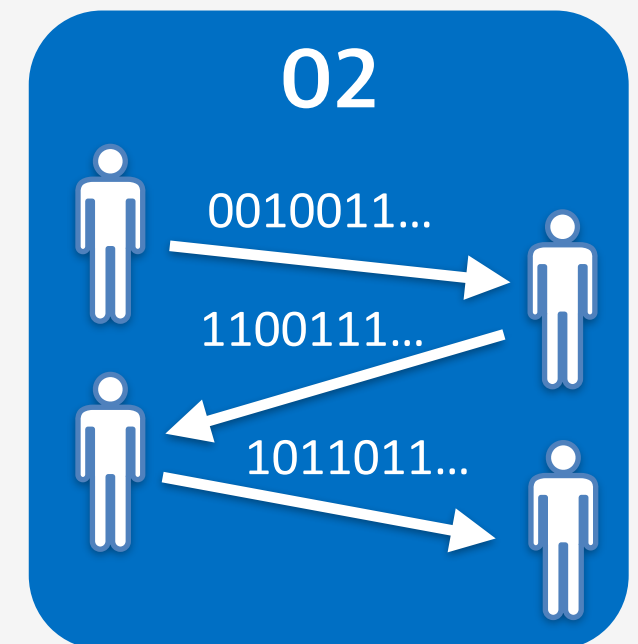
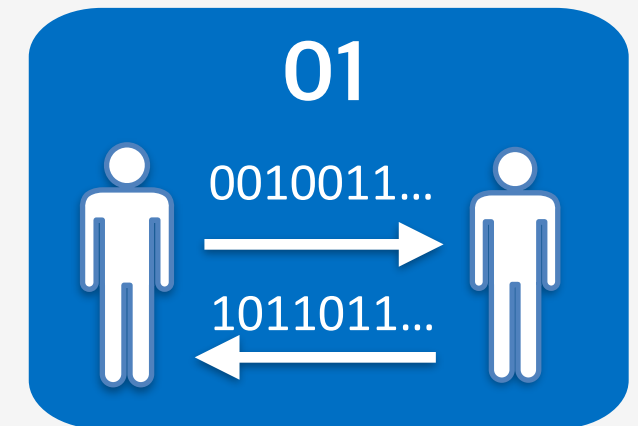


## 01 | Sequential communication model between Two parties

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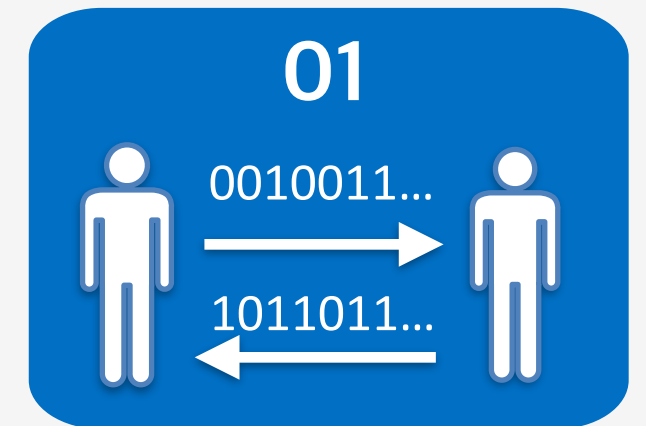
## 02 | Sequential communication model among $t$ out of $n$ parties

- It allows corruption up to  $t-1$  party
- Pre-processing for secret sharing is required



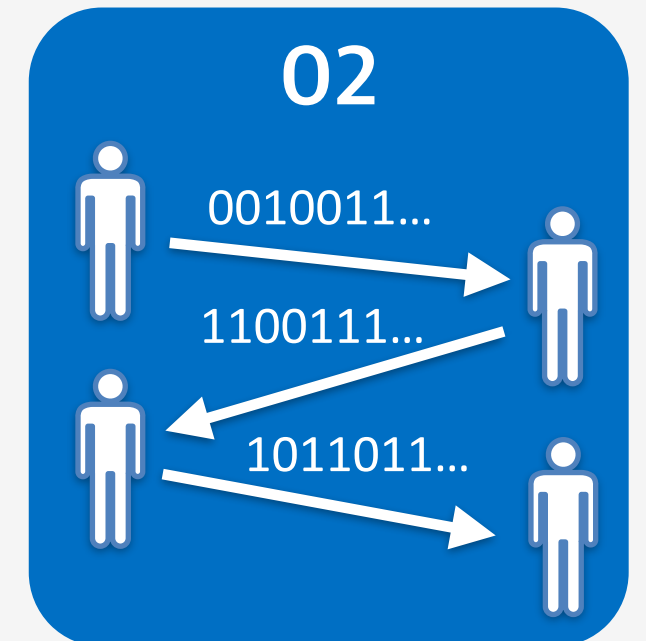
## 01 | Sequential communication model between Two parties

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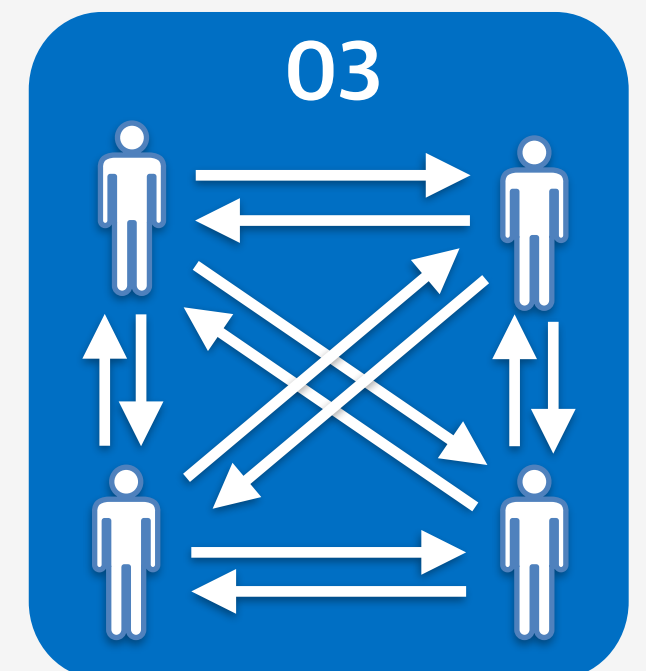
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- It allows corruption up to t-1 party
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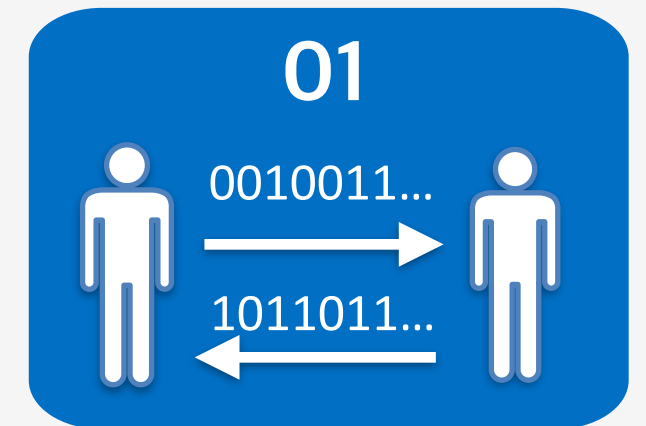
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- It allows corruption up to t-1 party
- Broadcast messages with traditional MPC



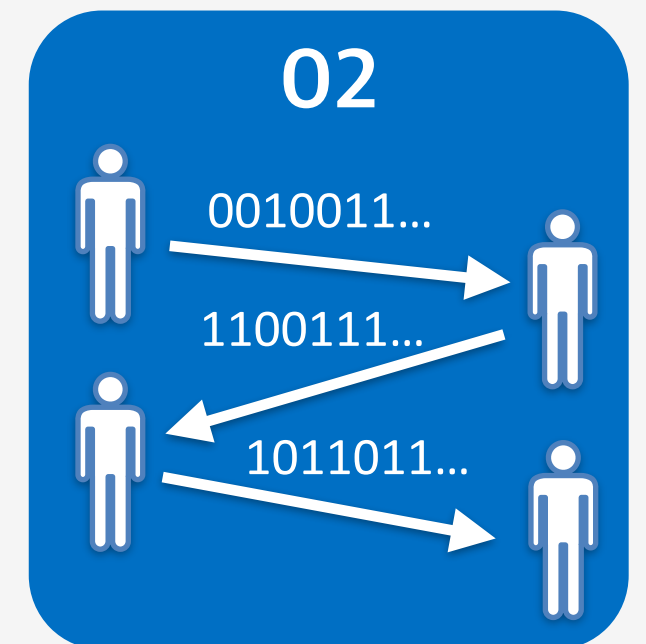
## 01 | Sequential communication model between Two parties

- It allows 1 party corruption



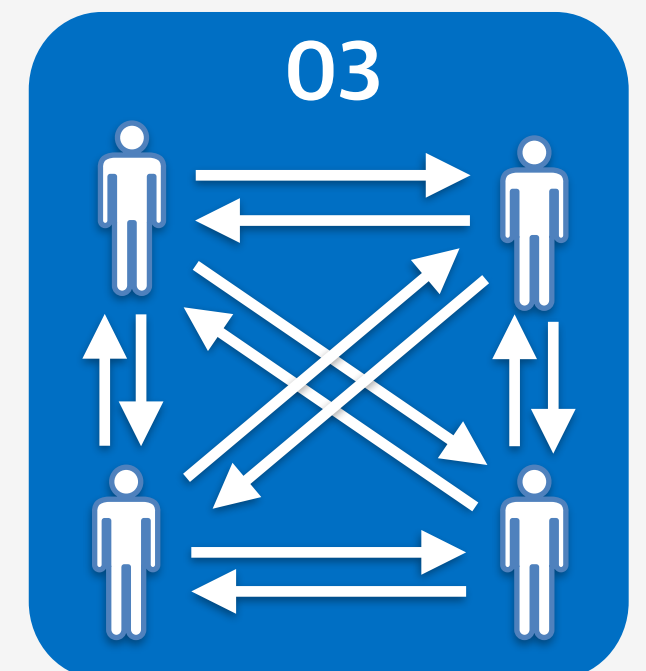
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## Threshold Interactive Mercurial Signatures

$$\underline{\text{TSign}(M, \text{sk})} \rightarrow \sigma$$

$$\sigma = (Z, Y, \hat{Y}) = \left( \left( \prod_{i=1}^{\ell} M_i^{x_i} \right)^y, G^{\frac{1}{y}}, \hat{G}^{\frac{1}{y}} \right)$$

$$\text{ConvertSig}(\text{vk}, M, \sigma ; \rho) \rightarrow \tilde{\sigma}$$

$$\underline{\text{ConvertVK}(\text{vk} ; \rho)} \rightarrow \tilde{\text{vk}}$$

$$\tilde{\text{vk}} = \text{vk}^{\rho} = (\hat{G}^{x_1 \rho}, \dots, \hat{G}^{x_{\ell} \rho})$$

Change Sign to TSign with 2 Party Interactive Protocol

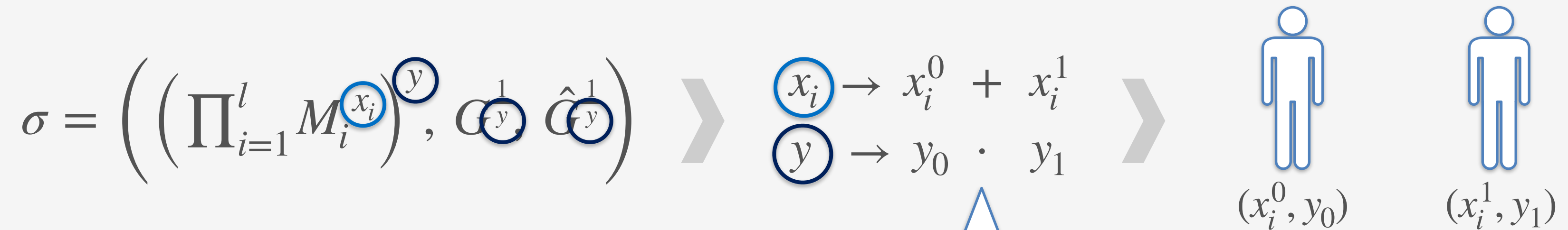


**Verification and Conversion method** in the original are adapted directly

... To keep the flexibility for applications using Mercurial Signatures

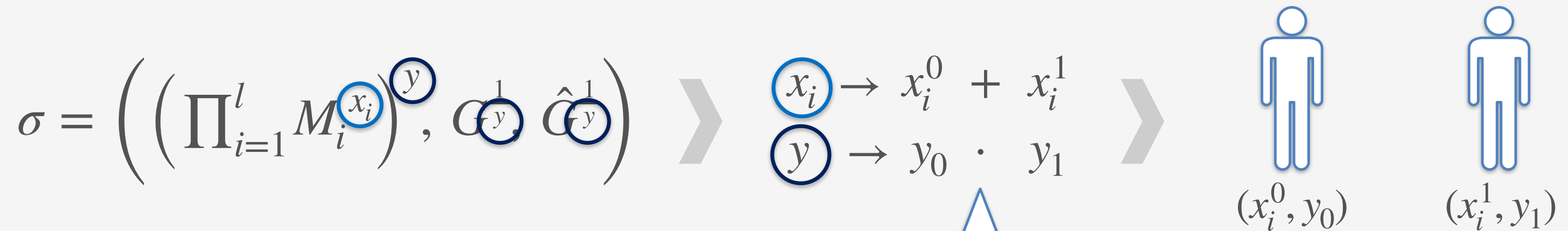
$\tilde{\text{sk}}$   
 $x_{\ell}^{\rho}$

Key is shared **additively** / Ephemeral Randomness is shared **multiplicatively**



This multiplicative sharing makes easier to add randomness one by one in the Sequential stream

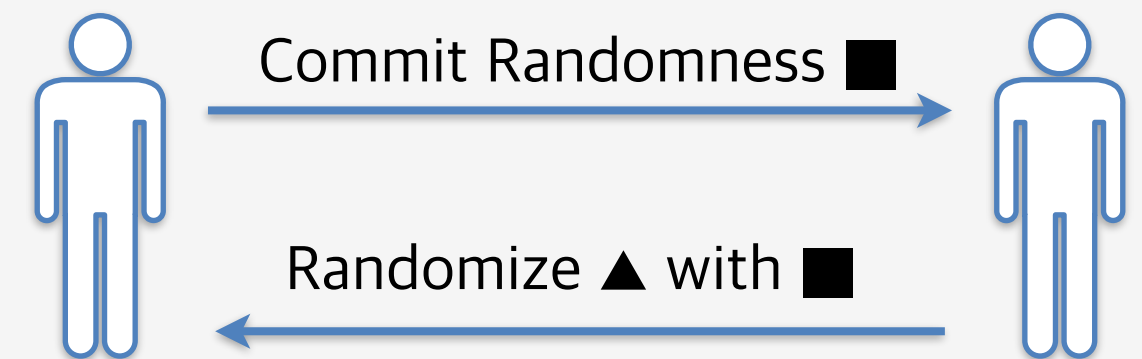
Key is shared **additively** / Ephemeral Randomness is shared **multiplicatively**



This multiplicative sharing makes easier to add randomness one by one in the Sequential stream

## Blinded Computation

... Blinding local computation using other party's random factor



$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$Z_0 = \prod_{i=1}^{\ell} M_i^{x_i^0}$$

$Z_0$

$$y_1 \xleftarrow{\$} \mathbb{Z}_p^*, Y_1 = G^{y_1}, \hat{Y}_1 = \hat{G}^{y_1}$$

$$Z_1 = \left( Z_0 \cdot \prod_{i=1}^{\ell} M_i^{x_i^1} \right)^{y_1}$$

$Z_1, Y_1, \hat{Y}_1$

$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, Y = Y_1^{y_0}, \hat{Y} = \hat{Y}_1^{y_0}$$

$$Z = Z_1^{y_0}$$

Return  $\sigma = (Z, Y, \hat{Y})$

# Problem : Naive protocol

$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$Z_0 = \prod_{i=1}^{\ell} M_i^{x_i^0}$$

$Z_0$  is computed with only deterministic values

 $Z_0$ 

$$y_1 \xleftarrow{\$} \mathbb{Z}_p^*, Y_1 = G^{y_1}, \hat{Y}_1 = \hat{G}^{y_1}$$

$$Z_1 = \left( Z_0 \cdot \prod_{i=1}^{\ell} M_i^{x_i^1} \right)^{y_1}$$

 $Z_1, Y_1, \hat{Y}_1$ 

$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, Y = Y_1^{y_0}, \hat{Y} = \hat{Y}_1^{y_0}$$

$$Z = Z_1^{y_0}$$

Return  $\sigma = (Z, Y, \hat{Y})$

It makes a difficulty for setting secure simulator  
It is required to blind  $Z_0$  without harming protocol



$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, \quad Y_0 = G^{y_0}, \quad \hat{Y}_0 = \hat{G}^{y_0}$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

Commitment of  $y_0$  (using ZK)

$$\text{Return } \sigma = (Z, Y, \hat{Y})$$

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$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, Y_0 = G^{y_0}, \hat{Y}_0 = \hat{G}^{y_0}$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

$$r \xleftarrow{\$} \mathbb{Z}_p^*, y_1 \xleftarrow{\$} \mathbb{Z}_p^*$$

$$K_1, \pi_1^{(1)}$$

$$K_1 = Y_0^r \cdot \prod_{i=1}^{\ell} M_i^{x_i^1}, Y = Y_0^{y_1}, \hat{Y} = \hat{Y}_0^{y_1}$$

Commitment of  $y_1$  (using ZK)

+

Partial signature is blinded with  $Y_0^r = G^{ry_0}$

Return  $\sigma = (Z, Y, \hat{Y})$

Return  $\sigma = (Z, Y, \hat{Y})$

$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

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$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, Y_0 = G^{y_0}, \hat{Y}_0 = \hat{G}^{y_0}$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

$$r \xleftarrow{\$} \mathbb{Z}_p^*, y_1 \xleftarrow{\$} \mathbb{Z}_p^*$$

$$K_1, \pi_1^{(1)}$$

$$K_1 = Y_0^r \cdot \prod_{i=1}^{\ell} M_i^{x_i^1}, Y = Y_0^{\frac{1}{y_1}}, \hat{Y} = \hat{Y}_0^{\frac{1}{y_1}}$$

$$Z_0 = \left( K_1 \cdot \prod_{i=1}^{\ell} M_i^{x_i^0} \right)^{y_0}$$

$$Z_0, \pi_0^{(2)}$$

Partial signing and randomizing

$$\text{Expansion... } Z_0 = G^{\frac{r}{y_0} \cdot y_0} \cdot \left( \prod_{i=1}^{\ell} M_i^{x_i^0 + x_i^1} \right)^{y_0}$$

Return  $\sigma = (Z, Y, Y)$

Return  $\sigma = (Z, Y, \hat{Y})$

$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

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$$y_0 \xleftarrow{\$} \mathbb{Z}_p^*, Y_0 = G^{y_0}, \hat{Y}_0 = \hat{G}^{y_0}$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

$$r \xleftarrow{\$} \mathbb{Z}_p^*, y_1 \xleftarrow{\$} \mathbb{Z}_p^*$$

$$K_1, \pi_1^{(1)}$$

$$K_1 = Y_0^r \cdot \prod_{i=1}^{\ell} M_i^{x_i^1}, Y = Y_0^{\frac{1}{y_1}}, \hat{Y} = \hat{Y}_0^{\frac{1}{y_1}}$$

$$Z_0 = \left( K_1 \cdot \prod_{i=1}^{\ell} M_i^{x_i^0} \right)^{y_0}$$

$$Z_0, \pi_0^{(2)}$$

$$Z = (Z_0 \cdot G^{-r})^{y_1} \quad \text{Return } \sigma = (Z, Y, \hat{Y})$$

$$\text{Return } \sigma = (Z, Y, \hat{Y})$$

Offsetting blinded part and randomizing

$$\text{Expansion... } Z = \left( G^{r-r} \cdot \prod_{i=1}^{\ell} M_i^{x_i^0 + x_i^1} \right)^{y_0 y_1}$$

Corr.

Sim. with  $\text{Sign}(\text{sk}, \cdot)$

$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$(Y_0, \hat{Y}_0, \pi_0^{(1)}) \leftarrow \mathcal{A}(\text{st})$$

$$(Z', Y', \hat{Y}') \leftarrow \text{Sign}(\text{sk}, M)$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

$$K_1 \stackrel{\$}{\leftarrow} \mathbb{G}, Y \leftarrow Y', \hat{Y} \leftarrow \hat{Y}'$$

$$K_1, \pi_1^{(1)}$$

$$\pi_1^{(1)} \leftarrow \text{ZKPoK} . \text{Sim}(Z_1, Y_0, M)$$

$$(Z_0, \pi_0^{(2)}) \leftarrow \mathcal{A}(\text{st}, K_1, \pi_1^{(1)})$$

$$Z_0, \pi_0^{(2)}$$

$$Z \leftarrow Z' \quad \sigma = (Z, Y, \hat{Y})$$

$$\sigma, \pi_1^{(1)}$$

$$\pi_1^{(2)} \leftarrow \text{ZKPoK} . \text{Sim}(Z, Z_0, Y, Y_0)$$

$$\text{Return } \sigma = (Z, Y, \hat{Y})$$

$$\text{Return } \sigma = (Z, Y, \hat{Y})$$

Sim. with  $\text{Sign}(\text{sk}, \cdot)$

Corr.

$$P_0 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_0 = \{x_i^0\}_{i \in [\ell]}$$

$$P_1 : M = \{M_i\}_{i \in [\ell]} \in (\mathbb{G})^\ell \quad \text{sk}_1 = \{x_i^1\}_{i \in [\ell]}$$

$$(Z', Y', \hat{Y}') \leftarrow \text{Sign}(\text{sk}, M)$$

$$Y_0 \leftarrow Y', \hat{Y}_0 \leftarrow \hat{Y}'$$

$$\pi_0^{(1)} \leftarrow \text{ZKPoK} . \text{Sim}(Y_0, \hat{Y}_0)$$

$$Y_0, \hat{Y}_0, \pi_0^{(1)}$$

$$(K_1, \pi_0^{(1)}) \leftarrow \mathcal{A}(\text{st}, Y_0, \hat{Y}_0, \pi_0^{(1)})$$

$$K_1, \pi_1^{(1)}$$

$$r \leftarrow \text{ZKPoK} . \text{Ext}(\pi_1^{(1)}); Z_0 \leftarrow Z'G^r$$

$$\pi_0^{(2)} \leftarrow \text{ZKPoK} . \text{Sim}(Z_0, K_1, M, Y_0)$$

$$Z_0, \pi_0^{(2)}$$

$$(\sigma, \pi_0^{(1)}) \leftarrow \mathcal{A}(\text{st}, Z_0, \pi_0^{(2)})$$

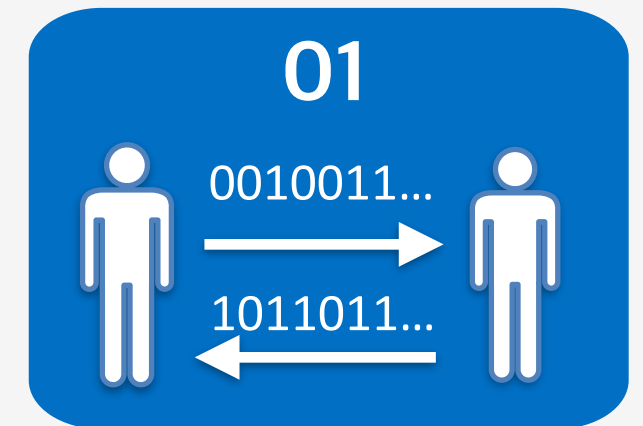
$$\sigma, \pi_1^{(1)}$$

$$\text{Return } \sigma = (Z, Y, \hat{Y})$$

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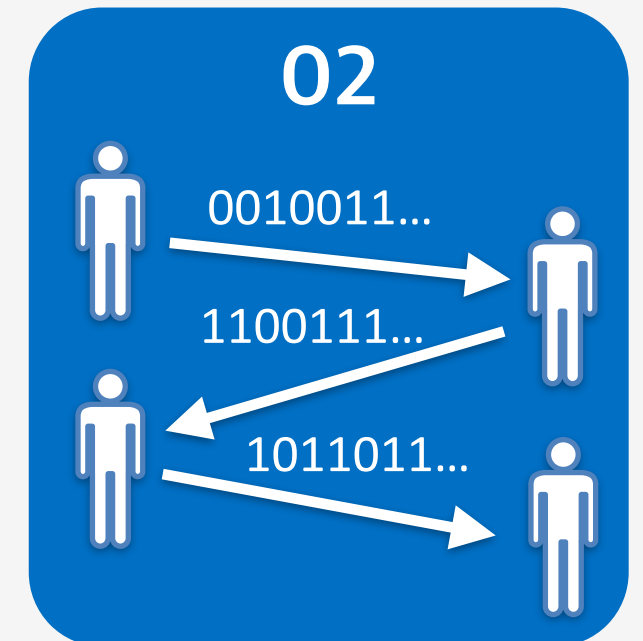
## 01 | Sequential communication model between Two parties

- It allows 1 party corruption



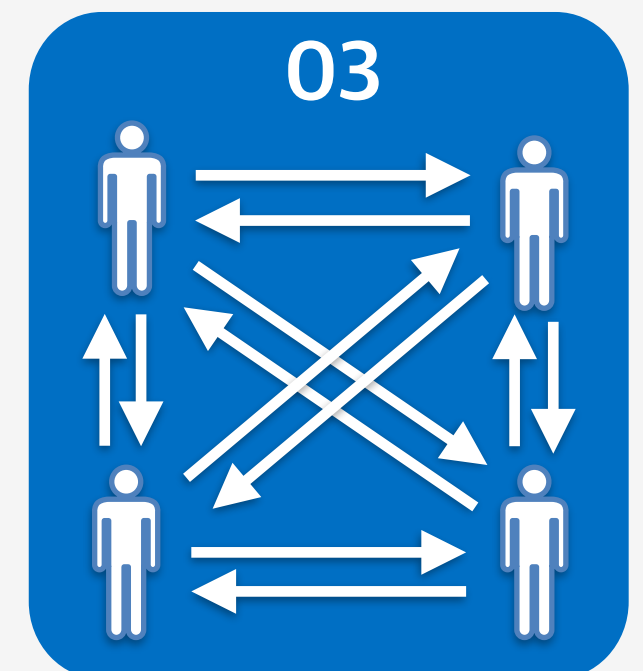
## 02 | Sequential communication model among t out of n parties

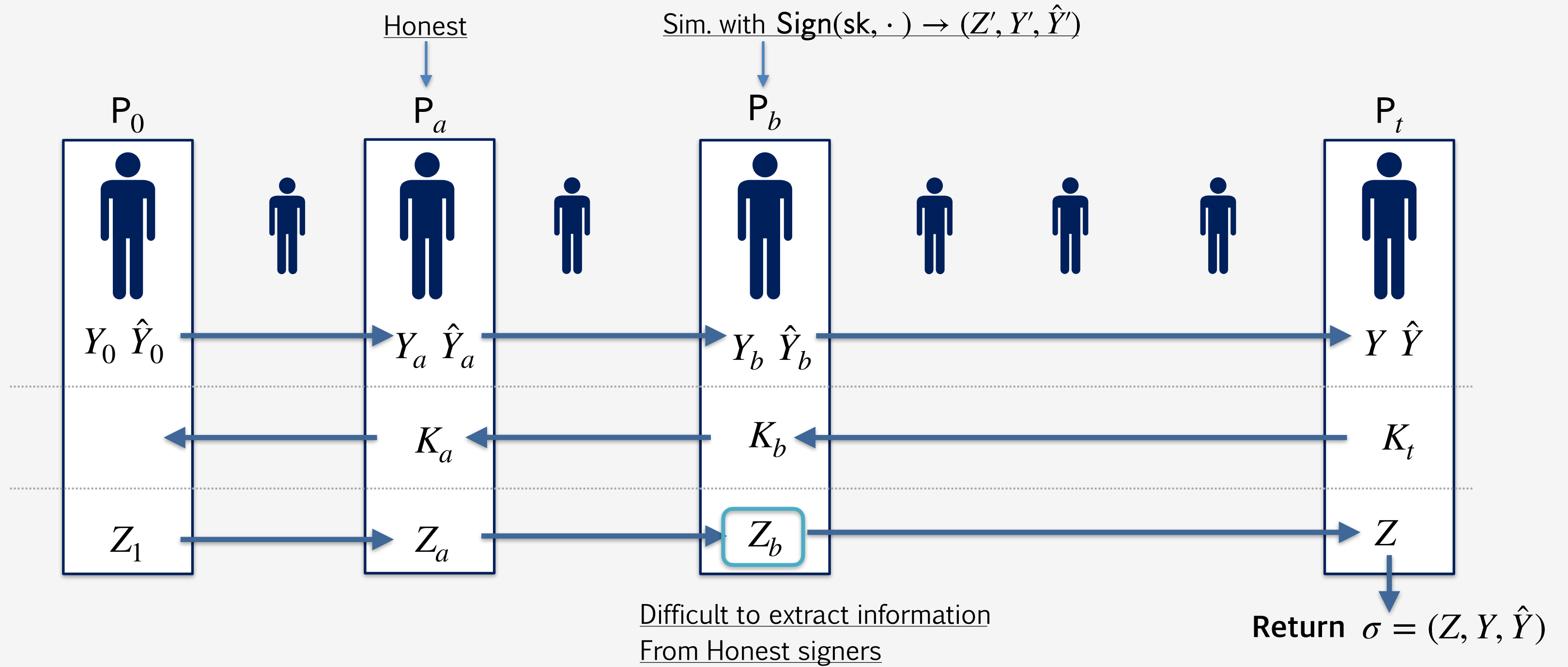
- It allows up to t-1 party corruption
- Pre-processing for secret sharing is required



## 03 | Synchronized communication model among t out of n parties

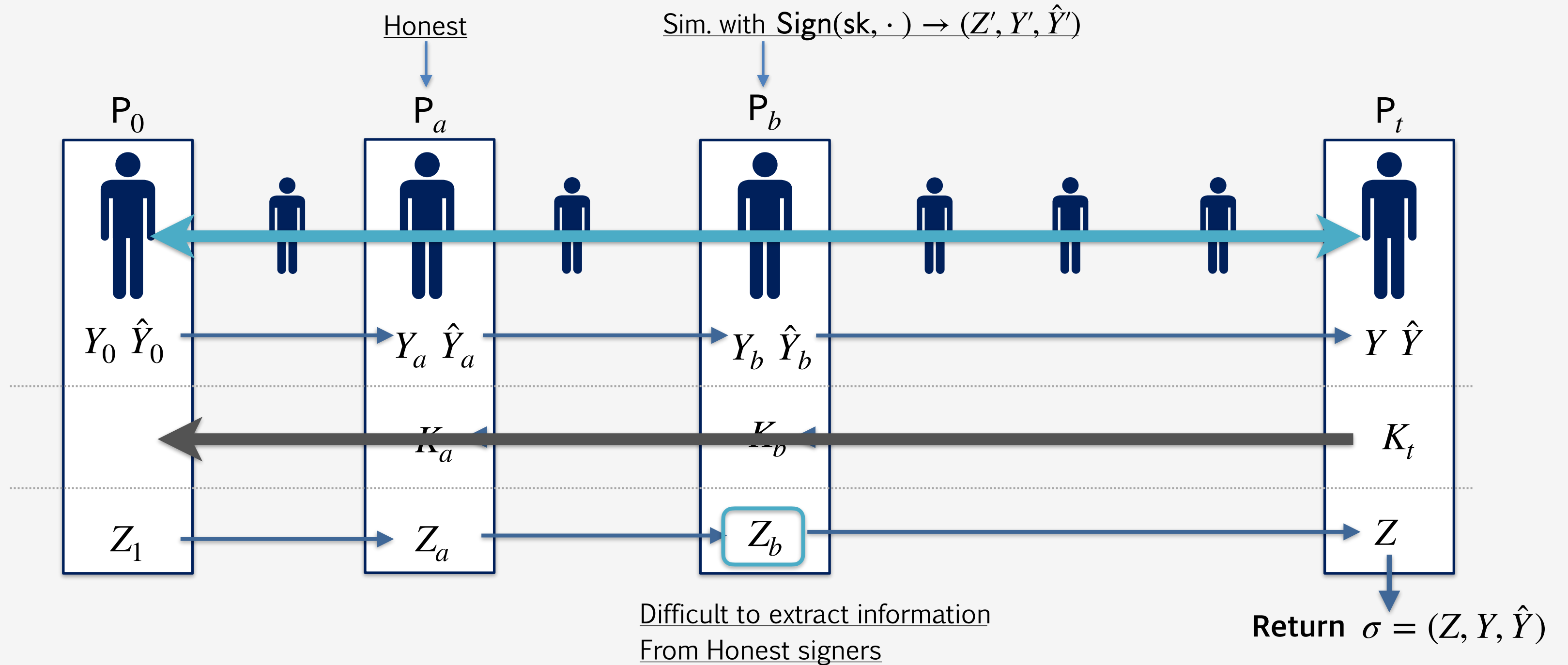
- It allows up to t-1 party corruption
- Broadcast messages with traditional MPC





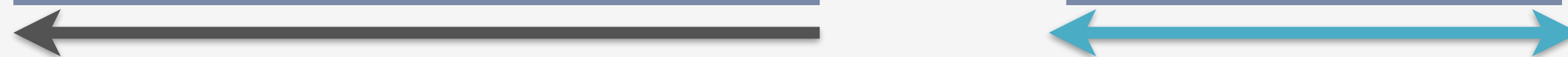
To construct the simulator for intermediators, another blinding trick is required





To construct the simulator for intermediators, another blinding trick is required

... Zero-Sharing over Public Channel (including pre-processing phase)



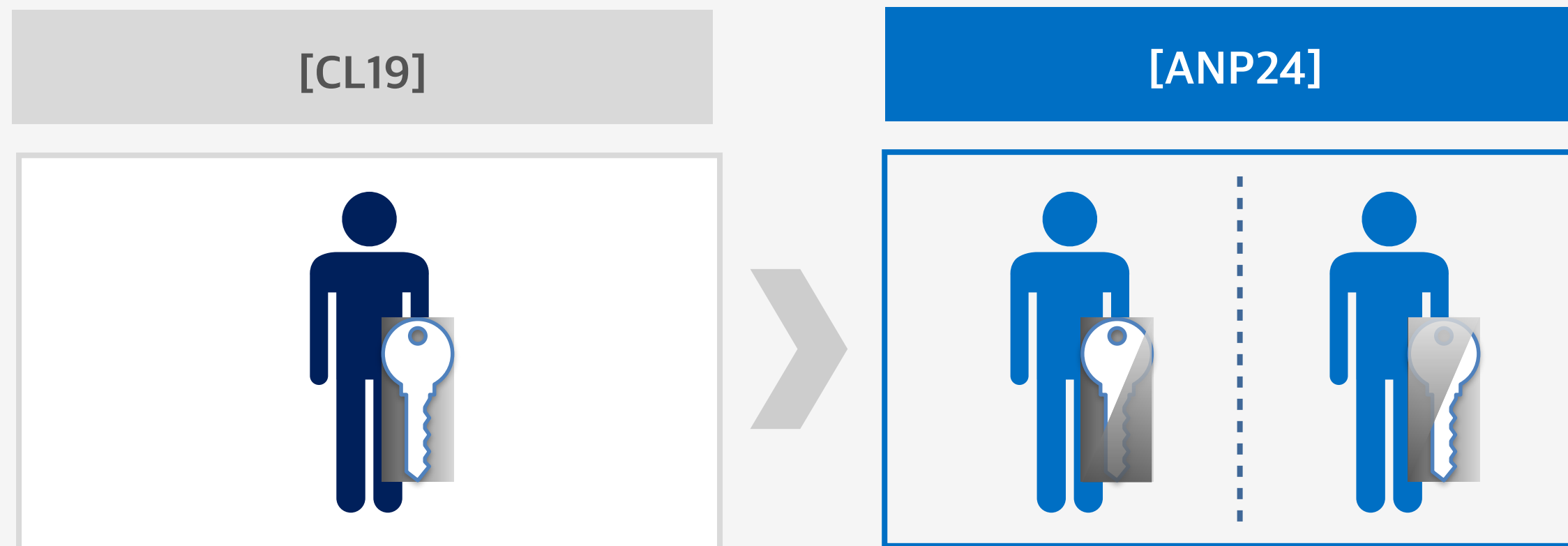
The cost is proportional to the size of the message and the number of parties

Scheme	# of Parties	Message Size 2	Message Size 5	Message Size 10
Mercurial Signatures [FHS19]	1	0.3	0.4	0.5
Sequential Communication in Two Parties	2	3.9	6.2	10.1
Sequential Communication in t out of n	5	13.3	19.3	29.6
Sequential Communication in t out of n	10	28.0	40.8	60.5

( Unit : millisecond )

## ✔ Contribution of our work

- > Extension for Mercurial Signatures for Distributed Parties (with threshold)
  1. Provides distributed trust of the root authority for delegatable credential system
  2. Improves privacy for standard anonymous credentials



- > Implementation of our scheme to show its reasonable cost



- > More Applications

  - ... e.g. Delegatable Anonymous Credentials System

- > Stronger security

  - ... e.g. Asynchronous and non-erasable Communication Model,  
Security for Adaptive Corruption



The latest version of our paper (<https://eprint.iacr.org/2024/625>)

Artifact of Implementation is accepted by IACR