Strongly Secure Universal Thresholdizer

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IST Austria

Threshold Cryptography - Motivation

Distributes privileged operation amongst multiple parties

↓

no single point of security failure

ThPKE, ThSignature, ThIBE, ThTraitorTracing, etc.

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Interactive /non-interactive

classical / post-quantum assumptions

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Adds thresholdizing functionality to several cryptographic primitives

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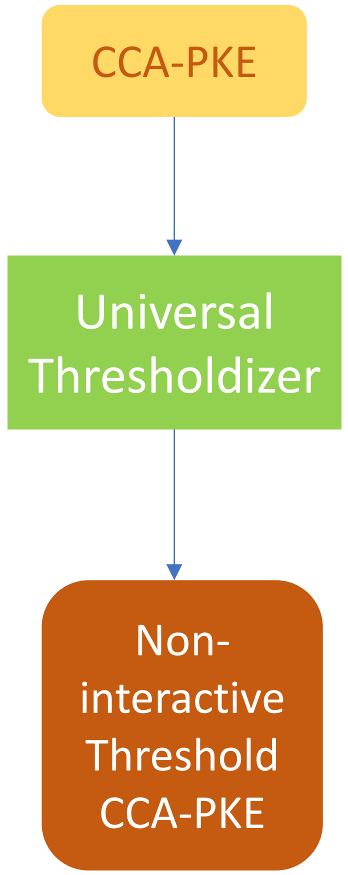

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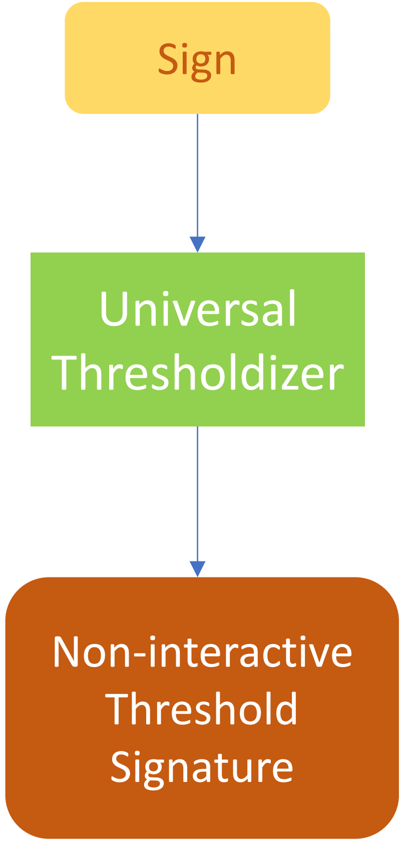

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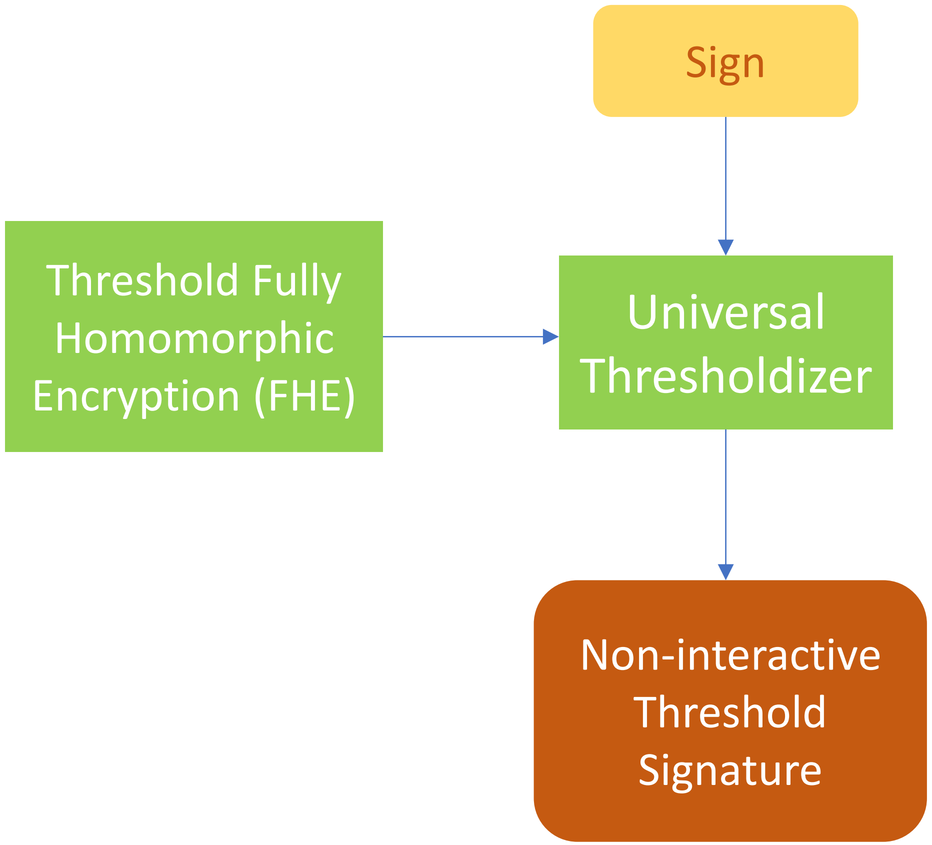

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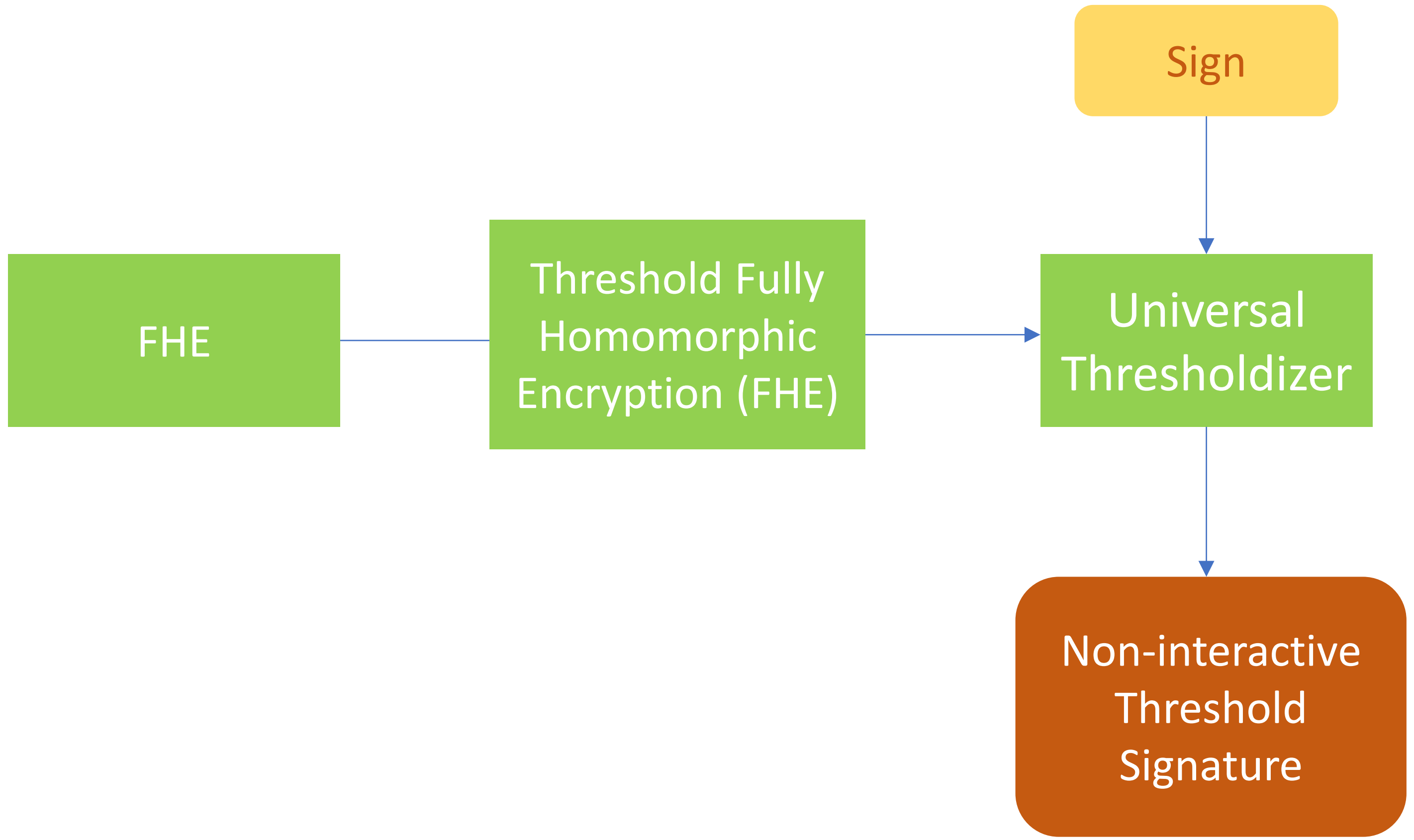

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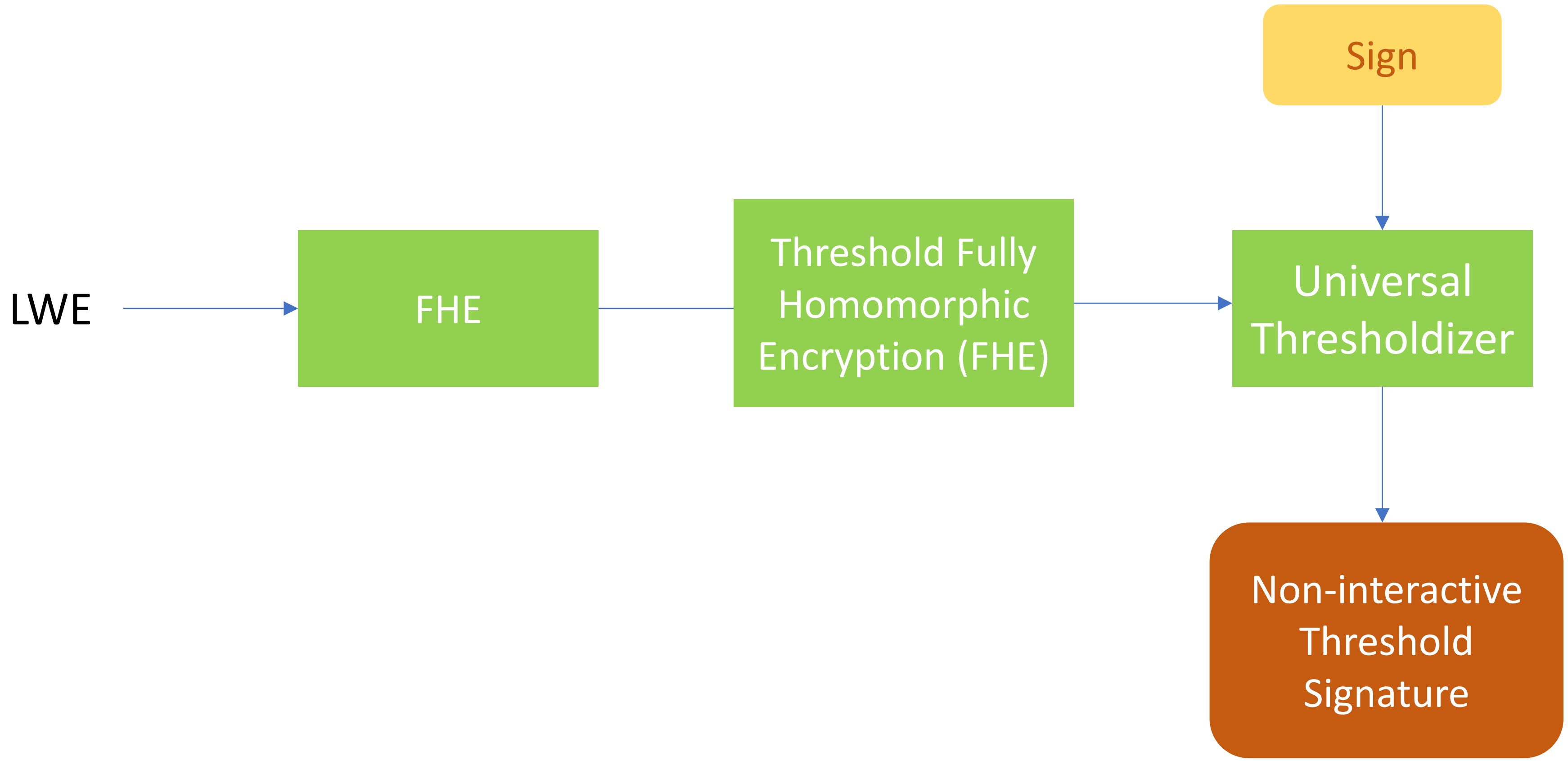

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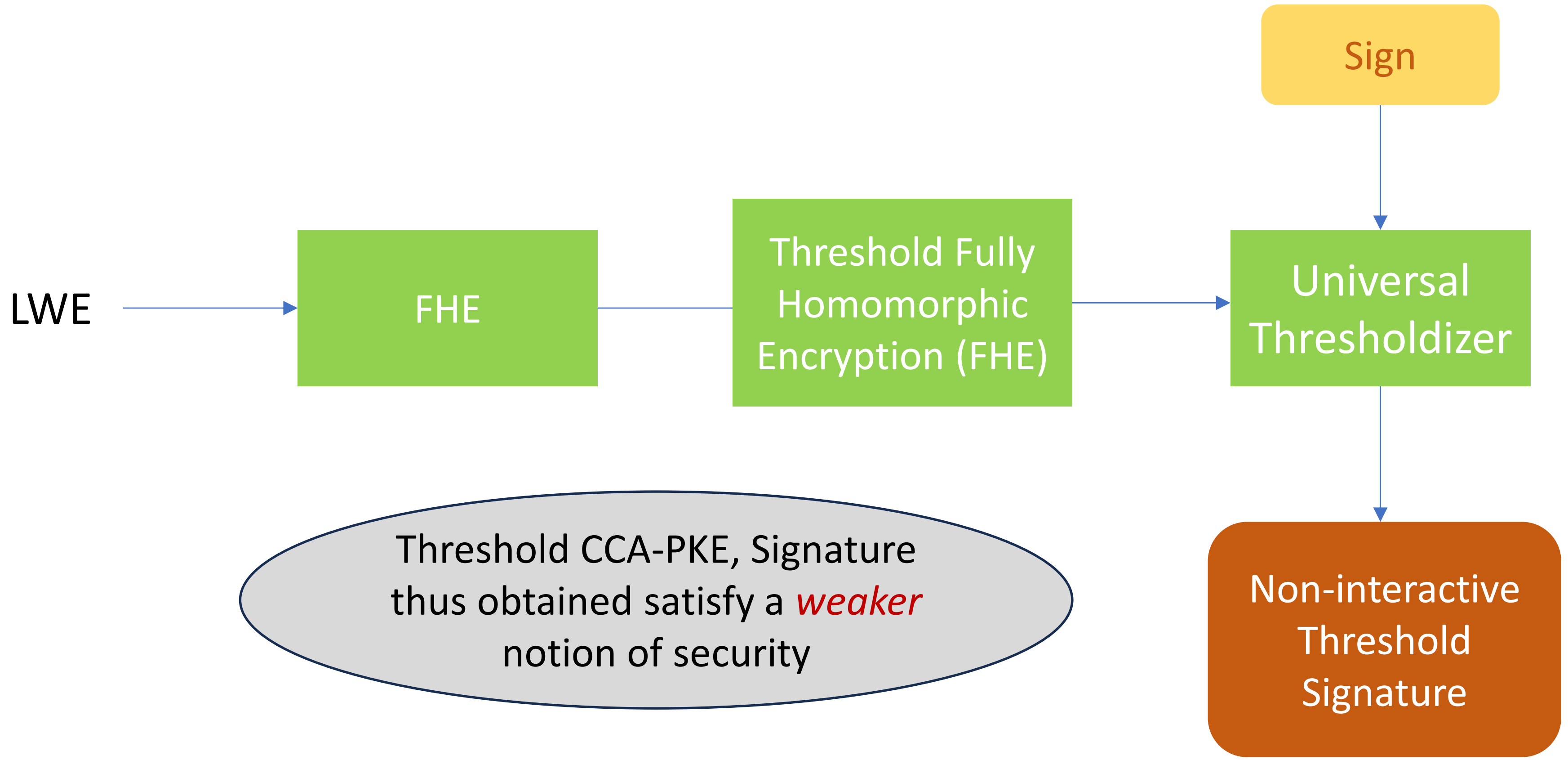

Our Contributions

- Define and build universal thresholdizer (UT) and threshold FHE (TFHE) with *stronger* security notions
  - Needed to achieve *stronger* security for primitives thresholdized using UT

- Using our universal thresholdizer we get the first non-interactive lattice based threshold signature scheme with the stronger security

- Also define various security notions for Threshold Signature and relations between them

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Related to (partial) adaptivity

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  - Needed to achieve *stronger* security for primitives thresholdized using UT

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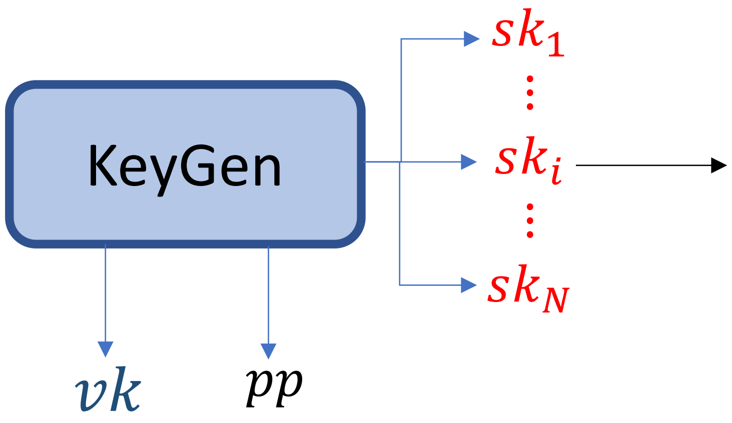
Threshold Signature Definition [BGG+18]



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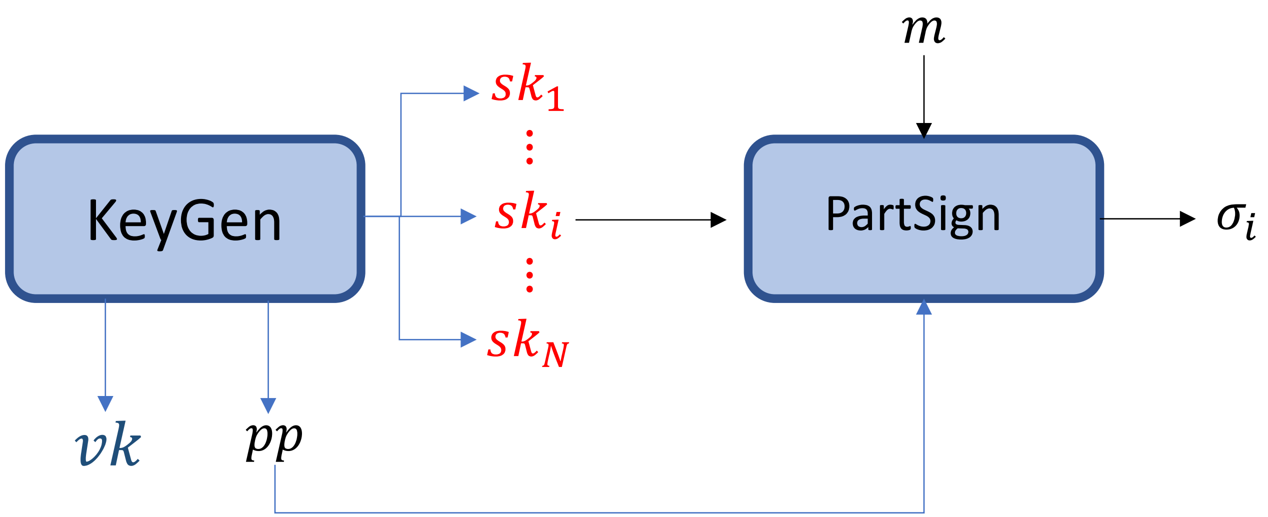

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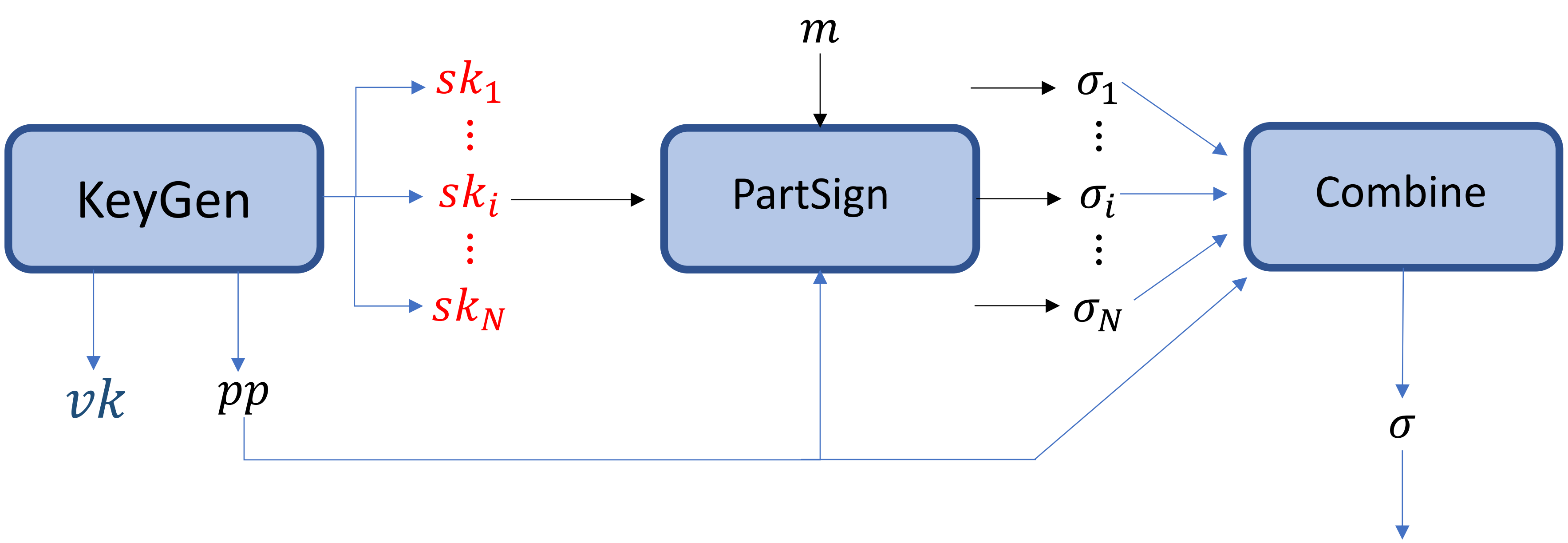

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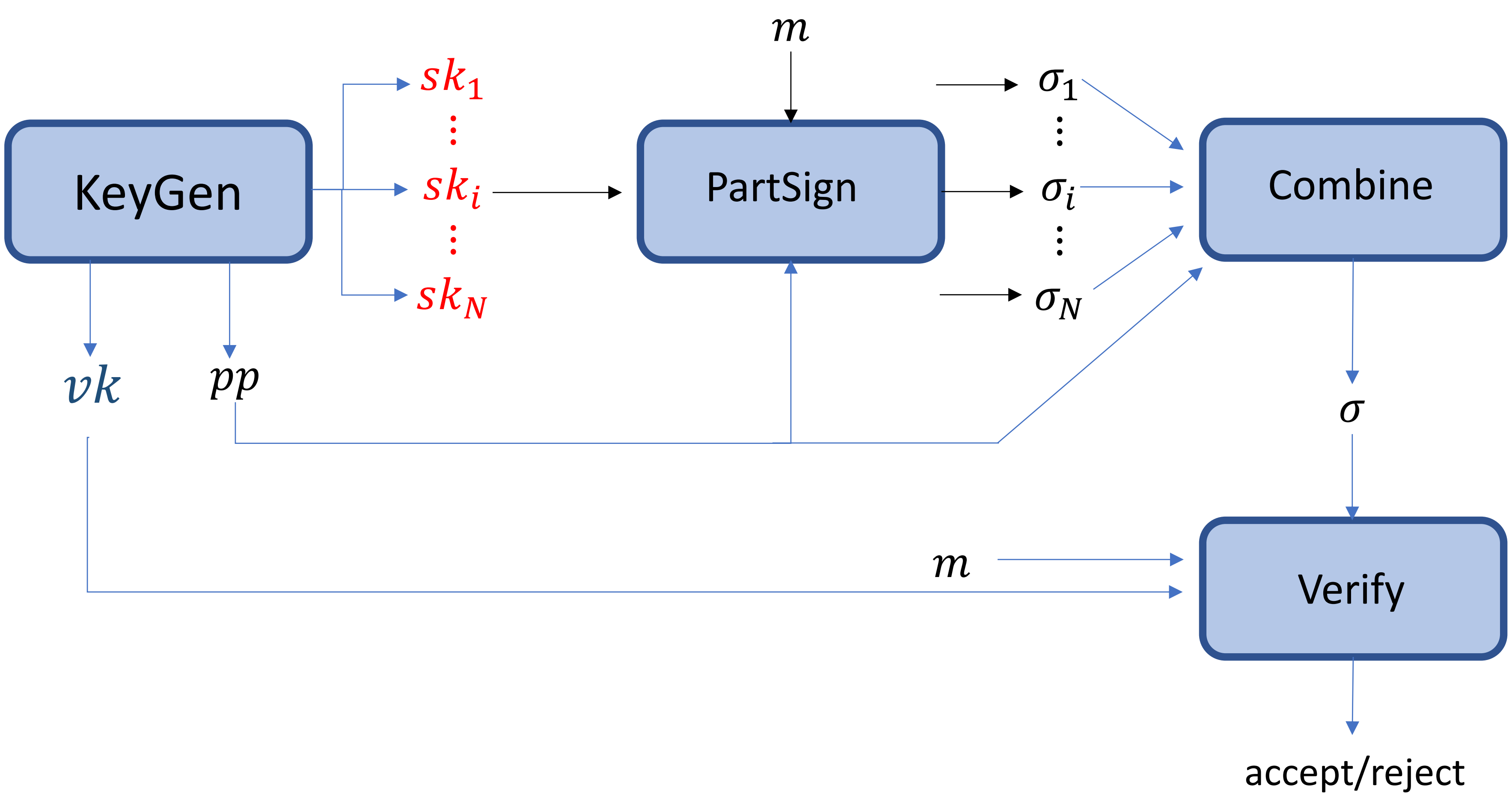

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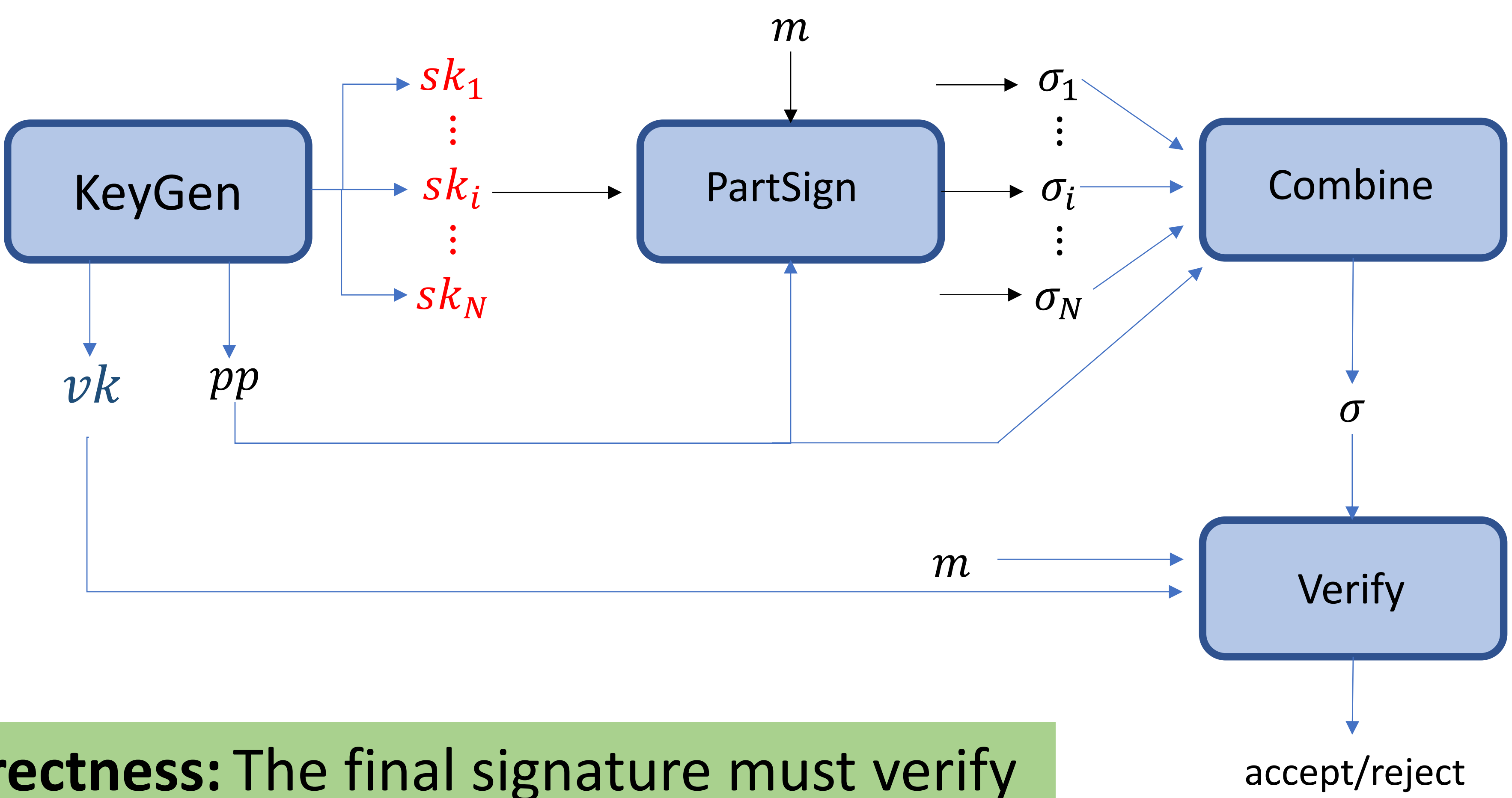

Correctness: The final signature must verify

Threshold Signature Security

No polynomial time adversary must be able to generate a signature on any message m^* even given

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✓ Partial signing keys from upto $t-1$ parties of adversary's choice

Threshold Signature Security

No polynomial time adversary must be able to generate a signature on any message m^* even given

- ✓ Partial signing keys from upto $t - 1$ parties of adversary's choice
- ✓ Partial/complete signatures on any number of other messages of adversary's choice

Threshold Signature Security

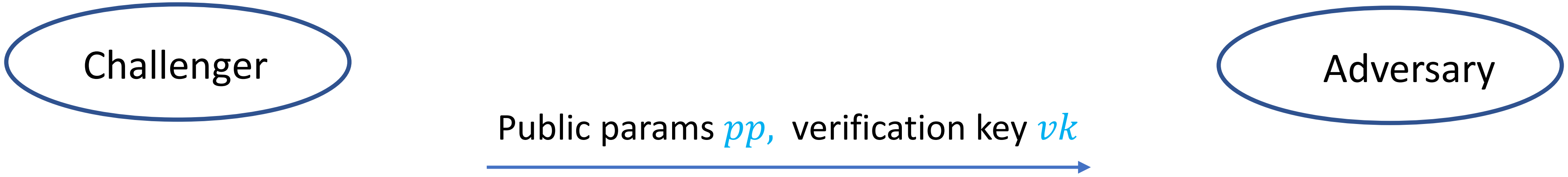
t-out-of-N access structure

Challenger

Adversary

Threshold Signature Security

t-out-of-N access structure



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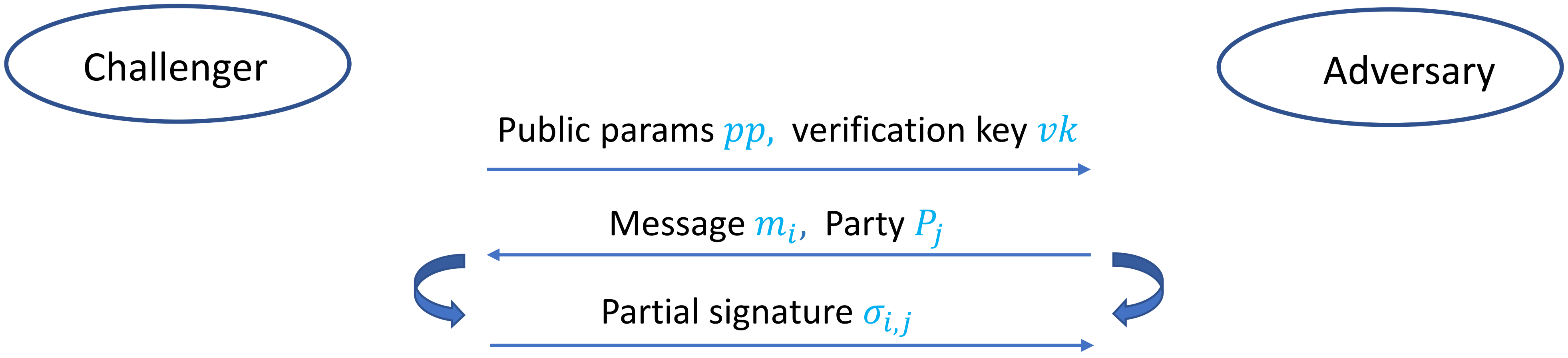

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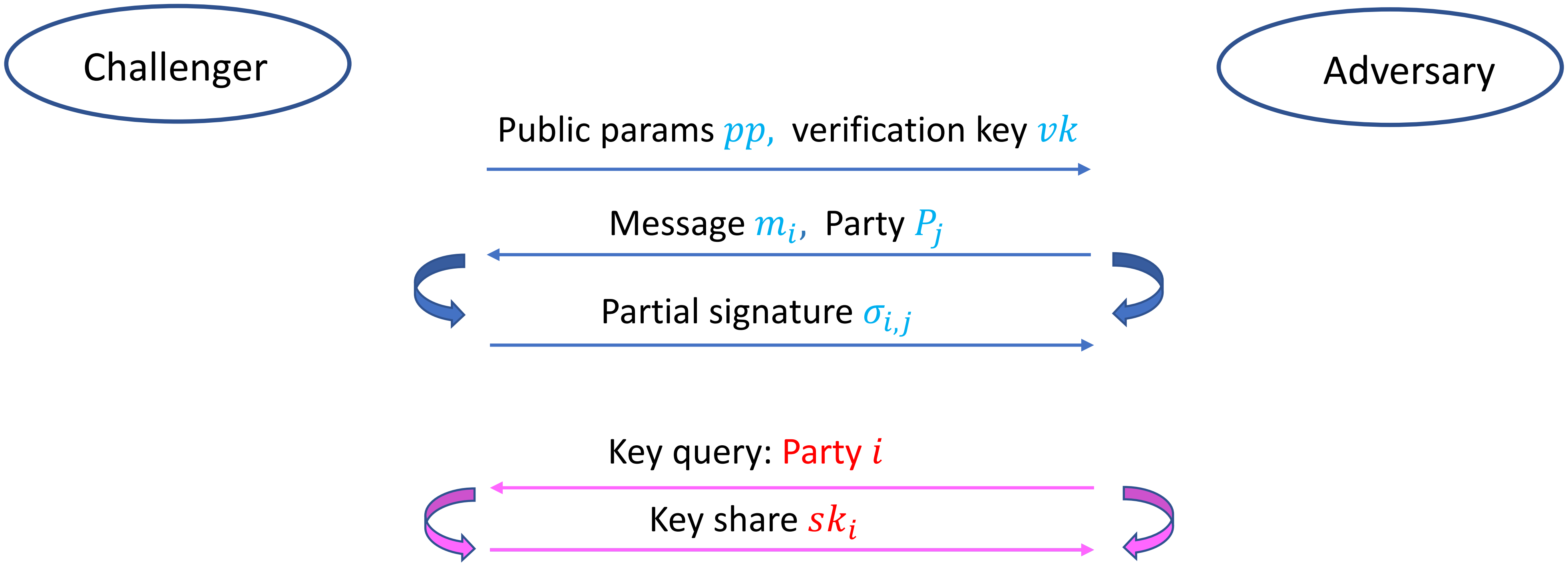

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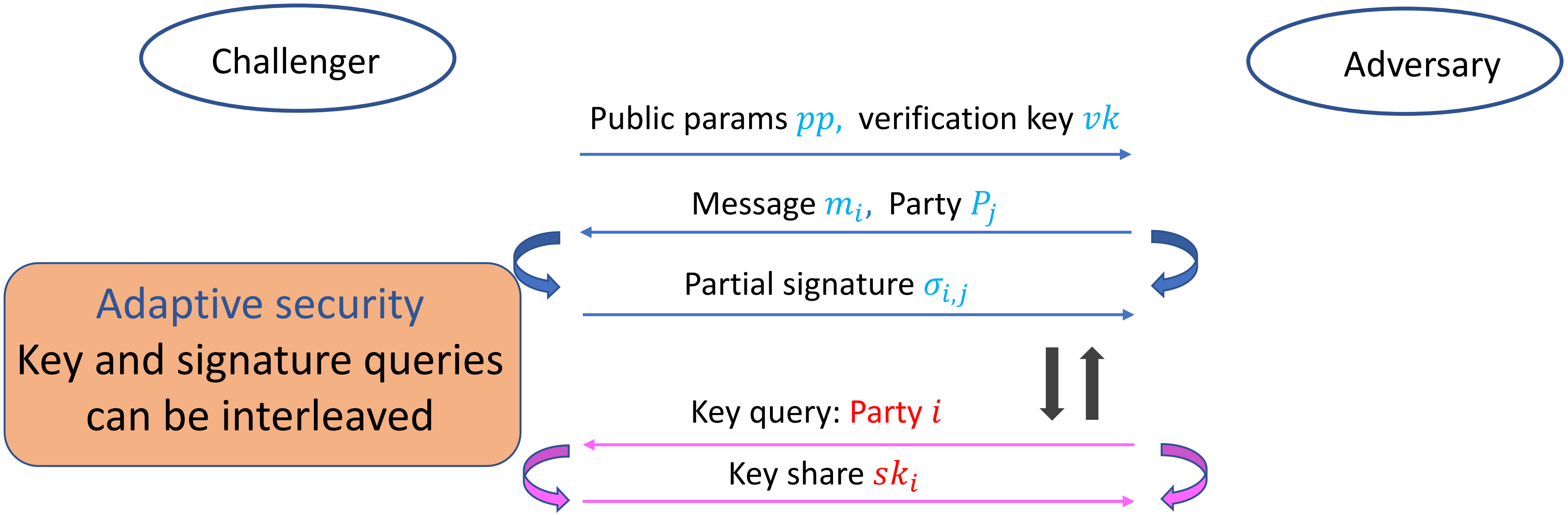

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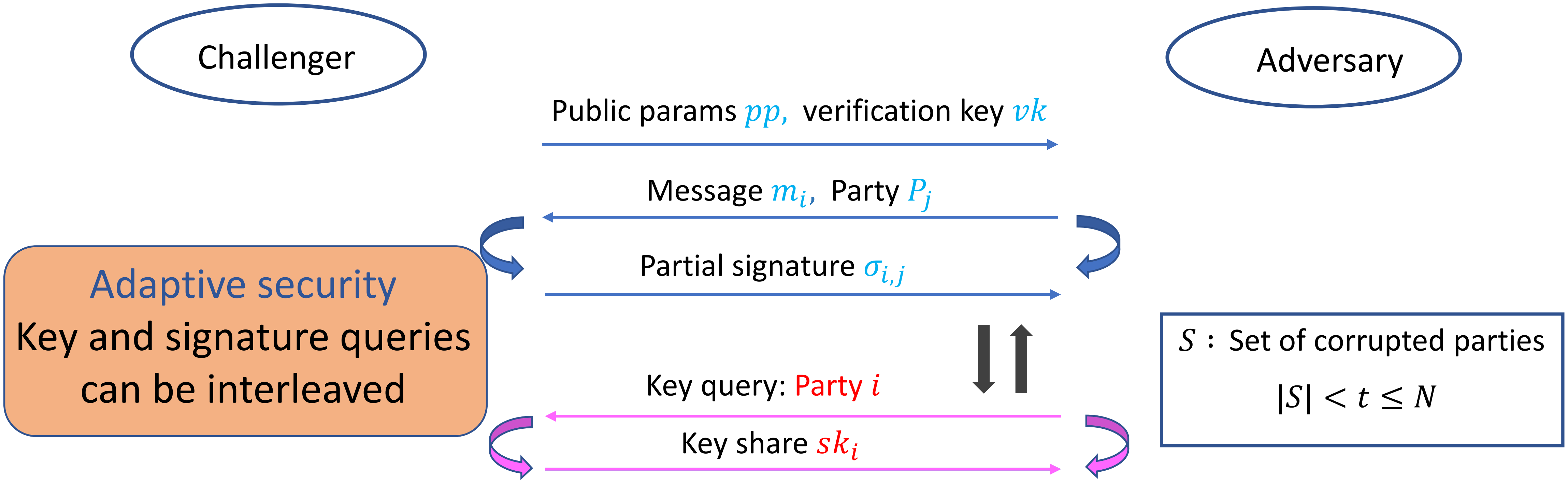

Threshold Signature Security

t-out-of-N access structure

Challenger

Adversary

Public params pp, verification key vk →

← Message m_i, Party P_j

Partial signature $\sigma_{i,j}$ →

Adaptive security
Key and signature queries can be interleaved

← Key query: Party i

Key share sk_i →

S : Set of corrupted parties

$|S| < t \leq N$

← Message-signature (m^*, σ^*)

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The adversary wins if σ^* is a valid signature on m^* and a signature on m^* cannot be trivially computed

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Generally hard to achieve

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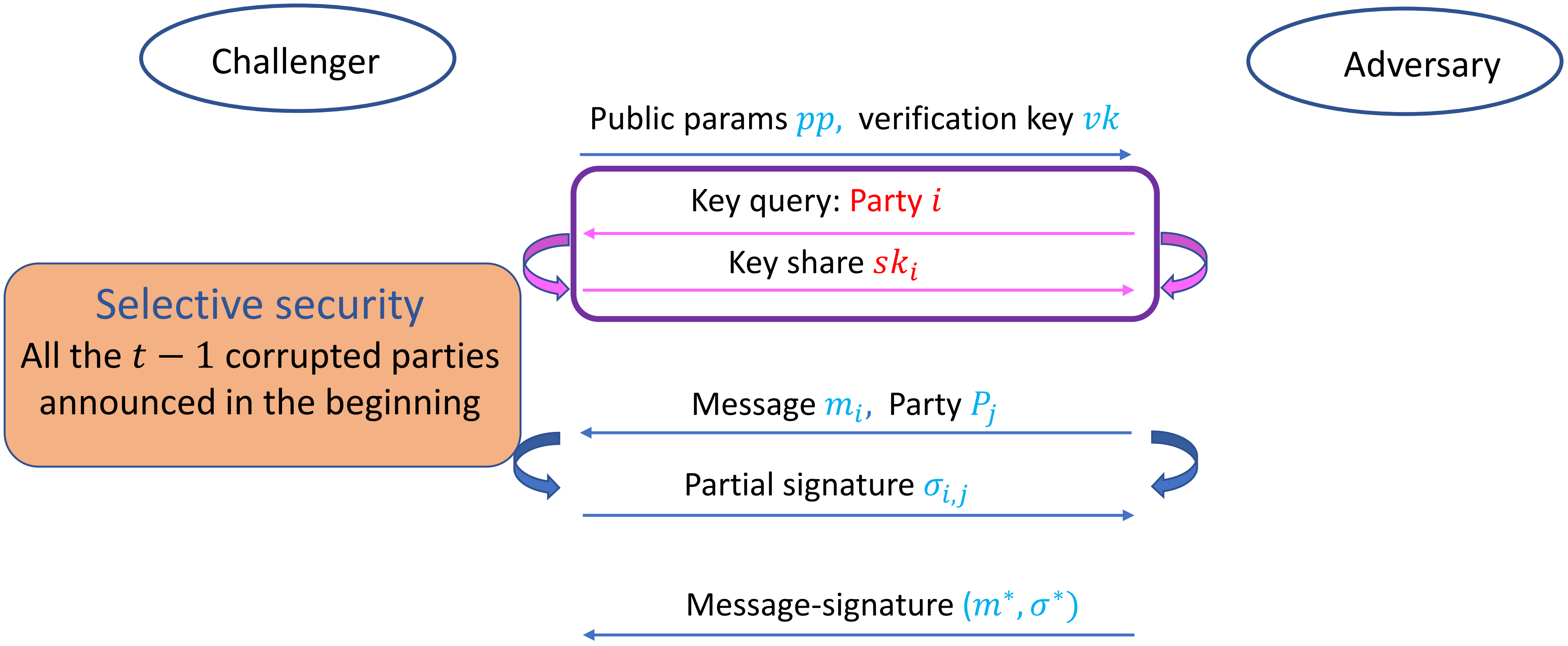

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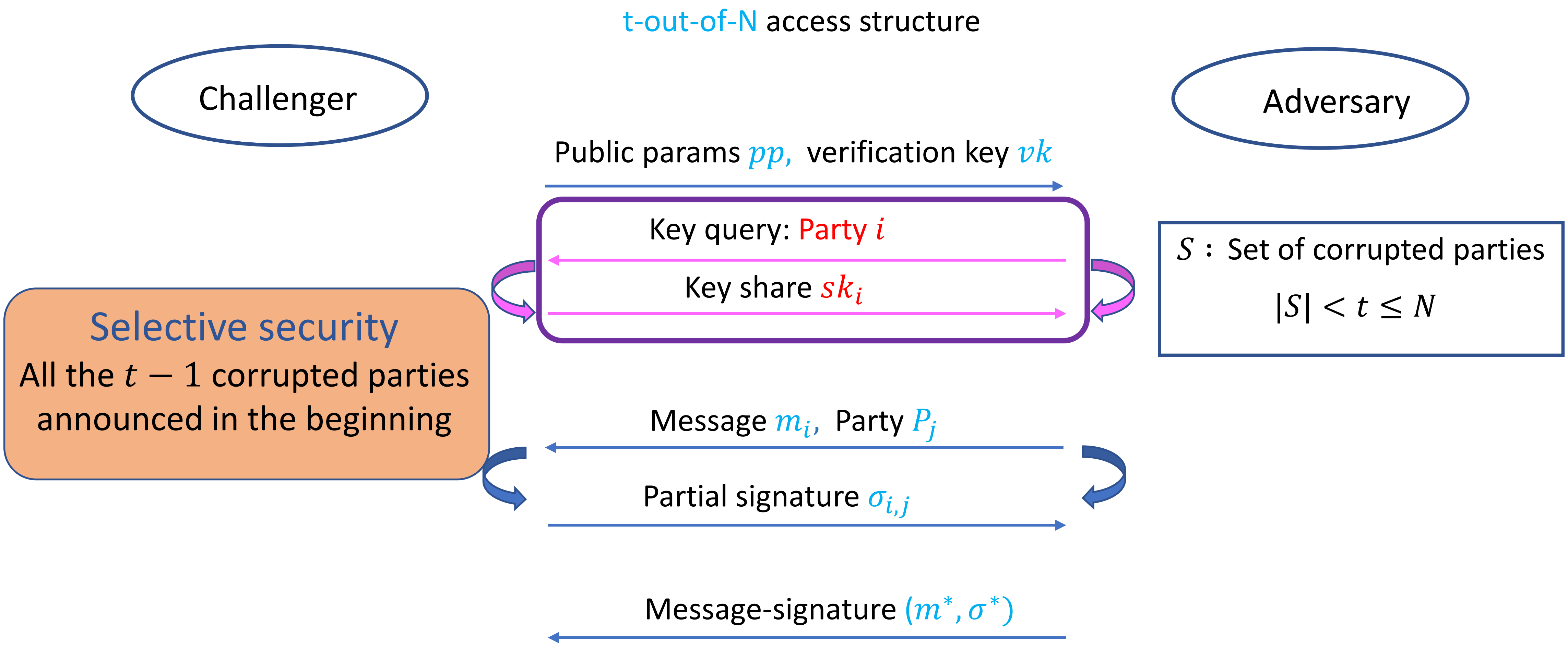

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Selective security
All the $t - 1$ corrupted parties announced in the beginning

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- σ^* is a valid signature on m^* and
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Selective security

All the $t-1$ corrupted parties announced in the beginning

Enforces strict selectivity in choosing the $t-1$ parties

Message m_i, Party P_j

Partial signature $\sigma_{i,j}$

Message-signature (m^*, σ^*)

The adversary wins if

- σ^* is a valid signature on m^* and
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Threshold Signature Security Definitions

Adaptive key queries

Part Sign on m^* ✓

At least as strong (adaptivity) as

Selective key queries [BGG+18]
– all $t-1$ key queries in the beginning of the game

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Our Construction

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Key Homomorphic
PRF (KHPRF)

+

BGG+18

Our Construction

$F(K_1, x) + F(K_2, x)$
$= F(K_1 + K_2, x)$

Obs: $F(0, x) = 0$

Key Homomorphic
PRF (KHPRF)

+

BGG+18

[BGG+18] Construction of Threshold Signatures

Building Blocks

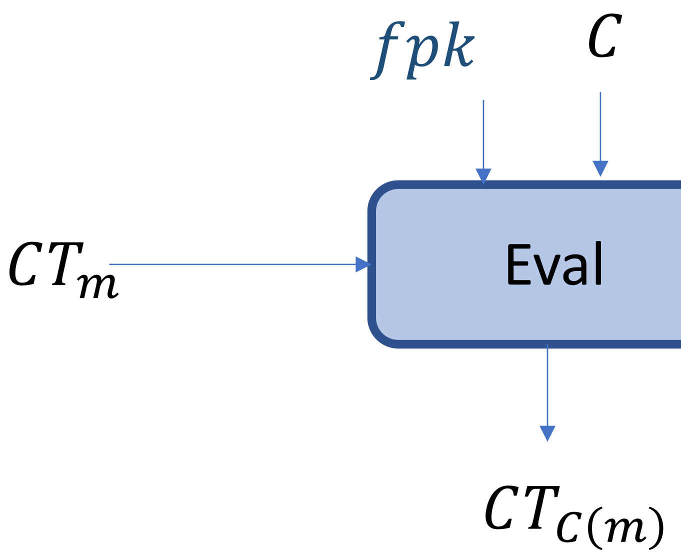
Standard signature scheme $(keys\text{: } sigvk, sigsk)$

FHE scheme $(keys\text{: } fpk, fsk)$

A Linear secret sharing scheme

Fully Homomorphic Encryption

Same as public key encryption scheme with added functionality



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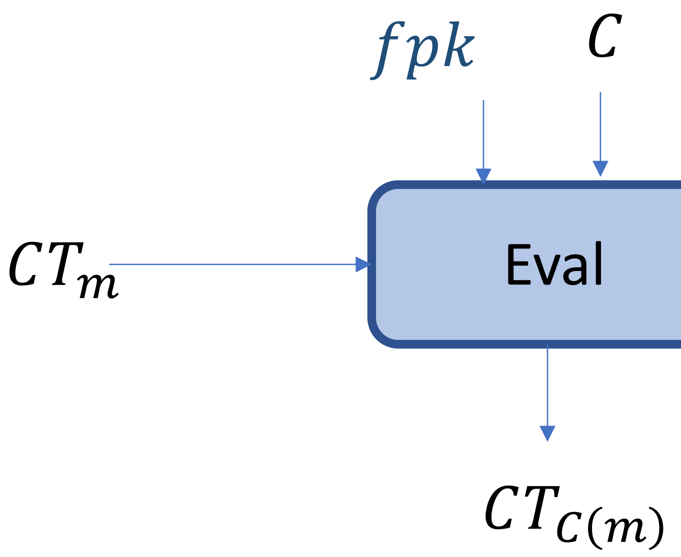
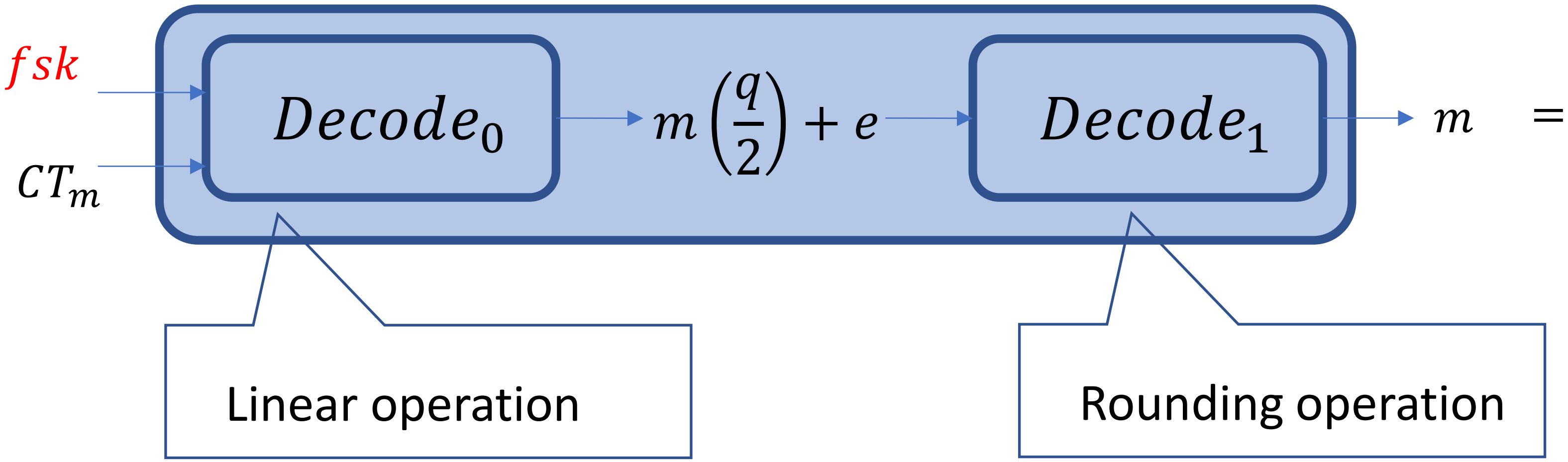


Public operation
Deterministic

Special Fully Homomorphic Encryption

Secret key fsk is a vector

Decrypt



$= 0 \;\; if\ m\left(\frac{q}{2}\right) + e < \frac{q}{2};$

1 otherwise

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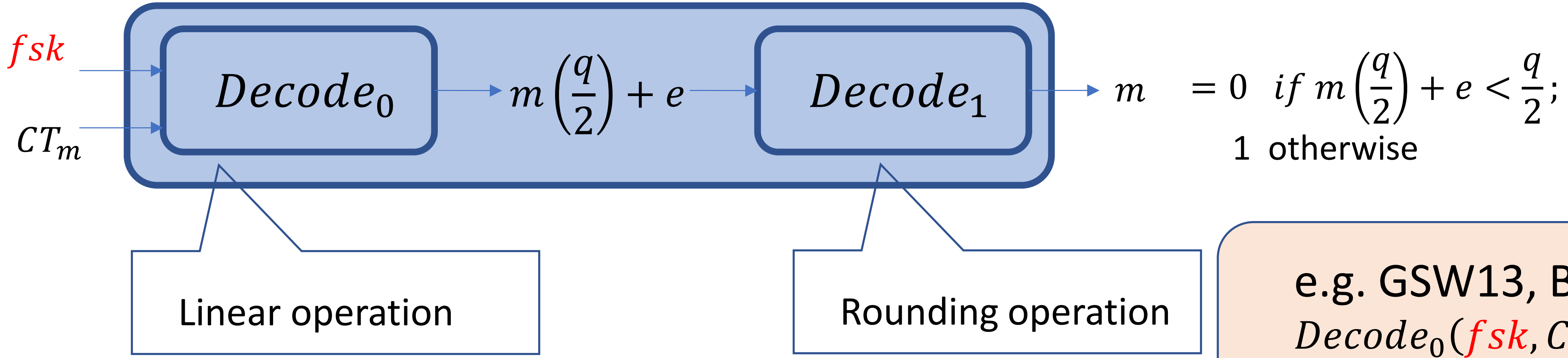

e.g. GSW13, BV11
$Decode_0(fsk, CT_m)$
$= \langle fsk, CT_m \rangle$

Usefulness of Linearity of $Decode_0$

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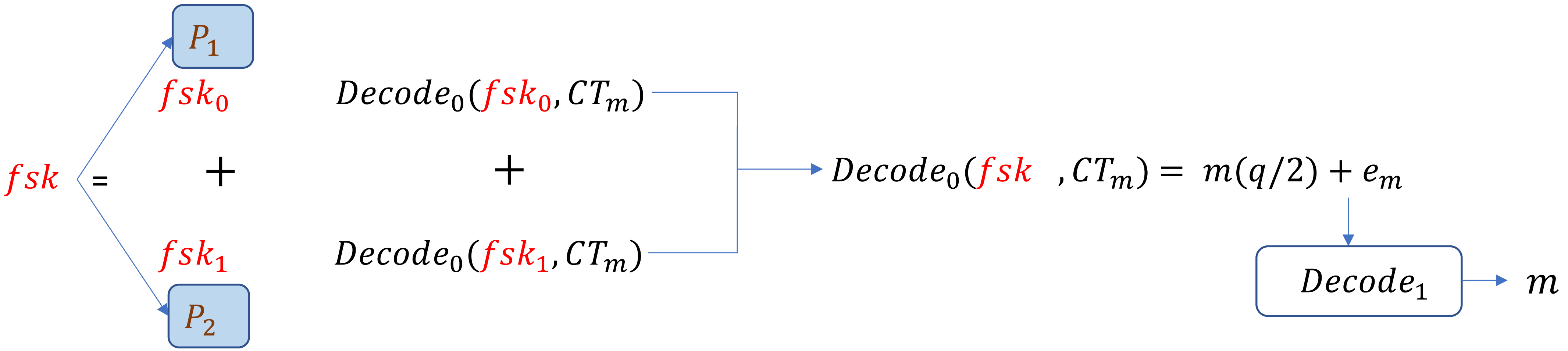

Construction Overview [BGG+18]

for 2-out-of-2 scheme

Key shares

PartSign(m)

Combine

Public params (pp)

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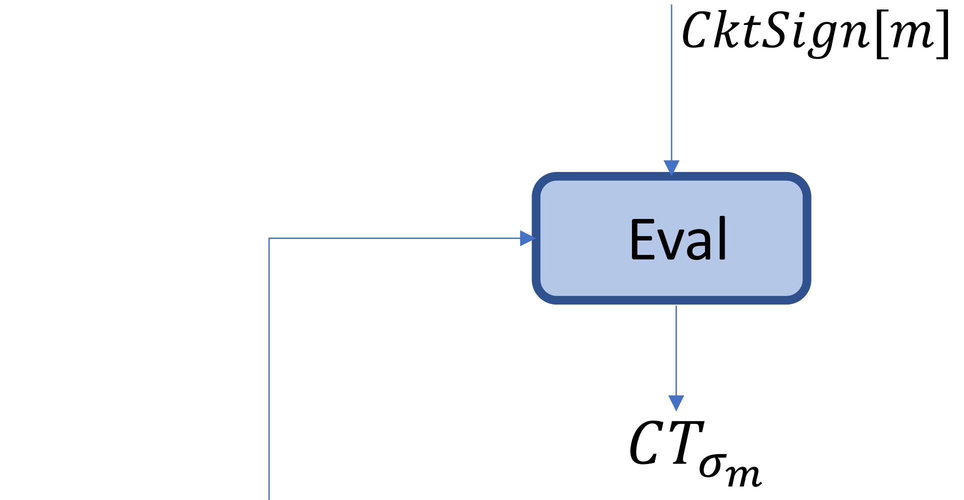

Public params (pp)

$fpk,\quad CT_{sigsk}$

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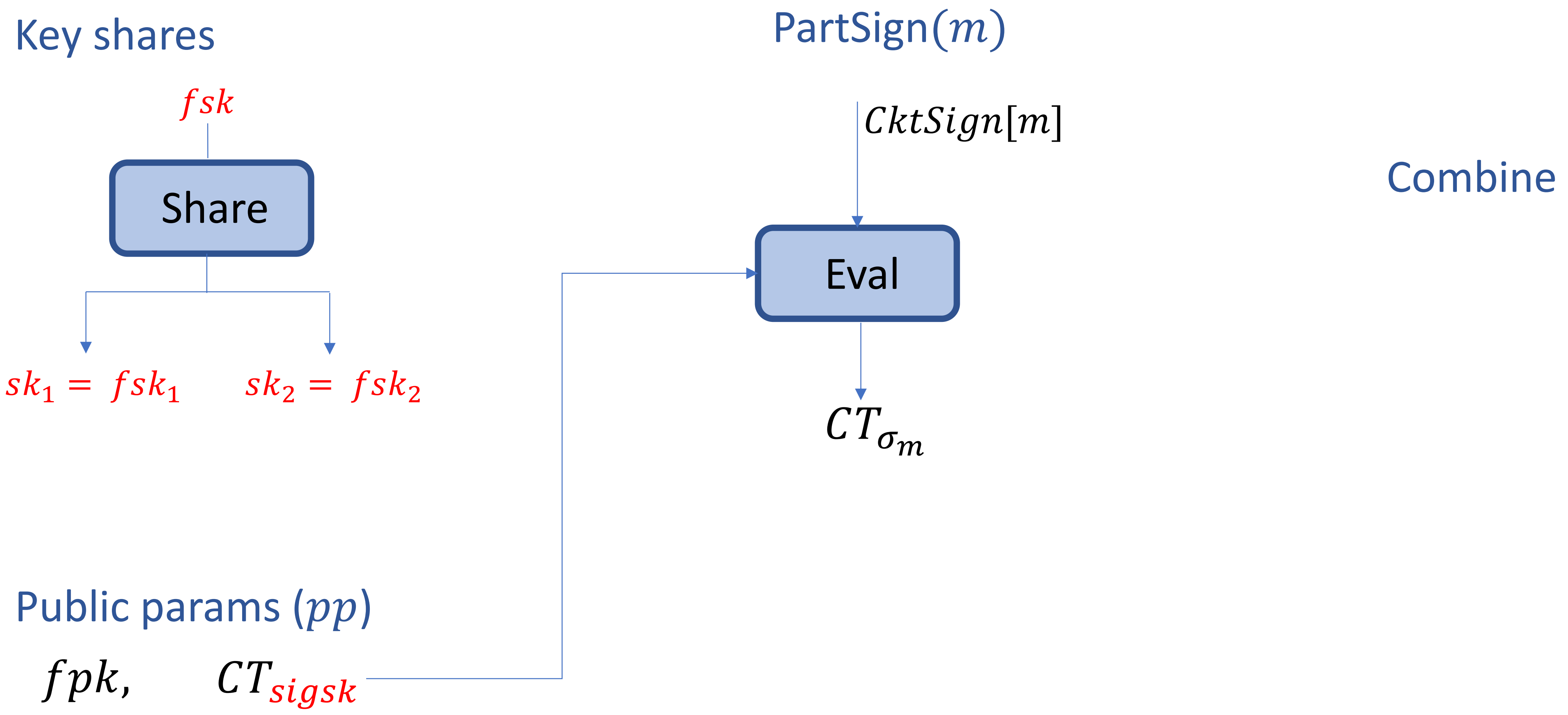

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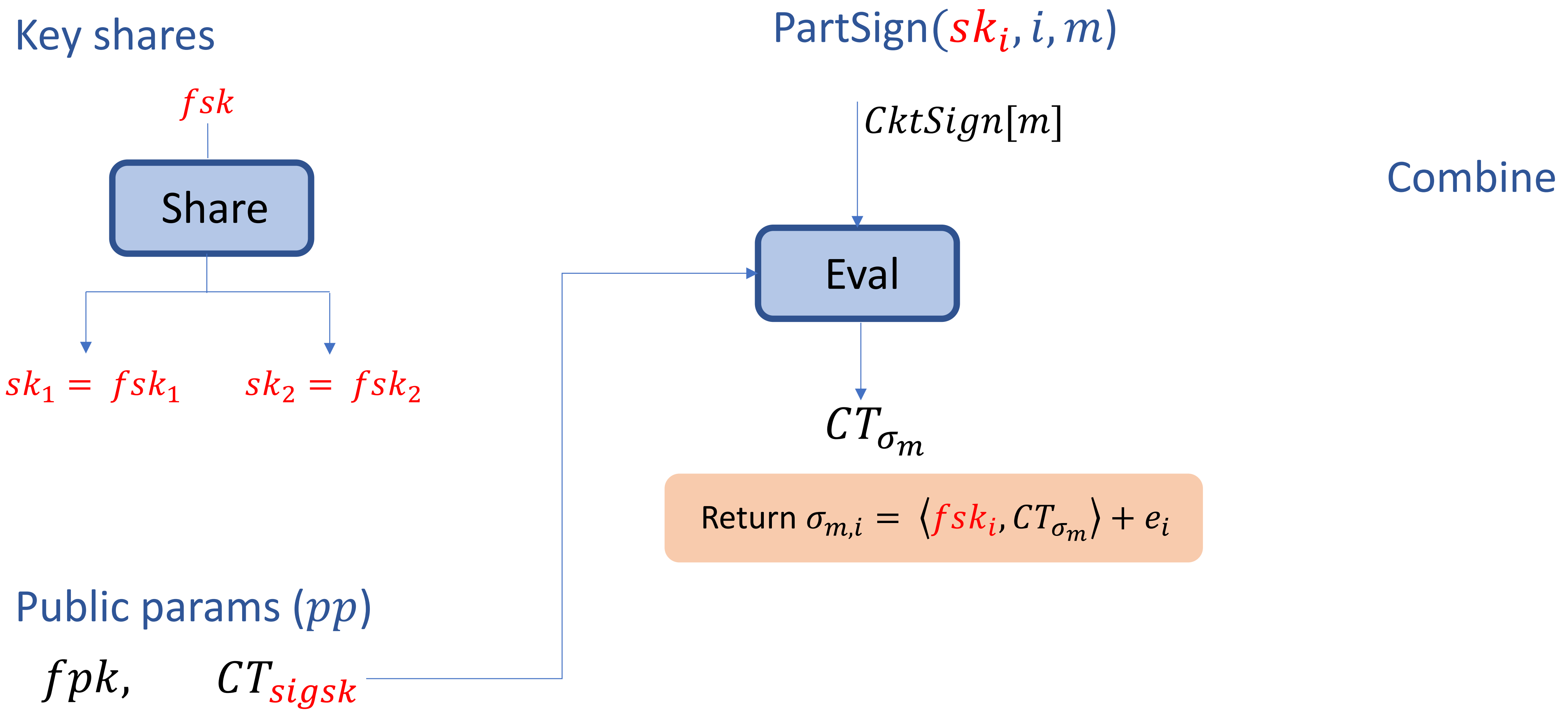

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for 2-out-of-2 scheme

Key shares

fsk

Share

$sk_1 = fsk_1$ $sk_2 = fsk_2$

Public params (pp)

$fpk,$ CT_{sigsk}

PartSign(sk_i, i, m)

$CktSign[m]$

Eval

CT_{σ_m}

Return $\sigma_{m,i} = \langle fsk_i, CT_{\sigma_m} \rangle + e_i$

Combine

$$\sigma_m = \text{round}(\sigma_{m,1} + \sigma_{m,2})$$

Security Sketch

(Selective Security, No PartSign on m^*)

Let the adversary gets partial signing key from P1

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H0:
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The honest game

H1: $mpk = CT_{sigsk};$ $\sigma_{m,2} = \lfloor q/2 \rceil \sigma_m - \langle fsk_1, CT_{\sigma_m} \rangle$

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Reduction to Sign Security in H2

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Sign-Challenger | Reduction | TSig-Adversary

Sign-Challenger → Reduction: svk

Reduction:

Sample (fpk, fsk); discard fsk

Sample fsk_1 randomly

Compute CT_0

Reduction → TSig-Adversary: svk, CT_0, fsk_1

TSig-Adversary → Reduction: PartSign Query: $(m, 2)$

Reduction → Sign-Challenger: Sign query: m

Sign-Challenger → Reduction: σ_m

Reduction:

Compute CT_{σ_m} using HE.Eval with CT_0

Compute $\sigma_{m,2} = \lfloor q/2 \rceil \sigma_m - \langle fsk_1, CT_{\sigma_m} \rangle$

Reduction → TSig-Adversary: $\sigma_{m,2}$

TSig-Adversary → Reduction: m^*, σ^*

Reduction → Sign-Challenger: m^*, σ^*

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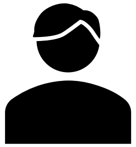
Selective Key Query, PartSign on m^* allowed

Let us consider 2-out-of-2 scheme. The adversary does not issue any key query



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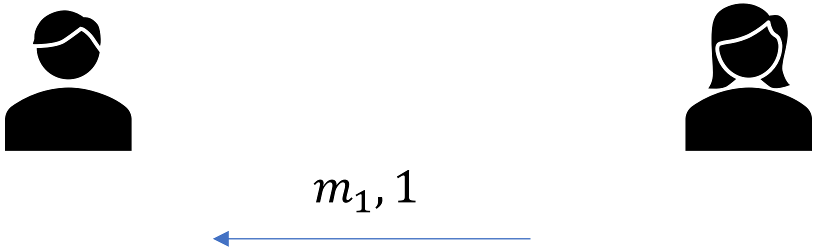

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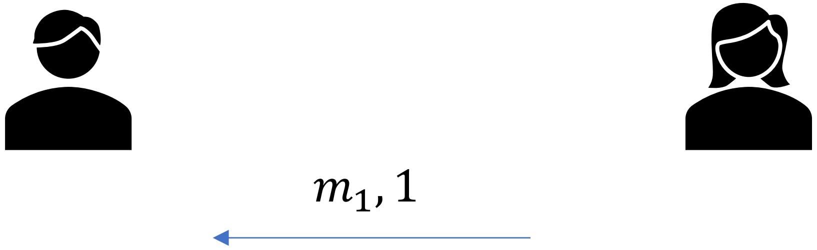

How to compute $\sigma_{m_1,1}$?

- Using fsk_1 as $\langle fsk_1, CT_{\sigma_{m_1}} \rangle$ or
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Challenges in Proving Stronger Security

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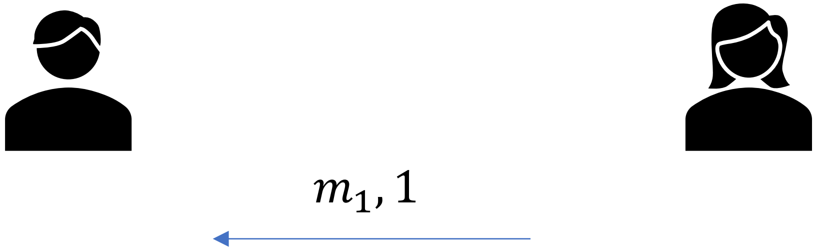

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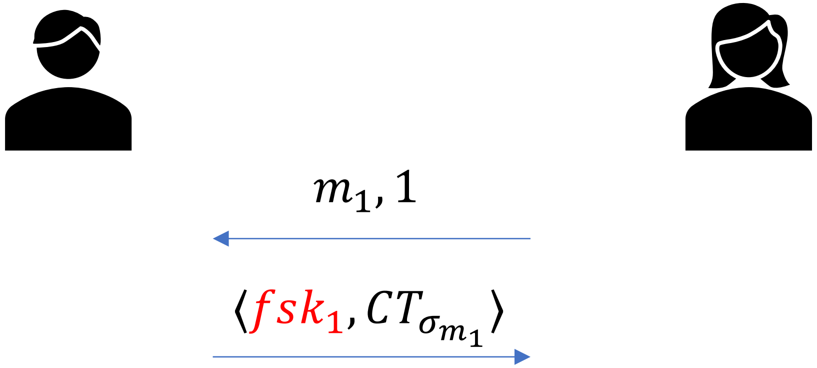

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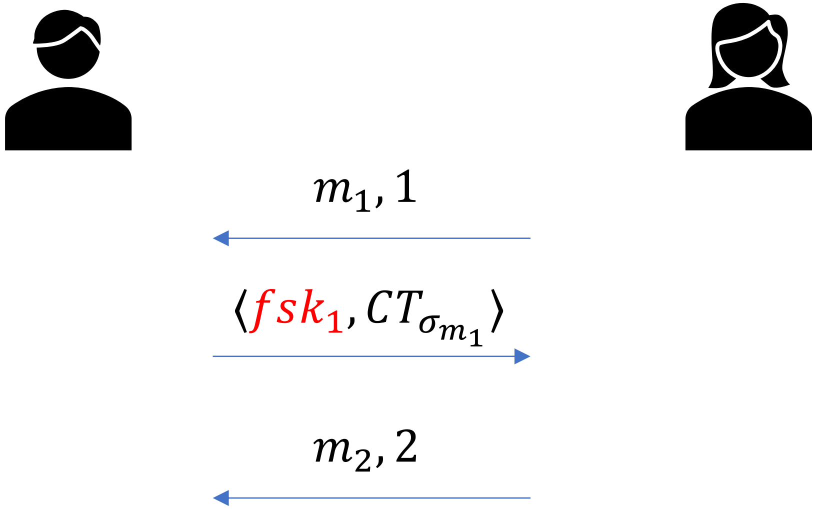

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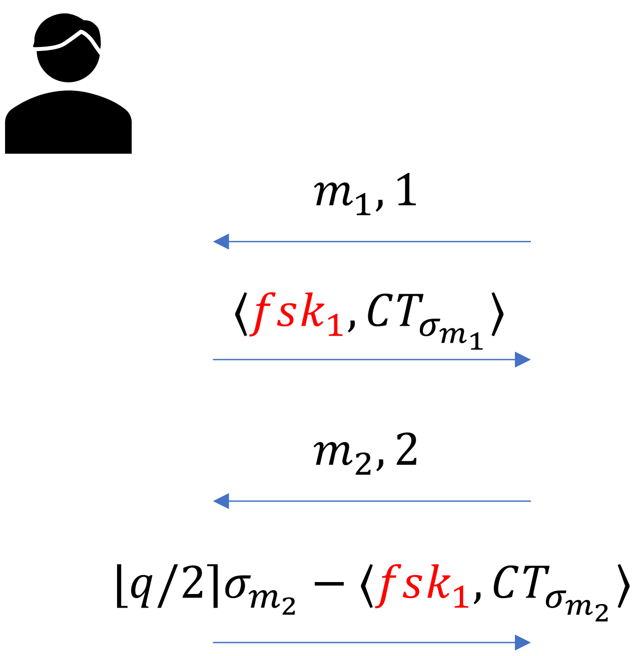

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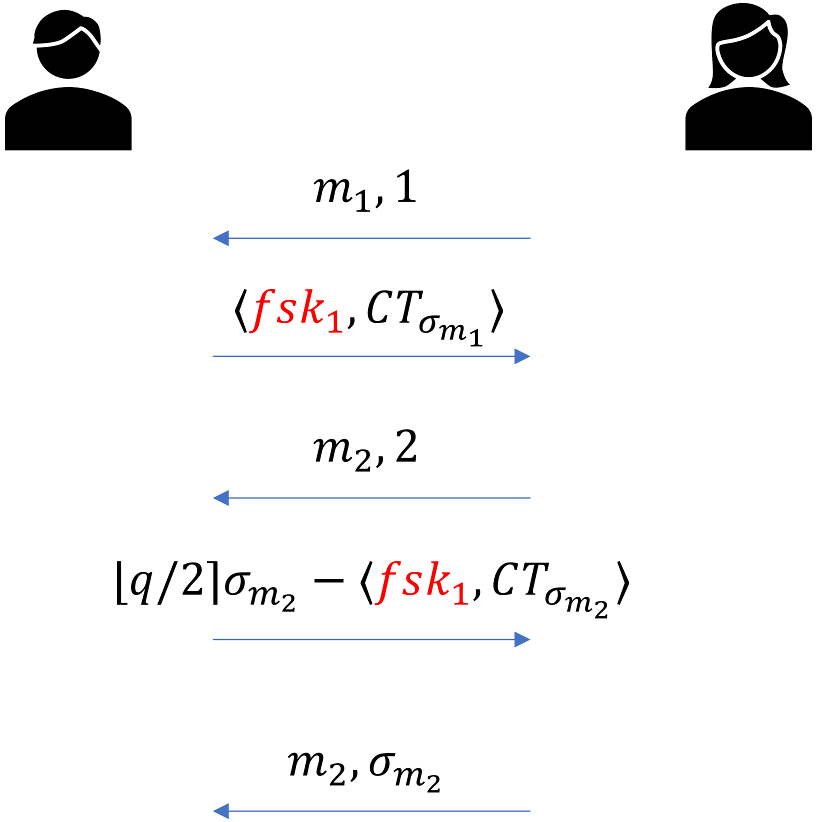

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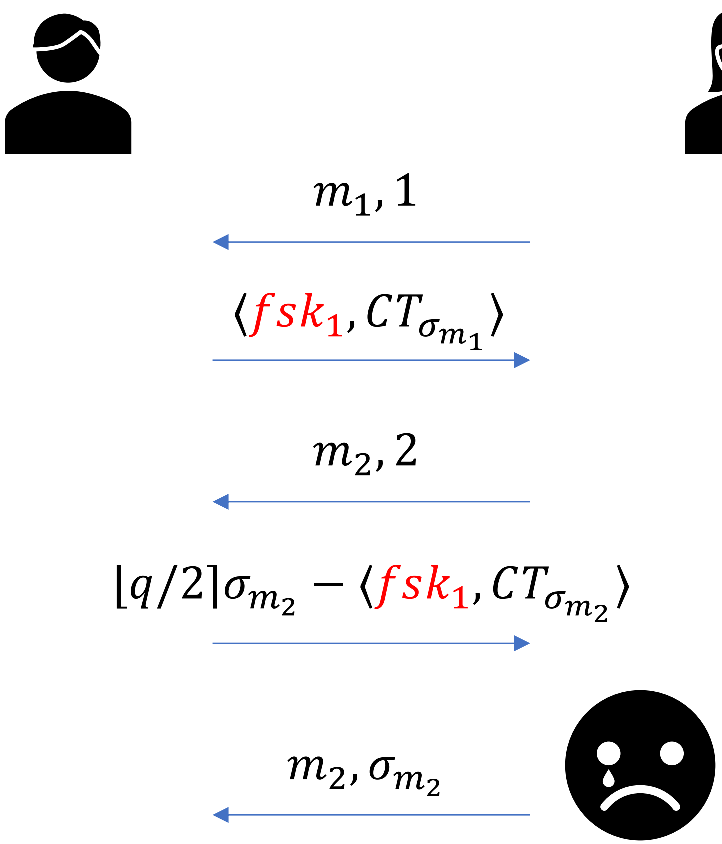

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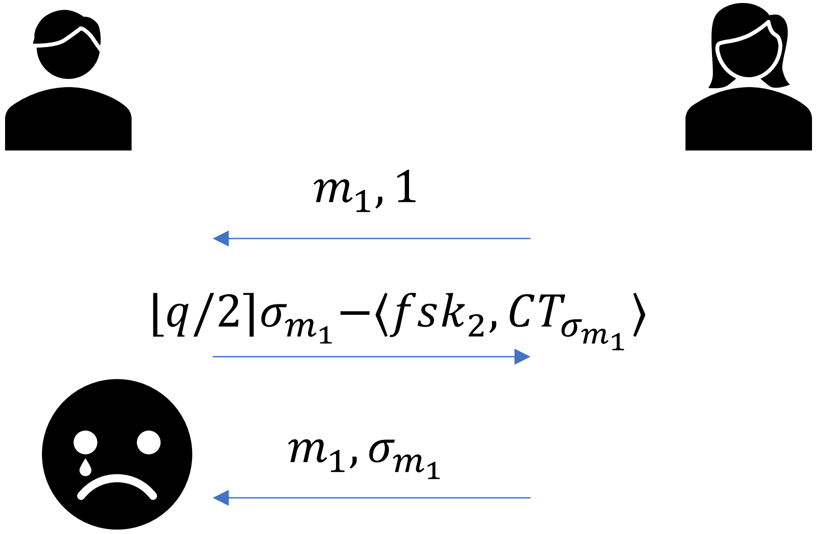

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Attempting Different Hybrid Policy

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For each message, answer the first PartSign(m, i) query with a randomly sampled $\sigma_{m,i}$

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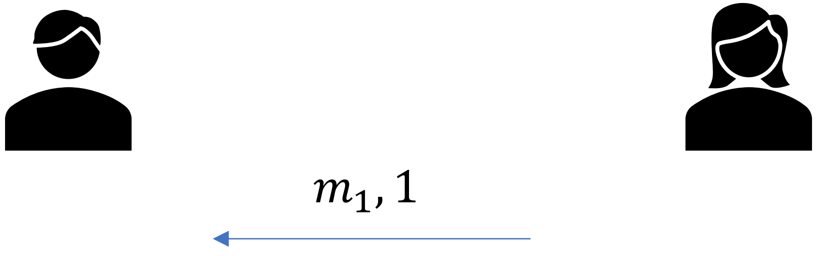

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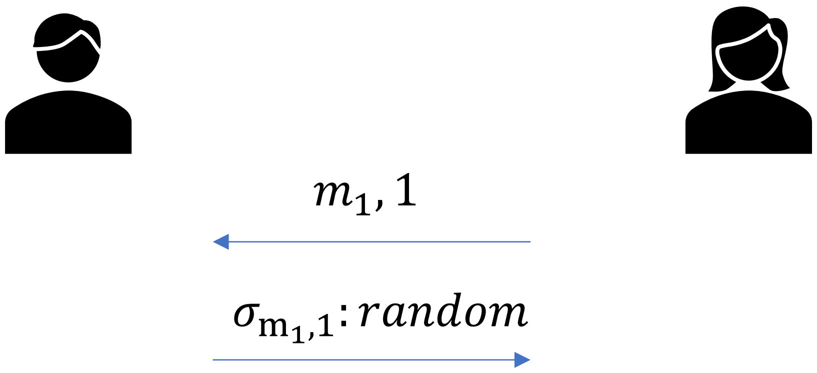

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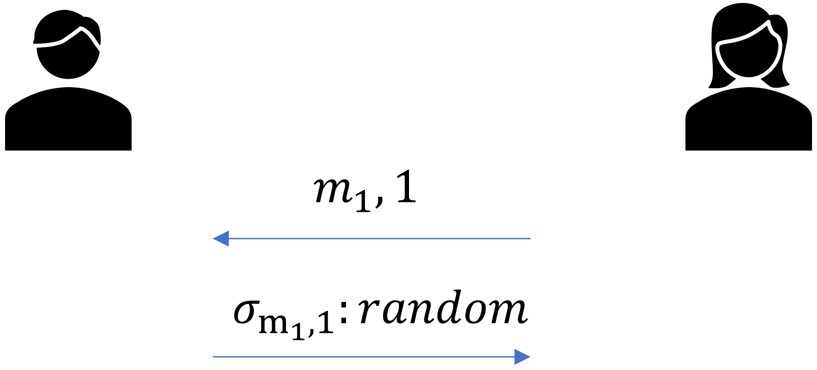

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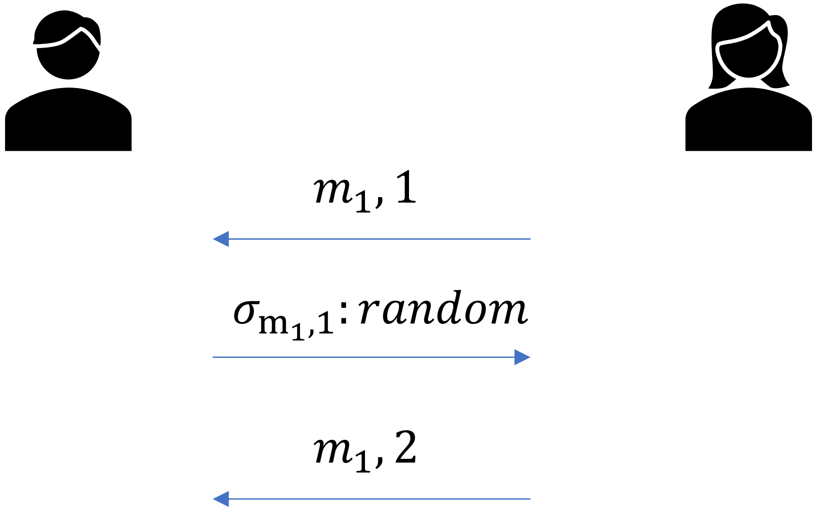

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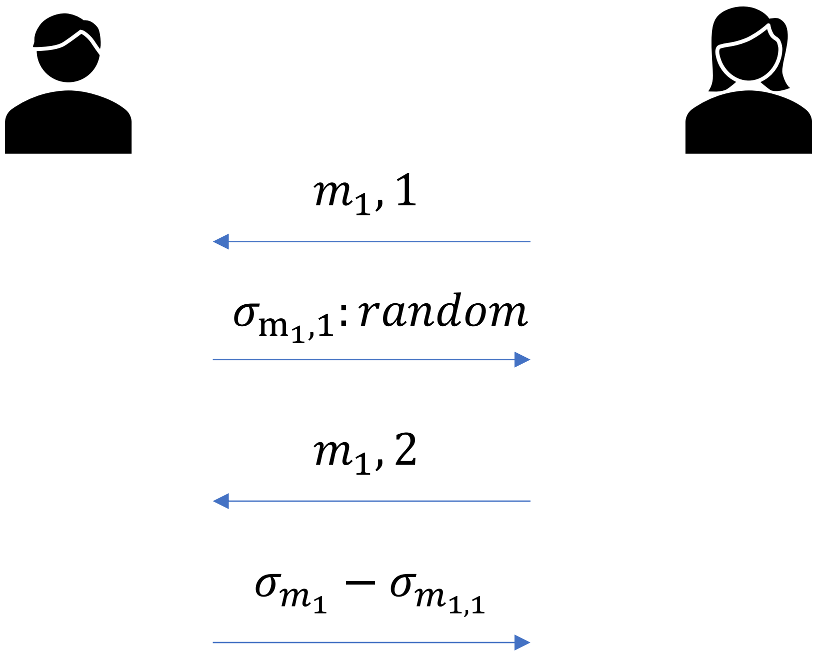

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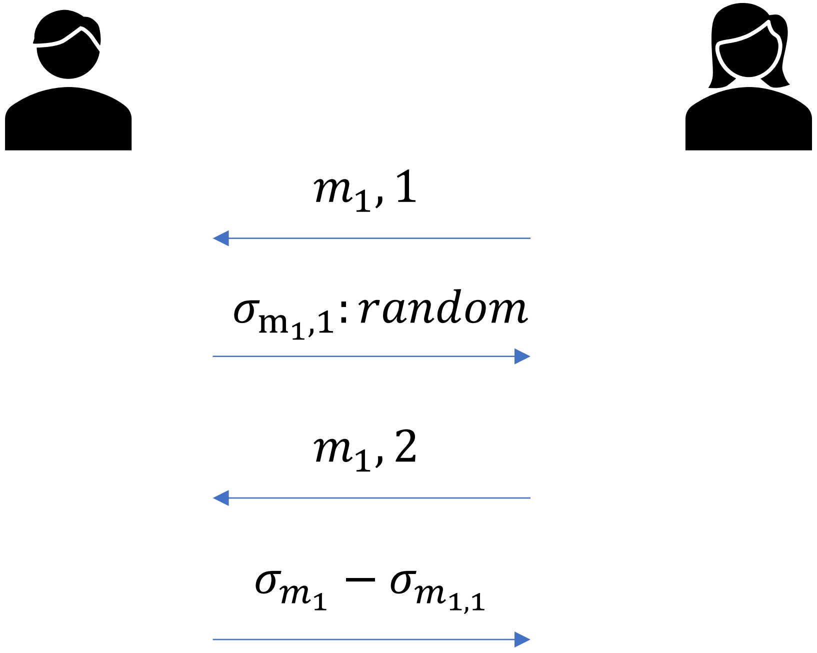

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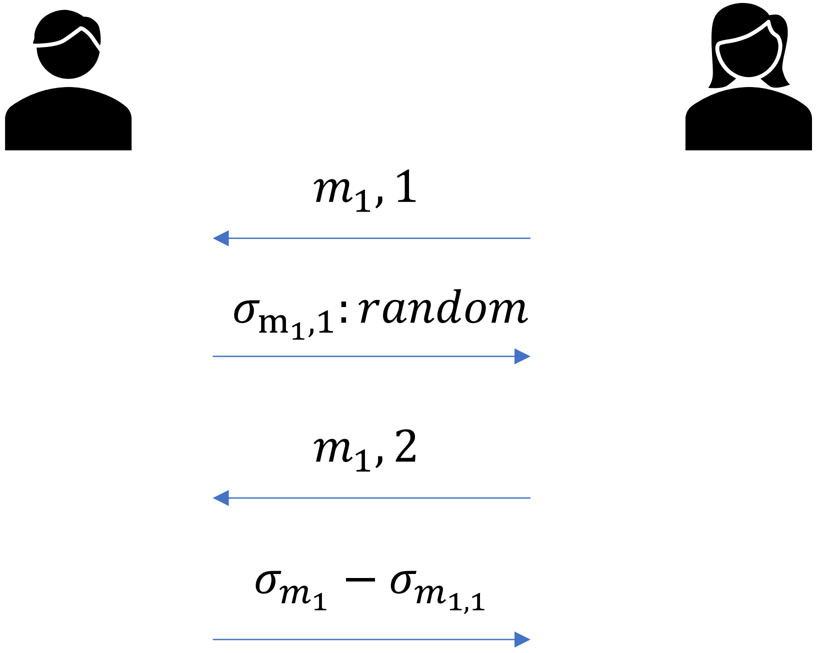

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Problem: Cannot argue the following indistinguishability

$$\langle fsk_i, CT_{\sigma_m} \rangle \approx random$$

Our Solution

PartSign(pp, fsk_1, m)

$$\sigma_{m,1} = Decode_0(fsk_1, CT_{\sigma_m}) + e_1' + \mathrm{r}_{\mathrm{m,1}}$$

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$\sigma_{m,2} = Decode_0(fsk$

Key Homomorphic PRF:
$F(K_1, x) + F(K_2, x) = F(K_1 + K_2, x)$

Obs: $F(0, x) = 0$

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Final Working Solution

PartSign(pp, fsk_1, m)

$\sigma_{m,1} \;=\; Decode_0\left(fsk_1, CT_{\sigma_m}\right) + e_1' + F(K_1, m)$

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$K_1 + K_2 = 0$

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$(x, F(K_1, x), F(K_2, x)) \approx (x, r_1, r_2)$, where both K_1 and K_2 as well as r_1 and r_2 are secret shares of 0

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H2: $\sigma_{m,1} = r'_{m,1}\ (random) + e_1'$ $\quad \sigma_{m,2} = r'_{m,2} = \left\lfloor \frac{q}{2} \right\rfloor \sigma_m - r'_{m,1} + e_2'$

H3: $mpk = CT_0$ instead of CT_{sigsk}

Reduction to Sign security in H3

Remarks and Conclusions

- Lattice based KHPRF do not satisfy exact homomorphism

$$F(K_1, x) + F(K_2, x) = F(K_1 + K_2, x) + \delta$$

We use flooding to hide δ

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⇓

Stronger UT

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Stronger UT

Stronger threshold signature/ CCA-PKE, etc

Thank You