Randomness in Private Sequential Stateless Protocols

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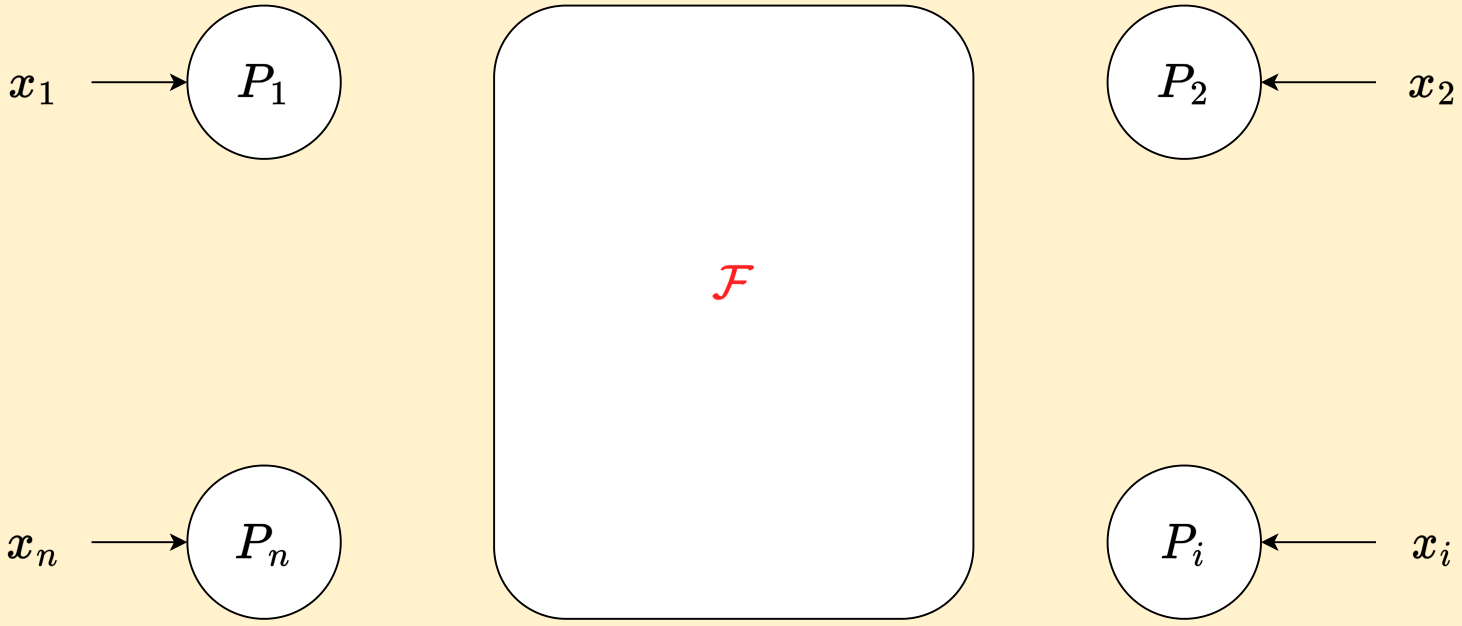
Secure multi-party computation



No party should learn anything apart from their own input and output of the function.

Randomness in MPC

Randomness: Central resource in cryptography, especially in unconditional cryptography

- Randomness efficient computation (and lower bounds).
  - Kushilevitz, Mansour (PODC 1996),
  - Kushilevitz, Ostrovsky, Rosén (STOC 1996)
  - Canetti, Kushilevitz, Ostrovsky, Rosén (PODC 1997)
  - Gal, Rosén (STOC 2003)
  - Jakoby, Liskiewicz, Reischuk (STACS 2003)
  - Blundo, Galdi, Persiano (2007)
  - Kushilevitz, Ostrovsky, Prouff, Rosén, Thillard, Vergnaud (TCC - 2019)
  - Goyal, Ishai, Song (CRYPTO 2022)
  - Couteau, Rosén (Asiacrypt 2022)

Randomness in MPC

Kushilevitz, Ostrovsky, Rosén. (STOC 1996)

Randomness complexity of 1-private computation

Circuit complexity

1-privacy: Semi-honest corruption of any one party

Our Results

Randomness complexity of private sequential stateless (PSS) computation

Branching program complexity

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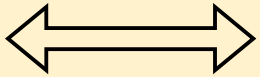
Branching program complexity

f has a speak-once PSS protocol with constant randomness



f has an read-once constant-width branching program

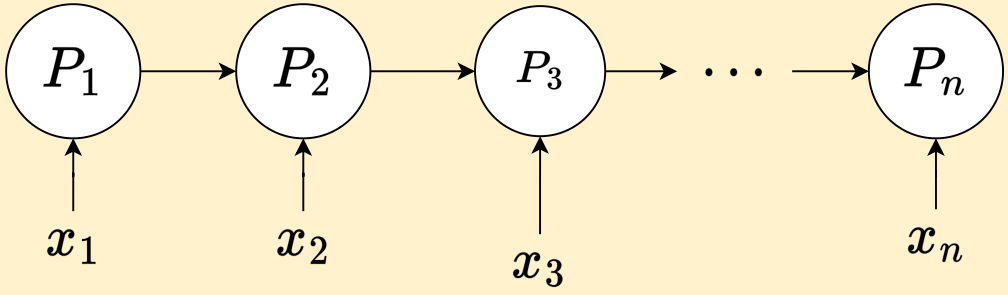
$$f : \{0,1\}^n \to \{0,1\}$$

Extends to speak-$O(k)$ PSS and read-$O(k)$ BP

where k is independent of the input size n

Sequential Model

Speak-once



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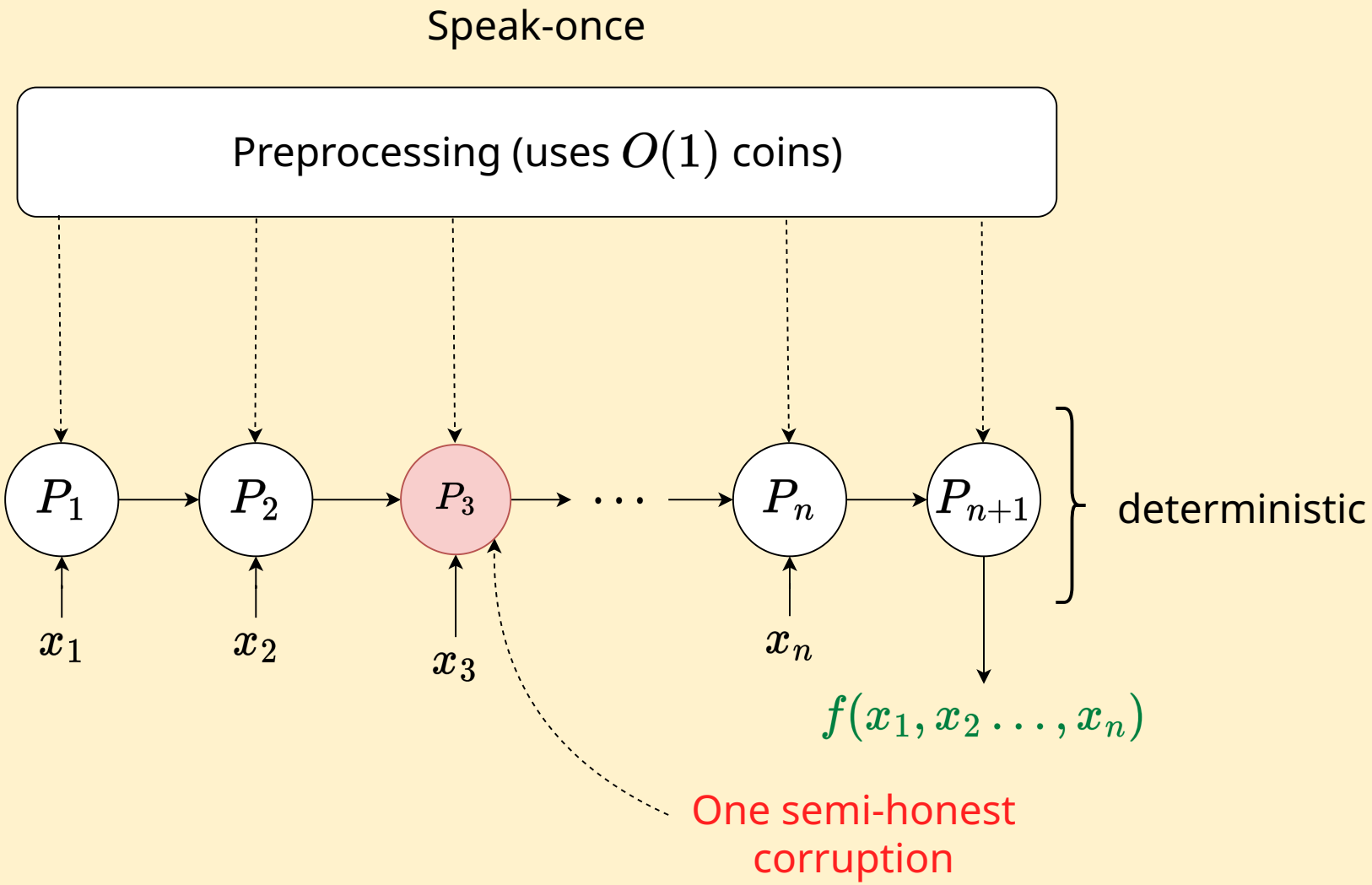

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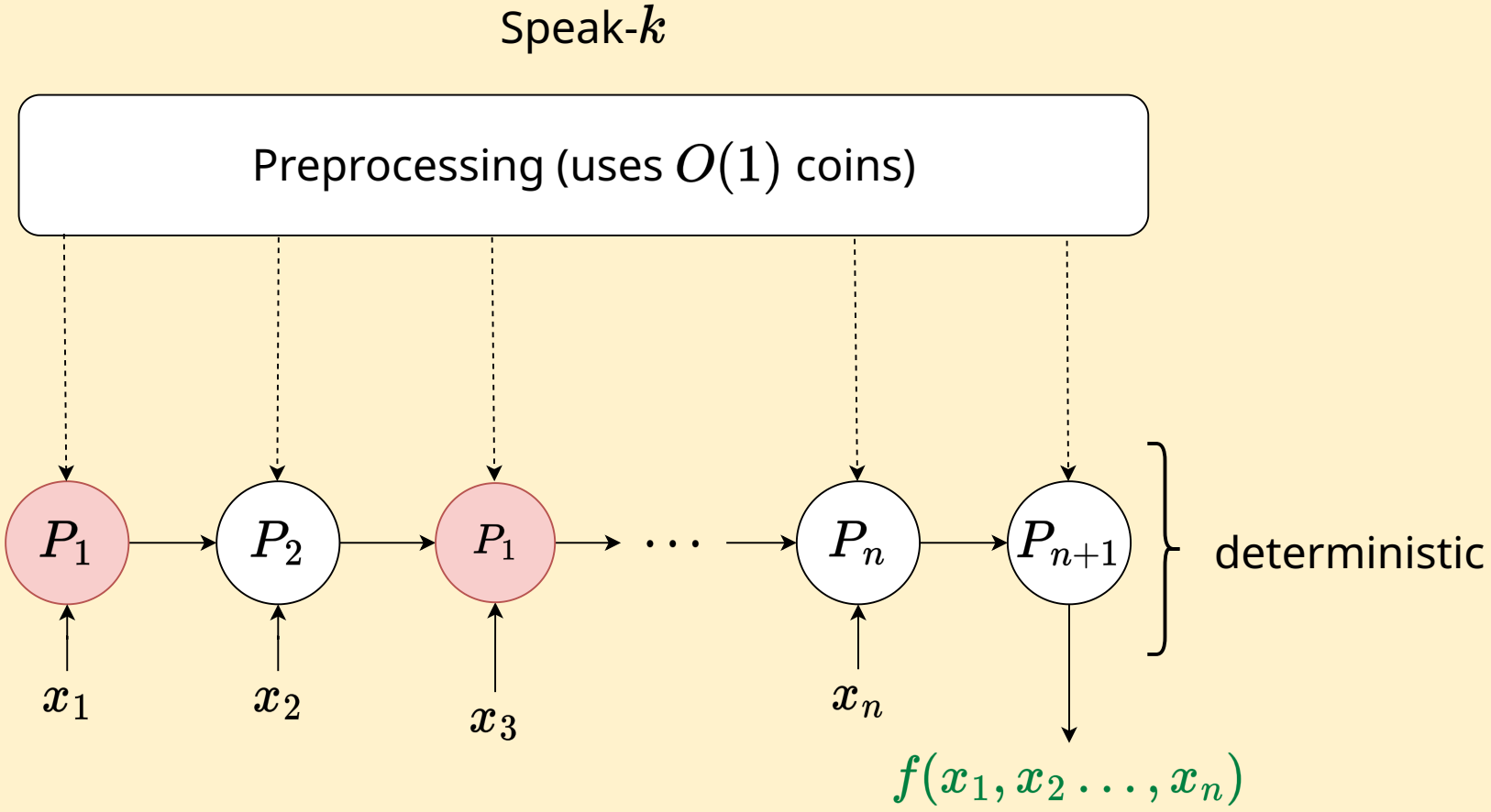
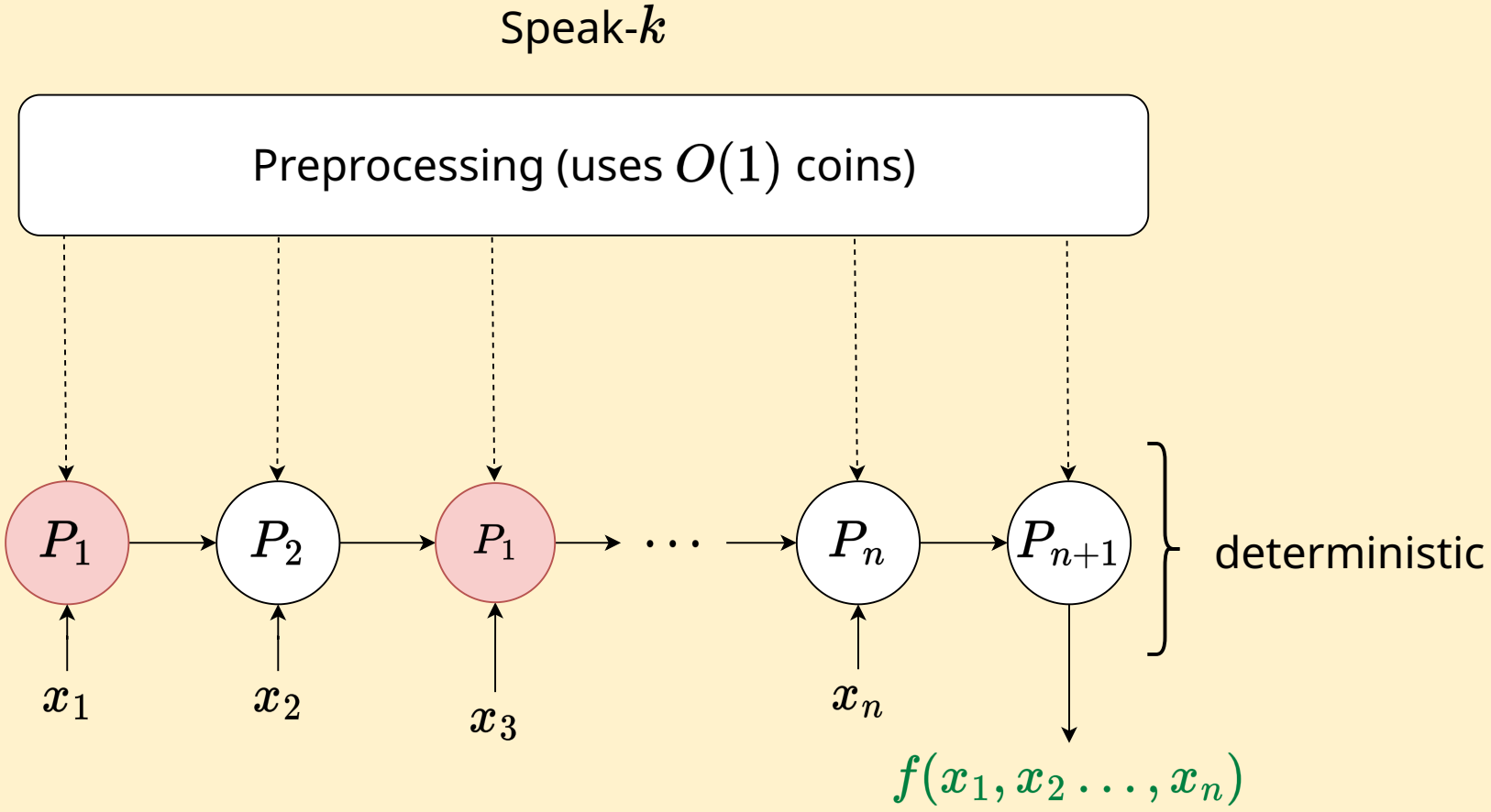


Sequential Model



Stateless: Parties do not maintain states between rounds when they speak

Motivation

Other simple models (having star topology)

- Private simultaneous messaging
- Non-interactive secure computation
- Conditional disclosure of secrets

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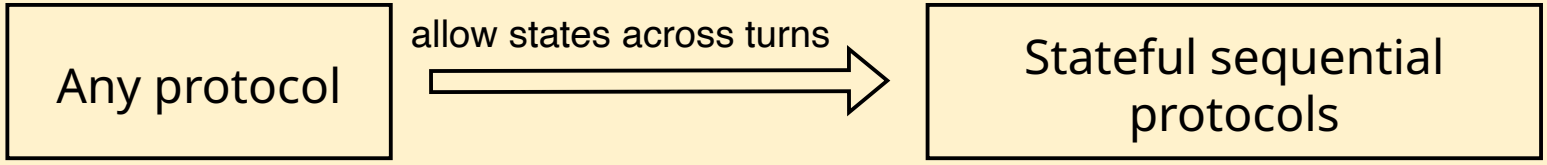
Why stateless?



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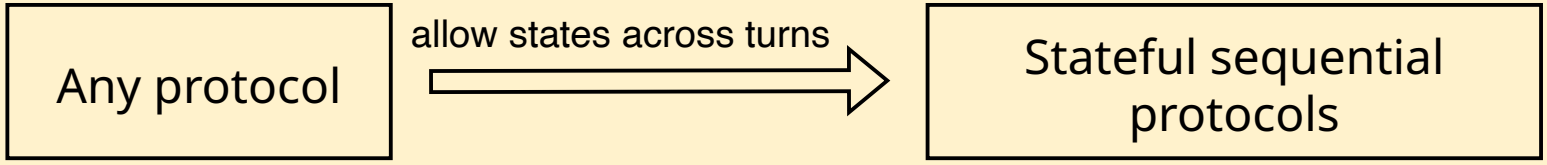
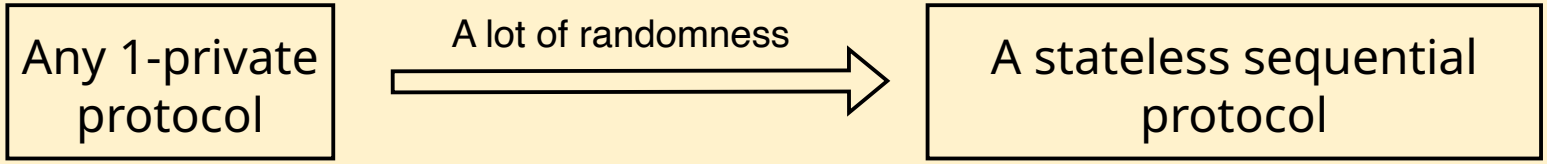
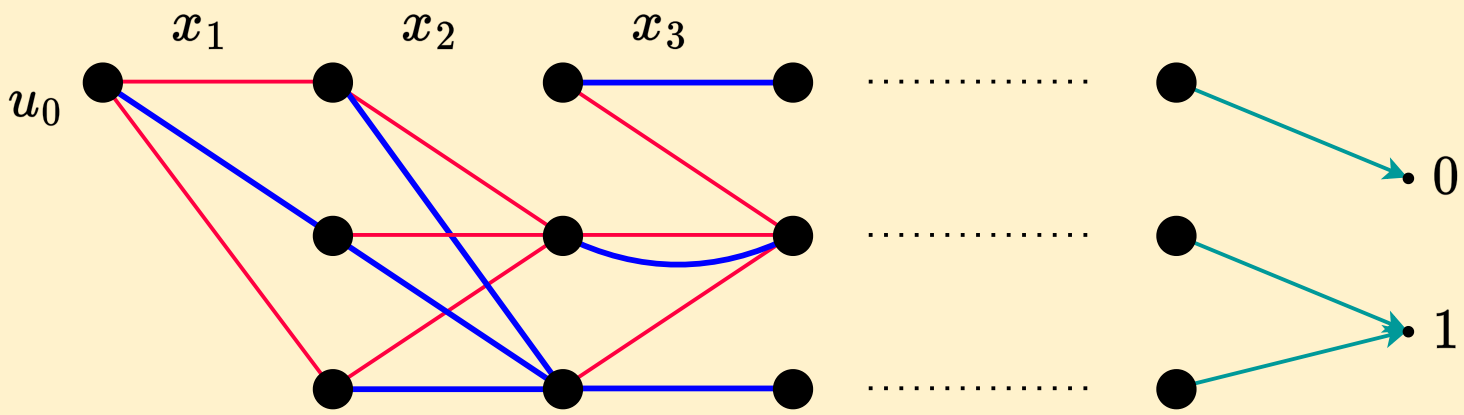


Why is restricting randomness interesting



Read-once branching programs



$\mathtt{Tr}_0^{(1)}$ $\mathtt{Tr}_0^{(2)}$ $\mathtt{Tr}_0^{(3)}$

$\mathtt{Tr}_1^{(1)}$ $\mathtt{Tr}_1^{(2)}$ $\mathtt{Tr}_1^{(3)}$

φ

Width $w = 3$ here

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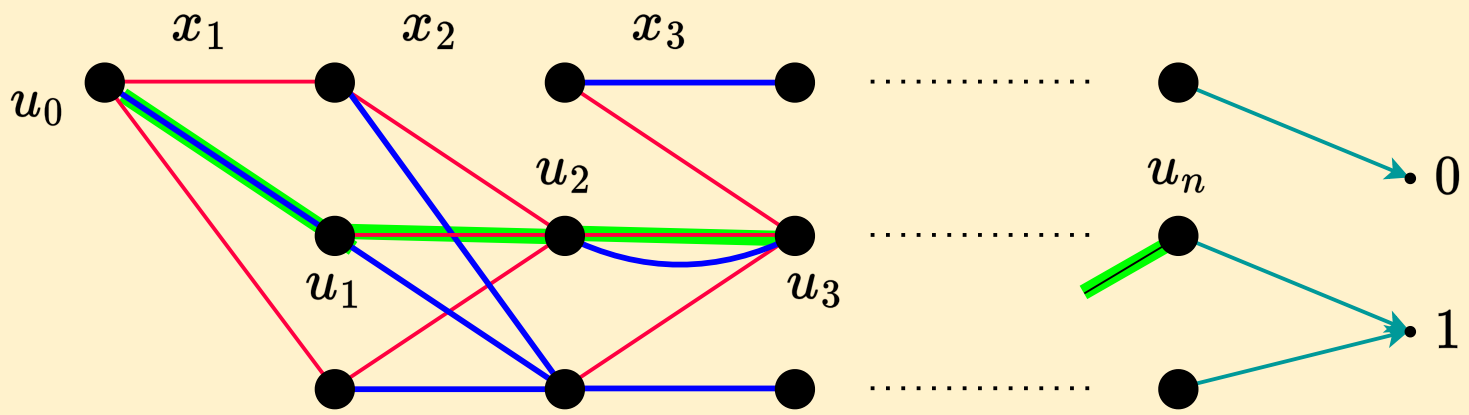

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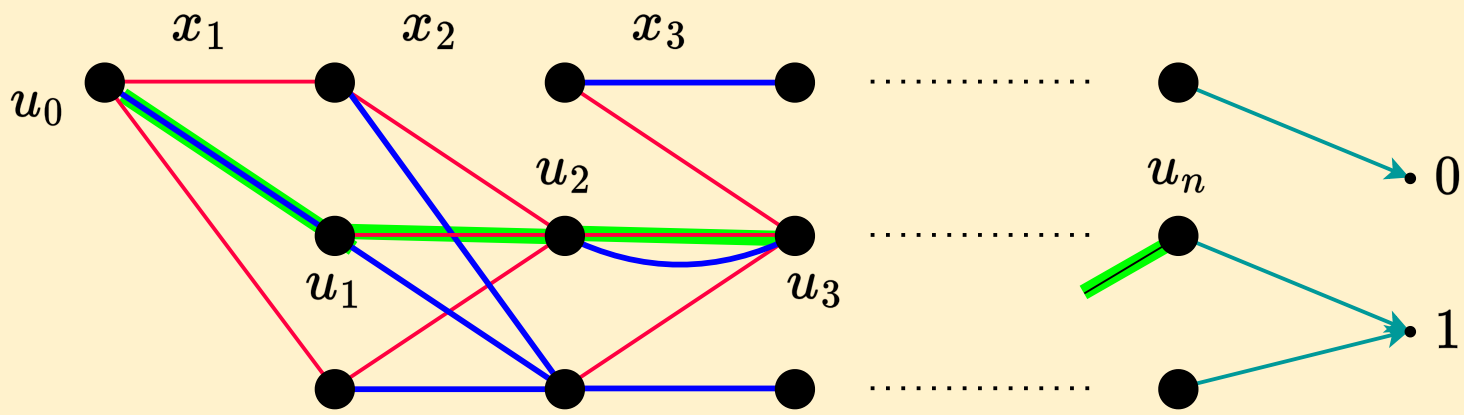

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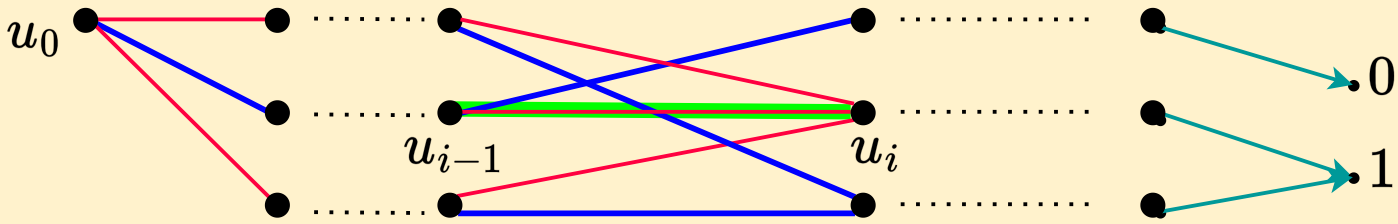
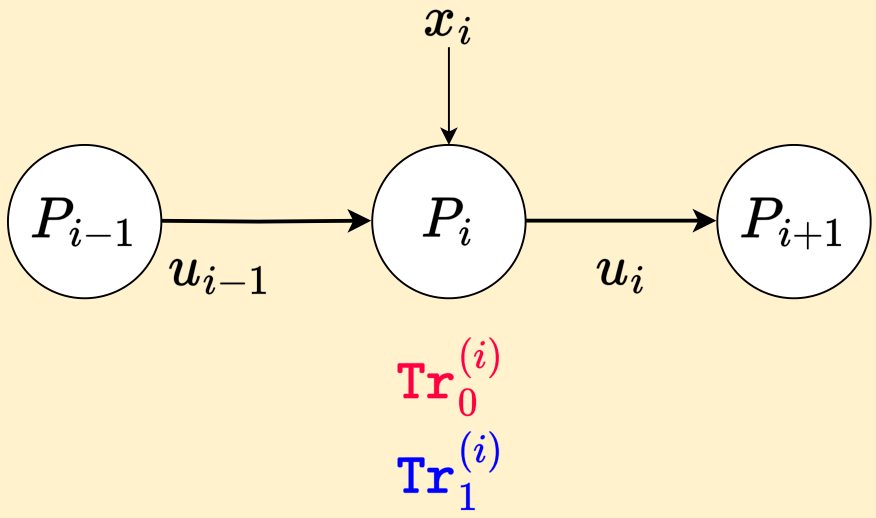
$$f(x_1, \ldots, x_n) = \varphi(u_n)$$

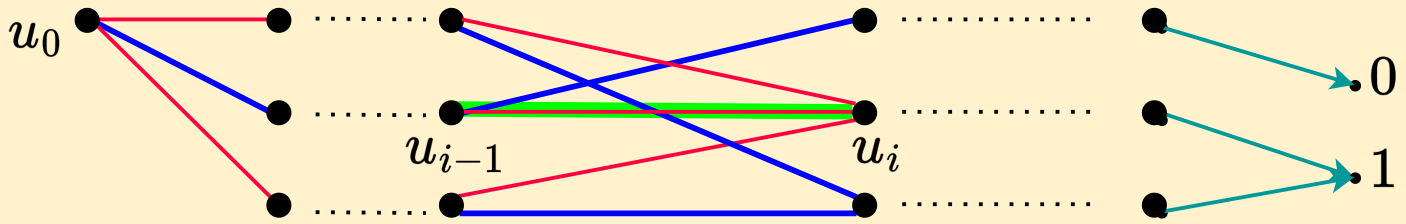
Protocol for evaluating a branching program insecurely





P_i receives u_{i-1}, which is insecure

Secure evaluation - Attempt 1



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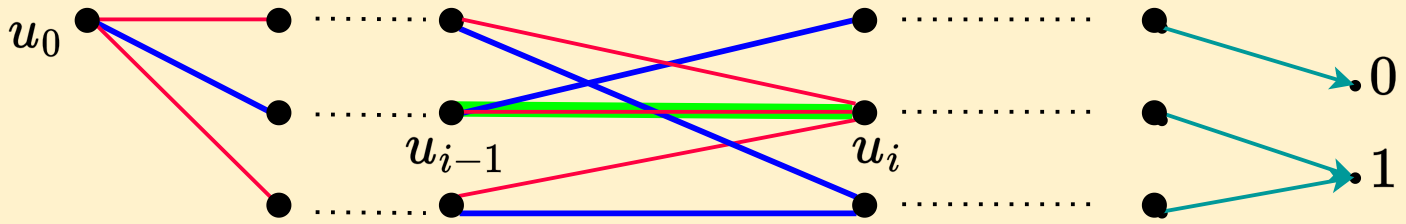
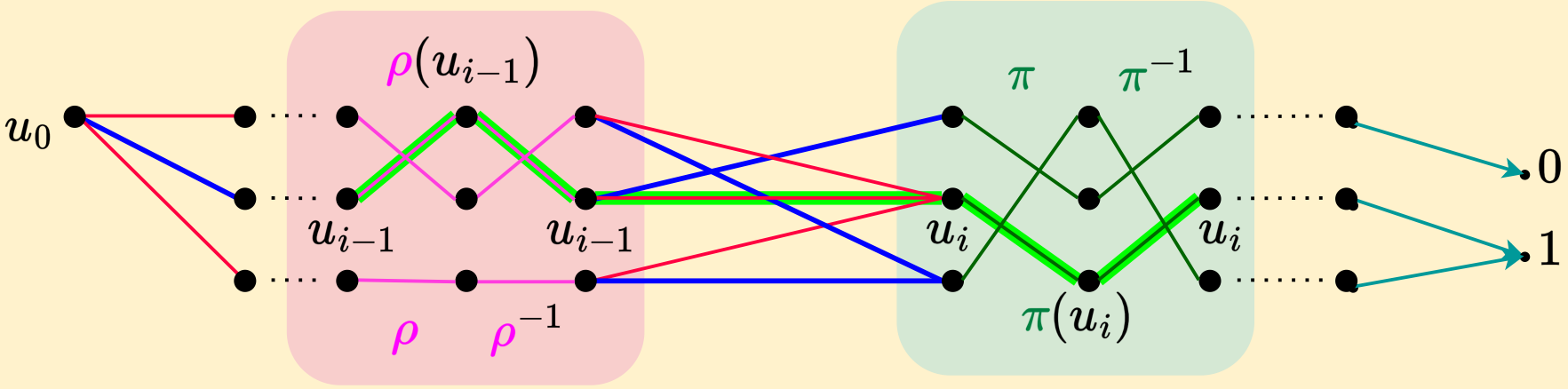
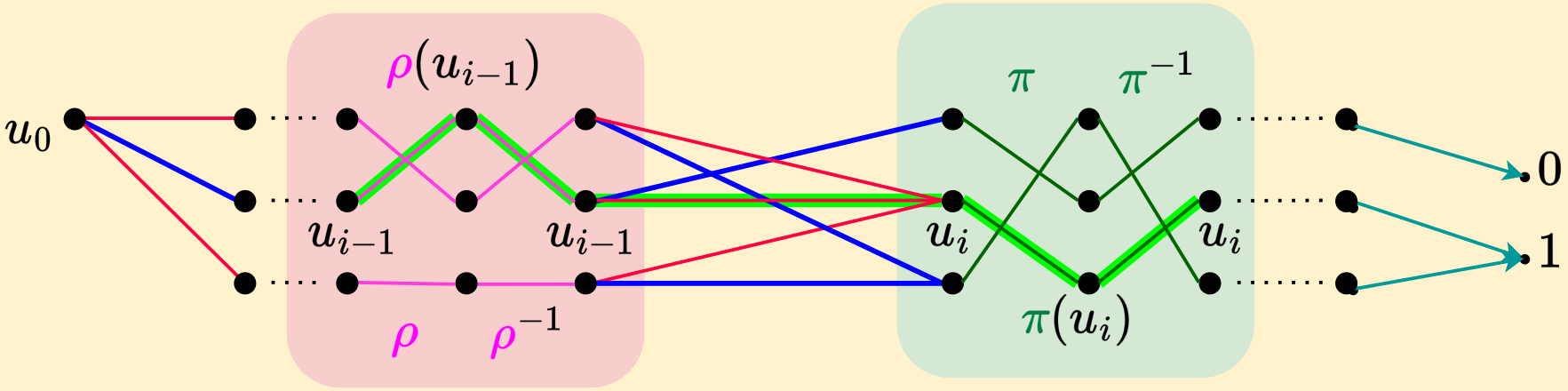




π, ρ are uniformly chosen permutations

Secure evaluation - Attempt 1



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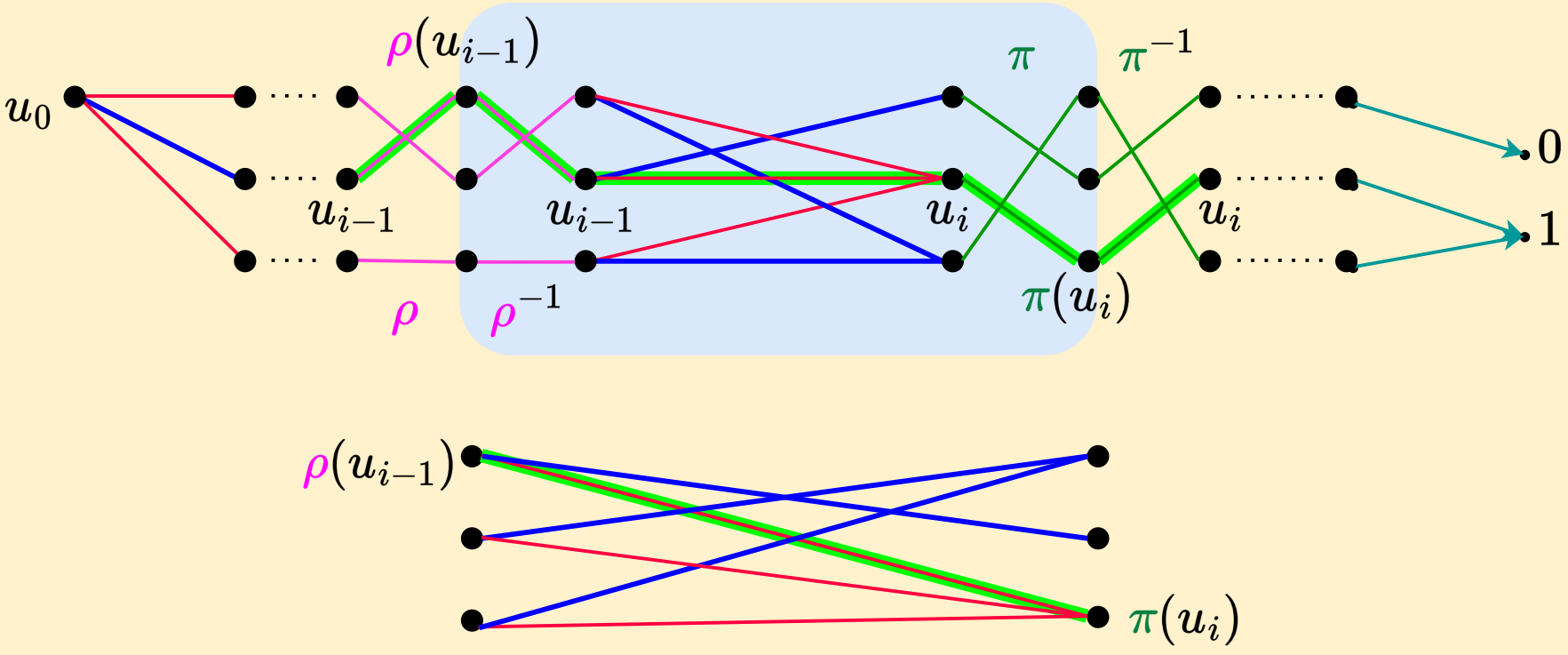

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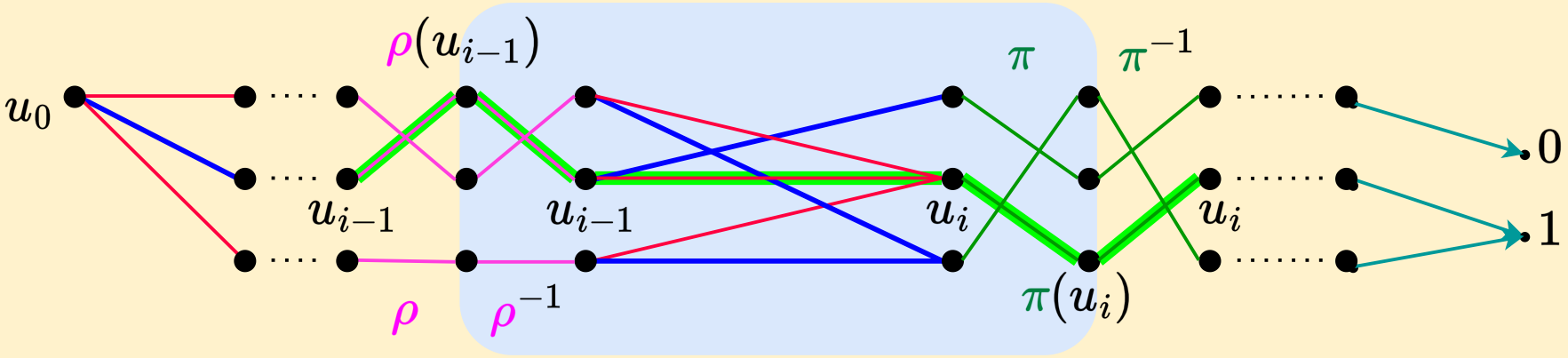

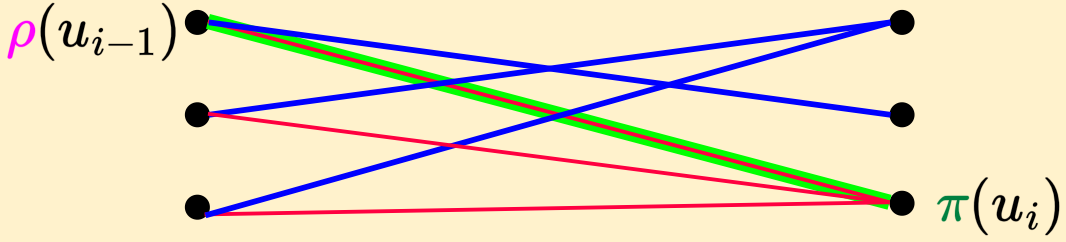

$\pi\ \mathtt{Tr}_0^{(i)}\ \rho^{-1}$

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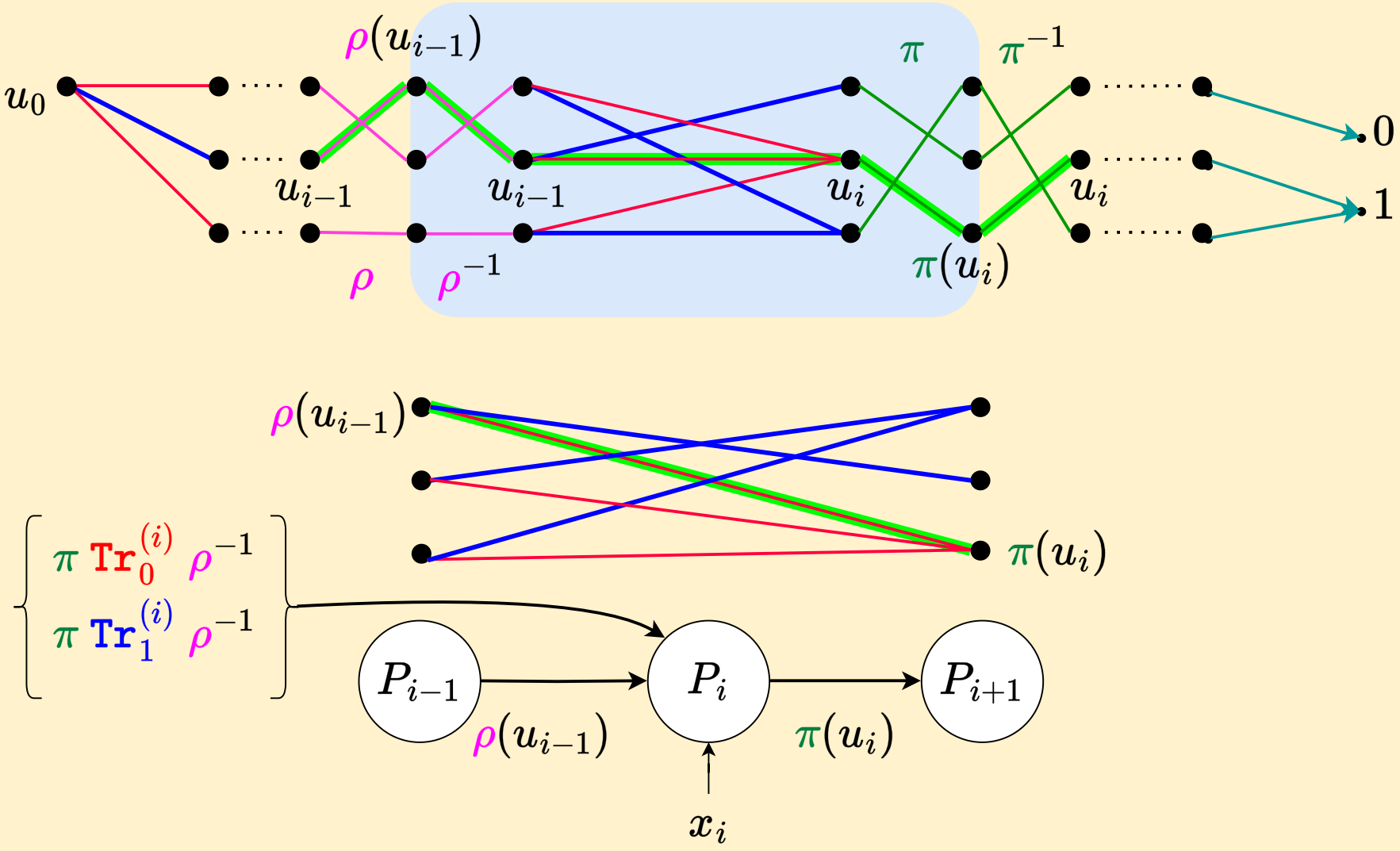

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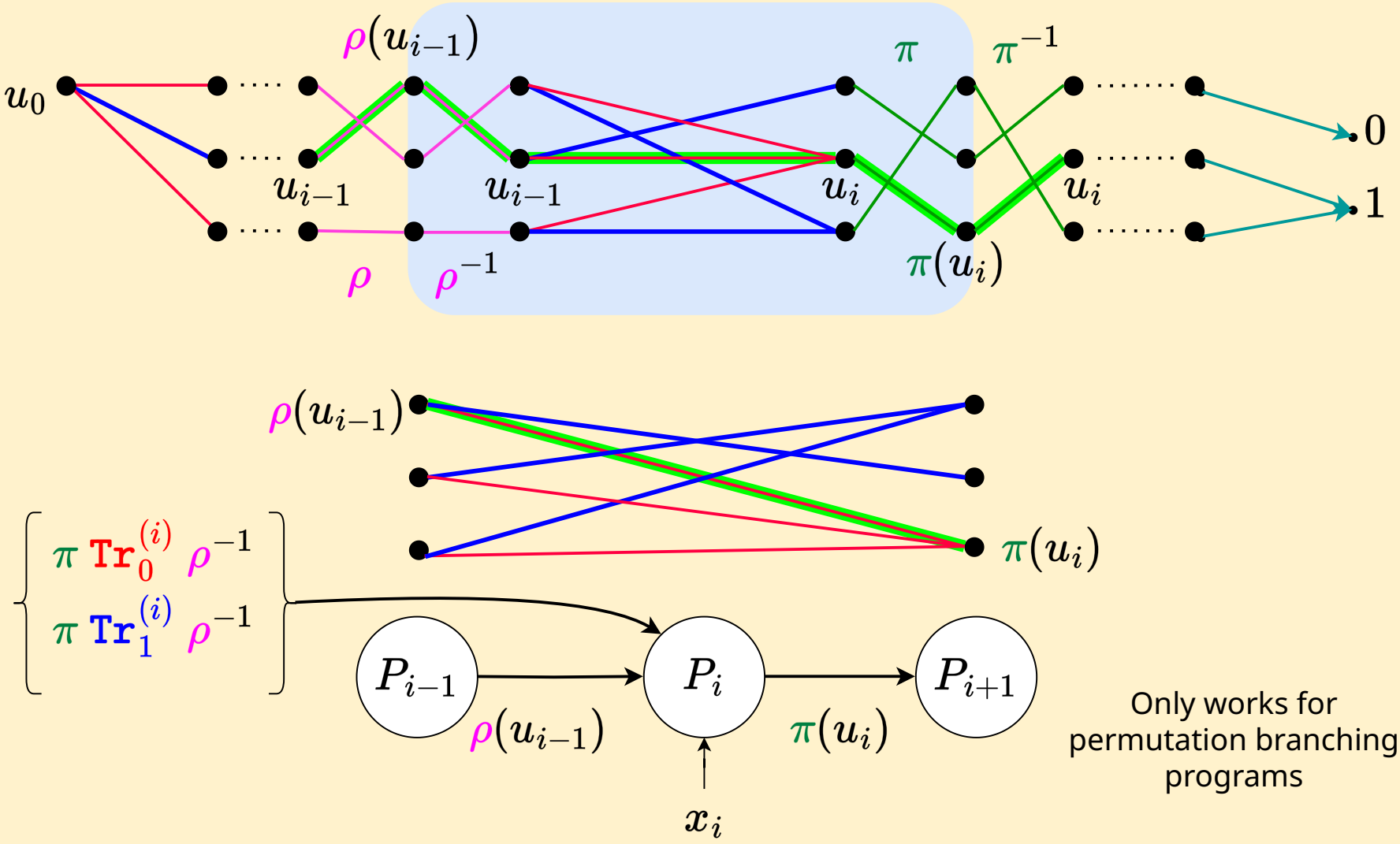
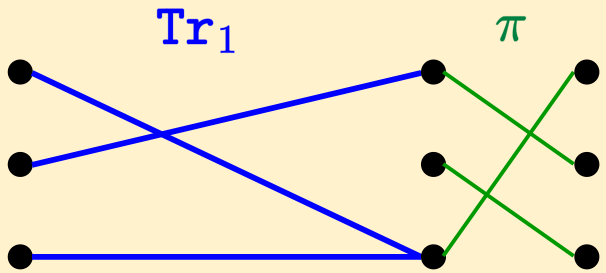
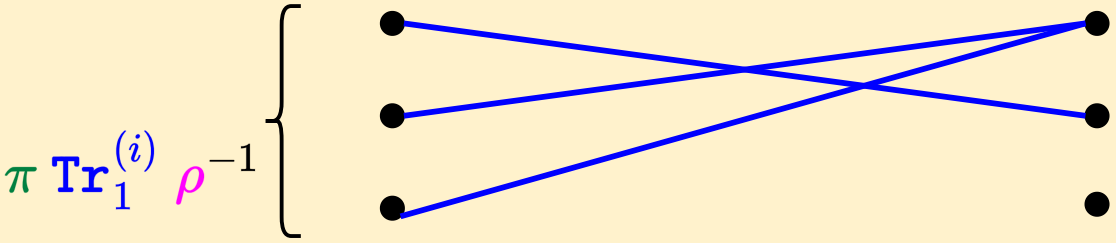


Only works for permutation branching programs

Secure evaluation - Attempt 1 fails most of the times





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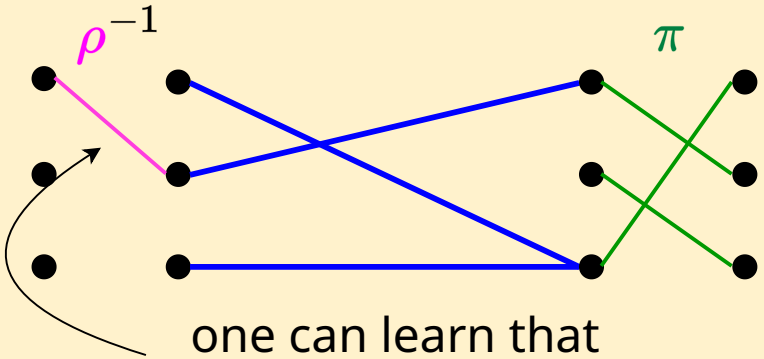

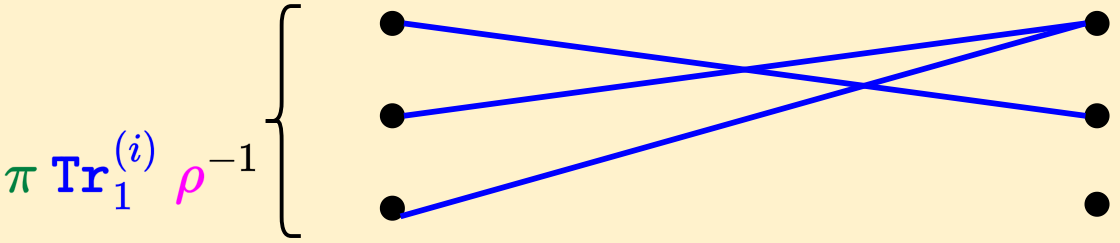

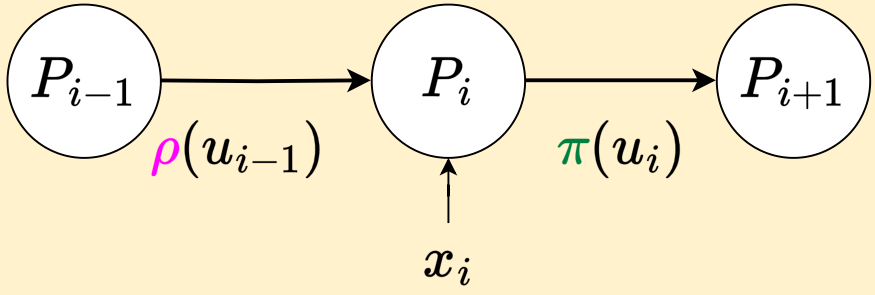

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- Workaround - Use two different sets of masks - π_0, π_1 instead of π
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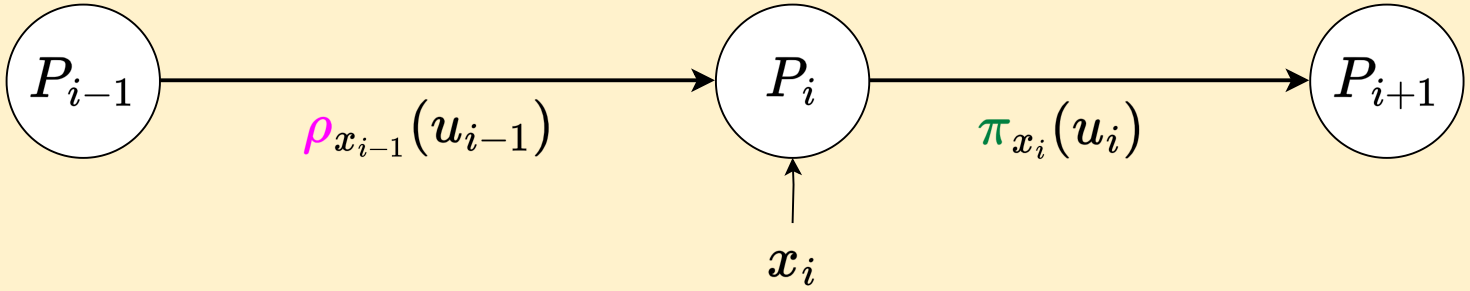

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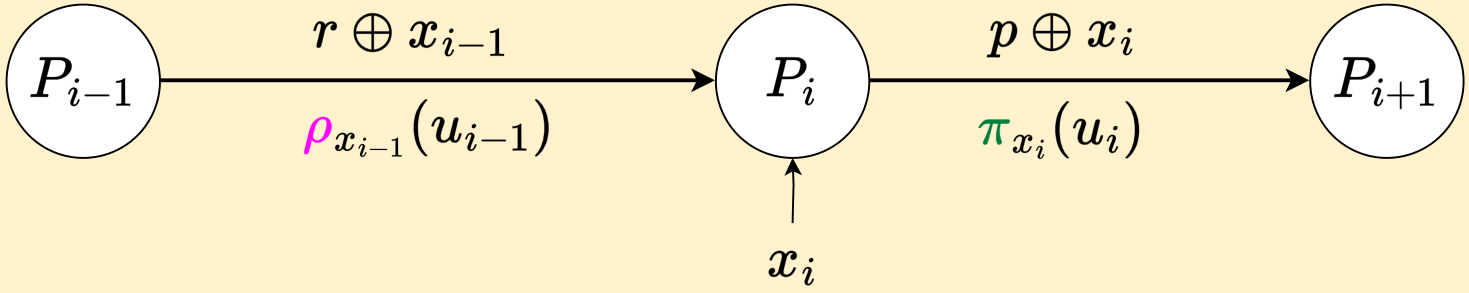
P_i needs to map $\rho_{x_{i-1}}(u_{i-1})$ to $\pi_{x_i}(u_i)$

As part of the preprocessing, P_i will be given 4 maps indexed by $(r \oplus x_{i-1}, x_i)$



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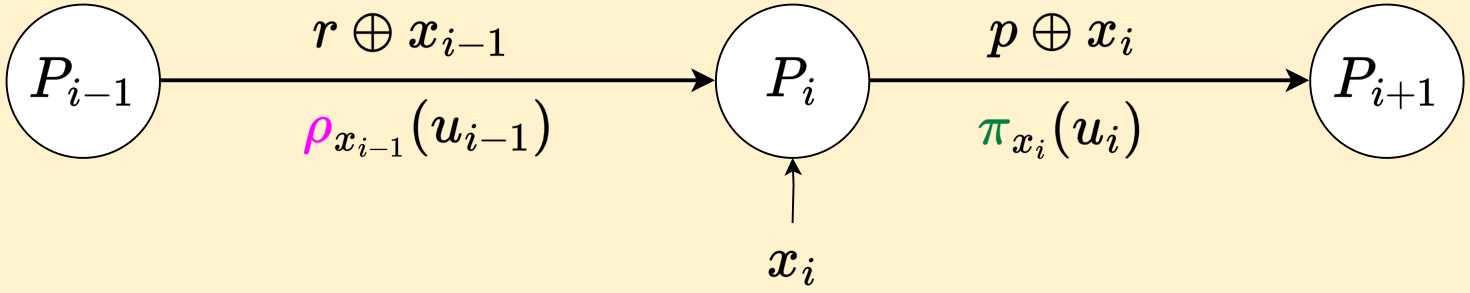

Secure for a class of BP called Strongly Regular Branching Programs!

Strongly regular branching program

- Our PSS protocol is secure for all SRBPs
- Examples: AND, XOR, Inner product

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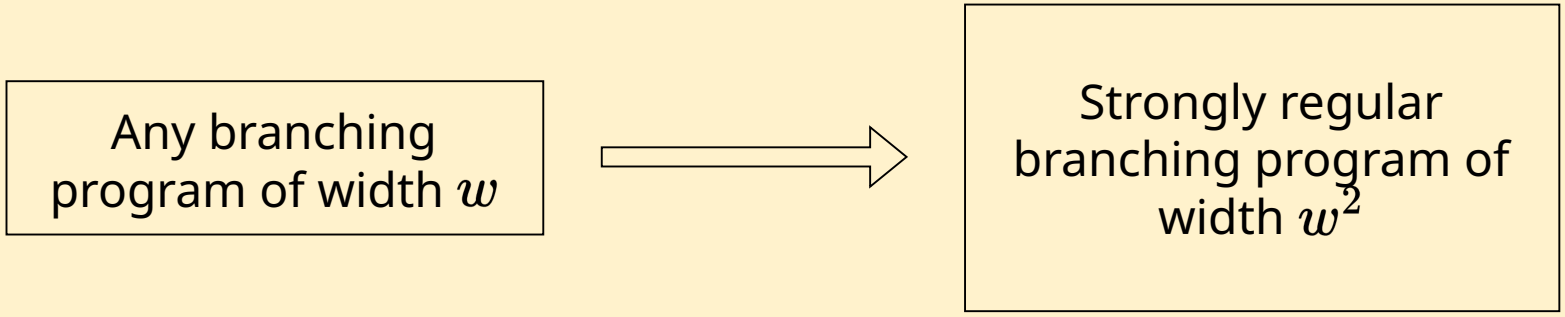

Randomness cost of this protocol

Read-once SRBP

Randomness required: $\pi_0, \rho_0, \pi_1, \rho_1, r, p$

$$r, p \leftarrow \{0, 1\}$$

$$\pi_0, \rho_0, \pi_1, \rho_1 \leftarrow Permutation[w]$$

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$$O(w \log w)$$

Constant since w is constant

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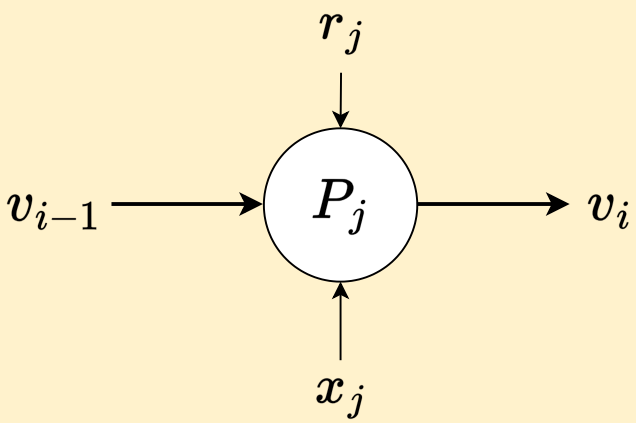
Read-k SRBP

$O(kw \log w)$ for read-k

Any width-w read-k BP has a speak-$O(k)$ PSS protocol with $O(kw^2 \log w)$ randomness complexity

PSS protocols to branching programs



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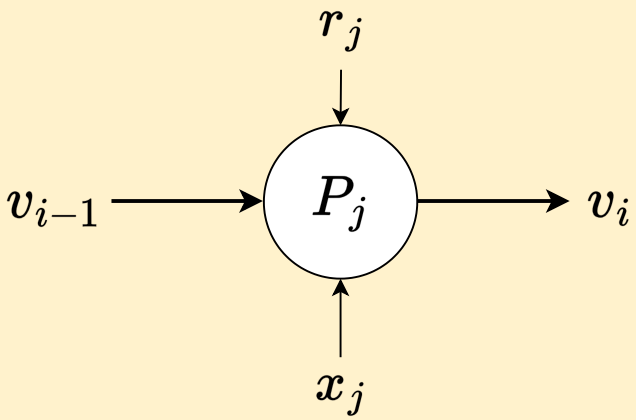
PSS protocols to branching programs



Step 1:

$$v_i = \mathsf{Nxt}(i, v_{i-1}, r_j, x_j)$$

$$\mathtt{Tr}_0^{(t)}(v_{i-1}) := \mathsf{Nxt}(i, v_{i-1}, r_j^*, 0) \qquad v_{i-1} \in M_{i-1} \qquad v_i \in M_i$$

$$\mathtt{Tr}_1^{(t)}(v_{i-1}) := \mathsf{Nxt}(i, v_{i-1}, r_j^*, 1) \qquad \mathtt{Tr}_0^{(t)}, \mathtt{Tr}_1^{(t)} : M_{i-1} \to M_i$$

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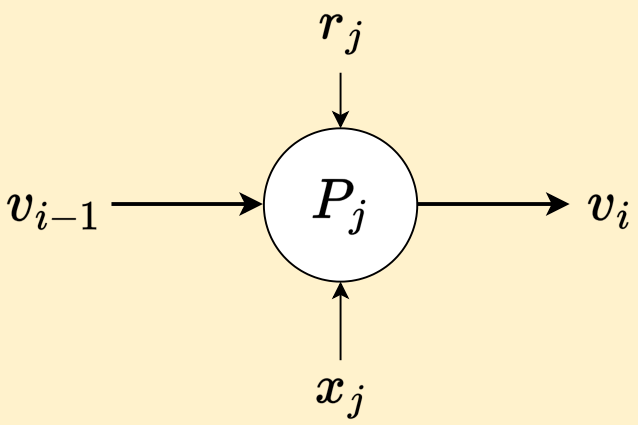

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Step 2:

Bound $|M_i| \leq 2^{R+2}$

message space for ith message over all inputs and randomness

Following [KOR96]

constant

Summary

New simple model for MPC: PSS

SRBP: Interesting subclass of branching programs

Implications to MPC in other models:

- Simpler protocol for computing AND without preprocessing.
- For odd number of parties, randomness cost of AND matches state-of-the-art [CR22]

[CR22] Geoffroy Couteau and Adi Rosén. Random sources in private computation, ASIACRYPT 2022,