Mind the Bad Norms

Revisiting Compressed Oracle-based Quantum Indistinguishability Proofs

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Introduction

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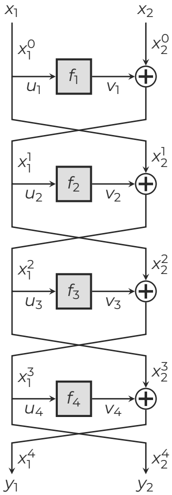
- Classical proofs $\Rightarrow$ Quantum proofs?

The Luby-Rackoff Construction



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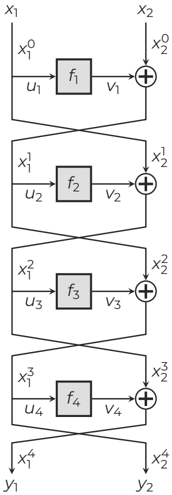

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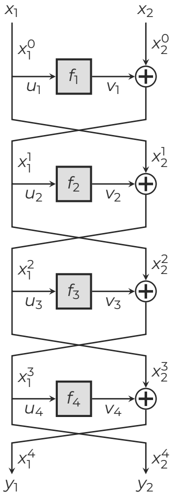

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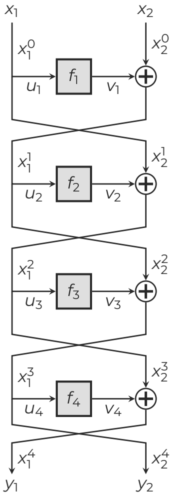

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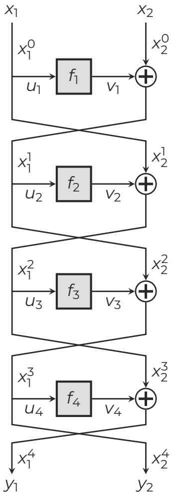

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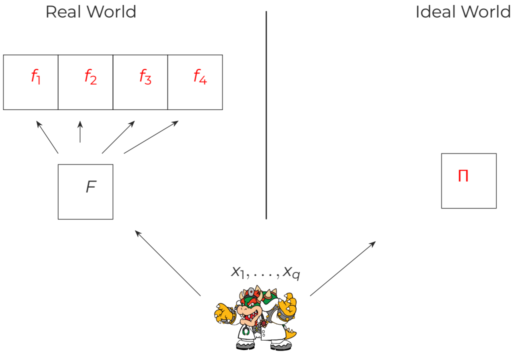
  2. qCPA proof for LR4 [HI19] - we revisit this proof and identify some challenges.

Quantum CPA Distinguishing Game



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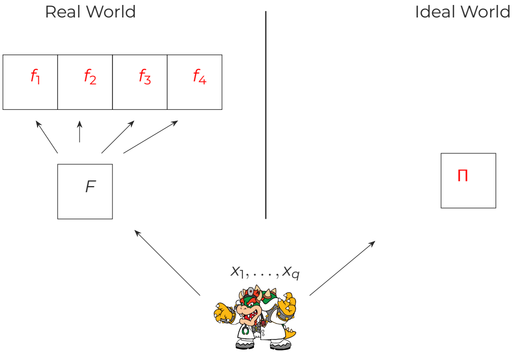

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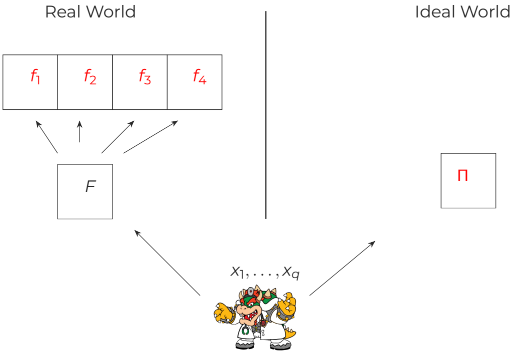

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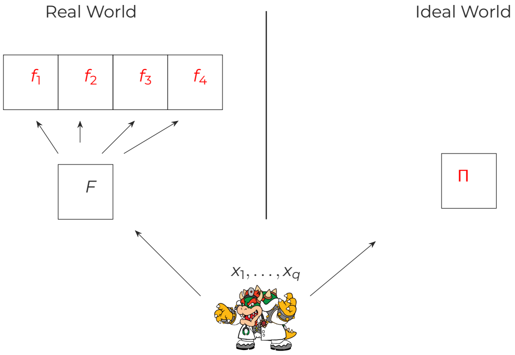

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- **Assumptions:**
    1. primitives f_1, f_2, f_3, f_4 are random;
    2. **Q2 Model:** allow quantum (superposition) queries.

Quantum CPA
Proof of LR4 [HI19]

Quantum Implementation of LR4 [HI19]

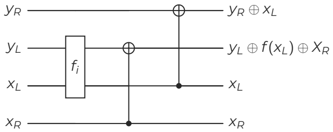


Figure 2: Round i of LR4 - O_{f_i}

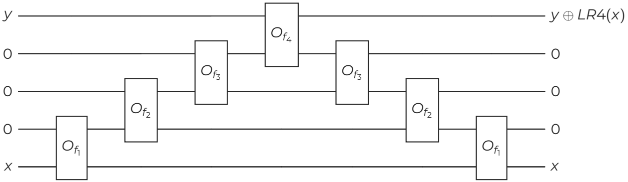
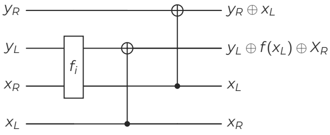


Figure 3: LR4

- **Action** = a call to the unitary O_{f_i}.
- Each O_{f_i} maintains a state - **Database**.

Modified LR4 - LR4'



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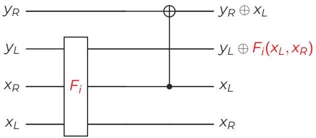

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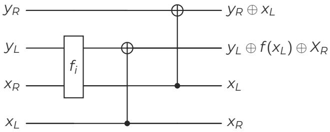

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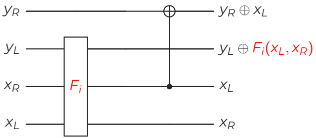

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- LR4' = LR4 with O_{F_i} instead of O_{f_i} for $i = 3, 4 \Rightarrow$ LR4' IND from Π
- **Hybrid Distance:** enough to bound distance from LR4 to LR4'.

Two-Domain Distance (TDD) Technique [BCEJ23]

- **Single Compressed Oracle:** Record all intermediate functions with random $\Gamma : \{0,1\}^{4+2nq} \to \{0,1\}^n$ where for $i \leq 4, j = 3, 4$

$$f_i(x) = \Gamma([8+i]_2 || x || 0 \ldots 0),$$

$$F_j(x_1, x_2, x_3) = \Gamma([10+j]_2 || x_1 || x_2 || x_3 || 0 \ldots 0).$$

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- **Bad Databases:** defined as $d^{\mathbf{R}}$ (resp. $d^{\mathbf{I}}$) with a collision on inputs to f_3 (resp. F_3) or f_4 (resp. F_4).
- **1-to-1 mapping:** for any good database $d^{\mathbf{R}}$, $d^{\mathbf{R}} \mapsto [d^{\mathbf{R}}]_{\mathbf{I}}$.

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- **Example:** bound *"bad"* norm of $O_{f_1}|\left(\psi_g^{\leq(j-1)}\right)\rangle$ (ideal world).
- **Simplification:** let $BAD = \{\beta : d^{\mathbf{I}} \cup (x_1, \beta)_1 \text{ is bad}\}$ then

$$||\mathrm{BN}||^2 \leq \frac{|BAD|}{2^n}.$$

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- **Correct claim:** $||\mathrm{BN}||^2 = O(1)$.

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- **Answer:** *No*. For any $r \geq 4$, creating a collision on $f_i \Rightarrow$ leads to trivialization of norm.
- **Underlying Issue:** lack of oracle's knowledge of adversarial query pattern.

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- **Non-Adaptive Setting:** includes Simon's non-adaptive version [BHNP$^+$19].

Non-Adaptive Proof for LR4

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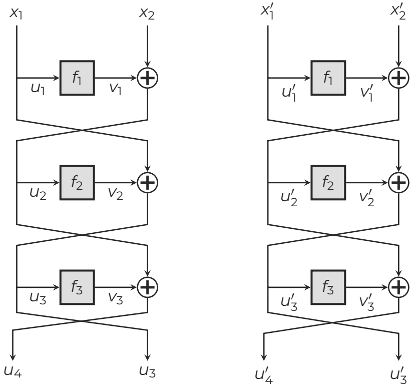

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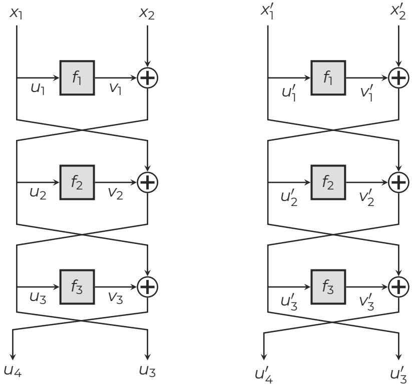

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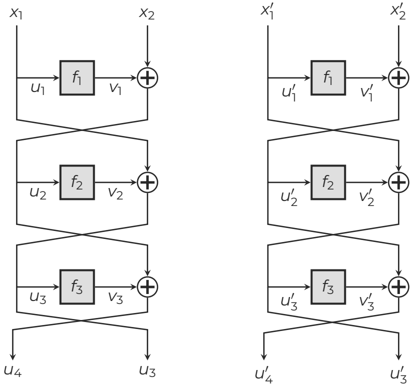

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- From the TDD Framework:

$$\mathbf{Adv}_{LR4}^{qNCPA}(A) \leq O\left(\sqrt{\frac{q^6}{2^n}}\right).$$

The Problem with the Adaptive Setting

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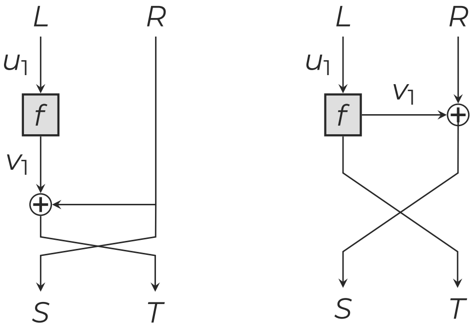
- TNT and LRWQ [BCEJ23] $\rightarrow$ bounds deteriorate to $O(2^{n/5})$.

The Misty Constructions

Misty-L vs Misty-R



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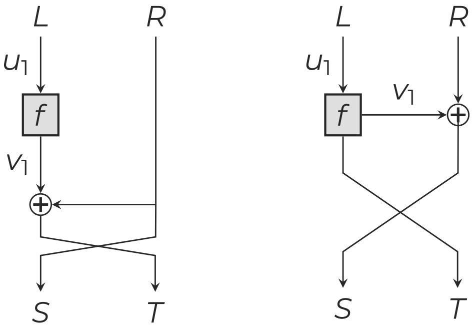

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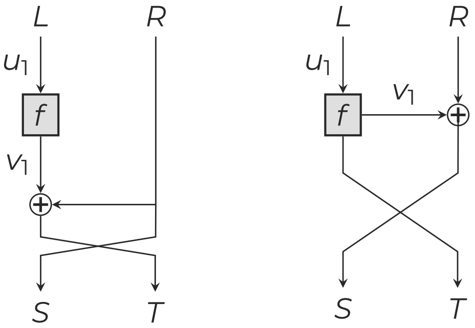

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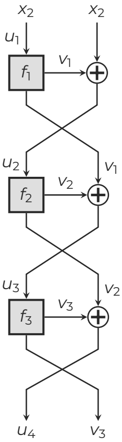
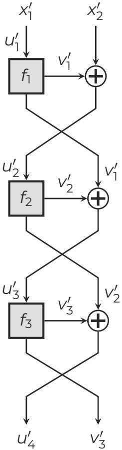
- **In this work:** we show qCPA (adaptive) proofs in the TDD framework.

Bad Events for 4 Rounds Misty-R





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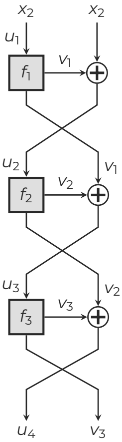

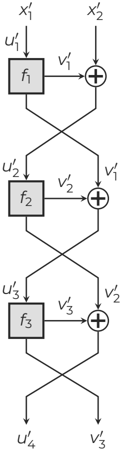

- **Bad Events:** $\exists$ collision on input to f_3 or f_4.
- **Difference from LR4:** "bad" events are dependent on v_1, v_2, v_3.

Conclusions

TDD Framework Quantum (N)CPA Proofs

Scheme	Calls	Model	Bound
Luby-Rackoff	4	qNCPA	$O(2^{n/6})$ (Section 5)
Misty-R	4	qCPA	$O(2^{n/5})$
Misty-L	5	qCPA	$O(2^{n/7})$
LRWQ [HI21]	3	qCPA	$O(2^{n/5})$ [BCEJ23]
TNT [BGGS]	3	qCPA	$O(2^{n/5})$ [BCEJ23]

Quantum BB - $O(2^{n/3})$ queries [Zha13].

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   - Non-adaptive qCPA proof for LR4 up to $O(2^{n/6})$ quantum queries - dummy call + TDD framework.
2. We provide qCPA proofs for the Misty constructions using TDD framework:
   - 4 rounds Misty-R up to $O(2^{n/5})$ quantum queries,
   - 5 rounds Misty-L up to $O(2^{n/7})$ quantum queries.

Future Work

- **Permutation Compressed Oracle:** permutation = product of transpositions [MMW24].

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- **Proofs in TDD framework [BCEJ23]:** define a property which makes schemes provable in the framework?
- **Tightening proofs:** new proof techniques? better bounds? seems hard!

Thank you!

References i

- Ritam Bhaumik, BenoIt Cogliati, Jordan Ethan, and Ashwin Jha.
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