Robust Quantum Public-Key Encryption

(With Applications to Quantum Key Distribution)

Giulio Malavolta (Bocconi University & MPI-SP)
Michael Walter (Ruhr University Bochum)

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https://arxiv.org/pdf/2304.02999.pdf

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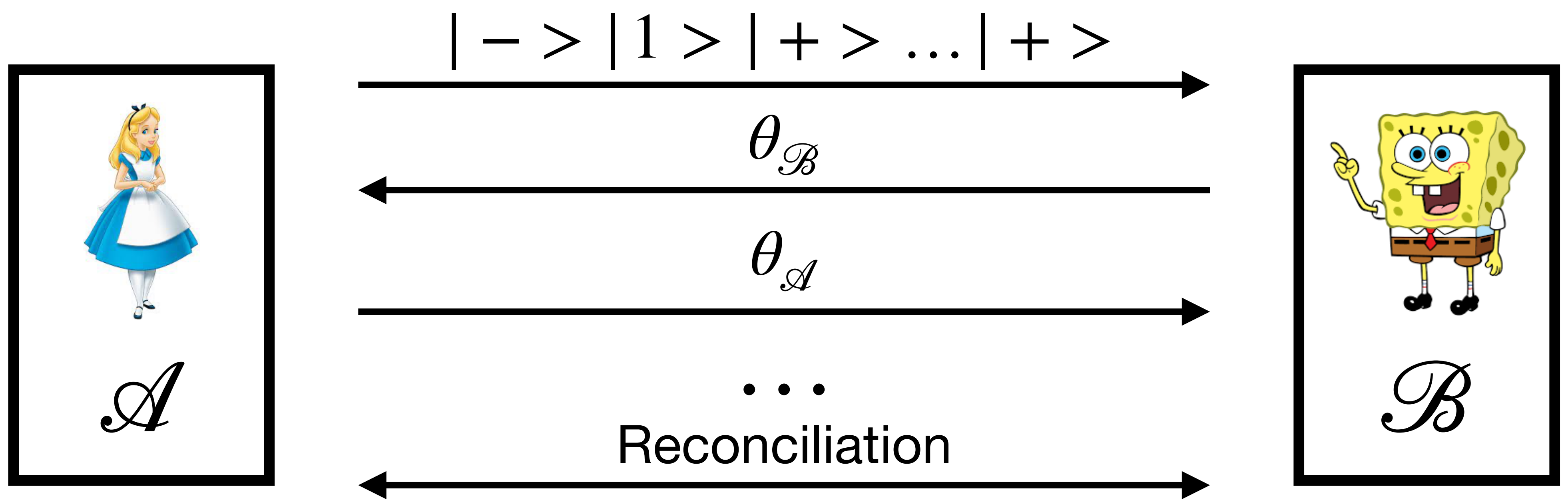

[BB84]

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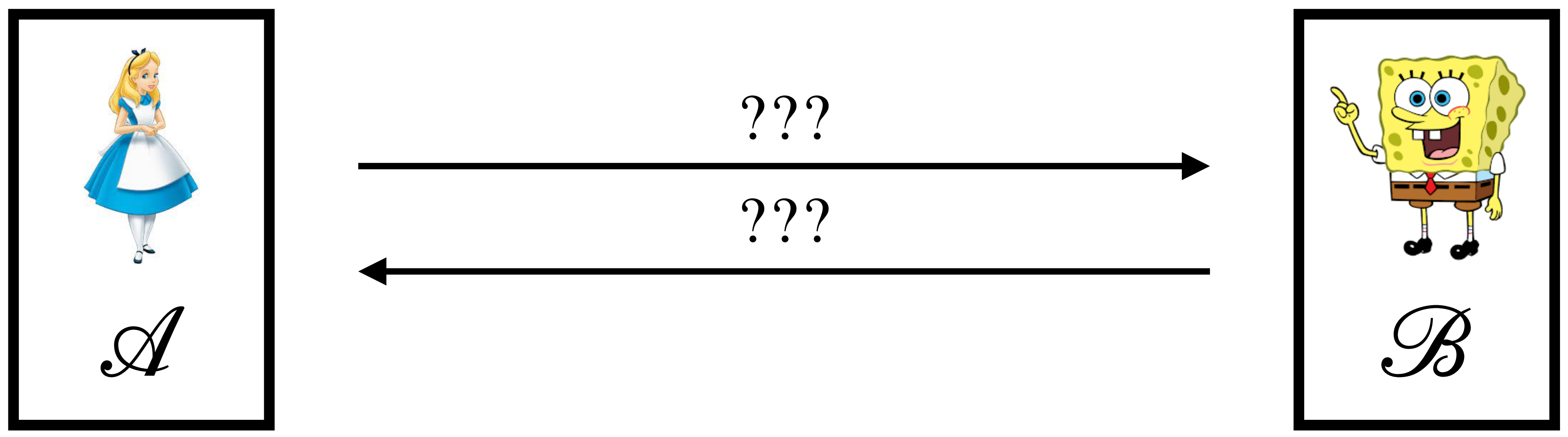

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Theorem II: If quantum-secure *one-way functions* exists, there exists standard QPKE with *computational* security.

- Classically, one-way functions are (widely believed to be) insufficient to construct PKE

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Roadmap

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- Part III: Conclusions

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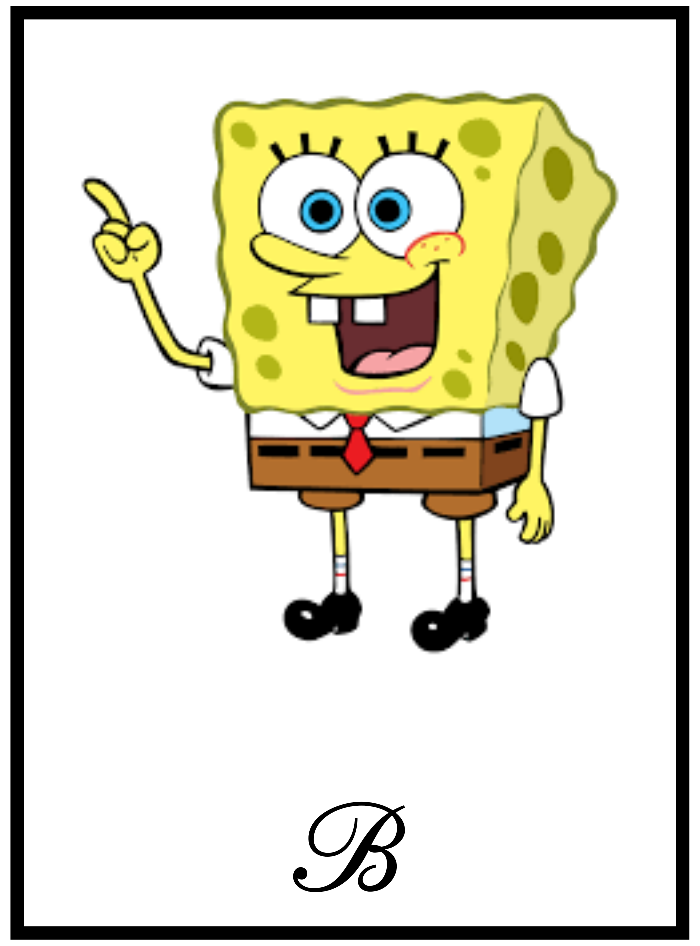

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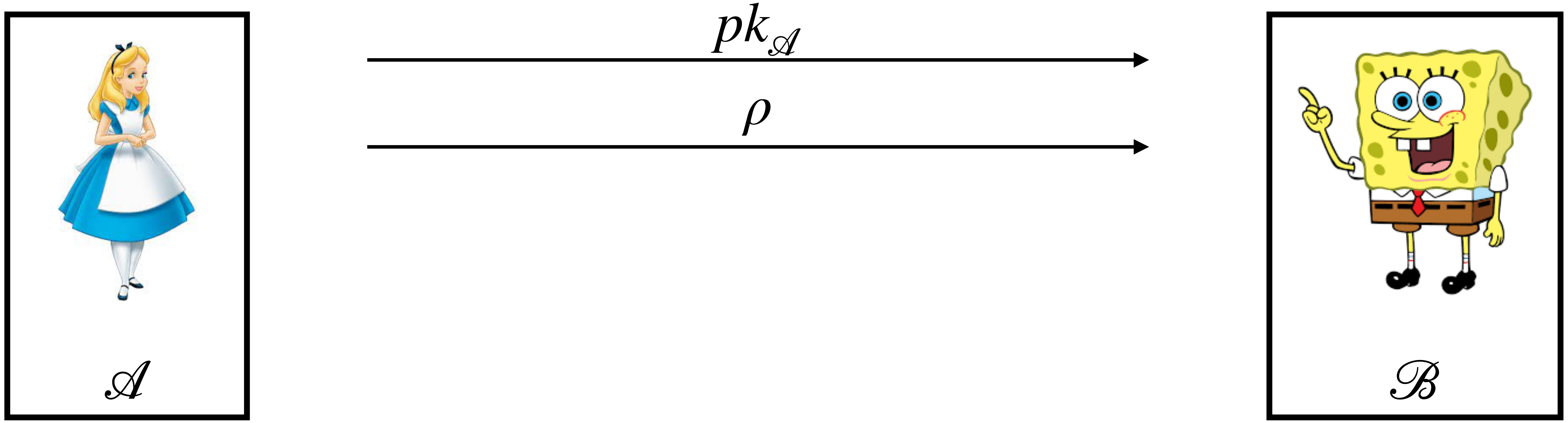

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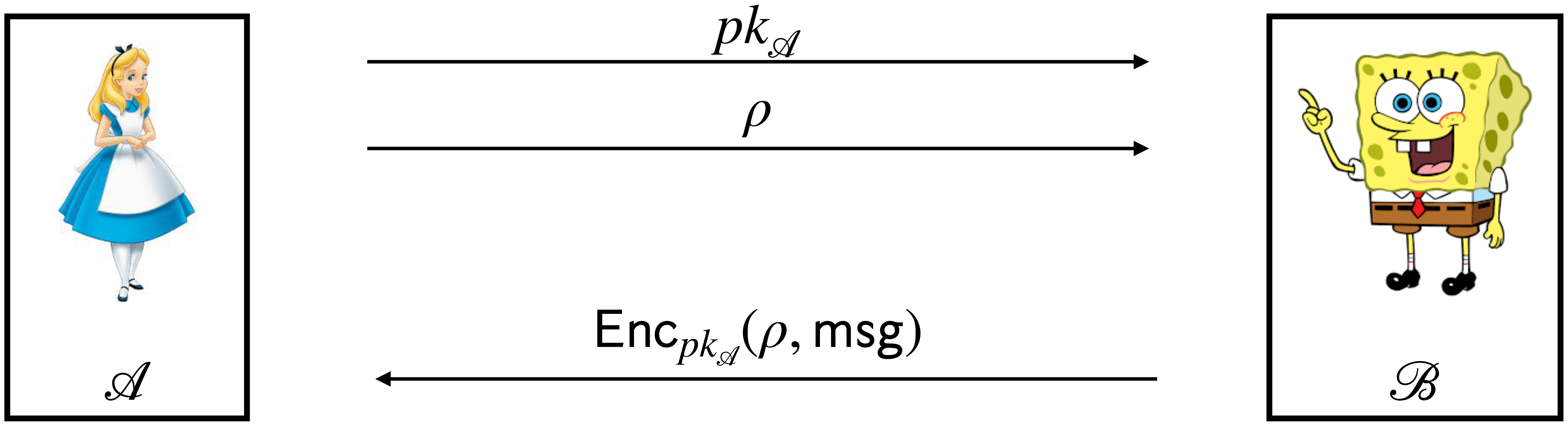

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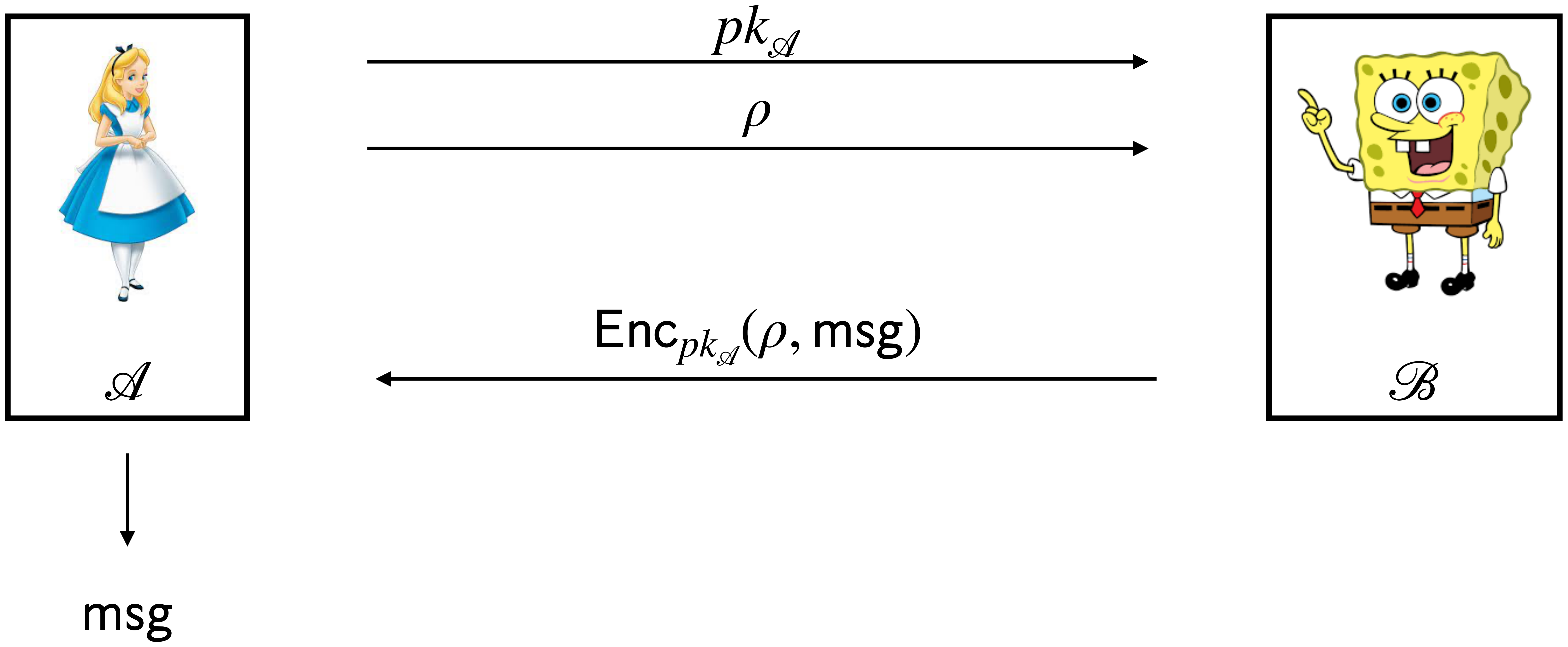

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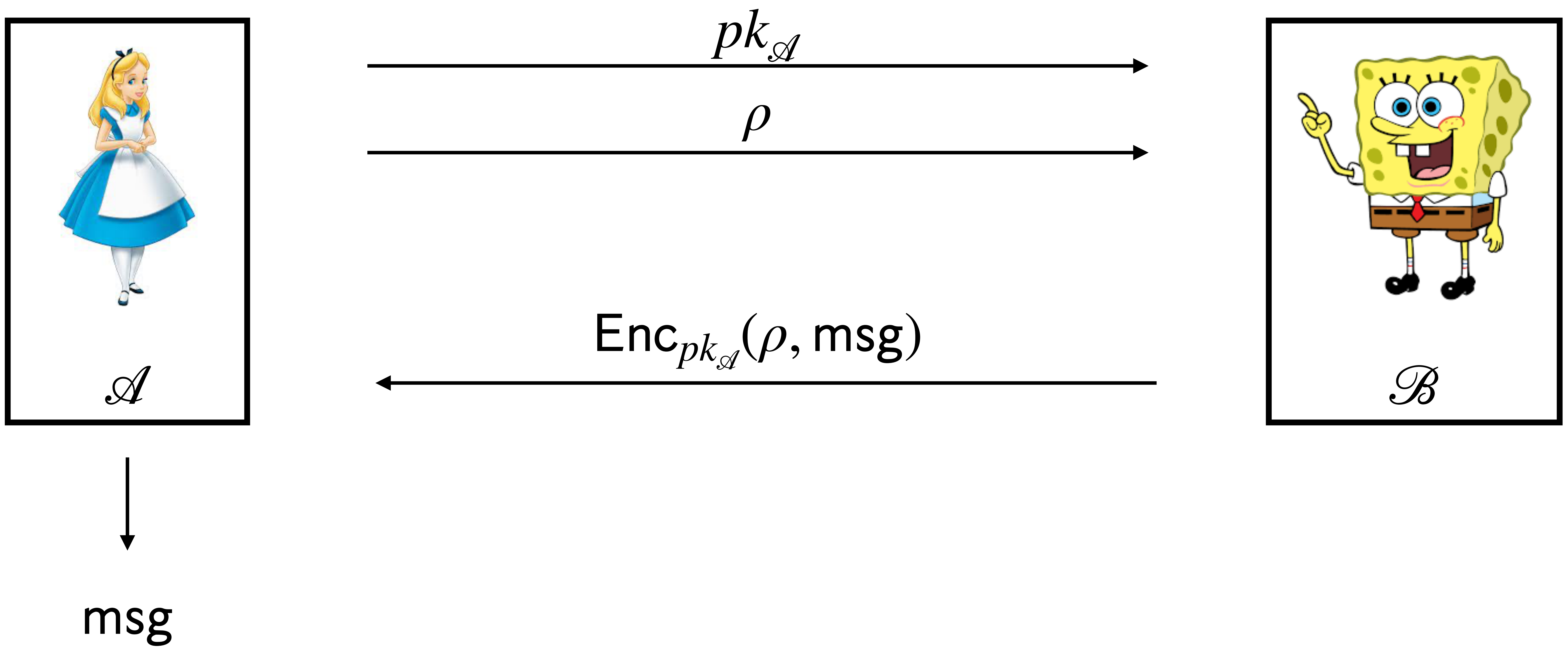

*Suffices for QKD (see paper)

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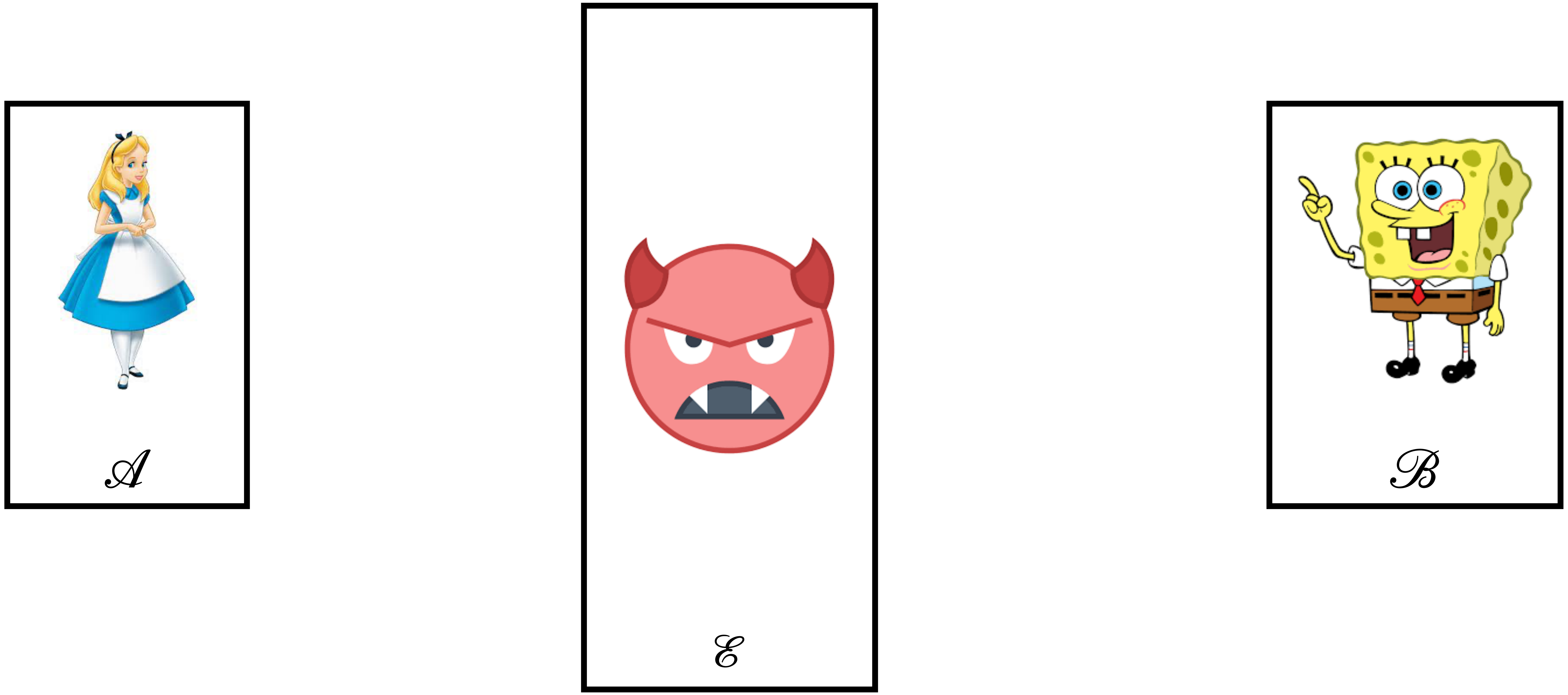

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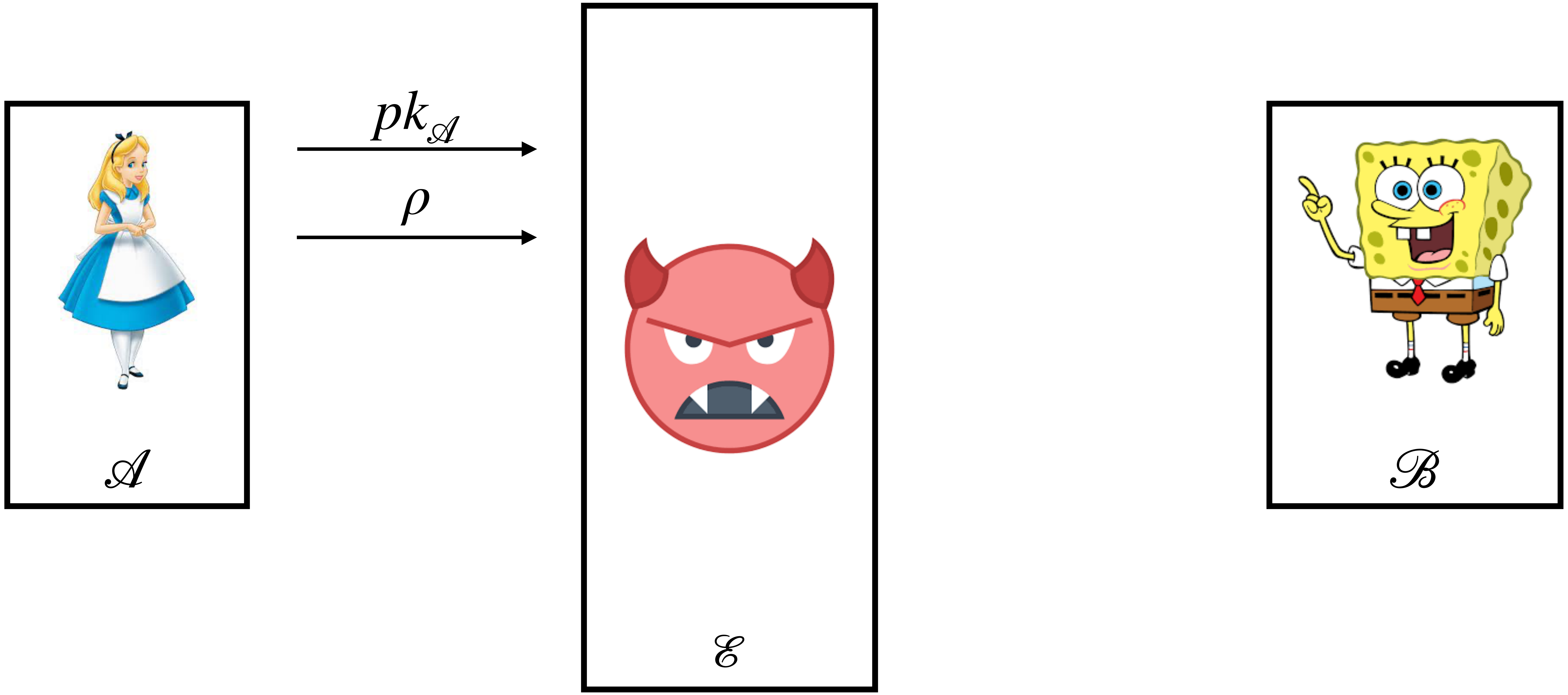

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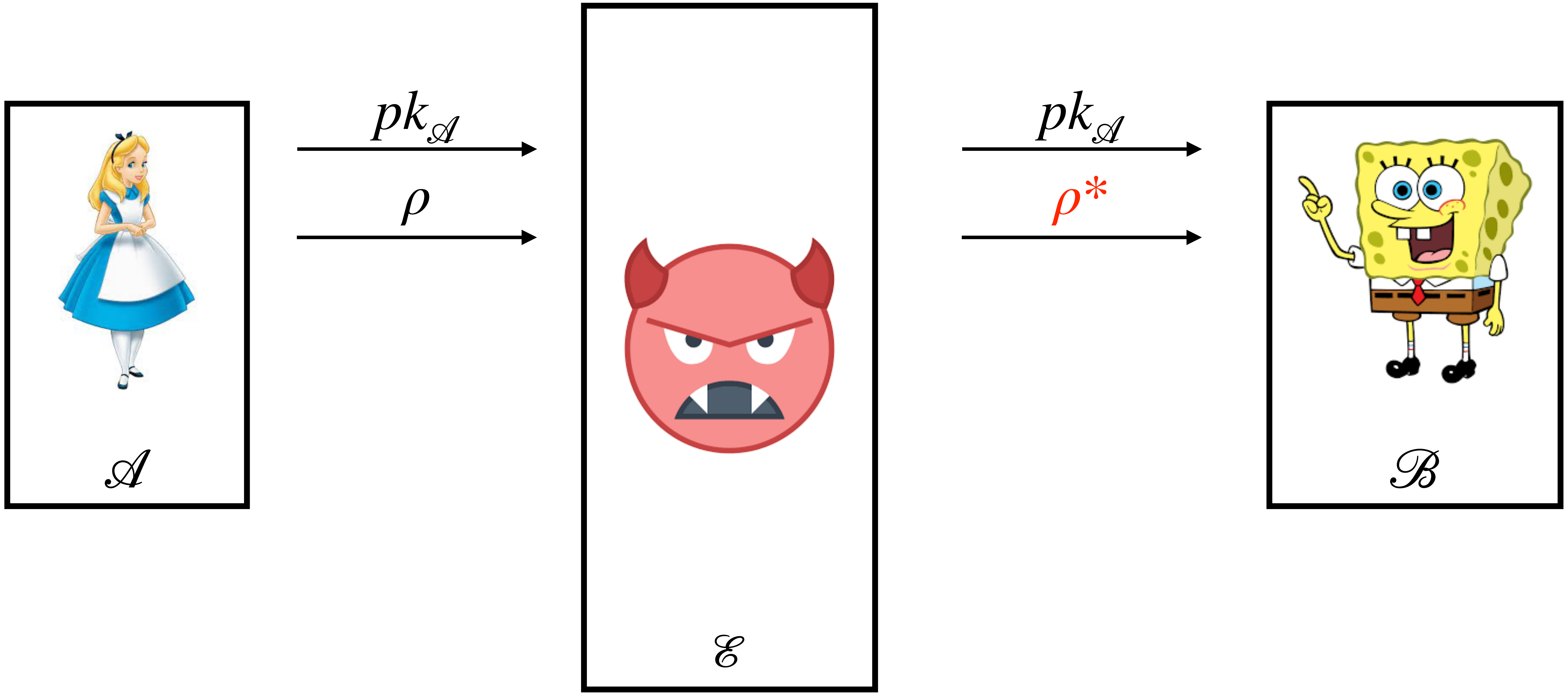

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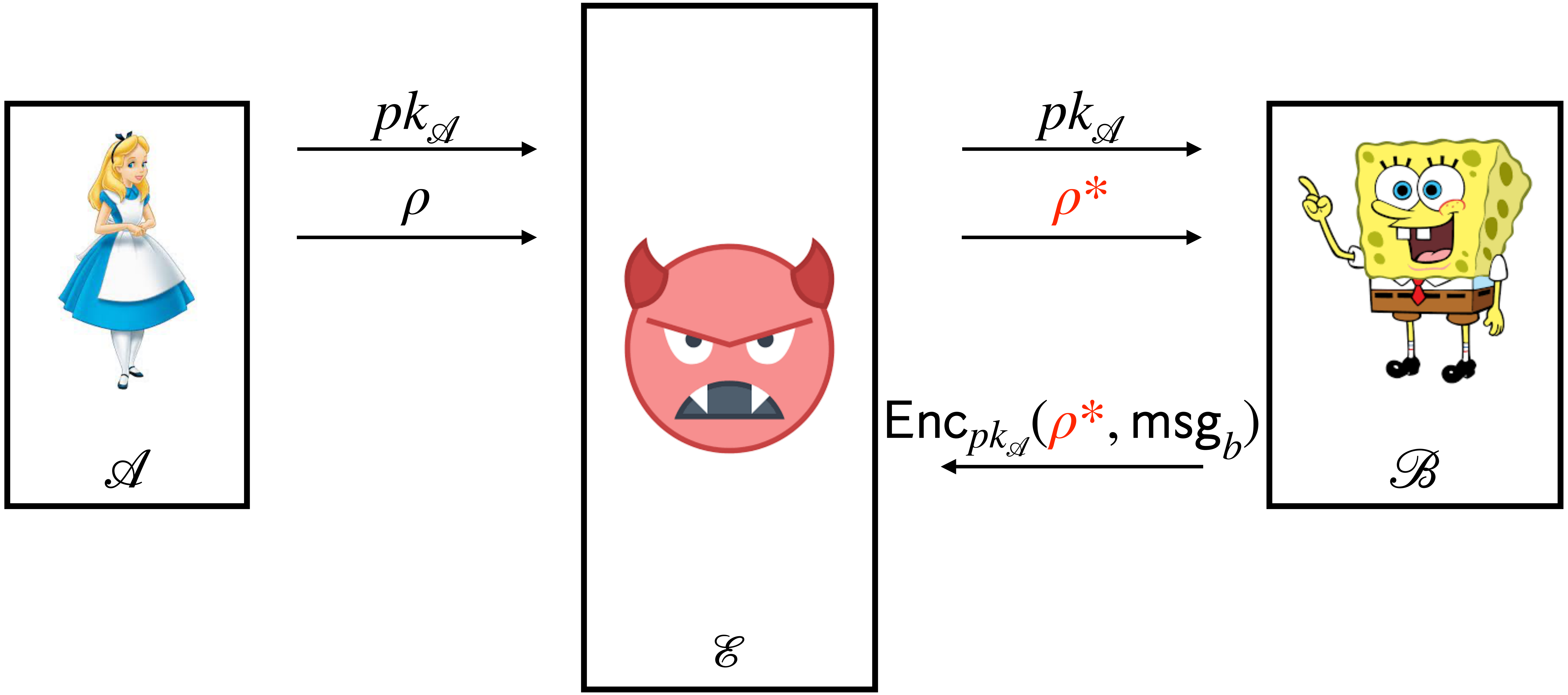

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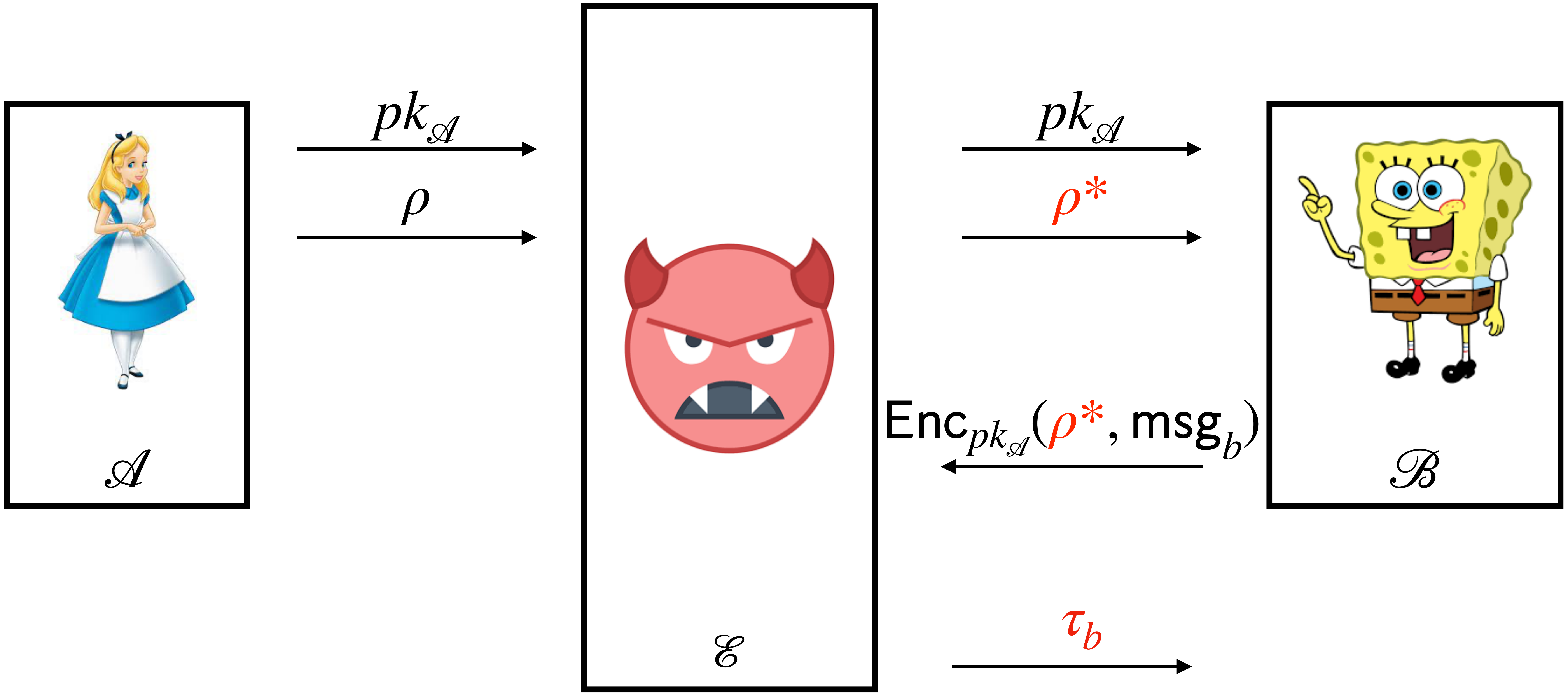

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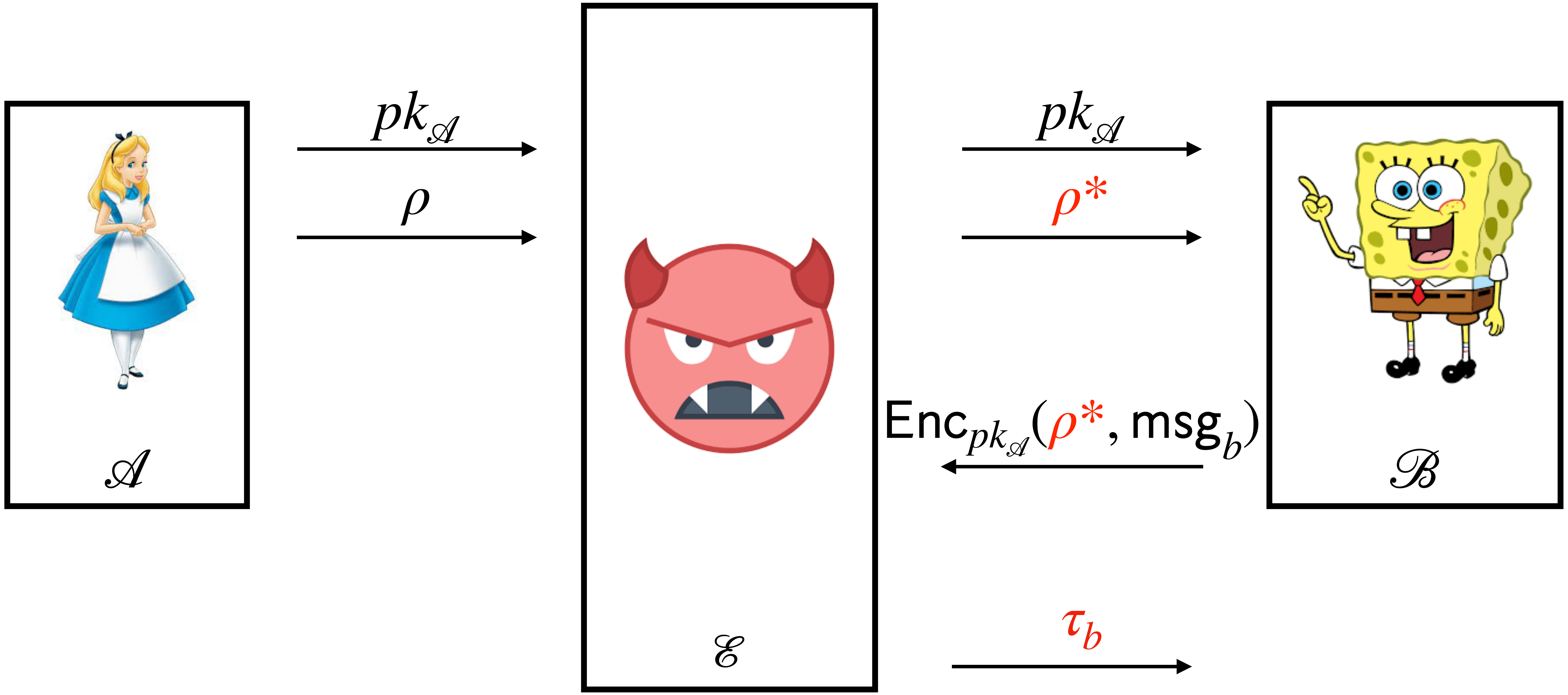

$\text{TD}(\tau_0, \tau_1) \approx 0$

Part II: The Protocol*

*Everlasting variant (see paper for the computational one)

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- Exists iff one-way functions exist [Lam79]

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- Alice can recover d_1, and consequently msg, since she knows s and (σ_0, σ_1)

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