Solving the Tensor Isomorphism Problem on Special Orbits with low rank points

Cryptanalysis and repair of an Asiacrypt 2023 commitment scheme

Valerie Gilchrist, **Laurane Marco**, Christophe Petit, Gang Tang (eprint 2024/337)

CRYPTO 2024 - August 22nd, 2024

EPFL LASEC

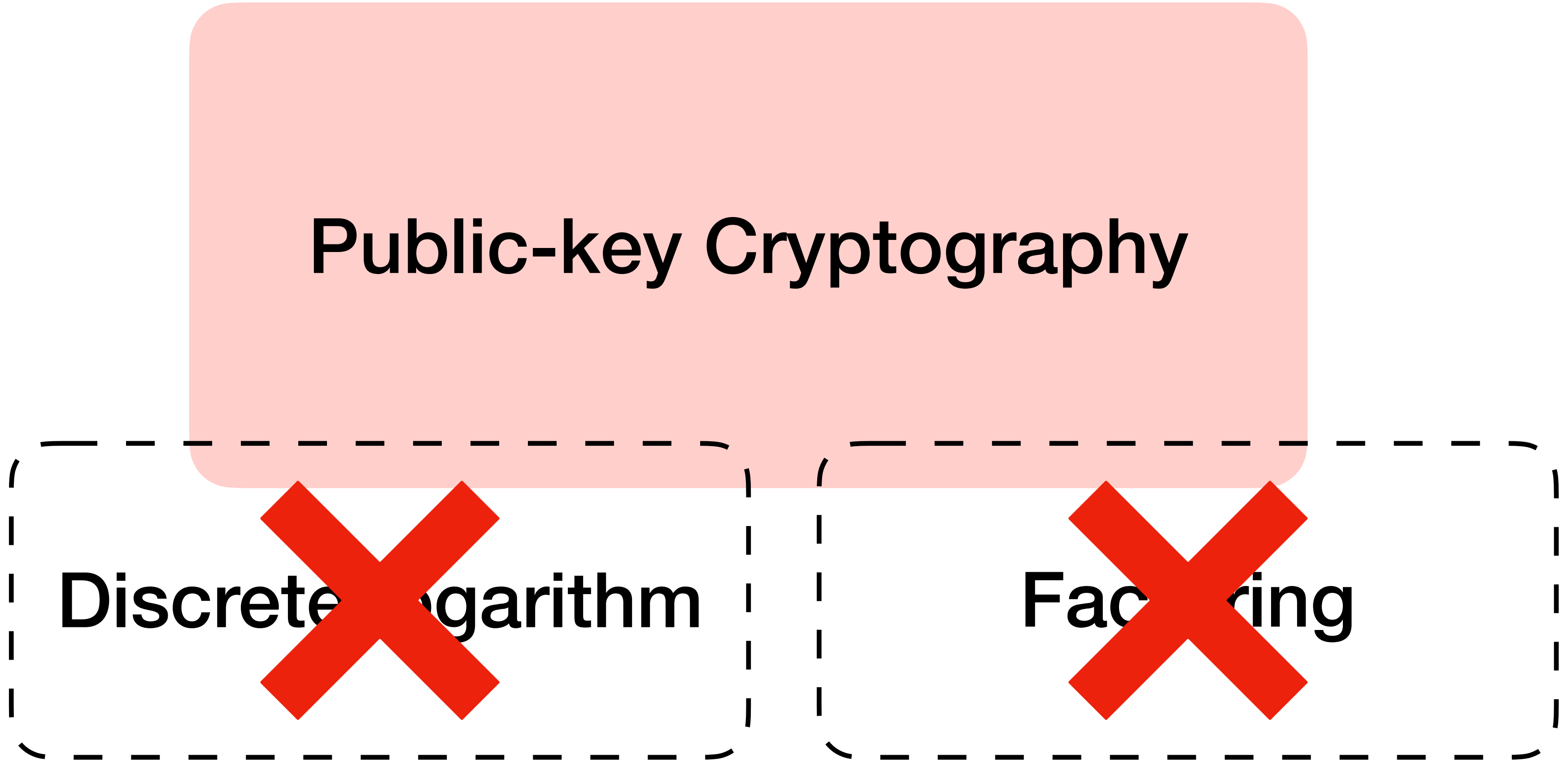
Post-quantum cryptography

Public-key Cryptography

Discrete logarithm

Factoring

Post-quantum cryptography



Solving the TIP on special orbits with low rank points

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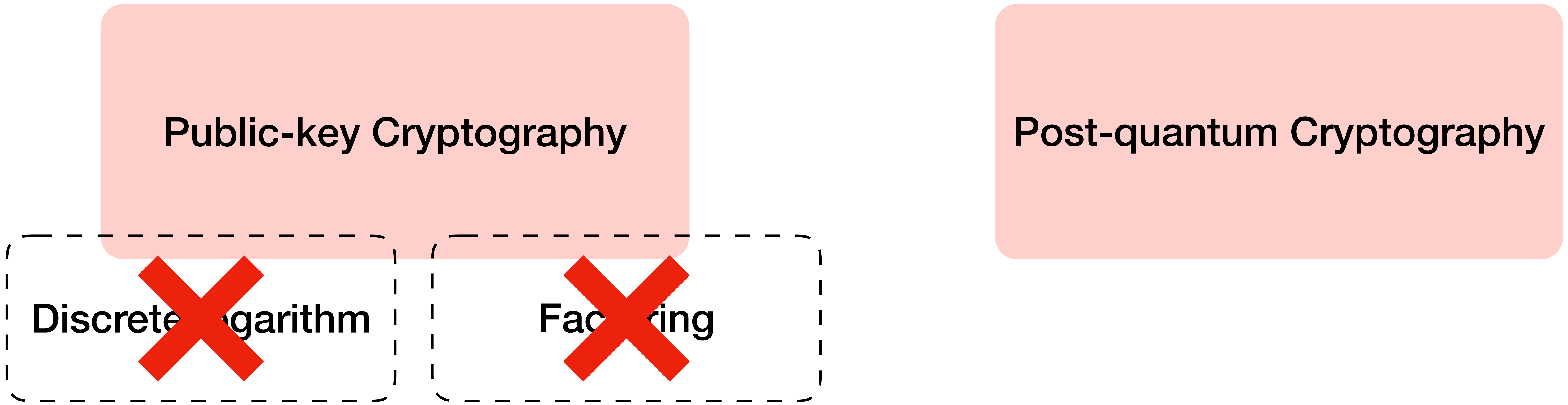

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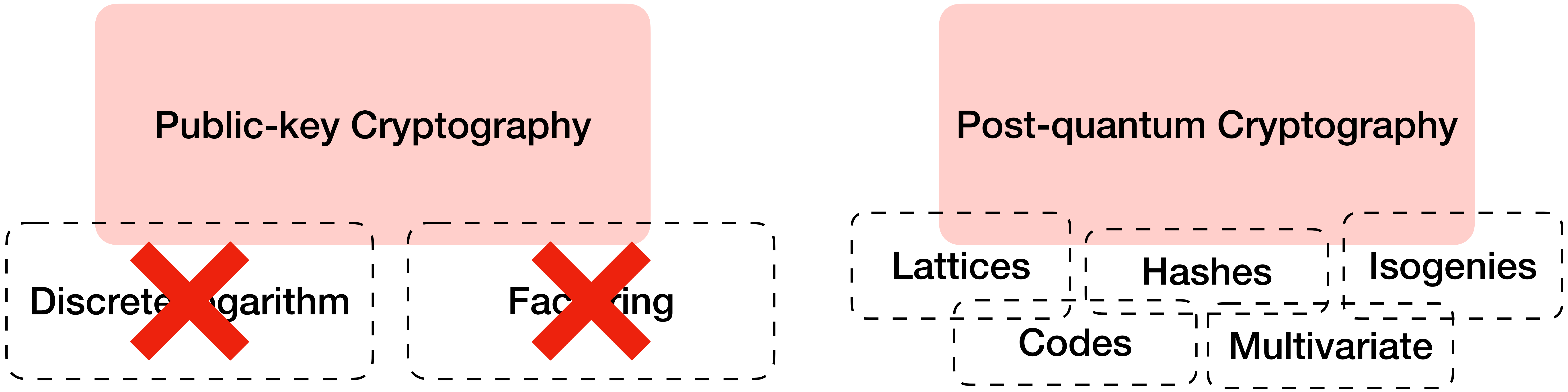

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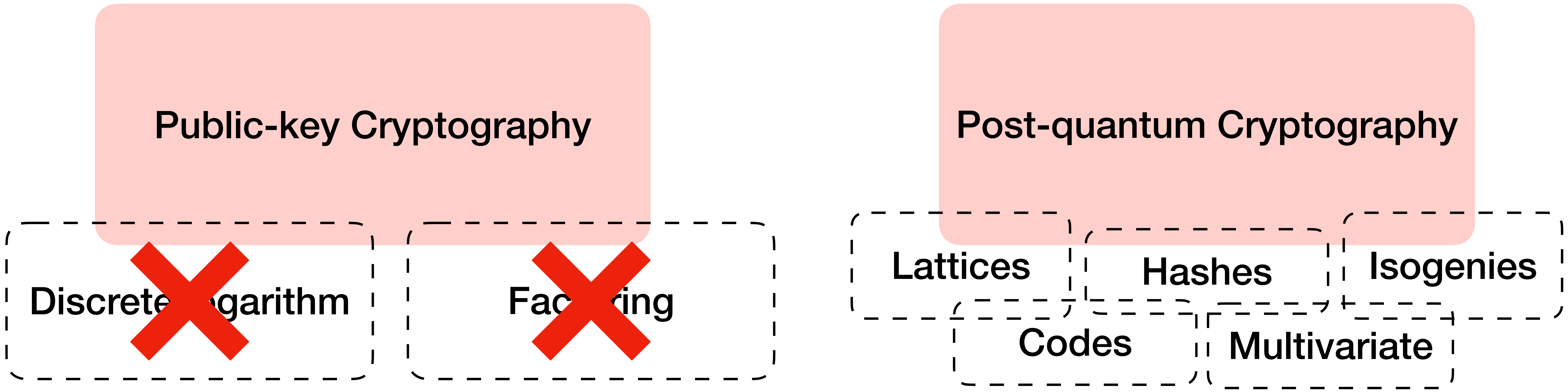

Diversity matters!!!

→ several recent attacks (Rainbow, SIDH/SIKE, …)

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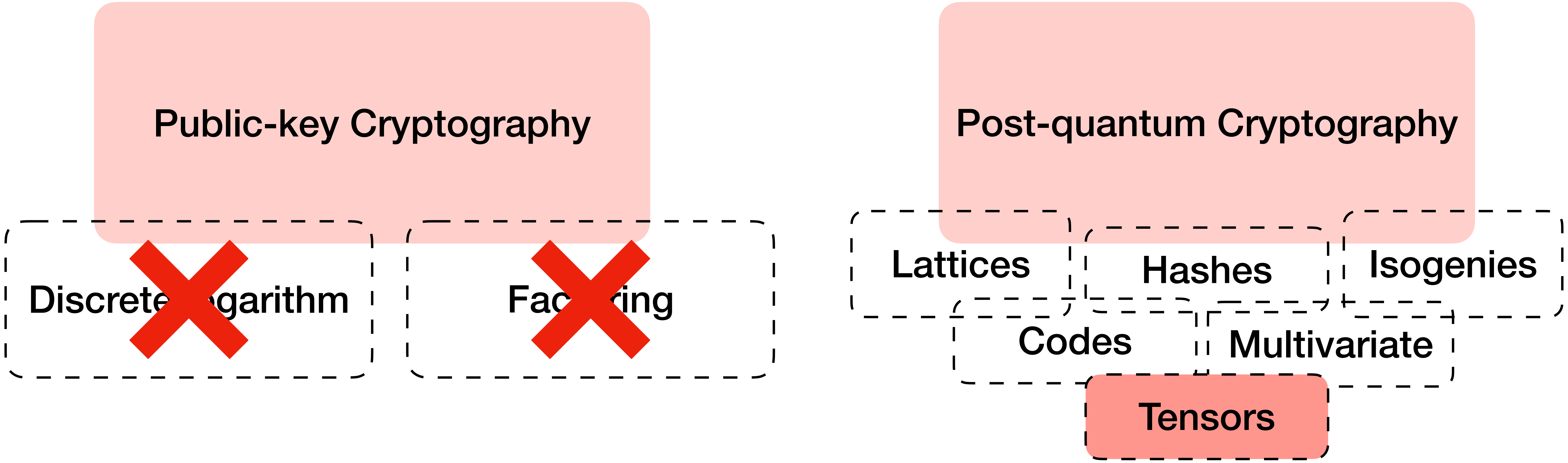

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Recent proposal: **commitment scheme** from **tensor**-based hard problems

Outline

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Cryptanalysis of the tensor-based commitment scheme of D’Alconzo, Flamini, Gangemi (Asiacrypt 23’)

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→ **Breaks** the **hiding** property of the commitment
→ Extension to a **polynomial time attack** on the **computational version**

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Repair
→ **Alternative commitment scheme** from random tensors

Tensor-based cryptography

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3-tensors: $v \in \mathbb{F}_q^n \otimes \mathbb{F}_q^n \otimes \mathbb{F}_q^n$ can be written as

$$v = \sum_{i,j,k=1}^{n} v(i,j,k) e_i \otimes e_j \otimes e_k$$

or as a **list of matrices** $[M_1, \ldots M_n]$, $M_i \in M(n,q)$

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Example: $(1,0,0,1) \otimes (0,2,0,2) \otimes (3,0,4,0)$ in $F_{11}^4 \otimes F_{11}^4 \otimes F_{11}^4$ can be written as

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 6 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 6 & 0 & 8 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 6 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 6 & 0 & 8 & 0 \end{bmatrix}$$

Tensor-based cryptography

Group action:

$G = GL(n,q) \times GL(n,q) \times GL(n,q)$ acts on $\mathbf{V} = \mathbb{F}_q^n \otimes \mathbb{F}_q^n \otimes \mathbb{F}_q^n$

$$\star : G \times \mathbf{V} \to \mathbf{V}$$

$$(A,B,C), \sum_{i,j,k} v(i,j,k) e_i \otimes e_j \otimes e_k \mapsto \sum_{i,j,k} v(i,j,k) Ae_i \otimes Be_j \otimes Ce_k$$

Studied by Ji, Qiao, Song, Fun (TCC 19'), Grochow, Qiao (ITICS 21')

Tensor-based cryptography

Hard problems

Tensor-based cryptography

Hard problems

Decisional Tensor Isomorphism Problem (dTIP) :

Given two **random** tensors $v_0, v_1 \in \mathbf{V}$ **decide** whether there exists $(A, B, C) \in GL(n) \times GL(n) \times GL(n)$ such that

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Computational Tensor Isomorphism Problem (cTIP) :
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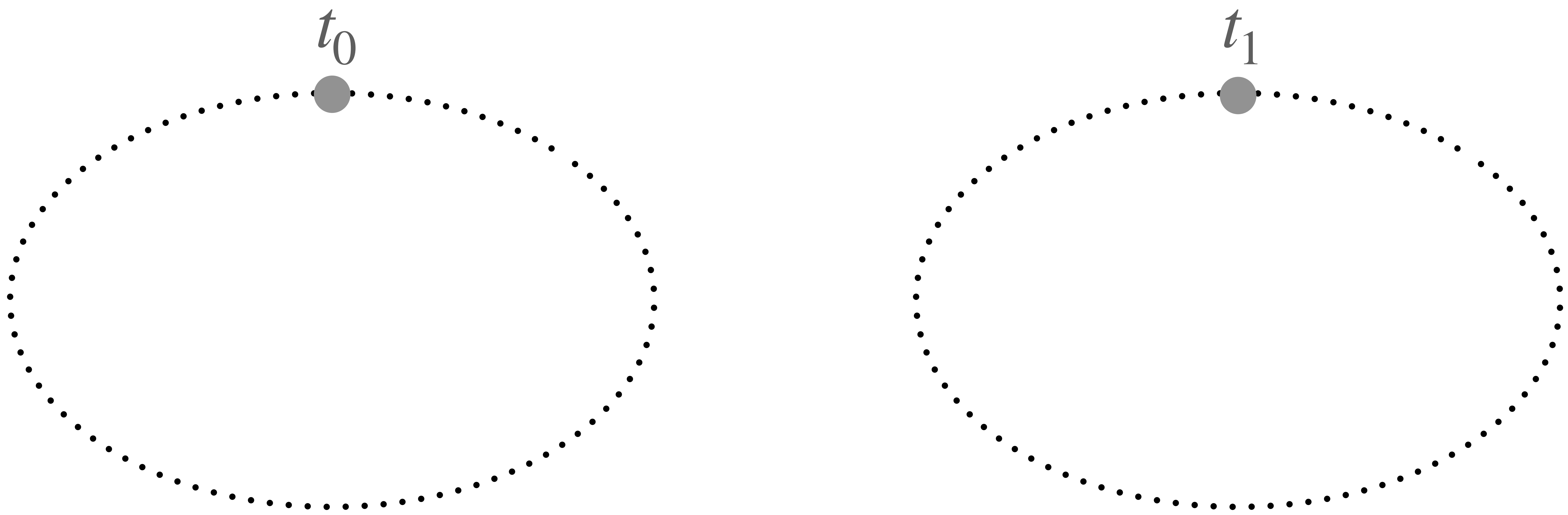
Equivalent to:
- trilinear form equivalence problem
- matrix code equivalence problem (MEDS, NIST signature call).

Bit commitment scheme from tensors

D'alconzo, Flamini, Gangemi (Asiacrypt 2023)



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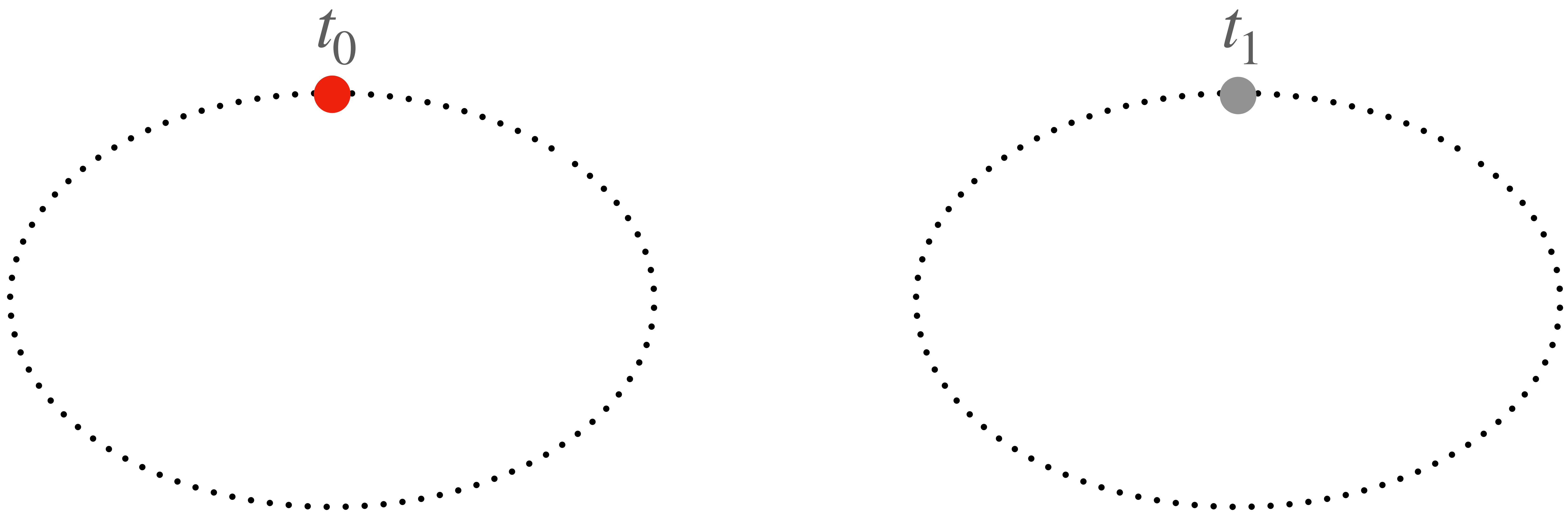

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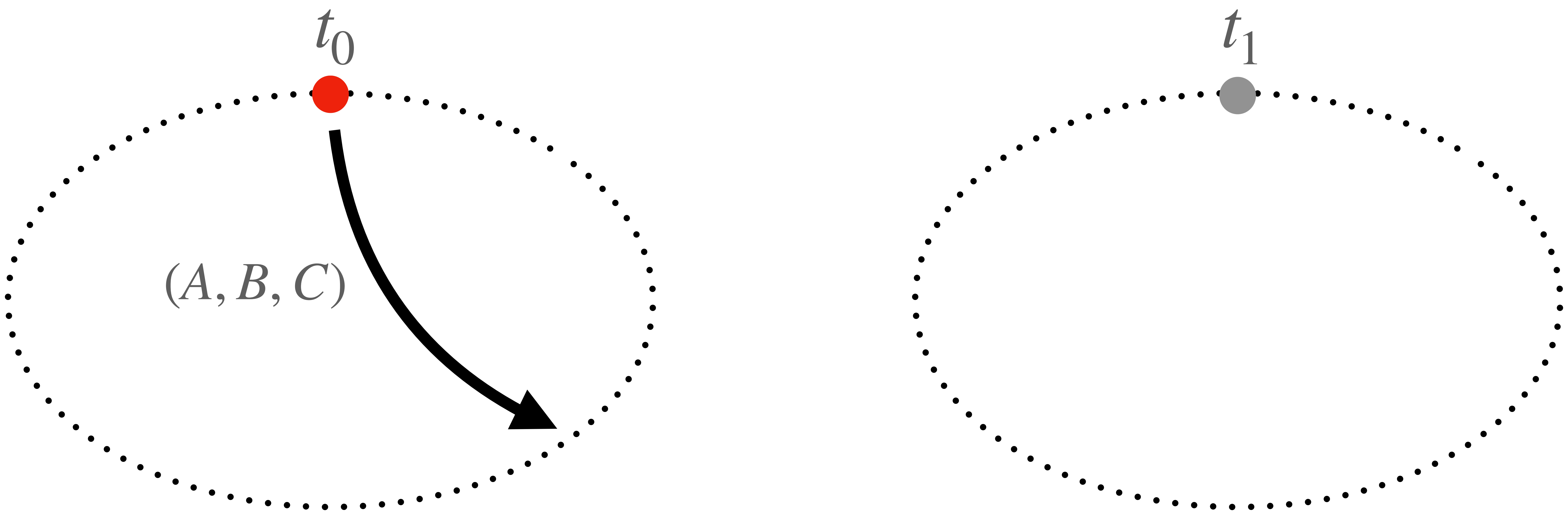

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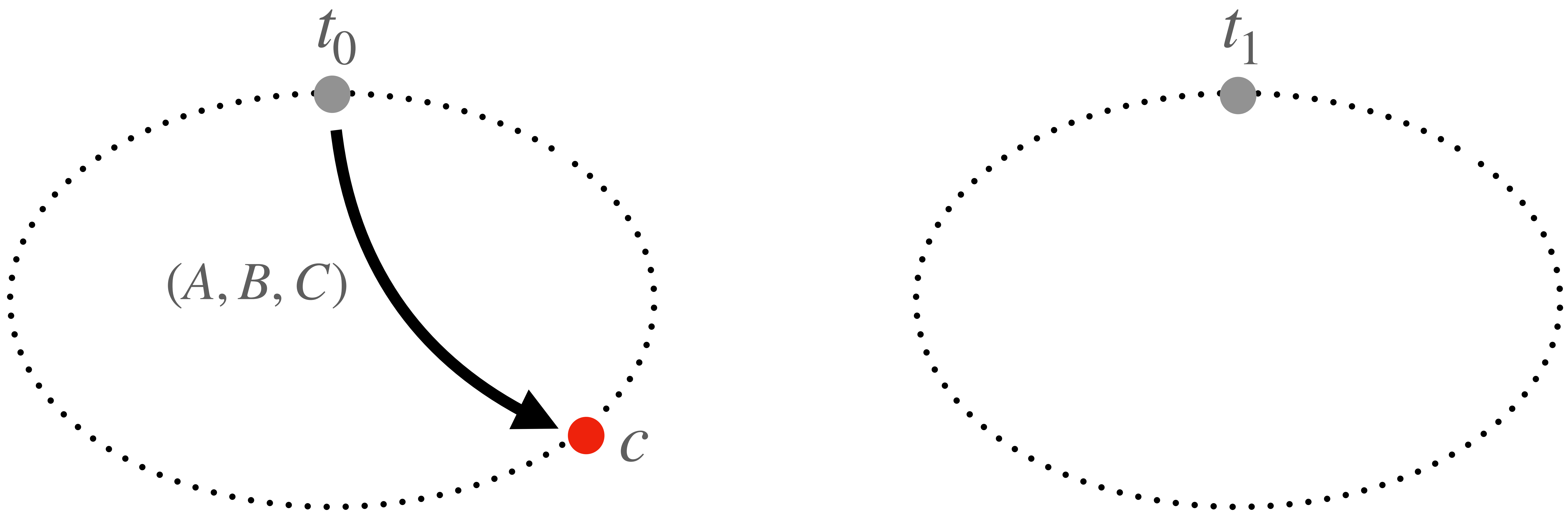

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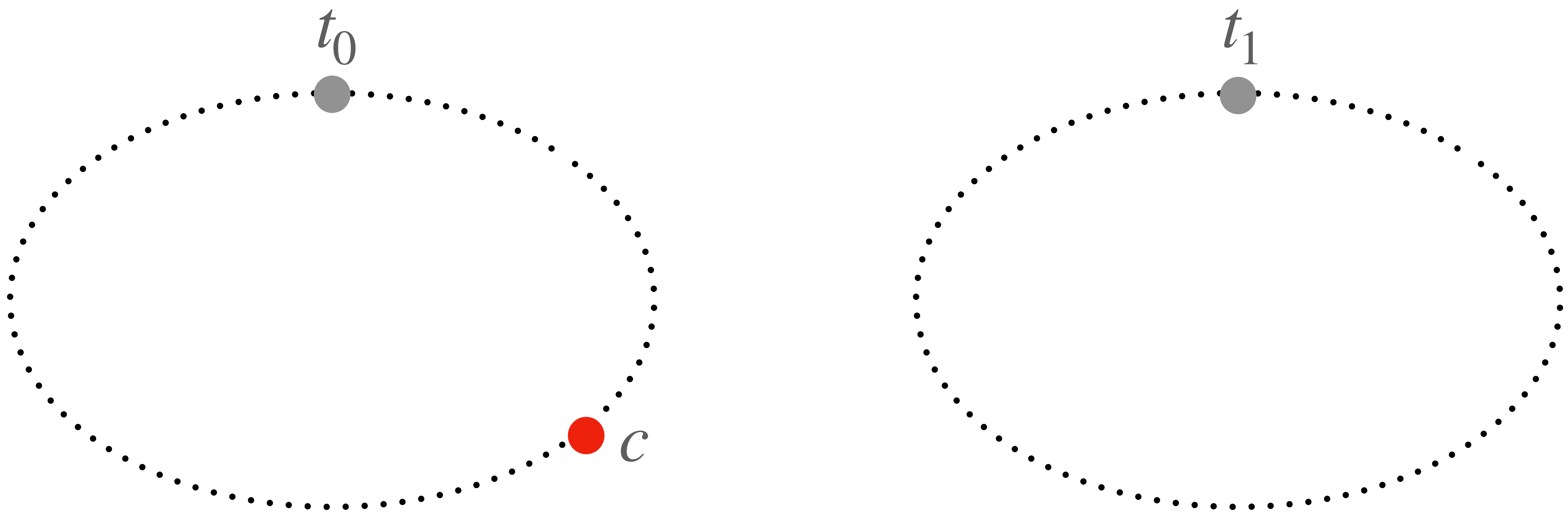

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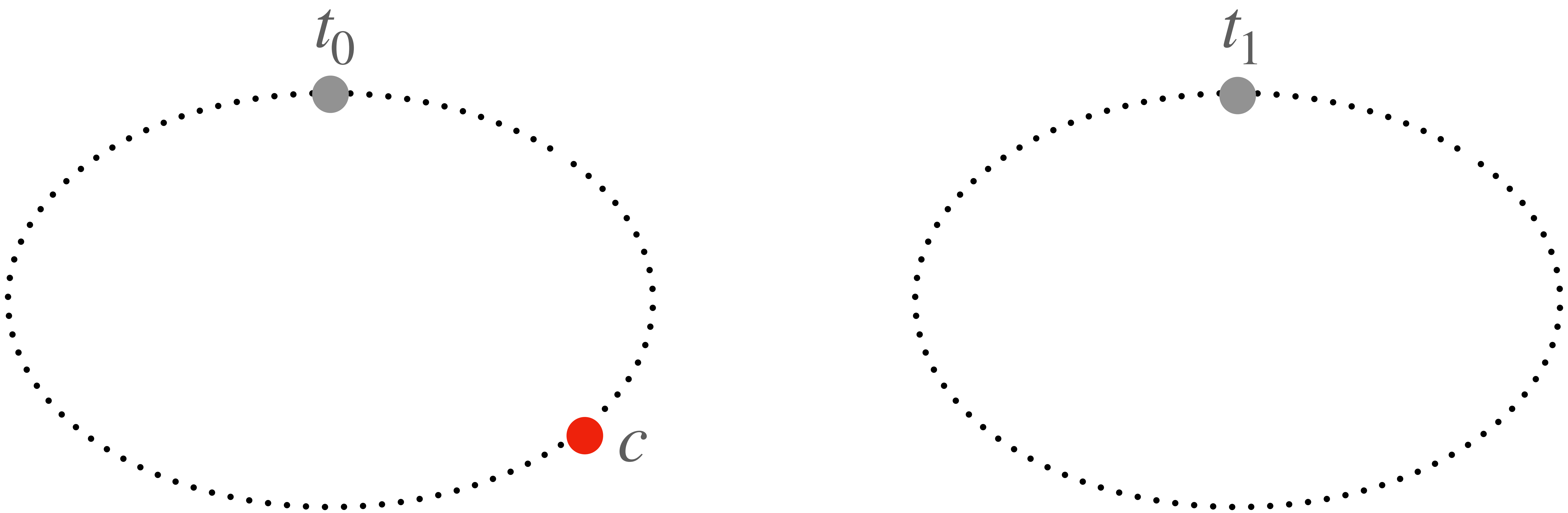

Requirements:

→ t_0, t_1 **must** be in **different orbits**

→c **must** look random

Tensor rank

Rank 1 tensor: $v = a \otimes b \otimes c, a, b, c \in \mathbb{F}_q^n$.

Rank of a tensor: minimal r such that

$v = \sum_{i=1}^{r} w_i$ with w_i rank 1.

→ hard to compute for **random** tensors!*

*Håstadt (J. Algorithms), Hilar, Lim (J. ACM), Schaefer, Stefankovic (Theory Compute. System.)

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Lemma: For $(A, B, C) \in G, v \in \mathbf{V}$, we have $rank((A, B, C) \star v) = rank(v)$

$rank(t_0) = n$ and $rank(t_1) = n - 1$ → different orbits!

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	Sender		Receiver
Commit	$g \xleftarrow{\$} G$ com $\leftarrow g \star t_b$	com $\longrightarrow$	

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Open		b, g $\longrightarrow$	$g^{-1} \star$ com $= t_b$?

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Effect of special orbits

$n = 4,\ q = 11$

$$t_b: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-b \end{bmatrix}$$

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$$\left(\begin{bmatrix}5&2&9&2\\5&8&9&10\\0&2&7&1\\6&8&10&9\end{bmatrix}, \begin{bmatrix}10&4&7&7\\10&10&4&1\\0&1&7&4\\9&6&0&9\end{bmatrix}, \begin{bmatrix}7&7&7&2\\8&8&1&3\\9&0&2&3\\6&8&10&3\end{bmatrix}\right) \star t_0$$

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Example: $u = (u_1, u_2, u_3, u_4) \in \mathbb{F}_{11}^4$, rank of u in t_0 (resp. t_1) is the rank of

$$M_0 = \begin{bmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & u_3 & 0 \\ 0 & 0 & 0 & u_4 \end{bmatrix}$$

resp. $M_1 = \begin{bmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & u_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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Lemma: The group action preserves the number of points of a given rank.

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Attack on dTIP: rank-0 points

Example: $u = (u_1, u_2, u_3, u_4) \in \mathbb{F}_{11}$

$rank_{t_0}(u) = 0 \Leftrightarrow rank(M_0) = 0 \Leftrightarrow u = 0 \in F_{11}$

Hence t_0 **has no non-trivial rank-0 points**

t_1 has **1** rank-0 point (e_n), up to scalar multiplication

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Lemma: Let $t_b = \sum_{i=1}^{n-b} e_i \otimes e_i \otimes e_i$**,** The rank-$0$ points of t_b form a vector space of **dimension** b

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Goal: Given $c = (A, B, C) \star t_b$, recover b.

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Write $c = [G_1, \ldots G_n]$
Solve the linear system $\alpha_1 G_1 + \ldots + \alpha_n G_n = 0$ for $\alpha \in \mathbb{F}_q^n$
If there is a solution $b = 1$, else $b = 0$

→**A couple seconds on a laptop** 😊 (at most $O(n^4)$ operations)

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Q: How likely is it for a tensor to have low rank points?

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t_0 has **n rank-1 points** $\{e_1, \ldots, e_n\}$
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→ Compute them (MinRank).

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Permutation matrices leave t_0 invariant

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💡 Use **rank-1 points** and knowledge of the **stabiliser subgroup** to get a **unique solution**

1. Rank 1 points of t_0
t_0 has **n rank-1 points** $\{e_1, \ldots, e_n\}$ (up to scalars)

c **will also have** n **rank 1 points** $\{a_1, \ldots, a_n\}$
→ Compute them (MinRank).

→ **Recover** A

Lemma : Given $\{a_1, \ldots, a_n\}$ and $\{e_1, \ldots, e_n\}$ there exists an ordering σ of $\{a_1, \ldots, a_n\}$ and a matrix A such that for each i, $a_{\sigma(i)} = e_i A^{-1}$

Permutation matrices leave t_0 invariant

Attack on cTIP: wrapping up

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Theorem : We can recover a valid (A, B, C) in $O(n^6)$ operations

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→ No structure on the tensors!
→ No new assumptions!

To conclude

Solving the Tensor Isomorphism Problem for special orbits with low rank points: Cryptanalysis and repair of an Asiacrypt 2023 commitment scheme

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Distinguish the committed bit and **compute** (A, B, C)

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Propose an alternative commitment scheme from **random** tensors

Thanks!

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